

# Quantizing neural networks

## Efficient Deep Learning - Session 3



**IMT Atlantique**  
Bretagne-Pays de la Loire  
École Mines-Télécom

## Sessions

- 1 Intro Deep Learning,
- 2 Data Augmentation and Self Supervised Learning,
- 3 Quantization,
- 4 Pruning,
- 5 Factorization,
- 6 Distillation,
- 7 Embedded Software and Hardware for DL,
- 8 Presentations for challenge.

## Sessions

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- 3 **Quantization,**
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- 8 Presentations for challenge.

# Today's Summary

## 1 Objectives

## 2 Quantization : Basics

- Floating Point
- Integers, Fixed Point
- Quantization

## 3 Quantization : Neural Networks

- Quantization Post Training
- Quantization Aware Training

## 4 Quantization in Pytorch

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# Motivation

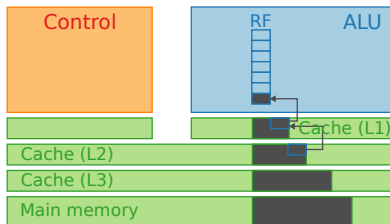
- 1 Reduce model size
  - Fewer bits → Reduced memory footprint
- 2 Decrease memory access
  - GPU & CPU : reduce Cache usage
- 3 Computational complexity

**Table:** Performance on the ImageNet dataset and complexities

Network	Alexnet	Inceptionv1	ResNet50	ResNet152
Top-5 error	16.4%	6.7%	5.25%	4.49%
Num. Weights	61M	7M	25.5M	63.75M
Num. MAC	724M	1.43G	3.9G	11.31G

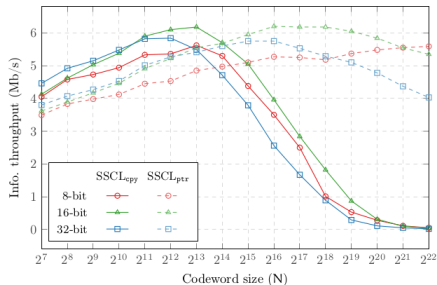
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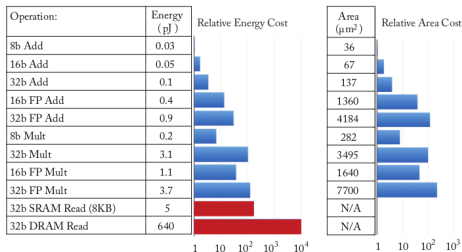
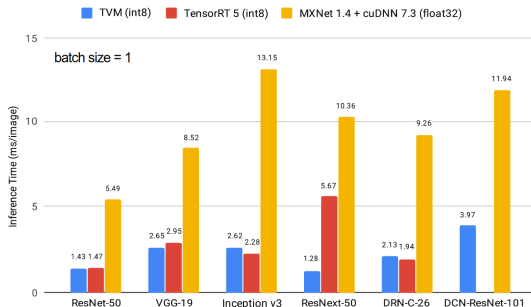


Figure 7.1: The area and energy cost for additions and multiplications at different precision, and memory accesses in a 45 nm process. The area and energy scale different for multiplication and addition. The energy consumption of data movement (red) is significantly higher than arithmetic operations (blue). (Figure adapted from [121].)

# Motivation

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From : <https://github.com/vinx13/tvm-cuda-int8-benchmark/>

## 1 Objectives

## 2 Quantization : Basics

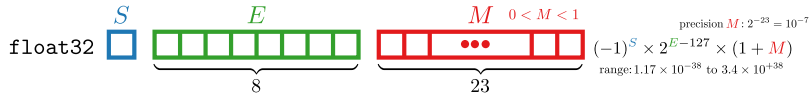
- Floating Point
- Integers, Fixed Point
- Quantization

