TÉCNICO LISBOA

Program Synthesis

An overview of logic-based approaches

Ricardo Brancas

Presentation Outline

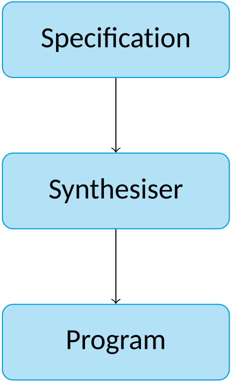
1. What is it?
2. Logic Background
3. Anatomy of a Synthesiser
4. A modern synthesiser: Synquid

Program Synthesis

Specification

Program

Program Synthesis



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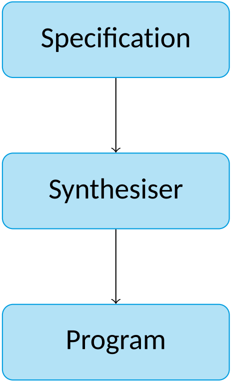

Input/Output Examples
First-order Logic Specification
Hoare Logic Specification
Separation Logic Specification
...

Formal Verification

How can we check if a program is correct (wrt. to the specification)?

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We can use a logic formulation.

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First-order Logic

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A superset of propositional logic, adding predicates, functions and quantifiers.

Propositional Logic

$P \land Q$

$Q \implies P \lor R$

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FOL Decidability

First-order logic is undecidable.

Decidability

Decidable Problem

A problem is decidable if if there is a way of deriving the correct answer.

Undecidable Problem

Complementary, if a problem is undecidable then it is *provably impossible* to create an algorithm that always derives the correct answer.

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Then how can we check a first-order formula?

Satisfiability Modulo Theories

Satisfiability Modulo Theory (SMT)

A decision problem on decidable subsets of first-order logic.

Such subsets are called theories.

Theory: Equality with Uninterpreted Functions

$$(f(b) = d) \wedge (f(a) + f(b) = d) \wedge (a = d)$$

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Theory: Linear Arithmetic

$3x + 2y < 3$

$x + y + z = 45$

Satisfiability Modulo Theories - Models

A model can be seen as a mapping from variables to constants/functions, and represents a solution of the formula.

Example: Model for $\{3x + 2y < 3\}$

$\{x \mapsto 0, y \mapsto 1\}$

Other Useful Theories for PL

- Bit vectors
- Arrays
- Pointer logic
- Quantified Theories

Formal Verification

How can we check if a program is correct (wrt. to the specification)?

We can use a logic formulation.

Simple Hoare Triple

```
{ X = 3 }
Y ::= X - 2;;
X ::= X - 1
{ X = 2 ∧ Y = 1}
```

SMT Formula

$$
\begin{aligned}
& X_0 = 3 \\
\wedge\ & Y_0 = (X_0 - 2) \\
\wedge\ & X_1 = (X_0 - 1) \\
\wedge\ & X_1 = 2 \wedge Y_0 = 1
\end{aligned}
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SMT Formula

$$X_0 = 3$$
$$\land \; Y_0 = (X_0 - 2)$$
$$\land \; X_1 = (X_0 - 1)$$
$$\land \; X_1 = 2 \land Y_0 = 1$$

We say that a program is correct, if the corresponding SMT formula is satisfiable.

Presentation Outline

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Enumeration

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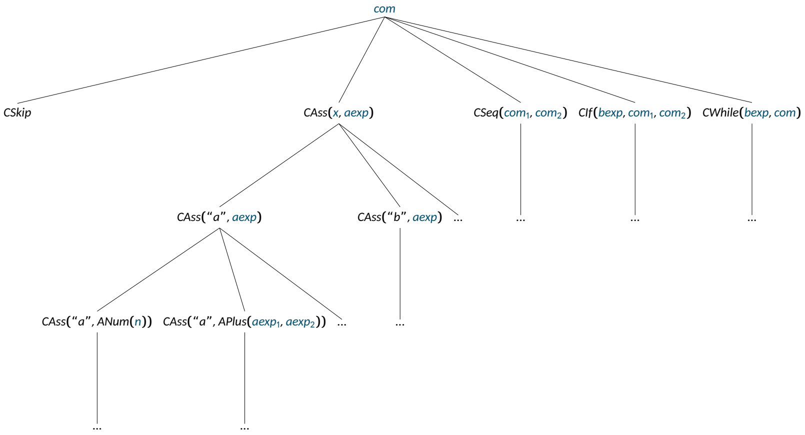
Problem

The space of possible programs, of fixed length, is exponentially large. It is impossible to just check all programs.

Generating Programs (Imp language)

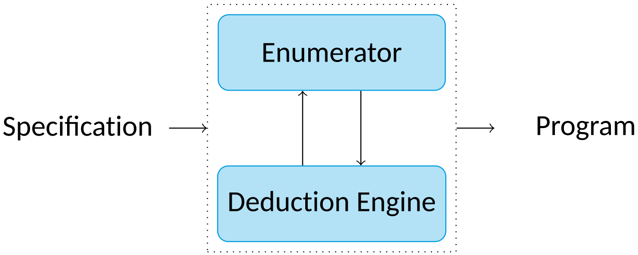


(Simple) Deduction Engine

The job of the deduction engine is:

- To prune the search when incorrect programs are generated.

The simplest synthesiser



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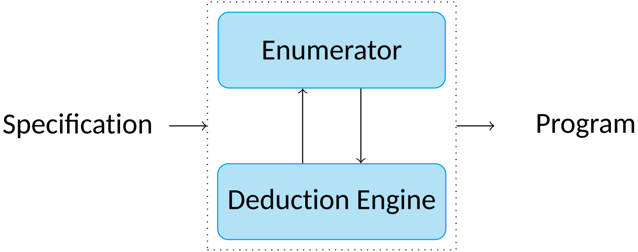

Problem

This idea is very simple, but it has a big problem: most of the time is spent testing very similar programs that will never lead to a solution.

Possible Improvements

- Domain Specific Languages (DSL)
- Partial evaluation
- Conflict-driven learning
- Type theory
- Synthetic separation logic
- ...

Presentation Outline

Synquid

by Nadia Polikarpova

Let's look at a modern synthesiser:

- bidirectional synthesis
- liquid types

Bidirectional synthesis

Liquid Types

Example: `replicate`

Taking a number n and some object α,
return a list containing n copies of α.

Specification

```
replicate :: n:Nat ↦ x:α ↦ {ν:List α | len ν = n}
replicate = ??
```

Example: `replicate`

Auxiliary components

```
zero :: {ν:Int | ν = 0}
inc :: x:Int ↦ {ν:Int | ν = x+1}
dec :: x:Int ↦ {ν:Int | ν = x-1}
leq :: x:Int ↦ y:Int ↦ {ν:Bool | ν = x≤y}
neq :: x:Int ↦ y:Int ↦ {ν:Bool | ν = x≠y}
```