## .04-03 .0

# ღ Quantizing neural networks





# Quantizing neural networks

#### Efficient Deep Learning - Session 3



# Course organisation

#### Sessions

- 1 Intro Deep Learning,
- 2 Data Augmentation and Self Supervised Learning,
- 3 Quantization,
- 4 Pruning,
- 5 Factorization,
- 6 Distillation,
- **T** Embedded Software and Hardware for DL,
- Presentations for challenge.

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-Course organisation

Course organisation

Intro Deep Learning,

Data Augmentation and Self Supervised Learning,

- Quantization,
  Pruning,
- Factorization,
  Distillation,
- Distillation,
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- 8 Presentations for challenge.

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-Course organisation

Intro Deep Learning,

Data Augmentation and Self Supervised Learning,

Pruning.

Factorization Distillation.

Embedded Software and Hardware for DL, Presentations for challenge.

# Today's Summary

- 1 Objectives
- 2 Quantization : Basics
  - Floating Point
  - Integers, Fixed Point
  - Quantization
- 3 Quantization: Neural Networks
  - Quantization Post Training
  - Quantization Aware Training
- 4 Quantization in Pytorch

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— Today's Summary

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■ Objectives

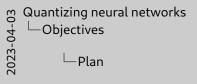
Guantization : Basics
• Floating Point
• Integers, Fixed Point
• Quantization

Quantization : Neural Networks
 Quantization Post Training
 Quantization Aware Training

Quantization in Pytorch

# Plan

- Objectives
- 2 Quantization : Basic
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- Reduce model size
  - Fewer bits → Reduced memory footprint
- Decrease memory access
  - GPU & CPU : reduce Cache usage
- Computational complexity

#### Table: Performance on the ImageNet dataset and complexities

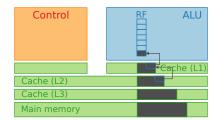
Network	Alexnet	Inceptionv1	ResNet50	ResNet152
Top-5 error	16.4%	6.7%	5.25%	4.49%
Num. Weights	61M	7M	25.5M	63.75M
Num. MAC	724M	1.43G	3.9G	11.31G

04-03	Quantizing neural networks  Objectives		
2023-	—Motivation		



Reducing the memory footprint can be beneficial particularly in the case of embedded systems. On an ESP32, which is commonly used for low power applications, the memory footprint is limited to 520KB.

- 1 Reduce model size
  - Fewer bits → Reduced memory footprint
- Decrease memory access
  - GPU & CPU : reduce Cache usage
- Computational complexity

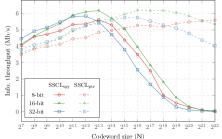


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Objectives
Motivation



The data bandwidth between the cache and the GPU or the CPU is limited. This can be the bottleneck of the implementation, limiting the throughput or the latency. Therefore, reducing the cache usage is also beneficial for the performance of neural network processing.

- 1 Reduce model size
  - Fewer bits → Reduced memory footprint
- Decrease memory access
  - GPU & CPU : reduce Cache usage
- Computational complexity







This is an example outside of the scope of Deep Learning. This is a simulation of a channel decoder inside a communication system. On the x-axis is the size of the codeword, which is the size of the main array of data that is processed by the algorithm. On the y-axis is the throughput of the system. The algorithm "SSCLcpy" is there an example of a cache issue, where the array becomes too big to fit in the first level of caches, with a dramatic decrease of the throughput after a certain array size.

- 1 Reduce model size
  - Fewer bits → Reduced memory footprint
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- Computational complexity

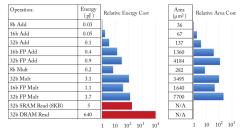
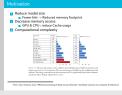


Figure 7.1: The area and energy cost for additions and multiplications at different precision, and memory accesses in a 45 nm process. The area and energy scale different for multiplication and addition. The energy consumption of data movement (red) is significantly higher than arithmetic operations (blue). (Figure adapted from [121].)

