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Research Article

A Swarm Robotic Exploration Strategy Based on an Improved Random Walk Method

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An environment can be searched far more efficiently if the appropriate search strategy is used. Because of the limited individual abilities of swarm robots, namely, local sensing and low processing power, random searching is the main search strategy used in swarm robotics. The random walk methods that are used most commonly are Brownian motion and Lévy flight, both of which mimic the self-organized behavior of social insects. However, both methods are somewhat limited when applied to swarm robotics, where having the robots search repeatedly can result in highly inefficient searching. Therefore, by analyzing the characteristics of swarm robotic exploration, this paper proposes an improved random walk method in which each robot adjusts its step size adaptively to reduce the number of repeated searches by estimating the density of robots in the environment. Simulation experiments and experiments with actual robots are conducted to study the effectiveness of the proposed method and evaluate its performance in an exploration mission. The experimental results presented in this paper show that an area is covered more efficiently using the proposed method than it is using either Brownian motion or Lévy flight.

1. Introduction

A complex problem in robotics and one that has received widespread attention is area exploration, which is used for various tasks including planetary exploration [1], search and rescue [2], foraging for food [3], and nanoscale drug delivery [4]. In area exploration, the core research issue is how to traverse an unknown area effectively. In a very large environment, it is relatively inefficient to have just one robot traverse the entire area. Instead, the exploration should be done using a multirobot approach, and swarm robots are used widely for this type of area exploration because of their robustness, flexibility, and scalability [5]. Most existing search methods depend on delicate systems of sensors (e.g., odometers and ultrasound radar) and sophisticated mapping algorithms [6, 7]. However, swarm robots, with their limited individual abilities (i.e., local sensing and low processing power), do not support complex localization and mapping, and instead they generally use a random walk (RW) as the area-exploration strategy. RWs can be divided into two categories, namely, (i) uncorrelated RWs, where the direction moved at each step is completely random, and (ii) correlated RWs, where there is a correlation between successive step orientations [8]. The main difference between the two types of RW is that in a correlated RW each step orientation is influenced by either the previous direction or the direction toward a given target. The RW methods studied herein refer mainly to uncorrelated RWs, the most commonly used being Brownian motion (BM) and Lévy flight (LF).

BM is the random motion of particles that are suspended in a fluid, and it results from the particles colliding with the fast-moving molecules of the fluid [9]. This pattern of motion involves a particle alternating its position randomly from one domain to another. BM is a continuous-time stochastic process and is usually described by the Wiener process. Because it is relatively easy to realize, BM has been used widely in robotics for random searching [10–12]. Each robot is regarded as a particle whose step size is normally distributed and each of whose movements is in an isotropically random direction. Wagner et al. [13] used robots with no sensory inputs to cover the gray area by means of BM; even though this method is not optimal, it has the advantages of (i) requiring no sensors and (ii) being relatively inexpensive and tolerant. Furthermore, a novel RW method based on BM has been proposed to improve the area coverage ratio by allowing the motion of each robot to be influenced by landmarks installed in the environment [14].

An LF is an RW by which the walker can travel a large distance by taking many short steps and the occasional long step [15]. The step size has a power-law distribution, and a robot using LF is more likely to reach a remote area than is one that uses BM. Generally speaking, searching for a target and foraging can both be viewed as exploration missions. The foraging behaviors of many creatures in nature resemble LF, such as (i) the flight trajectories of albatrosses when foraging [16, 17], (ii) the intermittent foraging flight trajectories of fruit flies [18], and (iii) the flight trajectories of pelagic seabirds [19]. As Viswanathan et al. [20] noted, when the target sites are sparse and can be visited any number of times, an inverse-square power-law distribution of flight lengths, corresponding to LF, is an optimal strategy. When the target sites are abundant, simple Brownian motion is sufficiently efficient [21]. Fricke et al. evaluated the effectiveness of a LF search strategy and used a genetic algorithm to map the relationship between the search parameters and target configurations [22]. For the exploration missions of swarm robotics, many studies have used LF as the search strategy to improve the searching efficiency (SE). Fujisawa and Dobata showed that the LF search strategy maximized the SE of swarm robots by using pheromone to communicate with each other [23]. Schroeder et al. proposed a control law that combines a virtual pheromone and LF for efficient area coverage [24].

