

Particle in a box

$$x \in [0, L]$$

$$\hat{H}\Psi_n(x) = E_n\Psi_n(x)$$

$$\hat{H} = \frac{1}{2m} \hat{p}^2 = -\frac{\hbar}{2m} \frac{d^2}{dx^2}$$

then,

$$\Psi_n(x) = c e^{ikx}$$

$$E_n = \frac{\hbar^2 k^2}{2m}$$

$$\Psi_n(x) = \Psi_n(x+L) \rightarrow \text{Periodic boundary condition}$$

Also

$$\exp(iKL) = 1 \rightarrow KL = 2\pi n$$

$$k = \frac{2\pi n}{L} \quad ; \quad n = 0, \pm 1, \pm 2, \dots$$

$$\frac{2\pi}{L} \rightarrow 0 \quad \text{if} \quad L \rightarrow \infty$$

$$\sum_{\vec{k}} f(\vec{k}) \rightarrow \int \frac{d\vec{k}}{\frac{2\pi}{L}} f(\vec{k}) = \frac{L}{2\pi} \int d\vec{k} f(\vec{k})$$

In 3D

$$\Psi(\vec{r}) = c \exp(i\vec{k} \cdot \vec{r})$$

$$\vec{k} = \frac{2\pi}{L_1} m_1 \hat{e}_x + \frac{2\pi}{L_2} m_2 \hat{e}_y + \frac{2\pi}{L_3} m_3 \hat{e}_z$$

$$E_{\vec{k}} = \frac{\hbar^2 \vec{k}^2}{2m} \quad \text{and} \quad \sum_{\vec{k}} f(\vec{k}) \rightarrow \frac{V}{(2\pi)^3} \int d^3\vec{k} f(\vec{k})$$

$$E_j := E_{\vec{k}, \sigma} = \frac{\hbar^2 k^2}{2m}$$

$$j := (\vec{k}, \sigma)$$

$$E_j := E_{\vec{k}, \sigma} = \frac{\hbar^2 k^2}{2m} - \mu_B H_{\sigma}$$

$$\sigma = \pm 1$$

Bohr magneton

$$\hat{H}_{\text{mol}} = \hat{H}_{\text{tr}} + \hat{H}_{\text{el}} + \hat{H}_{\text{rot}} + \hat{H}_{\text{vib}} + \dots \quad \text{molecules}$$

$$E_j = E_{\vec{k}, J, n} = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2}{2I} J(J+1) + \hbar \omega \left(n + \frac{1}{2} \right)$$