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Correction

—Objectives

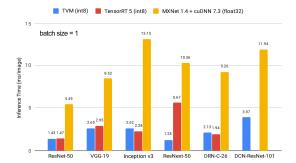
—Motivation



These are the relative energy cost of doing different operations on a 45 nm ASIC. The energy cost of floating point operations is higher than the energy cost of integer operations. It takes also more space on the chip, which means that there is a higher parallelization potential with fixed point operations than with floating point ones.

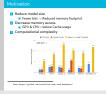
IMT-Atlantique Quantizing neural networks 4/20

- 1 Reduce model size
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  - GPU & CPU : reduce Cache usage
- Computational complexity



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Cobjectives

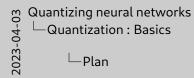
Motivation



This is a concrete example of the benefits of quantization on the inference time of different neural networks, on an NVIDIA GTX 1080, using different framewoks. The inference time is measured in milliseconds. Different frameworks are used, MXNet and cuDNN for the floating point implementation, and TVM and tensorRT for the fixed point implementation. For each neural network, inference time is lower with int8 than with float32.

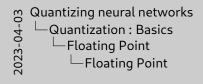
# Plan

- 1 Objectives
- 2 Quantization : Basics
  - Floating Point
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- **3** Quantization: Neural Networks
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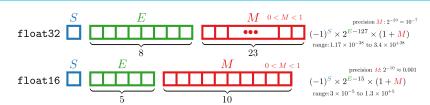








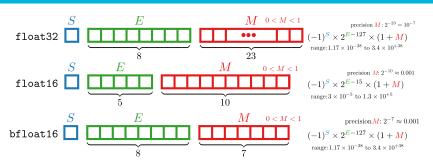






Quantizing neural networks
Quantization: Basics
Floating Point
Floating Point





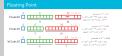


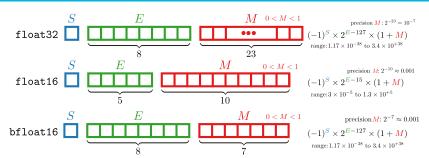
Quantizing neural networks

Quantization: Basics

Floating Point

Floating Point





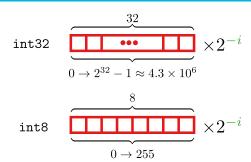
- To add two FP numbers:
  - Shift M according to E (int shift  $n_E$  bits)
  - Add M (int add  $n_M$  bits)
  - Normalize (0 < M < 1)
- To multiply two FP numbers:
  - Multiply M (int mult  $n_M$  bits)
  - Add E (int mult  $n_E$  bits)
  - Normalize (0 < M < 1)



Quantizing neural networks
Quantization: Basics
Floating Point
Floating Point

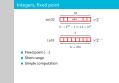


# Integers, fixed point



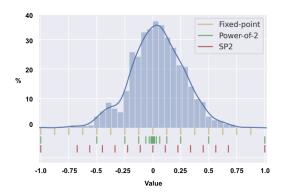
- Fixed point (-i)
- Short range
- Simple computation

Quantizing neural networks
Quantization: Basics
Integers, Fixed Point
Integers, fixed point

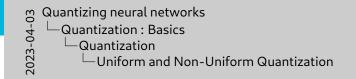


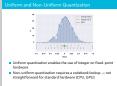
On the other side, the main way to represent fractional numbers with integers is the fixed point representation. i is the number of fractional bits. The range of the representation is  $[-2^{i-1}, 2^{i-1} - 1]$ . The representation is simple, but the range is limited.

#### Uniform and Non-Uniform Quantization



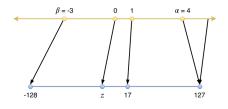
- Uniform quantization enables the use of integer on fixed-point hardware
- Non-uniform quantization requires a codebook lookup  $\rightarrow$  not straightforward for standard hardware (CPU, GPU)

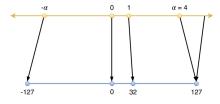




There are other ways to use integers to represent real numbers. The different values of the integers can represent arbitrary real numbers in a uniform or non-uniform way.

