# 2024-02-21

# Quantizing neural networks





# Quantizing neural networks

#### Efficient Deep Learning - Session 3



# Course organisation

#### Sessions

- 1 Intro Deep Learning,
- 2 Data Augmentation and Self Supervised Learning,
- 3 Quantization,
- 4 Pruning,
- 5 Factorization,
- 6 Distillation,
- **T** Embedded Software and Hardware for DL,
- Presentations for challenge.

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-Course organisation

Course organisation

Sessions Intro Deep Learning,

Data Augmentation and Self Supervised Learning,

Quantization,

Pruning,
Factorization,

Distillation,
Embedded Software and Hardware for DL,
Presentations for challenge.

02

# Course organisation

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02

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Sessions Intro Deep Learning,

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☐ Quantization, ☐ Pruning,

Factorization,
 Distillation,
 Embedded Software and Hardware.

Embedded Software and Hardware for DL,
 Presentations for challenge.

# Today's Summary

- Objectives
- 2 Quantization : Basics
  - Floating Point
  - Integers, Fixed Point
  - Quantization
- 3 Quantization: Neural Networks
  - Quantization Post Training
  - Quantization Aware Training
- 4 Quantization in Pytorch

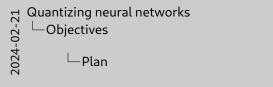
Quantizing neural networks

Today's Summary



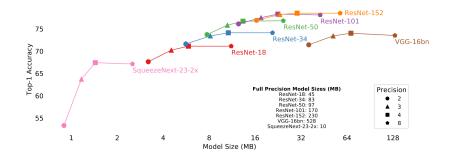
# Plan

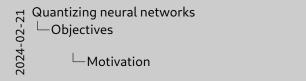
- Objectives
- **Quantization**: Basic
  - Floating Point
  - Integers, Fixed Point
  - Quantization
- Quantization : Neural Network
  - Quantization Post Training
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- 4 Quantization in Pytorch

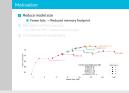




- 1 Reduce model size
  - Fewer bits → Reduced memory footprint
- Decrease memory access
  - GPU & CPU : reduce Cache usage
- Computational complexity



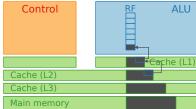




Reducing the memory footprint can be beneficial particularly in the case of embedded systems. On an ESP32, which is commonly used for low power applications, the memory footprint is limited to 520KB.

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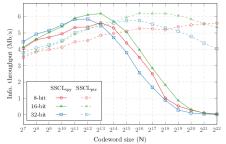


Quantizing neural networks
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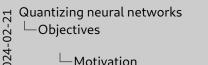


The data bandwidth between the cache and the GPU or the CPU is limited. This can be the bottleneck of the implementation, limiting the throughput or the latency. Therefore, reducing the cache usage is also beneficial for the performance of neural network processing.

- Reduce model size
  - Fewer bits → Reduced memory footprint
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  - GPU & CPU : reduce Cache usage
- Computational complexity









This is an example outside of the scope of Deep Learning. This is a simulation of a channel decoder inside a communication system. On the x-axis is the size of the codeword, which is the size of the main array of data that is processed by the algorithm. On the y-axis is the throughput of the system. The algorithm "SSCLcpy" is there an example of a cache issue, where the array becomes too big to fit in the first level of caches, with a dramatic decrease of the throughput after a certain array size.

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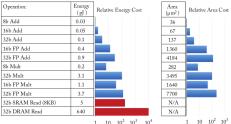


Figure 7.1: The area and energy cost for additions and multiplications at different precision, and memory accesses in a 45 nm process. The area and energy scale different for multiplication and addition. The energy consumption of data movement (red) is significantly higher than arithmetic operations (blue). (Figure adapted from [121].)

| Energy (pf) | Relative Energy Cost | Area (µm²) | Selative Area Cost | Area (µm²) | Selative Area Cost | Cost (µm²) | Selative Area Cost (µm²) | Selative



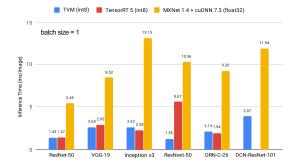
These are the relative energy cost of doing different operations on a 45 nm ASIC. The energy cost of floating point operations is higher than the energy cost of integer operations. It takes also more space on the chip, which means that there is a higher parallelization potential with fixed point operations than with floating point ones.

