# Application of game theory to the capacity allocation problems in Internet Service Providers markets

Dariusz Gasior
Institute of Computer Science,
Wroclaw University of Technology
Wybrzeze Wyspianskiego 27, 50-370 Wroclaw, Poland
tel.: +48 71 320 35 83
Email: dariusz.gasior@pwr.edu.pl

Maciej Drwal
Institute of Computer Science,
Wroclaw University of Technology
Wybrzeze Wyspianskiego 27, 50-370 Wroclaw, Poland
tel.: +48 71 320 35 83
Email: dariusz.gasior@pwr.edu.pl

***Abstract*—In this paper the mathematical model of Internet Service Providers market is introduced. We assume that users' preferences are described in terms of the utility which reflects their willingness to pay. Each Internet Service Provider allocate his links' capacities among users to maximize his total profit. Users choose the Internet Service Provider which offers them the highest transmission rate. We model such a situation with game theoretical framework. This approach allows to examine in detail the interdependency between teletraffic market participants (network operators). We present an algorithm which enables finding all Nash equilibria of the proposed game. Introduced method is illustrated with a numerical example.**

***Keywords*—algorithmic game theory, network flow control, Internet Service Provider, network operators**

## I. Introduction

Nowadays, telecommunication and data delivery services are provided to end users via a hierarchical structure of operator networks, owned by Internet Service Provider (ISP) companies. Usually, such company owns or leases a communication infrastructure, consisting of wired links or wireless channels, which connects groups of end users to edge routers, capable of forwarding data packets to any network in the Internet. The maximum allowed capacity of links (i.e. maximum bandwidth or transmission rate) depends on the technology used to deploy them (fiber optics links, radio interface standards, etc.). However, due to the diversity of transmission patterns generated by distinct users and applications, the proper provisioning of bandwidth in packet networks becomes a key issue in optimizing performance. In order to improve the last mile bandwidth management and provide clients with flexible services, ISP companies allocate their capacity to particular local area networks, and in return, yield monetary profits for providing them with the global Internet connectivity. ISP can participate in the interconnection with other networks on the Internet via either settlement-free peering (mutual agreement to exchange of data packets) or by purchasing the access from other ISPs. The exchanged or purchased capacity need to be appropriately distributed for servicing individual transmissions (packet flows) in order to meet demands of end users. These demands are usually modeled in the subject literature in terms of utility functions, which relate the level of user satisfaction with their allocated transmission rate, reflecting users' willingness-to-pay for subscribed services [1], [2]. Several models include stricter bandwidth requirements (termed Quality of Service requirements) [3].

End users, content providers and network operators in the Internet share communication, computational and storage resources, with different goals in mind, translated into different notions of utility. On the global scale their actions are characterized by selfishness, i.e. each participant wishes to maximize only its own utility. However, assuming their behavior needs to be rational in order to reach their goals, it is necessary to take into consideration the mutual influence of their decisions. These aspects of network utility maximization evoke the use of game theory [4].

## II. Related work

The main difference of our proposal between the approach currently explored the most in the related game-theoretic literature is the departure from modeling network users as players, in order to model resource providers as players. We focus on considering game

978-1-4799-6515-1/14/$31.00 ©2014 IEEE

solutions not only from the point of view of resources consumers (users), but also the resource providers (operators, ISPs).

Below we summarize the current state of the art in this area and indicate some of the open problems. In [5] authors introduced models of profit-maximizing service providers. They showed that Wardrop equilibrium [6] rate allocation may differ from socially optimal one, and proposed oligopolistic and monopolistic pricing schemes for simple parallel network structures that lead to efficient allocations.

In [7] network pricing models including end users and ISPs were extended with content provider participation, resulting in a three-way interaction. It is proposed how to charge not only service subscribers but also content publishers, in order to improve total network utility. Work [8] presents a rudimentary modeling framework of demand and cost in ISP transit market, which allows to determine how to structure tiered contracts for selling connectivity in order to gain optimal profits. It was shown that the networks with lower traffic costs (e.g. covering shorter distances) require fewer tiers to achieve highest profit

The important class of games concerning allocation problems in networks (not necessarily communication networks) is called congestion games. Typically congestion games are applied to routing problems in computer networks, where the sources (users) are interpreted as players deciding on the selection of paths to transmit data at a given rate [9]. The players strategy consists in deciding how to split this rate among all possible paths from the source to the destination, or, if flows are unsplittable, which routes to use for transmission. In [10] authors propose a methodology of architecting noncooperative games for network resource allocation problem, which may improve overall system performance during provisioning and operating phase of network lifecycle. The solution is obtained for a parallel link network structure. It is shown that for such a case, the occurrence of the Braess paradox [11] may be avoided. Some of these results are extended for a general networks. In [12] the congestion model for the rate allocation problem is presented. The variant of a one-link network is analyzed, and it is shown that for such case, the Price of Anarchy is no greater than 4/3. Similarly, the extension for general networks is briefly discussed.