### Affine and Scale Quantization





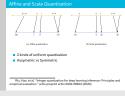
(a) Affine quantization

(b) Scale quantization

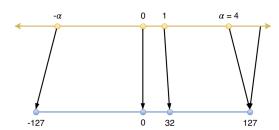
- 2 kinds of uniform quantization
- Assymetric vs Symmetric

Wu, Hao, et al. "Integer quantization for deep learning inference: Principles and empirical evaluation." arXiv preprint arXiv:2004.09602 (2020).

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Quantization : Basics
Quantization
Quantization
Affine and Scale Quantization



#### Scale Quantization



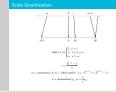
$$\operatorname{clip}(x,l,u) \begin{cases} l, & x < l \\ x, & l \le x \le u \\ u, & x > u \end{cases}$$

$$s = \frac{2^{b-1} - 1}{2^{b}}$$

$$x_q = \text{quantize}(x,b,s) = \text{clip}(\text{round}(s\cdot x), -2^{b-1}+1, 2^{b-1}-1)$$
 
$$\hat{x} = \text{dequantize}(x_q,s) = \frac{1}{s}x_q$$



Quantizing neural networks
Quantization: Basics
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Quantization
Scale Quantization



The upper arrow represents the real axis, the lower one represent the quantized axis. Scale quantization is a category of uniform quantization, where real number 0 is represented as the integer 0. It is also symmetric, as there is the same number of positive and negative values on the quantized axis. Only one parameter is then needed to define the quantization, which is the real number  $\alpha$  that corresponds to the maximum integer value. The axes are then split uniformly in  $2^b-1$  intervals. The quantize and dequantize functions are defined accordingly.

#### Scale Quantization

$$y_{ij} = \sum_{k=1}^{p} x_{ik} \cdot w_{kj} \approx$$

$$\sum_{i=1}^{p} \text{dequantize}(x_{q,ik}, s_{q,ik}) \cdot \text{dequantize}(w_{q,kj}, s_{w,kj}) =$$

$$\sum_{k=1}^{p} \frac{1}{s_{x,ik}} x_{q,ik} \cdot \frac{1}{s_{w,kj}} w_{q,kj}$$

And, in order to use integer multiplication, the scaling factor s must be independent of k:

$$\frac{1}{s_{x,i} \cdot s_{w,j}} \sum_{k=1}^{p} x_{q,ik} \cdot w_{q,kj}$$

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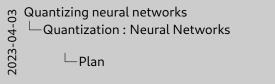
Quantizing neural networks
Quantization: Basics
Quantization
Quantization
Scale Quantization



This is the important part about scale quantization. When the scaling factor is constant over a part of the data (weights and activations), the scaling to go back to floating point values can be done a posteriori, after all of the computations have been done with integers.

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# Estimating the impact of quantization

#### Impact on weights

Signal-to-Quantization Noise Ratio metric.

 $W_k$ : weight number index k in the set.

 $\hat{W}_k$ : quantized weight index k in the set.

L: number of element in the set.

$$\mathsf{SQNR}(\hat{W}) = \frac{\sum_{k=0}^{L-1} |W_k|^2}{\sum_{k=0}^{L-1} \underbrace{|W_k - \hat{W}_k|^2}_{\mathsf{quantization error}}}$$

Generally expressed in dB:  $SQNR_{dB} = 10log_{10}(SQNR)$ 

#### Impact on network performance

Directly measure the accuracy of the network. For instance: Top-1 or Top-5 errors.

Quantizing neural networks
Quantization: Neural Networks
Quantization Post Training
Estimating the impact of quantization



The SQNR metric is used to estimate the impact of quantization on the weights. It is a measure of the noise introduced by the quantization. The lower the SQNR, the more noise is introduced. The noise is measured in dB. The higher the SQNR, the better the quantization. SQNR can be used to evaluate the impact of quantization on the weights. The advantage of post training quantization is that it is easy to implement. The disadvantage is that it is not possible to reduce the number of bits without a loss of accuracy.

## Quantization Post Training: Weights

Start by considering weights with a few number of bits n. Quantize  $\to$  measure accuracy  $\to$  increase the number of bits and repeat.

### Different weight sets can be considered

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation  $\rightarrow$  better accuracy.

Depends on how weights are stored in hardware (parallel accesses).

