A Solution of Scattered Field of Particle in Electromagnetic Beam Based on Beam Series Expansion

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***Abstract*—Based on the orthogonalities among the spherical vector wave functions, the expressions of scattering fields are developed as the particle being irradiated by the zeroth-order field and the first-order fields of x, y, and z. A general relation between the expansion coefficients of scattering field and incident field is presented. With the example of the elliptical beam, the scattering property of a particle in beam is investigated. Upon analyzing of the effects of the beam waist, irradiating distance, etc., on scattering property, the validity of the algorithm used is demonstrated. The results show that the beam waist may improve the particle's identification property and the particle has a strong scattering property in both forward and backward directions. The method used is simple and provides a new way for investigating the scattering fields from particles in electromagnetic beams.**

***Index Terms*—Electromagnetic beam, Gaussian beam, scattering.**

I. INTRODUCTION

THE technology of photoelectric beam detection [1]–[5] now has become an indispensable means in the fields of space, defense, and civilian industry. Laser beam detection, the beam imaging system of laser radar, etc., are gradually becoming a research focus. The beam scattering property of a particle and the interaction between the particle and the beam are subjects that need to be well solved in these realms. The scattering characteristics [6] of a sphere irradiated by a plane electromagnetic wave have been studied in detail, which provides an evaluation standard for general particle scattering. The scattering characteristics for the spherical particles in the Gaussian beam and other beams have been investigated by many scholars [7]–[9]. The effects of factors such as the scattering property of beam waist on the particle were analyzed. Additionally, the interior field and near field for a homogeneous sphere in the presence of a high-order Gaussian beam were expressed [10] by using the expansion of the spherical wave function. In the presence of a focused Laguerre-Gaussian beam, Zhao [11] studied the particle scattering property and derived the scattering coefficients. It was found that expanding the beam with the spherical wave function and unifying the coordinate system were the very effective methods for studying the scattering property of a pair of spheres [12]. The method of expressing the beam in series [13], [14] to study Rayleigh scattering and rough surface scattering achieves a high accuracy in calculation. Although there are two ways to study the particle scattering property, namely expansion of the spherical vector wave function and the complete numerical calculation, the former one usually expands coefficients of the incident wave with the vector wave functions, this coefficients are dependent on the expression of the incident wave and sometimes the numerical integration must be employed. Those coefficients are not universal. The latter is not economical with its result being confined by the factors such as beam shape and the concrete programming.

In this article the electromagnetic beam will be expanded into Taylor series to investigate the particle field distribution and particle scattering property. Firstly, the terms in the Taylor series will be expressed with the spherical wave function, which is of the generality without being confined by beam shape. The expanding coefficients of the incident wave x_n y_n and z_n etc., will be well developed as well. These coefficients are found to be dependent only on the particle's radius and beam partial derivatives. Secondly, the respective scattering fields corresponding to the incident wave of x_n y_n and z_n will be presented in the analytical expressions. Finally, with the elliptical Gaussian beam taken as an example, the first-order scattering property of a particle will be investigated in detail and the validity of the proposed method will be demonstrated. The impacts of the beam waist and the frequency on scattering characteristics will be simulated. In the following analysis the $e^{j\omega t}$ time dependence is assumed and suppressed throughout.

II. SCATTERED FIELD OF A PARTICLE IN PHOTOELECTRIC BEAM

A. A General Expression of Scattering Field

Under the irradiation of a plane electromagnetic wave, the scattering field of a spherical particle was obtained from the well-known Mie theory. In the photoelectric beam, the scattering field by a particle is determined by the particle radius, dielectric constant, and beam shape. To make the obtained result general, we assume that the incident wave beam is expressed in the particle coordinate system, polarized in the x-direction and propagating along the z-direction, namely

$$\mathbf{E} = f(x,y,z)\hat{\mathbf{x}}e^{-jkz}$$

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When the expression above is expanded at the origin of the system, it is obtained as

$$\mathbf{E} = \hat{\mathbf{x}} \left[f(0) + \frac{1}{1!} \left[x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z} \right] \times f(x,y,z) + \cdots \right] e^{-jkz}. \quad (1)$$

The first term in (1) is a plane E.M. wave with an amplitude of $f(0,0,0)$. It is reduced to

$$\mathbf{E}_0 = \hat{\mathbf{x}} f_0 e^{-jkz}. \quad (2)$$

This term may be expanded with the spherical vector wave function [15] as

$$\mathbf{E} = \sum_{mni} \left[a^i_{mn}\mathbf{M}^i_{mn} + b^i_{mn}\mathbf{N}^i_{mn} \right]$$
$$\mathbf{H} = \frac{j}{\eta}\sum_{mni} \left[a^i_{mn}\mathbf{N}^i_{mn} + b^i_{mn}\mathbf{M}^i_{mn} \right].$$