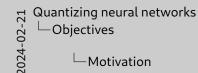
Quantizing neural networks

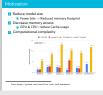
Motivation

-Objectives

- 1 Reduce model size
  - Fewer bits → Reduced memory footprint
- Decrease memory access
  - GPU & CPU : reduce Cache usage
- Computational complexity



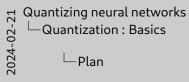




This is a concrete example of the benefits of quantization on the inference time of different neural networks, on an NVIDIA GTX 1080, using different framewoks. The inference time is measured in milliseconds. Different frameworks are used, MXNet and cuDNN for the floating point implementation, and TVM and tensorRT for the fixed point implementation. For each neural network, inference time is lower with int8 than with float32.

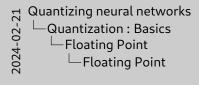
# Plan

- 1 Objectives
- 2 Quantization : Basics
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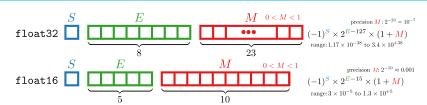








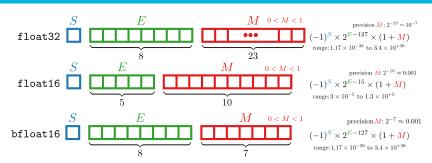






Quantizing neural networks
—Quantization : Basics
—Floating Point
—Floating Point

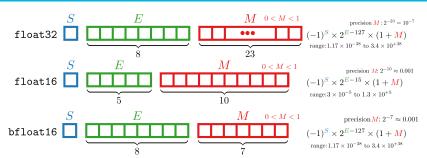






Quantizing neural networks
—Quantization : Basics
—Floating Point
—Floating Point





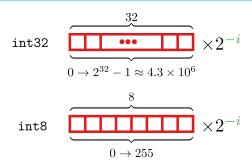
- To add two FP numbers:
  - Shift M according to E (int shift  $n_E$  bits)
  - Add M (int add  $n_M$  bits)
  - Normalize (0 < M < 1)
- To multiply two FP numbers:
  - Multiply M (int mult  $n_M$  bits)
  - Add E (int mult  $n_E$  bits)
  - Normalize (0 < M < 1)



Quantizing neural networks
—Quantization : Basics
—Floating Point
—Floating Point



# Integers, fixed point



- Fixed point (-i)
- Short range
- Simple computation

Quantizing neural networks

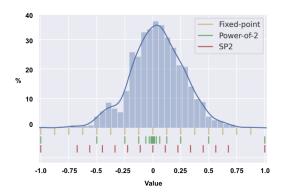
Quantization: Basics
Integers, Fixed Point
Integers, fixed point

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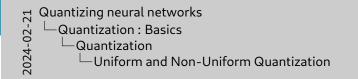


On the other side, the main way to represent fractional numbers with integers is the fixed point representation. i is the number of fractional bits. The range of the representation is  $[-2^{i-1}, 2^{i-1} - 1]$ . The representation is simple, but the range is limited.

### Uniform and Non-Uniform Quantization



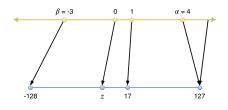
- Uniform quantization enables the use of integer on fixed-point hardware
- Non-uniform quantization requires a codebook lookup  $\rightarrow$  not straightforward for standard hardware (CPU, GPU)

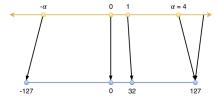




There are other ways to use integers to represent real numbers. The different values of the integers can represent arbitrary real numbers in a uniform or non-uniform way.

