Combinatorial evaluation of pairs partition permutations

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I. Introduction:

In 2-Way superscalar SMT processor with four hardware threads, the input instructions of the issue scheduler are eight with layout of a 2x4 instructions image, as shown in fig. 1. Each clock cycle two instructions of the eight, i.e., instructions pair, is issue scheduled to the decode stage. The question is how many permutations of instructions image coverage by four instructions pairs (under constraint of in-order processor) exist. The issue scheduling instructions image coverage's four pairs constitute a coverage series. A series issue scheduling order of any coverage's pairs matters but a single coverage where all pairs contain instructions of the same thread. The permutations number is 52 where the combinatorial evaluation is given at section II.

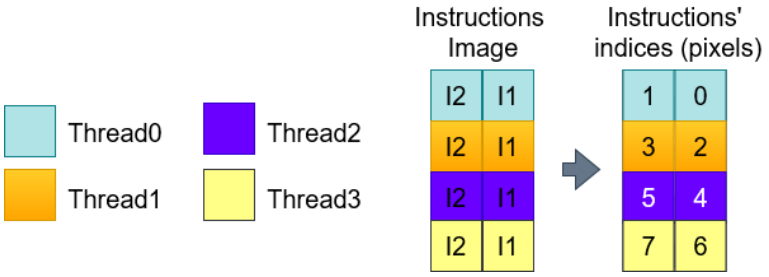

Fig.1: Output of four Instruction Queues as an Instructions Image size of 2x4 pixels. Instruction Image indices 0,1 i.e., II(0,1) are the IQ0's output channels 1,2 respectively (each IQ has two output channels). II(2,3) are the IQ1's output channels 1,2 respectively. II(4,5) are the IQ2's output channels 1,2 respectively. II(6,7) are the IQ3's output channels 1,2 respectively.

II. Combinatorial evaluation:

Combinatorial evaluation of instructions image coverage is composed from five permutations groups. Each group has a peculiar combinatorial pattern and from it we extract partial permutations.

a. Combinatorial pattern 1:

Fig.2 depicts the combinatorial pattern 1 which contains nine permutations shown in Fig.3. The pattern of choosing two pairs from the right column contains 3 permutations {II(0,2), II(4,6)}, {II(0,4), II(2,6)}, {II(0,6), II(2,4)}. The pattern of choosing two pairs from the left column contains 3 permutations {II(1,3), II(5,7)}, {II(1,5), II(3,7)}, {II(1,7), II(3,5)}. The two columns together contains $3 \cdot 3 = 9$ permutations. Fig.3 shows the unfolded permutations of combinatorial pattern 1.

