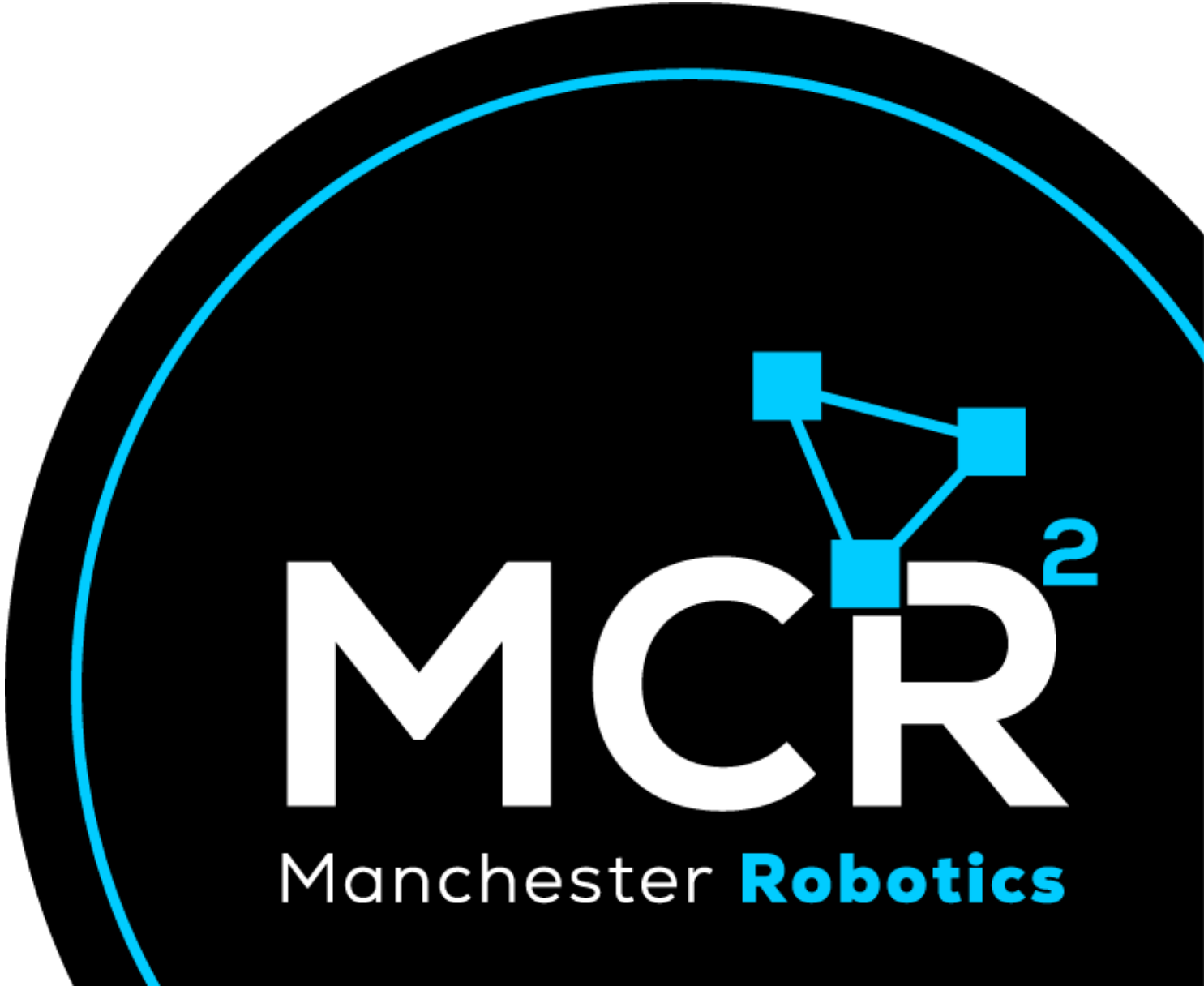
{Learn, Create, Innovate};

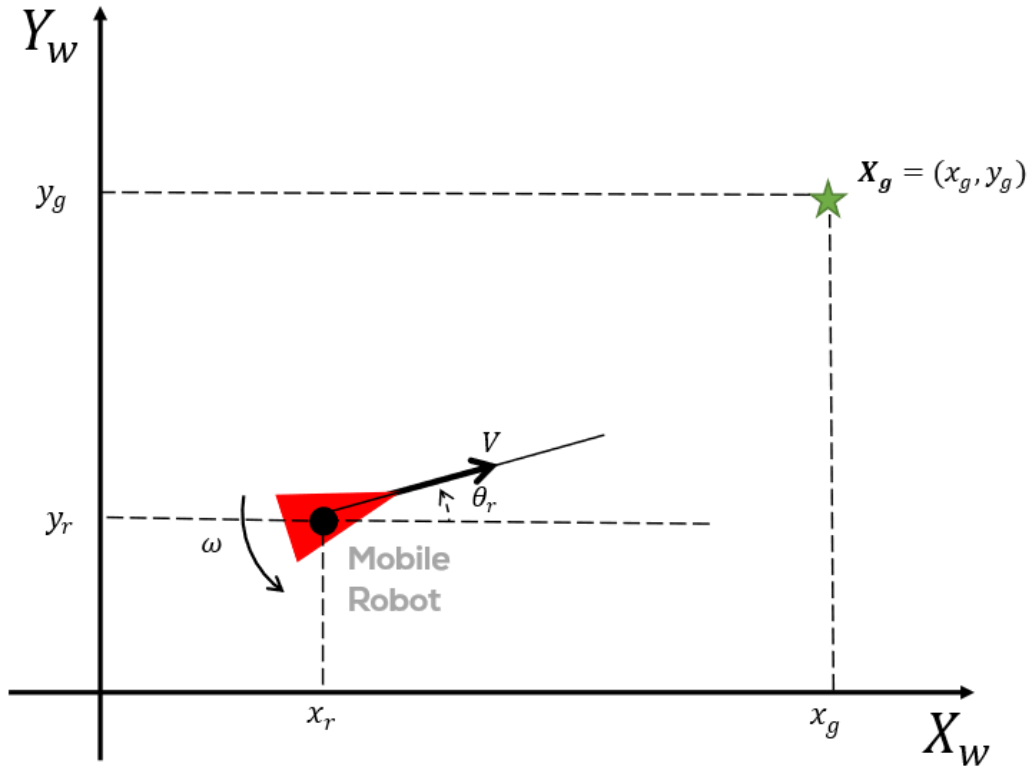
Autonomous Systems

Mobile Robot: Motion Control



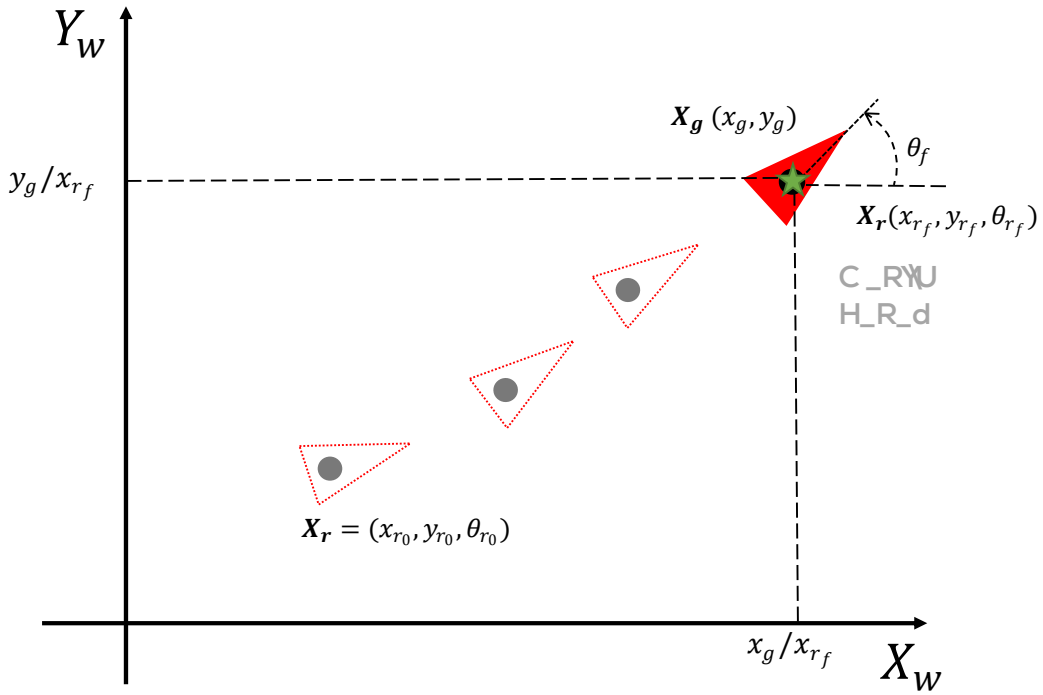
Motion Control

- The **motion control** for a mobile robot deals with the task of finding the control inputs that need to be applied to the robot such that a predefined goal can be reached in a finite amount of time.
- Control of differential drive robots has been studied from several points of view but essentially falls into one of the following three categories: **point-to-point navigation (or point stabilisation)**, **trajectory tracking**, and **path following**.



