Einstein solid 2

$$E\{n_j\} = \sum_{j=1}^{N} \left(n_j + \frac{1}{2}\right)\hbar\omega$$

$$Z = \sum_{\{n_j\}} \exp\left(-\beta E\{n_j\}\right) = \sum_{n_1,\ldots,n_N} \exp\left[-\beta \sum_{j=1}^{N} \left(n_j + \frac{1}{2}\right)\hbar\omega\right]$$

$$= \left\{\sum_{n=0}^{\infty} \exp\left[-\left(n + \frac{1}{2}\right)\beta\hbar\omega\right]\right\}^N = Z_1^N$$

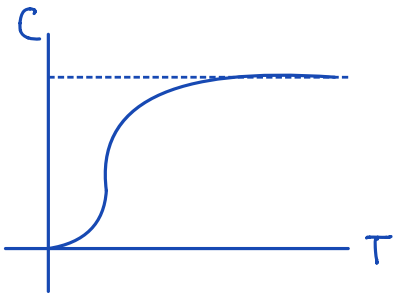
$$Z_1 = \sum_{n=0}^{\infty} \exp\left[-\left(n + \frac{1}{2}\right)\beta\hbar\omega\right] = \sum_{n=0}^{\infty} \exp(-n\beta\hbar\omega)\exp\left(-\frac{1}{2}\beta\hbar\omega\right)$$

$$= \exp\left(-\frac{1}{2}\beta\hbar\omega\right)\frac{1}{1-\exp(-\beta\hbar\omega)}$$

$$f = -\frac{1}{\beta}\lim_{N\to\infty}\frac{1}{N}\ln(Z) = \frac{1}{2}\hbar\omega + k_B T \ln\left(1 - \exp\left(-\frac{\hbar\omega}{k_B T}\right)\right)$$

$$s = -\frac{\partial f}{\partial T} = -k_B \ln\left(1 - \exp\left(-\frac{\hbar\omega}{k_B T}\right)\right) + k_B\left(\frac{\hbar\omega}{k_B T}\right)\frac{\exp\left(-\frac{\hbar\omega}{k_B T}\right)}{\left(1 - \exp\left(-\frac{\hbar\omega}{k_B T}\right)\right)}$$

$$c = T\frac{\partial s}{\partial T} = k_B\left(\frac{\hbar\omega}{k_B T}\right)^2 \frac{\exp\left(\frac{\hbar\omega}{k_B T}\right)}{\left(1 - \exp\left(-\frac{\hbar\omega}{k_B T}\right)\right)^2}$$



$$u = -\frac{1}{N}\frac{\partial}{\partial\beta}\ln(Z) = -\frac{\partial}{\partial\beta}\ln(Z_1)$$

$$= \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1}$$