

# Quantizing neural networks

## Efficient Deep Learning - Session 3



**IMT Atlantique**  
Bretagne-Pays de la Loire  
École Mines-Télécom

## Sessions

- 1 Intro Deep Learning,
- 2 Data Augmentation and Self Supervised Learning,
- 3 Quantization,
- 4 Pruning,
- 5 Factorization,
- 6 Distillation,
- 7 Embedded Software and Hardware for DL,
- 8 Presentations for challenge.

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2024-02-06

Quantizing neural networks

└ Course organisation

Course organisation

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## Today's Summary

## 1 Objectives

## 2 Quantization : Basics

- Floating Point
- Integers, Fixed Point
- Quantization

### 3 Quantization : Neural Networks

- Quantization Post Training
- Quantization Aware Training

## 4 Quantization in Pytorch

## 1 Objectives

- Floating Point
- Integers, Fixed Point
- Quantization

- Quantization Post Training
- Quantization Aware Training

## 1 Reduce model size

- Fewer bits → Reduced memory footprint

## 2 Decrease memory access

- GPU & CPU : reduce Cache usage

## 3 Computational complexity

**Table:** Performance on the ImageNet dataset and complexities

Network	Alexnet	Inceptionv1	ResNet50	ResNet152
Top-5 error	16.4%	6.7%	5.25%	4.49%
Num. Weights	61M	7M	25.5M	63.75M
Num. MAC	724M	1.43G	3.9G	11.31G

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## Quantizing neural networks

### Objectives

### Motivation

Reducing the memory footprint can be beneficial particularly in the case of embedded systems. On an ESP32, which is commonly used for low power applications, the memory footprint is limited to 520KB.

- Reduce model size
  - Fewer bits → Reduced memory footprint
- Decrease memory access
  - GPU & CPU : reduce Cache usage
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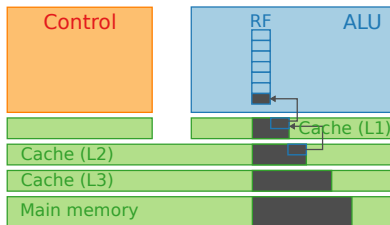
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## Quantizing neural networks

### Objectives

### Motivation

The data bandwidth between the cache and the GPU or the CPU is limited. This can be the bottleneck of the implementation, limiting the throughput or the latency. Therefore, reducing the cache usage is also beneficial for the performance of neural network processing.

Motivation

- Reduce model size
  - Fewer bits → Reduced memory footprint
- Decrease memory access
  - GPU & CPU : reduce Cache usage
- Computational complexity



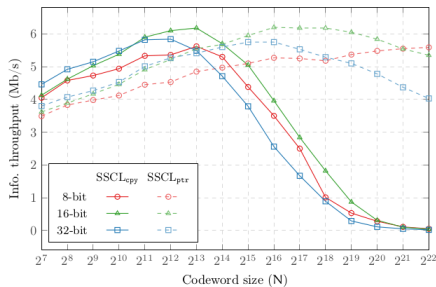
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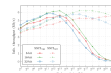
## 3 Computational complexity



### Objectives

### Motivation

- Reduce model size
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- Computational complexity



This is an example outside of the scope of Deep Learning. This is a simulation of a channel decoder inside a communication system. On the x-axis is the size of the codeword, which is the size of the main array of data that is processed by the algorithm. On the y-axis is the throughput of the system. The algorithm "SSCL<sub>cpy</sub>" is there an example of a cache issue, where the array becomes too big to fit in the first level of caches, with a dramatic decrease of the throughput after a certain array size.



# Motivation

## 1 Reduce model size

- Fewer bits → Reduced memory footprint

## 2 Decrease memory access

- GPU & CPU : reduce Cache usage

## 3 Computational complexity

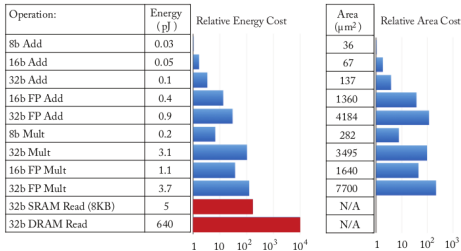


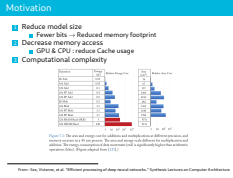
Figure 7.1: The area and energy cost for additions and multiplications at different precision, and memory accesses in a 45 nm process. The area and energy scale different for multiplication and addition. The energy consumption of data movement (red) is significantly higher than arithmetic operations (blue). (Figure adapted from [121].)