Quantizing neural networks
Quantization: Neural Networks
Quantization Post Training
Quantization Post Training: Weights

Start by considering weight his with a few number of bits ...

Start by considering weight with a few number of bits and report.

Conferent weight sets can be considered.

If Whole retwood,

If per layer,

If per never,

Fave sets geometation – better accuracy, Expends on the department of the closes of the consideration o

# **Quantization Post Training: Activation**

Start by considering activations with a few number of bits n. Quantize  $\to$  measure accuracy  $\to$  increase the number of bits and repeat.

## Also different strategies

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation  $\rightarrow$  better accuracy.

Depends on how activations are stored (parallel accesses).

Quantizing neural networks
Quantization : Neural Networks
Quantization Post Training
Quantization Post Training : Activation

Ouantization Post Training s'Activation

Start by considering substaines with a few number of bits a.
Guartize — measure accuracy — increase the number of bits and
repeat.

Also different strategies

# Whole network,
p per law,
pp are pure.

Experimentation — better accuracy.
Departed on the victualism are started (parallel accesses).

# Quantization Aware Training

- Quantize Forward
- Quantize Backward & Forward
- Weights
- Weights & Activations

Quantizing neural networks
—Quantization : Neural Networks
—Quantization Aware Training
—Quantization Aware Training

2 Quantization Aware Training

If Quantize Forward

If Quantize Backward & Forward

If Weights & Activations

# Quantization Aware Training

- Quantize Forward
- Quantize Backward & Forward
- Weights
- Weights & Activations

- Quantization Aware Techniques yield way better accuracy
- Especially for extremely low-bit precision (2-3-4 bit precision)

Quantizing neural networks
Quantization: Neural Networks
Quantization Aware Training
Quantization Aware Training

