DATA-DRIVEN MEAN-FIELD GAME APPROXIMATION FOR A POPULATION OF ELECTRIC VEHICLES

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ABSTRACT

For a population of electric vehicles (EVs) we design a data-driven mean-field game and provide analysis of approximated mean-field equilibrium points based on a receding horizon approach. The model involves stochastic disturbances on the data that drive the game. Some numerical studies illustrate the efficacy of the proposed strategies.

Index Terms— Mean-field games, Smart-grid

1. INTRODUCTION

In the spirit of dynamic demand management [1, 2, 3, 4], the main contribution of this paper is the design of a mean-field game for a population of EVs. We introduce the mean-field equilibrium and investigate ways in which we can obtain an approximation of such equilibrium point via simple calculations. Mean-field games were formulated in [5] and independently also in [6, 7]. The main idea is to turn the game into a sequence of infinite-horizon receding horizon optimization problems that each EV solves online. The online computation is driven by data made available at regular sampling intervals. We provide analysis of the asymptotic stability of the microscopic and macroscopic dynamics. The physical interpretation of the results corresponds to a level of charge and charging mode for each EV which converge to pre-defined reference values. We show through simulations that such convergence properties are guaranteed also when the EVs dynamics are affected by an additional stochastic disturbance which we model using a Brownian motion. Finally, we show the impact of measurement noise on the overall dynamics. Such noise is added to the estimate of the grid frequency and could be also viewed as the consequence of attacks on the part of hackers aiming at compromising the data. An expository work on stochastic analysis and stability is [8].

1.1. Notation

The symbol $\mathbb{E}$ indicates the expectation operator. We use ∂_x and ∂^2_{xx} to denote the first and second partial derivatives with respect to x, respectively. Given a vector $x \in \mathbb{R}^n$ and a matrix $a \in \mathbb{R}^{n\times n}$ we denote by $\|x\|_a^2$ the weighted two-norm $x^T ax$. The symbol $a_{i\bullet}$ means the ith row of a given matrix a. We denote by $Diag(x)$ the diagonal matrix in $\mathbb{R}^{n\times n}$ whose entries in the main diagonal are the components of x. We denote by $dist(X, X^*)$ the distance between two points X and X^* in $\mathbb{R}^n$. We denote by $\Pi_{\mathcal{M}}(X)$ the projection of X onto set $\mathcal{M}$. The symbol ":" denotes the Frobenius product. We denote by $]\xi, \zeta[$ the open interval for any pair of real numbers $\xi \le \zeta$.

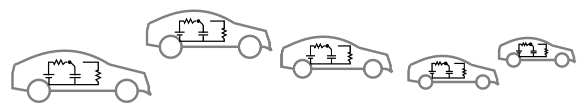

Fig. 1. Population of EVs. assimilated to capacitors

2. POPULATION OF ELECTRIC VEHICLES (EVS)

In a population of electric vehicles (EVs), each EV is modeled by a continuous state, which represents the level of charge $x(t)$, and by a binary state $\pi_{on}(t) \in \{0, 1\}$, which indicates whether the EV is charging (on state) or discharging (off state) at time $t \in [0, T]$. Here $[0, T]$ is the time horizon window. By setting to on state, the level of charge increases exponentially up to a fixed maximum level of charge x_{on} whereas in the off position the level of charge decreases exponentially up to a minimum level of charge x_{off}. Then, the level of charge of each EV in $[0, T)$ is given by:

$$\dot{x}(t) = \begin{cases} -\alpha(x(t) - x_{on}) & \text{if } \pi_{on}(t) = 1 \\ -\beta(x(t) - x_{off}) & \text{if } \pi_{on}(t) = 0 \end{cases}, \qquad (1)$$

where $x(0) = x$ represents the boundary condition at initial time, and where the α, β are the rates of charging and discharging respectively. Both are given positive scalars.

In the spirit of stochastic models as provided in [9, 10], we assume that each EV can be in states on or off in accordance to probabilities $\pi_{on} \in [0, 1]$ and $\pi_{off} \in [0, 1]$. The control input corresponds to the transition rate u_{on} from state off to state on and the transition rate u_{off} from state on to state