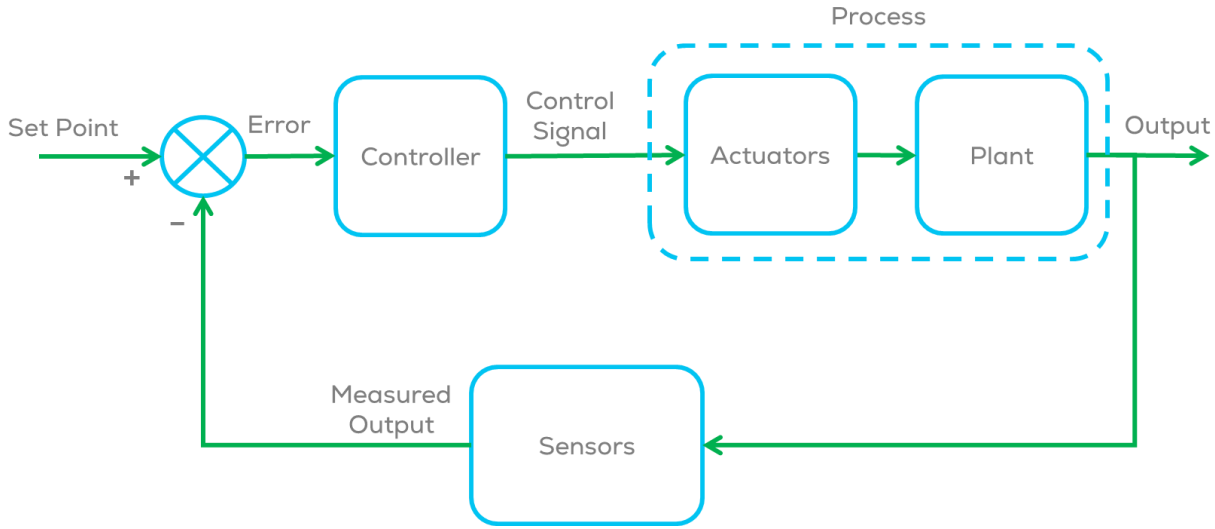
Motion Control: Point Stabilisation

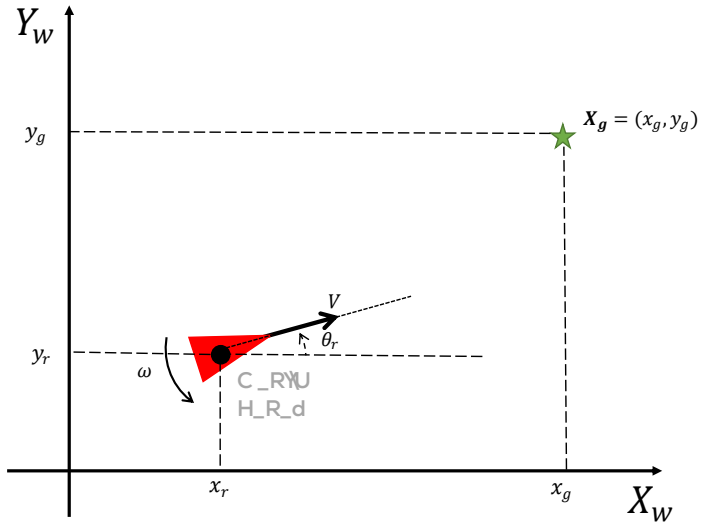
- The objective here is to drive the robot to a desired fixed state, say a fixed position and orientation.
- When the vehicle has nonholonomic constraints, point stabilisation presents a true challenge to control systems.
    - Since that goal cannot be achieved with smooth time-invariant state-feedback control laws.
- This control technique will be used in this course.



The Control System

- When Designing a Controller a question arises…
- How can we transform the classical feedback control diagram, to fit the purpose of controlling the position of a mobile robot using Point Stabilisation?
- … We start by defining the main things of the control diagram: Set Point, Measured Output, Control Signal, etc…

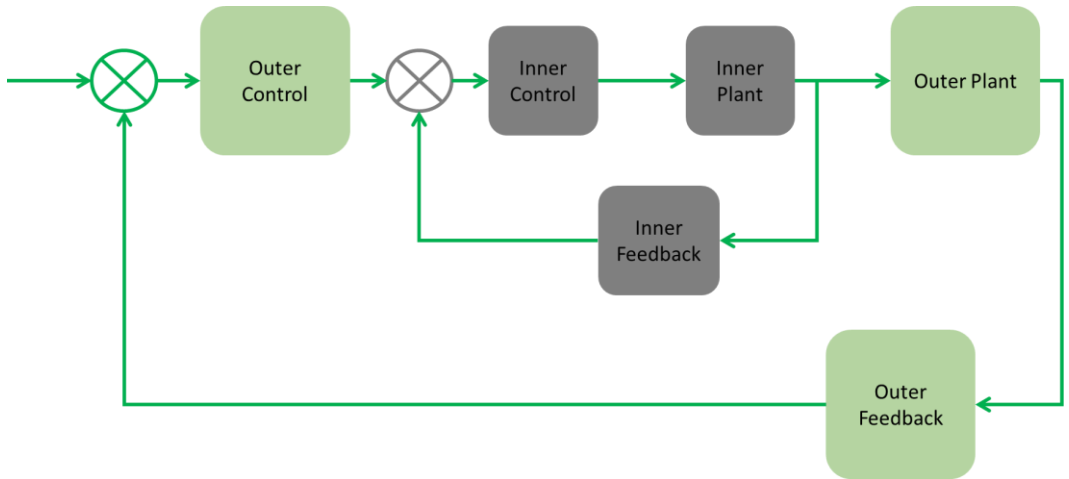


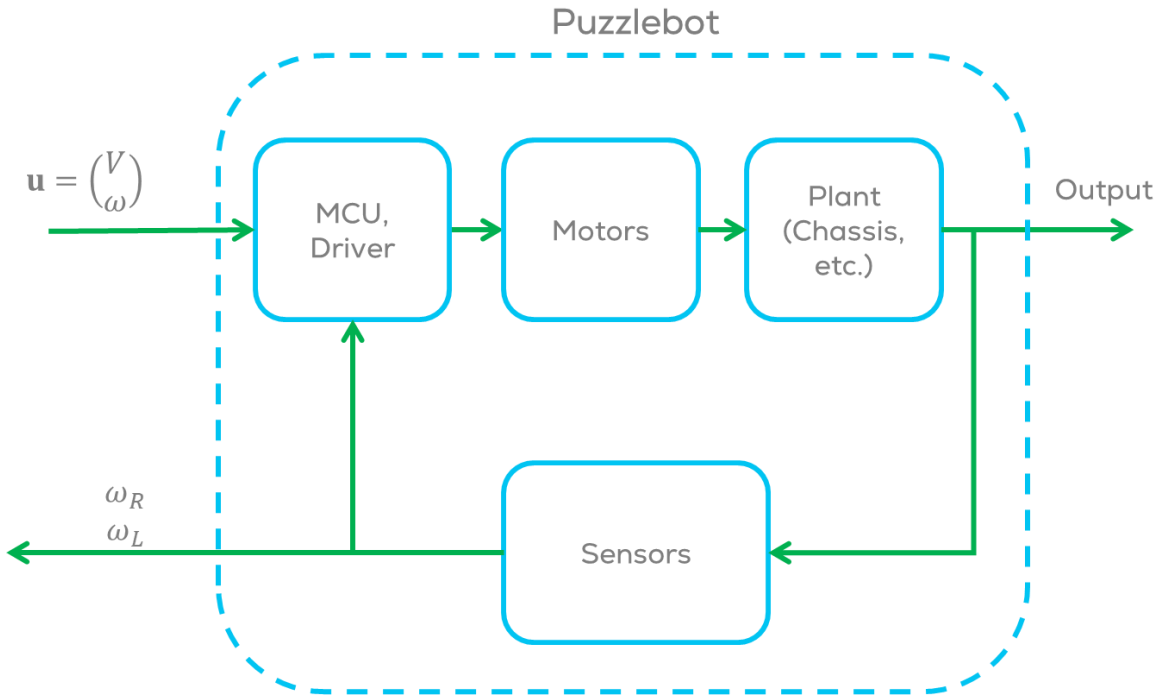


The Mobile Robot

Internal Controllers

- Mobile Robots usually contain internally some controllers to regulate the speed of the motors.
- These controllers must be taken into consideration when trying to control the robot using an external controller (cascade control)

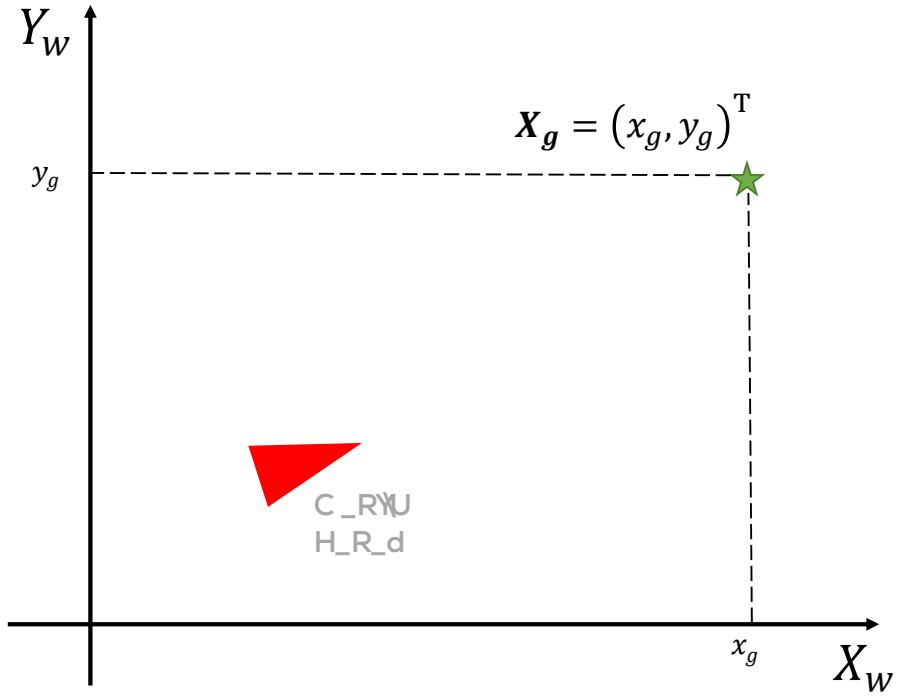




Point Stabilisation

Set Point

- The set point/goal can be defined in the simplest way as a set of coordinates in a multidimensional space. For instance, if the robot is moving in a two-dimensional space the goal is:
- $\boldsymbol{X}_g = \begin{pmatrix} x_g \\ y_g \end{pmatrix}$ if the position of the robot needs to be controlled, or
- $\boldsymbol{X}_g = \begin{pmatrix} x_g \\ y_g \\ \theta_g \end{pmatrix}$ if both position and heading need to be controlled.