Bottleneck games are a similar class of routing games, in which a different payoff function is used [9]. Although the Nash equilibria for such games usually exist, their performance (estimated via Price of Anarchy values) is usually poor. In [13] two types of bottleneck games are considered, for splittable and unsplittable flows. It is also shown that for both proposed games the Price of Anarchy is unbounded. However, it is proven that under some mild conditions the Nash equilibrium is socially optimal. Work [14] considers both congestion game and bottleneck game, in the application to the routing problem. It also proposes a new routing game specifically for the elastic flows. All three approaches are compared.

In [3] the approach to resource allocation for the networks with quality of service (QoS) based on Differentiated Services darchitecture is proposed. The sources (flows) are players. They choose one QoS class and the transmission rate in this chosen class. The players payoffs are proportional to the transmission rate if their QoS requirements are satisfied and zero otherwise. For the proposed noncooperative game, a simple algorithm computing Nash equilibrium is presented. The extension of this concept is given in [15]. The joint problem of QoS routing and capacity allocation problem was also considered. In the proposed game, two groups of players are introduced, namely capacity players (each related to one link) and network users (each related to one pair of source and destination). Each capacity player divides its capacity among given Class of Services to minimize overall congestion over the associated link. On the other hand each user splits their traffic among all available paths so as to maximize a degree of satisfaction.

In [16] the bandwidth allocation problem in the virtual networks environment is considered. The problem is presented in terms of the noncooperative game between service providers (virtual network owners) seen as players. The strategy of a player is determined by virtual links capacities and flow rates in the particular virtual network. The utility and cost functions constitute payoffs. The constraints concerning limited amount of physical links capacities (bandwidths) are substituted with a congestion cost, which is one of the addends of the cost function. Authors prove the existence of Nash equilibrium for such a game. An iterative algorithm is proposed, converging to the equilibrium, based on the best response method.

Furthermore, another type of games called auctions [11] seems very suitable for computer network applications [17]. Classic Vickerey-Clarke-Groves (VCG) mechanism, together with the so-called Kelly mechanism (based on results obtained in [1]), is used for the network resource allocation. In the capacity allocation problem is stated as an auction game between flows (users), seen as buyers, and network operator, seen as an auctioneer. A distributed algorithm to find efficient Nash equilibrium is proposed. The presented mechanism is described as VCG-like, since, on the contrary to the classic VCG auction, it does not require a full valuation function.

Currently, the game-theoretical framework is also extensively studied in the context of wireless networks. Bandwidth allocation problem for a class of wireless networks categorized under corresponding OSI Layers (namely: physical, data link, network and transport layers) are presented in [18].

More detailed surveys of game theoretical applications in various network resource allocation problems may be also found in [22].

## III. Problem formulation

The fundamental problem of the ISPs resource managing consist in determining such a capacity allocation which maximizes total profit of the network operators. The network users (clients) pay for particular transmission rates assigned to their flows. On the other hand, Internet Service Providers pay some operating cost proportional to the amount of used resources (links' capacities).

Let us introduce the following notation:

$M$ - number of network operators or Internet Service Providers,

$N$ - number of users (flows),

$L$ - number of links,

$C_{l,m}$ - capacity of $l$th link owned by $m$th operator,

$x_m^{(n)}$ - $m$th operator's allocation for $n$th user, $x_m = [x_m^{(n)}]$,

$w_n$ - weight parameter of $n$th user reflecting his willingness to pay,

$u_n : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}$ - utility function of $n$th user, concave, non-decreasing, non-negative twice differentiable.

$h_m$ - unit operating cost paid by Internet Service Provider for capacity allocation.

$a_{mnl}$ - routing variables, defined as follows:

$$a_{mnl} = \begin{cases} 1 & n\text{th user uses } l\text{th link of } m\text{th operator,} \\ 0 & \text{otherwise.} \end{cases}$$

$K_m$ - set of users served by $m$th network operator.

