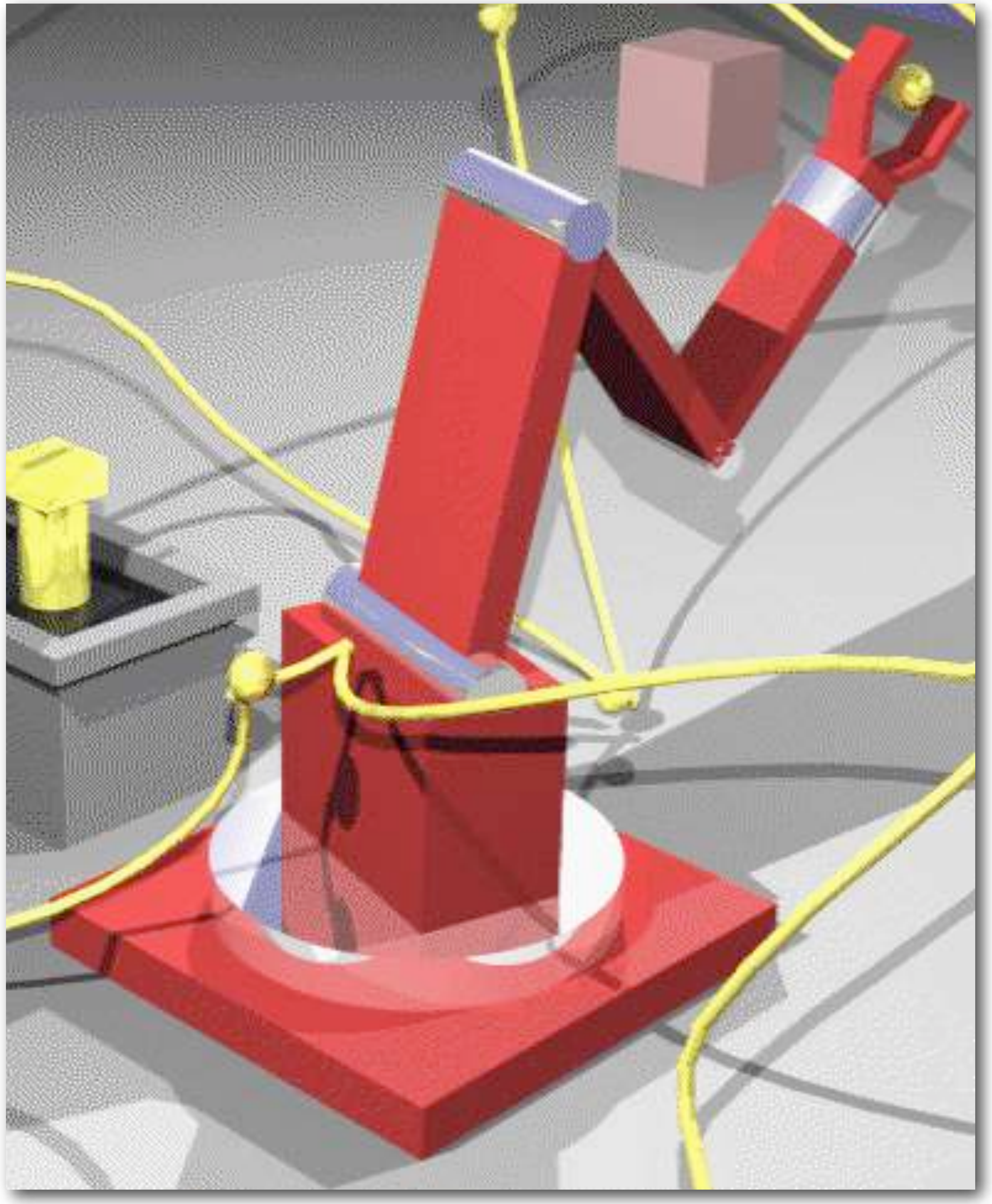
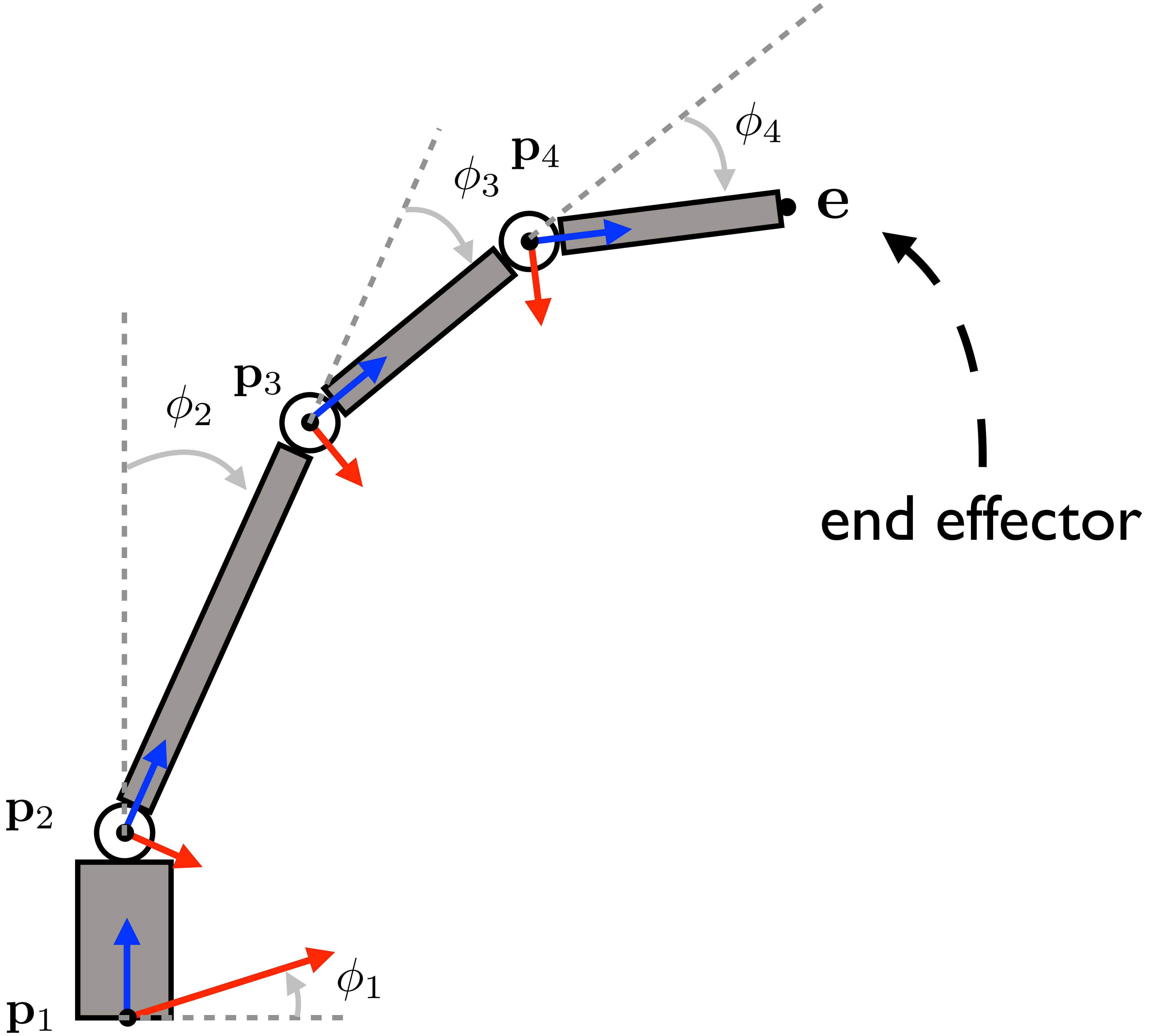
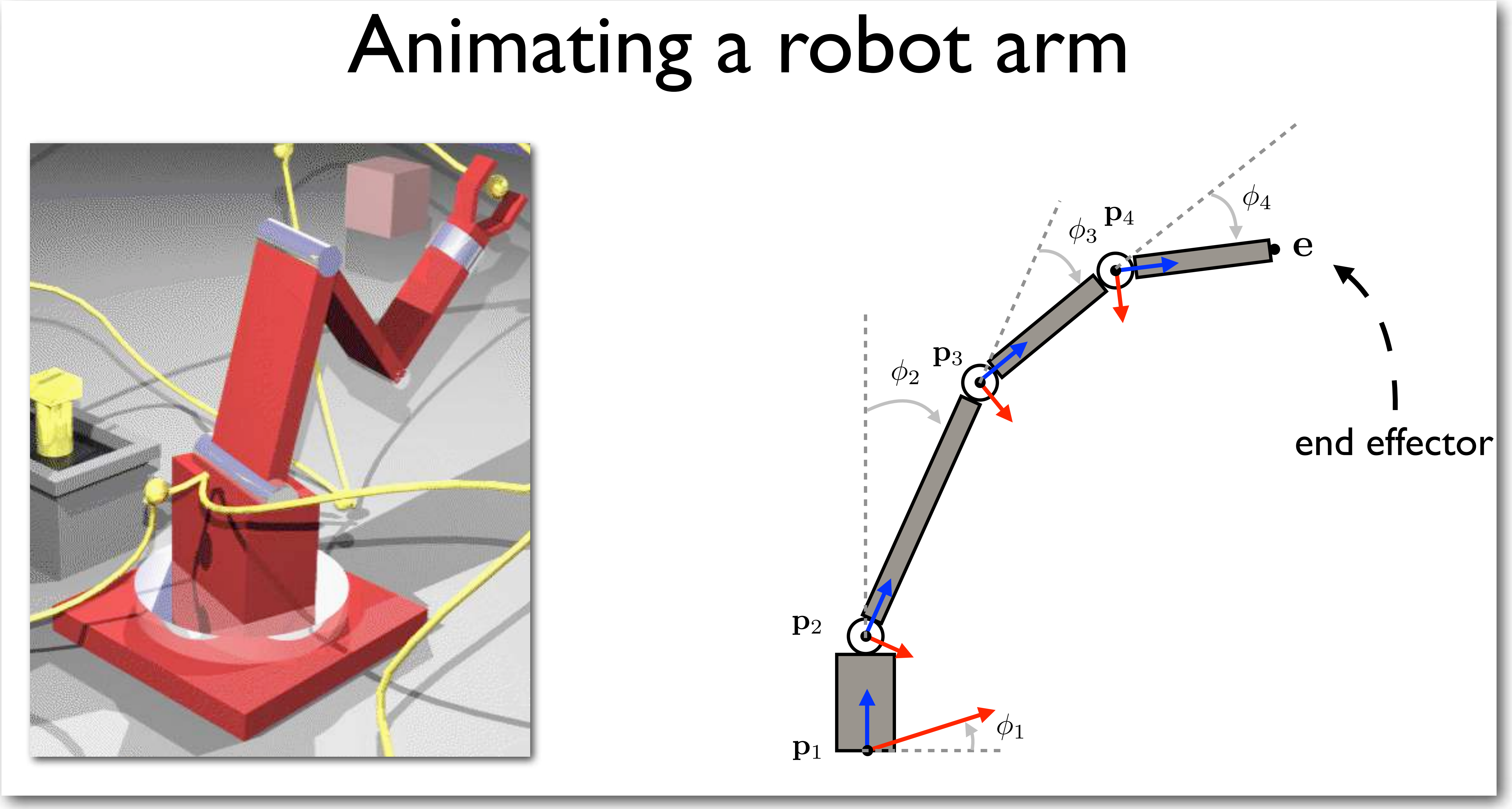
Animating a robot arm: Part 5





