

Assignment

Consider the following 3D equations defined on $\Omega = (-1, 1) \times (-1, 1) \times (-1, 1)$:

$$\begin{aligned}\frac{\partial F_1}{\partial x} &= \sigma \left(\frac{1}{6} \sum_{i=1}^6 F_i - F_1 \right), \\ -\frac{\partial F_2}{\partial x} &= \sigma \left(\frac{1}{6} \sum_{i=1}^6 F_i - F_2 \right), \\ \frac{\partial F_3}{\partial y} &= \sigma \left(\frac{1}{6} \sum_{i=1}^6 F_i - F_3 \right), \\ -\frac{\partial F_4}{\partial y} &= \sigma \left(\frac{1}{6} \sum_{i=1}^6 F_i - F_4 \right), \\ \frac{\partial F_5}{\partial z} &= \sigma \left(\frac{1}{6} \sum_{i=1}^6 F_i - F_5 \right), \\ -\frac{\partial F_6}{\partial z} &= \sigma \left(\frac{1}{6} \sum_{i=1}^6 F_i - F_6 \right).\end{aligned}$$

The boundary conditions are

$$\begin{aligned}F_1(-1, y, z) &= F_b(y, z), & F_3(x, -1, z) &= F_b(x, z), & F_5(x, y, -1) &= F_b(x, y), \\ F_2(1, y, z) &= F_4(x, 1, z) = F_6(x, y, 1) &= 0,\end{aligned}$$

where

$$F_b(p, q) = \begin{cases} 1, & \text{if } |p| \leq 0.2 \text{ and } |q| \leq 0.2, \\ 0, & \text{otherwise.} \end{cases}$$

Solve this system of equations numerically for $\sigma = 0.1, 1, 10, 100$. Write a report including

1. An introduction to the numerical method used to solve the problem.
2. The numerical settings.
3. Numerical solutions represented by figures.
4. Plots showing the convergence of the iterative method.

Submit your report and your codes to Canvas no later than 1 March 2024.