The values of the subscript and the superscript in the expressions above are

$$n = 0,1,2,\ldots\ldots$$
$$m = 0,1,2,\ldots,n$$
$$i = 1 \quad \text{or } 2.$$

The spherical vector wave functions are written as

$$\mathbf{M}^i_{mn} = \frac{1}{\sin\theta} z_n(kr)\frac{\partial Y^i_{mn}}{\partial\phi}\hat{\mathbf{u}}_\theta - z_n(kr)\frac{\partial Y^i_{mn}}{\partial\theta}\hat{\mathbf{u}}_\varphi$$
$$\mathbf{N}^i_{mn} = \frac{n(n+1)}{kr} z_n(kr) Y^i_{mn}\hat{\mathbf{u}}_r + \frac{1}{kr}\frac{d}{dr}[rz_n(kr)]\frac{\partial Y^i_{mn}}{\partial\theta}\hat{\mathbf{u}}_\theta + \frac{1}{kr\sin\theta}\frac{d}{dr}[rz_n(kr)]\frac{\partial Y^i_{mn}}{\partial\varphi}\hat{\mathbf{u}}_\varphi$$
$$Y^i_{mn} = P^m_n(\cos\theta)\begin{bmatrix} \cos m\varphi, i=1 \\ \sin m\varphi, i=2 \end{bmatrix}.$$

In the equations above, $z_n(kr)$ may be the spherical Bessel function $j_n(kr)$, the spherical Hankel function $h^{(1)}_n(kr)$ or the $h^{(2)}_n(kr)$. On the spherical surface of radius a, the coefficients of expanding expression are determined by

$$a^i_{mn} = k_{mn}\int_0^\pi\int_0^{2\pi} \mathbf{M}^i_{mn}(a,\theta,\phi)\cdot\mathbf{E}_t(a,\theta,\phi)\sin\theta d\theta d\phi$$
$$b^i_{mn} = l_{mn}\int_0^\pi\int_0^{2\pi} \mathbf{N}^i_{mn}(a,\theta,\phi)\cdot\mathbf{E}_t(a,\theta,\phi)\sin\theta d\theta d\phi$$
$$k_{mn} = \frac{1}{[z_n(ka)]^2}\frac{2n+1}{2\pi(1+\delta_{m0})n(n+1)}\frac{(n-m)!}{(n+m)!}$$
$$l_{mn} = \frac{1}{\left\{kr\frac{d}{dr}[rz_n(ka)]\right\}^2_{r=a}} \times \frac{2n+1}{2\pi(1+\delta_{m0})n(n+1)}\frac{(n-m)!}{(n+m)!}. \quad (3)$$

In expression (3), the subscript t denotes the tangential component of the electric field. After changing expression (2) with the spherical coordinates, we obtain

$$\mathbf{E}_0 = \left(\hat{\mathbf{u}}_r\sin\theta\cos\phi + \hat{\mathbf{u}}_\theta\cos\theta\cos\phi - \hat{\mathbf{u}}_\phi\sin\phi\right) f_0 e^{-jkr\cos\theta}.$$

Putting the expression above into expression (3) yields

$$a^i_{mn} = k_{mn} f_0 \int_0^\pi\int_0^{2\pi} \left[\frac{1}{\sin\theta} z_n(ka)\frac{\partial Y^i_{mn}}{\partial\varphi}\cos\theta\cos\varphi + z_n(ka)\frac{\partial Y^i_{mn}}{\partial\theta}\sin\varphi \right]\sin\theta e^{-jka\cos\theta} d\theta d\phi.$$

When $i = 1$, we obtain

$$a^1_{mn} = k_{mn} f_0 \int_0^\pi\int_0^{2\pi} \left[\frac{-m}{\sin\theta} z_n(ka) P^m_n(\cos\theta)\cos\theta\cos\phi\sin m\phi + z_n(ka)\frac{dP^m_n(\cos\theta)}{d\theta}\sin\phi\cos m\phi \right]\sin\theta e^{-jka\cos\theta} d\theta d\varphi. \quad (4)$$