## Affine and Scale Quantization





(a) Affine quantization

(b) Scale quantization

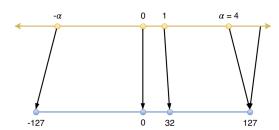
- 2 kinds of uniform quantization
- Assymetric vs Symmetric

Wu, Hao, et al. "Integer quantization for deep learning inference: Principles and empirical evaluation." arXiv preprint arXiv:2004.09602 (2020).

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Quantization: Basics
Quantization
Affine and Scale Quantization



### Scale Quantization



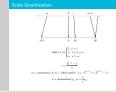
$$\operatorname{clip}(x,l,u) \begin{cases} l, & x < l \\ x, & l \le x \le u \\ u, & x > u \end{cases}$$

$$s = \frac{2^{b-1} - 1}{2^{c}}$$

$$x_q = \text{quantize}(x,b,s) = \text{clip}(\text{round}(s\cdot x), -2^{b-1}+1, 2^{b-1}-1)$$
 
$$\hat{x} = \text{dequantize}(x_q,s) = \frac{1}{s}x_q$$



Quantizing neural networks
Quantization: Basics
Quantization
Scale Quantization



The upper arrow represents the real axis, the lower one represent the quantized axis. Scale quantizaiton is a category of uniform quantization, where real number 0 is represented as the integer 0. It is also symmetric, as there is the same number of positive and negative values on the quantized axis. Only one parameter is then needed to define the quantization, which is the real number  $\alpha$  that corresponds to the maximum integer value. The axes are then split uniformly in  $2^b-1$  intervals. The quantize and dequantize functions are defined accordingly.

#### Scale Quantization

$$y_{ij} = \sum_{k=1}^{p} x_{ik} \cdot w_{kj} \approx$$

$$\sum_{l=1}^{p} \operatorname{dequantize}(x_{q,ik}, s_{q,ik}) \cdot \operatorname{dequantize}(w_{q,kj}, s_{w,kj}) =$$

$$\sum_{k=1}^{p} \frac{1}{s_{x,ik}} x_{q,ik} \cdot \frac{1}{s_{w,kj}} w_{q,kj}$$

And, in order to use integer multiplication, the scaling factor s must be independent of k :

$$\frac{1}{s_{x,i} \cdot s_{w,j}} \sum_{k=1}^{p} x_{q,ik} \cdot w_{q,kj}$$



Quantizing neural networks
Quantization: Basics
Quantization
Scale Quantization



This is the important part about scale quantization. When the scaling factor is constant over a part of the data (weights and activations), the scaling to go back to floating point values can be done a posteriori, after all of the computations have been done with integers.

-Plan

- Floating Point
- Integers, Fixed Point
- Quantization

3 Quantization: Neural Networks

- Quantization Post Training
- Quantization Aware Training

# Quantization Post Training: Weights

#### Quantize a set of parameters

- Select the number of quantization bits,
- Select the set of parameters to quantize,
- Determine range of values,
- Determine the scaling factor (and zero if affine).

### Different weight sets can be considered

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation  $\rightarrow$  better accuracy.

Quantizing neural networks 02-Quantization: Neural Networks Quantization Post Training -Quantization Post Training: Weights

Determine the scaling factor (and zero if affine)

Whole network,

per laver.

per neuron. Finer sets segmentation → better accuracy.

# **Quantization Post Training: Activation**

#### Quantize a set of parameters

- Select the number of quantization bits,
- Select the set of activations to quantize,
- Determine range of values according to the training set or a subset,
- Determine the scaling factor (and zero if affine).

### Different weight sets can be considered

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation  $\rightarrow$  better accuracy.

Quantizing neural networks

Quantization: Neural Networks
Quantization Post Training
Quantization Post Training: Activation

itization Post Training : Activation

Quantize a set of parameters

- Select the set of activations to quantize,
   Determine range of values according to the training set subset.
- Determine the scaling factor (and zero if affine).
  - Different weight sets can be con

    Whole network,
- per layer,
- Finer sets segmentation → better accuracy.

