TÉCNICO LISBOA

Program Synthesis

An overview of logic-based approaches

Ricardo Brancas

Presentation Outline

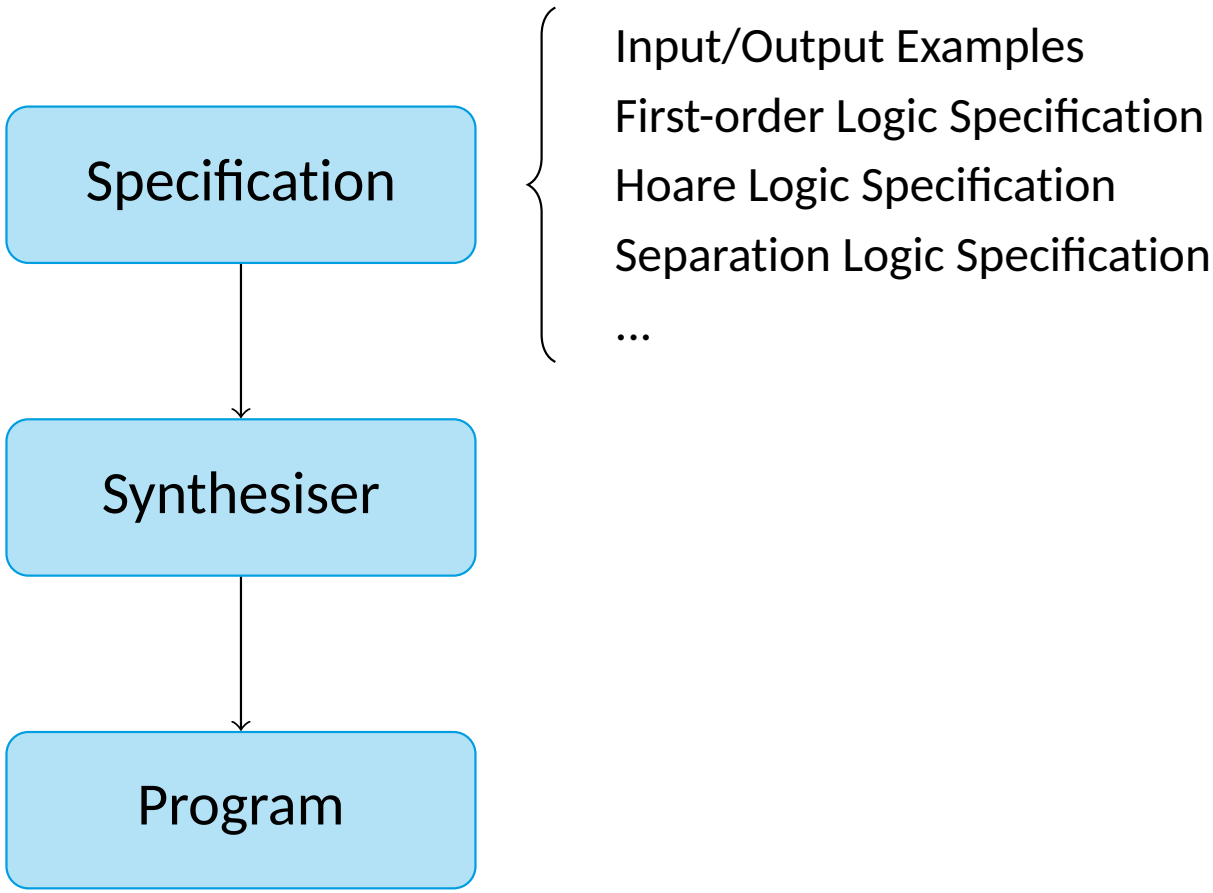
1. What is it?
2. Logic Background
3. A very simple synthesiser
4. A modern synthesiser: Synquid

Program Synthesis

Specification

Program

Program Synthesis



Formal Verification

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First-order Logic I

First-order Logic (FOL)

A superset of propositional logic, adding predicates, functions and quantifiers.

Propositional Logic

$P \wedge Q$

$Q \implies P \vee R$

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First-order Logic

$$\forall x.\, \mathit{Panda}(x) \implies \mathit{Mammal}(x)$$

$$\neg \exists x.\, \mathit{Panda}(x) \land \neg \mathit{Panda}(\mathit{father\text{-}of}(x))$$

First-order Logic II

Validity & Satisfiability

A formula is satisfiable if it is True for some assignment to the variables/functions.

