

ON THE POSITIVE CORRELATION BETWEEN INCOME INEQUALITY AND  
UNEMPLOYMENT

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## ON THE POSITIVE CORRELATION BETWEEN INCOME INEQUALITY AND UNEMPLOYMENT

Rubens Penha Cysne\*

**Abstract**—Two papers published in this journal (Jantti, 1994, and Mocan, 1999), among others, find empirical evidence that "increases in structural unemployment have a substantial aggravating impact on income inequality." The main point of this work is to show that standard job-search models can help us understand this empirical regularity. As a byproduct of the analysis, the paper also provides a closed-form general expression that enables direct calculation of the Gini coefficient of wage-income inequality as a function of any arbitrary initial distribution of wage offers. Three numerical examples illustrate the results.

### I. Introduction

THE fact that unemployment raises inequality is well documented in the literature. As put by Mocan (1999), citing many different studies, "the consensus has been that income inequality is countercyclical in behavior, i.e., increases in unemployment

worsen the relative position of low-income groups." Using U.S. time series data ranging from 1948 to 1994, this author finds that "increases in structural unemployment have a substantial aggravating impact on income inequality." Jantti (1994), using U.S. data ranging from 1948 to 1989, as well as Cardoso (1993) and Cardoso, Barros, and Urani (1995), using Brazilian data, arrive at the same conclusion. Nolan (1986) measured the impact of changes on the level of unemployment on the U.K. size distribution of annual income using cross-section data of the Family Expenditure Survey. He documented that unemployment leads to a shift in the shape of the income distribution, with a rise in the top decile, the effect of unemployment on the deterioration of the income distribution being very significant.

The literature on income distribution, though, still lacks theoretical formalizations able to deliver such results in a setting in which consumers maximize utility intertemporally, subject to uncertainty. To show that standard job-search models can help us in the understanding of this empirical regularity is the main objective of this paper.

An important point to be understood is that the inequality measures to which we refer here are not those obtained over a certain period of time (lifetime measures or long-run measures).

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Typical measurements of inequality are not carried out in this way.<sup>1</sup> The inequality measures in which we are interested here are those usually reported by researchers, which are based on cross-sectional distributions of income.

The main point of this work is to show that the stationary cross-sectional income distribution generated by standard search models share with these reported empirical assessments the property of a positive correlation between unemployment and the Gini coefficient of inequality. Therefore, understanding the mechanisms by which such a correlation emerges in a job-search model can be useful for understanding the possible channels by means of which higher unemployment rates are associated with higher measured cross-sectional inequalities.

As a byproduct of the analysis, the paper also provides a general expression that allows for a direct calculation of the Gini coefficient of wage-income inequality as a function of any arbitrary distribution of wage offers, of the probability that a worker is laid off, and of the probability that an unemployed worker does not find a job offer in the next period.

The paper proceeds as follows: section II presents the basic model. Section III calculates the Gini coefficient. Section IV uses the results of section III to show how a positive correlation between unemployment and inequality can be established and displays the results with three examples. Section V concludes.

## II. The Model

Our purpose here is not to develop a new model, but to show that standard job-search models can be useful in understanding the empirical regularities mentioned in the previous section. In order to do so, we slightly generalize a version of McCall's (1970) job-search model presented in Stokey and Lucas (1989, c. 10). This generalization is carried out by allowing for the possibility that a worker does not get a definite job offer in each period. This point is important because some recent papers in the literature have placed a renewed emphasis on the job finding probability (which is clearly related to the job offer probability considered here). Shimer (2005), for example, referring to U.S. data, states that "virtually all of the increase in unemployment and decrease in employment during the 1991 and 2001 recessions was a consequence of a reduction in the job finding probability."

In the real world, unemployed workers may simply receive no job offers in some periods. The explicit consideration of this additional source of uncertainty, with respect to the basic model cited above, implies a lower reservation wage and a different invariant distribution of wages in the economy. The cross-sectional average wage and the Gini coefficient of wage inequality are also affected. All these facts are captured by the parameter  $\alpha$ , to be introduced below. The less general version of the model corresponds to the case in which  $\alpha = 0$ .

The fact that the main steps in the deduction of the stationary distribution of wage offers in the economy are similar to those detailed in Stokey and Lucas (1989) allows us to go straight to the main points of the paper.

From now on, consider a economy populated by a continuum of workers. This economy can be imagined as a small economy in which all workers are contracted by foreign firms. For  $0 < D < \infty$ , consider the measurable space  $(\Omega, \mathcal{F}, M)$  and, in this space, the measure  $m_w$

induced by the wage-offer function  $w: \Omega \rightarrow [0, D]$ . In the induced space  $([0, D], \mathcal{B}_{[0,D]}, m_w)$ , denote by  $F_w(t)$  the distribution function that  $(m_w - a.e.)$  uniquely determines the measure  $m_w: F_w(k) := m_w([w \leq k])$ .

**Assumption 1.**  $F_w(\cdot)$  is absolutely continuous in  $[0, D]$  with<sup>2</sup>  $F'_w(\cdot) = f_w(\cdot)(m_w - a.e.)$ .

From now on, suppose that assumption 1 is always valid (except in example 1, which deals with a point-mass distribution). Note that this condition only requires the distribution of wage offers to be representable by a density function, a trivial fact in applied search and distribution theory.

