Quantum Computing Explained Solutions

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1 Introduction

This document has the intention to show the solutions to the exercises presented in Quantum Computing Explained by David McMahon, the exercies are explained and in some cases R or Python code is presented.

2 A brief introduction to information theory

1.1 How many bits are necessary to represent the alphabet using a binary code if we only allow uppercase characters? How about if we allow both uppercase and lowercase character?

Solution
Let's consider that n is the number of characters in an alphabet, we would need x bits so that

$$2^x \geq n$$
$$x \geq \log_2 n$$

The english alphabet has 26 characters, so $n = 26$, therefore

$$\log_2 26 \approx 4.7$$

And since we can't have 4.7 bits we have to take the next integer, that is 5.

$$\therefore \text{ We need 5 bits}$$

How about if we allow both uppercase and lowercase characters? This implies that $n = 26 * 2 = 52$ characters. We could solve same equations as above, or we can just remember that

$$2^{x+1} = 2 * 2^x$$

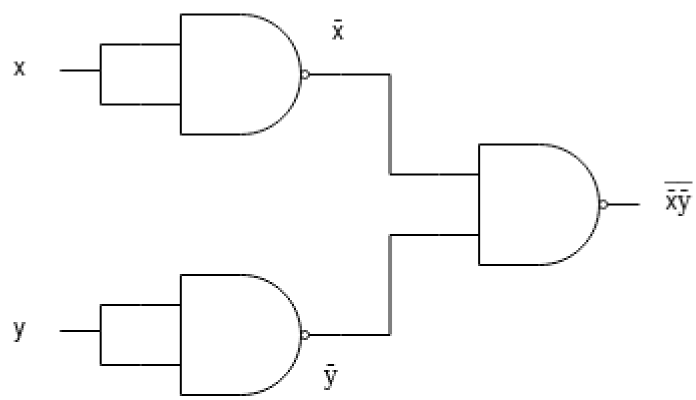
Knowing this we can see that

$$2^6 \geq 52$$
$$64 \geq 52$$
$$\therefore \text{ We need 6 bits}$$

1.2 Describe how you can create an OR gate using NOT gates and AND gates.

Solution
To create a 1 bit OR gate we need 2 NOT gates and a NAND gate (which is an AND gate followed by a NOT gate), this gates have to be connected as follows

(the not gates are going to be created with a NAND gate which receives the signal two times)



We can see that the result of the circuit is $\overline{\overline{x}\,\overline{y}}$. To develop this we have to remember one Boole's Algebra rule

$$\overline{xy} = \overline{x} + \overline{y}$$

Using this rule we can see that

$$\overline{\overline{x}\,\overline{y}} = \overline{\overline{x}} + \overline{\overline{y}}$$
$$\overline{\overline{x}} + \overline{\overline{y}} = x + y$$

1.3 A kilobyte is 1024 bytes. How many messages can it store?

Solution
This is a very straight forward solution, all that we have to remember is

$$1byte = 8bits$$
$$1024bytes = 1024 * 8 = 8192bits$$
$$\therefore\ 1kilobyte \text{ can store } 2^{8192}messages$$

1.4 What is the entropy associated with the tossing of a fair coin?

Solution
Let's remember that a fair coin implies the following

$$P(\{heads\}) = P(\{tails\}) = 0.5$$

So, being X the event of tossing a fair coin

$$
\begin{gathered}
H(X) = -\sum p_i \log_2 p_i \\
= -(0.5 \log_2 0.5 + 0.5 \log_2 0.5) \\
= -(-0.5 - 0.5) \\
= 1
\end{gathered}
$$

1.5 Suppose that X consists of the characters A, B, C, D that occur in a signal with respective probabilities 0.1, 0.4, 0.25, 0.25, what is the Shanon's entropy?

Solution
Given the probabilities

$$
\begin{gathered}
P(\{A\}) = 0.1 \\
P(\{B\}) = 0.4 \\
P(\{C\}) = 0.25 \\
P(\{D\}) = 0.25
\end{gathered}
$$

The Shanon's entropy is the following

$$
\begin{gathered}
H(X) = -\sum p_i log_2 p_i \\
= -(0.1 \log_2 0.1 + 0.4 \log_2 0.4 + 0.25 \log_2 0.25 + 0.25 \log_2 0.25) \\
= -(0.332 - 0.528 - 0.5 - 0.5) \\
= 1.86
\end{gathered}
$$

1.6 A room full of people has incomes distributed in the following way

$n(25.5) = 3$
$n(30) = 5$
$n(42) = 7$
$n(50) = 3$
$n(63) = 1$
$n(75) = 2$
$n(90) = 1$

What is the most probable income? What is the average income? What is the variance of this distribution?

Solution
Starting by the most probable income, we can calculate the probability of all incomes, but we don't have to, knowing that the total people $n = 22$, we can see that the probability of an income is

$$\frac{n_j}{n}$$

Where n_j is the count of occurences of that income, so the higher the occurences the higher the probability, knowing that the higher occurence is 7, $n(42)$ is the

most probable income with a value of $\frac{7}{22}$.
The average income is given by the expresion

$$\mu = \sum jp_j$$
$$= 25.5(0.13) + 30(0.22) + 42(0.31) + 50(0.13) + 63(0.04) + 75(0.09) + 90(0.04)$$
$$= 44.25$$

The variance of the distribution can be calculated in many ways, but I'm going to use the next formula

$$\sigma^2 = \frac{\sum X^2}{N} - \mu^2$$

Where X is the random variable, N is the number of observations, and μ is the mean of the distribution. Therefore, the variance is

$$\sigma^2 = \frac{1}{22}(1950.75 + 4500 + 12348 + 7500 + 3969 + 11250 + 8100) - 44.25^2$$
$$= 2255.3522 - 1958.0625 = 297.289$$