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## Quantizing neural networks

### Objectives

### Motivation



These are the relative energy cost of doing different operations on a 45 nm ASIC. The energy cost of floating point operations is higher than the energy cost of integer operations. It takes also more space on the chip, which means that there is a higher parallelization potential with fixed point operations than with floating point ones.

# Motivation

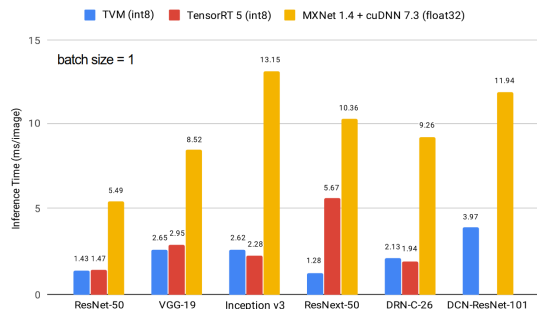
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From : <https://github.com/vinx13/tvm-cuda-int8-benchmark/>

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## Quantizing neural networks

### Objectives

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Motivation

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From: <https://github.com/vinx13/tvm-cuda-int8-benchmark/>

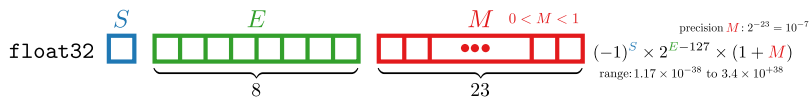
This is a concrete example of the benefits of quantization on the inference time of different neural networks, on an NVIDIA GTX 1080, using different frameworks. The inference time is measured in milliseconds. Different frameworks are used, MXNet and cuDNN for the floating point implementation, and TVM and tensorRT for the fixed point implementation. For each neural network, inference time is lower with int8 than with float32.

## 2 Quantization : Basics

- Floating Point
- Integers, Fixed Point
- Quantization

- Quantization Post Training
- Quantization Aware Training

# Floating Point



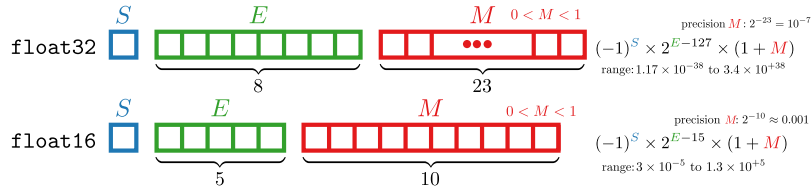
## Quantizing neural networks

- Quantization : Basics
  - Floating Point
    - Floating Point



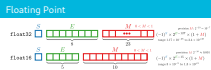
Here are three different floating point representations.  $S$  is the sign,  $E$  is the exponent, and  $M$  is the mantissa. The first representation is the IEEE 754 standard, which is the most common one. The second one is the float16 representation format, which uses only 16 bits. The third one is the bfloat16 representation format, which uses only 16 bits, but with different numbers of bits in the mantissa and in the exponent. It has been designed to work well for neural network training. Performing operations like addition or multiplication of floating point numbers is more complex than with integers.

# Floating Point



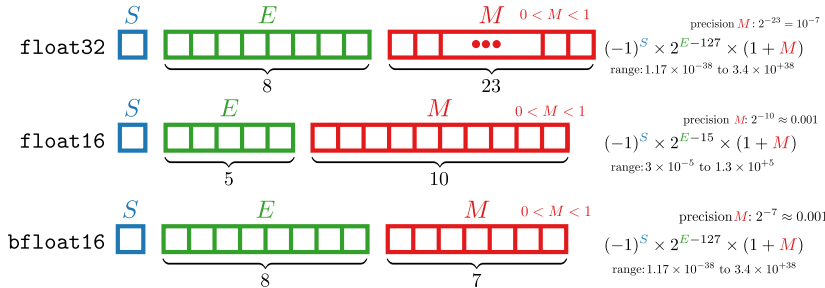
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- Quantizing neural networks
  - Quantization : Basics
    - Floating Point
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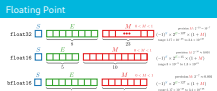
# Floating Point