The analysis of the job search can be made as a function of just two states regarding the consumer's optimization problem: call it state "w" and state "0." State w corresponds to a job offer of w at hand, and state 0 to no job offer. In state w the worker can accept or turn down the offer. If he (she) accepts it, by assumption he stays employed with that wage until he is laid off, which can happen, in each period, with probability  $\theta$  (the "separation probability"). If he does not accept the offer or if he gets no offer, he remains in state 0.<sup>3</sup> Being in state 0, the only thing he can do is to wait again for a job offer in the next period, which happens with probability  $1 - \alpha$  (the "job-offer finding rate"). By assumption,

$$\begin{cases} 0 < \theta < 1 \\ 0 \leq \alpha < 1 \end{cases} \quad (1)$$

The job offers are independent and drawn according to the measure  $m_w$ , which is supposed to be known by all workers. Consumers maximize the expected present value of their consumption:

$$E \left( \sum_{t=0}^{\infty} \beta^t c_t \right), \quad 0 < \beta < 1.$$

With  $v(w)$  stating for the value function, and  $\mathcal{A}, \mathcal{R}$ , respectively, for "accept" and "reject," the recursive version of the consumer's problem is given by the maximization of

$$v(w) = \max_{\mathcal{A}, \mathcal{R}} \{w + \beta[(1 - \theta)v(w) + \theta v(0)], v(0)\},$$

where (using the dummy variable  $t$ ),

$$v(0) = \beta \left[ (1 - \alpha) \int_{[0,D]} v(t) dF_w(t) + \alpha v(0) \right].$$

The reservation wage ( $\bar{w}$ ) in this economy is implicitly determined by

$$\bar{w} = \frac{\beta(1 - \alpha)}{1 - \beta(1 - \theta)} \int_{[\bar{w}, D]} (t - \bar{w}) dF_w(t). \quad (2)$$

Offers between zero and the reservation wage are not accepted by the worker.

<sup>2</sup> To simplify the notation, we denote density functions by small letters and the associated distribution functions by the respective capital letters.

<sup>3</sup> The introduction of an unemployment compensation  $c$  would add one more parameter (besides  $\theta$ ,  $\alpha$ , and  $\beta$ ) to the model, leading to an increase of the reservation wage and the average wage. The basic results of the paper, though, are not affected by this change.

<sup>1</sup> Friedman (1962, p. 171) and Flinn (2002), among others, point out the distinction between cross-sectional measures of income inequality and differences in long-run income status.

Make  $A = [\bar{w}, D]$  the acceptance region. Note that

$$m_w(A) = 1 - F(\bar{w}) \quad (3)$$

and that, since  $\bar{w}$  is a function of both  $\theta$  and  $\alpha$ , the same happens to  $m_w(A)$ .

At any fixed point of time, the fraction of people earning wages in the range  $s, s + ds$  has an invariant cross-sectional (mixed) measure trivially determined by

$$f_p(s)ds = \begin{cases} \frac{\theta}{\theta + (1-\alpha)m_w(A)} & \text{if } s = 0 \\ 0 & \text{if } 0 < s < \bar{w} \\ \frac{(1-\alpha)dF_w(s)}{\theta + (1-\alpha)m_w(A)} & \text{if } \bar{w} \leq s < D \end{cases} \quad (4)$$

Let  $U$  stand for the unemployment rate. Then, from equation (4):

$$U := \frac{\theta}{\theta + (1-\alpha)m_w(A)}. \quad (5)$$

Using equation (4), the cross-sectional average wage of all workers (employed and unemployed) in the economy is<sup>4</sup>

$$s_A = \int_{[\bar{w}, D]} \frac{(1-\alpha)tdF_w(t)}{\theta + (1-\alpha)m_w(A)}, \quad (6)$$

where  $\bar{w}$  is implicitly determined by equation (2).

The average wage is affected by the parameters  $\theta$  and of  $\alpha$  both directly and through their effect on the reservation wage. Considering both effects together, it is easy to see that

$$\frac{ds_A}{d\theta} < 0, \quad \frac{ds_A}{d\alpha} < 0. \quad (7)$$

To obtain this result, first use the implicit function theorem to conclude that

$$\frac{\partial \bar{w}}{\partial \theta} = \frac{-\beta \bar{w}}{1 - \beta[1 - \theta - (1-\alpha)(1 - F(\bar{w}))]} < 0 \quad (8)$$

and

$$\frac{\partial \bar{w}}{\partial \alpha} = \frac{-\bar{w}}{[1 - \alpha][1 + \frac{\beta(1-\alpha)(1 - F(\bar{w}))}{1 - \beta(1 - \theta)}]} < 0. \quad (9)$$

Next, observe that,<sup>5</sup> since  $\int_{[\bar{w}, D]} \frac{tdF_w(t)}{m_w(A)} = E_{F_w}(w | [w \geq \bar{w}])$ ,

$$\frac{\partial s_A}{\partial \bar{w}} > 0. \quad (10)$$

Use equations (8), (9), and (10) to conclude that the indirect effect (through the reservation wage) of either  $\theta$  or  $\alpha$  over  $s_A$  is negative.

<sup>4</sup> In this economy each single worker is subject to a infinite cycle of employment, layoff, and unemployment. The wage average can be understood as the average of any worker's wage throughout his infinite lifetime or, by the law of large numbers, also as the cross-sectional average of the wage of all workers in the economy, employed and unemployed, at each point in time.

<sup>5</sup> Making  $E_{F_w}$  denote the conditional expectation with respect to the measure  $m_w$ .

Since the direct effect (see equation [6]) is trivially negative, (7) is true.