Overview

- A 2-D robot arm
- Forward kinematics



Each part is represented in their local frame of reference. Starting with first frame, i.e., Frame 1, we create the other Frame matrix using the previous one as starting point.

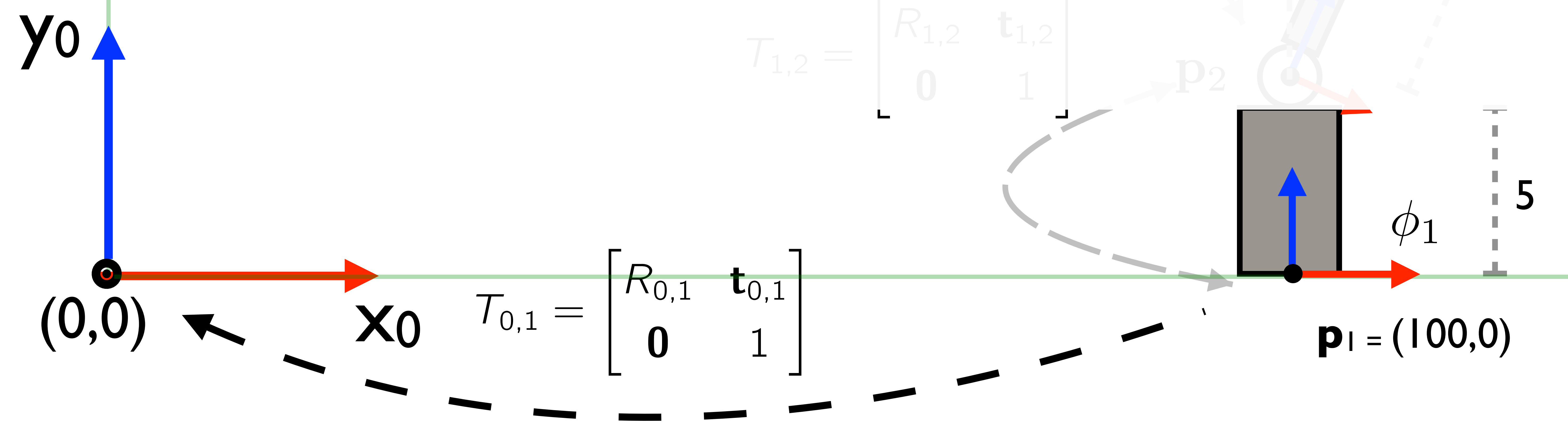
Frame 1 (part 1)

Rotate by $\phi_1 = 0$

Translation by $\mathbf{t}_{0,1} = (100, 0)^{\mathsf{T}}$

$$T_{0,1} = \begin{bmatrix} R_{0,1} & \mathbf{t}_{0,1} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 100 \\ \sin\phi_1 & \cos\phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 1 is at Frame 1.
Frame 1 has zero rotation and is translated by 100 along the x-axis.