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FOL Decidability

First-order logic is undecidable.

Decidability

Decidable Problem

A problem is decidable iff there is a way of deriving the correct answer.

Undecidable Problem

Complementary, if a problem is undecidable then it is *provably impossible* to create an algorithm that always derives the correct answer.

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Complementary, if a problem is undecidable then it is *provably impossible* to create an algorithm that always derives the correct answer.

As such, it is impossible to determine if any given formula is valid.
So how can we solve this?

Satisfiability Modulo Theories

Satisfiability Modulo Theory (SMT)

A decision problem on decidable subsets of first-order logic.

Such subsets are called theories.

Theory: Presburger arithmetic - $\{\mathbb{N}, +\}$

$3x + 2y < 3$

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Theory: Equality with Uninterpreted Functions

$f(b) = d \ \wedge \ f(a) = d \ \wedge \ a = d$

Satisfiability Modulo Theories - Models

A model can be seen as a mapping from variables to constants/functions, and represents a solution of the formula.

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$\{x \longmapsto 0, y \longmapsto 1\}$

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Example: Model for $\{f(b) = d \ \wedge \ f(a) = d \ \wedge \ a = d\}$

$\{a \mapsto *_1,\ b \mapsto *_2,\ d \mapsto *_1,\ f \mapsto \lambda x.\, *_1\}$

Other Useful Theories for PL

- Bit vectors
- Arrays
- Pointer logic
- Quantified Theories
- …

Different theories can also be combined while retaining decidability.

