

Is it possible that in just a few lines I can achieve what I failed to achieve in my many books and articles? Surely not. Nevertheless, this preface affords me the opportunity, and I shall make the attempt. It may be that misunderstandings which persist in the face of refutations dispersed or scattered over some hundreds of pages can be resolved once and for all if all the arguments are pre-emptively piled up against them.

My thesis, paradoxically, and a little provocatively, but nonetheless genuinely, is simply this:

PROBABILITY DOES NOT EXIST.

The abandonment of superstitious beliefs about the existence of Phlogiston, the Cosmic Ether, Absolute Space and Time,..., or Fairies and Witches, was an essential step along the road to scientific thinking. Probability, too, if regarded as something endowed with some kind of objective existence, is no less a misleading misconception, an illusory attempt to exteriorize or materialize our true probabilistic beliefs.

In investigating the reasonableness of our own modes of thought and behaviour under uncertainty, all we require, and all that we are reasonably entitled to, is consistency among these beliefs, and their reasonable relation to any kind of relevant objective data ('relevant' in as much as subjectively deemed to be so). This is Probability Theory. In its mathematical formulation we have the Calculus of Probability, with all its important off-shoots and related theories like Statistics, Decision Theory, Games Theory, Operations Research and so on.

This point of view is not bound up with any particular philosophical position, nor is it incompatible with any such. It is strictly *reductionist* in a methodological sense, in order to avoid becoming embroiled in philosophical controversy.

Probabilistic reasoning –always to be understood as subjective– merely stems from our being uncertain about something. It makes no difference whether the uncertainty relates to an unforeseeable future, or to an unnoticed past, or to a past doubtfully reported or forgotten; it may even relate to something more or less knowable (by means of a computation, a logical deduction, etc.) but for which we are not willing or able to make the effort; and so on.

Moreover, probabilistic reasoning is completely unrelated to general philosophical controversies, such as Determinism versus Indeterminism, Realism versus Solipsism—including the question of whether the world ‘exists’, or is simply the scenery of ‘my’ solipsistic dream. As far as Determinism and Indeterminism are concerned, we note that, in the context of gas theory or heat diffusion and transmission, whether one interprets the underlying process as being random or strictly deterministic makes no difference to one’s probabilistic opinion. A similar situation would arise if one were faced with forecasting the digits in a table of numbers; it makes no difference whether the numbers are random, or are some segment—for example, the 2001st to the 3000th digits— of the decimal expansion of π (which is not ‘random’ at all, but certain; possibly available in tables and, in principle, computable by you).

The only relevant thing is uncertainty—the extent of our own knowledge and ignorance. The actual fact of whether or not the events considered are in some sense *determined*, or known by other people, and so on, is of no consequence.

The numerous, different, opposed attempts to put forward particular points of view which, in the opinion of their supporters, would endow Probability Theory with a 'nobler' status, or a 'more scientific' character, or 'firmer' philosophical or logical foundations, have only served to generate confusion and obscurity, and to provoke well-known polemics and disagreements—even between supporters of essentially the same framework.

The main points of view that have been put forward are as follows.

The *classical* view, based on physical considerations of symmetry, in which one should be obliged to give the same probability to such 'symmetric' cases. But which symmetry? And, in any case, why? The original sentence becomes meaningful if reversed: the symmetry is probabilistically significant, in someone's opinion, if it leads him to assign the same probabilities to such events.

The *logical* view is similar, but much more superficial and irresponsible inasmuch as it is based on similarities or symmetries which no longer derive from the facts and their actual properties, but merely from the sentences which describe them, and from their formal structure or language.

The *frequentist* (or *statistical*) view presupposes that one accepts the classical view, in that it considers an *event* as a class of *individual events*, the latter being 'trials' of the former. The individual events not only have to be 'equally probable', but also 'stochastically independent'...(these notions when applied to individual events are virtually impossible to define or explain in terms of the frequentist interpretation). In this case, also, it is straightforward, by means of the subjective approach, to obtain, under the appropriate conditions, in a

perfectly valid manner, the results aimed at (but unattainable) in the statistical formulation. It suffices to make use of the notion of exchangeability. The result, which acts as a bridge connecting this new approach with the old, has been referred to by the objectivists as 'de Finetti's representation theorem'.

It follows that all the three proposed definitions of 'objective' probability, although useless *per se*, turn out to be useful and good as valid auxiliary devices when included as such in the subjectivistic theory.