# Quantization Aware Training

- Train the network while quantizing the weights and / or the activations,
- Quantization Aware Techniques yield way better accuracy,
- Especially for extremely low-bit precision (2-3-4 bit precision).

Quantizing neural networks

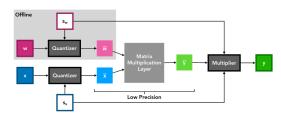
—Quantization: Neural Networks
—Quantization Aware Training
—Quantization Aware Training

Train the network white quantizing the weights and f or the activation.

Branch Rowse Techniques yield way better accuracy.

Begacially for extremely low-bit precision (2-3-4 bit precision).

# Learned Step Size Quantization



- s quantizer step size
- lacksquare  $Q_P$  and  $Q_N$ , the number of positive and negative quantization levels

$$\bar{v} = \lfloor clip(v/s, -Q_N, Q_P) \rceil, \tag{1}$$

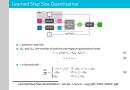
$$\hat{v} = \bar{v} \times s. \tag{2}$$

■ s is learned with:

$$\frac{\partial \hat{v}}{\partial s} = \begin{cases} -v/s + \lfloor v/s \rceil & \text{if } -Q_N < v/s < Q_P \\ -Q_N & \text{if } v/s \le -Q_N \\ Q_P & \text{if } v/s \ge Q_P \end{cases}$$
(3)

Learned Step Size Quantization - https://arxiv.org/pdf/1902.08153.pdf

Quantizing neural networks
—Quantization : Neural Networks
—Quantization Aware Training
—Learned Step Size Quantization



LSQ is a technique that allows to learn the step size s of the quantizer. The step size is learned with a gradient descent algorithm. Weights quantization can be done offline, which means that no additional computation is required during inference.

**Algorithm 1** SGD training with BinaryConnect. C is the cost function for minibatch and the functions binarize (w) and clip(w) specify how to binarize and clip weights. L is the number of layers.

**Require:** a minibatch of (inputs, targets), previous parameters  $w_{t-1}$  (weights) and  $b_{t-1}$  (biases), and learning rate  $\eta$ .

**Ensure:** updated parameters  $w_t$  and  $b_t$ .

#### 1. Forward propagation:

 $w_b \leftarrow \text{binarize}(w_{t-1})$ 

For k = 1 to L, compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$ 

#### 2. Backward propagation:

Initialize output layer's activations gradient  $\frac{\partial C}{\partial a_{r}}$ 

For k = L to 2, compute  $\frac{\partial C}{\partial a_{k-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $w_b$ 

#### 3. Parameter update:

Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{db_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$ 

$$w_t \leftarrow \operatorname{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$$
$$b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$$

IMT-Atlantique

Courbariaux, Matthieu, Yoshua Bengio, and Jean-Pierre David.
"Binaryconnect: Training deep neural networks with binary weights during propagations." Advances in neural information processing systems. 2015. https://arxiv.org/pdf/1511.00363.pdf

Quantizing neural networks

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Quantizing neural networks
Quantization : Neural Networks
Quantization Aware Training
Quantization while Learning - Binary Connect

identity function.

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Binaryconnect: Training deep neural networks with binary weights during

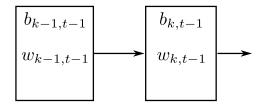
ropagations." Advances in neural information processing systems, 2015.