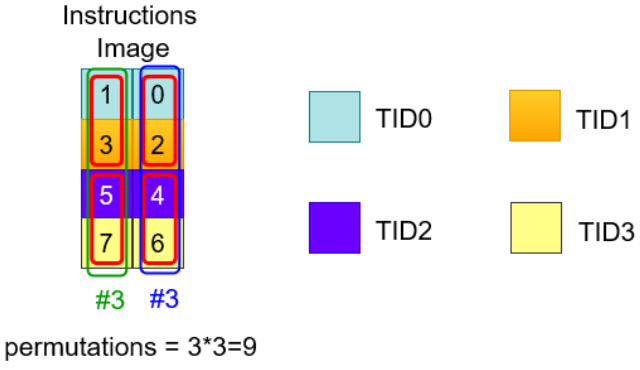

Fig.2: The combinatorial pattern 1

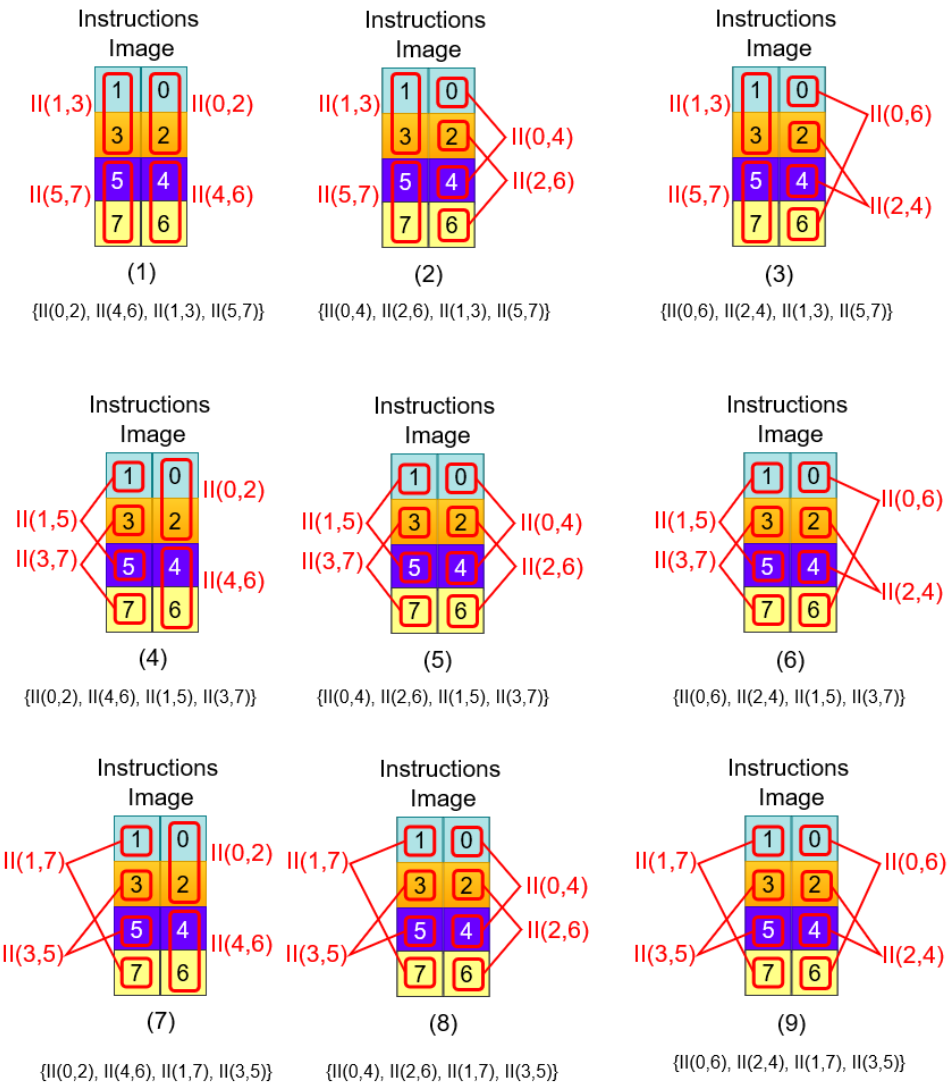

Fig.3: The unfolded permutations of combinatorial pattern 1.

b. Combinatorial pattern 2:

Fig.4 depicts the combinatorial pattern 2 which contains only a single permutation where all pairs contain instructions of the same thread.

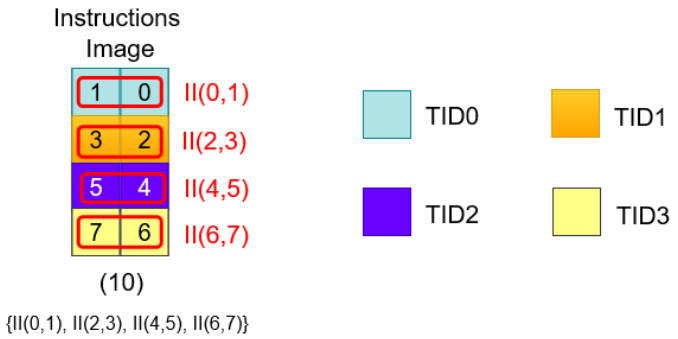

Fig.4: The combinatorial pattern 2

c. Combinatorial pattern 3:

Fig.5 depicts the combinatorial pattern 3 which contains six permutations shown in Fig.6. The pattern of choosing two pairs of the same thread each and the rest two pairs are of different threads has six options. Fig.6 shows the unfolded permutations of combinatorial pattern 3.