### III. The Gini Coefficient

Given the income density function (4), the fraction of the population earning income less than or equal to  $s$  is given by the distribution function:

$$F_p(s) := \int_0^s f_p(t)dt = \begin{cases} \frac{\theta}{\theta + (1-\alpha)m_w(A)} & \text{if } 0 \leq s < \bar{w} \\ \frac{\theta}{\theta + (1-\alpha)m_w(A)} + \int_{[\bar{w}, s]} \frac{(1-\alpha)dF_w(t)}{\theta + (1-\alpha)m_w(A)} & \text{if } \bar{w} \leq s \leq D \end{cases}, \quad (11)$$

and the fraction of income earned by workers with income less or equal to  $s$  by:

$$F_s(s) := \frac{1}{s_A} \int_{[0, s]} t dF_p(t) \quad (12)$$

$$= \frac{1}{s_A} \left[ sF_p(s) - \int_{[0, s]} F_p(t)dt \right] \quad (13)$$

$$= \frac{1}{s_A} \left[ s \int_{[0, s]} f_p(t)dt - \int_{[0, s]} \left[ \int_{[0, u]} f_p(t)dt \right] du \right].$$

Above, the second line uses integration by parts and the third uses the definition of the distribution function  $F_p(\cdot)$  displayed in equation (11).

The Lorenz curve can be represented (see, for example, Levine & Singer, 1970) by the function  $F_s(F_p)$ .  $F_p$  in this representation stands for a real variable taking values in  $[0, 1]$ . Since both  $F_s$  and  $F_p$  are functions of  $s$ , the Lorenz curve can be plotted by making  $s$  run from 0 to  $D$  and by generating and plotting simultaneous values for  $F_s$  and  $F_p$ . This usual parametric procedure is used in example 2 below as an alternative calculation of the final closed-form expression for the Lorenz curve obtained using the analytical results derived in this paper. For the interested reader, Kendall and Stuart (1963) is an example of a basic text on income inequality that derives the relationship between the income density function and the Lorenz curve.

The Gini coefficient ( $G$ ) is a ratio between two areas. The first area is the one between the 45° line and the Lorenz curve<sup>6</sup>  $F_s(F_p)$ . The second area is the one above the abscissa and below the 45° line (therefore, equal to 1/2, since in both cases the abscissa runs from 0 to 1).

Denote by  $A_L(0, W)$ ,  $W \in [0, D]$  the area under the Lorenz curve when wages run from 0 to  $W$ . By the definition of the Lorenz curve,

<sup>6</sup> The Lorenz curve plots the cumulative percentage of total income ( $F_s$ ) when the population is ordered by the size of their income (the abscissa standing for the  $F_p$ —poorest fraction of the population). Note that, parametrically,  $F_p$  runs from 0 to 1 when wages run from 0 to  $D$ .

$$\begin{aligned}
A_L(0, W) &:= \int_{[F_p(0), F_p(W)]} F_s(F_p) dF_p \\
&= \int_{[0, W]} F_s(s) \frac{dF_p}{ds} ds \\
&= \int_{[0, W]} F_s(s) f_p(s) ds.
\end{aligned} \tag{14}$$

Above, the second line uses a change-of-variable rule for the Stieltjes measure  $F_p$  and the third the definition of  $dF_p$ . Given equation (14), one can express the Gini coefficient of income distribution as

$$G = 1 - 2A_L(0, D). \tag{15}$$

From equation (15), it is easy to see that an increase of the income inequality can be characterized by a decrease of the area under the Lorenz curve.

The following lines pursue a general expression for the Lorenz curve and for the Gini coefficient as a function of any arbitrary distribution of wage offers  $F_w(\cdot)$ . First, in proposition 1, we use a general procedure detailed by Levine and Singer (1970) to show that the area under the Lorenz curve can be directly expressed in terms of  $f_p(s)$  only. Proposition 2 then uses the result of proposition 1 and the expression of  $f_p(s)$  given by equation (4) to determine the area under the Lorenz curve and, by equation (15), the Gini coefficient of wage distribution as well. By means of proposition 2, the Gini coefficient is directly expressed in closed form for any arbitrary initial distribution of wage offers  $F_w(\cdot)$ , a first result of the paper.

**Proposition 1.** *Consider the economy described in section II, with any arbitrary initial distribution of wage offers  $F_w(\cdot)$ , and suppose that assumption 1 is valid. Then: (i) the cross-sectional distribution of wage incomes at any point in time,  $f_p$ , and the average wage,  $s_A$ , are given, respectively, by equations (4) and (6) and; (ii) the area under the Lorenz curve in this economy can be expressed as*

$$A_L(0, D) = \frac{1}{s_A} \int_{[\bar{w}, D]} \left[ \int_{[\bar{w}, s]} tf_p(t) dt \right] f_p(s) ds. \tag{16}$$

**Proof.** Item (i) has already been proved.

To prove (ii), note that in this economy  $F_s(F_p) = 0$  for  $0 \leq F_p \leq \frac{\theta}{\theta + (1-\alpha)m_w(A)}$ . This happens because a fraction  $\frac{\theta}{\theta + (1-\alpha)m_w(A)}$  of the population is unemployed, thereby having a wage equal to 0. Parametrically, as a function of the wage of the economy, this region is covered when the realized wage ( $s$ ) varies in  $[0, \bar{w}]$ .

