Bayesian statistics

Lecture 3

Rianne de Heide, February 22, 2024

Exercise 1

Find Jeffreys' prior for mean parameter in a Gaussian model with known variance σ^2.

Exercise 2

Find Jeffreys' prior for variance parameter in a Gaussian model with known mean μ.

Exercise 3

Suppose we have samples $x_1, x_2, \ldots, x_5 = 0,0,1,1,0$ from a Bernoulli distribution with parameter $\theta \in (0,1)$.

a) Calculate the MLE.

Suppose we impose the restriction that $\theta \in \{0.2,0.5,0.7\}$.

b) Calculate the MLE.

Now we are going to be Bayesian. Suppose we have a prior $\pi(0.2) = 0.1, \pi(0.5 = 0.01), \pi(0.7) = 0.89$.

c) What is the MAP estimator?

Exercise 3 part 3

d) Show that we can make the MAP whatever we like, by finding a prior over $\{0.2, 0.5, 0.7\}$so that the MAP is 0.2, another so that it is 0.5, and another so that it is 0.7.

e) Typically for the Bernoulli distribution, if we use MAP, we want to be able to get any value in $[0,1]$, not just the ones in a finite set such as above. So we need a continuous prior distribution with range $[0,1]$. We assign the Bernoulli parameter a beta prior: $\theta \sim Beta(\alpha, \beta)$. Recall that the mode of a $Beta(\alpha, \beta)$random variable is $(\alpha - 1)/((\alpha - 1) + (\beta - 1))$. Suppose x^n are iid Bernoulli with unknown parameter. Show that the posterior has a $Beta(k + \alpha, n - k + \beta)$ distribution, and find the MAP estimator.

f) If we use a Beta(1,1) - a uniform - distribution as prior, how would the MAP and MLE compare?

Exercise 3 part 3

g) Since the posterior is also a Beta distribution, we call Beta the conjugate prior to the Bernoulli distribution. Interpret how the prior parameters α and β affect our MAP estimate.

h) As the number of samples goes to infinity, what is the relationship between MLE and MAP? What does this say about our prior when n is small, or n is large?

i) Would you prefer MLE or MAP? Why?

Exercise 4

Credible intervals are not unique. Come up with a simple posterior distribution and show more than one valid 95% credible interval.