TÉCNICO
LISBOA

Program Synthesis

An overview of logic-based approaches

Ricardo Brancas

Presentation Outline

1. What is it?
2. Logic Background
3. A very simple synthesiser
4. A modern synthesiser: Synquid

Program Synthesis

Specification

Program

Program Synthesis

Specification

↓

Synthesiser

↓

Program

- Input/Output Examples
- First-order Logic Specification
- Hoare Logic Specification
- Separation Logic Specification
- ...

Formal Verification

How can we check if a program is correct (wrt. to the specification)?

Formal Verification

How can we check if a program is correct (wrt. to the specification)?

We can use a logic formulation.

Presentation Outline

1. What is it?
2. Logic Background
3. A very simple synthesiser
4. A modern synthesiser: Synquid

First-order Logic

First-order Logic (FOL)

A superset of propositional logic, adding predicates, functions and quantifiers.

Propositional Logic

$P \wedge Q$

$Q \implies P \vee R$

First-order Logic

First-order Logic (FOL)

A superset of propositional logic, adding predicates, functions and quantifiers.

First-order Logic

$\forall x. P(x)$

$\exists y \forall x. Q(y, x) \land P(f(y))$

First-order Logic

First-order Logic (FOL)

A superset of propositional logic, adding predicates, functions and quantifiers.

First-order Logic

$\forall x. P(x)$

$\exists y \forall x. Q(y, x) \wedge P(f(y))$

FOL Decidability

First-order logic is undecidable.

Decidability

Decidable Problem

A problem is decidable if if there is a way of deriving the correct answer.

Undecidable Problem

Complementary, if a problem is undecidable then it is *provably impossible* to create an algorithm that always derives the correct answer.

Decidability

Decidable Problem

A problem is decidable if if there is a way of deriving the correct answer.

Undecidable Problem

Complementary, if a problem is undecidable then it is *provably impossible* to create an algorithm that always derives the correct answer.

Then how can we check a first-order formula?

Satisfiability Modulo Theories

Satisfiability Modulo Theory (SMT)

A decision problem on decidable subsets of first-order logic.

Such subsets are called theories.

Theory: Equality with Uninterpreted Functions

$$(f(b) = d) \land (f(a) + f(b) = d) \land (a = d)$$

Satisfiability Modulo Theories

Satisfiability Modulo Theory (SMT)

A decision problem on decidable subsets of first-order logic.

Such subsets are called theories.

Theory: Equality with Uninterpreted Functions

$(f(b) = d) \wedge (f(a) + f(b) = d) \wedge (a = d)$

Theory: Linear Arithmetic

$3x + 2y < 3$

$x + y + z = 45$

Satisfiability Modulo Theories - Models

A model can be seen as a mapping from variables to constants/functions, and represents a solution of the formula.

Example: Model for $\{3x + 2y < 3\}$

$\{x \mapsto 0, y \mapsto 1\}$

Other Useful Theories for PL

- Bit vectors
- Arrays
- Pointer logic
- Quantified Theories

Formal Verification

How can we check if a program is correct (wrt. to the specification)?

We can use a logic formulation.

Simple Hoare Triple

```
{ X = 3 }
Y ::= X - 2;;
X ::= X - 1
{ X = 2 ∧ Y = 1}
```

Formal Verification

How can we check if a program is correct (wrt. to the specification)?

We can use a logic formulation.

Simple Hoare Triple

```
{ X = 3 }
Y ::= X - 2;;
X ::= X - 1
{ X = 2 ∧ Y = 1}
```

SMT Formula

$$X_0 = 3$$
$$\land \; Y_0 = (X_0 - 2)$$
$$\land \; X_1 = (X_0 - 1)$$
$$\land \; X_1 = 2 \land Y_0 = 1$$

Formal Verification

How can we check if a program is correct (wrt. to the specification)?
We can use a logic formulation.

Simple Hoare Triple

```
{ X = 3 }
Y ::= X - 2;;
X ::= X - 1
{ X = 2 ∧ Y = 1}
```

SMT Formula

$$
\begin{aligned}
& X_0 = 3 \\
\wedge\ & Y_0 = (X_0 - 2) \\
\wedge\ & X_1 = (X_0 - 1) \\
\wedge\ & X_1 = 2 \wedge Y_0 = 1
\end{aligned}
$$

We say that a program is correct, if the corresponding SMT formula is satisfiable.

Presentation Outline

1. What is it?
2. Logic Background
3. A very simple synthesiser
4. A modern synthesiser: Synquid

Generating Programs

Now we can check if a given program is correct. But how do we generate them?

Generating Programs

Now we can check if a given program is correct. But how do we generate them?

Enumeration

We can use the grammar of the language to generate (enumerate) all possible programs.

Generating Programs

Now we can check if a given program is correct. But how do we generate them?