Although an exploration mission can be completed using either BM or LF, several deficiencies remain. BM is better for local searching and LF is better for global searching. For a balance between local searching and global searching, Deshpande et al. proposed a control law for efficient area coverage in a robot swarm by using a pheromone and by switching adaptively between BM and LF [25]. Sutantyo combined LF and an artificial potential field to improve the SE, with the potential field generating a repulsive force between pairs of robots, thereby dispersing neighboring robots [26]. Palmieri proposed using a weighted RW to find multiple paths among dynamical obstacles to improve the performance of robot navigation [27]. To explore the correlation between RW methods and the environment, Dimidov et al. used a swarm of Kilobots to search for a static target in different environments; the experimental results revealed which type of RW was best suited to each experimental scenario [28].

All the aforementioned methods achieved a certain degree of progress, but there still exist some problems. For one, BM and LF are used mainly for exploration missions conducted by a single robot. With swarm robots, having too many robots perform the exploration mission concurrently not only produces more instances of physical interference but also causes repeated searches, thereby reducing the SE markedly. The existing random search methods are therefore less efficient for swarm robotic exploration missions. Furthermore, the existing methods are based mostly on pheromones to simulate the foraging behavior of an ant colony, but such methods have considerable limitations in practical applications. Instead, what is proposed herein is an improved RW method in which each robot adjusts its step size adaptively by estimating the density of robots in the environment. The average time interval between two instances of physical interference is used to estimate the robot density; the shorter the time interval, the higher the density. When the robot density is relatively high, each robot searches a relatively small region with a relatively short step size; when the robot density is relatively low, each robot searches a relatively large region with a relatively long step size. To maintain a certain minimum distance between any two robots, one robot will turn directly away from another robot if obstacle avoidance occurs between them. Eventually, by adjusting their step sizes adaptively and by controlling their searching directions, the swarm robots become distributed evenly in the environment and each robot searches its own local area, thereby improving the SE.

The rest of this paper is organized as follows. Section 2 reviews the classical RW methods and Section 3 introduces the improved RW method. Section 4 reports on experiments that were conducted to assess the effectiveness of the improved RW method. Finally, Section 5 presents the conclusions and suggestions for future work.

2. Random Walk Methods

When no environmental information can be obtained, a random search is a basic search strategy for both animals and robots, especially for swarm robots that have limited individual abilities (i.e., local sensing and low processing power) and do not support more-complex search strategies. The commonly used RW methods are BM and LF.

2.1. Brownian Motion. BM describes the random motion of particles suspended in a fluid that is caused by the interactions between the particles. BM can be used to guide the random motion of robots and it has been widely used in robotics. In an exploration mission, a robot moves ahead by a given step size that is produced by the BM and then turns to a direction chosen randomly from the search space. In practice, BM can be viewed as a continuous-time stochastic process and can be described by the Wiener process. Mathematically, the Wiener process W_t is characterized by the following four properties:

(1) $W_0 = 0$;

(2) W_t is almost surely continuous;

(3) W_t has independent increments: if $0 \le s_1 < t_1 \le s_2 < t_2$, then $W_{t_1} - W_{s_1}$ and $W_{t_2} - W_{s_2}$ are independent random variables;

(4) W_t has Gaussian increments: $W_{t+u} - W_t \sim \mathcal{N}(0, u)$, where $\mathcal{N}(0, u)$ denotes the normal distribution with zero expectation and variance u.

In an RW by a single robot, the step size of the robot has a normal distribution with an expected value of zero and a variance of $u = 1$. To show how a single robot moves on an RW, a simulation experiment was performed in a fixed area.

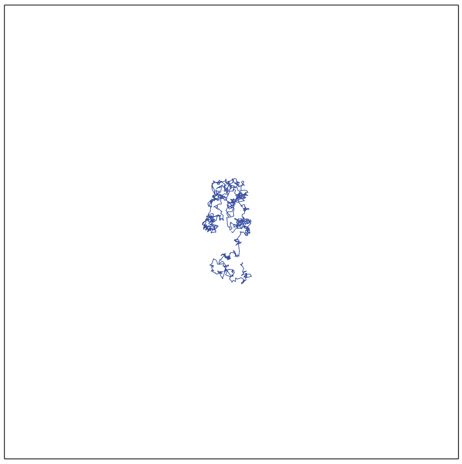

Figure 1: Random walk (RW) of one robot with Brownian motion (BM).