Thus, the capacity allocation problem for $m$th network operator may be formulated as follows:

$$\max_{x_m} \sum_{n \in K_m} u_n(x_m^{(n)}) - \sum_{n=1}^{N} h_m x_m^{(n)} \tag{1}$$

such that:

$$\forall_l \sum_{n=1}^{N} a_{m,n,l} u_n(x_m^{(n)}) \leq C_{l,m} \tag{2}$$

$$\forall_n x_m^{(n)} \geq 0 \tag{3}$$

## IV. Operators Market Game: Capacity Allocation Trading

In this paper, we consider a problem of capacity allocation of Internet Service Providers competing in the network operators market. We assume that each network user has his preferences defined in the terms of the utility function, which determines the price he is willing to pay for particular transmission rate. The ISPs offer to each user particular amount of capacity. The user choose the provider which offers the highest transmission rate and pay accordingly to his utility function.

In consequence, the set of the users served by $m$th operator $K_m$ is not given a priori and fixed. Instead, this set depends on the all other operators' decisions.

Therefore, we introduce the following game to model the aforementioned situation in the network operators market:

Players: $\mathcal{M} = \{1, 2, \ldots, M\}$,

Player strategy:

$s_m^{(n)}$ - $m$th operator's allocation for $n$th flow,

$\mathbf{s}_m = [s_m^{(1)}, s_m^{(2)}, \ldots, s_m^{(N)}]^T$ - strategy of $m$th operator,

$\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_M]^T$ - strategy profile,

Player's strategy space: $\mathcal{S}_m = \{\mathbf{s}_m : \forall_l \sum_{n=1}^{N} a_{lmn} s_m^{(n)} \leq C_{lm}\}$

Payoff function: $F_m(\mathbf{S}) = \sum_{n\in B_m(\mathbf{S})} u_n(s_m^{(n)}) - \sum_{n=1}^{N} h_m x_m^{(n)}$ where $B_m(\mathbf{S}) = \Big\{ n : \forall_{m'<m} u_n(s_m^{(n)}) \geq u_n(s_{m'}^{(n)}) \wedge \forall_{m'>m} u_n(s_m^{(n)}) > u_n(s_{m'}^{(n)}) \Big\}$ is a set of users for which $m$th operator offers the highest utility in strategy profile $\mathbf{S}$; in case of equal offers of two or more operators, we assume that a flow prefers the operator of higher index.

## V. Nash equilibria

### A. Best response strategy

One of the most important properties of the game are its equilibria. It must be recalled that the strategy profile which constitutes the pure Nash equilibrium consist only of the best response strategies. So, we introduce the algorithm for the computation of the best response strategy for $\mathbf{S}_{-m}$.

Let us define local optimization problem for $m$th operator conditioned by the resource allocations as $\mathbf{S}_{-m}$ determined by other operators:

$$\text{maximize} \; \sum_{n=1}^{N} u_n(x_n) - \sum_{n=1}^{N} h_m x_m^{(n)} \tag{4}$$

$$\text{subject to: } \; \mathbf{x} \in D_m, \tag{5}$$

where:

$$\mathbf{x} = [x_1, x_2, \ldots, x_N]^T,$$

$$D_m = \Big\{ \mathbf{x} : \forall_n \Big( x_n = 0 \vee \forall_{m' \neq m} x_n > s_{m'}^{(n)} \Big) \wedge \forall_l \sum_{n=1}^{N} x_n \leq C_{lm} \Big\}.$$

Let us denote a feasible solution of this problem by $\mathbf{x}(\mathbf{S}_{-m})$. This solution is the best response strategy.

### B. Existance of the pure Nash equilibrium

It must be notice that it is not obvious if there exist any pure Nash equilibrium. Furthermore, even if we are aware of pure Nash equilibrium existence, it is not trivial how to find it. In this section, we present two theorems which enables determination of the pure Nash equilibria. Theorem 1 applies only for some simple class of the considered problem, while Theorem 2 may be used for any case.

**Theorem 1** (Sufficient condition of the existence of pure Nash equilibrium)

*Let $L = 1$ and $C_{1,1} \leq C_{1,2} \leq \ldots \leq C_{1,M}$.*

*If the optimal solution of problem (4)–(5) for $M$th operator satisfies $x_n^*(\mathbf{S}_{-M}) \geq C_{M-1}$ for all $n$, then $\mathbf{S} = (\mathbf{x}^*(\mathbf{S}_{-M}), \mathbf{S}_{-M})$ is a pure Nash equilibrium.*

We omit the proof of this theorem since it is quite obvious. The $M$th operator cannot improve his payoff since it is the optimal solution of problem (4)–(5). On the other hand, all other providers may not offer higher transmission rate since they have not sufficient link capacity.