Expression (4) is an integral one, in which the function $P^m_n(\cos\theta)$ is defined as

$$P^m_n(t) = \frac{(1-t^2)^{m/2}}{2^n n!}\frac{d^{m+n}}{dt^{m+n}}(t^2-1)^n$$

where $t = \cos\theta$. The integral variables θ and φ are independent of each other. We may firstly integrate expression (4) with respect to variable φ, but it is known that the trigonometric functions have orthogonality, namely

$$\int_0^{2\pi} \sin m\varphi\cos n\varphi d\varphi = 0.$$

Thus, $a^1_{mn} = 0$ is obtained. When $i = 2$, we obtain

$$a^2_{mn} = k_{mn} f_0 \int_0^\pi\int_0^{2\pi} \begin{bmatrix} \frac{m}{\sin\theta} z_n(ka) P^m_n(\cos\theta)\cos\theta\cos\phi\cos m\phi \\ + z_n(ka)\frac{dP^m_n(\cos\theta)}{d\theta}\sin\phi\sin m\phi \end{bmatrix} \times \sin\theta e^{-jka\cos\theta} d\theta d\varphi.$$

When the subscript $m \neq 1$, we also conclude that $a^2_{mn} = 0$.

When $m = 1$

$$a^2_{1n} = k_{mn} f_0 \int_0^\pi\int_0^{2\pi} \begin{bmatrix} \frac{1}{\sin\theta} z_n(ka) P^1_n(\cos\theta)\cos\theta\cos\phi\cos\phi \\ + z_n(ka)\frac{dP^1_n(\cos\theta)}{d\theta}\sin\phi\sin\phi \end{bmatrix} \times \sin\theta e^{-jka\cos\theta} d\theta d\varphi.$$

Since

$$\int_0^{2\pi} \cos^2\varphi d\varphi = \int_0^{2\pi} \sin^2\varphi d\varphi = \pi$$

we obtain

$$a^2_{1n} = \pi k_{1n} f_0 \int_0^\pi \begin{bmatrix} \frac{1}{\sin\theta} z_n(ka) P^1_n(\cos\theta)\cos\theta \\ + z_n(ka)\frac{dP^1_n(\cos\theta)}{d\theta} \end{bmatrix} \sin\theta e^{-jka\cos\theta} d\theta.$$

Let $z_n(ka) = j_n(ka)$ and $t = \cos\theta$. We obtain

$$d\theta = -(1-t^2)^{-\frac{1}{2}} dt.$$

In the integrating expression of a_{1n}^2 above, the spherical Bessel function is a constant and may be placed in front of the integral. It is changed as

$$a_{1n}^2 = \pi k_{1n} j_n(ka) f_0 \int_0^{\pi} \begin{bmatrix} \frac{1}{(1-t^2)^{\frac{1}{2}}} P_n^1(t) t \\ -\frac{dP_n^1(t)}{dt}(1-t^2)^{\frac{1}{2}} \end{bmatrix} \times (1-t^2)^{\frac{1}{2}} e^{-jkat} \left[-(1-t^2)^{-\frac{1}{2}} dt\right]$$

After further computation, we obtained

$$a_{1n}^2 = \pi k_{1n} j_n(ka) f_0 \int_1^{-1} \left[\frac{dP_n^1(t)}{dt}(1-t^2)^{\frac{1}{2}} - \frac{1}{(1-t^2)^{\frac{1}{2}}} P_n^1(t) t\right] e^{-jkat} dt$$

From the literature [15], we may obtain the relation

$$(1-t^2)\frac{dP_n^1(t)}{dt} = (n+1) t P_n^1(t) - n P_{n+1}^1(t).$$

When the relation above is substituted in a_{1n}^2, the following expression is obtained by exchanging the upper and lower integral limits and expanding e^{-jkat}into the Taylor series

$$\begin{aligned} a_{1n}^2 &= \pi k_{1n} f_0 j_n(ka) s_n^2 \\ s_n^2 &= n \int_{-1}^{1} \frac{\left[P_{n+1}^1(t) - t P_n^1(t)\right]}{(1-t^2)^{\frac{1}{2}}} \\ &\quad \times \left[1 - jkat - \frac{k^2a^2}{2} t^2 + \frac{jk^3a^3}{6} t^3 + \cdots\right] dt. \end{aligned}$$

The integration in above is independent with the particle parameter. Using the same process, we easily obtain the following

$$\begin{aligned} b_{mn}^1 &= 0 \quad m \neq 1 \\ b_{1n}^1 &= \pi l_{1n} \frac{f_0}{kr} \frac{d}{dr}[r j_n(kr)] s_n^1 \\ s_n^1 &= \int_{-1}^{1} \left[\frac{P_n^1(t)(1-(n+1)t^2) + nt P_{n+1}^1(t)}{(1-t^2)^{\frac{1}{2}}}\right] \\ &\quad \times \left[1 - jkat - \frac{k^2a^2}{2} t^2 + \cdots\right] dt. \end{aligned}$$