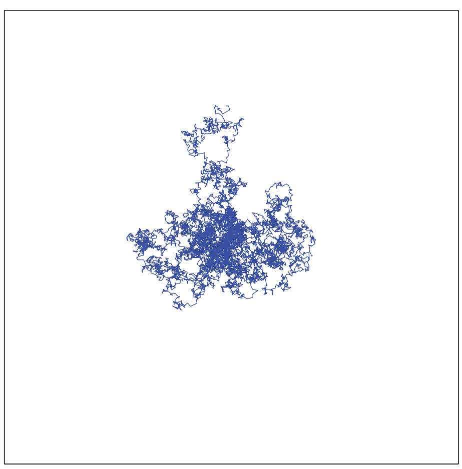

Figure 2: RWs of 10 robots with BM.

Figure 1 shows the trajectory of the robot over a given time; the robot tends to search around its original area, which is good for a local search but poor for a global search. Figure 2 shows the trajectories of 10 robots over a given time; with RWs, the robots produce too many repeated searches, which reduces the SE greatly.

2.2. Lévy Flight. An LF is an RW in which the step size has a heavy-tailed probability distribution that can be expressed as follows [20]:

$$P(s) = s^{-\lambda}, \tag{1}$$

where s is the step size with $1 < \lambda \leq 3$. LF generates a smaller step size with high frequency and occasionally a larger step size. In an exploration mission, this occasional larger step size allows the robot to reach the full range of the search space to complete a global search, whereas a robot with the smaller step size tends to complete a local search.

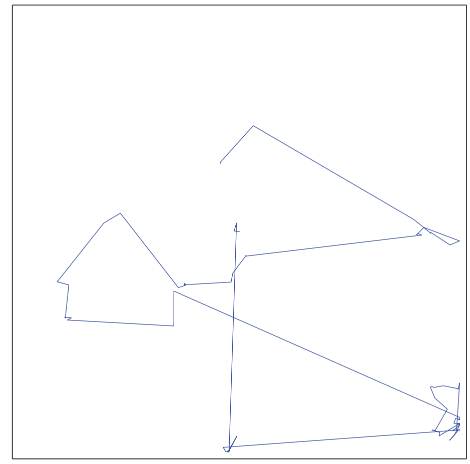

Figure 3: RW of one robot with Lévy flight (LF). The step size was generated according to (2) with $\beta = 1.5$.

This paper uses the method proposed by Mantegna [29] to calculate the LF step size, namely,

$$s = \frac{u}{|v|^{1/\beta}}, \tag{2}$$

where $\beta \in [0.3, 1.99]$; u and v are two normal stochastic variables with standard deviations σ_u and σ_v, respectively:

$$\begin{aligned} u &\sim N\left(0, \sigma_u^2\right), \\ v &\sim N\left(0, \sigma_v^2\right), \end{aligned} \tag{3}$$

$$\begin{aligned} \sigma_u &= \left\{ \frac{\Gamma(1+\beta)\sin(\pi\beta/2)}{\Gamma\left[(1+\beta)/2\right] 2^{(\beta-1)/2}\beta} \right\}^{1/\beta}, \\ \sigma_v &= 1, \end{aligned} \tag{4}$$

where $\Gamma(z)$ is the gamma function.

Figure 3 shows that a robot using LF can better complete the exploration mission than one using BM. However, Figure 4 shows that having multiple robots performing the exploration mission concurrently results in many repeated searches. Therefore, although LF leads to better searching, some problems remain. In the experiments, the robots used infrared sensors to measure the proximity of objects up to 6 cm away. If the proximity between a robot and the boundary was less than 6 cm, then the robot turned to another direction to avoid the boundary; consequently, the robots never collided with the boundary.