Frame 1 (part 1)

Rotate by $\phi_1 = 0$

Translation by $\mathbf{t}_{0,1} = (100, 0)^T$

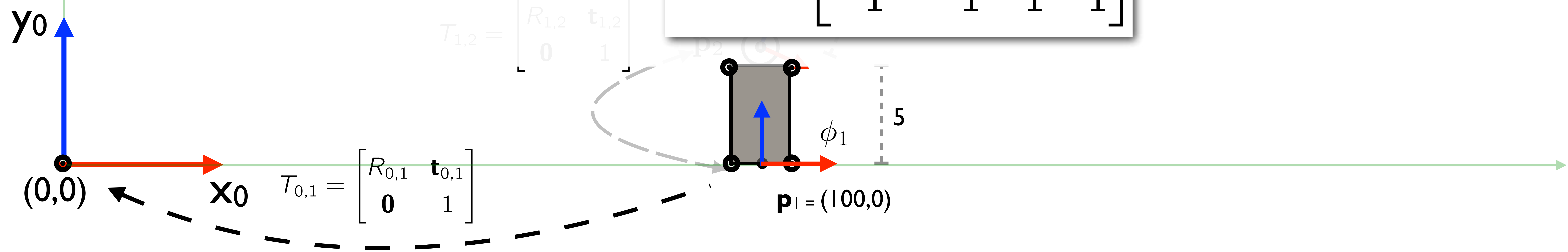
$$T_{0,1} = \begin{bmatrix} R_{0,1} & \mathbf{t}_{0,1} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 100 \\ \sin\phi_1 & \cos\phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 1 has its coordinates described in terms of Frame 1. These coordinates are listed in matrix P1. The superscript notation with curly bracket labels the frame of reference.

To draw Part 1, we must transform its local coordinates to global coordinates first before we call the plot or drawing function.

Part 1 coords w.r.t. Frame 1

$$P_1^{\{1\}} = \begin{bmatrix} -3 & -3 & 3 & 3 \\ 0 & 5 & 5 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



Frame 1 (part 1)

Rotate by $\phi_1 = 0$

Translation by $\mathbf{t}_{0,1} = (100, 0)^\mathsf{T}$

$$T_{0,1} = \begin{bmatrix} R_{0,1} & \mathbf{t}_{0,1} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 100 \\ \sin\phi_1 & \cos\phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can now call the plot/drawing functions on the converted coordinates of Part 1 to display the part at the correct location in global coordinates.

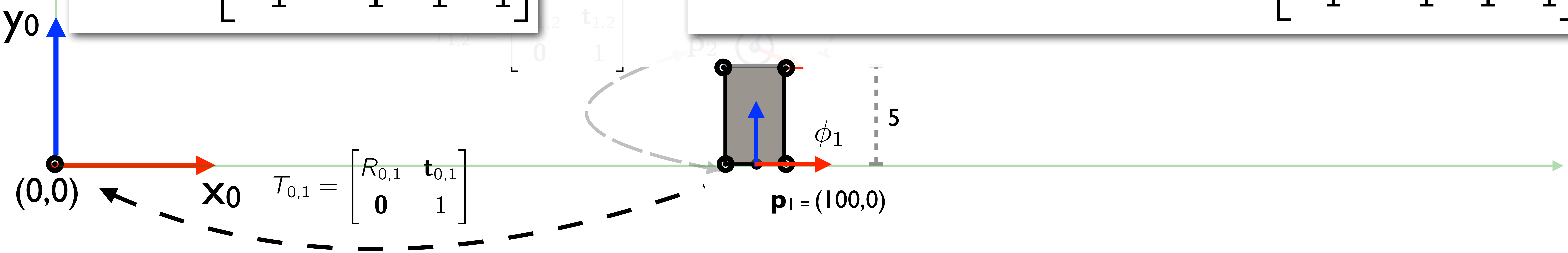
Next slide, we will look at the Part 2 of the articulated arm.

Part 1 coords w.r.t. Frame 1

$$P_1^{\{1\}} = \begin{bmatrix} -3 & -3 & 3 & 3 \\ 0 & 5 & 5 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Part 1 coords w.r.t. Frame 0

$$P_1^{\{0\}} = T_{0,1}\, P_1^{\{1\}} = \begin{bmatrix} R_{0,1} & \mathbf{t}_{0,1} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} -3 & -3 & 3 & 3 \\ 0 & 5 & 5 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

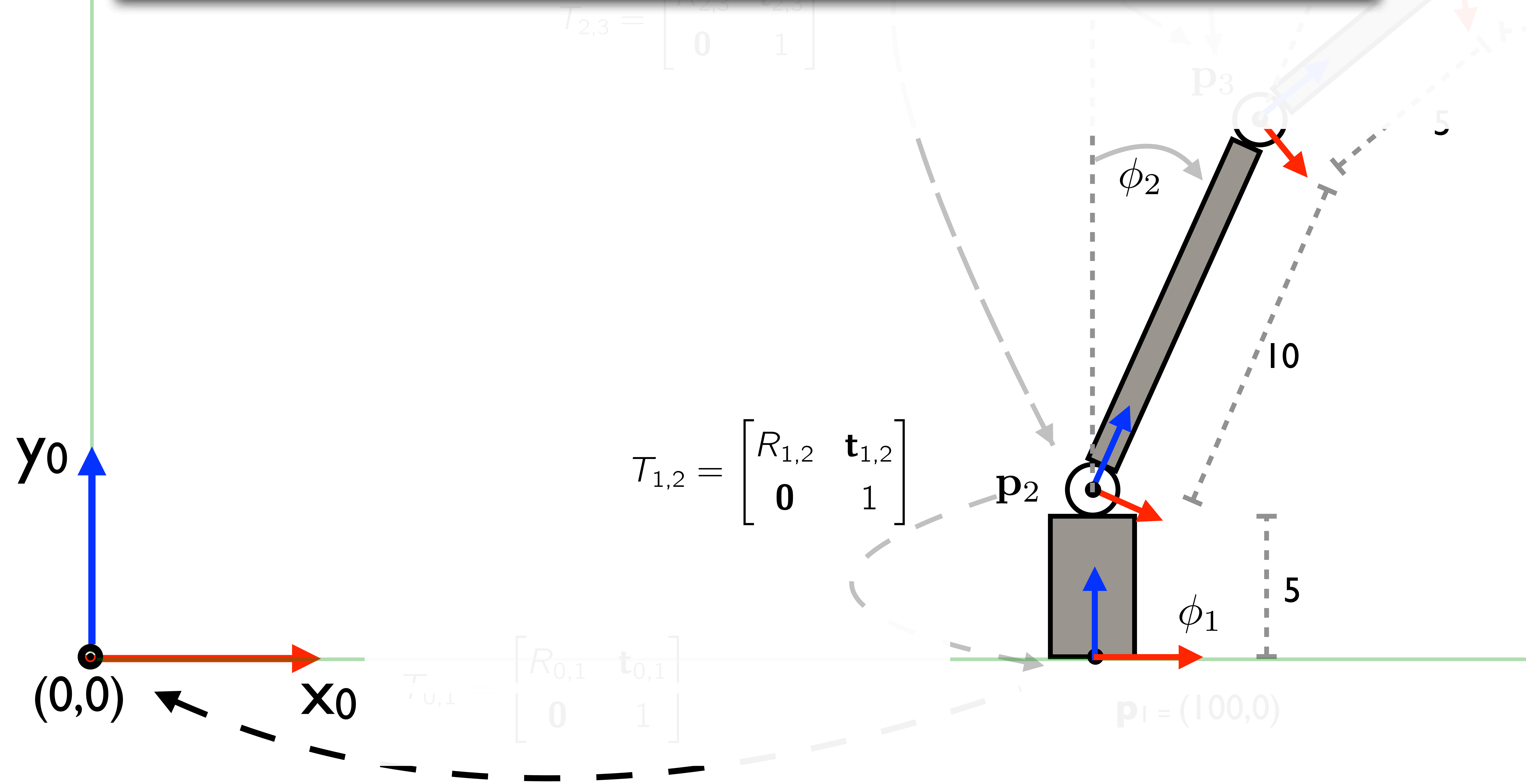


Frame 2 (part 2)