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off. Under the assumption that $\dot{\pi}_{on}(t) + \dot{\pi}_{off}(t) = 0$, one can consider only one the dynamics for π_{on}. Then, let us denote $y(t) = \pi_{on}(t)$. For any x, y in the

$$\text{“set of feasible states”} \quad \mathcal{S} :=]x_{off}, x_{on}[\times]0,1[,$$

we obtain the following dynamical system

$$\begin{array}{rcl} \dot{x}(t) & = & \Big(y(t)\Big[-\alpha(x(t)-x_{on})\Big] \\ & & +(1-y(t))\Big[-\beta(x(t)-x_{off})\Big] \\ & =: & f(x(t),y(t)),\ t\in[0,T), \quad x(0)=x, \\ \dot{y}(t) & = & \Big(u_{on}(t)-u_{off}(t)\Big) \\ & =: & g(u(t)),\ t\in[0,T), \quad y(0)=y. \end{array} \tag{2}$$

A macroscopic description of the model can be obtained by introducing the probability density function $m : [x_{on}, x_{off}] \times [0,1] \times [t,T] \to [0,+\infty[, (x,y,t) \mapsto m(x,y,t)$, for which it holds $\int_{x_{on}}^{x_{off}} \int_{[0,1]} m(x,y,t)dxdy = 1$ for every t in $[0,T)$. Furthermore, let

$$m_{on}(t) := \int_{x_{on}}^{x_{off}} \int_{[0,1]} y m(x,y,t) dxdy.$$

Analogously, let $m_{off}(t) := 1 - m_{on}(t)$. At every time t the grid frequency depends linearly on the discrepancy between the percentage of EVs in state *on* and a nominal value. We refer to such deviation to as *error* and denote as $e(t) = m_{on}(t) - \overline{m}_{on}$. Here $\overline{m}_{on}$ is the nominal value. Note that the more EVs are in state on if compared with the nominal value, the more the the grid frequency presents negative deviation from the nominal value). In other words the grid frequency depends on the mismatch between the power supplied and consumed. For sake of simplicity, henceforth we assume that the power supply is constant and equal to the nominal power consumption all the time.

2.1. Forecasting based on Holt's model

The information on the error $e(t)$ is relevant to let the EVs adjust their best response charging policies. This information is available at discrete times t_k, and after receiving a new data the players first forecast the next value of the error and based on that they compute their best-response strategies over an infinite planning horizon, namely $T \to \infty$.

Once the forecasted error is obtained, the players assume that this value remains fixed throughout the planning horizon. Note that this assumption is mitigated if the interval between consecutive samples is sufficiently small. Once the sequence of optimal controls is obtained, the players implement their first controls until a new sample becomes available. In other words, the players implement a *receding horizon* technique which consists in a multi-step ahead *action* horizon. In the following we denote the length of the interval between consecutive samples as $\delta = t_{k+1} - t_k$. In the following we can take $\delta = 1$ without loss of generality.

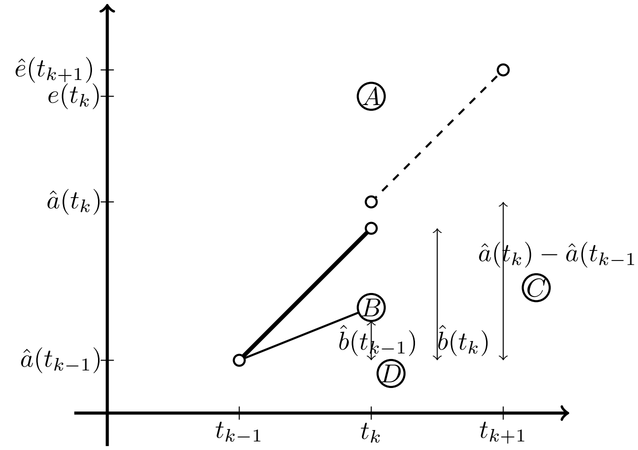

Fig. 2. One iteration of the forecasting method based on the Holt's model

Forecasting is based on the Holt's model, which assumes that the underlying model for the error is linear with respect to time, and has permanent component

$$\mu(t_k) = a(t_k) + b(t_k)t_k.$$

For the error we then have

$$e(t_k) = \mu(t_k) + \epsilon(t_k) = a(t_k) + b(t_k)t_k + \epsilon(t_k)$$

When the new observation $e(t_k)$ becomes availabe, the estimate of the constant is updated as follows by

$$\hat{a}(t_k) = \hat{\alpha} e(t_k) + (1-\hat{\alpha})\Big(\hat{a}(t_{k-1}) + \hat{b}(t_{k-1})\Big)$$

Similarly, the estimate for the slope is obtained as

$$\hat{b}(t_k) = \hat{\beta}(\hat{a}(t_k) - \hat{a}(t_{k-1})) + (1-\hat{\beta})\hat{b}(t_{k-1})$$