2.3. Other Random Walk Methods. To improve the SE, some researchers have proposed other RW methods. For example, Sutantyo et al. proposed the combination of LF and an artificial potential field for multirobot explorations [26]. The LF generates the step size of the movement, while the artificial potential field improves the dispersion efficiency during deployment by generating a repulsive force between pairs of robots to disperse neighboring robots.

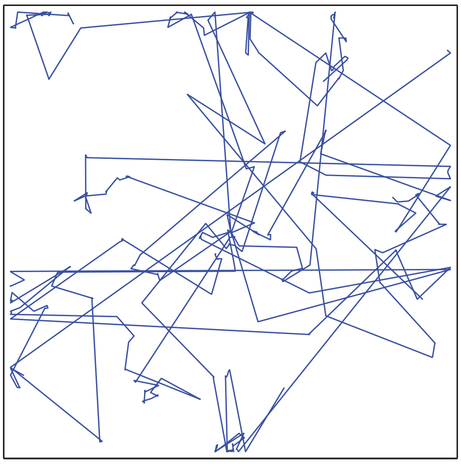

Figure 4: RWs of 10 robots with LF. The step size was generated according to (2) with $\beta = 1.5$.

Most other RWs are based on a virtual pheromone that serves as an indirect means of communication for swarm robots. When searching an environment, a robot deposits a pheromone that is detected by the other robots, along with its relative concentration. In an RW, a robot moves in an isotropically random direction. However, by using pheromone communication, a robot decides where to move based on the information conveyed by the pheromone, thereby not only improving the SE but also reducing the amount of physical interference between robots. Fujisawa et al. showed that the LF search strategy enhances the SE of swarm robots by using pheromone to communicate with other robots [23, 30].

However, the RW methods that involve an artificial potential field or a virtual pheromone are difficult to implement in practical applications. Consequently, only the basic RWs (i.e., BM and LF) are studied herein and are compared with the proposed improved RW.

3. Improved Random Walk Method

Because the existing RW methods have relatively low SE and are not particularly suited to swarm robotic exploration, the present paper proposes an improved RW method for completing such an exploration mission.

The best way to improve the SE is to reduce the number of repeated searches. These can be divided into two categories, namely, (i) those produced by the same robot (as in Figure 1) and (ii) those produced by other robots (as in Figures 2 and 4). Regarding category (i), because a robot with a small step size produces many repeated searches, the number of these can be reduced by increasing the step size. Indeed, Figure 3 shows that a robot with a large step size produces fewer repeated searches. Regarding category (ii), when many robots search an environment concurrently, BM and LF both produce many more repeated searches. Repeated searches are inevitable when two robots are close together, as can be seen in Figure 2; when the two robots are farther apart, the one with the larger step size can still produce many category (ii) repeated searches, as can be seen in Figure 4. To reduce the number of such repeated searches, the robots should therefore maintain a certain separation and the step size should be set to an appropriate value. In this way, if the swarm robots can be distributed evenly in the environment so that each robot searches only its local area, there will be fewer repeated searches.

Following this line of reasoning, an RW method is proposed herein based on the density of robots in the environment. When the robot density is high, each robot should search a small area with a small step size; when the robot density is low, each robot should search a large area with a large step size. To ensure that each robot occupies its own separate area, a robot that encounters another robot turns in the opposite direction when obstacle avoidance occurs between them. By adjusting the step size of each robot and controlling the direction of obstacle avoidance, the proposed method not only distributes the robots evenly in the environment but also causes them to search locally with an adaptive step size.

In an exploration mission, the area of the environment is usually unknown and the number of robots can change. Moreover, because of the limited sensory capabilities of swarm robots, the robot density cannot be calculated directly. However, the higher the robot density is, the more instances of physical interference (e.g., obstacle avoidance between robots) arise. Unfortunately, because the robots do not communicate with each other in the present case, an individual robot cannot assess the amount of physical interference, which is a global variable. Instead, the robot density is estimated from the average time interval ($\overline{t}$) between two instances of physical interference for a single robot; the smaller this average time interval, the higher the robot density.

