Landau diamagnetism

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2$$

Hamiltonian of a charged particle in the magnetic field.

Classical case:

$$z_1 = \int d^3\vec{r} \int d^3\vec{p} \exp\left[-\frac{\beta}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2\right]$$

making a change of variables

$$\vec{p} \to \vec{p} - \left(\frac{q}{c}\right)\vec{A}$$

then,

$$z_1 = \int d^3\vec{r} \int d^3\vec{p} \exp\left(-\frac{\beta}{2m}\vec{p}^2\right)$$

There is no effect in the classical case!

Purely quantum phenomenon!

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{H})$$

Lorentz force.

then,

$$\omega = \frac{q\omega}{mc}$$

Rotation frequency of a particle in a uniform field.

Let, $\vec{H} = H\vec{K}$

$$\vec{A} = xH\vec{j}$$

Landau norm

then,

$$\mathcal{H} = \frac{1}{2m}\left[p_x^2 + \left(p_y - \frac{qH}{c}x\right)^2 + p_z^2\right]$$

Let

$$\Psi = \exp(ik_y y)\exp(ik_z z)\varphi(x)$$

$$\mathcal{H}\Psi = E\Psi$$

then,

$$\frac{1}{2m}\left[p_x^2 + \left(\hbar k_y - \frac{q\mathcal{H}}{c}x\right)^2\right]\varphi(x) = \left(E - \frac{\hbar^2 k_z^2}{2m}\right)\varphi(x)$$

Harmonic Oscillator!

$$\omega = \frac{|q|\,H}{mc}$$ Oscillator frequency.

therefore,

$$\left[\frac{1}{2m}(p'_x)^2 + \frac{1}{2}m\omega^2(x')^2\right] = \hbar\omega\left(n+\frac{1}{2}\right)\varphi_n(x')$$

thus,

$$\varphi_n(x') = H_n(x')\exp\left(-\frac{1}{2}\alpha(x')^2\right)$$

$H_n :=$ Hermite polynomials

$$\alpha := \frac{|q|\,H}{\hbar c}$$

the energy is

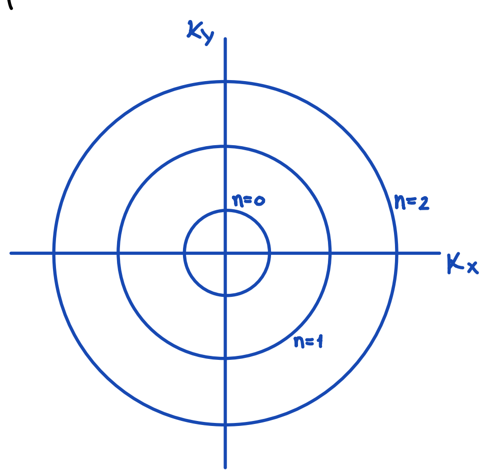
$$E = E(n, K_z) = \frac{\hbar^2 K_z^2}{2m} + \hbar\omega\left(n + \frac{1}{2}\right)$$

$$\Psi = \Psi^{K_y K_z}(x,y,z) = \exp(iK_y y)\exp(iK_z z)\,\varphi\left(x - \frac{1}{\alpha}K_y\right)$$

In the absence of magnetic field we know

$$E_{free} = \frac{\hbar^2}{2m}(K_x^2 + K_y^2 + K_z^2)$$

In the presence of magnetic field the states the electrons collapse



$$K_x^2 + K_y^2 = \frac{2m}{\hbar^2}\hbar\omega\left(n+\frac{1}{2}\right)$$

$$= \frac{2eH}{\hbar c}\left(n+\frac{1}{2}\right)$$

$e :=$ Elementary charge

level $n=0$.

$$K^2 = K_x^2 + K_y^2 = \frac{eH}{\hbar c} \longrightarrow \# = \frac{\pi \frac{eH}{\hbar c}}{\left(\frac{2\pi}{L}\right)^2}$$

Number of states inside the circle $n=0$

Then,

Spin

$$\frac{2\pi L^2 eH}{4\pi^2 \frac{h}{2\pi} c} = \frac{eHL^2}{hc}$$

level $n=1$

$$2\left(\frac{L}{2\pi}\right)^2 \pi \left(\frac{2eH}{c\hbar}\right)\left(\frac{3}{2}\right) = \frac{3eHL^2}{hc}$$

level $n=2$

$$2\left(\frac{L}{2\pi}\right)^2 \pi \left(\frac{2eH}{c\hbar}\right)\left(\frac{5}{2}\right) = \frac{5eHL^2}{hc}$$

level $n=3$

$$2\left(\frac{L}{2\pi}\right)^2 \pi \left(\frac{2eH}{c\hbar}\right)\left(\frac{7}{2}\right) = \frac{7eHL^2}{hc}$$

$g = 2\frac{eHL^2}{hc} \rightarrow$ degeneration factor

$$\mathcal{E} = \mathcal{E}(n, K_z, \delta) = \frac{\hbar^2 K_z^2}{2m} + \hbar\omega\left(n+\frac{1}{2}\right)$$

$K_z = -\infty, \dots, \infty$ in intervals of $2\pi/L$

$n = 0, 1, 2, \dots \qquad \delta = 1, 2, \dots, g.$

$$\ln(\Xi) = \sum_j \ln\left\{ 1 + z \exp(-\beta E_j) \right\}$$

$$\ln(\Xi) = \frac{2eHL^2}{hc} \sum_{n=0}^{\infty} \frac{L}{2\pi} \int_{-\infty}^{\infty} dk_z \ln\left\{ 1 + z \exp\left[-\frac{\beta\hbar^2 k_z^2}{2m} - \frac{\beta\hbar e H}{mc}\left(n + \frac{1}{2}\right) \right] \right\}$$

let $z \ll 1 \rightarrow \ln(1+x) \simeq x \rightarrow$ Gaussian

$$\ln(\Xi) = \frac{2eHL^2}{hc} \frac{L}{2\pi} z \left(\frac{2\pi m}{\beta\hbar} \right)^{1/2} \sum_{n=0}^{\infty} \exp\left(-\frac{\beta\hbar e H}{mc}\left(n + \frac{1}{2}\right) \right)$$

then,

$$\ln(\Xi) = \frac{eHL^3}{hc\lambda} z \frac{1}{\sinh(\beta\mu_B H)}$$

$$\mu_B = \frac{e\hbar}{2mc} \qquad \lambda = \frac{h}{(2\pi m k_B T)^{1/2}}$$

$$N = z \frac{\partial}{\partial z} \ln(\Xi) = \frac{eHL^3}{hc\lambda} z \left[\sinh(\beta\mu_B H) \right]^{-1}$$

$$M = -\frac{\partial \phi}{\partial H} = \frac{1}{\beta} \frac{\partial}{\partial H} \ln(\Xi)$$

$$= \frac{eL^3 z}{\beta hc\lambda} \left\{ \frac{1}{\sinh(\beta\mu_B H)} - \frac{\beta\mu_B H \cosh(\beta\mu_B H)}{\sinh^2(\beta\mu_B H)} \right.$$

solving for z in N and substituying in M

$$M = -N \mu_B \mathcal{L}(\beta\mu_B H)$$

$$\mathcal{L}(x) := \coth(x) - \frac{1}{x}$$ Langevin function