Finally, the forecast of the error at time t_{k+1} obtained at time t_k is given by

$$\hat{e}(t_{k+1}) = \hat{a}(t_k) + \hat{b}(t_k)$$

The forecasting iteration at time t_{k-1} using the Holt's model is illustrated in Fig. 2. The new level $\hat{a}(t_k)$ is in the convex hull between the last obtained sample $e(t_k)$ (indicated by circle A) and the forecasted value from previous iteration $\hat{a}(t_{k-1}) + \hat{b}(t_{k-1})$ (indicated by circle B). Likewise, the new slope $\hat{b}(t_k)$ is in the convex hull between the last observed slope $\hat{a}(t_k) - \hat{a}(t_{k-1})$ (indicated by circle C) and the previous forecast $\hat{b}(t_{k-1})$ (indicated by circle D).

2.2. Receding horizon

The running cost for each player introduces the dependence on the distribution $m(x,y,t)$ through the error $\hat{e}(t_{k+1})$ and is given below:

$$
\begin{aligned}
&c(\hat{x}(\tau,t_k),\hat{y}(\tau,t_k),\hat{u}(\tau,t_k),\hat{e}(t_{k+1}))\\
&=\tfrac{1}{2}\Big(q\hat{x}(\tau,t_k)^2+r_{on}\hat{u}_{on}(\tau,t_k)^2+r_{off}\hat{u}_{off}(\tau,t_k)^2\Big)\\
&\qquad+\hat{y}(\tau,t_k)(S\hat{e}(t_{k+1})+W),
\end{aligned}
\tag{3}
$$

where q, r_{on}, r_{off}, and S are opportune positive scalars.

In the running cost (3) we have four terms. One term penalizes the deviation of the EVs' levels of charge from the nominal value, which we set to zero. Setting the nominal level of charge to a nonzero value would simply imply a translation of the origin of the axes. This penalty term is then given by $\frac{1}{2}qx(\tau,t_k)^2$.

A second term penalizes fast switching and is given by $\frac{1}{2}r_{on}u_{on}(\tau,t_k)^2$; this cost is zero when either $u_{on}(\tau,t_k)=0$ (no switching) and is maximal when $u_{on}(\tau,t_k)=1$ (probability 1 of switching). The same reasoning applies to the term $\frac{1}{2}r_{off}u_{off}(\tau,t_k)^2$. A positive error forecast $e(t_{k+1})>0$, means that demand exceeds supply. Therefore, with the term $y(\tau,t_k)Se(\tau,t_k)$ we penalize the EVs that are in state on when power consumption exceeds the power supply ($e(\tau,t_k)>0$). On the contrary, when the power supply exceeds power consumption, the error is negative, i.e. $e(\tau,t_k)>0$, and the related term $y(\tau,t_k)Se(\tau,t_k)$ penalizes the EVs that are in state off. The last term is $y(\tau,t_k)W$ and is a cost on the power consumption; when the EV is in state on the consumption is W. We also consider a terminal cost $g:\mathbb{R}\to[0,+\infty[, x\mapsto g(x)$ to be yet designed.

Let the following update times be given, $t_k=t_0+\delta k$, where $k=0,1,\ldots$. Let $\hat{x}(\tau,t_k)$, $\hat{y}(\tau,t_k)$ and $\hat{e}(t_{k+1})$, $\tau\geq t_k$ be the predicted state of player i and of the error $e(t)$ for $t\geq t_k$, respectively. The problem we wish to solve is the following one.

For all players and times $t_k, k=0,1,\ldots$, given the initial state $x(t_k)$, and $y(t_k)$ and $\hat{e}(t_{k+1})$ find

$$
\hat{u}^\star(\tau,t_k)=\arg\min\mathcal{J}(\hat{x}(t_k),\hat{y}(t_k),\hat{e}(t_{k+1})),\hat{u}(\tau,t_k)),
$$

where

$$
\begin{aligned}
&\mathcal{J}(\hat{x}(t_k),\hat{y}(t_k),\hat{e}(t_{k+1}),\hat{u}(\tau,t_k))\\
&=\lim_{T\to\infty}\int_{t_k}^{T}c(\hat{x}(\tau,t_k),\hat{y}(\tau,t_k),\hat{u}(\tau,t_k),\hat{e}(t_{k+1}))d\tau
\end{aligned}
\tag{4}
$$

subject to the following constraints:

$$
\begin{aligned}
\dot{\hat{x}}(\tau,t_k) &= \Big(\hat{y}(\tau,t_k)\Big[-\alpha(\hat{x}(\tau,t_k)-x_{on})\Big]\\
&\quad +(1-\hat{y}(\tau,t_k))\Big[-\beta(\hat{x}(\tau,t_k)-x_{off})\Big]\\
&=: f(\hat{x}(\tau,t_k),\hat{y}(\tau,t_k)),\ \tau\in[t_k,T),\\
\dot{\hat{y}}(\tau,t_k) &= \Big(\hat{u}_{on}(\tau,t_k)-\hat{u}_{off}(\tau,t_k)\Big)\\
&=: g(\hat{u}(\tau,t_k)),\ \tau\in[t_k,T),
\end{aligned}
\tag{5}
$$