To calculate the area under the Lorenz curve when the wage assumes values between 0 and a certain threshold  $W \in [0, D]$  use equation (14) and the expression for  $F_s(s)$  given by equation (13):

$$\begin{aligned}
A_L(0, W) &= \frac{1}{s_A} \int_{[0, W]} \left[ s \int_{[0, s]} f_p(t) dt \right. \\
&\quad \left. - \int_{[0, s]} \left[ \int_{[0, u]} f_p(t) dt \right] du \right] f_p(s) ds.
\end{aligned}$$

To nicely simplify this expression, use integration by parts:

$$- \int_{[0, s]} \left[ \int_{[0, u]} f_p(t) dt \right] du = -s \int_{[0, s]} f_p(t) dt + \int_{[0, s]} tf_p(t) dt. \tag{17}$$

Therefore, one can write, using equation (17),

$$A_L(0, W) = \frac{1}{s_A} \int_{[0, W]} \left[ \int_{[0, s]} tf_p(t) dt \right] f_p(s) ds.$$

Since the only mass of  $f_p$  for  $0 \leq s < \bar{w}$  is concentrated at  $s = 0$ ,

$$\int_{[0, s]} tf_p(t) dt = \int_{[\bar{w}, s]} tf_p(t) dt. \tag{18}$$

Therefore:

$$A_L(0, W) = \frac{1}{s_A} \int_{[0, W]} \left[ \int_{[\bar{w}, s]} tf_p(t) dt \right] f_p(s) ds.$$

Since, as we have seen above,  $A_L(0, W) = 0$  for  $W \in [0, \bar{w}]$  the expression above can be written as equation (16). ■

The first main result of this paper is presented as proposition 2 below. Note that the result applies regardless of the underlying distribution of wage offers  $F_w(\cdot)$ . In proposition 2, the reservation wage  $\bar{w}$  is determined by equation (2).

**Proposition 2.** *Consider the economy described in section II and take assumption 1 for granted. Then the total area under the Lorenz curve associated with the long-run wage distribution is given by:*

$$A_L(0, D) = \frac{(1-\alpha) \int_{[\bar{w}, D]} \left[ \int_{[\bar{w}, s]} t dF_w(t) \right] dF_w(s)}{[\theta + (1-\alpha)(1-F(\bar{w}))] \int_{[\bar{w}, D]} t dF_w(t)} \tag{19}$$

and the Gini coefficient of income distribution by:

$$G = 1 - 2 \frac{(1-\alpha) \int_{[\bar{w}, D]} \left[ \int_{[\bar{w}, s]} t dF_w(t) \right] dF_w(s)}{[\theta + (1-\alpha)(1-F(\bar{w}))] \int_{[\bar{w}, D]} t dF_w(t)}. \tag{20}$$

**Proof.** The first part of the proof follows from equations (4), (6), and (16). The second part follows from equation (15). ■

#### IV. Unemployment and Inequality

This section uses the general expression of the Gini coefficient derived above to provide one possible way of understanding the empirical regularity (the positive correlation between inequality and unemployment) detailed in the introduction. Assumption 2 below is made in order to avoid unnecessary algebrisms derived from second-order variations of the reservation wage with respect to the parameters  $\alpha$  and  $\theta$ .



**Assumption 2.** The reservation wage lies below the infimum of the support of the distribution of wages (call it  $a_L$ ) for all values of  $\beta$ ,  $\alpha$ , and  $\theta$  under consideration.<sup>7</sup>

Proposition 3 delivers the second and main point of the paper.

**Proposition 3.** Suppose that an economy is characterized as detailed above and as in section II, and that assumptions 1 and 2 are valid for all values of the parameters under consideration (a case when both conditions are not valid is treated in example 1 below; two cases in which condition 2 is not valid are given by examples 2 and 3 below). Then, regardless of the initial distribution of wage offers, the Gini coefficient of income distribution is an increasing function of the probability of layoff ( $\theta$ ) and an increasing function of the probability that the worker does not get a job offer ( $\alpha$ ). Moreover, since the unemployment rate  $U$  in this case is trivially an increasing function of  $\theta$  and  $\alpha$ , increases in any one of these parameters generate a positive correlation between unemployment and inequality.

**Proof.** Under assumption 2  $F_w(\bar{w}) = 0$ , in which case equation (20) can be written as

$$G = 1 - 2 \frac{(1 - \alpha) \int_{[a_L, 1]} \left[ \int_{[a_L, s]} t dF_w(t) \right] dF_w(s)}{(\theta + (1 - \alpha)) \int_{[a_L, D]} t dF_w(t)}. \quad (21)$$

Using equation (5), the unemployment rate now reads

$$U = \frac{\theta}{\theta + (1 - \alpha)}. \quad (22)$$

The proposition is proved by simply observing that  $G$  and  $U$ , determined, respectively, by equations (21) and (22), are both increasing functions of  $\theta$  and of  $\alpha$ . ■

The examples to be presented below show that assumption 2 is not necessary for the result of proposition 3 to be true.

The positive correlation between unemployment and the separation probability  $\theta$  can be proved independently of assumption 2, provided that one considers values of  $\theta$  close enough to 0. The result is shown below as proposition 4.

**Proposition 4.** Suppose that an economy is characterized as detailed above and as in section II, and that condition 1 is valid. Then, for values of  $\theta$  low enough, regardless of the initial distribution of wage offers, the unemployment rate  $U(\theta, \alpha) := \frac{\theta}{\theta + (1 - \alpha)(1 - F(\bar{w}))}$  is an increasing function of the parameters  $\theta$  and  $\alpha$ .