In the proposed RW method, when a robot either moves forward by a given step size or encounters another robot, it calculates its step size as

$$S_t = \begin{cases} v * \overline{t} + k * S_{t-1}, & \Delta t \geq \overline{t} \\ v * \overline{t} - k * S_{t-1}, & \Delta t < \overline{t}, \end{cases} \tag{5}$$

where S_t is the step size with which the robot should move, S_{t-1} is the step size that the robot calculated last time, v is the speed at which the robot moves, and k $(0 < k < 1)$ is an adjustment factor used to regulate the contribution of the previous step size S_{t-1}. The variable $\overline{t}$ is the average time interval between two instances of physical inference and is used to estimate the robot density in the entire search area; $\overline{t}$ changes with time and is updated when obstacle avoidance occurs. The variable Δt is the time between the current instance of physical inference and the previous one and is used to estimate the robot density in the local search area. When $\Delta t \geq \overline{t}$, the local robot density is lower than the global one; in this case, the local area contains fewer robots and therefore the robot in question should use a larger step size to search its area. When $\Delta t < \overline{t}$, the local robot density is higher than the global one; in this case, the local area contains

Table 1: Comparison of proposed RW method with other methods for different numbers of robots. Mean: average coverage ratio; Std.: standard deviation.

Methods	Brownian motion		Lévy flight		Our method	
	Mean	Std.	Mean	Std.	Mean	Std.
1	1.10%	9.31×10^{-4}	4.28%	4.52×10^{-3}	5.25%	2.63×10^{-3}
10	3.45%	3.47×10^{-3}	33.02%	1.11×10^{-2}	40.81%	1.10×10^{-2}
20	4.59%	2.91×10^{-3}	54.05%	1.35×10^{-2}	63.57%	1.13×10^{-2}
30	5.51%	4.17×10^{-3}	68.45%	1.07×10^{-2}	77.12%	1.59×10^{-2}

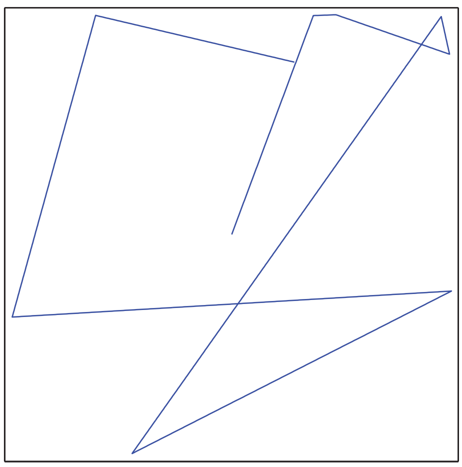

Figure 5: RW of one robot with the proposed method.

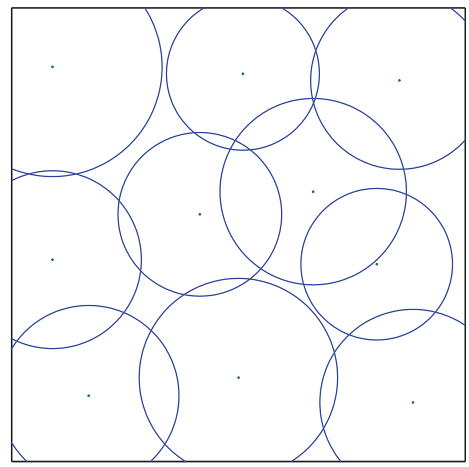

Figure 7: Distribution of robots and step size of each robot after exploration mission.

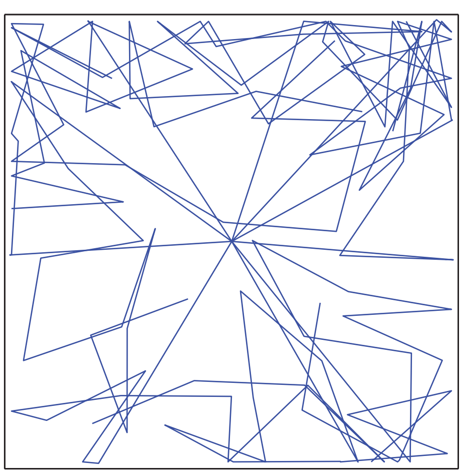

Figure 6: RWs of 10 robots with proposed method.

more robots and therefore the robot in question should use a smaller step size to search its area.