## 3 Quantization : Neural Networks

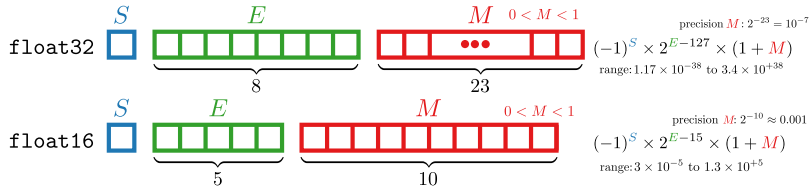
- Quantization Post Training
- Quantization Aware Training

## 4 Quantization in Pytorch

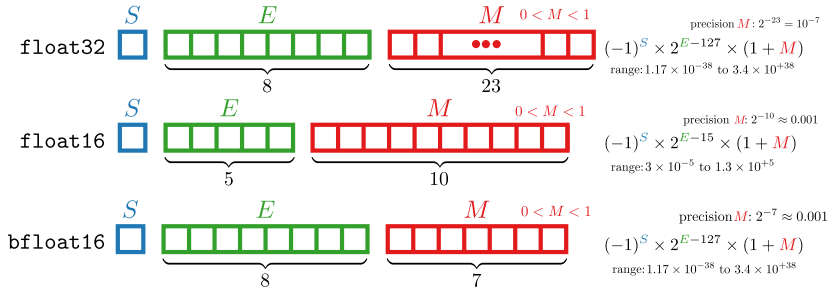
# Floating Point



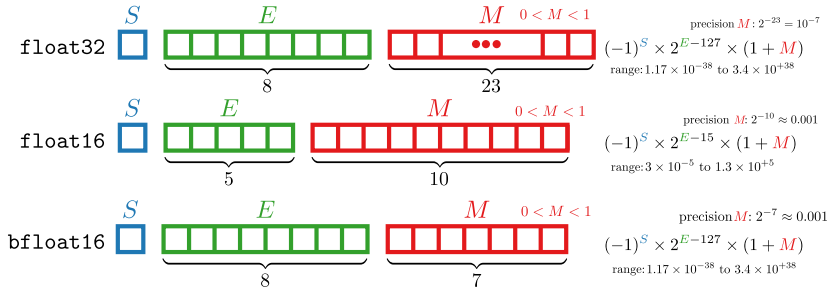
# Floating Point



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# Floating Point



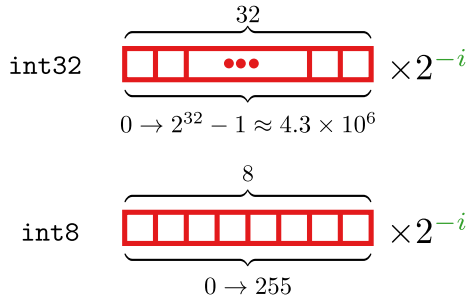
## ■ To add two FP numbers:

- Shift  $M$  according to  $E$  (int shift  $n_E$  bits)
- Add  $M$  (int add  $n_M$  bits)
- Normalize ( $0 < M < 1$ )

## ■ To multiply two FP numbers:

- Multiply  $M$  (int mult  $n_M$  bits)
- Add  $E$  (int mult  $n_E$  bits)
- Normalize ( $0 < M < 1$ )

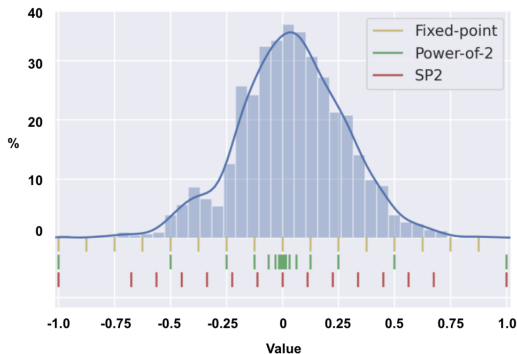
# Integers, fixed point



- Fixed point ( $-i$ )
- Short range
- Simple computation

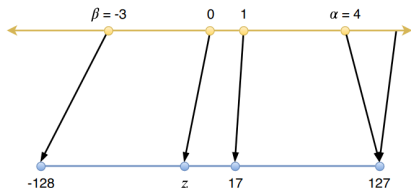


# Uniform and Non-Uniform Quantization

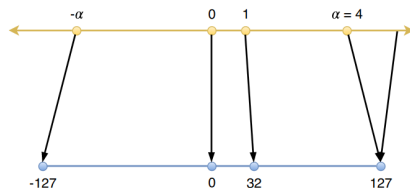


- Uniform quantization enables the use of integer on fixed-point hardware
- Non-uniform quantization requires a codebook lookup → not straightforward for standard hardware (CPU, GPU)