**Theorem 2** (Necessary and sufficient condition of the existence of pure Nash equilibrium)

*A strategy profile $\mathbf{S}$ is a pure Nash equilibrium if and only if*

$$\forall_m \mathbf{s}_m = \mathbf{x}^*(\mathbf{S}_{-m}),$$

*where $\mathbf{x}^*(\mathbf{S}_{-m})$ is an optimal solution of problem (4)–(5).*

*Proof:* For the sake of contradiction, let us assume that $\mathbf{S}$ is a pure Nash equilibrium, and there exists such operator $m$, that there exists flow $n$, for which this operator allocates no less that other operators with lower indexes and strictly more than operators with higher indexes, and such allocation is not an optimal solution of problem (4)–(5). Then by changing $m$th operator's allocation to an optimal solution of problem (4)–(5), the payoff improves, while the allocation still preserve the aforementioned conditions. Consequently, the assumed allocation $\mathbf{S}$ cannot be a Nash equilibrium. Moreover, the definition implies that the strategy profile is a Nash equilibrium if each player is playing their best response to the strategies that all other players are actually playing. ■

### C. Algorithm for the Nash equilibria computation

The idea of algorithm finding Nash equilibria given in Algorithm 1 is as follows. We assume some initial assignment clients to the network operator and solve the optimal allocation problem for given assignment. For every operator, we try to improve his payment with finding best responses to the allocations which have been just determined. According to the Theorem 2, if best responses for all operators are equal to their initial allocation then such a solution constitutes Nash equilibrium. We repeat this procedure for all possible initial assignments.

One should notice, that the empty set may be returned by Algorithm 1. It means that the particular game has no pure Nash equilibrium.

**Algorithm 1** Finding Nash equilibria

Set $E \leftarrow \emptyset$.

Let $\mathbf{V}$ be a sequence of all possible initial clients assignments to the network operator, i.e.:

$$\mathbf{V} = [V_i], V_i = [V_i^m],$$

where $V_i^m$ is the set fulfilling:

$$V_i^m \subseteq \{1, 2, ..., N\}$$

$$\bigcup_m V_i^m = \{1, 2, ..., N\} \wedge \forall_{p,q:p\neq q} V_i^p \cap V_i^q = \emptyset$$

consisting of all possible partitioning of the set $\{1, 2, ..., N\}$.

**for all** $V_i$ in $\mathbf{V}$ **do**:

  **for all** $m$ **do**:

    Determine $\mathbf{s}_m$ as the solution of the optimization problem: (1) - (3), assuming $K_m = V_i^m$.

  **end for**

  **for all** $m$ **do**:

    Determine $\mathbf{x}^*(\mathbf{S}_{-m})$ solving (4)-(5).

  **end for**

  **if** solution differs from solution obtained in previous step **then**

    $E \leftarrow E \cup \{\}$

  **end if**

**end for**

**return** $E$ (the set of all pure Nash equilibria).

## VI. Numerical example

We conducted experiment for the following numerical data: $L = 1, N = 3, M = 2, w_1 = 1, w_2 = 2, w_3 = 3, u_n(x_n) = w_n\sqrt{x_n}, C_1 = 1, C_2 = 2, h_1 = 0,5, h_2 = 0,5$. The run of the Algorithm 1 is presented in Table I and II. In both tables, we denoted $F_m(\mathbf{x}_m(\mathbf{S}_{-m}), \mathbf{S}_{-m}) = \hat{F}_m(\mathbf{x}_m)$ for simplicity. In the considered example there is no Nash equilibrium (NE) for the proposed game. Since in each algorithm step at least one of the ISPs may improve his payoff with changing his strategy.