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Quantization Aware Training

Quantization Foward

Quantize Backward & Forward

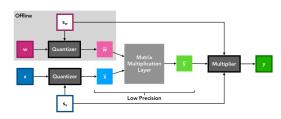
Weights & Activations

Weights & Activations
```

Quantization Aware Techniques yield way better accurac

■ Especially for extremely low-bit precision (2-3-4 bit precision)

## Learned Step Size Quantization



- s quantizer step size
- lacksquare  $Q_P$  and  $Q_N$ , the number of positive and negative quantization levels

$$\bar{v} = \lfloor clip(v/s, -Q_N, Q_P) \rceil, \tag{1}$$

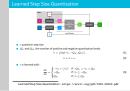
$$\hat{v} = \bar{v} \times s. \tag{2}$$

■ s is learned with:

$$\frac{\partial \hat{v}}{\partial s} = \begin{cases} -v/s + \lfloor v/s \rceil & \text{if } -Q_N < v/s < Q_P \\ -Q_N & \text{if } v/s \le -Q_N \\ Q_P & \text{if } v/s \ge Q_P \end{cases}$$
(3)

Learned Step Size Quantization - https://arxiv.org/pdf/1902.08153.pdf

Quantizing neural networks
Quantization: Neural Networks
Quantization Aware Training
Learned Step Size Quantization



LSQ is a technique that allows to learn the step size s of the quantizer. The step size is learned with a gradient descent algorithm. Weights quantization can be done offline, which means that no additional computation is required during inference.

**Algorithm 1** SGD training with BinaryConnect. C is the cost function for minibatch and the functions binarize(w) and clip(w) specify how to binarize and clip weights. L is the number of layers.

**Require:** a minibatch of (inputs, targets), previous parameters  $w_{t-1}$  (weights) and  $b_{t-1}$  (biases), and learning rate  $\eta$ .

**Ensure:** updated parameters  $w_t$  and  $b_t$ .

#### 1. Forward propagation:

$$w_b \leftarrow \text{binarize}(w_{t-1})$$

For k = 1 to L, compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$ 

#### 2. Backward propagation:

Initialize output layer's activations gradient  $\frac{\partial C}{\partial a_{r}}$ 

For k = L to 2, compute  $\frac{\partial C}{\partial a_{k-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $w_b$ 

#### 3. Parameter update:

Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{db_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$ 

$$w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$$

$$b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$$

Courbariaux, Matthieu, Yoshua Bengio, and Jean-Pierre David.
"Binaryconnect: Training deep neural networks with binary weights during propagations." Advances in neural information processing systems. 2015. https://arxiv.org/pdf/1511.00363.pdf

☐ Quantiza ☐ Quant Binary Connect

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Quantizing neural networks

Quantization: Neural Networks

Quantization Aware Training

Quantization while Learning - Binary Connect

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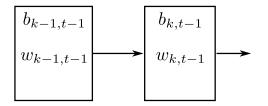
Courbariaux, Matthieu, Yoshua Bengio, and Jean-Pierre David.
"Binaryconnect: Training deep neural networks with binary weights during propagations." Advances in neural information processing systems. 2015. https://arxiv.org/pdf/1511.00363.pdf

Binary Connect is a technique that allows to train a network with binary weights. The weights are binarized during the forward pass and then the gradients are backpropagated. The weights are updated with a gradient descent algorithm. During training, the weights are binarized, which means that the network is not differentiable. To overcome this issue, Straight Through Estimator (STE) is used. It consists of replacing the gradient of the binarization function by the gradient of the identity function.

#### 1. Forward propagation:

 $w_b \leftarrow \text{binarize}(w_{t-1})$ 

For k = 1 to L, compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$ 



Quantizing neural networks

Quantization: Neural Networks
Quantization Aware Training
Quantization while Learning - Binary Connect

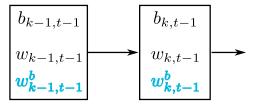
Quantization white Learning = Binary

I. Forward propagation:  $w_i = b \text{ total}(w_{i+1}, w_i)$   $w_i = b \text{ total}(w_{i+1}, w_i)$   $w_i = b \text{ total}(w_{i+1}, w_i)$   $w_{i+1} = b \text{ total}(w_{i+1}, w_i)$ 

-40

#### 1. Forward propagation:

$$w_b \leftarrow \text{binarize}(w_{t-1})$$



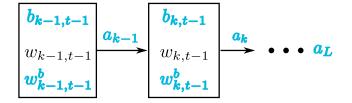
$$w^b = \begin{cases} +1, & \text{if } w \ge 0\\ -1, & \text{otherwise} \end{cases}$$

Quantizing neural networks Quantization: Neural Networks -Quantization Aware Training 2023--Quantization while Learning - Binary Connect



#### 1. Forward propagation:

 $w_b \leftarrow \text{binarize}(w_{t-1})$ For k = 1 to L, compute  $a_k$  knowing  $a_{k-1}$ ,  $w_b$  and  $b_{t-1}$ 



Quantizing neural networks

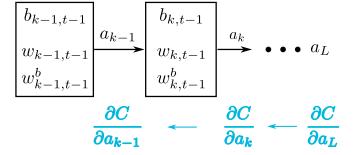
Quantization: Neural Networks
Quantization Aware Training