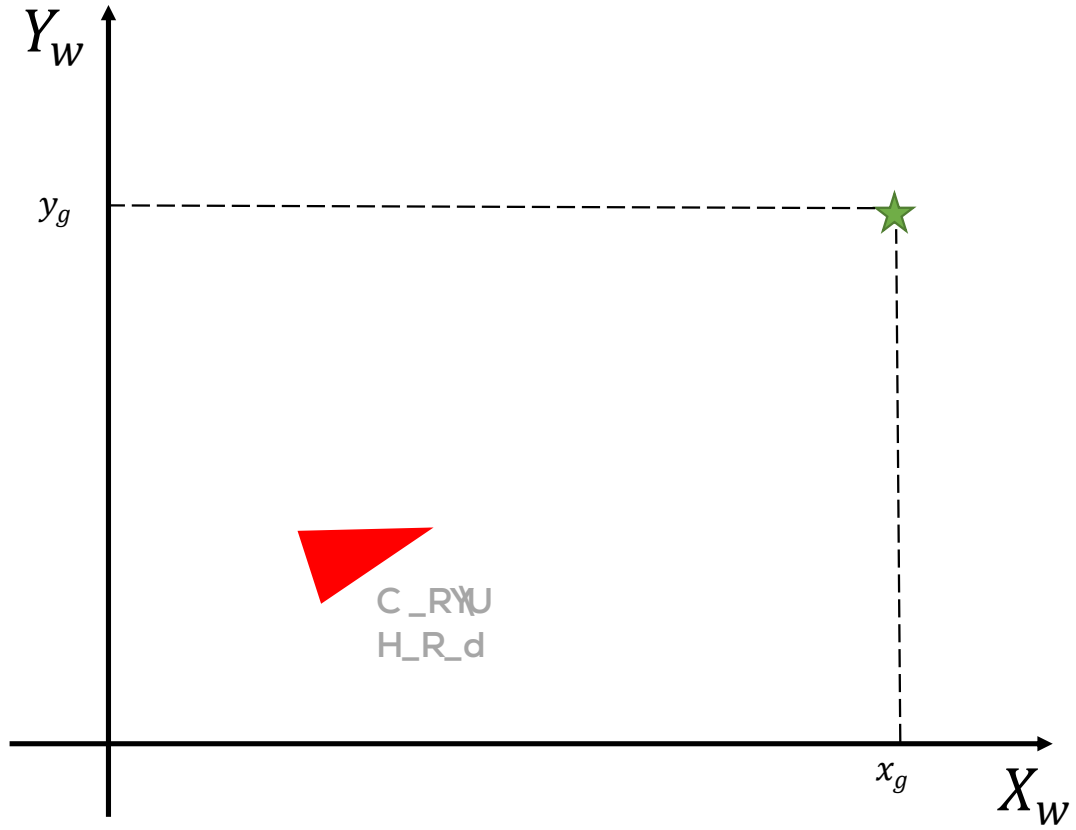
Point Stabilisation

Control Inputs

- The inputs to the robot are the linear and angular velocity of the robot

$$\mathbf{u} = \begin{pmatrix} V \\ \omega \end{pmatrix}.$$

- The linear velocity of the robot, $\mathbf{V}$, is always oriented in the direction of x axis of the robot reference frame because of the non-holonomic constraint.
- The inputs are transformed into wheel velocities by the motors.



Point Stabilisation

Error

- To Error is the difference between the set point and the measurement.

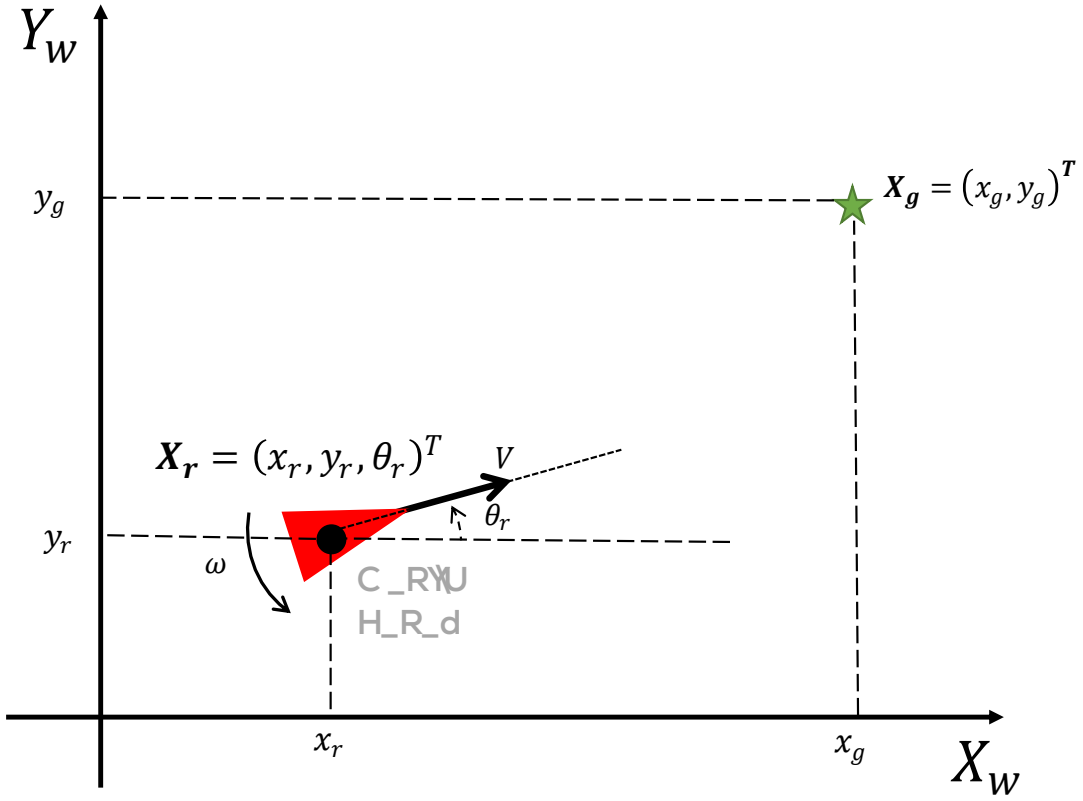
- In this case the set point was stablished to be $\boldsymbol{X_g} = \begin{pmatrix} x_g \\ y_g \\ \theta_g \end{pmatrix}$

- Therefore, for a non-holonomic robot moving in a 2D environment the error must be defined as:

$$\boldsymbol{e} = \boldsymbol{X_g} - \boldsymbol{X_r}$$

- Where, $\boldsymbol{X_r}$ represents the robot's pose represented by the vector

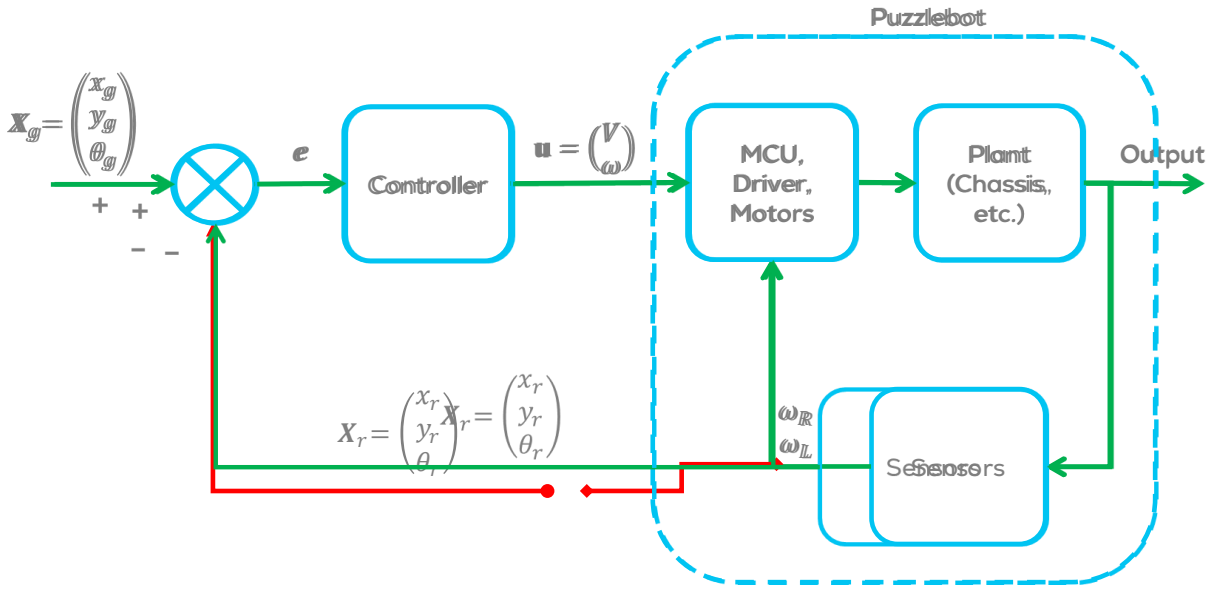
$$\boldsymbol{X_r} = \begin{pmatrix} x_r \\ y_r \\ \theta_r \end{pmatrix}.$$



The Control System

Control

- The control diagram can then be changed as follows…
- Just one problem…
- In reality, the Sensors of a Robot do not provide the position of the robot…
- Usually, the sensors of the mobile robot provide the information about the velocity of each wheel i.e., ω_R and ω_L.
- Another question arises… Can I use the information of the wheel's speed to get the position of my robot?
- YES… it's called Localisation!



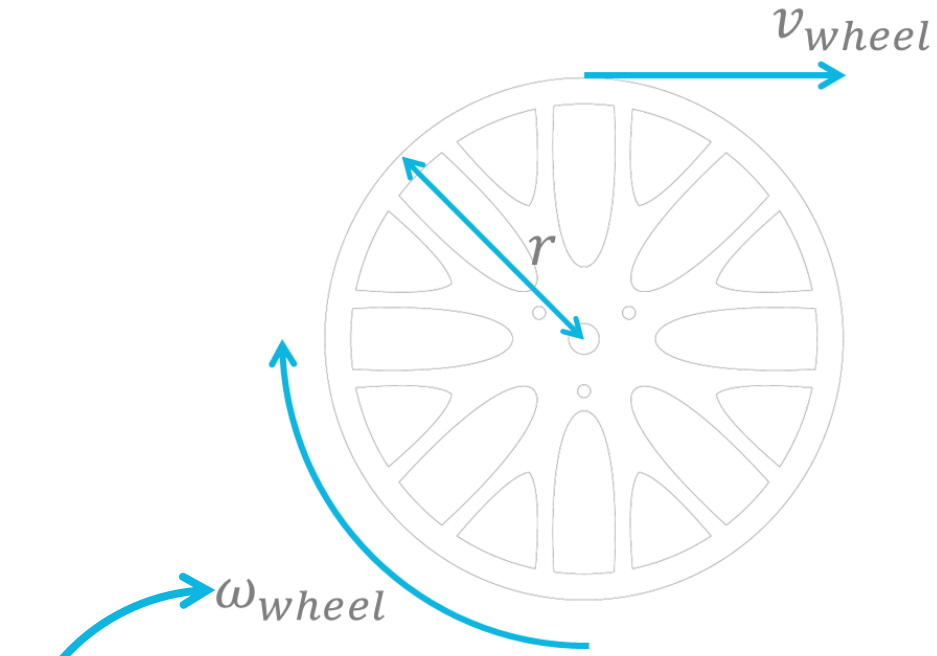
Localisation

Tangential Velocity

- The first step to get the position of a robot, is to get the tangential velocity of the wheel.
- The tangential speed of a wheel is given by

$$v_{wheel} = r \cdot \omega_{wheel}$$

- Where r is the radius and ω_{wheel} is the angular speed of the wheel.
- For this case, the robot has two wheels, therefore two tangential velocities must be calculated.
- On the Puzzlebot (and most robots) this is given by the topics `/wr` and `/wl`