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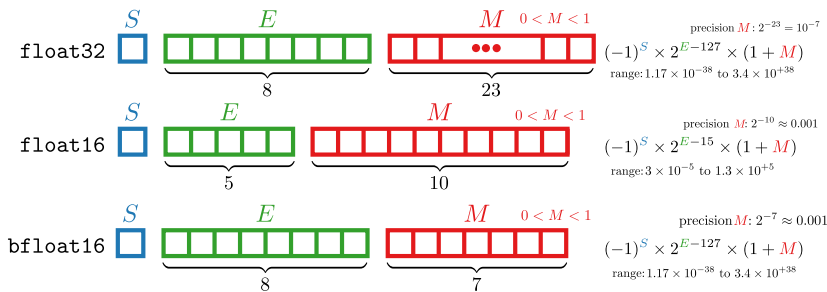
## Quantizing neural networks

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# Floating Point



## ■ To add two FP numbers:

- Shift  $M$  according to  $E$  (int shift  $n_E$  bits)
- Add  $M$  (int add  $n_M$  bits)
- Normalize ( $0 < M < 1$ )

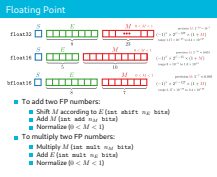
## ■ To multiply two FP numbers:

- Multiply  $M$  (int mult  $n_M$  bits)
- Add  $E$  (int mult  $n_E$  bits)
- Normalize ( $0 < M < 1$ )

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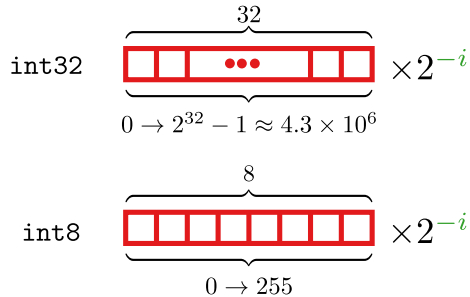
## Quantizing neural networks

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# Integers, fixed point



- Fixed point ( $-i$ )
- Short range
- Simple computation

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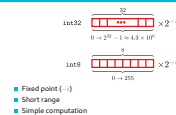
## Quantizing neural networks

### └ Quantization : Basics

#### └ Integers, Fixed Point

##### └ Integers, fixed point

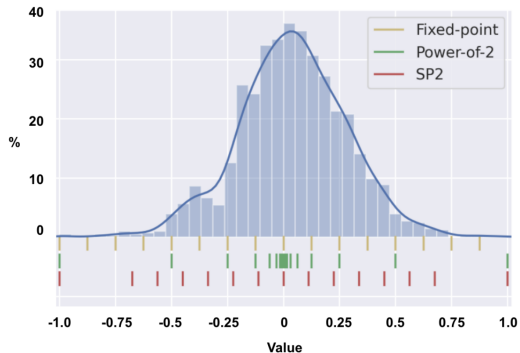
Integers, fixed point



On the other side, the main way to represent fractional numbers with integers is the fixed point representation.  $i$  is the number of fractional bits. The range of the representation is  $[-2^{i-1}, 2^{i-1} - 1]$ . The representation is simple, but the range is limited.



# Uniform and Non-Uniform Quantization



- Uniform quantization enables the use of integer on fixed-point hardware
- Non-uniform quantization requires a codebook lookup → not straightforward for standard hardware (CPU, GPU)

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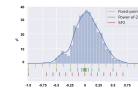
Quantizing neural networks

└ Quantization : Basics

└ Quantization

└ Uniform and Non-Uniform Quantization

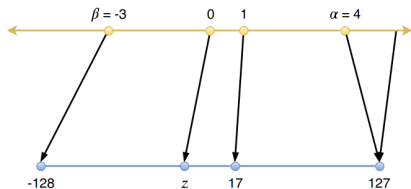
Uniform and Non-Uniform Quantization



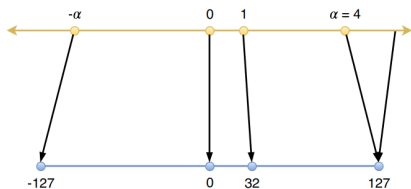
- Uniform quantization enables the use of integer on fixed-point hardware
- Non-uniform quantization requires a codebook lookup → not straightforward for standard hardware (CPU, GPU)

There are other ways to use integers to represent real numbers. The different values of the integers can represent arbitrary real numbers in a uniform or non-uniform way.