Quantization while Learning - Binary Connect



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#### 2. Backward propagation:

Initialize output layer's activations gradient  $\frac{\partial C}{\partial a_L}$ For k=L to 2, compute  $\frac{\partial C}{\partial a_{k-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $w_b$ 

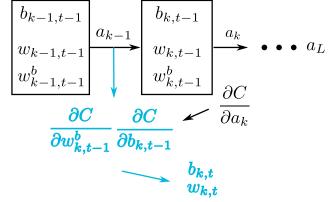


Quantizing neural networks
Quantization : Neural Networks
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Quantization while Learning - Binary Connect



#### 3. Parameter update:

Compute  $\frac{\partial C}{\partial w_b}$  and  $\frac{\partial C}{db_{t-1}}$  knowing  $\frac{\partial C}{\partial a_k}$  and  $a_{k-1}$   $w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$   $b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$ 





Quantizing neural networks

Quantization: Neural Networks
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Quantization while Learning - Binary Connect



04-03

2023-(

## Binarization: Stochastic vs Deterministic

Deterministic

$$w_b = \begin{cases} +1, & \text{if } w \ge 0 \\ -1, & \text{otherwise} \end{cases}$$

Stochastic

$$w_b = \begin{cases} +1, & \text{with probability } p = \sigma(w) \\ -1, & \text{with probability } 1-p \end{cases}$$

avec

$$\sigma(x) = \text{clip}(\frac{x+1}{2}, 0, 1) = \max(0, \min(1, \frac{x+1}{2}))$$



Quantizing neural networks
Quantization : Neural Networks
Quantization Aware Training
Binarization : Stochastic vs Deterministic



Two different ways to binarize the weights exist. The first one is deterministic, which means that the weights are binarized with a threshold. The second one is stochastic, which means that the weights are binarized with a probability.

## Binarization: Stochastic vs Deterministic

Deterministic

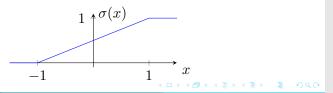
$$w_b = \begin{cases} +1, & \text{if } w \ge 0 \\ -1, & \text{otherwise} \end{cases}$$

Stochastic

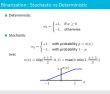
$$w_b = \begin{cases} +1, & \text{with probability } p = \sigma(w) \\ -1, & \text{with probability } 1-p \end{cases}$$

avec

$$\sigma(x) = \text{clip}(\frac{x+1}{2}, 0, 1) = \max(0, \min(1, \frac{x+1}{2}))$$

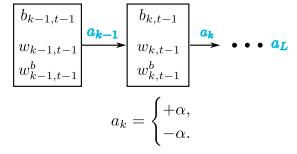


Quantizing neural networks
Quantization : Neural Networks
Quantization Aware Training
Binarization : Stochastic vs Deterministic



Two different ways to binarize the weights exist. The first one is deterministic, which means that the weights are binarized with a threshold. The second one is stochastic, which means that the weights are binarized with a probability.

# Quantization while Learning - Binary Weighted network (XNOR-NET)



Rastegari, Mohammad, et al. "Xnor-net: Imagenet classification using binary convolutional neural networks." European conference on computer vision. Springer, Cham, 2016. https://arxiv.org/pdf/1603.05279.pdf



Quantizing neural networks
Quantization: Neural Networks
Quantization Aware Training
Quantization while Learning - Binary
Weighted network (XNOR-NET)

Dauntziaton while Learning - Binary Weighted network DKNOR-NETI

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# Quantization while Learning - Binary Weighted network (XNOR-NET)

	Network Variations	Operations used in Convolution	Memory Saving (Inference)	Computation Saving (Inference)	Accuracy on ImageNet (AlexNet)
Standard Convolution	Real-Value Inputs  0.11 - 0.21 0.34 0.25 0.61 0.52 0.68	+,-,×	1x	1x	%56.7
Binary Weight	Real-Value Inputs  0.11 - 0.21 0.340.25 0.61 0.52  Binary Weights  1 - 1 11 1	+,-	~32x	~2x	%56.8
BinaryWeight Binary Input (XNOR-Net)	Binary Inputs  1 -11   Binary Weights  1 -1 1   1 -1   1 -1   1 -1   1 -1   1   1	XNOR , bitcount	~32x	~58x	%44.2

Rastegari, Mohammad, et al. "Xnor-net: Imagenet classification using binary convolutional neural networks." European conference on computer vision. Springer, Cham, 2016. https://arxiv.org/pdf/1603.05279.pdf

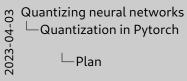


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# Plan

- 1 Objectives
- 2 Quantization: Basic
  - Floating Point
  - Integers, Fixed Point
  - Quantization
- 3 Quantization : Neural Network
  - Quantization Post Training
  - Quantization Aware Training
- 4 Quantization in Pytorch





# Quantization in Pytorch

- Dynamic Quantization
- Static Quantization
- **3** Quantization Aware Training

#### https:

//pytorch.org/blog/introduction-to-quantization-on-pytorch/

And for our need: https://pytorch.org/tutorials/prototype/fx\_graph\_mode\_ptq\_static.html



Quantizing neural networks

Quantization in Pytorch

Quantization in Pytorch

2023



Quantization in Pytorch is a new feature. It is still in beta version. A group of students from last year worked on this topic. They implemented a post quantization algorithm in order to quantize the network with 8-bit weights and activations.