To apply the series is truncated by k^4a^4, which is accurate enough for usage. The partial expressions are shown as follows:

$$\begin{aligned} s_1^2 &= 0.2k^2a^2 + 0.0119k^4a^4, \\ s_2^2 &= \mathrm{j}0.8ka - j0.0571\mathrm{k}^3\mathrm{a}^3 \\ s_3^2 &= 0.7286k^2a^2 - 0.0377k^4a^4 \\ s_4^2 &= -j0.0423k^3a^3, \\ s_5^2 &= -1.125k^2a^2 + 0.0255k^4a^4 \\ s_1^1 &= 2.6667 - 0.8jka + 0.0825jk^3a^3 \\ s_2^1 &= -0.2143k^2a^2 + 0.0139k^4a^4 \\ s_3^1 &= 0.9143jka - 0.0762jk^3a^3 \\ s_4^1 &= 0.4894k^2a^2 - 0.0274k^4a^4 \\ s_5^1 &= -0.0346jk^3a^3. \end{aligned}$$

Therefore (2) may be rephrased as

$$\begin{aligned} \mathbf{E} &= \sum_n \left[a_{1n}^2 \mathbf{M}_{1n}^2 + b_{1n}^1 \mathbf{N}_{1n}^1\right] \\ \mathbf{H} &= \frac{j}{\eta} \sum_n \left[a_{1n}^2 \mathbf{N}_{1n}^2 + b_{1n}^1 \mathbf{M}_{1n}^1\right]. \end{aligned} \tag{5}$$

In expression (5), the function $z_n(kr)$ denotes the spherical Bessel function $j_n(kr)$.

According to expression (5), the scattering field and the internal field may be respectively written as

$$\begin{aligned} \mathbf{E}_s &= \sum_n \left[a_{1ns}^2 \mathbf{M}_{1n}^2 + b_{1ns}^1 \mathbf{N}_{1n}^1\right] \\ \mathbf{H}_s &= \frac{j}{\eta} \sum_n \left[a_{1ns}^2 \mathbf{N}_{1n}^2 + b_{1ns}^1 \mathbf{M}_{1n}^1\right] \end{aligned} \tag{6}$$

$$\begin{aligned} \mathbf{E}_i &= \sum_n l\left[a_{1ni}^2 \mathbf{M}_{1n}^2 + b_{1ni}^1 \mathbf{N}_{1n}^1\right] \\ \mathbf{H}_i &= \frac{jm}{\eta} \sum_n \left[a_{1ni}^2 \mathbf{N}_{1n}^2 + b_{1ni}^1 \mathbf{M}_{1n}^1\right] \end{aligned} \tag{7}$$

In expression (6), function $z_n(kr)$ symbolizes $h_n^{(2)}(kr)$, and in expression (7) it is the spherical Bessel function $j_n(mkr)$. From expressions (5) to (7), the expanding coefficients of the scattering field are obtained as follows

$$\begin{aligned} a_{1ns}^2 &= a_{1n}^2 f_1(n,a) \\ f_1(n,a) &= \frac{j_n(ka)\frac{d}{dr}\left[r j_n(kmr)\right] - j_n(kma)\frac{d}{dr}\left[r j_n(kr)\right]}{j_n(kma)\frac{d}{dr}\left[r h_n^{(2)}(kr)\right] - h_n^{(2)}(ka)\frac{d}{dr}\left[r j_n(kmr)\right]} \end{aligned} \tag{8}$$

$$\begin{aligned} b_{1ns}^1 &= b_{1n}^1 f_2(n,a) \\ f_2(n,a) &= \frac{j_n(ka)\frac{d}{dr}\left[r j_n(kmr)\right] - m^2 j_n(kma)\frac{d}{dr}\left[r j_n(kr)\right]}{m^2 j_n(kma)\frac{d}{dr}\left[r h_n^{(2)}(kr)\right] - h_n^{(2)}(ka)\frac{d}{dr}\left[r j_n(kmr)\right]} \end{aligned} \tag{9}$$

For the sake of simplicity, $(d/dr)[rh_n^{(2)}(kr)]$ in both expressions (8) and (9) denote derivative at the point $r = a$. The relation between the coefficients of the scattering field and those of the incident field is determined by the summation index n only.