Rotate by ϕ_1
Translation by $\mathbf{t}_{1,2} = (0, 5)^\mathsf{T}$

$$T_{1,2} = \begin{bmatrix} R_{1,2} & \mathbf{t}_{1,2} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \cos\phi_2 & -\sin\phi_2 & 0 \\ \sin\phi_2 & \cos\phi_2 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 2, w.r.t. Frame 1, is translated along the y-axis by an amount equal to the length of Part 1 which is 5. Here, I will ignore the circle joints that are shown in the diagram. When you draw your robot, you can account for the circle joints in your calculations. Hint: make the circle joint a fixed part of Parts 2, 3, and 4 so the circle rotates with the part.



Frame 2 (part 2)

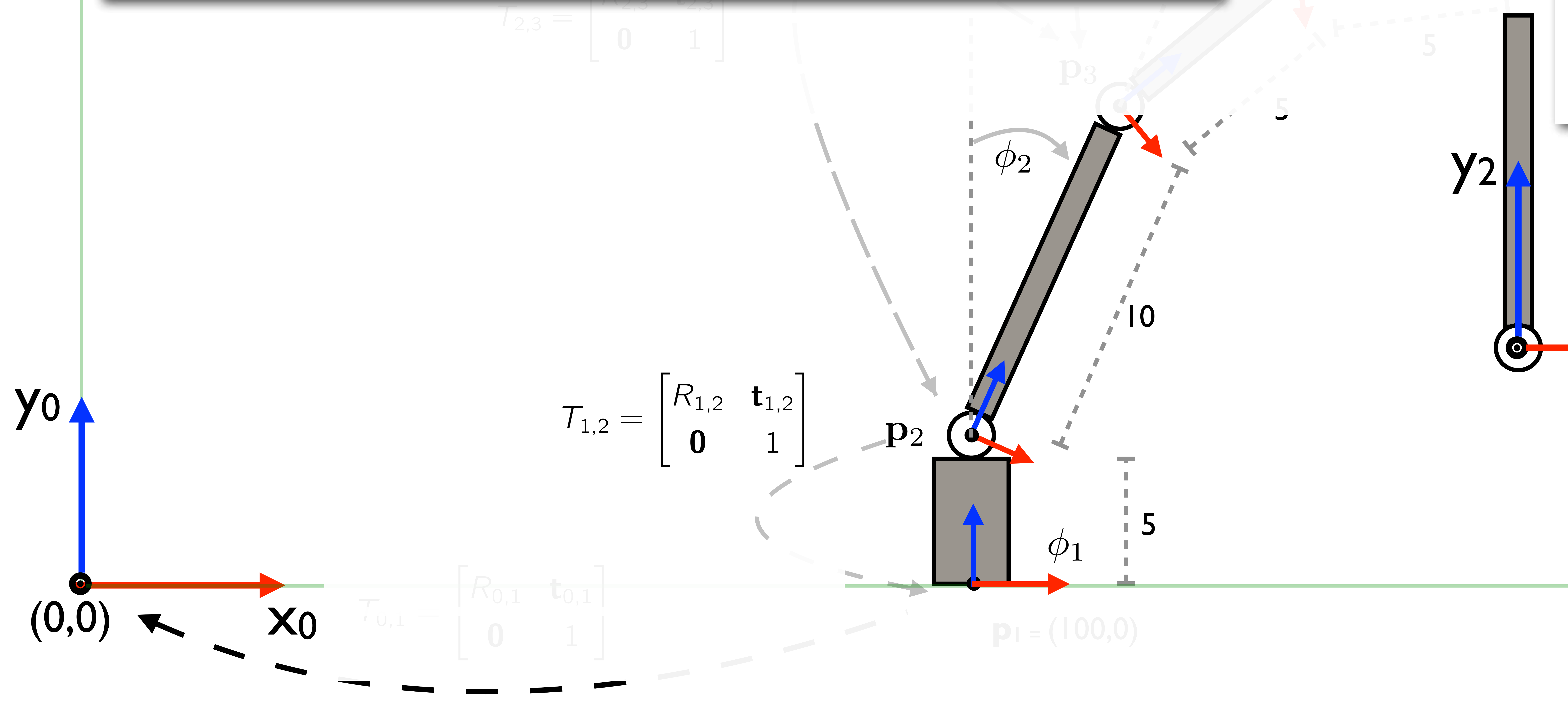
Rotate by ϕ_1

Translation by $\mathbf{t}_{1,2} = (0, 5)^{\mathsf{T}}$

$$T_{1,2} = \begin{bmatrix} R_{1,2} & \mathbf{t}_{1,2} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \cos\phi_2 & -\sin\phi_2 & 0 \\ \sin\phi_2 & \cos\phi_2 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

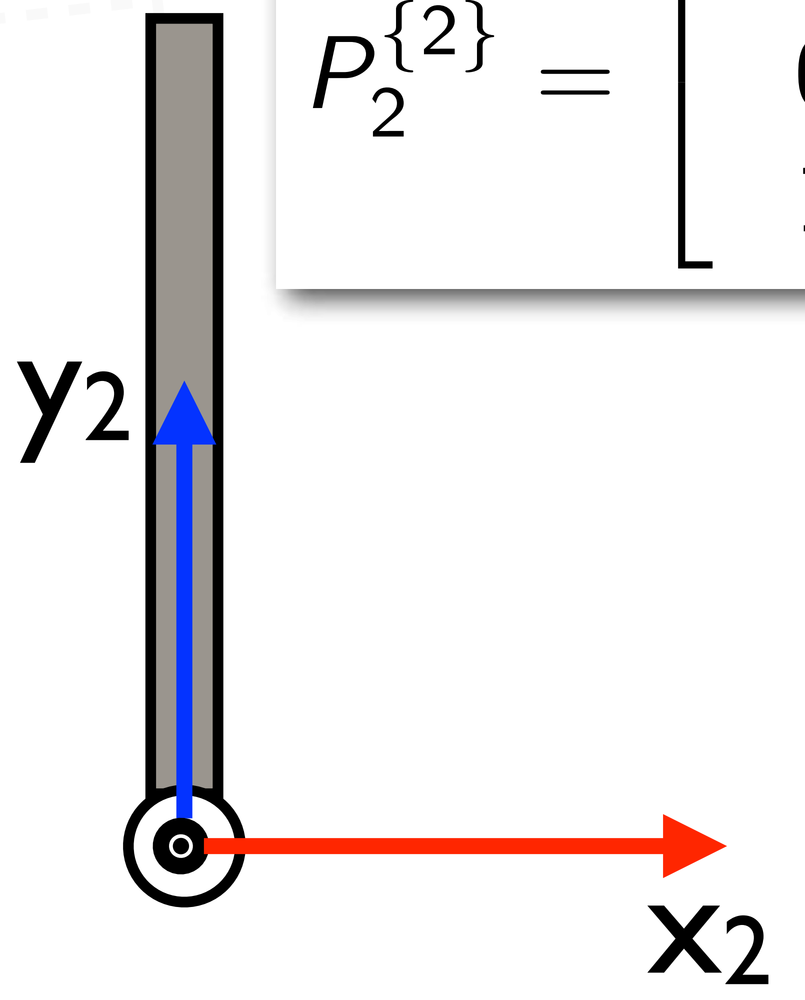
Initially, at its rest or neutral position, Part 2 is not yet rotated by its joint angle. So, we use the coordinates of part two in Frame 2 in its original neutral position

But, these coordinates are given w.r.t. Frame 2 and not Frame 0. As a result, we cannot plot/draw the object directly using them. We must transform the coordinates to the global coordinate system.



