Exercises on Laplace Transform

- Fecha de entrega 12 de ago en 23:59
- Puntos 100
- Preguntas 15
- Disponible 5 de ago en 23:30 - 19 de ago en 23:59
- Límite de tiempo Ninguno
- Intentos permitidos Ilimitados

Instrucciones

The following quiz includes some exercises to evaluate the student's Laplace transform knowledge. The student can take this quiz as many times as necessary while verifying its answers. Hints and comments are provided in each exercise to remind the student how to approach the problem. It is recommended that students review the recorded material (in Spanish) on Laplace Transform, specifically the prepared videos on

- Laplace Transform on special functions (**here** (https://drive.google.com/file/d/1AGJewqrqY3TLQOWVr-TbGi88nVES15a_/view?usp=share_link)),
- Examples of Laplace Transforms (**here** (https://drive.google.com/file/d/1NHtNXRsr6Rk7gdOwf6RuWYJoWoMYuo0X/view?usp=sharing))
- Inverse Laplace Transform (**here** (https://drive.google.com/file/d/1c2G1YasJpJwuBETCngIcD1WrmDhtZSdy/view?usp=share_link))

Good luck!

Volver a realizar el examen

Historial de intentos

	Intento	Hora	Puntaje
MÁS RECIENTE	Intento 1	128 minutos	100 de 100

Puntaje para este intento: 100 de 100

Entregado el 9 de ago en 13:24

Este intento tuvo una duración de 128 minutos.

Pregunta 1

8 / 8 pts

Assume that $g(t) = 0$, for all $t < 0$. Compute the Laplace transform of the following function:

$$g(t) = e^{-at}\sin(\omega t + \varphi)$$

for $t \geq 0$ and constant values ω and φ.

G(s)=$\frac{\sin(\varphi)(s-a)+\cos(\varphi)\omega}{(s-a)^2+\omega^2}$

¡Correcto!

○ G(s)=$\frac{\sin(\varphi)(s+a)+\cos(\varphi)\omega}{(s+a)^2+\omega^2}$

The easy way is the following. Setting $f(t)=\sin(\omega t+\varphi)$ we can obtain that $F(s)=\frac{\sin(\varphi)s+\cos(\varphi)\omega}{s^2+\omega^2}$. Then, by using the frequency shift property we obtain that

$$G(s)=\mathcal{L}\left\{e^{-at}f(t)\right\}=\frac{\sin(\varphi)(s+a)+\cos(\varphi)\omega}{(s+a)^2+\omega^2}$$

○ G(s)=$\frac{\sin(\varphi)s+\cos(\varphi)\omega}{s^2+\omega^2}$

○ There is no correct answer!

Pregunta 2

6 / 6 pts

Assume that $f(t)=0$, for all $t<0$. Compute the Laplace transform of the following function:

$$f(t)=a$$

for $t\geq 0$ and a any constant.

Hint: Evaluate the Laplace transform of the function $F(s)$, that is $F(s)=\int_0^\infty f(t)e^{-st}dt$

¡Correcto!

○ $F(s)=\frac{a}{s}$

$F(s)=\int_0^\infty f(t)e^{-st}dt=\int_0^\infty e^{-st}dt=-\frac{a}{s}|_0^\infty=\frac{1}{s}$

○ $F(s)=\frac{1}{s+a}$

○ $F(s)=\frac{a}{s^2}$

○ $F(s)=\frac{a}{(s+a)^2}$

Pregunta 3

8 / 8 pts

Assume that $g(t)=0$, for all $t<0$. Compute the Laplace transform of the following function:

$$g(t)=\sin(\omega t+\phi)$$

for $t\geq 0$.

a) Use the Maclaurin series expansion of $sin(\omega t+\phi)$ and reduce the resulting expression by using the Maclaurin series expansion of $\cos(\omega t)$ and $\sin(\omega t)$. Check this result with the equivalent geometric identity.

b) Set $\omega=2$ and $\phi=\frac{\pi}{4}$ in the result of point (a).

c) Apply the Laplace Transform to the resulting expression in point (b).

The response to this exercise is the answer to point (c).

☐ $\sin(\frac{\pi}{4})\frac{s}{s^2+4}$-$\cos(\frac{\pi}{4})\frac{2}{s^2+4}$

¡Correcto!

- [x] $\frac{\sqrt{2}}{2}\frac{s+2}{s^2+4}$
- [ ] $\frac{\sqrt{2}}{2}\frac{s-2}{s^2+4}$

You should notice first that $g(t) = \sin(\phi)\cos(\omega t) + \cos(\phi)\sin(\omega t)$. Thus, by applying the Laplace Transform it will give

$$H(s) = \sin(\phi)\mathcal{L}\left\{\cos(\omega t)\right\} + \cos(\phi)\left\{\sin(\omega t)\right\}$$

, the result then follows by setting $\omega = 2$ and $\phi = \frac{\pi}{4}$ and reducing the expression.