Let us consider the second example. Assuming $C_1 = 0.1$ while the rest of the numerical data remains unchanged in comparison to the first example. We obtain Nash equilibrium directly from Theorem 1. Namely,

TABLE I. Results of algorithm 1 for ISP 1.

| i | $V_1^i$ | $\mathbf{s}_1$ | $F_1(\mathbf{s})$ | $\mathbf{x}_1(\mathbf{S}_{-1})$ | $F_1(\mathbf{x}_1)$ | NE |
|---|---|---|---|---|---|---|
| 1 | $\{1,2,3\}$ | $\begin{bmatrix}0.07\\0.28\\0.64\end{bmatrix}$ | 3.24 | $\begin{bmatrix}0.07\\0.28\\0.64\end{bmatrix}$ | 3.24 | No |
| 2 | $\{2,3\}$ | $\begin{bmatrix}0\\0.31\\0.69\end{bmatrix}$ | 3.1 | $\begin{bmatrix}0\\0.31\\0.69\end{bmatrix}$ | 3.1 | No |
| 3 | $\{1,3\}$ | $\begin{bmatrix}0.1\\0\\0.9\end{bmatrix}$ | 2.66 | $\begin{bmatrix}0.1\\0\\0.9\end{bmatrix}$ | 2.66 | No |
| 4 | $\{1,2\}$ | $\begin{bmatrix}0.2\\0.8\\0\end{bmatrix}$ | 1.74 | $\begin{bmatrix}0.2\\0.8\\0\end{bmatrix}$ | 1.74 | No |
| 5 | $\{3\}$ | $\begin{bmatrix}0\\0\\1\end{bmatrix}$ | 2.5 | $\begin{bmatrix}0\\0\\1\end{bmatrix}$ | 2.5 | No |
| 6 | $\{2\}$ | $\begin{bmatrix}0\\1\\0\end{bmatrix}$ | 1.5 | $\begin{bmatrix}0.2\\0.8\\0\end{bmatrix}$ | 1.74 | No |
| 7 | $\{1\}$ | $\begin{bmatrix}1.0\\0\\0\end{bmatrix}$ | 0.5 | $\begin{bmatrix}0.2\\0.8\\0\end{bmatrix}$ | 1.74 | No |
| 8 | $\emptyset$ | $\begin{bmatrix}0\\0\\0\end{bmatrix}$ | 0 | $\begin{bmatrix}0.2\\0.8\\0\end{bmatrix}$ | 1.74 | No |

strategies: $\mathbf{s}_1 = \begin{bmatrix}0\\0\\0\end{bmatrix}$, $\mathbf{s}_2 = \begin{bmatrix}0.14\\0.57\\1.29\end{bmatrix}$ constitutes the Nash equilibrium.

## VII. Conclusions

In the presented paper we proposed the game to model the Internet Service Provider market. We presented the algorithm which allows finding all pure Nash equilibria in the introduced game. We illustrated elaborated method with numerical examples. We showed that the game under consideration may have no Nash equilibrium. The further works will include an examination of the game properties, especially Price of Anarchy, Price of Stability and Pareto-optimality.

## References


[1] F. Kelly, A. Maulloo, and D. Tan, “Rate control for communication networks: shadow prices, proportional fairness and

TABLE II. RESULTS OF ALGORITHM 1 FOR ISP 2.

| i | $V_2^i$ | $\mathbf{s}_2$ | $F_2(\mathbf{s})$ | $\mathbf{x}_2(\mathbf{S}_{-2})$ | $F_2(\mathbf{x}_2)$ | NE |
|---|---|---|---|---|---|---|
| 1 | $\emptyset$ | $\begin{bmatrix}0\\0\\0\end{bmatrix}$ | 0 | $\begin{bmatrix}0.14\\0.57\\1.29\end{bmatrix}$ | 4.29 | No |
| 2 | $\{1\}$ | $\begin{bmatrix}2\\0\\0\end{bmatrix}$ | 0.41 | $\begin{bmatrix}0.14\\0.57\\1.29\end{bmatrix}$ | 4.29 | No |
| 3 | $\{2\}$ | $\begin{bmatrix}0\\2\\0\end{bmatrix}$ | 1.83 | $\begin{bmatrix}0.14\\0.57\\1.29\end{bmatrix}$ | 4.29 | No |
| 4 | $\{3\}$ | $\begin{bmatrix}0\\0\\2\end{bmatrix}$ | 3.24 | $\begin{bmatrix}0.2\\0.8\\1\end{bmatrix}$ | 4.23 | No |
| 5 | $\{1,2\}$ | $\begin{bmatrix}0.4\\0.6\\0\end{bmatrix}$ | 2.16 | $\begin{bmatrix}0.14\\0.57\\1.29\end{bmatrix}$ | 4.29 | No |
| 6 | $\{1,3\}$ | $\begin{bmatrix}0.2\\0\\1.8\end{bmatrix}$ | 3.47 | $\begin{bmatrix}0.1\\1\\1.9\end{bmatrix}$ | 4.16 | No |
| 7 | $\{2,3\}$ | $\begin{bmatrix}0\\0.62\\1.38\end{bmatrix}$ | 4.1 | $\begin{bmatrix}0\\0.62\\1.38\end{bmatrix}$ | 4.1 | No |
| 8 | $\{1,2,3\}$ | $\begin{bmatrix}0.14\\0.57\\1.29\end{bmatrix}$ | 4.29 | $\begin{bmatrix}0.14\\0.57\\1.29\end{bmatrix}$ | 4.29 | No |


stability," *Journal of the Operational Research society*, vol. 49, no. 3, pp. 237–252, 1998.