# Affine and Scale Quantization



(a) Affine quantization



(b) Scale quantization

- 2 kinds of uniform quantization
- Assymetric vs Symmetric

Wu, Hao, et al. "Integer quantization for deep learning inference: Principles and empirical evaluation." arXiv preprint arXiv:2004.09602 (2020).

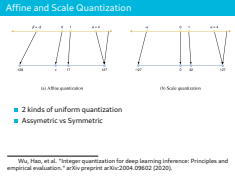
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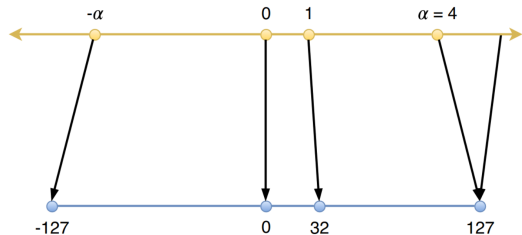
└ Quantization : Basics

└ Quantization

└ Affine and Scale Quantization



# Scale Quantization



$$\text{clip}(x, l, u) = \begin{cases} l, & x < l \\ x, & l \leq x \leq u \\ u, & x > u \end{cases}$$

$$s = \frac{2^{b-1} - 1}{\alpha}$$

$$x_q = \text{quantize}(x, b, s) = \text{clip}(\text{round}(s \cdot x), -2^{b-1} + 1, 2^{b-1} - 1)$$

$$\hat{x} = \text{dequantize}(x_q, s) = \frac{1}{s} x_q$$

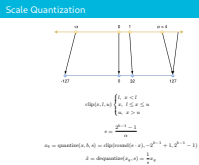
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## Quantizing neural networks

└ Quantization : Basics

└ Quantization

└ Scale Quantization



The upper arrow represents the real axis, the lower one represent the quantized axis. Scale quantization is a category of uniform quantization, where real number 0 is represented as the integer 0. It is also symmetric, as there is the same number of positive and negative values on the quantized axis. Only one parameter is then needed to define the quantization, which is the real number  $\alpha$  that corresponds to the maximum integer value. The axes are then split uniformly in  $2^b - 1$  intervals. The *quantize* and *dequantize* functions are defined accordingly.

$$y_{ij} = \sum_{k=1}^p x_{ik} \cdot w_{kj} \approx$$

$$\sum_{k=1}^p \text{dequantize}(x_{q,ik}, s_{q,ik}) \cdot \text{dequantize}(w_{q,kj}, s_{w,kj}) =$$

$$\sum_{k=1}^p \frac{1}{s_{x,ik}} x_{q,ik} \cdot \frac{1}{s_{w,kj}} w_{q,kj}$$

And, in order to use integer multiplication, the scaling factor  $s$  must be independent of  $k$  :

$$\frac{1}{s_{x,i} \cdot s_{w,j}} \sum_{k=1}^p x_{q,ik} \cdot w_{q,kj}$$

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## Quantizing neural networks

### └ Quantization : Basics

#### └ Quantization

#### └ Scale Quantization

$$w_{ij} = \sum_{k=1}^p x_{ik} \cdot w_{kj} \approx \sum_{k=1}^p \text{dequantize}(x_{q,ik}, s_{q,ik}) \cdot \text{dequantize}(w_{q,kj}, s_{w,kj}) =$$

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This is the important part about scale quantization. When the scaling factor is constant over a part of the data (weights and activations), the scaling to go back to floating point values can be done a posteriori, after all of the computations have been done with integers.

## 1 Objectives

## 2 Quantization : Basics

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## 3 Quantization : Neural Networks

- Quantization Post Training
- Quantization Aware Training

## 4 Quantization in Pytorch

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Quantizing neural networks

└─ Quantization : Neural Networks

└─ Plan

Plan

■ Objectives

■ Quantization : Basics

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■ Quantization : Neural Networks

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■ Quantization in Pytorch

# Estimating the impact of quantization

## Impact on weights

Signal-to-Quantization Noise Ratio metric.