When the incident wave is

$$\mathbf{E}_x = \hat{\mathbf{x}} f_x x e^{-jkz}$$

where f_x denotes a partial derivative at origin with respect to the x coordinate, with the method available, we also may obtain the corresponding electromagnetic field as

$$\begin{aligned} \mathbf{E}_x &= \sum_n \left[a_{2nx}^2 \mathbf{M}_{2n}^2 + b_{2nx}^1 \mathbf{N}_{2n}^1\right] \\ \mathbf{H}_x &= \frac{j}{\eta} \sum_n \left[a_{2nx}^2 \mathbf{N}_{2n}^2 + b_{2nx}^1 \mathbf{M}_{2n}^1\right] \end{aligned} \tag{10}$$

The respective coefficients are as follows:

$$\begin{aligned} a_{2nx}^2 &= \pi f_x k_{2n} a j_n(ka) s_{nx}^2 \\ s_{nx}^2 &= \left(1 + \frac{n}{2}\right) \int_{-1}^{1} \left[t P_n^2(t) - P_{n-1}^2(t)\right] \\ &\quad \times \left[1 - jkat - \frac{k^2a^2}{2} t^2 + \cdots\right] dt \end{aligned}$$

$$b_{2nx}^1 = \frac{\pi l_{2n} f_x}{2k}\left\{\frac{d[rj_n(kr)]}{dr}\right\}_{r=a} s_{nx}^1$$
$$s_{nx}^1 = \int_{-1}^{1}\left[P_n^2(t)(2+nt^2) - t(n+2)P_{n-1}^2(t)\right] \times \left[1 - jkat - \frac{k^2a^2}{2}t^2 + \cdots\right]dt$$

Partial simulations of expression s_{nx}^2 and expression s_{nx}^1 in the integration above are given as

$$\begin{aligned}
s_{2x}^2 &= 2\lfloor -0.8jka + 0.0571jk^3a^3\rfloor\\
s_{3x}^2 &= 2.5[-0.4572k^2a^2 + 0.0254k^4a^4]\\
s_{4x}^2 &= 0.381jk^3a^3\\
s_{5x}^2 &= 0.0808k^4a^4, s_{1x}^2 = 0,\ s_{nx}^2 = 0, \quad n \geq 6\\
s_{2x}^1 &= 9.6 - 3.6571jka + 0.3556jk^3a^3\\
s_{3x}^1 &= -1.7381k^2a^2 + 0.1019k^4a^4\\
s_{4x}^1 &= 3.8095jka - 0.2424jk^3a^3\\
s_{5x}^1 &= 3.1572k^2a^2 - 0.1591k^4a^4, s_{1x=0}^1 = 0
\end{aligned}$$

According to expression (7), we can write the scattering field and the internal field with the spherical vector wave function. Using the boundary condition of the electromagnetic field on the surface of the particle, the coefficients of scattering field are obtained as

$$a_{2nxs}^2 = a_{2nx}^2 f_1(n,a) \tag{11}$$
$$b_{2nxs}^1 = b_{2nx}^1 f_2(n,a). \tag{12}$$

When the incident wave is

$$\mathbf{E}_y = \hat{\mathbf{x}} f_y y e^{-jkz}$$

where f_y is a partial derivative of the beam at the origin with respect to the y coordinate, the expansion of the above is

$$\begin{aligned}
\mathbf{E}_y &= \sum_n \left[a_{2ny}^1\mathbf{M}_{2n}^1 + b_{2ny}^2\mathbf{N}_{2n}^2\right]\\
\mathbf{H}_y &= \frac{j}{\eta}\sum_n \left[a_{2ny}^1\mathbf{N}_{2n}^1 + b_{2ny}^2\mathbf{M}_{2n}^2\right].
\end{aligned} \tag{13}$$

The coefficients are given by

$$a_{2n}^1 = k_{2n} a\pi f_y j_n(ka) s_{ny}^1$$
$$s_{ny}^1 = \left(1 + \frac{n}{2}\right)\int_{-1}^{1}\left[P_{n-1}^2(t) - tP_n^2(t)\right] \times \left[1 - jkat - \frac{k^2a^2}{2}t^2 + \cdots\right]dt$$
$$b_{2n}^2 = \frac{\pi f_y l_{2n}}{2k}\left\{\frac{d[rj_n(kr)]}{dr}\right\}_{r=a} s_{ny}^2$$
$$s_{ny}^2 = \int_{-1}^{1}\left[P_n^2(t)(2+nt^2) - t(n+2)P_{n-1}^2(t)\right] \times \left[1 - jkat - \frac{k^2a^2}{2}t^2 + \cdots\right]dt.$$