# Affine and Scale Quantization



(a) Affine quantization

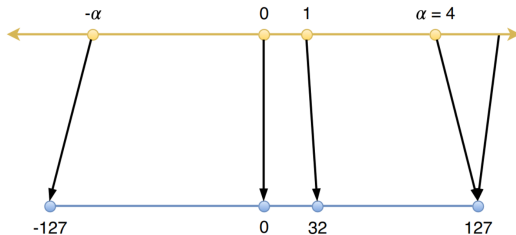


(b) Scale quantization

- 2 kinds of uniform quantization
- Assymmetric vs Symmetric

Wu, Hao, et al. "Integer quantization for deep learning inference: Principles and empirical evaluation." arXiv preprint arXiv:2004.09602 (2020).

# Scale Quantization



$$\text{clip}(x, l, u) = \begin{cases} l, & x < l \\ x, & l \leq x \leq u \\ u, & x > u \end{cases}$$

$$s = \frac{2^{b-1} - 1}{\alpha}$$

$$x_q = \text{quantize}(x, b, s) = \text{clip}(\text{round}(s \cdot x), -2^{b-1} + 1, 2^{b-1} - 1)$$

$$\hat{x} = \text{dequantize}(x_q, s) = \frac{1}{s} x_q$$

$$y_{ij} = \sum_{k=1}^p x_{ik} \cdot w_{kj} \approx$$

$$\sum_{k=1}^p \text{dequantize}(x_{q,ik}, s_{q,ik}) \cdot \text{dequantize}(w_{q,kj}, s_{w,kj}) =$$

$$\sum_{k=1}^p \frac{1}{s_{x,ik}} x_{q,ik} \cdot \frac{1}{s_{w,kj}} w_{q,kj}$$

And, in order to use integer multiplication, the scaling factor  $s$  must be independent of  $k$  :

$$\frac{1}{s_{x,i} \cdot s_{w,j}} \sum_{k=1}^p x_{q,ik} \cdot w_{q,kj}$$

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# Estimating the impact of quantization

## Impact on weights

Signal-to-Quantization Noise Ratio metric.

$W_k$ : weight number index  $k$  in the set.

$\hat{W}_k$ : quantized weight index  $k$  in the set.

$L$ : number of element in the set.

$$\text{SQNR}(\hat{W}) = \frac{\sum_{k=0}^{L-1} |W_k|^2}{\sum_{k=0}^{L-1} \underbrace{|W_k - \hat{W}_k|^2}_{\text{quantization error}}}$$

Generally expressed in dB:  $\text{SQNR}_{\text{dB}} = 10\log_{10}(\text{SQNR})$

## Impact on network performance

Directly measure the accuracy of the network. For instance: Top-1 or Top-5 errors.

# Quantization Post Training : Weights

Start by considering weights with a few number of bits  $n$ .

Quantize → measure accuracy → increase the number of bits and repeat.

## Different weight sets can be considered

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation → better accuracy.

Depends on how weights are stored in hardware (parallel accesses).

# Quantization Post Training : Activation

Start by considering **activations** with a few number of bits  $n$ .  
Quantize → measure accuracy → increase the number of bits and repeat.

## Also different strategies

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation → better accuracy.  
Depends on how **activations** are stored (parallel accesses).