When there is only one robot, the proposed RW method becomes simply the linear search method. Figure 5 shows that with the linear search method, a robot makes fewer repeated searches. Figure 6 shows the trajectories of 10 robots performing the exploration mission.

Figure 7 shows that, by adjusting the step size adaptively, the proposed method distributes the robots evenly in the environment. As the exploration mission proceeds, the step sizes of the robots gradually converge. Moreover, because each robot searches only its local area, there are fewer repeated searches.

4. Experiments and Results

4.1. Simulation Experiments. To test the performance of the proposed RW method, simulation experiments were performed on the Webots platform. Figure 8 shows a screenshot of the simulation interface at the beginning of a simulation experiment. The effectiveness of the proposed RW method was evaluated by comparing its results with those of BM and LF. After each simulation experiment, MATLAB was used to process the image and calculate the coverage ratio (i.e., the ratio of the explored area to the total area).

For adequate comparison, we performed simulation experiments with one, 10, 20, and 30 robots. Each experiment lasted for 10 min (600 s), and the experimental area was a square with length of side $L = 20.0$ m. At the beginning of each experiment, all the robots were placed in the central area. Each robot moved with a speed of $v = 0.1$ m/s, and the parameter k was set as 0.1. The purpose of the area exploration was efficient area coverage, with the coverage ratio being used to evaluate the effectiveness of the proposed method. Each robot had a limited detection range, and the domain within the detection range was considered to have been covered only if the footprint of the robot passed over it, the footprint being the area covered by the robot. The coverage ratio is the ratio of

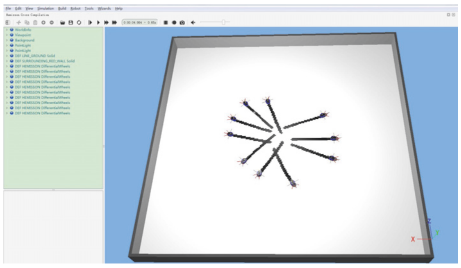

Figure 8: Exploration mission performed by 10 robots in simulation environment.

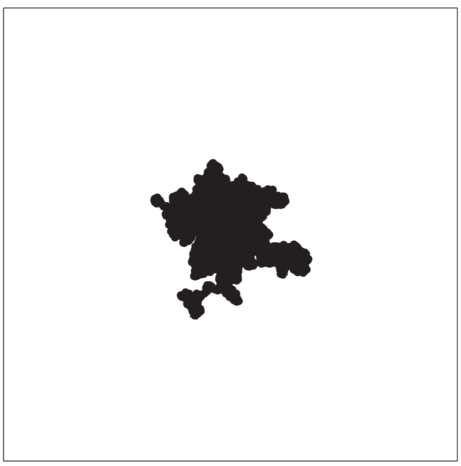

Figure 9: Result of exploration mission using 10 robots with BM. Black denotes explored area.

the explored area to the total area. For each RW method, the results of 20 simulation experiments were averaged to obtain the average coverage ratio.

Figures 9–11 show the results of exploration missions with different RW methods. Because BM generates a relatively small step size, the robots with BM did well in searching locally (Figure 9), but they did not reach other areas and thus many repeated searches were produced, thereby reducing the SE. Figure 10 shows that the robots with larger step sizes reached other areas to reduce the number of repeated searches, but the swarm robots did not become uniformly distributed and the SE remains to be improved. Figure 11 shows that the robots with adaptive step sizes not only reached other areas but also became uniformly distributed in the environment. Consequently, the proposed RW method resulted in far fewer repeated searches.

Table 1 gives the mean and standard deviation of the coverage ratio for each method for different numbers of robots. In each case, the proposed method achieved a much higher coverage ratio, performing significantly better than the other RW methods. Moreover, the standard deviation of each coverage ratio is relatively small, meaning that the SE of each of the three RW methods is stable.

To study how the initial positions of the robots influence the subsequent exploration mission, we also ran simulation experiments in which all the robots were initially distributed randomly in the environment. Figures 12–14 show the results of exploration missions with different RW methods. Because BM does well in a local search, the randomly distributed robots made fewer repeated searches (Figure 12). Because of the larger step size, there was little influence on the robots with LF (Figure 13). By adjusting the step size adaptively, the proposed method again distributed the robots uniformly in the environment (Figure 14), meaning that distributing the robots at random initially does not affect the SE of the proposed method.