The following are some results of the numerical integration

$$\begin{aligned}
s_{2y}^1 &= 1.6jka - 0.1142jk^3a^3\\
s_{3y}^1 &= 2.1427k^2a^2 - 0.0635k^4a^4\\
s_{4y}^1 &= -0.3809jk^3a^3, s_{5y}^1 = -0.0808k^4a^4\\
s_{1y}^1 &= 0, \quad s_{ny}^1 = 0, \quad n \geq 6\\
s_{2y}^2 &= 9.6 - 3.6571jka + 0.3556jk^3a^3\\
s_{3y}^2 &= -1.7381k^2a^2 + 0.1019k^4a^4\\
s_{4y}^2 &= 3.8095jka - 0.2424jk^3a^3\\
s_{5y}^2 &= 3.1572k^2a^2 - 0.1591k^4a^4, \quad s_{1y}^2 = 0.
\end{aligned}$$

According to expression (10), we can also write the scattering field and the internal field with the spherical vector wave function. Then by using the boundary condition of the electromagnetic field on the surface of the particle, the coefficients of the scattering field are obtained as

$$a_{2nys}^1 = a_{2ny}^1 f_1(n,a) \tag{14}$$
$$b_{2nys}^2 = b_{2ny}^2 f_2(n,a). \tag{15}$$

When the incident wave is

$$\mathbf{E}_z = \hat{\mathbf{x}} f_z z e^{-jkz}$$

where the f_z is a partial derivative of the beam at the origin with respect to the z coordinate. The field $\mathbf{E}_z$ can be expanded as follows with utilizing the orthogonality of the trigonometric functions

$$\begin{aligned}
\mathbf{E}_z &= \sum_n \left[a_{1nz}^2\mathbf{M}_{1n}^2 + b_{1nz}^1\mathbf{N}_{1n}^1\right]\\
\mathbf{H}_z &= \frac{j}{\eta}\sum_n \left[a_{1nz}^2\mathbf{N}_{1n}^2 + b_{1nz}^1\mathbf{M}_{1n}^1\right]\\
a_{1n}^2 &= k_{1n} a f_z \pi j_n(ka) s_{nz}^2\\
s_{nz}^2 &= (1+n)\int_{-1}^{1}\frac{\left[t^2P_n^1(t) - tP_{n-1}^1(t)\right]}{(1-t^2)^{\frac{1}{2}}} \times \left[1 - jkat - \frac{k^2a^2}{2}t^2 + \cdots\right]dt\\
b_{1n}^1 &= \frac{\pi f_z l_{1n}}{k}\left\{\frac{d[rj_n(kr)]}{dr}\right\}_{r=a} s_{nz}^1\\
s_{nz}^1 &= \int_{-1}^{1}\frac{\left[t(1+nt^2)P_n^1(t) - (1+n)t^2P_{n-1}^1(t)\right]}{(1-t^2)^{\frac{1}{2}}} \times \left[1 - jkat - \frac{k^2a^2}{2}t^2 + \cdots\right]dt
\end{aligned} \tag{16}$$

The following are some results of the numerical integration

$$\begin{aligned}
s_{1z}^2 &= 1.3334 - 0.2666jka + 0.0159jk^3a^3\\
s_{2z}^2 &= -0.0215k^2a^2 + 0.0030k^4a^4\\
s_{3z}^2 &= 0.6856\mathrm{j}ka - 0.0635jk^3a^3\\
s_{4z}^2 &= 0.3083k^2a^2 - 0.0185k^4a^4\\
s_{5z}^2 &= -0.0289jk^3a^3\\
s_{1z}^1 &= -0.1357k^2a^2 + 0.0083k^4a^4\\
s_{2z}^1 &= 2.4000 - 0.3429jka + 0.0444jk^3a^3\\
s_{3z}^1 &= 0.0547k^2a^2 - 0.0014k^4a^4\\
s_{4z}^1 &= 0.6349jka - 0.0673jk^3a^3\\
s_{5z}^1 &= 0.2667k^2a^2 - 0.0146k^4a^4.
\end{aligned}$$

On the surface of the particle, by using the boundary condition of electromagnetic field, the scattering coefficients are

$$a^2_{1nzs} = a^2_{1nz} f_1(n, a) \tag{17}$$

$$b^1_{1nzs} = b^1_{1n} f_2(n, a). \tag{18}$$

When a general electromagnetic wave beam irradiates the particle, from expressions of (5), (6), (8), (9), (11), (12), (14), and (15), we may obtain the total scattering field as

$$\mathbf{E}_s = \sum_{n=1} \begin{bmatrix} a^2_{1ns}\mathbf{M}^2_{1n} + b^1_{1ns}\mathbf{N}^1_{1n} + a^2_{2nxs}\mathbf{M}^2_{2n} + b^1_{2nxs}\mathbf{N}^1_{2n} \\ + a^1_{2nys}\mathbf{M}^1_{2n} + b^2_{2nys}\mathbf{N}^2_{2n} + a^2_{1nzs}\mathbf{M}^2_{1n} + b^1_{1nzs}\mathbf{N}^1_{1n} \end{bmatrix}. \tag{19}$$