**Proof.** Using the chain rule,

$$\frac{dU(\theta, \alpha)}{d\theta} = \frac{\partial U(\theta, \alpha)}{\partial \theta} + \frac{\partial U(\theta, \alpha)}{\partial \bar{w}} \frac{\partial \bar{w}}{\partial \theta}. \quad (23)$$

Note that  $\partial U(\theta, \alpha)/\partial \theta > 0$  and that, given the definition of  $f_w(t) := \partial F(t)/\partial t$ ,

$$\frac{\partial U(\theta, \alpha)}{\partial \bar{w}} = \frac{\theta(1 - \alpha)f_w(\bar{w})}{[\theta + (1 - \alpha)(1 - F(\bar{w}))]^2} > 0. \quad (24)$$

Remember equation (8). It follows that, for values of  $\theta$  low enough, the second term in equation (23) is close enough to 0, the signal of  $\frac{dU(\theta, \alpha)}{d\theta}$  being the same as that of  $\frac{\partial U(\theta, \alpha)}{\partial \theta}$ . Now we turn to the signal of  $\frac{dU(\theta, \alpha)}{d\alpha}$ . Use equation (23) again, with  $\alpha$  replacing  $\theta$ . As before, note that  $\frac{\partial U(\theta, \alpha)}{\partial \alpha} > 0$  and remember equation (9). Next, use equation (24) again to conclude that for values of  $\theta$  low enough, the signal of  $\frac{dU(\theta, \alpha)}{d\alpha}$  is the same as that of  $\frac{\partial U(\theta, \alpha)}{\partial \alpha}$ , which is positive. ■

**Example 1.** (Point-mass distribution).<sup>8</sup> Suppose the measure  $m_w$  is given by a point-mass distribution at the point  $\hat{w} \in (0, D)$ . Note that assumptions 1 and 2 do not apply here. Assumption 1 does not apply because  $F_w$  is not representable by a density function in  $[0, D]$ . Assumption 2 does not apply because  $\bar{w} = \hat{w}$  (the first offer will be accepted). In terms of the previous definitions, we trivially have

$$F_p(s) = \begin{cases} \frac{\theta}{\theta + 1 - \alpha} & \text{if } 0 \leq s < \hat{w} \\ 1 & \text{if } s = \hat{w} \end{cases}, \quad (25)$$

$$F_s(s) = \begin{cases} 0 & \text{if } 0 \leq s < \hat{w} \\ 1 & \text{if } s = \hat{w} \end{cases}, \quad (26)$$

$$s_A = \frac{(1 - \alpha)\hat{w}}{\theta + 1 - \alpha}. \quad (27)$$

In the present case there are two groups of workers. The first one has mass  $\frac{\theta}{\theta + 1 - \alpha}$  and a fraction of income equal to 0. The other group, with remaining mass  $\frac{1 - \alpha}{\theta + 1 - \alpha}$ , has the total amount of the wage income. This leads to a Lorenz curve given by the following:

$$F_s(F_p) = \begin{cases} 0, & 0 \leq j < \frac{\theta}{\theta + 1 - \alpha} \\ -\frac{\theta}{1 - \alpha} + \frac{(\theta + 1 - \alpha)j}{1 - \alpha}, & \frac{\theta}{\theta + 1 - \alpha} \leq j \leq 1 \end{cases}. \quad (28)$$

Integrate  $F_s(F_p)$  with respect to the Lebesgue measure (since income is equally distributed among those who happen to be employed) and use equation (15) to obtain (remember (1) and (5))

$$G = U = \frac{\theta}{\theta + 1 - \alpha}. \quad (29)$$

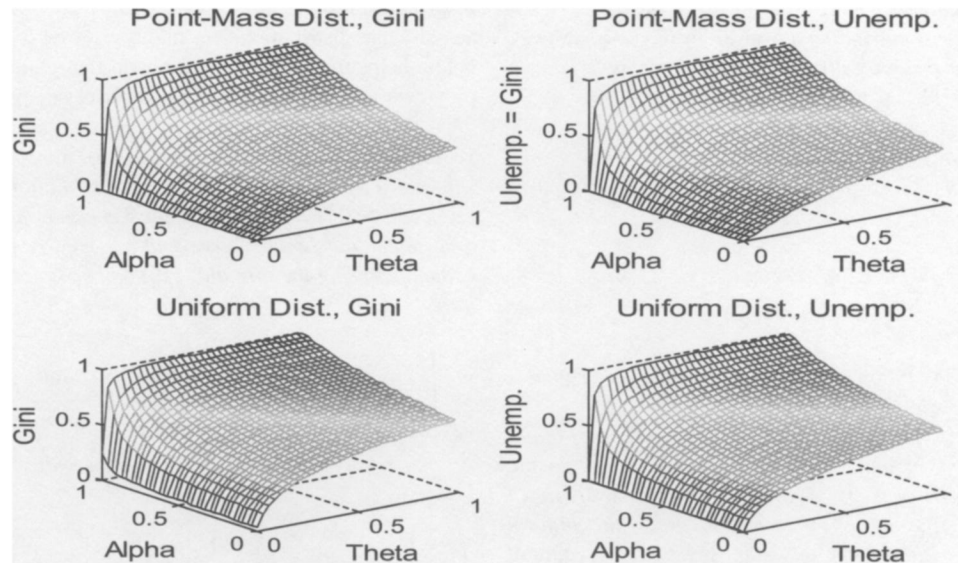
Note that the Gini coefficient in this case coincides with the rate of unemployment, a result that very nicely illustrates the main point investigated in this work: the positive correlation between these two variables.

From equation (1) it follows that  $0 < G < 1$ . To get some intuition about this result, make  $\theta \rightarrow 0$ . In this case, under the stationary distribution, all workers tend to be employed with wage  $\hat{w}$ , with both the rate of unemployment and the Gini coefficient tending toward 0. When  $\alpha = 0$  and  $\theta \rightarrow 1$ , on the other hand, under the stationary distribution one-half of the workers tends to be unemployed and the other half to be employed with wage  $\hat{w}$ . The average wage, of course, goes to  $\frac{1}{2}\hat{w}$  (as one could also infer from equation [27]) and so does

<sup>7</sup> Note that the reservation wage is a decreasing function of  $\theta$  and  $\alpha$  and that it is equal to 0 when  $\beta = 0$ .

<sup>8</sup> This example with a point-mass distribution was suggested to me by one of the referees, to whom I am thankful.

