Lightweight Virtual Analog Modeling

Stefano D'Angelo
Independent Researcher
Via P. Voso 22/B
Agropoli 84043, Italy
s@dangelo.audio

ABSTRACT

The purpose of this paper is to give theoretical and practical advice to DSP programmers willing to implement efficient real-time circuit simulation algorithms. It summarizes the fundamental concepts behind the most widespread circuit modeling techniques and gives the reader sufficient references to get started. Then it tackles on a certain number of topics that are often neglected in theoretical studies yet highly relevant in real-world applications. Hopefully, this paper might also serve as a reference and an invite for researchers to direct their efforts towards problems and solutions that have a concrete impact.

1. INTRODUCTION

Many audio processing algorithms developed as of today are to some degree inspired by or try to replicate real-world phenomena [1, 2, 3], whether directly sound-related or where sound manipulation happens as a byproduct of another process. In particular, modeling of analog circuits that generate or process sound-related signals sparked particular interest in the last decades, both in industry and academia [4, 5, 6].

Simulation or emulation of circuits for real-time sound-related applications is often called "Virtual Analog modeling" (VA modeling), and many approaches have been proposed to address it specifically [7, 8]. Obviously, each technique has its peculiar strengths and weaknesses, yet some specific and often neglected aspects tend to be of utmost importance in practical applications: computational cost, maintainability, and parameterization.

This paper very concisely reviews the most widespread white-box VA modeling frameworks in Section 2 and provides some practical modeling and implementation advice in Section 3. Section 4 concludes the paper.

2. VA MODELING

2.1 State-space methods

State-space (SS) representation can be used to describe analog electronic circuits [9, 10, 11, 12] as

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{Ci}, \tag{1}$$

$$\mathbf{i} = f(\mathbf{v}), \tag{2}$$

$$\mathbf{v} = \mathbf{Dx} + \mathbf{Eu} + \mathbf{Fi}, \tag{3}$$

$$\mathbf{y} = \mathbf{Lx} + \mathbf{Mu} + \mathbf{Ni}, \tag{4}$$

where $\mathbf{x}$ is the state vector, $\mathbf{u}$ is the input vector, $\mathbf{y}$ is the output vector, and $f()$ is a MIMO nonlinear mapping, t is the time variable, and $\mathbf{A}$ to $\mathbf{N}$ are matrices describing the linear behavior of the system.

Such a system can be discretized by means of common numerical integration schemes, such as the backward Euler method or the trapezoidal rule, and finally implemented, usually after further mathematical manipulation. In most cases, however, implicit nonlinear equations arise, which need to be solved numerically or otherwise approximated.

2.2 Wave digital filters

Wave digital filters (WDFs) represent another technique to model and discretize analog circuits [13, 14, 15, 16] which does not directly operate on Kirchoff variables (*i.e.*, voltages and currents), but rather on so-called *wave variables* defined as

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 & R_0 \\ 1 & -R_0 \end{pmatrix} \begin{pmatrix} V \\ I \end{pmatrix}, \tag{5}$$

where a is the incoming wave, b is the reflected wave, R_0 is a free parameter, and V and I are, respectively, the voltage and current of an electrical port. By this formulation, all LTI multiports (*e.g.*, resistors, capacitors, inductors, terminated voltage and current sources) can be translated into standalone and reusable DSP components that can be interconnected using a tree of *adaptors* implementing the topology of the circuit.

It is remarkable that a WDF tree represents, at the same time, the model and the DSP structure of the resulting algorithm. On the other hand, in their classic formulation, WDFs are only suitable for a limited set of topologies and up to one nonlinear element. Several solutions have been proposed to overcome such limitations [17, 18, 19, 20].