The above set of constraints involves the predicted state dynamics of the individual player and of the rest of the population through $\hat{e}(t_{k+1})$. The constraints also involve the boundary conditions at the initial time t_k. Note that a player restrains the error to be constant over the planning horizon.

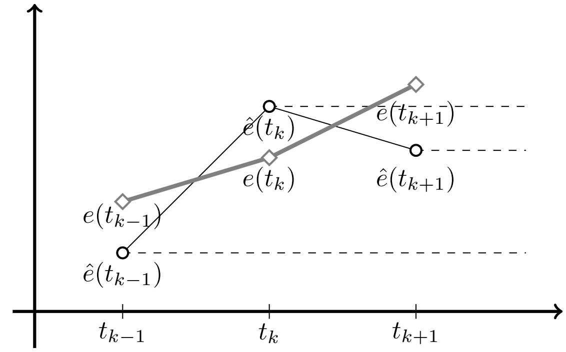

Fig. 3. One iteration of the forecasting method based on the Holt's model

At t_{k+1} a new sample data $e(t_{k+1})$ becomes available. Then the players update their best-response strategies, which we refer to as *receding horizon control policies*. Consequently, for the individual player, we obtain the *closed-loop* system

$$
\begin{aligned}
\dot{\hat{x}}(\tau,t_k) &= \Big(\hat{y}(\tau,t_k)\Big[-\alpha(\hat{x}(\tau,t_k)-x_{on})\Big]\\
&\quad +(1-\hat{y}(\tau,t_k))\Big[-\beta(\hat{x}(\tau,t_k)-x_{off})\Big]\\
&=: f(\hat{x}(\tau,t_k),\hat{y}(\tau,t_k)),\ \tau\in[t_k,T),\\
\dot{\hat{y}}(\tau,t_k) &= \Big(u^{RH}_{on}(\tau)-u^{RH}_{off}(\tau)\Big)\\
&=: g(u^{RH}(\tau)),\ \tau\in[t_k,T),
\end{aligned}
\tag{6}
$$

where the receding horizon control law $u^{RH}(\tau)$ satisfies

$$
u^{RH}(\tau)=\hat{u}^\star(\tau,t_k),\quad \tau\in[t_k,t_{k+1}).
$$

Figure 3 provides a graphical illustration of the receding horizon model. Given three consecutive samples $e(t_{k-1})$, $e(t_k)$ and $e(t_{k+1})$ (diamonds), the forecasts obtained from the Holt's method are $\hat{e}(t_{k-1})$, $\hat{e}(t_k)$ and $\hat{e}(t_{k+1})$ (circles). The dashed lines indicate that each value is kept fixed throughout the horizon when running the receding horizon optimization.

In [11], it was showed that the mean-field game can be approximated by a sequence of linear quadratic problems, one per each player. Note that the receding horizon problems constitute a sequence of consecutive approximations of the mean-field game. In [11] it was also shown that the an explicit solution can be obtained by solving three matrix equations. We refer to such solution as receding horizon equilibrium. This equilibrium is an approximation of the mean-field equilibrium. We use the same approach here to perform the following numerical analysis.

3. NUMERICAL STUDIES

Consider a population of 100 EVs, and set $n = 100$. Simulations are carried out with MATLAB on an Intel(R) Core(TM)2 Duo, CPU P8400 at 2.27 GHz and a 3GB of RAM. Simulations involve a number of iterations $T = 1000$. Parameters are set as follows. The step size $dt = 0.1$, the charging and discharging rates are $\alpha = \beta = 1$, the lowest and highest level of charge are $x_{on} = 1$, and $x_{off} = -1$, respectively; the penalty coefficients are $r_{on} = r_{off} = 20$, and $q = 10$, and the initial distribution is normal with zero-mean and standard deviation $std(m(0)) = 0.3$ for x and mean equal to 0.5 and standard deviation $std(m(0)) = 0.1$ for y.