$W_k$ : weight number index  $k$  in the set.

$\hat{W}_k$ : quantized weight index  $k$  in the set.

$L$ : number of element in the set.

$$\text{SQNR}(\hat{W}) = \frac{\sum_{k=0}^{L-1} |W_k|^2}{\underbrace{\sum_{k=0}^{L-1} |W_k - \hat{W}_k|^2}_{\text{quantization error}}}$$

Generally expressed in dB:  $\text{SQNR}_{\text{dB}} = 10\log_{10}(\text{SQNR})$

## Impact on network performance

Directly measure the accuracy of the network. For instance: Top-1 or Top-5 errors.

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## Quantizing neural networks

- Quantization : Neural Networks

- Quantization Post Training

- Estimating the impact of quantization

### Impact on weights

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### Impact on network performance

Directly measure the accuracy of the network. For instance: Top-1 or Top-5 errors.

The SQNR metric is used to estimate the impact of quantization on the weights. It is a measure of the noise introduced by the quantization. The lower the SQNR, the more noise is introduced. The noise is measured in dB. The higher the SQNR, the better the quantization. SQNR can be used to evaluate the impact of quantization on the weights. The advantage of post training quantization is that it is easy to implement. The disadvantage is that it is not possible to reduce the number of bits without a loss of accuracy.

# Quantization Post Training : Weights

Start by considering weights with a few number of bits  $n$ .  
Quantize → measure accuracy → increase the number of bits and repeat.

## Different weight sets can be considered

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation → better accuracy.  
Depends on how weights are stored in hardware (parallel accesses).

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## Quantizing neural networks

- └ Quantization : Neural Networks
  - └ Quantization Post Training
    - └ Quantization Post Training : Weights

- Whole network,
- per layer,
- per neuron.

## Quantization Post Training : Activation

Start by considering **activations** with a few number of bits  $n$ .

Quantize  $\rightarrow$  measure accuracy  $\rightarrow$  increase the number of bits and repeat.

## Also different strategies

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation  $\rightarrow$  better accuracy.

Depends on how **activations** are stored (parallel accesses).

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# Quantizing neural networks

## Quantization : Neural Networks

- Quantization Post Training

- Quantization Post Training : Activation



# Quantization Aware Training

- Quantize Forward
- Quantize Backward & Forward
- Weights
- Weights & Activations

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## Quantizing neural networks

## Quantization : Neural Networks

- Quantization Aware Training

- Quantization Aware Training

# Quantization Aware Training

- Quantize Forward
- Quantize Backward & Forward
- Weights
- Weights & Activations

- Quantization Aware Techniques yield way better accuracy
- Especially for extremely low-bit precision (2-3-4 bit precision)

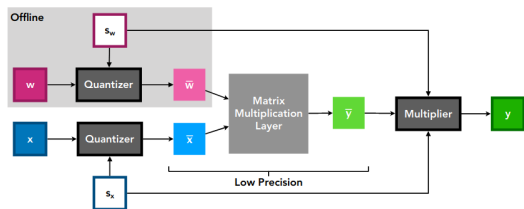
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- Quantizing neural networks
  - Quantization : Neural Networks
    - Quantization Aware Training
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- Quantize Forward
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# Learned Step Size Quantization



- $s$  quantizer step size
- $Q_P$  and  $Q_N$ , the number of positive and negative quantization levels

$$\bar{v} = \lfloor \text{clip}(v/s, -Q_N, Q_P) \rfloor, \quad (1)$$

$$\hat{v} = \bar{v} \times s. \quad (2)$$

- $s$  is learned with :

$$\frac{\partial \hat{v}}{\partial s} = \begin{cases} -v/s + \lfloor v/s \rfloor & \text{if } -Q_N < v/s < Q_P \\ -Q_N & \text{if } v/s \leq -Q_N \\ Q_P & \text{if } v/s \geq Q_P \end{cases} \quad (3)$$

LSQ is a technique that allows to learn the step size  $s$  of the quantizer. The step size is learned with a gradient descent algorithm. Weights quantization can be done offline, which means that no additional computation is required during inference.

