

a)

$$\mathcal{X} = \{A, B_1, B_2, C, D, E, F\}$$

$$\mathcal{A} = \{a, b, c\}$$

$$\mathcal{Z} = \{A, B, C, D, E, F\}$$

b)

$$P_a = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$P_b = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$P_c = \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$O_a = O_b = O_c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

c)

c)1

$$b' = \frac{b_t P_a}{\|b_t P_a\|}$$

$$b_t P_a = \begin{bmatrix} 0 & .5 & .5 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & .5 & .5 \end{bmatrix}$$

$$\implies b' = \frac{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & .5 & .5 \end{bmatrix}}{\left\| \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & .5 & .5 \end{bmatrix} \right\|} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & .5 & .5 \end{bmatrix}$$

c)2

$$b' = \frac{b_t P_b}{\|b_t P_b\|}$$

$$\begin{aligned}
b_t P_a &= [0 \quad .5 \quad .5 \quad 0 \quad 0 \quad 0 \quad 0] \times \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad .5 \quad .5] \\
\Rightarrow b' &= \frac{[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad .5 \quad .5]}{\|[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad .5 \quad .5]\|} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad .5 \quad .5]
\end{aligned}$$

c)3

$$\begin{aligned}
b' &= \frac{b_t P_c}{\|b_t P_c\|} \\
b_t P_a &= [0 \quad .5 \quad .5 \quad 0 \quad 0 \quad 0 \quad 0] \times \begin{bmatrix} 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&= [0 \quad 0 \quad 0 \quad .5 \quad .5 \quad 0 \quad 0] \\
\Rightarrow b' &= \frac{[0 \quad 0 \quad 0 \quad .5 \quad .5 \quad 0 \quad 0]}{\|[0 \quad 0 \quad 0 \quad .5 \quad .5 \quad 0 \quad 0]\|} = [0 \quad 0 \quad 0 \quad .5 \quad .5 \quad 0 \quad 0]
\end{aligned}$$