Two levels

$$N_1 \longrightarrow 0 \qquad N_2 \longrightarrow \varepsilon > 0 \qquad ; \quad N = N_1 + N_2$$

$$E = \varepsilon (N - N_1)$$

$$\Omega = \frac{N!}{N_1!\,(N-N_1)!} \longrightarrow \Omega(E,N) = \frac{N!}{\left(N - \frac{E}{\varepsilon}\right)!\left(\frac{E}{\varepsilon}\right)!}$$

$$\ln(\Omega(E,N)) = N\ln(N) - \cancel{N} - N_1\ln(N_1) + \cancel{N_1} - N_2\ln(N_2) + \cancel{N_2}$$

$$= N\ln(N) - N_1\ln(N_1) - N_2\ln(N_2)$$

$$= N\ln(N) - \left(N - \frac{E}{\varepsilon}\right)\ln\left(N - \frac{E}{\varepsilon}\right) - \frac{E}{\varepsilon}\ln\left(\frac{E}{\varepsilon}\right)$$

$$s(u) = \lim_{\text{Term}} \frac{1}{N} K_B \ln \Omega(E,N) = -K_B\left(1 - \frac{u}{\varepsilon}\right)\ln\left(1 - \frac{u}{\varepsilon}\right) - K_B \frac{u}{\varepsilon}\ln\left(\frac{u}{\varepsilon}\right)$$

$$\frac{1}{T} = \frac{\partial s}{\partial u} = \frac{K_B}{\varepsilon}\left[\ln\left(1 - \frac{u}{\varepsilon}\right) - \ln\left(\frac{u}{\varepsilon}\right)\right]$$

$$u = \frac{\varepsilon e^{-\beta\varepsilon}}{1 + e^{-\beta\varepsilon}} \qquad \beta := \frac{1}{K_B T}$$

$$c = \frac{\partial u}{\partial T} = K_B (\beta\varepsilon)^2 \frac{e^{-\beta\varepsilon}}{(1 + e^{-\beta\varepsilon})^2}$$