# Quantization while Learning - Binary Connect

**Algorithm 1** SGD training with BinaryConnect.  $C$  is the cost function for minibatch and the functions  $\text{binarize}(w)$  and  $\text{clip}(w)$  specify how to binarize and clip weights.  $L$  is the number of layers.

**Require:** a minibatch of (inputs, targets), previous parameters  $w_{t-1}$  (weights) and  $b_{t-1}$  (biases), and learning rate  $\eta$ .

**Ensure:** updated parameters  $w_t$  and  $b_t$ .

**1. Forward propagation:**

$w_b \leftarrow \text{binarize}(w_{t-1})$   
For  $k = 1$  to  $L$ , compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$

**2. Backward propagation:**

Initialize output layer's activations gradient  $\frac{\partial C}{\partial a_L}$   
For  $k = L$  to 2, compute  $\frac{\partial C}{\partial a_{k-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $w_b$

**3. Parameter update:**

Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{\partial b_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$   
 $w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$   
 $b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$

Courbariaux, Matthieu, Yoshua Bengio, and Jean-Pierre David.  
"Binaryconnect: Training deep neural networks with binary weights during propagations." Advances in neural information processing systems. 2015.  
<https://arxiv.org/pdf/1511.00363.pdf>

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## Quantizing neural networks

└ Quantization : Neural Networks

└ Quantization Aware Training

└ Quantization while Learning - Binary Connect

Binary Connect is a technique that allows to train a network with binary weights. The weights are binarized during the forward pass and then the gradients are backpropagated. The weights are updated with a gradient descent algorithm. During training, the weights are binarized, which means that the network is not differentiable. To overcome this issue, Straight Through Estimator (STE) is used. It consists of replacing the gradient of the binarization function by the gradient of the identity function.

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 $w_b \leftarrow \text{binarize}(w_{t-1})$   
For  $k = 1$  to  $L$ , compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$

**2. Backward propagation:**  
Initialize output layer's activations gradient  $\frac{\partial C}{\partial a_L}$   
For  $k = L$  to 2, compute  $\frac{\partial C}{\partial a_{k-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $w_b$

**3. Parameter update:**  
Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{\partial b_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$   
 $w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$   
 $b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$

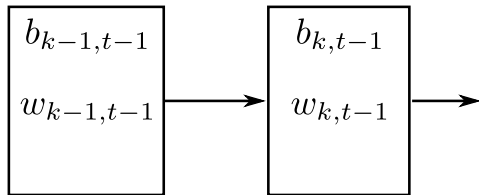
Courbariaux, Matthieu, Yoshua Bengio, and Jean-Pierre David.  
"Binaryconnect: Training deep neural networks with binary weights during propagations." Advances in neural information processing systems. 2015.  
<https://arxiv.org/pdf/1511.00363.pdf>

# Quantization while Learning - Binary Connect

## 1. Forward propagation:

$w_b \leftarrow \text{binarize}(w_{t-1})$

For  $k = 1$  to  $L$ , compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$



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## Quantizing neural networks

- Quantization : Neural Networks

- Quantization Aware Training

- Quantization while Learning - Binary Connect

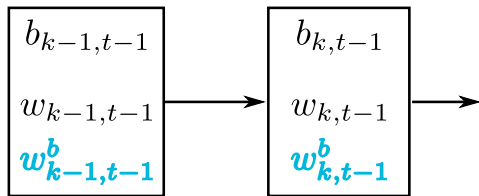


1. Forward propagation:  
 $a_k \leftarrow \text{binarize}(a_{k-1})$   
For  $t = 1$  to  $T$ , compute  $a_t$  knowing  $a_{t-1}$ ,  $w_b$  and  $b_{t-1}$

# Quantization while Learning - Binary Connect

## 1. Forward propagation:

$$w_b \leftarrow \text{binarize}(w_{t-1})$$



$$w^b = \begin{cases} +1, & \text{if } w \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

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## Quantizing neural networks

- Quantization : Neural Networks

- Quantization Aware Training

- Quantization while Learning - Binary Connect

1. Forward propagation:

$$w_b \leftarrow \text{binarize}(w_{t-1})$$



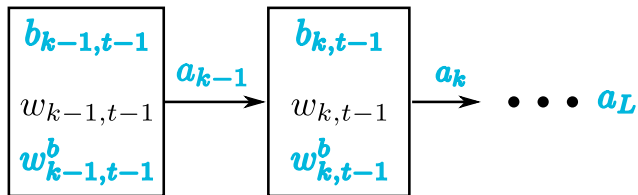
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# Quantization while Learning - Binary Connect