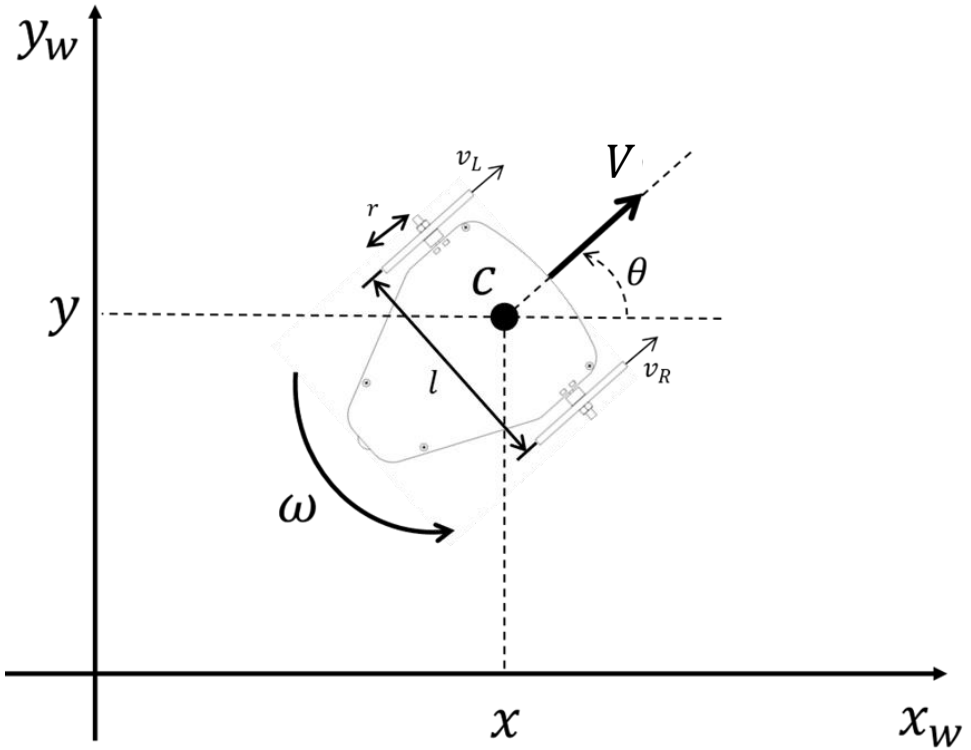

```
/puzzlebot/cmd_vel
/puzzlebot/joint_states
/puzzlebot/odom
/puzzlebot/wl
/puzzlebot/wr
```

Localisation

Robot Velocity

- Using the wheel velocities, it is possible to estimate the forward velocity V and the angular velocity ω.
- For this case, the resultant forward velocity $\mathbf{V}$ through C (centre of mass) may be reasoned as an average of the two forward wheel velocities given by

$$V = \frac{v_R + v_L}{2} = r\frac{\omega_R + \omega_L}{2}$$



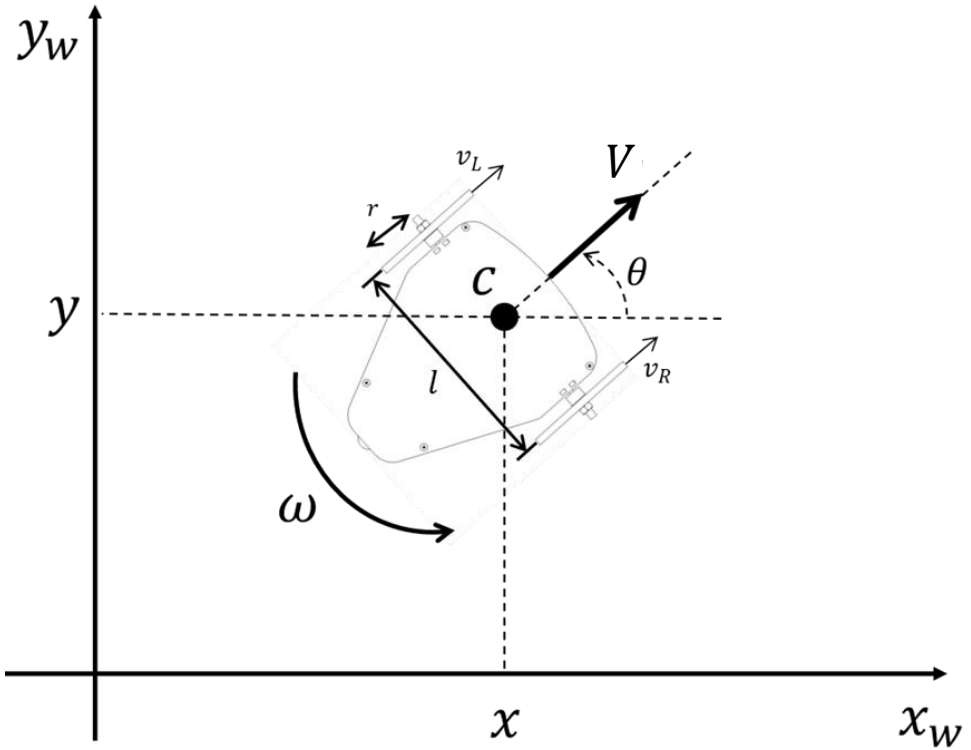
Differential Drive Robot
Puzzlebot ©.

Localisation

Robot Velocity

- The resultant angular velocity ω (steering velocity), may also be reasoned as proportional to the difference between wheel velocities but inversely proportional to distance between the wheels, i.e.,

$$\omega = \frac{v_R - v_L}{l} = r\frac{\omega_R - \omega_L}{l}$$



Differential Drive Robot
Puzzlebot ©.

Localisation

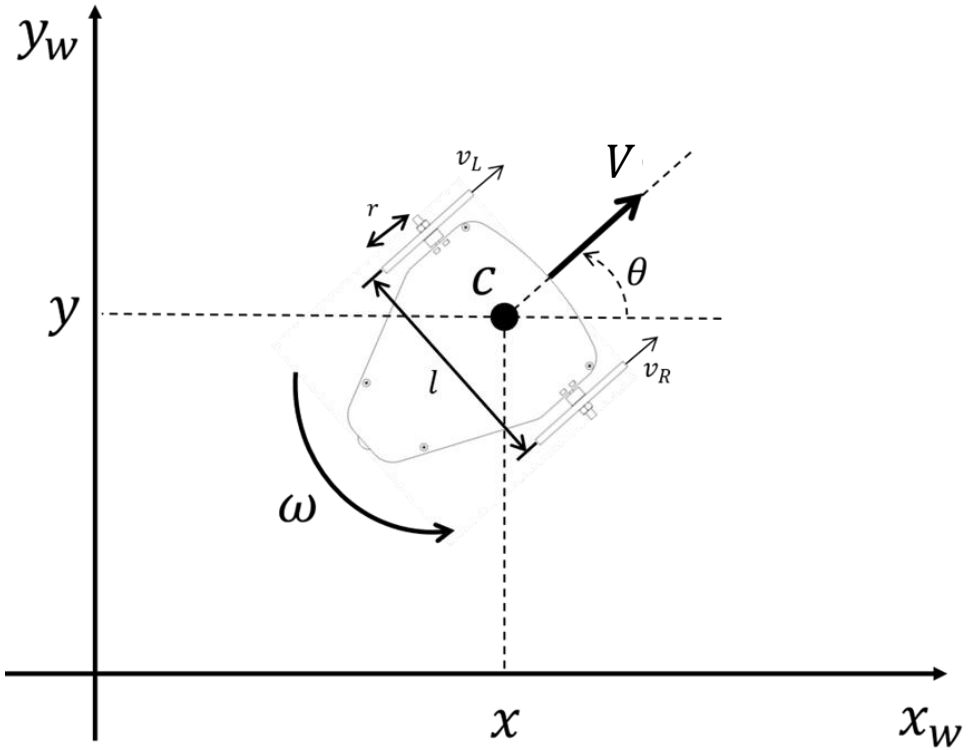
Robot Velocity

- Using the Differential Drive Kinematic Model, given by:

$$\begin{cases} \dot{x} = \mathrm{V} \cdot cos\theta \\ \dot{y} = \mathrm{V} \cdot sin\theta \\ \dot{\theta} = \omega \end{cases}$$

- It is possible to decompose the speed of the robot into its components

$$\begin{cases} \dot{x} = r\dfrac{\omega_R + \omega_L}{2} \cdot cos\theta \\ \dot{y} = r\dfrac{\omega_R + \omega_L}{2} \cdot sin\theta \\ \dot{\theta} = r\left(\dfrac{\omega_R - \omega_L}{l}\right) \end{cases}$$



Differential Drive Robot
Puzzlebot ©.