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Simple Hoare Triple

```
{ X = 3 }
Y ::= X - 2;;
X ::= X - 1
{ X = 2 ∧ Y = 1}
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SMT Formula

$$
\begin{aligned}
& X_0 = 3 \\
\wedge\ & Y_0 = (X_0 - 2) \\
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We say that a program is correct, if the corresponding SMT formula is satisfiable.

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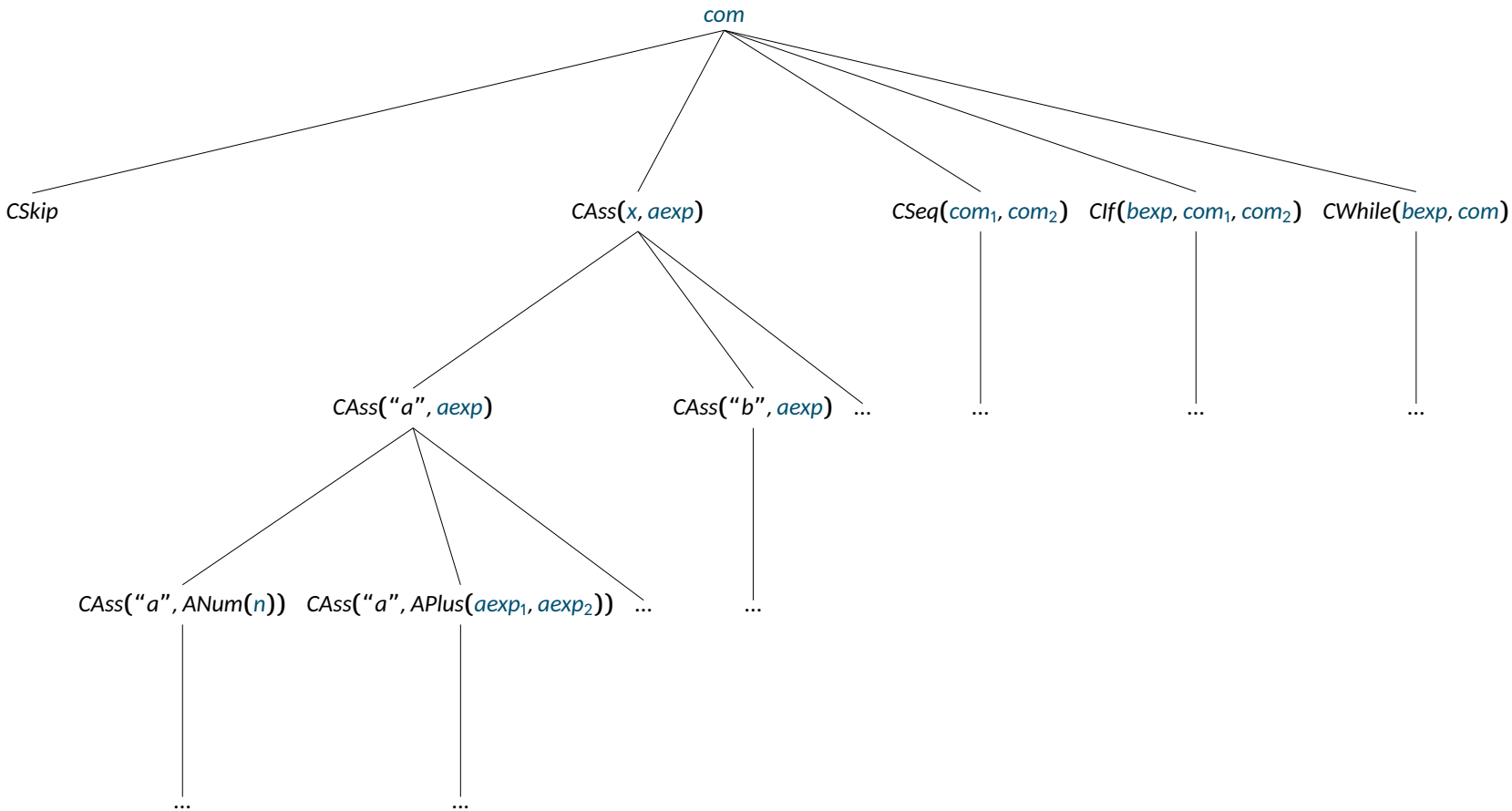
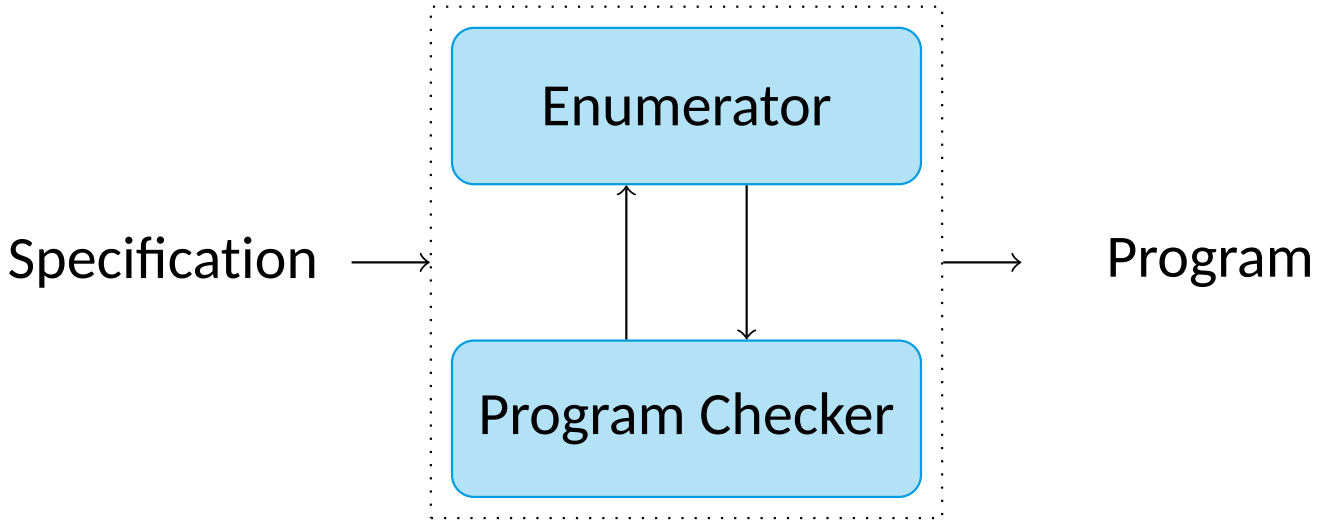
Problem

The space of possible programs of fixed length, is exponentially large. It is impossible to just check all programs.

Generating Programs (Imp language)



The simplest synthesiser



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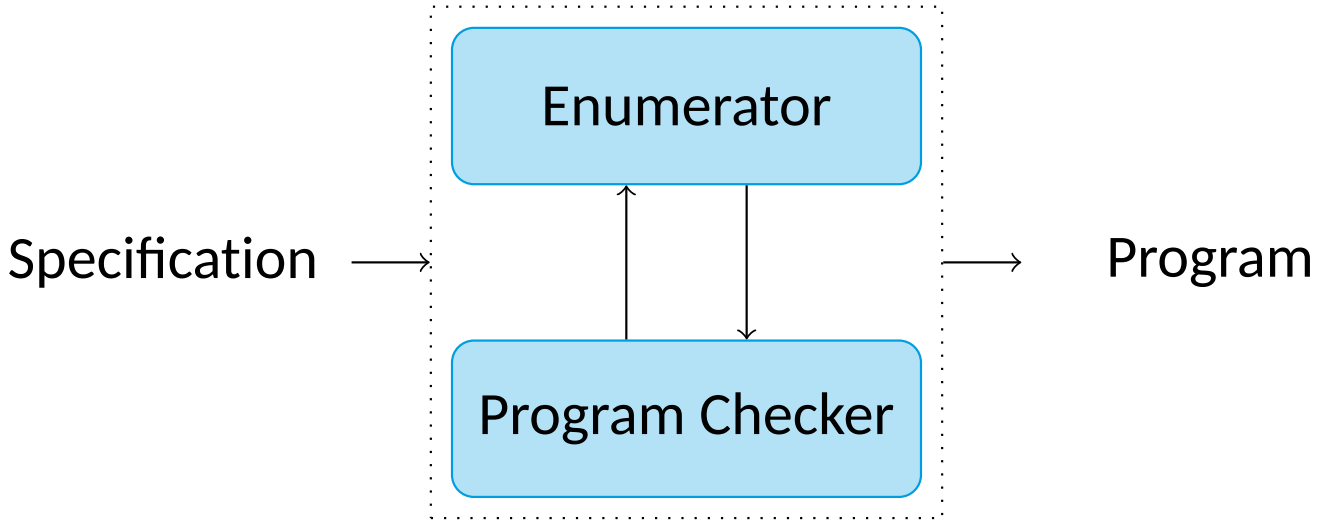

Problem

This idea is very simple, but it has a big problem: most of the time is spent testing very similar programs that will never lead to a solution.

Possible Improvements

- Domain Specific Languages (DSL)
- Partial evaluation
- Conflict-driven learning
- Type theory
- Synthetic separation logic
- …

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Synquid [1]

Synquid is a modern synthesiser that uses *polymorphic refinement types* in order to prune the search space and make deductions about the program.

Nadia Polikarpova

Logically Qualified Data Types

Liquid Types [2]

Liquid Types is a way for automatically deriving refinement types.

Refinement types

A combination of a “regular type” and a **refinement**.

A refinement is a logic restriction on the values of the type.

Logically Qualified Data Types

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A refinement is a logic restriction on the values of the type.

Example