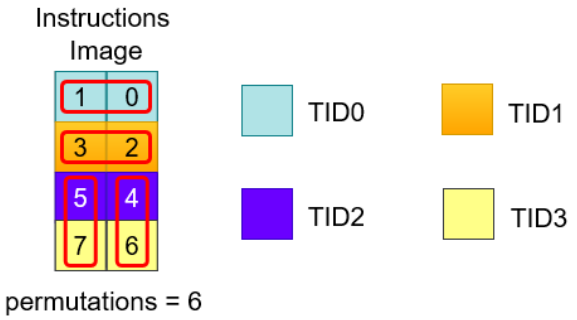

Fig.5: The combinatorial pattern 3

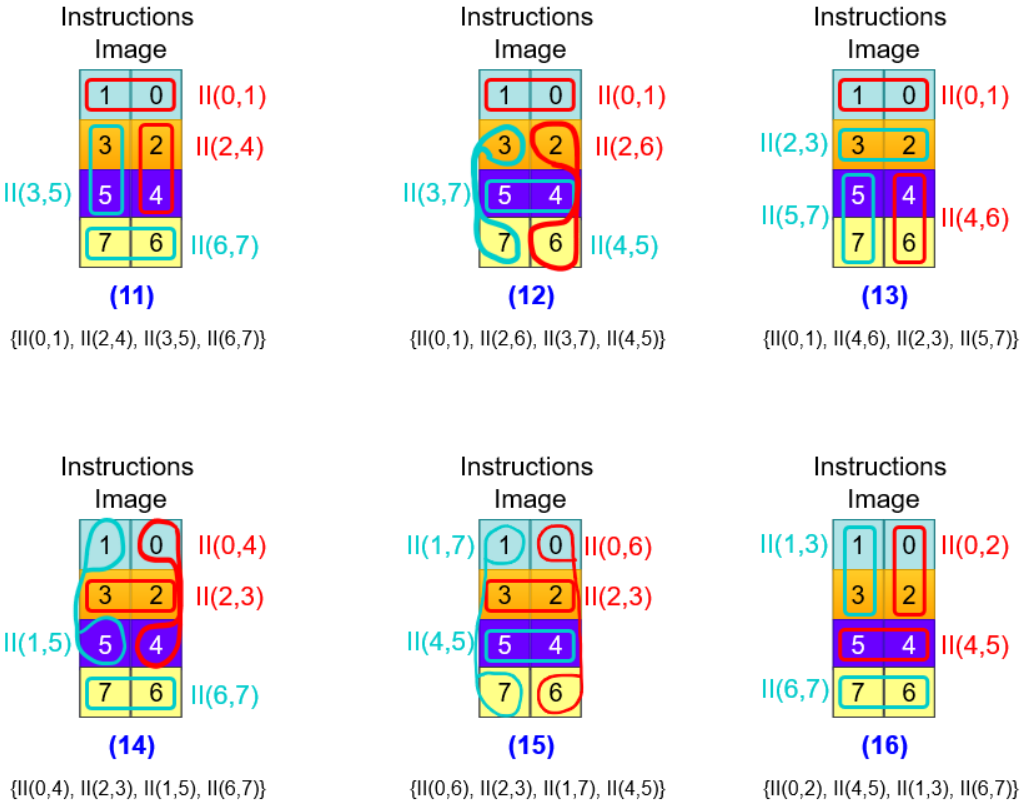

Fig.6: The unfolded permutations of combinatorial pattern 3.

d. Combinatorial pattern 4:

Fig.7 depicts the combinatorial pattern 4 which contains 24 permutations are shown in Fig.8, Fig.9. The pattern of choosing coverage of four pairs are as follows:

i. Choose one pair of the same thread has 4 options {II(0,1), II(2,3), II(4,5), II(6,7)}
ii. Then choose one pair of different threads from the right column, has 3 options.
iii. Then choose one diagonal pair which its first instruction is from the right column and the second from the left column, this phase has 2 options.
iv. The last pair is of different threads from the left column, this has one option.