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For  $k = 1$  to  $L$ , compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$



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## Quantizing neural networks

- Quantization : Neural Networks

- Quantization Aware Training

- Quantization while Learning - Binary Connect



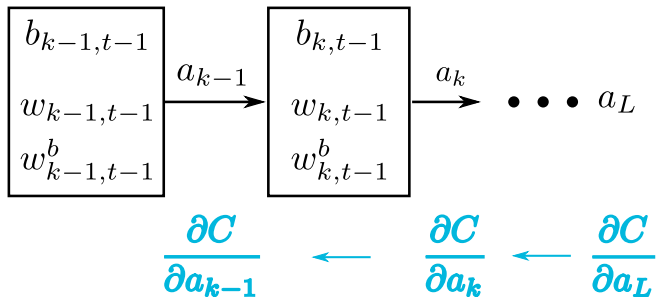
1. Forward propagation:  
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For  $t = 1$  to  $T$ , compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$

# Quantization while Learning - Binary Connect

## 2. Backward propagation:

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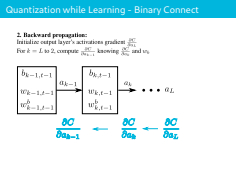
2024-02-06

## Quantizing neural networks

- Quantization : Neural Networks

- Quantization Aware Training

- Quantization while Learning - Binary Connect





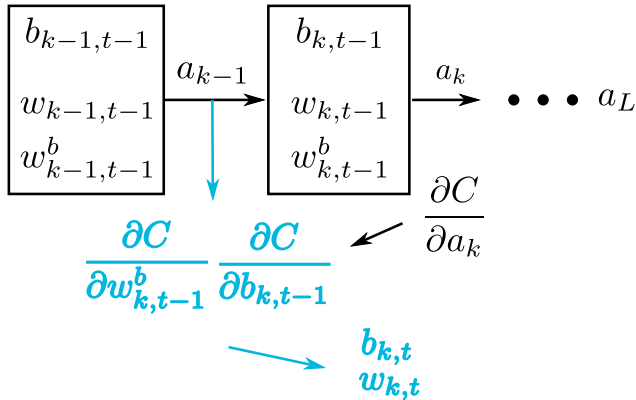
# Quantization while Learning - Binary Connect

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Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{\partial b_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$

$$w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$$

$$b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$$



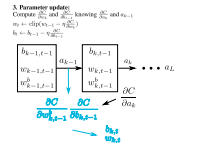
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## Quantizing neural networks

└ Quantization : Neural Networks

└ Quantization Aware Training

└ Quantization while Learning - Binary Connect



# Binarization : Stochastic vs Deterministic

## ■ Deterministic

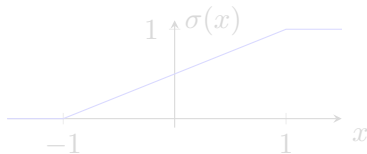
$$w_b = \begin{cases} +1, & \text{if } w \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

## ■ Stochastic

$$w_b = \begin{cases} +1, & \text{with probability } p = \sigma(w) \\ -1, & \text{with probability } 1 - p \end{cases}$$

avec

$$\sigma(x) = \text{clip}\left(\frac{x+1}{2}, 0, 1\right) = \max\left(0, \min\left(1, \frac{x+1}{2}\right)\right)$$



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## Quantizing neural networks

### └ Quantization : Neural Networks

### └ Quantization Aware Training

### └ Binarization : Stochastic vs Deterministic

Binarization : Stochastic vs Deterministic

#### ■ Deterministic

$$w_b = \begin{cases} +1, & \text{if } w \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

#### ■ Stochastic

$$w_b = \begin{cases} +1, & \text{with probability } p = \sigma(w) \\ -1, & \text{with probability } 1 - p \end{cases}$$

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Two different ways to binarize the weights exist. The first one is deterministic, which means that the weights are binarized with a threshold. The second one is stochastic, which means that the weights are binarized with a probability.