FIGURE 1.—POINT MASS AND UNIFORM DISTRIBUTION



the rate of unemployment (see equation [29]). The Gini coefficient is also equal to  $1/2$ . One-half of the population receives no income and the other half an income equal to  $\bar{w}$ .

The upper part of figure 1 shows the value of the Gini coefficient and of the unemployment rate for different values of  $\theta$  and  $\alpha$ . The positive correlation generated by the search model is clear, either when  $\theta$  increases or when  $\alpha$  increases.

The next example deals with a uniform distribution of wage offers. To illustrate the usefulness of the closed-form solution derived in this paper, the area under the Lorenz curve is calculated by two different methods. The first calculation uses the usual parametric procedure outlined in the literature.<sup>9</sup> It uses  $F_p(s)$  to obtain the inverse function  $s(F_p)$ . Next, this result is used to eliminate  $s$  in  $F_s(s)$ , leading to the Lorenz curve  $F_s(F_p)$ . By integrating this curve with respect to  $F_p$  one finds the area under the Lorenz curve and, subsequently, using equation (15), the Gini coefficient. The second calculation draws directly upon our formula (19). Of course, both methods lead to the same result. However, the second one is much more straightforward.

**Example 2 (Uniform distribution).** Suppose now that the measure  $m_w$  is given by the Lebesgue measure in  $[0, 1]$ . Note below that this measure does not obey assumption 2, that  $\bar{w} < a_L$  (here,  $a_L = 0$ ). Using equation (4),

$$f_p(s)ds = \begin{cases} \frac{\theta}{\theta + (1-\alpha)(1-\bar{w})}, & s = 0 \\ 0, & 0 < s < \bar{w} \\ \frac{(1-\alpha)ds}{\theta + (1-\alpha)(1-\bar{w})}, & \bar{w} \leq s \leq 1 \end{cases}, \quad (30)$$

which leads to the expression for the fraction of the population with income less than or equal to  $s$ ,

$$F_p(s) = \begin{cases} \frac{\theta}{\theta + (1-\alpha)(1-\bar{w})}, & 0 \leq s < \bar{w} \\ \frac{\theta + (1-\alpha)(s-\bar{w})}{\theta + (1-\alpha)(1-\bar{w})}, & \bar{w} \leq s \leq 1 \end{cases}, \quad (31)$$

and to

$$F_s(s) = \frac{1}{s_A} \int_0^s tf_p(t)dt = \begin{cases} 0 & \text{if } 0 \leq s < \bar{w} \\ \frac{(s^2 - \bar{w}^2)}{1 - \bar{w}^2} & \bar{w} \leq s \leq 1 \end{cases}. \quad (32)$$

Solve for  $s$  in the second term in equation (31) and substitute into equation (32) to get the expression for the Lorenz curve:

$$F_s(F_p) = \begin{cases} 0, & 0 \leq F_p < \frac{\theta}{\theta + (1-\alpha)(1-\bar{w})} \\ \frac{\left[ \frac{(F_p)(\theta + (1-\alpha)(1-\bar{w})) + \bar{w}(1-\alpha) - \theta}{1-\alpha} \right]^2 - \bar{w}^2}{(1-\bar{w}^2)}, & \frac{\theta}{\theta + (1-\alpha)(1-\bar{w})} \leq F_p \leq 1 \end{cases}, \quad (33)$$

To calculate the area under the Lorenz curve ( $A_L(0, 1)$ ), integrate with respect to  $F_p$ :

$$\begin{aligned} A_L(0, 1) &= \int_0^1 F_s(F_p) dF_p \\ &= \int_{\frac{\theta}{\theta + (1-\alpha)(1-\bar{w})}}^1 \frac{\left[ \frac{t(\theta + (1-\alpha)(1-\bar{w})) + \bar{w}(1-\alpha) - \theta}{1-\alpha} \right]^2 - \bar{w}^2}{(1-\bar{w}^2)} dt. \end{aligned}$$

Making  $u = \frac{t(\theta + (1-\alpha)(1-\bar{w})) + \bar{w}(1-\alpha) - \theta}{1-\alpha}$ , the above integral reads

$$A_L(0, 1) = \int_{\bar{w}}^1 \frac{(1-\alpha)(u^2 - \bar{w}^2)}{(1-\bar{w}^2)(\theta + (1-\alpha)(1-\bar{w}))} du.$$

By integration,

$$A_L(0, 1) = \frac{(1-\alpha)(1 + \bar{w} - 2\bar{w}^2)}{3(1 + \bar{w})(\theta + (1-\alpha)(1-\bar{w}))}. \quad (34)$$

<sup>9</sup> See, for example, Kendall and Stuart (1963).

By using equations (2) and (15), equation (34) leads to the closed-form solution to the Gini coefficient.

To compare this expression with the one given by our formula (19), and show that both expressions deliver the same result, note that, in this case, in equation (19),  $1 - F(\bar{w}) = 1 - \bar{w}$  and

$$\int_{[\bar{w}, s]} t dF_w(t) = \frac{s^2 - \bar{w}^2}{2},$$

$$\int_{[\bar{w}, 1]} ds \int_{[\bar{w}, s]} t dF_w(t) = \frac{1 - 3\bar{w}^2 + 2\bar{w}^3}{6},$$

from which equation (34) follows trivially from equation (19) in a much simpler and more direct way.