[2] M. Chiang, S. Low, A. Calderbank, and J. Doyle, "Layering as optimization decomposition: A mathematical theory of network architectures," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 255–312, 2007.

[3] K. Park, M. Sitharam, and S. Chen, "Quality of service provision in noncooperative networks with diverse user requirements," *Decision Support Systems*, vol. 28, no. 1, pp. 101–122, 2000.

[4] M. Osborne, *An introduction to game theory*. Oxford University Press, 2004.

[5] T. Basar and R. Srikant, "Revenue-maximizing pricing and capacity expansion in a many-users regime," in *INFOCOM 2002. Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 1. IEEE, 2002, pp. 294–301.

[6] J. R. Correa and N. E. Stier-Moses, "Wardrop equilibria," *Wiley Encyclopedia of Operations Research and Management Science*, 2011.

[7] P. Hande, M. Chiang, R. Calderbank, and S. Rangan, "Network pricing and rate allocation with content provider participation," in *INFOCOM 2009, IEEE*. IEEE, 2009, pp. 990–998.

[8] V. Valancius, C. Lumezanu, N. Feamster, R. Johari, and V. V. Vazirani, "How many tiers?: pricing in the internet transit market," *ACM SIGCOMM Computer Communication Review*, vol. 41, no. 4, pp. 194–205, 2011.

[9] T. Harks, M. Hoefer, M. Klimm, and A. Skopalik, "Computing pure nash and strong equilibria in bottleneck congestion games," *Proceedings of 18th European Symposium on Algorithms*, pp. 29–38, 2010.

[10] H. Kameda and E. Altman, "Inefficient noncooperation in networking games of common-pool resources," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 7, pp. 1260–1268, 2008.

[11] N. Nisan, *Algorithmic game theory*. Cambridge University Press, 2007.

[12] T. Roughgarden and É. Tardos, "How bad is selfish routing?" *Journal of the ACM (JACM)*, vol. 49, no. 2, pp. 236–259, 2002.

[13] R. Banner and A. Orda, "Bottleneck routing games in communication networks," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 6, pp. 1173–1179, 2007.

[14] F. Larroca and J. Rougier, "Routing games for traffic engineering," in *IEEE International Conference on Communications*, 2009, pp. 1–6.

[15] P. Fuzesi and A. Vidács, "Game theoretic analysis of network dimensioning strategies in differentiated services networks," in *Proceedings of IEEE International Conference on Communications*, vol. 2, 2002, pp. 1069–1073.

[16] Y. Zhou, Y. Li, G. Sun, D. Jin, L. Su, and L. Zeng, "Game theory based bandwidth allocation scheme for network virtualization," in *Proceedings of IEEE Global Telecommunications Conference GLOBECOM*, 2010.

[17] I. Koutsopoulos and G. Iosifidis, "Auction mechanisms for network resource allocation," in *Proceedings of 8th International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks*, 2010, pp. 554–563.

[18] D. Charilas and A. Panagopoulos, "A survey on game theory applications in wireless networks," *Computer Networks*, vol. 54, no. 18, pp. 3421–3430, 2010.

[19] K. Leyton-Brown and Y. Shoham, "Essentials of game theory: A concise multidisciplinary introduction," *Synthesis Lectures on Artificial Intelligence and Machine Learning*, vol. 2, no. 1, pp. 1–88, 2008.

[20] A. Tang and L. Andrew, "Game theory for heterogeneous flow control," in *Proceedings of 42nd Annual Conference on Information Sciences and Systems*, 2008, pp. 52–56.

[21] T. Trinh and S. Molnár, "A game-theoretic analysis of tcp vegas," *Quality of Service in the Emerging Networking Panorama*, pp. 338–347, 2004.

[22] E. Altman, T. Boulogne, R. El-Azouzi, T. Jimenez, and L. Wynter, "A survey on networking games in telecommunications," *Computers & Operations Research*, vol. 33, no. 2, pp. 286–311, 2006.