Part 2 coords w.r.t. Frame 2

$$P_2^{\{2\}} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & 10 & 10 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



Frame 2 (part 2)

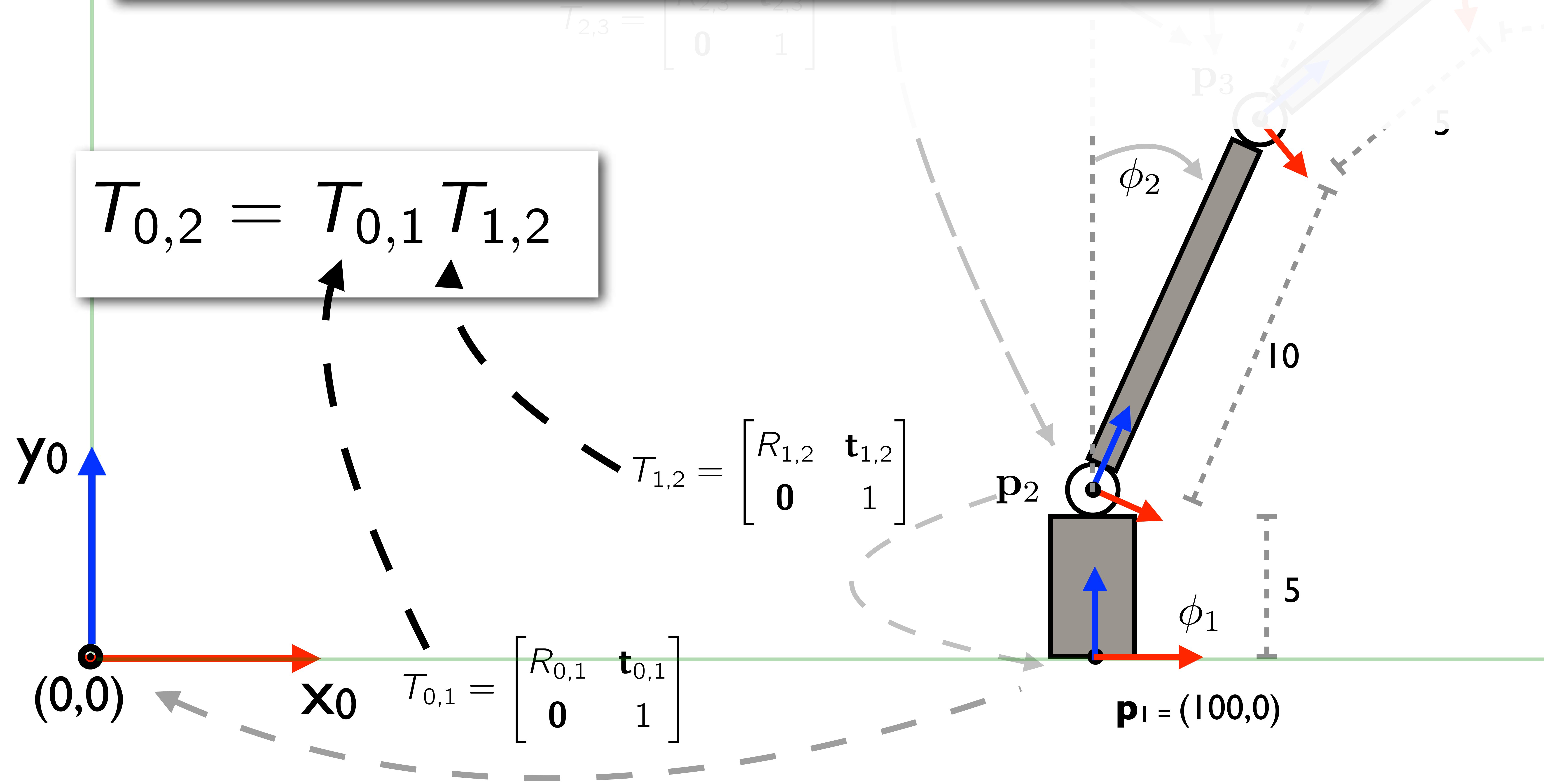
We need to calculate T_{0,2}. This calculation is done recursively, i.e., T_{0,2} = T{0,1} * T{1,2}.

Rotate by ϕ_1

Translation by $\mathbf{t}_{1,2} = (0, 5)^{\mathsf{T}}$

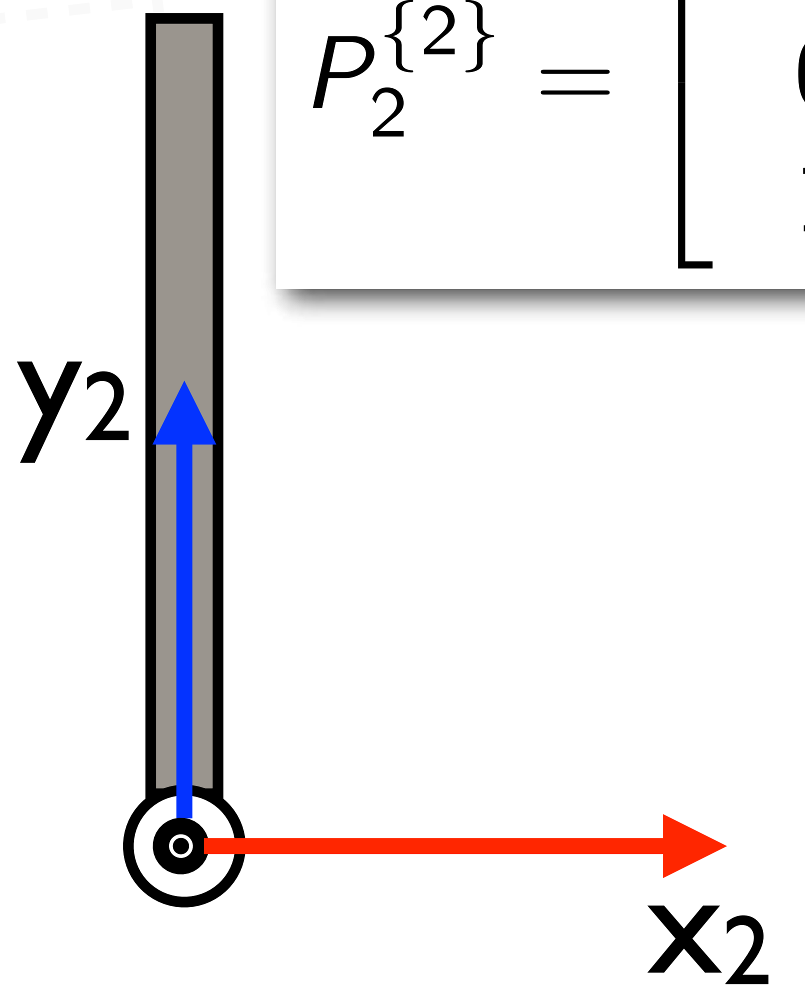
$$T_{1,2} = \begin{bmatrix} R_{1,2} & \mathbf{t}_{1,2} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \cos\phi_2 & -\sin\phi_2 & 0 \\ \sin\phi_2 & \cos\phi_2 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{0,2} = T_{0,1}\, T_{1,2}$$



Part 2 coords w.r.t. Frame 2

$$P_2^{\{2\}} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & 10 & 10 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



After converting the frames, P_2^{0} is now part 2 w.r.t. to frame 0 and we can draw the object at the correct location and orientation.

