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Question ~~3~~ 3

Soln

i) ~~Asset class~~

Date	Equity	Equity returns	Bond	~~Bond return~~	Bond return
06/06/2023	40	~~0.08~~	100		
Day 1	43.2	0.08	97.86	~~0.001008531~~	-0.0214
Day 2	44.5	0.030092593	99.8	~~0.000129567~~ 0.	0.019824239
Day 3	47.6	0.069662921	102.1	~~0.00075~~	0.023046092
Day 4	40.9	-0.140756303	101.9		-0.001958864
Day 5	38.34	-0.062591687	94.56		-0.072031403
Day 6	42.2	0.100678143	93.9		-0.006979695
Day 7	39.6	-0.061611374	96.67		0.029499468

$$\text{Returns} = \frac{\text{Final} - \text{Initial}}{\text{Initial}}$$

$$\text{Variance} = \sum_{i=1}^{n} \frac{(\text{Return}_i - \mu)^2}{n-1} \qquad \text{where } \mu = \frac{\sum x}{n}$$

$$\text{Equity return average} = \frac{0.015474293}{7} = 0.002210613$$

$$\text{Bond return average} = \frac{-0.030000164}{7} = -0.004285738$$

Equity return variance $= 0.008297731$

Bond return variance $= 0.001227467$

$$\sigma_e = \sqrt{0.008297731} = 0.091091881$$

$$\sigma_b = \sqrt{0.001227467} = 0.035035232$$

The standard deviation of a two-asset portfolio

$$\sigma_p = \sqrt{(w_1^2 \sigma_e^2) + (w_2^2 \sigma_b^2) + (2 w_1 w_2 \rho_{12} \sigma_e \sigma_b)}$$

where $\sigma_p \Rightarrow$ standard deviation of the portfolio

w_1 \$ $w_2 \Rightarrow$ weights of the two assets

σ_e \$ $\sigma_b \Rightarrow$ standard deviation of the returns of two assets

$\rho_{12} \Rightarrow$ Correlation between the returns of the assets

$$w_1 = \frac{800,000}{(800,000 + 700,000)} = 0.53333$$

$$w_2 = \frac{700,000}{(800,000 + 700,000)} = 0.46667$$

$$\rho_{12} = 0.1472$$

$$\Rightarrow \sigma_p = \sqrt{(0.5333^2 \cdot 0.091091881^2) + (0.46667^2 \cdot 0.0350353232^2) + (2 \cdot 0.5333 \cdot 0.46667 \cdot 0.1472 \cdot 0.091091881 \cdot 0.0350353232)}$$

$$\sigma_p = 0.002861404$$

$$VaR = \frac{\sigma_p}{z \text{ score}} = \frac{0.002861404}{1.645}$$

$$= 0.004707009$$

ii) Expected Shortfall

$$ES = VaR + \varphi(z) / (1 - \text{Confidence interval})$$

where $\varphi(z) = \dfrac{e^{-0.5z^2}}{\sqrt{2\pi}}$

$$= 0.004707009 + \frac{0.103}{1-0.95}$$

$$ES = 2.064707009$$

iii) Solution

252-day Stressed 95% VaR

portfolio volatility increased by 20% & 50%

~~$\neq$ 0.004707009 × 1.20 =~~

~~0.004707009 × 1.50 =~~

$\Rightarrow$ σ_P × percentage increase

$0.002861404 \times 1.20 = 0.003433684$

$0.002861404 \times 1.50 = 0.004292105$

$\therefore$ 252-day stressed 95% VaR with a 20% increase in portfolio volatility $= 0.003433684$

and

252-day stressed 95% VaR with a 50% increase in portfoli volatility $= 0.004292105$

252-day stressed Expected Shortfall

$$ES_{20} = 0.003433684 + \frac{0.103}{1-0.95}$$

$$ES_{20} = 2.06564841$$

$$ES_{50} = 0.004292105 + \frac{0.103}{1-0.95}$$

$$ES_{50} = 2.06706051$$