# Binarization : Stochastic vs Deterministic

## ■ Deterministic

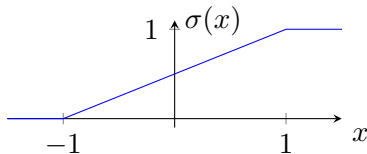
$$w_b = \begin{cases} +1, & \text{if } w \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

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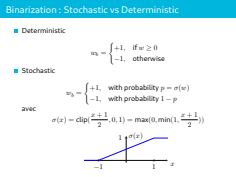
2024-02-06

## Quantizing neural networks

### └ Quantization : Neural Networks

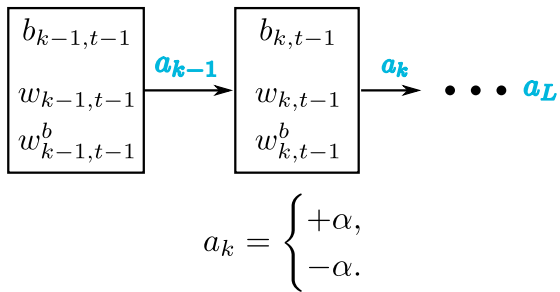
### └ Quantization Aware Training

### └ Binarization : Stochastic vs Deterministic



Two different ways to binarize the weights exist. The first one is deterministic, which means that the weights are binarized with a threshold. The second one is stochastic, which means that the weights are binarized with a probability.

# Quantization while Learning - Binary Weighted network (XNOR-NET)

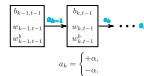


Rastegari, Mohammad, et al. "Xnor-net: Imagenet classification using binary convolutional neural networks." European conference on computer vision. Springer, Cham, 2016. <https://arxiv.org/pdf/1603.05279.pdf>

2024-02-06

## Quantizing neural networks

- Quantization : Neural Networks
  - Quantization Aware Training
    - Quantization while Learning - Binary Weighted network (XNOR-NET)



# Quantization while Learning - Binary Weighted network (XNOR-NET)

	Network Variations	Operations used in Convolution	Memory Saving (Inference)	Computation Saving (Inference)	Accuracy on ImageNet (AlexNet)
Standard Convolution	<div>Real-Value Inputs 0.11 -0.21 ... -0.34 -0.25 0.61 ... 0.52</div> <div>Real-Value Weights 0.12 -1.2 ... 0.41 -0.1 0.5 ... 0.68</div>	$+, -, \times$	1x	1x	%56.7
Binary Weight	<div>Real-Value Inputs 0.11 -0.21 ... -0.34 -0.25 0.61 ... 0.52</div> <div>Binary Weights 1 -1 ... 1 -1 1 ... 1</div>	$+, -$	~32x	~2x	%56.8
BinaryWeight Binary Input (XNOR-Net)	<div>Binary Inputs 1 -1 ... -1 -1 1 ... 1</div> <div>Binary Weights 1 -1 ... 1 -1 1 ... 1</div>	XNOR , bitcount	~32x	~58x	%44.2

Rastegari, Mohammad, et al. "Xnor-net: Imagenet classification using binary convolutional neural networks." European conference on computer vision. Springer, Cham, 2016. <https://arxiv.org/pdf/1603.05279.pdf>

2024-02-06

- Quantizing neural networks
  - Quantization : Neural Networks
    - Quantization Aware Training
      - Quantization while Learning - Binary Weighted network (XNOR-NET)

Quantization while Learning - Binary Weighted network (XNOR-NET)

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- Floating Point
- Integers, Fixed Point
- Quantization

- Quantization Post Training
- Quantization Aware Training

## 4 Quantization in Pytorch

# Quantization in Pytorch

- 1 Dynamic Quantization
- 2 Static Quantization
- 3 Quantization Aware Training

https:  
//pytorch.org/blog/introduction-to-quantization-on-pytorch/

And for our need : [https://pytorch.org/tutorials/prototype/fx\\_graph\\_mode\\_ptq\\_static.html](https://pytorch.org/tutorials/prototype/fx_graph_mode_ptq_static.html)

Quantization in Pytorch is a new feature. It is still in beta version. A group of students from last year worked on this topic. They implemented a post quantization algorithm in order to quantize the network with 8-bit weights and activations.