2.3 Port-Hamiltoninan systems

An analog circuit can also be modeled as a port-Hamiltonian system (PHS) [21, 22, 23] by the equation

$$\begin{pmatrix} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} \\ \hline \mathbf{w} \\ \hline \mathbf{y} \end{pmatrix} = \left(\begin{array}{c|c|c} \mathbf{J_x} & -\mathbf{K} & \mathbf{G_x} \\ \hline \mathbf{K}^T & \mathbf{J_w} & \mathbf{G_w} \\ \hline \mathbf{G_x}^T & \mathbf{G_w}^T & \mathbf{J_y} \end{array}\right) \begin{pmatrix} \nabla\mathcal{H}(\mathbf{x}) \\ \hline \mathbf{z}(\mathbf{w}) \\ \hline \mathbf{u} \end{pmatrix}. \tag{6}$$

PHS formulation is a subclass of SS representation with a special focus on energy balancing, and therefore naturally tends to the development of algorithms with remarkable stability. The downsides are, arguably, a higher level of theoretical complexity and higher resulting computational cost when compared to other options.

3. PRACTICAL CONSIDERATIONS

When developing digital models of analog circuits for the general public, the mechanical application of rules found in scientific literature rarely does directly lead to acceptable results. This section explores a few practical aspects of VA modeling.

3.1 Programming approach

Computational cost is a major concern in real-time systems, therefore DSP code is often written in languages such as C and C++ that are nowadays considered relatively low-level and for which highly-optimizing compilers are available, or even in machine-specific assembly. These languages, however, are based on imperative programming paradigms that are quite distant from the dataflow metaphor, which more naturally describes most DSP applications [24, 25]. Unsurprisingly, there have been some attempts at developing domain-specific languages for the task, and especially based off declarative syntaxes and coupled with a source-to-source compiler that emits C or C++ code [26, 27, 28, 29].

To the best of my knowledge, however, none of them has been widely adopted by the industry yet, and most existing codebases remain written in traditional general-purpose languages. Therefore, for the sake of code correctness and maintainability, I would recommend DSP programmers to rather concentrate on mathematical aspects, DSP structure design, and prototyping, and to defer the actual coding to the latest possible development stages. Furthermore, I would suggest to first translate prototypes and designs to production code in a very mechanical fashion, that is following clearly-defined patterns and avoiding language-provided abstractions, and only apply optimizations at the very end and only where needed. In the end, it is also highly recommendable to produce clear and exhaustive documentation and to keep it up to date.

3.2 Mathematical modeling

The derivation of workable models of analog circuits is a repetitive, tedious, and error-prone activity, which can luckily be largely automated. Traditionally, the go-to method for expressing a circuit in SS representation is Modified Nodal Analysis (MNA) [30], which can be found at the heart of circuit simulators such as SPICE [31], and which still influences recent automatic modeling approaches and tools [10, 11, 32]. Also PHSs can be automatically formulated from schematics [23], and indeed a tool called PyPHS [33] exists that goes as far as generating C++ code that simulates a given circuit. Finally, tools to ease the implementation of WDFs have also been proposed [26, 27, 29, 34].

The manipulation of model equations can easily become cumbersome to perform manually, hence the usage of mathematical software is nowadays indispensable. At the very least, one should have access to software for numerical analysis, symbolic computation, arbitrary-precision arithmetic, 2D/3D plotting, and a prototyping language.

Models should always be verified by comparison with a reliable circuit simulator, such as SPICE, and possibly with measurements of real-world devices. In this sense, while it is necessary to choose appropriate input test signals on a case-by-case basis, it is still also possible to employ well-documented output analysis techniques [35, 36]. In the end, it is also advisable to refrain from the temptation of excessively complicating a model in order to match measurements, as certain improvements might fall under the thresholds of component tolerances or human hearing.

3.3 Circuit separation

The computational complexity of digital models of nonlinear analog circuits tends to grow more-than-linearly with the number of components involved, hence large savings can be obtained by simply splitting a circuit into two or more subcircuits characterized by unidirectional control flow [8, 37, 38]. In most cases, this corresponds to an approximation that may or may not be acceptable for a given application, therefore careful evaluation is needed. The definition of an automated method for the task is still an open research topic [39]. Furthermore, an alternative approach that has been proposed is the joint simulation of adjacent couples of nonlinear subcircuits in a chain [40, 38].