Pregunta 4

6 / 6 pts

Assume that $h(t) = 0$, for all $t < 0$. Compute the Laplace transform of the following function:

$$h(t) = e^{-3t}$$

for $t \geq 0$ and a any constant.

Hint: Evaluate the Laplace transform of the function $H(s)$, that is $H(s) = \int_0^\infty h(t)e^{-st}dt$

- $H(s) = \frac{1}{s-3}$
- $H(s) = \frac{3}{s}$

¡Correcto!

- (selected) $H(s) = \frac{1}{s+3}$

$H(s) = \int_0^\infty e^{-3t}e^{-st}dt = \int_0^\infty e^{-(s+3)t}dt = -\frac{1}{s+3}e^{-(s+3)t}|_0^\infty = \frac{1}{s+3}$

- There is no correct answer

Pregunta 5

6 / 6 pts

Assume that $f(t) = 0$, for all $t < 0$. Compute the Laplace transform of the following function:

$$f(t) = t^2e^{-3t}$$

for $t \geq 0$.

¡Correcto!

- (selected) $F(s) = \frac{2}{(s+3)^3}$

This is correct. Probably you set $g(t) = t^2$ such that $G(s) = \frac{2}{s^3}$ and then applied the property of frequency shift to obtain that $F(s) = \mathcal{L}\left\{e^{-3t}g(t)\right\} = G(s+3)$

- $F(s) = \frac{6}{(s+3)^2}$
- $F(s) = \frac{3}{(s+3)^2}$

$F(s) = \frac{2}{(s+2)^3}$

Pregunta 6

6 / 6 pts

Find the inverse Laplace Transform of the following function:

$$F(s) = \frac{2s+4}{(s^2+4s+3)}$$

¡Correcto!

- $f(t) = e^{-3t} + e^{-t}$
- $f(t) = e^{3t} + e^{t}$
- $f(t) = e^{3t} - e^{t}$
- $f(t) = -e^{3t} - e^{t}$

Pregunta 7

6 / 6 pts

Assume that $h(t) = 0$, for all $t < 0$. Compute the Laplace transform of the following function:

$$h(t) = (1 + e^{2t})^2$$

for $t \geq 0$.

- $H(s) = \frac{1}{s} + \frac{2}{s+2} + \frac{1}{s+4}$
- $H(s) = \frac{1}{s} - \frac{2}{s-2} - \frac{1}{s-4}$
- $H(s) = \frac{1}{s} - \frac{2}{s+2} + \frac{1}{s-4}$

¡Correcto!

- $H(s) = \frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}$

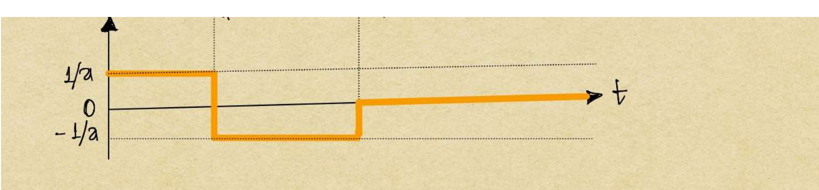
Pregunta 8

8 / 8 pts

Assume that $f(t) = 0$, for all $t < 0$. Compute the Laplace transform of the following function:

$$f(t) = \begin{cases} 1/a & 0 \leq t < a \\ -1/a & a \leq t \leq 2a \\ 0 & \text{otherwise} \end{cases}$$

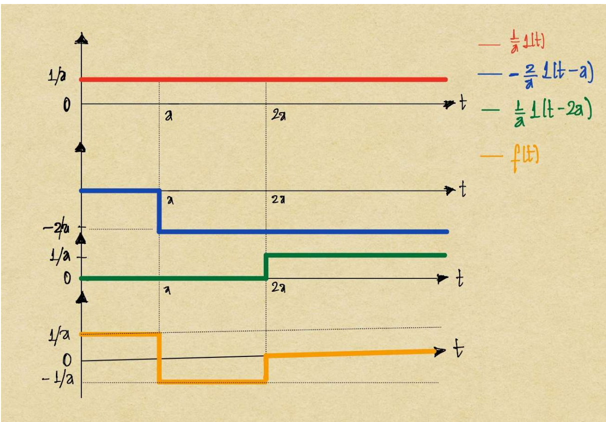
with a a positive constant. Hint: Use the following plot of function $f(t)$



¡Correcto!

- $F(s) = \frac{1}{as}(1 - 2e^{-as} + e^{-2as})$

You noticed that the function $f(t)$ can be expressed as the sum of unit steps such that $f(t) = \frac{1}{a}1(t) - \frac{2}{a}1(t-a) + \frac{1}{a}1(t-2a)$ and then apply the Laplace Transform, we drew similar picture for that class of functions in class.