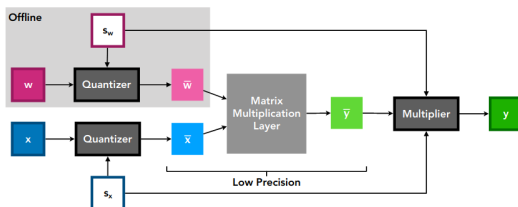
# Quantization Aware Training

- Quantize Forward
- Quantize Backward & Forward
- Weights
- Weights & Activations

# Quantization Aware Training

- Quantize Forward
  - Quantize Backward & Forward
  - Weights
  - Weights & Activations
- 
- Quantization Aware Techniques yield way better accuracy
  - Especially for extremely low-bit precision (2-3-4 bit precision)

# Learned Step Size Quantization



- $s$  quantizer step size
- $Q_P$  and  $Q_N$ , the number of positive and negative quantization levels

$$\bar{v} = \lfloor \text{clip}(v/s, -Q_N, Q_P) \rfloor, \quad (1)$$

$$\hat{v} = \bar{v} \times s. \quad (2)$$

- $s$  is learned with :

$$\frac{\partial \hat{v}}{\partial s} = \begin{cases} -v/s + \lfloor v/s \rfloor & \text{if } -Q_N < v/s < Q_P \\ -Q_N & \text{if } v/s \leq -Q_N \\ Q_P & \text{if } v/s \geq Q_P \end{cases} \quad (3)$$

# Quantization while Learning - Binary Connect

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**Algorithm 1** SGD training with BinaryConnect.  $C$  is the cost function for minibatch and the functions  $\text{binarize}(w)$  and  $\text{clip}(w)$  specify how to binarize and clip weights.  $L$  is the number of layers.

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**Require:** a minibatch of (inputs, targets), previous parameters  $w_{t-1}$  (weights) and  $b_{t-1}$  (biases), and learning rate  $\eta$ .

**Ensure:** updated parameters  $w_t$  and  $b_t$ .

**1. Forward propagation:**

$$w_b \leftarrow \text{binarize}(w_{t-1})$$

For  $k = 1$  to  $L$ , compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$

**2. Backward propagation:**

Initialize output layer's activations gradient  $\frac{\partial C}{\partial a_L}$

For  $k = L$  to 2, compute  $\frac{\partial C}{\partial a_{k-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $w_b$

**3. Parameter update:**

Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{\partial b_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$

$$w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$$

$$b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$$

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Courbariaux, Matthieu, Yoshua Bengio, and Jean-Pierre David.

"Binaryconnect: Training deep neural networks with binary weights during propagations." Advances in neural information processing systems. 2015.

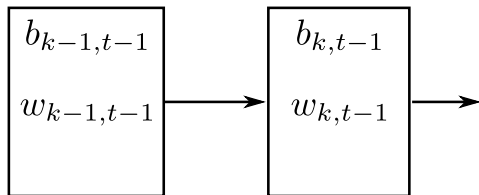
<https://arxiv.org/pdf/1511.00363.pdf>

# Quantization while Learning - Binary Connect

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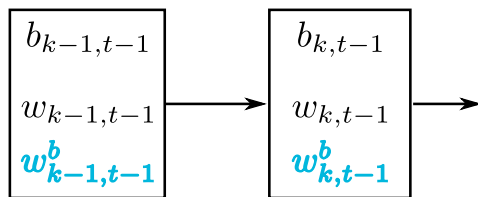
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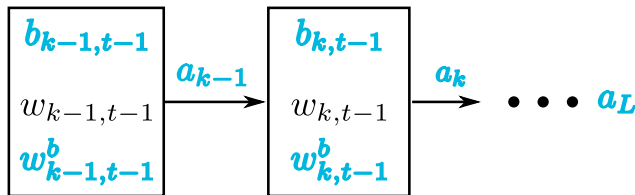
$$w^b = \begin{cases} +1, & \text{if } w \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

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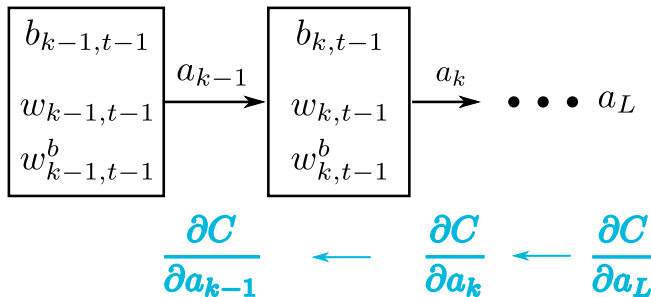