3.4 Delay-free loop implementation

Another major source of computational issues is the presence of implicit nonlinear equations, which naturally translate to delay-free loops in DSP terms. The numerical solution of such equations through root-finding methods is a central topic of numerical analysis, and indeed it is rather common to employ such iterative methods [41, 42, 43, 22].

Otherwise, it is possible to employ precomputed lookup tables [9, 19]. This approach is generally only viable as long as the dimensionality of such tables is low and the input ranges are known beforehand. Also, the use of tables may substantially increase cache pressure, thus potentially causing significant and hard-to-detect performance problems.

In the case of simple transcendental equations, which are often obtained when modeling circuits containing semiconductor devices, closed-form solutions or remarkably accurate approximations have been found [44, 45, 46, 47]. Furthermore, piecewise-linear models have also been proposed to obtain approximate solutions of delay-free loops [48, 49].

In some cases the problem was avoided by simply adding fictitious delay units in the feedback branches of delay-free loops [50, 51]. While extremely simple and efficient, usage of this method affects the frequency response of the system and often also leads to undesirable and nonobvious parameter coupling effects. If a good amount of oversampling is used, the outcome can actually be acceptable, otherwise a method has also been proposed to restore the original linear behavior and parameterization by adding extra compensation filters [52]. The stability and accuracy properties of such method, however, still await systematic study.

3.5 Further optimizations

Oversampling can be yet another source of inefficiency. Firstly, recent research has gone into alias reduction without oversampling [53, 54]. Then, it is fundamental to carefully choose oversampling ratios and resampling boundaries when using chains of oversampled structures. Finally, in many cases IIR resampling filters provide acceptable results and use less resources than FIR filters.

Beforehand knowledge on, e.g., signal and parameter ranges and frequency-related effects, can justify convenient approximations and substantial simplifications without affecting the output quality significantly. The computation of nonlinear functions can be sped up significantly by employing polynomial and rational approximations, leveraging binary number representations, or avoiding general-purpose routines. Also, limiting the computation of coefficients only to the moment in which a parameter value is changed and precomputing as many subexpressions as possible are strategies that allow for significant savings. Finally, while modern compilers are said to perform excellent code optimization, in my experience it is still possible to exploit hardware-specific knowledge and facilities, such as vector instructions, in order to reduce computational load even more.

4. CONCLUSIONS

The translation of analog circuits into actual production-ready VA models is a highly multidisciplinary activity. It requires knowledge not only regarding the specific topic, but also in mathematics, circuit analysis, computer science, DSP, psychoacoustics, as well as familiarity with specific software tools and a decent amount of self-discipline. The development of lightweight and maintainable VA models is possible by taking into account many different aspects, especially w.r.t. optimizations and approximations, and always being ready to explore non-systematic approaches. In other words, as of today, high-performance VA modeling is still be more of a craft than a predetermined and automatable industrial task.

5. REFERENCES

[1] J. O. Smith, "Physical modeling synthesis update," *Computer Music J.*, vol. 20, no. 2, pp. 44–56, 1996.

[2] V. Välimäki, J. Pakarinen, C. Erkut, and M. Karjalainen, "Discrete-time modelling of musical instruments," *Reports on Progress in Physics*, vol. 69, pp. 1–78, January 2006.

[3] V. Välimäki, J. D. Parker, L. Savioja, J. O. Smith, and J. S. Abel, "More than 50 years of artificial reverberation," in *Proc. AES 60th Conf.: DREAMS (Dereverberation and Reverberation of Audio, Music, and Speech)*, (Leuven, Belgium), February 2016.