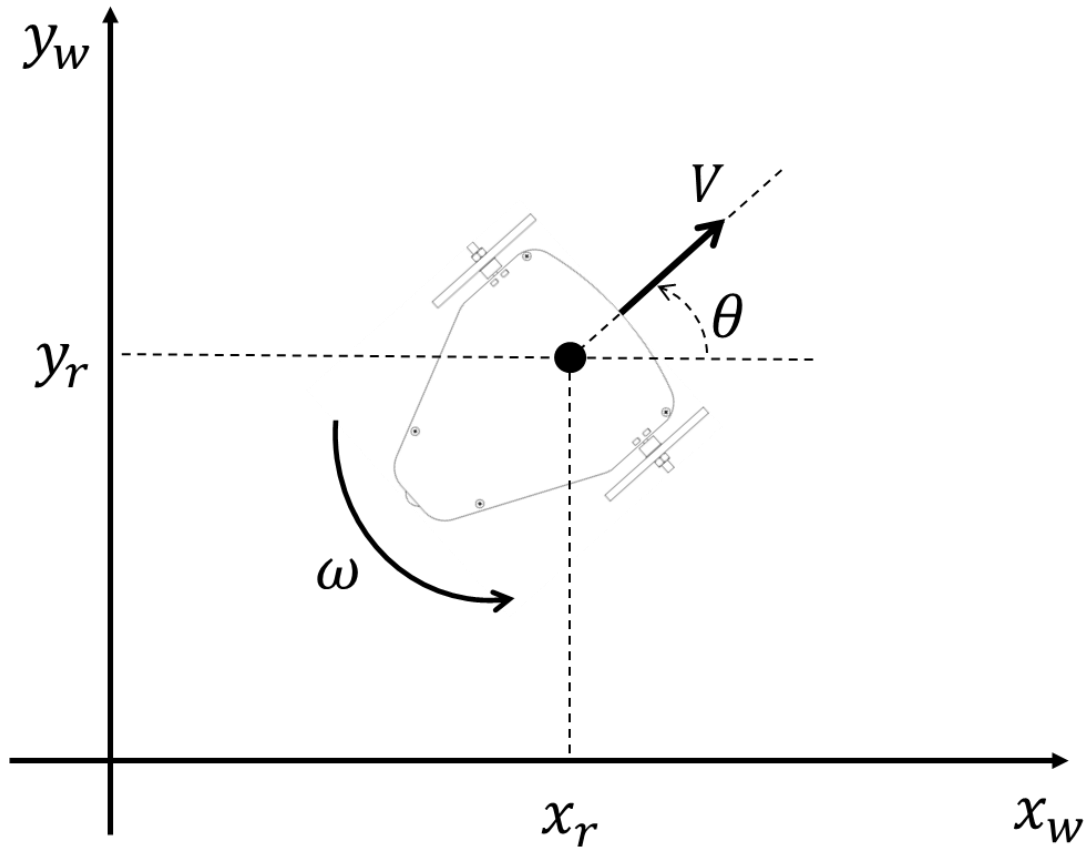
Localisation

Robot Position

- Discretising the model and solving it using Euler's Method:

$$\begin{cases} x_{r\,k+1} = x_{r\,k} + r\dfrac{\omega_R + \omega_L}{2}\cos\theta\ dt \\ y_{r\,k+1} = y_{r\,k} + r\dfrac{\omega_R + \omega_L}{2}\sin\theta\ dt \\ \theta_{r\,k+1} = \theta_{r\,k} + r\dfrac{\omega_R - \omega_L}{l}dt \end{cases}$$

- Where ω_R and ω_L are the speed given by the encoders
- Estimating the change in position a robot from its sensors is called "odometry".



Determining the Robot Position

$$\begin{bmatrix} \theta_{r\,k+1} \\ x_{r\,k+1} \\ y_{r\,k+1} \end{bmatrix} = \begin{bmatrix} \theta_{r\,k} \\ x_{r\,k} \\ y_{r\,k} \end{bmatrix} + \begin{bmatrix} r\dfrac{\omega_R - \omega_L}{l}dt \\ r\dfrac{\omega_R + \omega_L}{2}dt\cos\theta_k \\ r\dfrac{\omega_R + \omega_L}{2}dt\sin\theta_k \end{bmatrix}$$

Robot Location:
(x_k, y_k, θ_k): Pose of the robot at timestep k (m, m, rad). Stored in memory, initial value 0

Robot Constants:
r: Wheel radius = 0.05 m
l: Distance between robot wheels = 0.19 m

Measured variables
(ω_R, ω_L): Wheel velocity (rad/s)
dt: Time between samples (s)

Values of θ can grow unbounded so they must be contained within a single circle (wrap2pi):
Either:

$$-\pi \leq \theta < \pi$$

Or:

$$0 \leq \theta < 2\pi$$

Point Stabilisation Control

- For the sake of simplicity, in this course, the goal is defined only by a 2D set of coordinates

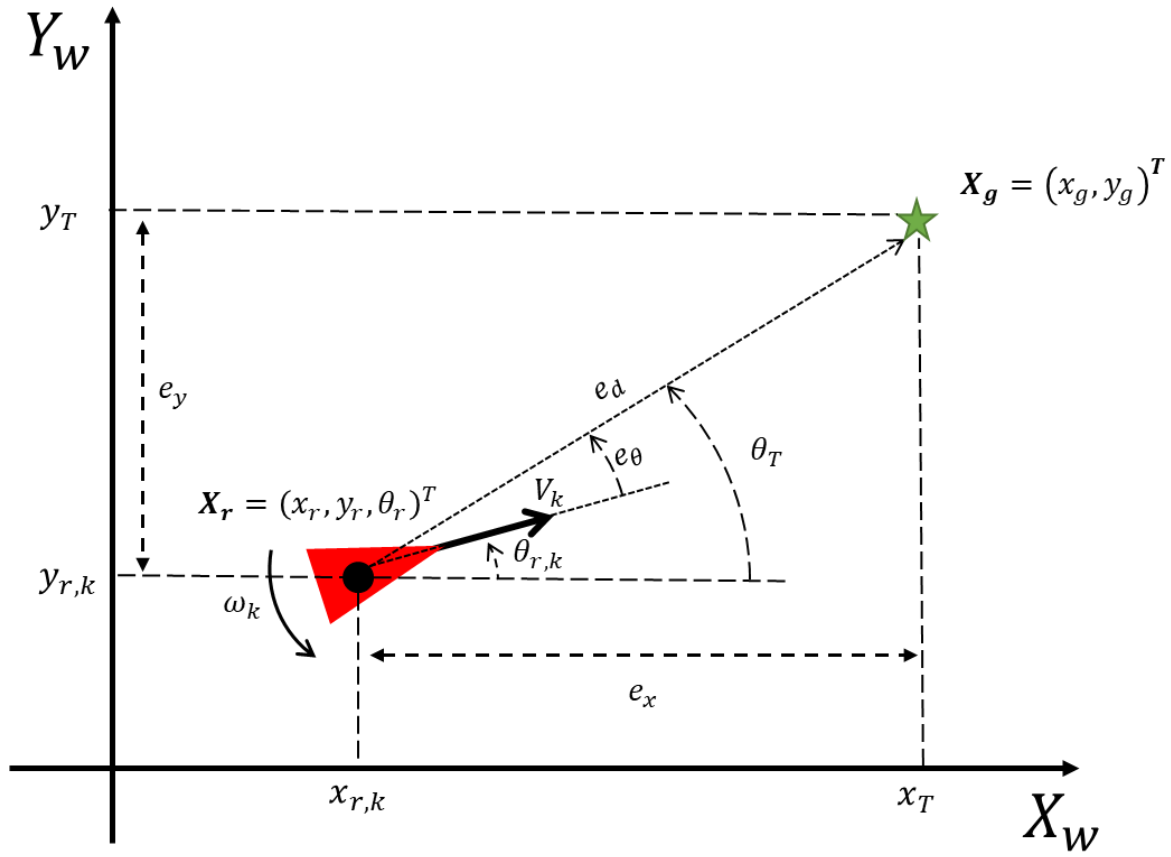
$$X_g = \begin{pmatrix} x_g \\ y_g \end{pmatrix}.$$

- Then compute the errors by using the goal coordinates and robot position as in the following:

$$e_x = x_g - x_r$$

$$e_y = y_g - y_r$$

$$e_\theta = atan2(e_y, e_x) - \theta_r$$



Point Stabilisation

The robot position is assumed to be known and can be computed using various localisation techniques as Dead Reckoning. The equations for the error can be represented in vector format as follows:

$$\boldsymbol{e} = \begin{pmatrix} e_x \\ e_y \\ e_\theta \end{pmatrix}$$

The general form of the control law can be written as:

$$\begin{pmatrix} V \\ \omega \end{pmatrix} = \boldsymbol{K} \begin{pmatrix} e_x \\ e_y \\ e_\theta \end{pmatrix},$$

where

$\boldsymbol{K} = \begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{pmatrix}$, is the control matrix.

Point Stabilisation

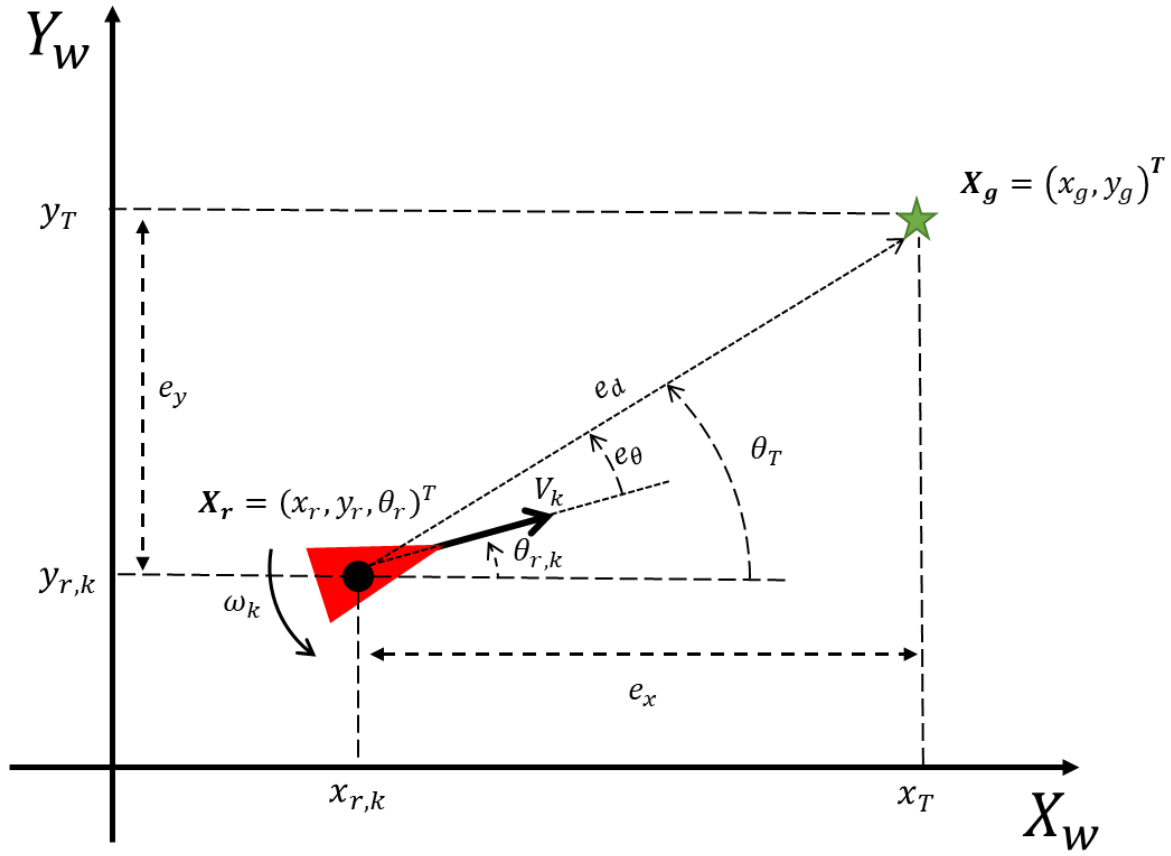
- For simplicity, the six controller gain parameters can be reduced to only two by defining the distance error:

$$e_d = \sqrt{e_x^2 + e_y^2}$$

- The control law can now be written as:

$$V = K_d e_d$$

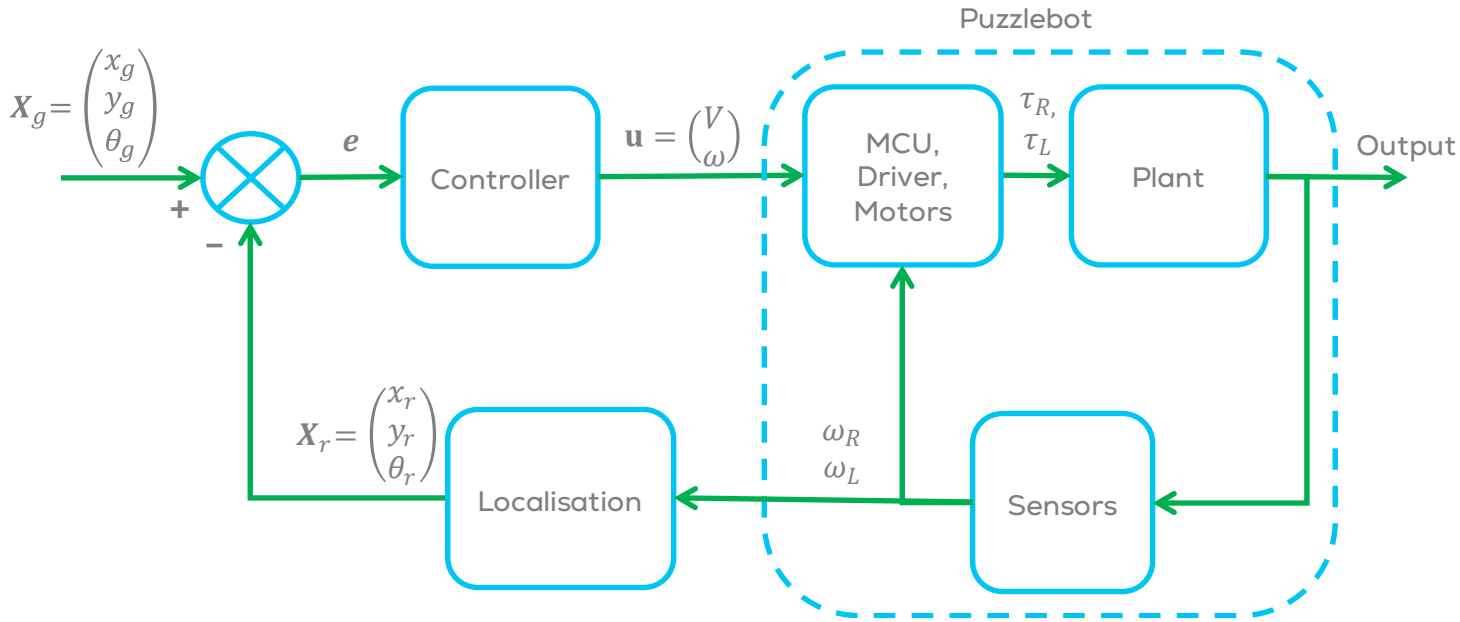
$$\omega = K_\theta e_\theta$$



The Control System

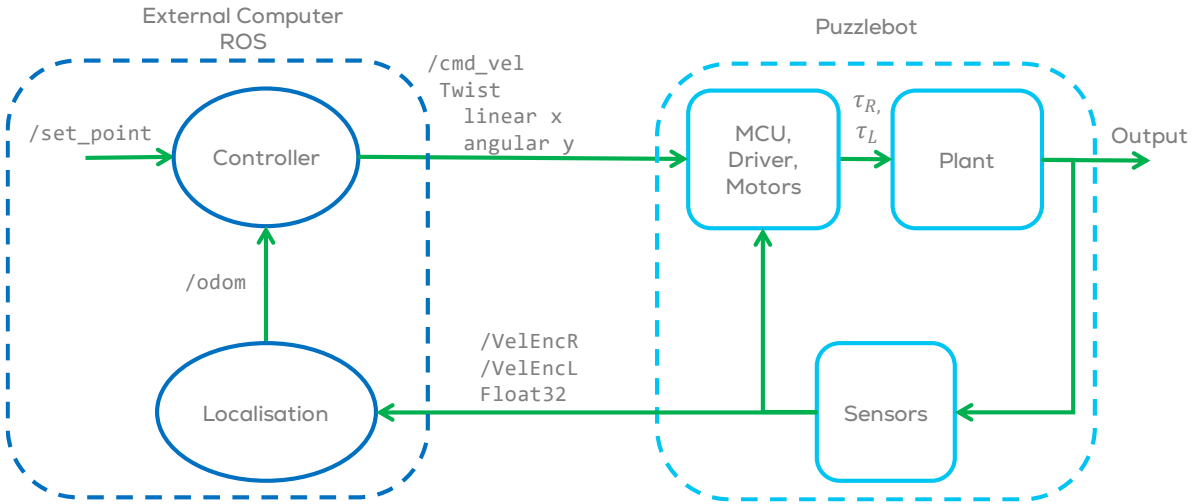
Control

- The control diagram can now be redefined as follows.
- It can be observed that the position can now be estimated using the encoder information.

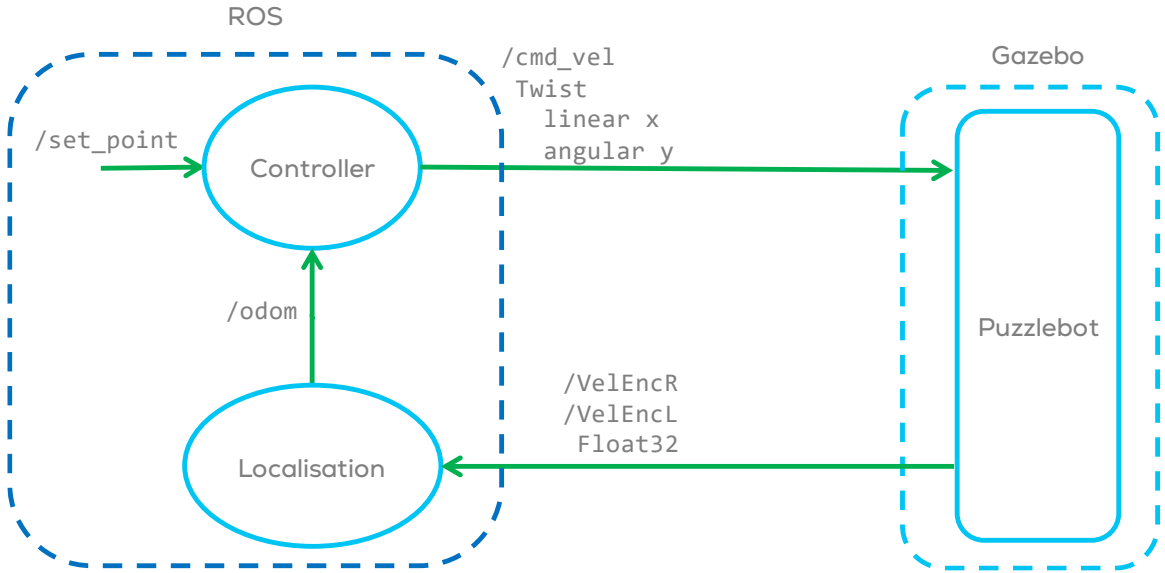


The Control System

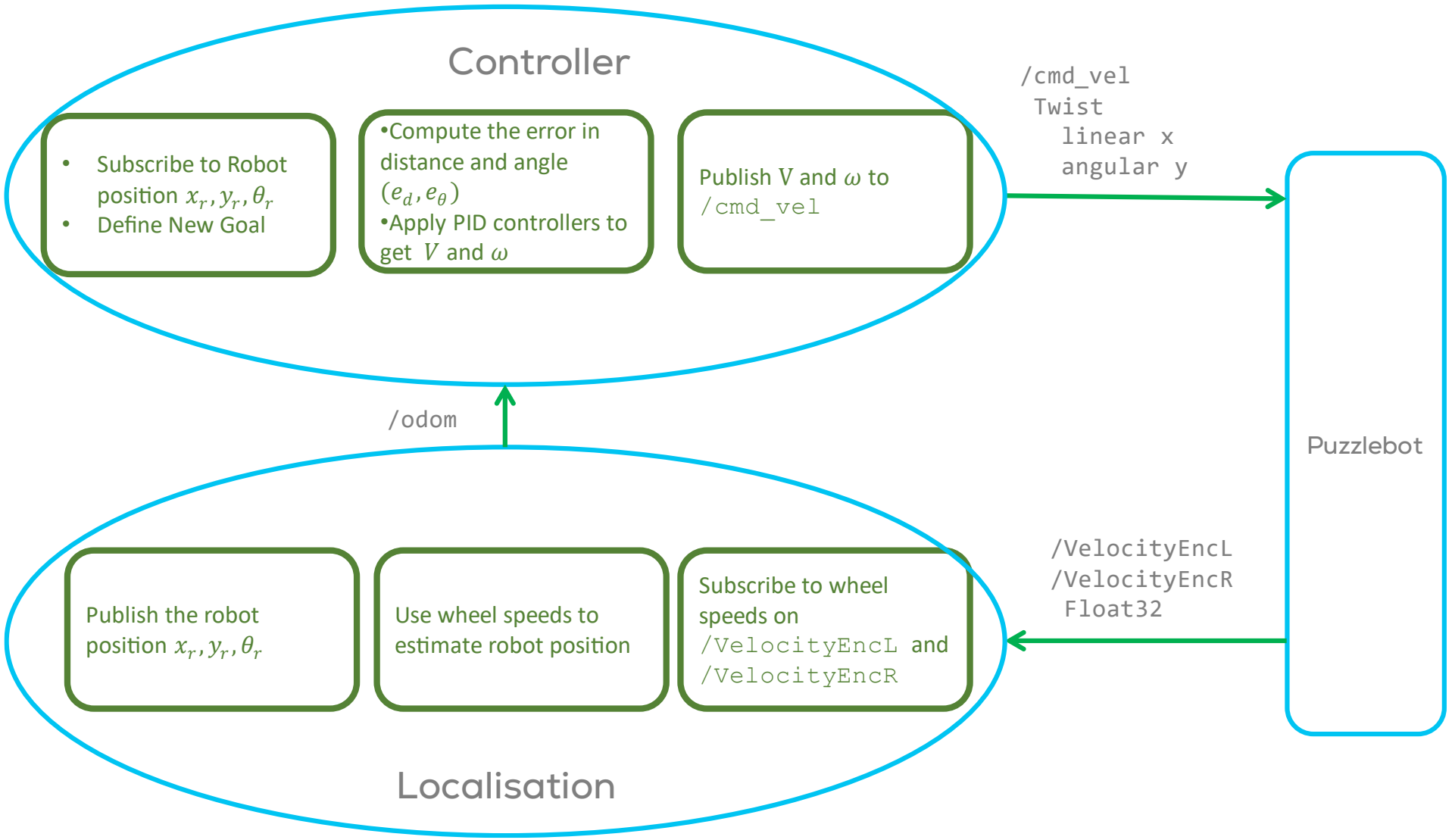
Real Robot



Gazebo sim



ROS setup


Controller
Subscribe to Robot position x_r, y_r, θ_r
Define New Goal
•Compute the error in distance and angle (e_d, e_θ)
•Apply PID controllers to get V and ω
Publish V and ω to /cmd_vel
/cmd_vel
Twist
linear x
angular y
Puzzlebot
/odom
Publish the robot position x_r, y_r, θ_r
Use wheel speeds to estimate robot position
Subscribe to wheel speeds on /VelocityEncL and /VelocityEncR
/VelocityEncL
/VelocityEncR
Float32
Localisation