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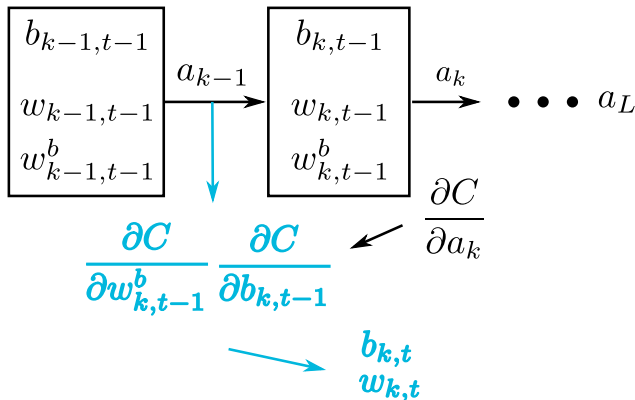
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## 3. Parameter update:

Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{\partial b_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$

$$w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$$

$$b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$$



# Binarization : Stochastic vs Deterministic

## ■ Deterministic

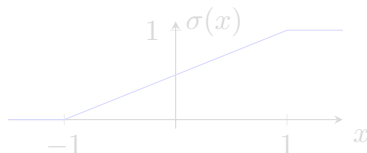
$$w_b = \begin{cases} +1, & \text{if } w \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

## ■ Stochastic

$$w_b = \begin{cases} +1, & \text{with probability } p = \sigma(w) \\ -1, & \text{with probability } 1 - p \end{cases}$$

avec

$$\sigma(x) = \text{clip}\left(\frac{x+1}{2}, 0, 1\right) = \max\left(0, \min\left(1, \frac{x+1}{2}\right)\right)$$



# Binarization : Stochastic vs Deterministic

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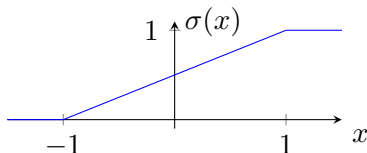
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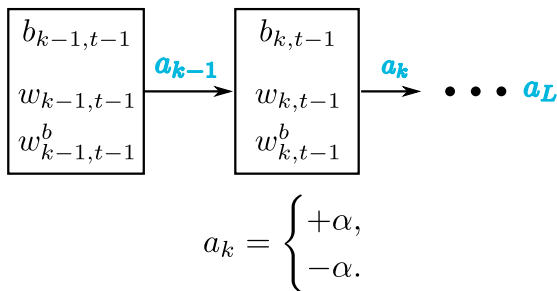
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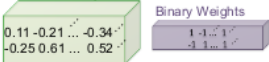



# Quantization while Learning - Binary Weighted network (XNOR-NET)



Rastegari, Mohammad, et al. "Xnor-net: Imagenet classification using binary convolutional neural networks." European conference on computer vision. Springer, Cham, 2016. <https://arxiv.org/pdf/1603.05279.pdf>

# Quantization while Learning - Binary Weighted network (XNOR-NET)

	Network Variations	Operations used in Convolution	Memory Saving (Inference)	Computation Saving (Inference)	Accuracy on ImageNet (AlexNet)
Standard Convolution		$+, -, \times$	1x	1x	%56.7
Binary Weight		$+, -$	$\sim 32x$	$\sim 2x$	%56.8
BinaryWeight Binary Input (XNOR-Net)		XNOR, bitcount	$\sim 32x$	$\sim 58x$	%44.2

Rastegari, Mohammad, et al. "Xnor-net: Imagenet classification using binary convolutional neural networks." European conference on computer vision. Springer, Cham, 2016. <https://arxiv.org/pdf/1603.05279.pdf>

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# Quantization in Pytorch

- 1 Dynamic Quantization
- 2 Static Quantization
- 3 Quantization Aware Training

`https://pytorch.org/blog/introduction-to-quantization-on-pytorch/`

And for our need : `https://pytorch.org/tutorials/prototype/fx_graph_mode_ptq_static.html`