[4] V. Välimäki, F. Fontana, J. O. Smith, and U. Zölzer, "Introduction to the special issue on virtual analog audio effects and musical instruments," *IEEE Trans. Audio, Speech, and Lang. Process.*, vol. 18, pp. 713–714, May 2010.

[5] V. Välimäki, S. Bilbao, J. O. Smith, J. S. Abel, J. Pakarinen, and D. P. Berners, "Virtual analog effects," in *DAFX: Digital Audio Effects, Second Edition* (U. Zölzer, ed.), pp. 473–522, Chichester, UK: Wiley, 2011.

[6] J. Pakarinen, V. Välimäki, F. Fontana, V. Lazzarini, and J. S. Abel, "Recent advances in real-time musical effects, synthesis, and virtual analog models," *EURASIP J. Advances Signal Process.*, vol. 2011, Article ID 940784, 15 pages, 2011.

[7] R. C. D. de Paiva, *Circuit modeling studies related to guitars and audio processing*. PhD thesis, Aalto University, Espoo, Finland, November 2013.

[8] S. D'Angelo, *Virtual analog modeling of nonlinear musical circuits*. PhD thesis, Aalto University, Espoo, Finland, November 2014.

[9] G. Borin, G. De Poli, and D. Rocchesso, "Elimination of delay-free loops in discrete-time models of nonlinear acoustic systems," *IEEE Trans. Audio, Speech, and Lang. Process.*, vol. 8, pp. 597–605, September 2000.

[10] D. Yeh, J. Abel, and J. Smith, "Automated physical modeling of nonlinear audio circuits for real-time audio effects—part I: Theoretical development," *IEEE Trans. Audio, Speech, and Lang. Process.*, vol. 18, pp. 728–737, May 2010.

[11] D. Yeh, "Automated physical modeling of nonlinear audio circuits for real-time audio effects—part II: BJT and vacuum tube examples," *IEEE Trans. Audio, Speech, and Lang. Process.*, vol. 20, pp. 1207–1216, May 2012.

[12] K. Dempwolf, M. Holters, and U. Zölzer, "Discretization of parametric analog circuits for real-time simulations," in *Proc. 13th Intl. Conf. Digital Audio Effects (DAFx-10)*, (Graz, Austria), September 2010.

[13] A. Fettweis, "Wave digital filters: Theory and practice," *Proc. IEEE*, vol. 74, pp. 270–327, February 1986.

[14] S. Bilbao, *Wave and scattering methods for numerical simulation*. John Wiley & Sons, 2004.

[15] G. De Sanctis and A. Sarti, "Virtual analog modeling in the wave-digital domain," *IEEE Trans. Audio, Speech, and Lang. Process.*, vol. 18, pp. 715–727, May 2010.

[16] K. J. Werner, *Virtual analog modeling of audio circuitry using wave digital filters*. PhD thesis, Stanford University, Stanford, USA, December 2016.

[17] S. Petrausch and R. Rabenstein, "Wave digital filters with multiple nonlinearities," in *Proc. 12th European Signal Process. Conf. (EUSIPCO 2004)*, (Wien, Austria), pp. 77–80, September 2004.

[18] D. Fränken, J. Ochs, and K. Ochs, "Generation of wave digital structures for networks containing multiport elements," *IEEE Trans. Circ. Systems I: Regular Papers*, vol. 52, pp. 586–596, March 2005.

[19] K. J. Werner, V. Nangia, J. O. Smith, and J. S. Abel, "Resolving wave digital filter with multiple/multiport nonlinearites," in *Proc. 18th Intl. Conf. Digital Audio Effects (DAFx-15)*, (Trondheim, Norway), November 2015.

[20] K. J. Werner, J. O. Smith, and J. S. Abel, "Wave digital filter adaptors for arbitrary topologies and multiport linear elements," in *Proc. 18th Intl. Conf. Digital Audio Effects (DAFx-15)*, (Trondheim, Norway), November 2015.