Fig.8, Fig.9 show the unfolded permutations of combinatorial pattern 4.

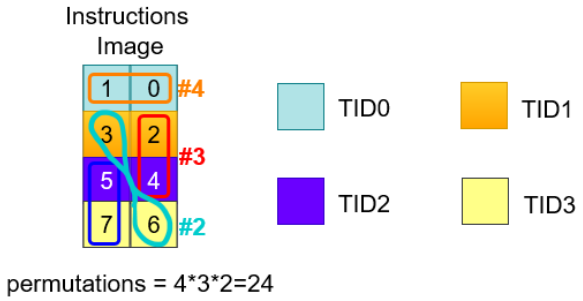

Fig.7: The combinatorial pattern 4.

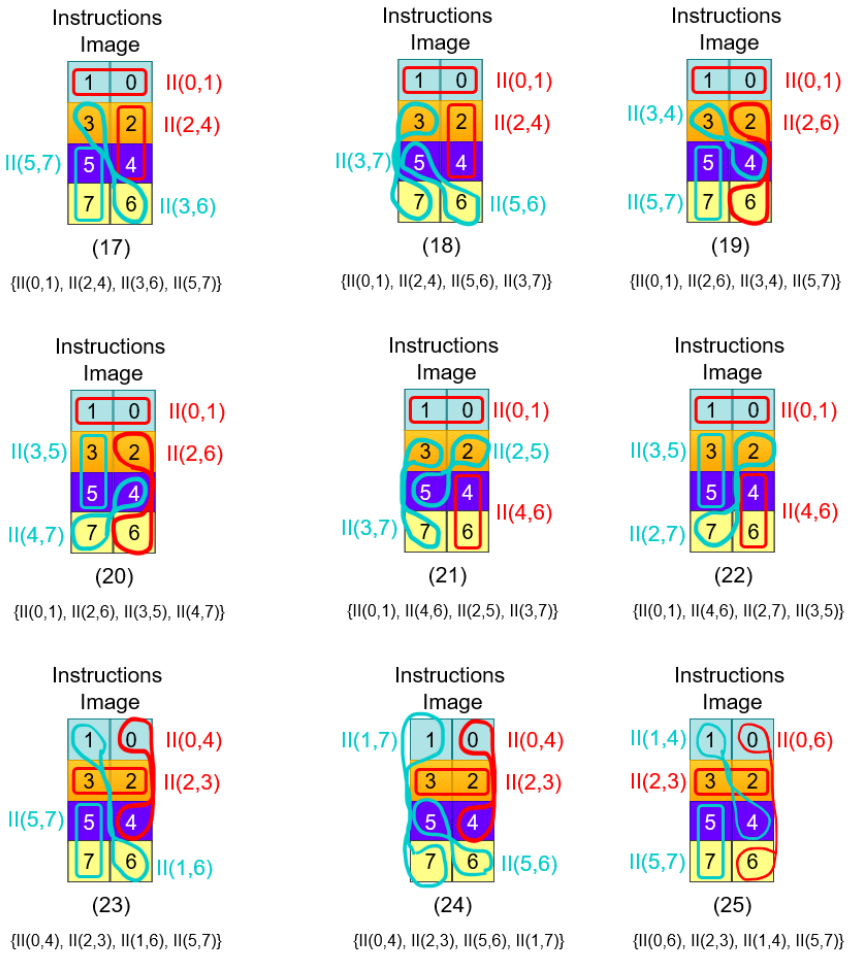

Fig.8: The unfolded first-nine permutations of combinatorial pattern 4.