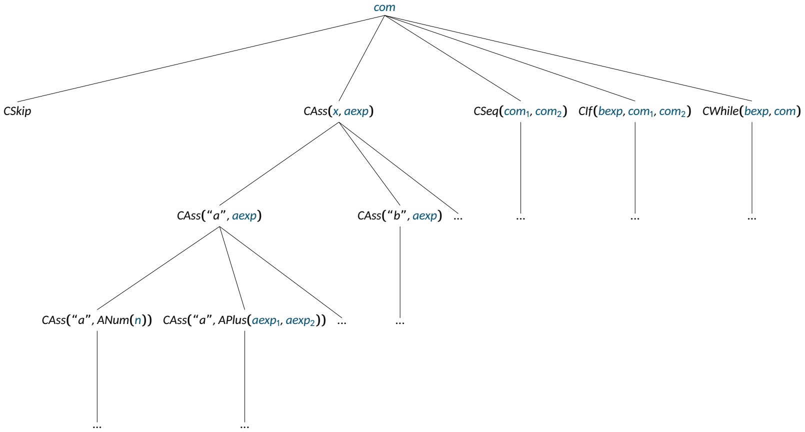
Enumeration

We can use the grammar of the language to generate (enumerate) all possible programs.

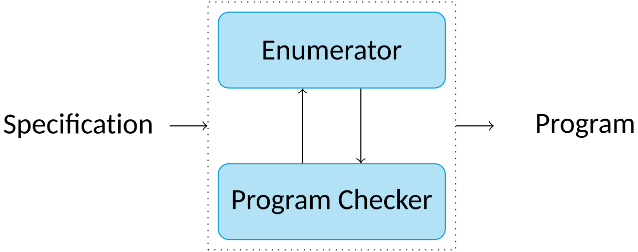
Problem

The space of possible programs of fixed length, is exponentially large. It is impossible to just check all programs.

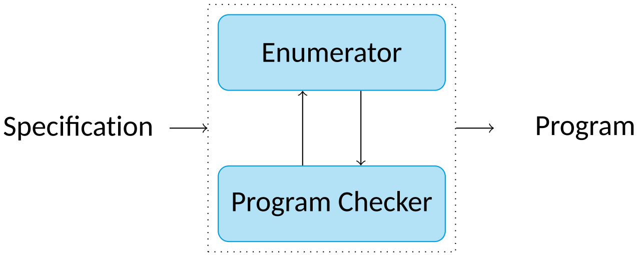
Generating Programs (Imp language)



The simplest synthesiser



The simplest synthesiser



Problem

This idea is very simple, but it has a big problem: most of the time is spent testing very similar programs that will never lead to a solution.

Possible Improvements

- Domain Specific Languages (DSL)
- Partial evaluation
- Conflict-driven learning
- Type theory
- Synthetic separation logic
- ...

Presentation Outline

1. What is it?
2. Logic Background
3. A very simple synthesiser
4. A modern synthesiser: Synquid

Synquid [1]

by Nadia Polikarpova

Synquid is a modern synthesiser that uses *polymorphic refinement types* in order to prune the search space and make deductions about the program.

Logically Qualified Data Types

Liquid Types [2]

Liquid Types is a way for automatically derivating refinement types.

Refinement types

A combination of a “regular type” and a **refinement**.

A refinement is a logic restriction on the values of the type.

Logically Qualified Data Types

Liquid Types [2]

Liquid Types is a way for automatically derivating refinement types.

Refinement types

A combination of a “regular type” and a **refinement**.

A refinement is a logic restriction on the values of the type.

Example

$\texttt{i :: \{}\nu\texttt{:Int | 1} \leq \nu \land \nu \leq \texttt{99\}}$

ν is the notation used to represent the value.

Example: `replicate`

Taking a number *n* and some object *x* of type *α*,
return a list containing *n* copies of *x*.

Specification

```
replicate :: n:Nat ↦ x:α ↦ {ν:List α | len ν = n}
replicate = ??
```

Example: `replicate`

Specification

```
replicate :: n:Nat ↦ x:α ↦ {ν:List α | len ν = n}
replicate = ??
```

Auxiliary components

```
zero :: {ν:Int | ν = 0}
inc :: x:Int ↦ {ν:Int | ν = x+1}
dec :: x:Int ↦ {ν:Int | ν = x-1}
leq :: x:Int ↦ y:Int ↦ {ν:Bool | ν = x≤y}
neq :: x:Int ↦ y:Int ↦ {ν:Bool | ν = x≠y}
```

References I

[1] Nadia Polikarpova, Ivan Kuraj, and Armando Solar-Lezama. "Program Synthesis from Polymorphic Refinement Types". In: *Proceedings of the 37th ACM SIGPLAN Conference on Programming Language Design and Implementation*. PLDI '16. New York, NY, USA: ACM, 2016, pp. 522–538. ISBN: 978-1-4503-4261-2. DOI: 10.1145/2908080.2908093.

[2] Patrick M. Rondon, Ming Kawaguci, and Ranjit Jhala. "Liquid Types". In: *Proceedings of the 29th ACM SIGPLAN Conference on Programming Language Design and Implementation*. PLDI '08. New York, NY, USA: ACM, 2008, pp. 159–169. ISBN: 978-1-59593-860-2. DOI: 10.1145/1375581.1375602.