[21] A. Falaize and T. Hélie, "Simulation of an analog circuit of a wah pedal: A port-Hamiltonian approach," in *Proc. 135th AES Convention*, (New York, USA), October 2013.

[22] N. Lopes, T. Hélie, and A. Falaize, "Explicit second-order accurate method for the passive guaranteed simulation of port-Hamiltonian systems," *IFAC-PapersOnLine*, vol. 48, no. 13, pp. 223–228, 2015.

[23] A. Falaize and T. Hélie, "Passive guaranteed simulation of analog audio circuits: A port-Hamiltonian approach," *Applied Sciences*, vol. 6, no. 10, 2016.

[24] E. A. Lee and D. G. Messerschmitt, "Synchronous data flow," *Proc. IEEE*, vol. 75, pp. 1235–1245, September 1987.

[25] K. Barkati and P. Jouvelot, "Synchronous programming in audio processing: A lookup table oscillator case study," *ACM Computing Surveys (CSUR)*, vol. 46, pp. 24:1–24:35, December 2013.

[26] M. Karjalainen, "BlockCompiler: Efficient simulation of acoustic and audio systems," in *Proc. 114th AES Convention*, (Amsterdam, Netherlands), March 2003.

[27] M. Karjalainen, "BlockCompiler - a research tool for physical modeling and DSP," in *Proc. 6th Intl. Conf. Digital Audio Effects (DAFx-03)*, (London, UK), September 2003.

[28] Y. Orlarey, D. Fober, and S. Letz, "FAUST: An efficient functional approach to DSP programming," *New Computational Paradigms for Computer Music*, vol. 290, 2009.

[29] S. D'Angelo, "Physics-based modeling for sound processing: Theory and implementation," Master's thesis, Politecnico di Torino, Turin, Italy, September 2010.

[30] C.-W. Ho, A. E. Ruehli, and P. A. Brennan, "The modified nodal approach to network analysis," *IEEE Trans. Circ. Systems*, vol. 22, pp. 504–509, June 1975.

[31] L. W. Nagel, *SPICE2: A computer program to simulate semiconductor circuits*. PhD thesis, University of California, Berkeley, USA, May 1975.

[32] M. Holters and U. Zölzer, "A generalized method for the derivation of non-linear state-space models from circuit schematics," in *Proc. 23rd European Signal Process. Conf. (EUSIPCO 2015)*, (Nice, France), pp. 1073–1077, IEEE, August 2015.

[33] "PyPHS." `http://pyphs.github.io/pyphs/`. Accessed September 16th, 2018.

[34] M. Rest, R. W. Dunkel, K. J. Werner, and J. O. Smith, "RT-WDF–a modular wave digital filter library with support for arbitrary topologies and multiple nonlinearities," in *Proc. 19th Intl. Conf. Digital Audio Effects (DAFx-16)*, (Brno, Czech Republic), September 2016.

[35] A. Farina, "Simultaneous measurement of impulse response and distortion with a swept-sine technique," in *Proc. 108th AES Convention*, (Paris, France), February 2000.

[36] J. Pakarinen, "Distortion analysis toolkit—a software tool for easy analysis of nonlinear audio systems," *EURASIP J. Advances Signal Process.*, vol. 2010, Article ID 617325, 13 pages, 2010.

[37] O. Bogason and K. J. Werner, "Modeling circuits with operational transconductance amplifiers using wave digital filters," in *Proc. 20th Intl. Conf. Digital Audio Effects (DAFx-17)*, (Edinburgh, UK), September 2017.

[38] J. Zhang and J. O. Smith, "Real-time wave digital simulation of cascaded vacuum tube amplifiers using modified blockwise method," in *Proc. 21th Intl. Conf. Digital Audio Effects (DAFx-18)*, (Aveiro, Portugal), September 2018.

[39] M. Holters and U. Zölzer, "Automatic decomposition of non-linear equation systems in audio effect circuit simulation," in *Proc. 20th Intl. Conf. Digital Audio Effects (DAFx-17)*, (Edinburgh, UK), September 2017.