- $F(s) = \frac{1}{as}(1 - 2e^{-as} - e^{-2as})$
- $F(s) = \frac{1}{as}(1 + 2e^{-as} + e^{-2as})$
- $F(s) = \frac{1}{as}(1 + 2e^{-as} - e^{-2as})$

Pregunta 9

6 / 6 pts

Assume that $g(t) = 0$, for all $t < 0$. Compute the Laplace transform of the following function:

$$g(t) = \begin{cases} \sin(t-5) & t \geq 5 \\ 0 & \text{otherwise} \end{cases}$$

Hint: Use the time shift property. To make sure that your result is correct, plot the function.

¡Correcto!

- $G(s) = \frac{e^{-5s}}{s^2+1}$
- $G(s) = \frac{1}{(s+5)^2+1}$
- None of them is a correct answer

Pregunta 10

8 / 8 pts

Assume that $h(t) = 0$, for all $t < 0$. Compute the Laplace transform of the following function:

$$h(t) = \begin{cases} 1 & t \geq 5 \\ 0 & \text{otherwise} \end{cases}$$

Hint: Use the time shift property. To make sure that your result is correct, plot the function.

¡Correcto!

- $H(s) = \frac{e^{-5s}}{s}$

Remember that $h(t) = 1(t-5)$ is a delayed unit step.

- $H(s) = \frac{1}{s+5}$
- $H(s) = \frac{1}{s-5}$

Pregunta 11

6 / 6 pts

Find the inverse Laplace Transform of the following function:

$$F(s) = \frac{5}{s+3}$$

¡Correcto!

- $f(t) = 5e^{-3t}$

Use tables of Laplace transform to find out what is this function in the time domain.

- $f(t) = 5e^{3t}$
- $f(t) = 3e^{-5t}$

Pregunta 12

6 / 6 pts

Find the inverse Laplace Transform of the following function:

$$X(s) = \frac{s+1}{s^2+s+1}$$

¡Correcto!

- $x(t) = e^{-0.5t}\cos(\sqrt{0.75}t) + \frac{0.5}{\sqrt{0.75}}e^{-0.5}t\sin(\sqrt{0.75}t)$
- $x(t) = e^{-0.5t}\cos(\sqrt{0.75}t) + \frac{0.5}{\sqrt{0.75}}e^{0.5}t\sin(\sqrt{0.75}t)$
- $x(t) = e^{0.5t}\cos(\sqrt{0.75}t) + \frac{0.5}{\sqrt{0.75}}e^{-0.5}t\sin(\sqrt{0.75}t)$
- $x(t) = e^{-0.5t}\cos(\sqrt{0.75}t) - \frac{0.5}{\sqrt{0.75}}e^{-0.5}t\sin(\sqrt{0.75}t)$

Pregunta 13

8 / 8 pts

Find the inverse Laplace Transform of the following function:

$$H(s) = \frac{5s+13}{s(s^2+4s+13)}$$

- $h(t) = 1 - e^{-2t} + e^{-2t}$

¡Correcto!

- $h(t) = 1 - e^{-2t}\cos(3t) + e^{-2t}\sin(3t)$
- $h(t) = 1 + e^{-2t}\cos(3t) + e^{2t}\sin(3t)$

Pregunta 14

6 / 6 pts

Find the inverse Laplace Transform of the following function:

$$G(s) = \frac{6s - 22}{2s^2 + 7s - 15}$$

- $g(t) = -e^{-\frac{3}{2}t} - 4e^{-5t}$
- $g(t) = -e^{-\frac{3}{2}t} + 4e^{-5t}$

¡Correcto!

- $g(t) = -e^{\frac{3}{2}t} + 4e^{-5t}$

Pregunta 15

6 / 6 pts

Assume that $\theta(t) = 0$, for all $t < 0$. Compute the Laplace transform of the following function:

$$\theta(t) = t^2$$

for $t \geq 0$.

¡Correcto!

- $\Theta(s) = \frac{2}{s^3}$

You could proceed in two ways, by completely solving the integral

$$\Theta(s) = \int_0^\infty t^2 e^{-st} dt = -\frac{t}{s}e^{-st}|_0^\infty + \frac{2}{s}\int_0^\infty te^{-st}dt = \frac{2}{s}\left[-\frac{t}{s}e^{-st}|_0^\infty + \frac{1}{s}\int_0^\infty e^{-st}dt\right] = \frac{2}{s^2}\left(-\frac{1}{s}e^{-st}\right)|_0^\infty = \frac{2}{s^3}$$

or using the result from tables.

- $\Theta(s) = \frac{2}{s^2}$
- $\Theta(s) = \frac{3}{s^3}$
- Ninguna de las anteriores

Puntaje del examen: 100 de 100