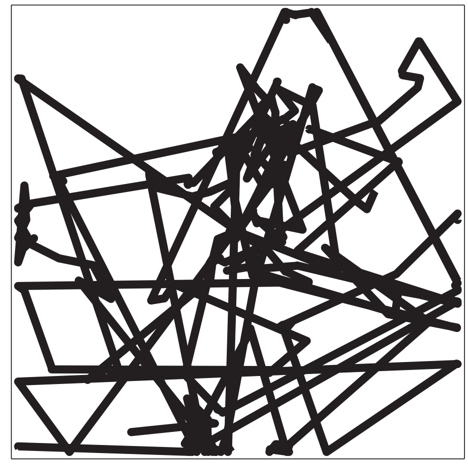

Figure 10: Result of exploration mission using 10 robots with LF. Black denotes explored area.

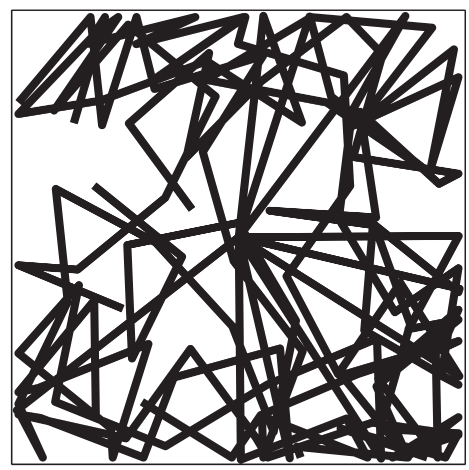

Figure 11: Result of exploration mission using 10 robots with proposed method. Black denotes explored area.

Table 2 gives the mean and standard deviation of the coverage ratio for each method with different numbers of robots. The proposed method still achieves the highest coverage ratio. Compared with Table 1, the exploration mission using BM obtained a higher coverage ratio. From the standard deviation of the coverage ratio, we can conclude that the three RW methods possess stable SE.

Table 2: Comparison of proposed RW method with other methods when robots are distributed at random initially. Mean: average coverage ratio; Std.: standard deviation.

Methods	Brownian motion		Lévy flight		Our method	
	Mean	Std.	Mean	Std.	Mean	Std.
1	1.08%	6.98×10^{-4}	4.40%	4.30×10^{-3}	5.00%	5.71×10^{-3}
10	5.38%	4.04×10^{-3}	33.59%	1.02×10^{-2}	41.34%	1.17×10^{-2}
20	9.73%	4.11×10^{-3}	55.54%	1.07×10^{-2}	62.77%	1.30×10^{-2}
30	13.72%	6.86×10^{-3}	69.34%	9.59×10^{-3}	77.59%	1.08×10^{-2}

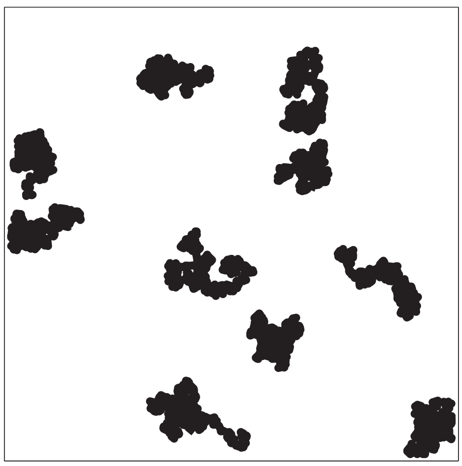

Figure 12: Result of exploration mission using 10 randomly distributed robots with BM. Black denotes explored area.

Figure 13: Result of exploration mission using 10 randomly distributed robots with LF. Black denotes explored area.