We may utilize the orthogonality of the spherical vector wave functions to obtain the scattering power of a spherical surface with radius r

$$P_s = \frac{1}{2\eta}\sum_n (t_1 + t_2 + t_3 + t_4) \tag{20}$$

where

$$t_1 = \left(\left|a^2_{1ns}\right|^2 + \left|a^2_{1nzs}\right|^2\right) r^2 h_n^{(2)}(kr) h_n^{(2)*}(kr) \frac{2n^2\pi(n+1)^2}{2n+1}$$

$$t_2 = \left(\left|a^2_{2nxs}\right|^2 + \left|a^1_{2nys}\right|^2\right) r^2 h_n^{(2)} \times (kr) h_n^{(2)*}(kr) \frac{2n\pi(n+1)}{2n+1}\frac{(n+2)!}{(n-2)!}$$

$$t_3 = \left(\left|b^2_{2nys}\right|^2 + \left|b^1_{2nxs}\right|^2\right) \frac{1}{k^2}\left\{\frac{d}{dr}\left[r h_n^{(n)}(kr)\right]\right\} \times \left\{\frac{d}{dr}\left[r h_n^{(n)}(kr)\right]\right\}^* \frac{2n\pi(n+1)}{2n+1}\frac{(n+2)!}{(n-2)!}$$

$$t_4 = \left(\left|b^1_{1nzs}\right|^2 + \left|b^1_{1ns}\right|^2\right) \frac{1}{k^2}\left\{\frac{d}{dr}\left[r h_n^{(n)}(kr)\right]\right\} \times \left\{\frac{d}{dr}\left[r h_n^{(n)}(kr)\right]\right\}^* \frac{2n^2\pi(n+1)^2}{2n+1}.$$

The scattering cross-section σ is easily obtained both by the scattering power and the power density of the incident wave beam. There is no need to discuss it here.

B. *The Scattering Property of a Particle in the Elliptical Gaussian Beam*

In [3], we may find the expression of the elliptical Gaussian beam in the particle's coordinate system. When this beam polarizes in the x-direction and propagates along the z-direction, it can be written as

$$f(0,0,0) = E_0\sqrt{-Q_{x0}Q_{y0}}e^{-ikd} \tag{21}$$

$$f_z = E_0 \frac{\frac{1}{kW_{0y}^2} + \frac{1}{kW_{0x}^2} - i\frac{4d}{k^2W_{0y}^2W_{0x}^2}}{\left(i\left(\frac{2d}{kW_{0y}^2} + \frac{2d}{kW_{0x}^2}\right) + \frac{4d^2}{k^2W_{0y}^2W_{0x}^2} - 1\right)^{\frac{3}{2}}} e^{-ikd}$$

$$f_x = 0 \quad f_y = 0 \tag{22}$$

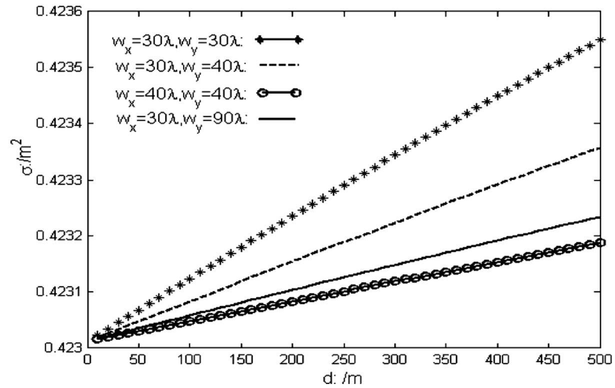

Fig. 1. Change of RCS with irradiating distance.