Activity 1

Estimating Robot's Speed

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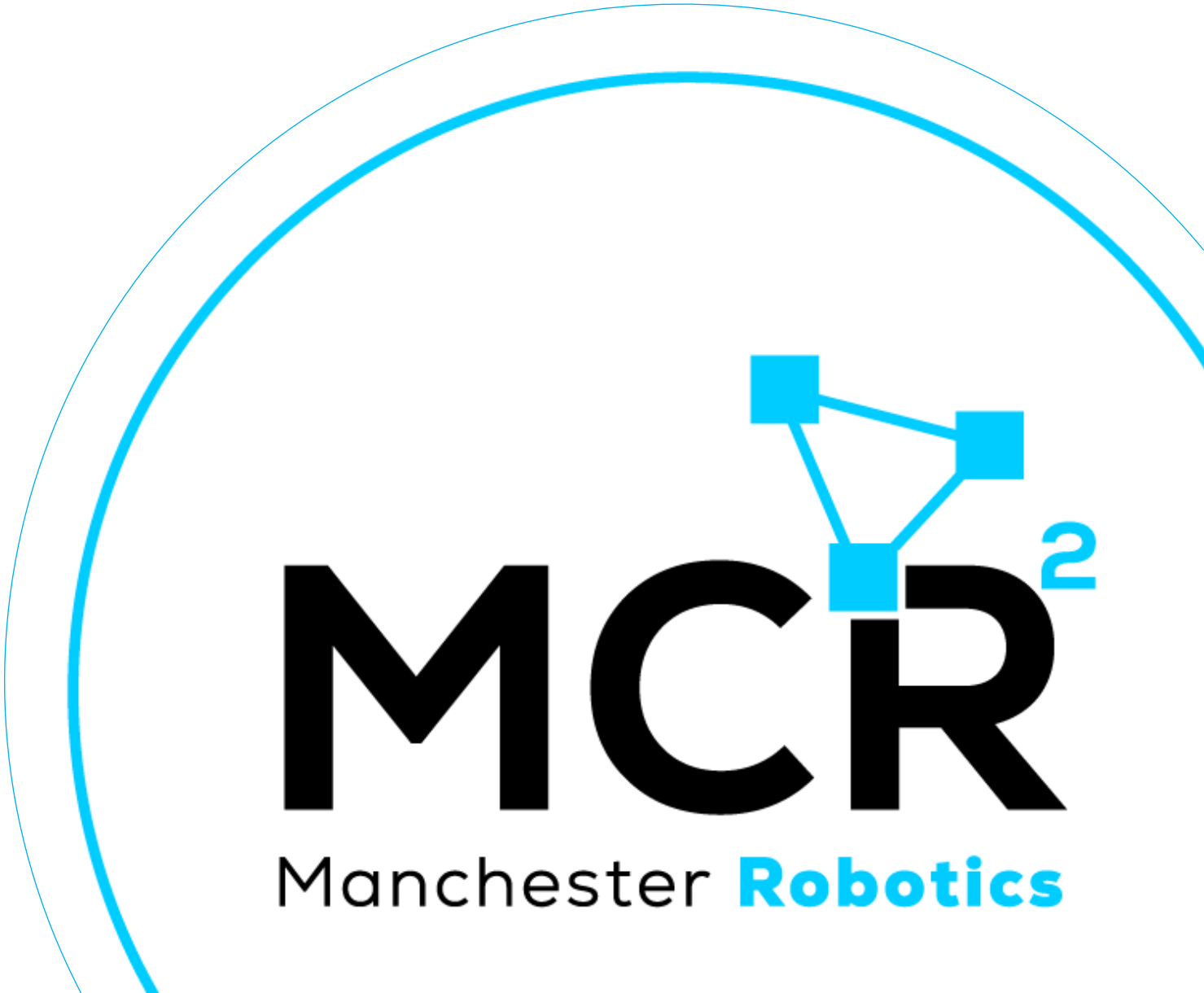
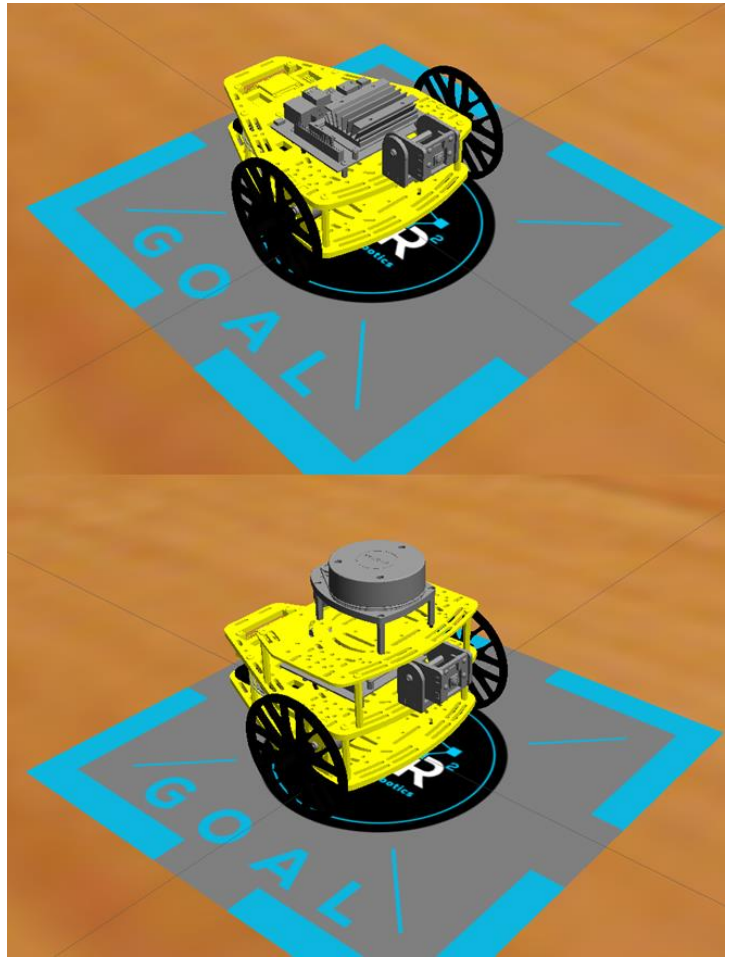


Introduction

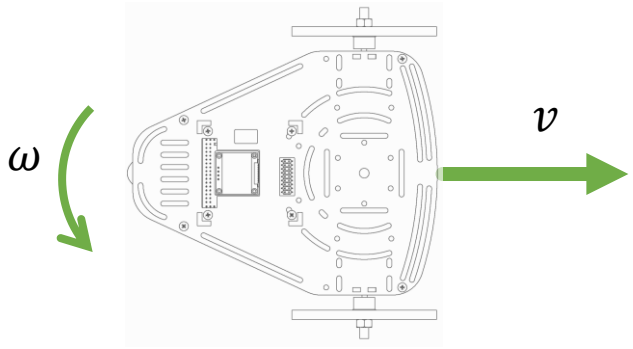
Introduction

- The following activity will help you to estimate the speed of the Puzzlebot
- This activity consist of creating a node that subscribes to the wheel speed of the robot and output the linear and angular speeds.
- This activity will use the Puzzlebot for testing.
- For more information about the Puzzlebot and how to use it please go to the presentation “MCR2_Puzzlebot_Jetson_Ed_ROS2”



Activity 1: Moving a Puzzlebot

Linear and Angular velocity estimation



Objectives:

- Create a ROS node that subscribes to the wheel's velocities and outputs the Puzzlebot's lienar and angular speeds.
- The Puzzlebotoutputs the wheel's velocities in the topics "`/VelocityEncR`" and "`/VelocityEncL`".
- The message used is a Float32

```
std_msgs/Float32

  float32 data
```

Activity 1: Moving a Puzzlebot

Instructions

For this project two options are given to the students, using the template or making a package from scratch.

Template:

- Download the template from Activity, add it to your workspace and compile it using Colcon.

Create a package:

- Make a new package called “Puzzlebot_localisation” as follows

```
$ ros2 pkg create --build-type ament_python puzzlebot_localisation --node-name puzzlebot_odometry --dependencies rclpy ros2launch python3-numpy std_msgs geometry_msgs nav_msgs --license Apache-2.0 --maintainer-name 'Mario Martinez' --maintainer-email 'mario.mtz@manchester-robotics.com'
```

- Give executable permission to the file

```
puzzlebot_odometry
```

```
$ cd ~/catkin_ws/src/markers/scripts/
$ sudo chmod +x activity1.py
```

```
 catkin_install_python(PROGRAMS scripts/activity1.py

   DESTINATION ${CATKIN_PACKAGE_BIN_DESTINATION})
```

Activity 1: Moving a Puzzlebot

Instructions

For this project two options are given to the students, using the template or making a package from scratch.

Template:

- Download the template from Activity 1, add it to your workspace and compile it using Colcon.