Fig.9: The unfolded last fifteen permutations of combinatorial pattern 4.

e. Combinatorial pattern 5:

Fig.10 depicts the combinatorial pattern 5 which contains 12 permutations are shown in Fig.11.

The pattern of choosing coverage of four pairs are as follows:

i. Choose one pair of different threads from the right column has 6 options {II(0,2), II(0,4), II(0,6), II(2,4), II(2,6), II(4,6)}

ii. Then choose one diagonal pair which its first instruction is from the right column and the second from the left column of threads which was selected in phase i, this phase has 2 options.

iii. Repeat step ii again, , this phase has one option.

iv. The last pair is of different threads from the left column, this has one option.

Fig.11 shows the unfolded permutations of combinatorial pattern 4.

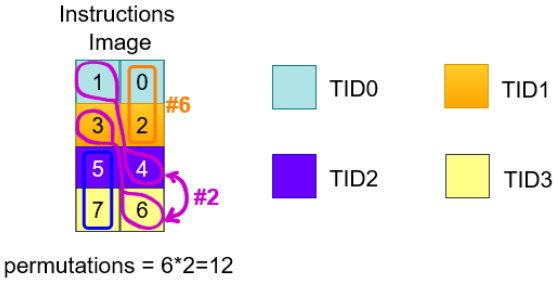

Fig.10: The combinatorial pattern 5.

III. Table of an unfolded list of all the 52 permutations:

Permutation Number	Pair 1		Pair 2		Pair 3		Pair 4	
	(left element	right element)	(left element	right element)	(left element	right element)	(left element	right element)
1	0	2	1	3	4	6	5	7
2	0	4	1	5	2	6	3	7
3	0	6	1	7	2	4	3	5
4	0	2	1	5	3	7	4	6
5	0	2	1	7	3	5	4	6
6	0	4	1	7	2	6	3	5
7	0	4	1	3	2	6	5	7
8	0	6	1	3	2	4	5	7
9	0	6	1	5	2	4	3	7
10	0	1	2	3	4	5	6	7
11	0	1	2	4	3	6	5	7
12	0	1	2	4	3	7	5	6
13	0	1	2	4	3	5	6	7
14	0	1	2	6	3	4	5	7
15	0	1	2	6	3	5	4	7
16	0	1	2	6	3	7	4	5
17	0	1	2	5	3	7	4	6
18	0	1	2	7	3	5	4	6
19	0	1	2	3	4	6	5	7
20	0	4	1	6	2	3	5	7
21	0	4	1	7	2	3	5	6
22	0	4	1	5	2	3	6	7
23	0	6	1	4	2	3	5	7
24	0	6	1	7	2	3	4	5
25	0	6	1	5	2	3	4	7
26	0	5	1	7	2	3	4	6
27	0	7	1	5	2	3	4	6
28	0	2	1	6	3	7	4	5
29	0	2	1	7	3	6	4	5
30	0	2	1	3	4	5	6	7
31	0	6	1	2	3	7	4	5

32	0	6	1	3	2	7	4	5
33	0	3	1	7	2	6	4	5
34	0	7	1	3	2	6	4	5
35	0	2	1	4	3	5	6	7
36	0	2	1	5	3	4	6	7
37	0	4	1	2	3	5	6	7
38	0	4	1	3	2	5	6	7
39	0	3	1	5	2	4	6	7
40	0	5	1	3	2	4	6	7
41	0	2	1	4	3	6	5	7
42	0	2	1	6	3	4	5	7
43	0	4	1	2	3	6	5	7
44	0	4	1	6	2	5	3	7
45	0	6	1	2	3	5	4	6
46	0	6	1	4	2	7	3	5
47	0	3	1	7	2	4	5	6
48	0	5	1	7	2	4	3	6
49	0	3	1	5	2	6	4	7
50	0	7	1	5	2	6	3	4
51	0	7	1	3	2	5	4	6
52	0	5	1	3	2	7	4	6