[40] J. Mačák, *Real-time digital simulation of guitar amplifiers as audio effects*. PhD thesis, Brno University of Technology, Brno, Czech Republic, 2012.

[41] F. Fontana, F. Avanzini, and D. Rocchesso, "Computation of nonlinear filter networks containing delay-free paths," in *Proc. 7th Intl. Conf. Digital Audio Effects (DAFx-04)*, (Naples, Italy), pp. 113–118, October 2004.

[42] F. Fontana and E. Bozzo, "Explicit fixed-point computation of nonlinear delay-free loop filter networks," *IEEE/ACM Trans. Audio, Speech, and Lang. Process.*, vol. 26, pp. 1884–1896, October 2018.

[43] M. J. Olsen, K. J. Werner, and J. O. Smith, "Resolving grouped nonlinearities in wave digital filters using iterative techniques," in *Proc. 19th Intl. Conf. Digital Audio Effects (DAFx-16)*, (Brno, Czech Republic), September 2016.

[44] J. Parker, "A simple digital model of the diode-based ring-modulator," in *Proc. 14th Intl. Conf. Digital Audio Effects (DAFx-11)*, (Paris, France), September 2011.

[45] R. C. D. de Paiva, S. D'Angelo, J. Pakarinen, and V. Välimäki, "Generation of wave digital structures for networks containing multiport elements," *IEEE Trans. Circ. Systems II: Express Briefs*, vol. 59, pp. 688–692, October 2012.

[46] J. Parker and S. D'Angelo, "A digital model of the Buchla lowpass-gate," in *Proc. 16th Intl. Conf. Digital Audio Effects (DAFx-13)*, (Maynooth, Ireland), September 2013.

[47] A. Bernardini, K. J. Werner, A. Sarti, and J. O. Smith, "Modeling nonlinear wave digital elements using the Lambert function," *IEEE Trans. Circ. Systems I: Regular Papers*, vol. 63, pp. 1231–1242, August 2016.

[48] A. Bernardini, K. J. Werner, A. Sarti, and J. O. Smith, "Modeling a class of multi-port nonlinearities in wave digital structures," in *Proc. 23rd European Signal Process. Conf. (EUSIPCO 2015)*, (Nice, France), pp. 664–668, August 2015.

[49] A. Bernardini and A. Sarti, "Canonical piecewise-linear representation of curves in the wave digital domain," in *Proc. 25th European Signal Process. Conf. (EUSIPCO 2017)*, (Kos, Greece), pp. 1125–1129, August 2017.

[50] A. Huovilainen, "Non-linear digital implementation of the Moog ladder filter," in *Proc. 7th Intl. Conf. Digital Audio Effects (DAFx-04)*, (Naples, Italy), pp. 61–64, October 2004.

[51] M. Karjalainen and J. Pakarinen, "Wave digital simulation of a vacuum-tube amplifier," in *Proc. Intl. Conf. Acoust., Speech, and Signal Process. (ICASSP 2006)*, vol. 5, (Tolouse, France), pp. 153–156, May 2006.

[52] S. D'Angelo and V. Välimäki, "Generalized moog ladder filter: Part iiexplicit nonlinear model through a novel delay-free loop implementation method," *IEEE/ACM Trans. Audio, Speech, and Lang. Process.*, vol. 22, pp. 1873–1883, December 2014.

[53] F. Esqueda, *Aliasing reduction in nonlinear audio signal processing*. PhD thesis, Aalto University, Espoo, Finland, May 2018.

[54] E. Paschou, F. Esqueda, V. Välimäki, and J. Mourjopoulos, "Modeling and measuring a Moog voltage-controlled filter," in *2017 Asia-Pacific Signal and Information Process. Association Annual Summit and Conference (APSIPA ASC)*, pp. 1641–1647, December 2017.