Create a package:

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puzzlebot_localisation --node-name puzzlebot_odometry --
dependencies rclpy ros2launch python3-numpy std_msgs
geometry_msgs nav_msgs --license Apache-2.0 --maintainer-
name 'Mario Martinez' --maintainer-email
'mario.mtz@manchester-robotics.com'
```

- Give executable permission to the file

```
$ cd ~/src/puzzlebot_localisation/puzzlebot_localisation/
$ sudo chmod +x puzzlebot_odometry.py
```

- Open the file “`puzzlebot_odometry.py`”

```python
import rclpy
import transforms3d
import numpy as np
import signal, os, time

   from rclpy import qos
from rclpy.node import Node
from std_msgs.msg import Float32
from geometry_msgs.msg import TransformStamped
from nav_msgs.msg import Odometry
from tf2_ros import TransformBroadcaster


   class DeadReckoning(Node):

       def __init__(self):
        super().__init__('dead_reckoning')

           #Set the parameters of the system
        self.X = 0.0
        self.Y = 0.0
        self.Th = 0.0
        self._l = 0.18
        self._r = 0.05
        self._sample_time = 0.01
        self.rate = 200.0

        # Internal state
        self.first = True
        self.start_time = 0.0
        self.current_time = 0.0
       self.last_time = 0.0

        #Variables to be used
        self.v_r = 0.0
        self.v_l = 0.0
        self.V = 0.0

        #Messages to be used
        self.wr = Float32()
        self.wl = Float32()
        self.odom_msg = Odometry()

        # Subscriptions
        self.sub_encR =
self.create_subscription(Float32,'VelocityEncR',self.encR_callback,qos.qos_pr
ofile_sensor_data)
        self.sub_encL =
self.create_subscription(Float32,'VelocityEncL',self.encL_callback,qos.qos_pr
ofile_sensor_data)

        # Publishers
        self.odom_pub = self.create_publisher(Odometry, 'odom',
qos.qos_profile_sensor_data)

        # Timer to update kinematics at ~100Hz
        self.timer = self.create_timer(1.0 / self.rate, self.run)  # 100 Hz

        self.get_logger().info("Localisation Node Started.")

    # Callbacks
    def encR_callback(self, msg):
        self.wr = msg

    def encL_callback(self, msg):
        self.wl = msg
```

```python
    def run(self):
        if self.first:
            self.start_time = self.get_clock().now()
            self.last_time = self.start_time
            self.current_time = self.start_time
            self.first = False
            return

        # Get current time and compute dt
        current_time = self.get_clock().now()
        dt = (current_time - self.last_time).nanoseconds * 1e-9  # Convert to seconds

        if dt > self._sample_time:
            #Wheel Tangential Velocities
            self.v_r = self._r  * self.wr.data
            self.v_l = self._r  * self.wl.data
            #Robot Velocities
            self.V = (1/2.0) * (self.v_r + self.v_l)
            self.Omega = (1.0/self._l) * (self.v_r - self.v_l)
            self.last_time = current_time
            self.publish_odometry()

    def publish_odometry(self):
        q1 = transforms3d.euler.euler2quat(0, 0, self.Th)
        self.odom_msg.header.stamp = self.get_clock().now().to_msg()
        self.odom_msg.header.frame_id = 'odom'
        self.odom_msg.child_frame_id = "base_footprint"
        self.odom_msg.pose.pose.position.x = self.X
        self.odom_msg.pose.pose.position.y = self.Y
        self.odom_msg.pose.pose.position.z = 0.0
        self.odom_msg.pose.pose.orientation.x = q1[1]
        self.odom_msg.pose.pose.orientation.y = q1[2]
        self.odom_msg.pose.pose.orientation.z = q1[3]
        self.odom_msg.pose.pose.orientation.w = q1[0]
        self.odom_msg.twist.twist.linear.x = self.V
        self.odom_msg.twist.twist.angular.z = self.Omega
        self.odom_pub.publish(self.odom_msg)
```

```python
    def stop_handler(self,signum, frame):
        """Handles Ctrl+C (SIGINT)."""
        self.get_logger().info("Interrupt received! Stopping node...")
        raise SystemExit



def main(args=None):

    rclpy.init(args=args)

    node = DeadReckoning()

    signal.signal(signal.SIGINT, node.stop_handler)

    try:
        rclpy.spin(node)
    except SystemExit:
        node.get_logger().info('SystemExit triggered. Shutting down
cleanly.')
    finally:
        node.destroy_node()
        rclpy.shutdown()

if __name__ == '__main__':
    main()
```

Activity 1: Results

- Save and recompile the project.
- Build the program using “colcon build”

```
$ colcon build
$ source install/setup.bash
```

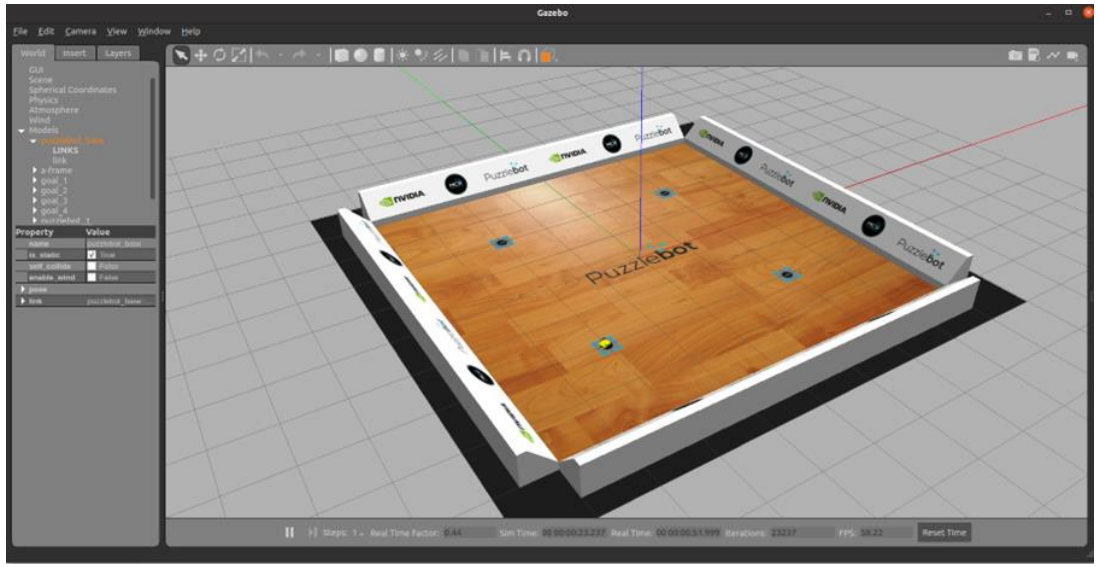
- Connect to the Puzzlebot via Wi-Fi
- If using Gazebo run the gazebo simulator, start the simulation as follows:

```
$ ros2 launch puzzlebot_gazebo gazebo_example_launch.py
```

- Open another terminal and run the Activity

```
$ ros2 run puzzlebot_localisation Puzzlebot_odometry
```

- Move the robot using the teleoperation node, and verify if the speed are correct.



Activity 2: Localisation Node

- Implement a ROS node that computes the robot location using the encoder data
  - It should subscribe to `/VelocityEncR` and `/VelocityEncR`, and publish the data to a suitable set of topics
  - The published messages could be a set floats, or you can use the Odom message (standard way).
  - Use the remote-control function “teleop_twist_keyboard” to test the position estimation.

Activity 3: The Controller

- Make another node to that computes e_d and e_θ.
- Use the remote-control function “teleop_twist_keyboard” to test the errors.
- Drive the robot around, checking that the angle to the target and the distance from the target are updated correctly
- Remember to wrap all angles (wrap to pi)

```
#wrap to pi function
def wrap_to_Pi(theta):
    result = np.fmod((theta + np.pi),(2 * np.pi))
    if(result < 0):
        result += 2 * np.pi
    return result - np.pi
```

Activity 3: The Controller

- Since the robot is inherently stable, a simple PID scheme should be sufficient.
- Start with a pair of proportional controllers:

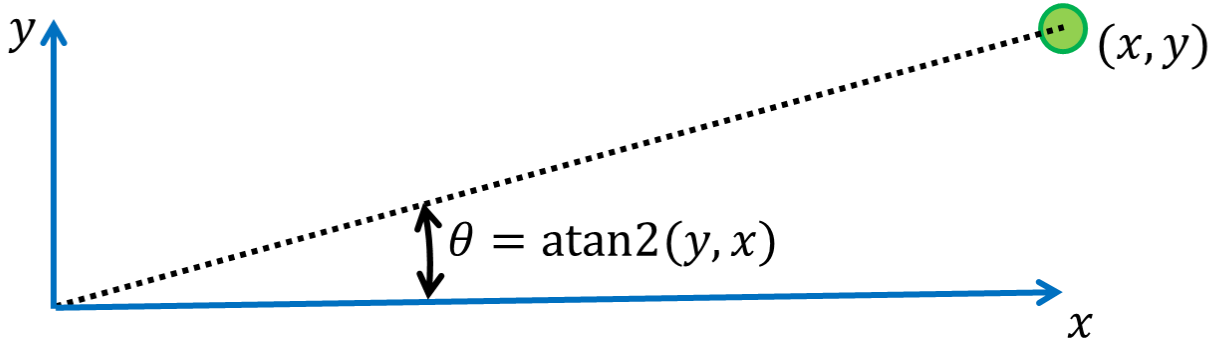
$$V = K_v e_d$$

$$\omega = K_\omega e_\theta$$

… and add integral and derivative elements if necessary?.

atan2

- The atan2 function is a special form of arctan or $\tan^{-1}$.
- It takes two arguments, y and x, and returns the angle to the x axis:



- It is included in most maths libraries, but it is recommended to use numpy, as numpy will be necessary later on in the course

```
import numpy

theta = numpy.arctan2(y,x)
```

Tips and Tricks

- Write and test your node with the Gazebo Simulator:
  - Use this to check the basics of your code are working correctly, such as the sign (+/-) of your controller parameters K_v and K_ω
  - Does the robot turn towards the goal?
  - Does the robot move towards or away from the goal?
- Tune one of the controllers at a time. You may find it easier to tune K_ω first, while setting your robot to move with a fixed forward speed.
- If in doubt, *lower* the value of the control constants.
- You may find it helpful to use a launch file to load your controller constants using parameters.

Accuracy

- It will not be possible to tune the controllers such that the robot moves perfectly into position.
  - You will need a threshold after which your algorithm decides it has successfully arrived.
  - Suggested initial threshold: 10 cm ($e_d < 0.1\ m$)
- Additionally, if you measure the position of the robot, it will likely not match up with the measurement computed from the encoders.
  - This is inevitable due to additive noise in the encoder readings.
  - The solution to this is to use sensors that can measure the position of the robot relative to its environment (another class).