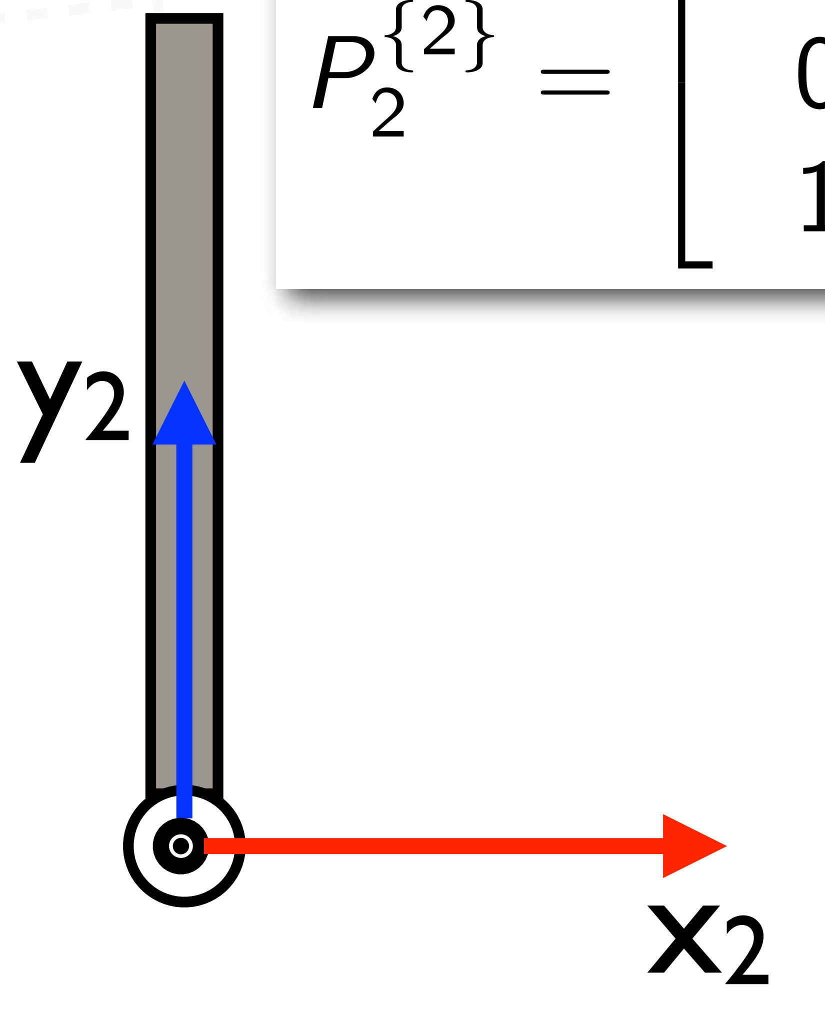
Repeat these steps for all parts to draw the whole arm.

Part 2 coords w.r.t. Frame 0

$$P_2^{\{0\}} = \underbrace{T_{0,1}\, T_{1,2}}_{T_{0,2}}\, P_2^{\{2\}} = \begin{bmatrix} R_{0,2} & \mathbf{t}_{0,2} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & 10 & 10 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Part 2 coords w.r.t. Frame 2

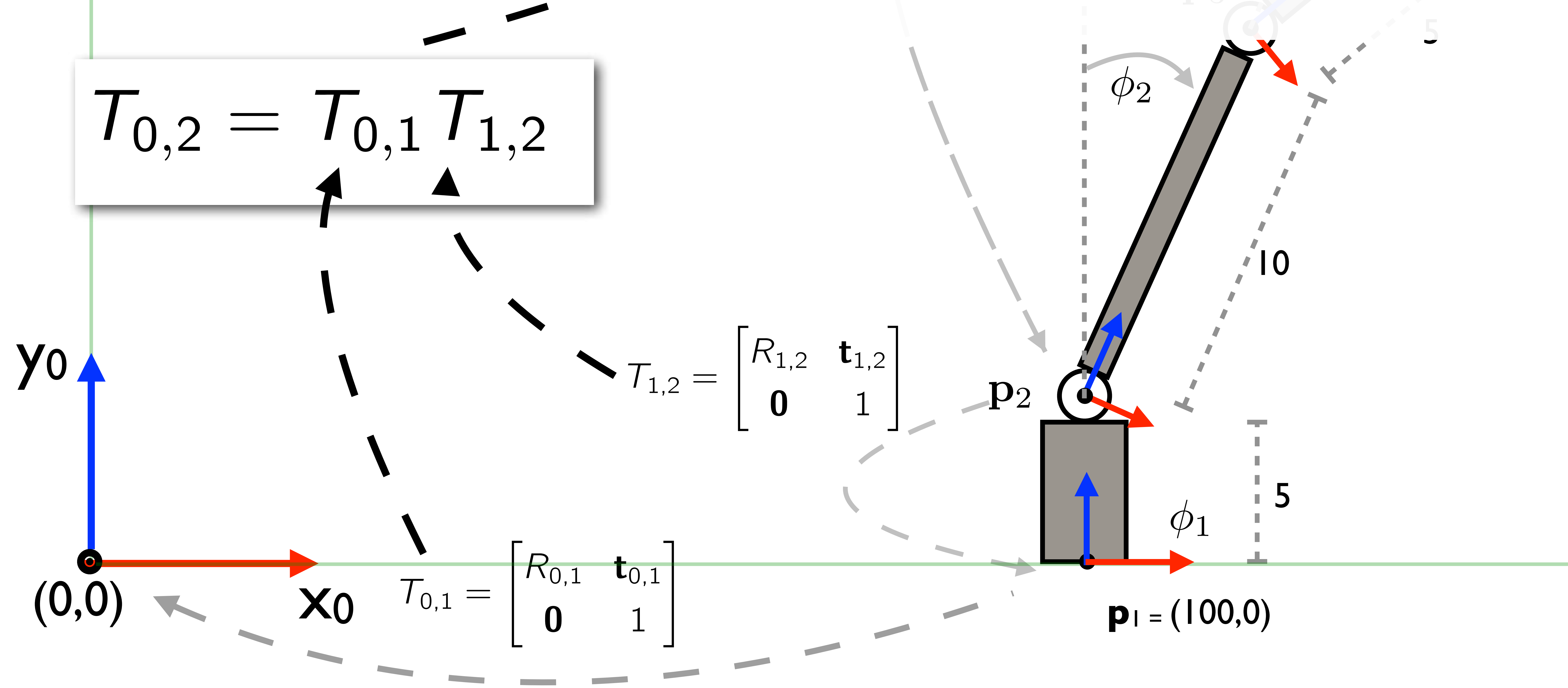
$$P_2^{\{2\}} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ 0 & 10 & 10 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$