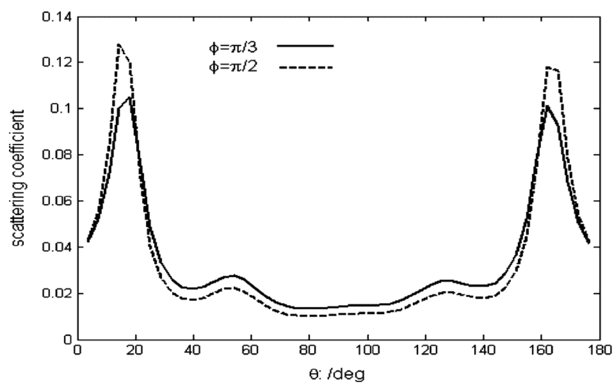

Fig. 2. Change of normalized RCS with angle.

Where

$$Q_{y0} = \left(i + \frac{2d}{kW_{0y}^2}\right)^{-1}, \quad Q_{x0} = \left(i + \frac{2d}{kW_{0x}^2}\right)^{-1}$$

We can obtain the scattering field from a particle in the elliptical Gaussian beam by combining expressions (18) (19) and (16). The followings are numerical results.

Fig. 1 shows the radar cross section (RCS) change with respect to the irradiating distance. The frequency is 300 MHz, the size of the particle is 0.35 m and the relative refractive index is 1.5 in the computation. The result shows that the RCS is proportional to the irradiating distance d. When a plane wave irradiates the particle, its RCS is the biggest, which is in agreement with [16]. In the case in which other conditions remain unchanged, the smaller the beam waist width is, the better focus effect it has. An excellent focus property will make the particle get more electromagnetic power from the exposure beam. Thus, the scattering field becomes strong, as shown in Fig. 1. In Fig. 2, according to the [16], the operating wave length is 0.6328 um and the radius of the particle is 30 um. This figure shows that the particle has a stronger scattering property in forward and backward directions. In Fig. 3 the following parameters are assumed: beam waist of 10 um, the radius of 9.929 um and the relative refractive index of 1.33. The dotted line is a result of this paper and the solid line is that of [16], the two kinds of line

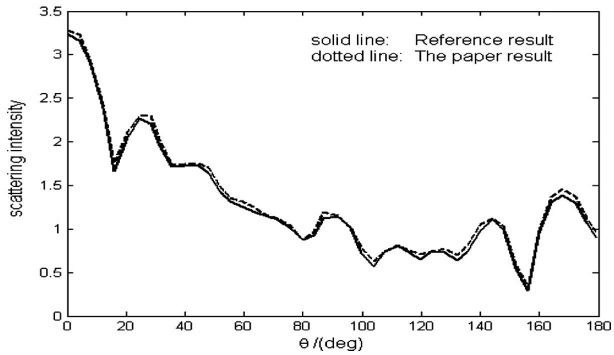

Fig. 3. A comparison with the reference.

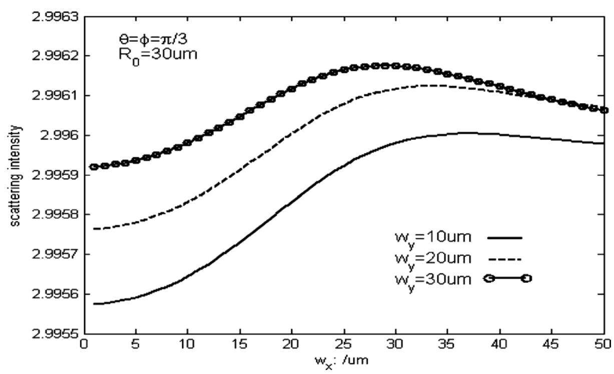

Fig. 4. An effect of the beam waist.

are in agreement with each other. This identical result demonstrates the validity of the algorithm used in this paper. Fig. 4 shows a relation between the relative scattering intensity and the beam waist. The scattering intensity of the particle is shown to be nonlinearly increased when the beam waist increases. It is well known that the bigger the beam waist is, the closer the beam to the plane electromagnetic wave is. Particles have the strongest scattering property in the plane electromagnetic wave.

III. Conclusion

Based on the orthogonalities of vector wave functions and the trigonometric functions, the scattering fields for the zeroth-order incident field and the first-order incident field for a general beam wave are completely developed. A general expression between the coefficients of the incident wave expressions and those of scattering field expressions was presented as well. In application, only the derivatives of the beam are needed to determine the scattering field entirely. The cumbersome process of directly expanding the electromagnetic beam with the vector spherical wave function is avoided. The scattering characteristics of the particle in the elliptical Gaussian beam are investigated. The influences of the beam waist, irradiating distance, etc., on the scattering cross section were analyzed and the scattering intensity was simulated. The validity of the method used was tested by comparing the result of this article with that in the [16]. The obtained results are not confined by the size of the particle and the frequency, which is significant in the investigations of scattering property for the particle or the target in electromagnetic wave beams.

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