Regarding equation (34), note the following. If  $\theta \rightarrow 0$ ,  $\alpha = 0$  and  $\bar{w} = 0$  (which happens in our search model, for instance in the degenerate case in which  $\beta = 0$ ),  $A_L(0, D) = \frac{1}{3}$ , the well-known value of the area under the Lorenz curve in the case of the uniform distribution. Through equation (15), this number leads to a value of the Gini coefficient equal to  $\frac{1}{3}$ . If  $\bar{w} = 0$  and  $\theta \neq 0$  or  $\alpha \neq 0$ ,  $A_L(0, 1) = \frac{\frac{1}{2}(1-\alpha)}{\theta + 1 - \alpha} < \frac{1}{3}$ , showing the positive impact on the Gini coefficient of the unemployment probability  $\theta$ .

Note also from equation (34) that the direct effect of  $\theta$  or  $\alpha$  over  $A_L$  is negative, and that  $\frac{\partial A_L}{\partial \bar{w}}|_{\theta=0} = \frac{1-2\bar{w}+\bar{w}^2}{K_1} > 0$  for  $\bar{w} \in [0, 1]$ ,  $K_1$  standing for a positive constant. Since, by equation (8), the reservation wage is a decreasing function of  $\theta$ , this implies that the area under the Lorenz curve decreases (and the Gini coefficient increases) when  $\theta$  increases from 0. Again, for low values of  $\theta$ , the same positive dependence of the Gini coefficient happens (using equation [9]) with regard to  $\alpha$ . On the other hand, given proposition 4, a positive correlation between the rate of unemployment and the Gini coefficient can also be proved to happen for values of  $\theta$  low enough. These two facts put together deliver a positive correlation between the Gini coefficient and the rate of unemployment which does not rely on the validity of assumption 2.

Furthermore, the numerical analysis carried out below shows that the assumption that  $\theta$  is low enough, used in the theoretical discussion above, as well as in the demonstration of proposition 4 does not happen to be necessary here.

Use equation (2) to get

$$\bar{w}(\theta, \alpha) = \frac{1 - \beta\alpha + \beta\theta}{\beta - \beta\alpha} - \sqrt{\left(\frac{1 - \beta\alpha + \beta\theta}{\beta - \beta\alpha}\right)^2 - 1}. \quad (35)$$

The final expression for the Gini coefficient can then be obtained by plugging equation (35) into (34) and using equation (15).

The two graphs in the lower part of figure 1 show the Gini coefficient and the unemployment rate when  $\beta = 0.98$  and the parameters  $\theta$  and  $\alpha$  assume values in  $(0, 1)$ . As in example 1, note that both the Gini coefficient and the rate of unemployment are increasing functions of  $\theta$  and of  $\alpha$ . Again, when either  $\theta$  and  $\alpha$  increase, a positive correlation between income inequality and unemployment is generated.

When  $\alpha \rightarrow 1$  and  $\theta > \epsilon > 0$ ,  $G \rightarrow 1$ , because a very small percentage of the unemployed workers happens to receive job offers. All the remaining workers have no offers and a wage equal to 0. Having  $\alpha > \epsilon > 0$  and  $\theta \rightarrow 1$ , though, does not imply  $G \rightarrow 1$ . Those workers who were not employed in the last period are allowed (with probability  $1 - \alpha$ ) to get new job offers and, possibly, to accept them.

The two previous examples have used two distributions (point-mass and uniform) that do not adequately incorporate the fact that actual wage distributions are highly skewed to the right. Example 3 below helps the reader to understand the interaction between unemployment and inequality more completely.

**Example 3** (Exponential distribution). Suppose now that the measure  $m_w$  is given by the exponential distribution, with density  $f(s) = \lambda e^{-\lambda s}$ ,  $\lambda > 0$ , with respect to the Lebesgue measure in  $[0, \infty)$ . The area under the Lorenz curve and the Gini coefficient can easily be calculated using the formulas (19) and (20) derived in this paper. We have

$$\int_{[\bar{w}, s]} t dF_w(t) = \int_{[\bar{w}, s]} \lambda t e^{-\lambda s} dt = \bar{w} e^{-\lambda \bar{w}} - s e^{-\lambda s} + \frac{1}{\lambda} (e^{-\lambda \bar{w}} - e^{-\lambda s}), \quad (36)$$

$$\int_{[\bar{w}, \infty)} \left[ \int_{[\bar{w}, s]} s dF_w(t) \right] dF_w(s) = e^{-2\lambda \bar{w}} \left[ \frac{\bar{w}}{2} + \frac{1}{4\lambda} \right].$$

$$1 - F(\bar{w}) = e^{-\lambda \bar{w}}. \quad (37)$$

Using these expressions in equation (19):

$$A_L(0, \infty) = \frac{(1 - \alpha) \left[ \frac{2\lambda \bar{w} + 1}{4(\lambda \bar{w} + 1)} \right]}{\theta e^{\lambda \bar{w}} + 1 - \alpha}. \quad (38)$$

In order to get some intuition about equation (38) and to compare this result with the previous ones, note the following. If  $\theta = 0$  and  $\bar{w} = 0$ ,  $A_L(0, D) = \frac{1}{4}$ , the well-known value of the area under the Lorenz curve in the case of the exponential distribution. This value does not depend on the parameter of the distribution  $\lambda$  and, through equation (20), leads to a value of the Gini coefficient equal to  $\frac{1}{2}$ . As one would expect given the skewness of the exponential distribution, the wage-income inequality here happens to be higher than that found in the case of the uniform distribution under the same circumstances.

On the other hand, if  $\bar{w} = 0$  and  $\theta \neq 0$  or  $\alpha \neq 0$ ,  $A_L(0, \infty) = \frac{\frac{1}{2}(1-\alpha)}{\theta + 1 - \alpha} < \frac{1}{4}$ , showing the positive impact on the Gini coefficient of the unemployment probability  $\theta$ . It is also easy to see that the area under the Lorenz curve decreases (and the Gini coefficient increases) when  $\theta$  moves upward departing from  $\theta = 0$  ( $\frac{dA_L}{d\theta}|_{\theta=0} > 0$ ). Given proposition 4, a positive correlation between income inequality and the rate of unemployment can once more be formally established for all values of these variables when either  $\theta$  or  $\alpha$  increases.