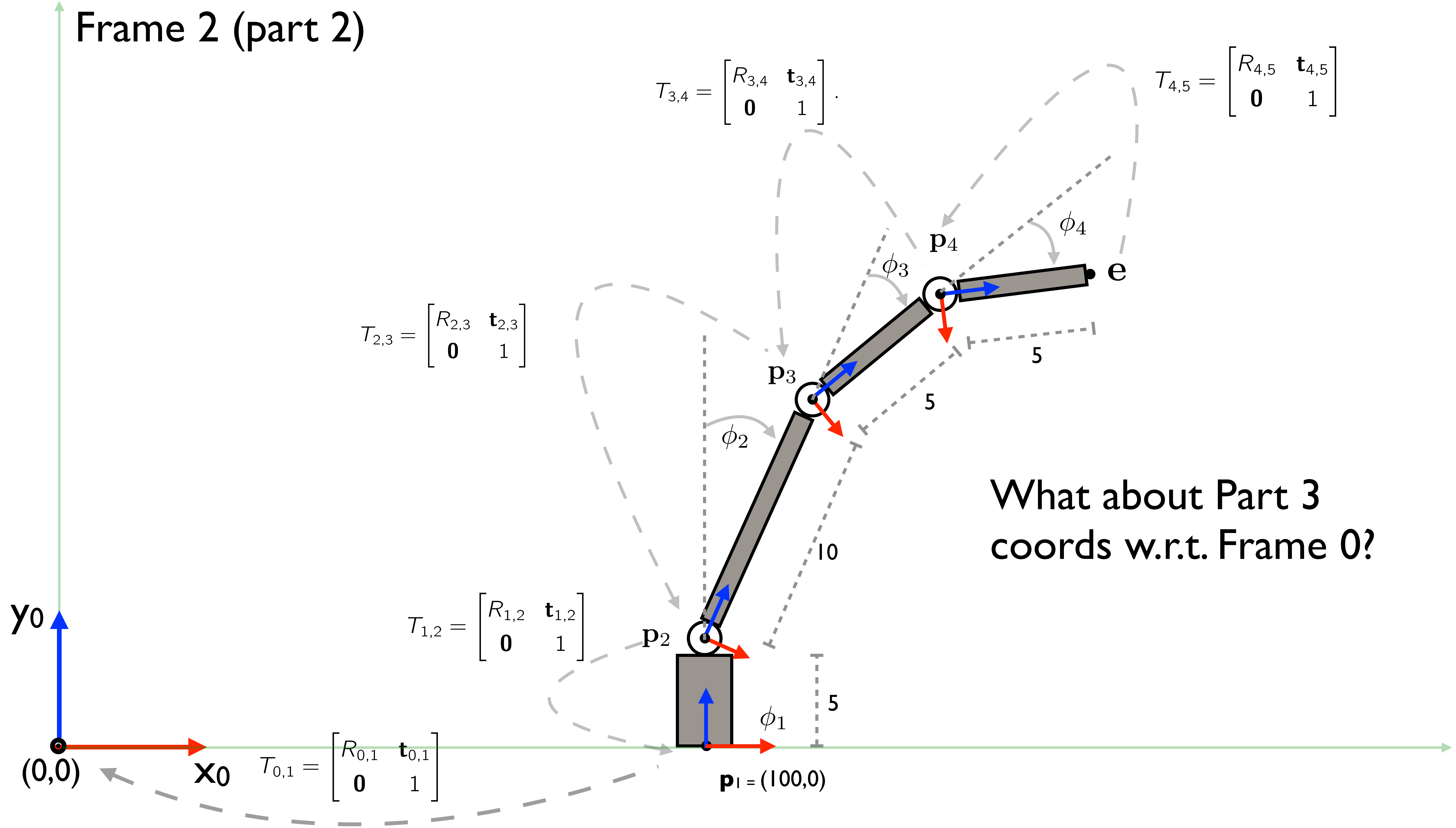
$$T_{0,2} = T_{0,1}\, T_{1,2}$$

$$T_{1,2} = \begin{bmatrix} R_{1,2} & \mathbf{t}_{1,2} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$T_{0,1} = \begin{bmatrix} R_{0,1} & \mathbf{t}_{0,1} \\ \mathbf{0} & 1 \end{bmatrix}$$



Frame 2 (part 2)



What about Part 3 coords w.r.t. Frame 0?

