Mean value & standard deviation

$u \longrightarrow$ Random variable

$M \longrightarrow$ # of values that may take u

$P_j = P(u_j) \longrightarrow$ Probability that happens u_j

$\sum_{j=1}^{M} P(u_j) = 1 \longrightarrow$ Normalization

$\bar{u} = \langle u \rangle := \sum_{j=1}^{M} u_j P(u_j) \longrightarrow$ Mean value

$$\overline{f(u)} = \langle f(u) \rangle := \sum_{j=1}^{M} f(u_j) P(u_j)$$

I. $\langle f(u) + g(u) \rangle = f(u) + g(u)$

II. $\langle c f(u) \rangle = c \langle f(u) \rangle$

$\Delta u := u - \langle u \rangle \longrightarrow$ mean value deviation

$\longrightarrow \langle \Delta u \rangle = \langle u - \langle u \rangle \rangle = \langle u \rangle - \langle \langle u \rangle \rangle = \langle u \rangle - \langle u \rangle = 0$

$\hookrightarrow$ first moment.

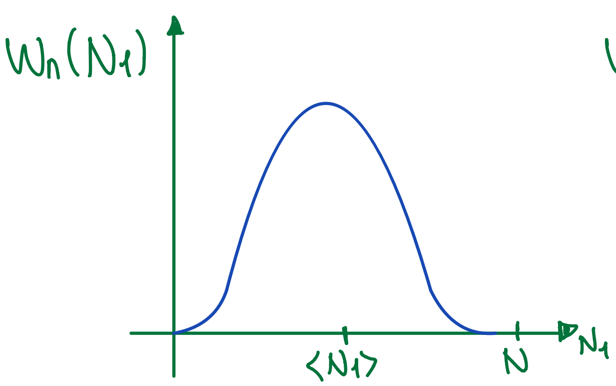
$(\Delta u)^2 = (u - \langle u \rangle)^2 \longrightarrow$ Quadratic deviation

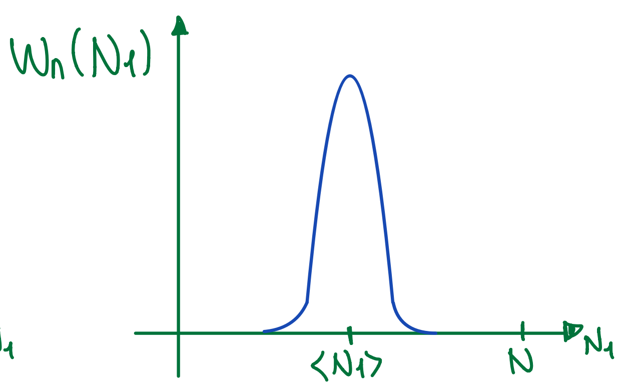
$$\longrightarrow \langle (\Delta u)^2 \rangle = \langle (u - \langle u \rangle)^2 \rangle = \langle u^2 - 2u\langle u \rangle + \langle u \rangle^2 \rangle$$

$$= \langle u^2 \rangle - 2\langle u \rangle \langle u \rangle + \langle u \rangle^2 = \langle u^2 \rangle - \langle u \rangle^2 \geq 0.$$

therefore, $\langle u^2 \rangle \geq \langle u \rangle^2 \longrightarrow$ Variance, Second moment, dispersion.

$\sqrt{\langle (\Delta u)^2 \rangle} \longrightarrow$ Standard deviation.





$\langle (\Delta u)^n \rangle = \langle (u - \langle u \rangle)^n \rangle \longrightarrow$ n-th moment with respect to the mean value.

For the random walk we get.

$$\langle N_1 \rangle = \sum_{N_1=0}^{N} N_1 W_N(N_1) = \sum_{N_1=0}^{N} N_1 \frac{N!}{N_1! \cdot N_2!} p^{N_1} q^{N_2}$$

for $N_2 = N - N_1$ and $q = 1-p$.

$$\langle N_1 \rangle = p \frac{\partial}{\partial p} \sum_{N_1=0}^{N} \frac{N!}{N_1! \cdot N_2!} p^{N_1} q^{N_2} = p \frac{\partial}{\partial p} (p+q)^N$$

$$= pN(p+q)^{N-1} = pN =: \mu$$

$$\langle N_2 \rangle = \langle N - N_1 \rangle = \langle N \rangle - \langle N_1 \rangle = N - pN = N(1-p) = q(N)$$

$$\langle N_1^2 \rangle = \sum_{N_1=0}^{N} N_1^2 W_N(N_1) = \left(p \frac{\partial}{\partial p}\right)\left\{\left(p \frac{\partial}{\partial p}\right)\left[\sum_{N_1=0}^{N} \frac{N!}{N_1! \cdot N_2!} p^{N_1} q^{N_2}\right]\right\}$$

$$= \left(p \frac{\partial}{\partial p}\right)\left\{ pN(p+q)^{N-1} \right\} = pN\ (p+q)^{N-1} + p^2 N(N-1)(p+q)^{N-2}$$

$$= pN + p^2 N(N-1)$$

$$\longrightarrow \langle (\Delta N_1)^2 \rangle = \langle N_1^2 \rangle - \langle N_1 \rangle^2 = pN + p^2 N(N-1) - p^2 N^2$$

$$= pN\left[1 + \cancel{p(N-1) - pN}\right] = pN[1-p] = qpN. =: \sigma^2$$

$$\longrightarrow \Delta N_1^* := \sqrt{\langle (\Delta N_1)^2 \rangle} = (qp)^{1/2} \sqrt{N} = \sigma$$

$$\longrightarrow \frac{\Delta N_1^*}{\langle N_1 \rangle} = \frac{(qp)^{1/2} \sqrt{N}}{pN} = \left(\frac{q}{p}\right)^{1/2} \left(\frac{1}{N}\right)^{1/2} \xrightarrow[N \to \infty]{} 0$$