For the optimal control we get

$$u^* = -R^{-1}B^T[PX + \Psi],$$

where P is calculated using the MATLAB in-built function `[P]=care(A,B,Q,R)`. This function takes the matrices as input and returns the solution P to the algebraic Riccati equation. Assuming $BR^{-1}B^T\Psi \approx C$ we get the closed-loop dynamics

$$X(t + dt) = X(t) + [A - BR^{-1}B^T P]X(t)dt.$$

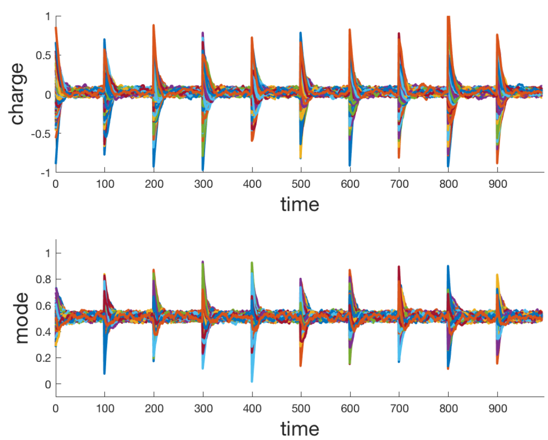

Fig. 4. Time plot of the state of each EV when no noise affects the measure of $e(t_k)$

Figure 4 displays the time plot of the state of each EV, namely level of charge $x(t)$ (top row) and charging mode $y(t)$ (bottom row) when no noise affects the measure of $e(t_k)$, namely $\epsilon(t_k) = 0$. Dynamics is affected by some Gaussian noise in $\mathcal{N}(0, 0.1)$. The simulation is carried out assuming that any 10 iterations a new sample is obtained and any 100 iterations initial states are re-initialized randomly.

Figure 5 displays the time plot of the state of each EV, namely level of charge $x(t)$ (top row) and charging mode $y(t)$ (bottom row) when Gaussian noise affects the measure of $e(t_k)$, namely $\epsilon(t_k) \in \mathcal{N}(0, 0.5)$. Due to the estimation noise, we observe oscillations of the level of charges around that value.

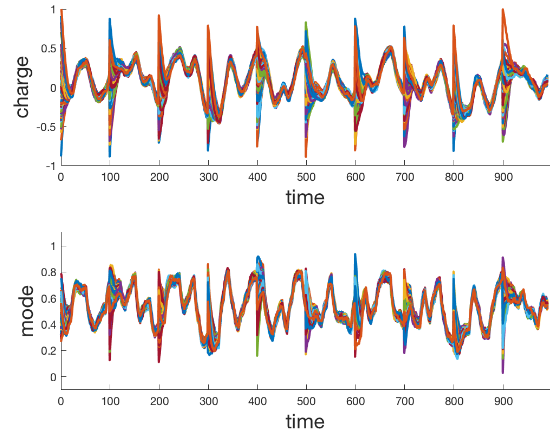

Fig. 5. Time plot of the state of each EV when Gaussian noise affects the measure of $e(t_k)$.

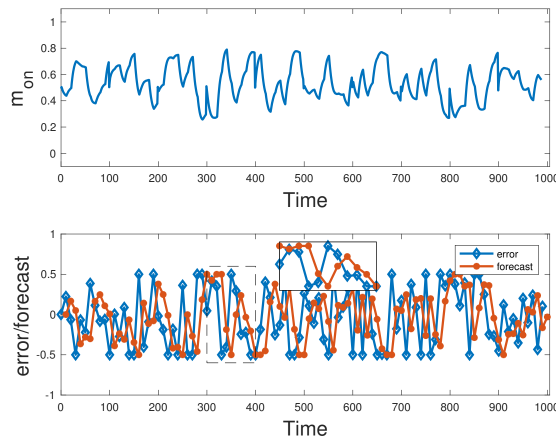

Fig. 6. Time plot of data $e(t_k)$ and forecasts $\hat{e}(t_{k+1})$.

Finally Fig. 6 displays the time plot of the observed data $e(t_k)$ and forecasts $\hat{e}(t_{k+1})$.

4. CONCLUDING REMARKS

The model proposed advances knowledge on mean field games with applications to a population of EVs. The model takes into account measurement disturbances. We have studied equilibria and designed stabilizing charging strategies. The estimation disturbance can be viewed as an attempt to compromise data. In the future, we will extend our study to dynamic pricing, coalitional production aggregation, and the design of incentives to stabilize aggregation of producers.

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