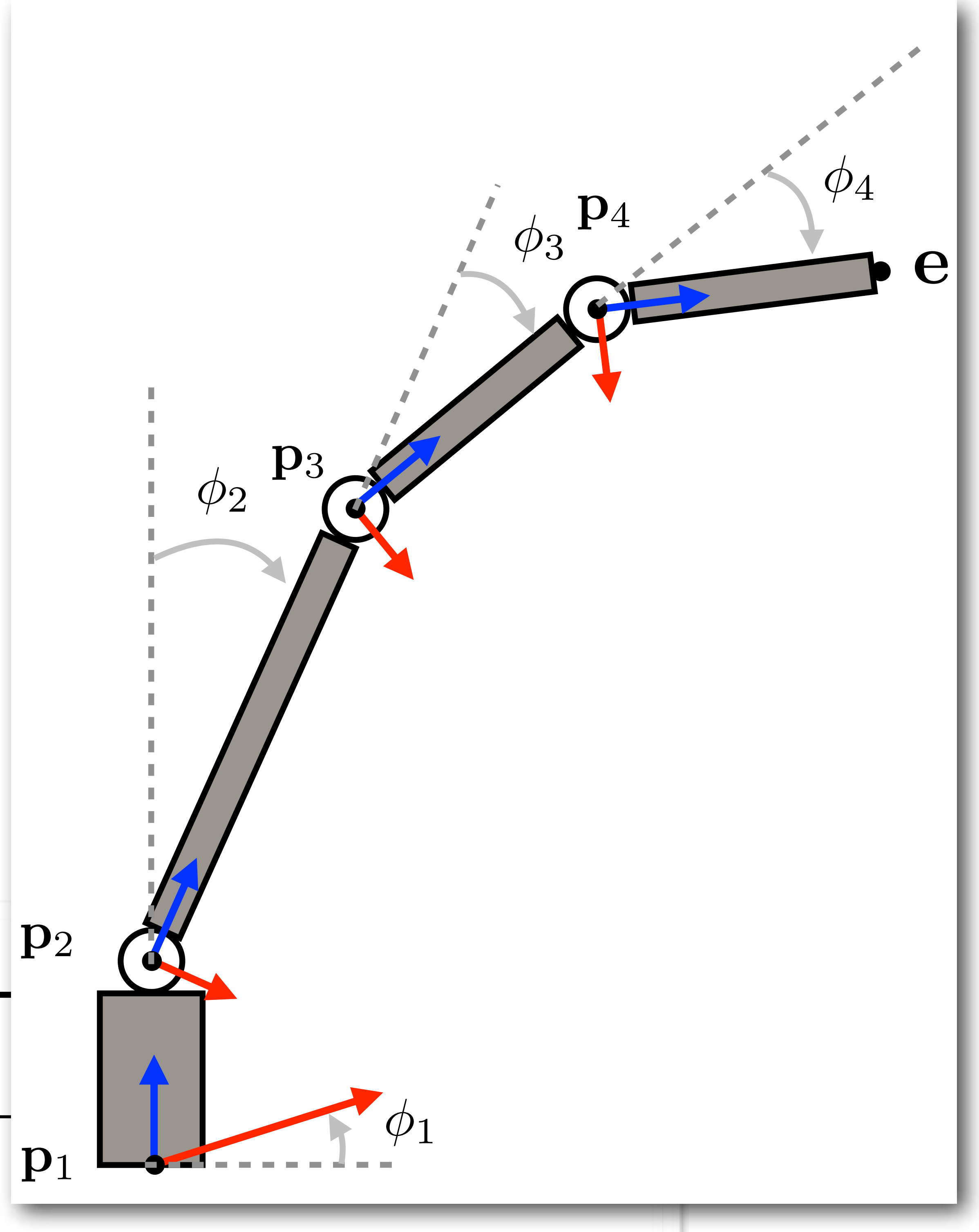
Forward kinematics

$$\mathbf{e} = f\left(\boldsymbol{\Phi}\right). \tag{1}$$

This function receives a set of joint angles (i.e., joint-angle configuration) as input and outputs the end-effector position. To implement this function, you need to calculate the matrix of last frame in the kinematic chain, which is given by:

$$T_{0,5} = T_{0,1}T_{1,2}T_{2,3}T_{3,4}T_{4,5} = \begin{bmatrix} R_{0,5} & \mathbf{p}_5 \\ \mathbf{0} & 1 \end{bmatrix}. \tag{2}$$

Forward kinematics



$$T_{0,5} = T_{0,1}T_{1,2}T_{2,3}T_{3,4}T_{4,5} = \begin{bmatrix} R_{0,5} & \mathbf{p}_5 \\ \mathbf{0} & 1 \end{bmatrix}$$

Algorithm 1 Forward-Kinematics Function (end-effector)

function **e**=f($\boldsymbol{\Phi}$)

Require: $l_1, \ldots, l_4$ ▷ Lengths of each part of the arm

Require: $T_{i-1,i}$, for $i = 1, \ldots, 5$. ▷ Local frame transformations

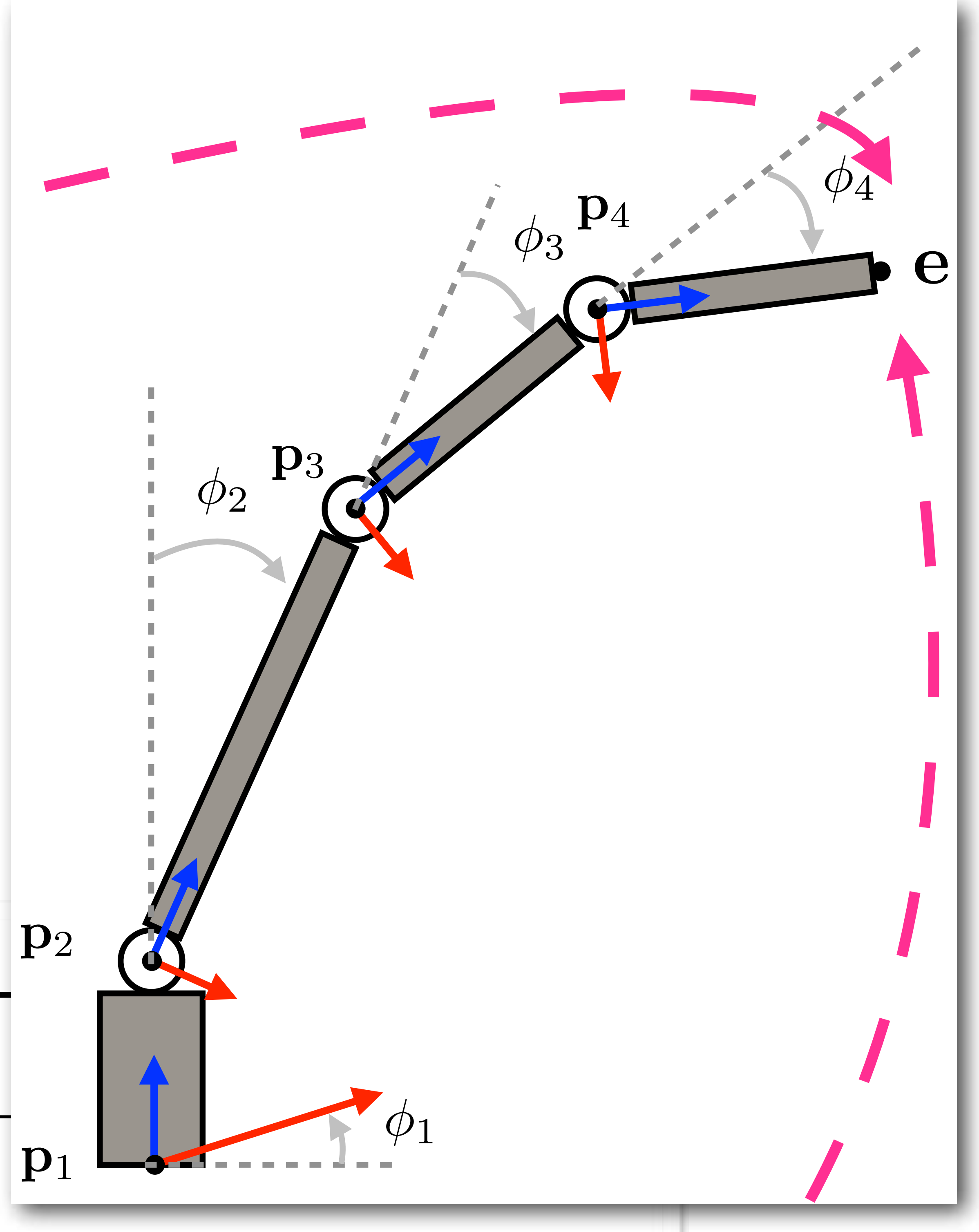
$T_{0,5} \leftarrow T_{0,1}T_{1,2}T_{2,3}T_{3,4}T_{4,5}$.

return $\mathbf{p}_5$ ▷ Translation component of $T_{0,5}$

end function

Forward kinematics

$$T_{0,5} = T_{0,1}T_{1,2}T_{2,3}T_{3,4}T_{4,5} = \begin{bmatrix} R_{0,5} & \mathbf{p}_5 \\ \mathbf{0} & 1 \end{bmatrix}$$



Algorithm 1 Forward-Kinematics Function (end-effector)

1: **function** $\mathbf{e}=\mathrm{f}(\boldsymbol{\Phi})$

Require: $l_1, \ldots, l_4$ ▷ Lengths of each part of the arm

Require: $T_{i-1,i}$, for $i = 1, \ldots, 5$. ▷ Local frame transformations

2:

3: $T_{0,5} \leftarrow T_{0,1}T_{1,2}T_{2,3}T_{3,4}T_{4,5}$.

return $\mathbf{p}_5$ ▷ Translation component of $T_{0,5}$

4: **end function**

Review

- A 2-D robot arm
- Forward kinematics

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