As in the previous example, let us now proceed to find the Gini coefficient in this case, now as a function of  $\theta$ ,  $\alpha$ , and  $\lambda$ . Using equation (38), the only thing we have to do is to calculate the reservation wage as a function of such parameters. From equation (2),  $\bar{w}$  is now implicitly determined by the following equation:

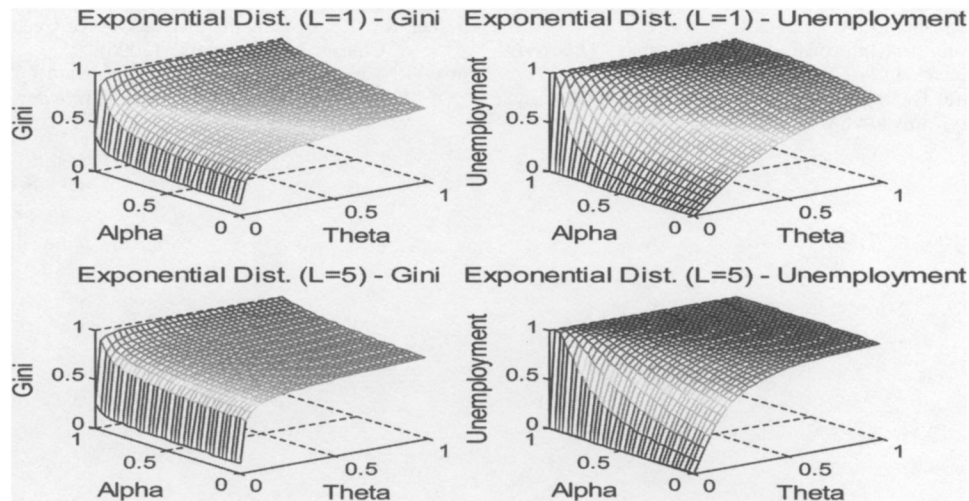
$$k(\bar{w}) = \bar{w} - \frac{\beta(1 - \alpha)e^{-\lambda \bar{w}}}{1 - \beta(1 - \theta)} = 0. \quad (39)$$

To obtain the final expression for the Gini coefficient, use the implicit solution of equation (39) in (38) and subsequently use equation (15).

Making, again,  $\beta = 0.98$ , the upper part of figure 2 presents the Gini coefficient and the rate of unemployment when  $\lambda = 1$ . The lower



FIGURE 2.—EXPONENTIAL DISTRIBUTION



part of figure 2 deals with the case in which  $\lambda = 5$ . In both cases,  $\theta$  and  $\alpha$  are allowed to assume values in  $(0, 1)$ . It is easy to see that the positive correlation between the Gini coefficient and the rate of unemployment is once more established for all values of  $\theta$  and  $\alpha$  when one of these parameters increases.

The examples outlined in this section, together with figures 1 and 2, lead to some interesting observations. First, as one would expect, for the same values of both  $\theta$  and  $\alpha$  the values of the Gini coefficient are apparently higher in the case of uniform distribution than in the case of point-mass distribution; and higher in the case of the exponential distribution of wage offers than in the case of uniform distribution.

Second, the dependence of the Gini coefficient and of the unemployment rate with respect to the two main parameters of the model is rather robust with respect to the distribution of wage offers. The general shape displayed in figures 1 and 2 is basically the same.

Third, in all cases it is clear that increases of either  $\theta$  or  $\alpha$  lead to positive correlations between inequality and unemployment, as shown by the empirical studies mentioned in the first section of this work.

Fourth, the assumption that the reservation wage is higher than the lower bound of the wage-offer distribution, used in the demonstration of proposition 3, does not seem to be important in practice. In all the examples presented, this assumption is not verified, but the positive correlation between the rate of unemployment and the Gini coefficient has been clearly established. The same happens with respect to the assumption that  $\theta$  is close enough to 0, used in the demonstration of proposition 4.

## V. Conclusions

In this paper we have seen that standard job-search models can be helpful in understanding the positive correlation between inequality and unemployment empirically assessed, among others, by Cardoso (1993), Cardoso et al. (1995), Nolan (1986), Jantti (1994), and Mocan (1999). In such models, the correlation is generated, independently of the initial distribution of job offers, by increases of either the probability of unemployment (separation rate) or the probability that an unemployed worker does not find a job offer.

As a byproduct of the analysis, the work has also provided a general expression that allows for a straightforward calculation of the

Gini coefficient of income inequality given any initial distribution of wage offers.

Three numerical examples have been used to illustrate the methods and the main results of the work.

Regarding future research, it should be pointed out that the model presented here is very stylized, and abstracts from general equilibrium concerns regarding the labor market. In the model, both the separation probability  $\theta$  and the job-offer finding rate  $(1 - \alpha)$  are exogenously given. Whereas such assumptions are convenient for analytical purposes, further research should try to extend the results obtained here to a general equilibrium framework. In particular, it would be interesting to endogenize the job-offer finding probability as a function of some other primitive variables in a more comprehensive model. Such an extended model could be useful for understanding empirical evidences based on inflow and outflow probabilities.

Lastly, in the standard job-search model used here, wage-income inequality is a synonym for income inequality, a simplification that may be dealt with in future work by the introduction of alternative sources of income.

The results derived here must be understood as subject to these simplifications.

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