4.2. Experiments with Actual Robots. To assess further the effectiveness of the proposed method, we conducted exploration missions using e-puck robots [31], which are open tools that are used extensively for swarm experiments. With a diameter of 6.8 cm and a height 5.3 cm, each e-puck robot is equipped with (i) eight infrared proximity sensors for detecting obstacles, (ii) one CMOS camera to look for objects, (iii) one three-dimensional accelerometer, (iv) three microphones, and (v) one loudspeaker. The environment was designed as a rectangular area of 1.2 m × 1.5 m, and each robot moved at a speed of $v = 0.05$ m/s. In these experiments, it would have been difficult to measure the explored area to assess the effectiveness of the proposed method. Instead, an alternative method was used in which, as shown in Figure 15, the e-puck robots searched for objects scattered in the arena. Normally, the most effective random search method would be the one that finds all the objects in the shortest time. However, because the purpose here was to test the SE of each RW method, whenever a robot found an object the latter was removed from the environment manually.

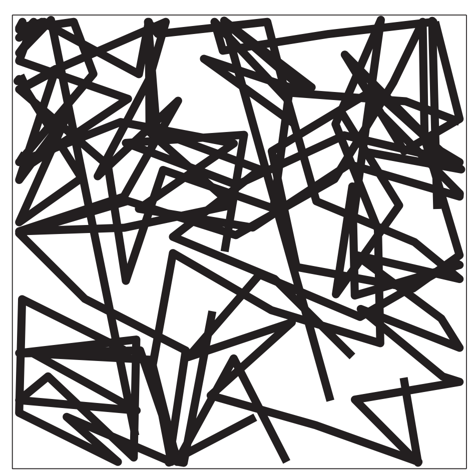

Figure 14: Result of exploration mission using 10 randomly distributed robots with proposed method. Black denotes explored area.

Figure 16 shows a photograph of an experiment with actual robots. After a period of time, the robots were distributed almost uniformly in the environment. During each of these experiments, the e-puck robots found the objects relatively quickly in the initial stage of the experiment because there were relatively many objects. However, with fewer objects, the robots took much longer to find them. All the objects were found in 5 min with the proposed RW method and 7 min with the LF method. However, the robots searched predominantly locally in the middle of the arena with the BM method because of its small step size, and it took 15 min for them to find seven objects. These experiments show that the proposed method resulted in all the objects being found in the least amount of time. It can therefore be concluded that, of BM, LF, and the proposed RW method, the latter is the most effective.

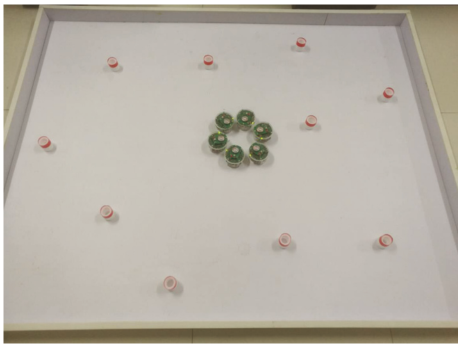

Figure 15: Photograph of arena used to conduct experiments with actual robots. At the beginning of each experiment, six e-puck robots in the waiting state were placed in the middle of the arena in which 10 objects were distributed randomly.

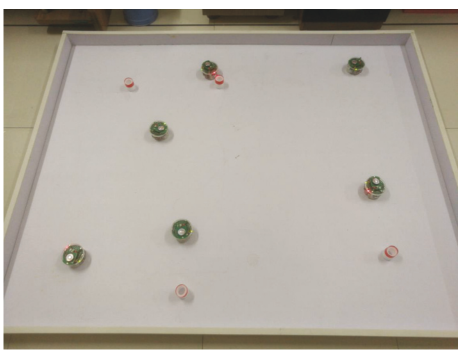

Figure 16: Photograph of experiment with actual robots using proposed method. After a period of time, the robots were distributed almost uniformly in the environment.

5. Conclusions and Future Work

This paper proposed an improved RW method based on the density of robots in the environment. Each swarm robot adjusts its step size adaptively to arrive at other areas, and the proposed method distributes the robots uniformly in the environment to reduce the number of repeated searches. The experimental results showed that the proposed method leads to the area being covered more efficiently than with either BM or LF, thereby improving the SE.

However, despite this, the efficiency of the proposed method still lacks theoretical analysis. Future work will therefore introduce the mathematical theory behind RWs in a straightforward manner and analyze theoretically the efficiency of the proposed RW method.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that no conflicts of interest exit in the submission of this manuscript.

Acknowledgments

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