Binary Connect is a technique that allows to train a network with binary weights. The weights are binarized during the forward pass and then the gradients are backpropagated. The weights are updated with a gradient descent algorithm. During training, the weights are binarized, which means that the network is not differentiable. To overcome this issue, Straight Through Estimator (STE) is used. It consists of replacing the gradient of the binarization function by the gradient of the

#### 1. Forward propagation:

 $w_b \leftarrow \text{binarize}(w_{t-1})$ 

For k = 1 to L, compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$ 



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—Quantization : Neural Networks

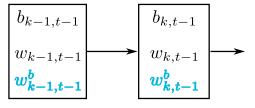
—Quantization Aware Training

—Quantization while Learning - Binary Connect



#### 1. Forward propagation:

$$w_b \leftarrow \text{binarize}(w_{t-1})$$



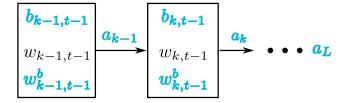
$$w^b = \begin{cases} +1, & \text{if } w \ge 0\\ -1, & \text{otherwise} \end{cases}$$

Quantizing neural networks
Quantization : Neural Networks
Quantization Aware Training
Quantization while Learning - Binary Connect



#### 1. Forward propagation:

 $w_b \leftarrow \text{binarize}(w_{t-1})$ For k=1 to L, compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$ 



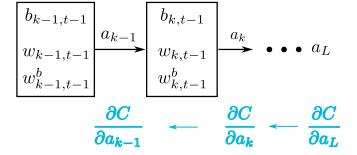
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Quantization: Neural Networks
Quantization Aware Training
Quantization while Learning - Binary Connect



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Initialize output layer's activations gradient  $\frac{\partial C}{\partial a_L}$ For k=L to 2, compute  $\frac{\partial C}{\partial a_{k-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $w_b$ 



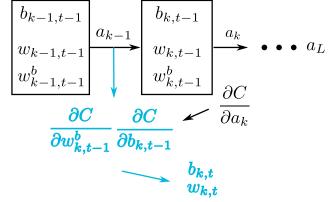
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Quantization: Neural Networks
Quantization Aware Training
Quantization while Learning - Binary Connect



#### 3. Parameter update:

Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{db_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$   $w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$   $b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$ 





Quantizing neural networks

Quantization: Neural Networks
Quantization Aware Training
Quantization while Learning - Binary Connect



# Binarization: Stochastic vs Deterministic

Deterministic

$$w_b = \begin{cases} +1, & \text{if } w \ge 0 \\ -1, & \text{otherwise} \end{cases}$$

Stochastic

$$w_b = \begin{cases} +1, & \text{with probability } p = \sigma(w) \\ -1, & \text{with probability } 1-p \end{cases}$$

avec

$$\sigma(x) = \operatorname{clip}(\frac{x+1}{2}, 0, 1) = \max(0, \min(1, \frac{x+1}{2}))$$



Quantizing neural networks
—Quantization: Neural Networks
—Quantization Aware Training
—Binarization: Stochastic vs Deterministic



Two different ways to binarize the weights exist. The first one is deterministic, which means that the weights are binarized with a threshold. The second one is stochastic, which means that the weights are binarized with a probability.

# Binarization: Stochastic vs Deterministic

Deterministic

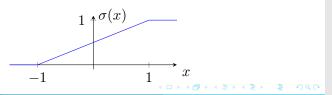
$$w_b = \begin{cases} +1, & \text{if } w \ge 0 \\ -1, & \text{otherwise} \end{cases}$$

Stochastic

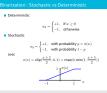
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—Quantization : Neural Networks
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—Binarization : Stochastic vs Deterministic



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Quantization in Pytorch

-Plan

- - Floating Point
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  - Quantization Aware Training
- 4 Quantization in Pytorch

# Quantization in Pytorch

- 1 Dynamic Quantization
- Static Quantization
- Quantization Aware Training

https://pytorch.org/docs/stable/quantization.html

And for our need:

 $\verb|https://pytorch.org/tutorials/prototype/pt2e_quant_ptq.html|\\$ 



Quantizing neural networks
—Quantization in Pytorch

Quantization in Pytorch



Quantization in Pytorch is a new feature. It is still in beta version. A group of students from last year worked on this topic. They implemented a post quantization algorithm in order to quantize the network with 8-bit weights and activations.