```
i :: {ν:Int | 1 ≤ ν ∧ ν ≤ 99}
```

ν is the notation used to represent the value.

Example: `replicate`

Taking a number *n* and some object *x* of type *α*,
return a list containing *n* copies of *x*.

Specification

```
replicate :: n:Nat ⟼ x:α ⟼ {ν:List α | len ν = n}
replicate = ??
```

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Auxiliary components

```
zero :: {ν:Int | ν = 0}
inc :: x:Int ⟼ {ν:Int | ν = x+1}
dec :: x:Int ⟼ {ν:Int | ν = x-1}
leq :: x:Int ⟼ y:Int ⟼ {ν:Bool | ν = x≤y}
neq :: x:Int ⟼ y:Int ⟼ {ν:Bool | ν = x≠y}
```

References I

[1] Nadia Polikarpova, Ivan Kuraj, and Armando Solar-Lezama. "Program Synthesis from Polymorphic Refinement Types". In: *Proceedings of the 37th ACM SIGPLAN Conference on Programming Language Design and Implementation*. PLDI '16. New York, NY, USA: ACM, 2016, pp. 522–538. ISBN: 978-1-4503-4261-2. DOI: 10.1145/2908080.2908093.

[2] Patrick M. Rondon, Ming Kawaguci, and Ranjit Jhala. "Liquid Types". In: *Proceedings of the 29th ACM SIGPLAN Conference on Programming Language Design and Implementation*. PLDI '08. New York, NY, USA: ACM, 2008, pp. 159–169. ISBN: 978-1-59593-860-2. DOI: 10.1145/1375581.1375602.