The above-mentioned 'representation theorem', together with every other more or less original result in my conception of probability theory, should not be considered as a discovery (in the sense of being the outcome of advanced research). Everything is essentially the fruit of a thorough examination of the subject matter, carried out in an unprejudiced manner, with the aim of rooting out nonsense.

And probably there is nothing new; apart, perhaps, from the systematic and constant concentration on the unity of the whole, avoiding piecemeal tinkering about, which is inconsistent with the whole; this yields, in itself, something new.

Something that may strike the reader as new is the radical nature of certain of my theses, and of the form in which they are presented. This does not stem from any deliberate attempt at radicalism, but is a natural consequence of my abandoning the reverential awe which sometimes survives in people who at one time embraced the objectivistic theories prior to their conversion (which hardly ever leaves them free of some residual).

It would be impossible, even if space permitted, to trace back the possible development of my ideas, and their relationships

with more or less similar positions held by other authors, both past and present. A brief survey is better than nothing, however (even though there is an inevitable arbitrariness in the selection of names to be mentioned).

I am convinced that my basic ideas go back to the years of High School as a result of my preference for the British philosophers Locke, Berkeley and, above all, Hume! I do not know to what extent the Italian school textbooks and my own interpretations were valid: I believe that my work based on exchangeability corresponds to Hume's ideas, but some other scholars do not agree. I was also favourably impressed, a few years later, by the ideas of Pragmatism, and the related notions of operational definitions in Physics. I particularly liked the Pragmatism of Giovanni Vailati—who somehow 'Italianized' James and Peirce—and, as for operationalism, I was very much struck by Einstein's relativity of 'simultaneity', and by Mach and (later) Bridgman.

As far as Probability is concerned, the first book I encountered was that of Czuber. (Before 1950—my first visit to the USA—I did not know any English, but only German and French.) For two or three years (before and after the 'Laurea' in Mathematics, and some application of probability to research on Mendelian heredity), I attempted to find valid foundations for all the theories mentioned, and I reached the conclusion that the classical and frequentist theories admitted no sensible foundation, whereas the subjectivistic one was fully justified on a normative-behaviouristic basis. I had some indirect knowledge of De Morgan, and found that some of Keynes' ideas were in partial agreement with mine; some years later I was informed of the similar approach that had been adopted by F. P. Ramsey.

Independent ideas, which were more or less similar, were put forward later by Harold Jeffreys. B. O. Koopman, and I. J.

Good (with some beautiful new discussion which illustrated the totally illusory nature of the so-called objective definitions of probability). I could add to this list the name of Rudolf Carnap, but this would be not altogether proper in the light of his own vivid, subjective behaviouristic interpretation. (Richard Jeffreys, in publishing Carnap's posthumous works, seems convinced of his underlying subjectivism.) A singular position is occupied by Robert Schlaifer, who arrived at the subjectivistic approach directly and with impressive freshness and originality, with little knowledge of previous work in the field. A similar thing, although in a different sense, may be said of George Pólya, who discussed *plausible reasoning* in mathematics in the sense of the probability (subjective, of course) of a supposed theorem being true, given the state of mind of the mathematician, and its (Bayesian) modification when new information or ideas appear. The following statement of his is most remarkable: 'It seems to me more philosophical to consider the general idea of *plausible reasoning* instead of its isolated particular cases' like *inductive* (and analogical) *reasoning*. (There have been so many vain attempts to build a theory of induction without beliefs—like a theory of elasticity without matter.)"

A very special mention must be reserved, however, for Leonard J. Savage and Dennis V. Lindley, who escaped from the objectivistic school, after having grown up in it, by a gradual discovery of its ambiguities with the clarity of the subjectivistic theory, and the latter's suitability for every kind of practical or theoretical problem. I have often had the opportunity of profitable exchanges of ideas with them, and, in the case of Savage, of actual collaboration, I wrote briefly of Savage's invaluable contributions as a dedication to my book *Probability, Induction and Statistics*, which appeared a few months after his sudden and premature death.

One should note, however, that, even with such close colleagues, agreement ought not to be absolute, on every detail. For example, not all agree with the rejection of countable-additivity.

Finally, having mentioned several of the authors who are more or less connected with the subjectivistic (and Bayesian) point of view, I feel an obligation to recall three great men—the first two, unfortunately, no longer with us—who, although they all shared an opposed view about our common subject, were always willing to discuss, and were extraordinarily friendly and helpful on every occasion. I refer to Guido Castelnuovo, Maurice Fréchet and Jerzy Neyman.

Rome, 16 July 1973

Bruno de Finetti

<https://www.everand.com/read/338056001/Theory-of-Probability-A-critical-introductory-treatment>