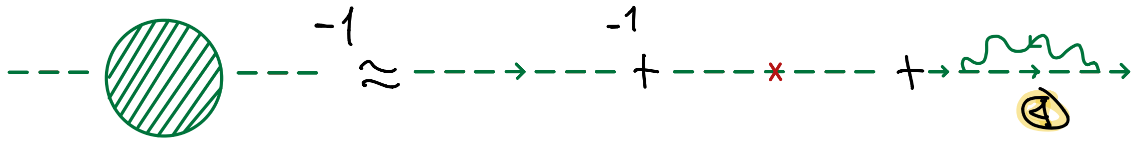
Yang-Mills renormalization to a loop (counterterms)

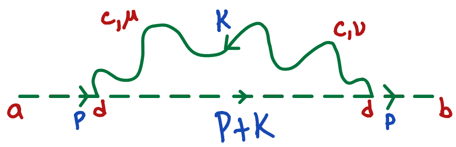
$N_n = 2 \longrightarrow D = 2$

$\tilde{\Gamma}^{(2)}_{ab}$:



$= i\delta_{ab} p^2 \Delta Z_n$

④ =



$$= \hat{g}^2 M^{\varepsilon} C_1 \delta_{ab} \int \frac{d^{2\omega}K}{(2\pi)^{2\omega}} \frac{D_F^{\mu\nu}(K)(p+K)_\mu p_\nu}{(p+K)^2 + i\epsilon}$$

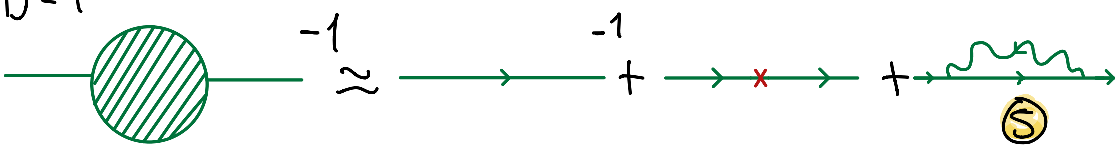
$$= \frac{2i\hat{g}^2 C_1 \delta_{ab}}{16\pi^2} \frac{1}{\varepsilon} p^2 \left(\frac{3}{4} - \frac{5}{4}\xi \right) + (\text{finite})$$

therefore,

$$\Delta Z_n = \frac{-2\hat{g}^2 C_1}{64\pi^2} \frac{1}{\varepsilon} (3 - 5\xi)$$

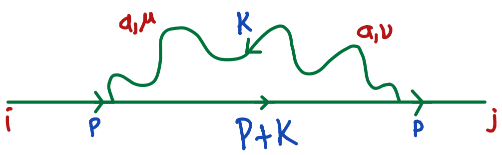
$N_F = 2$; $D = 1$

$\tilde{\Gamma}^{(2)}_{ab}$:



$= i(\Delta Z_\psi \not{p} - m K_m)\delta_{ij}$

⑤ =



$$\sim \sum_a (T_a T_a)_{ij} = C_3 \delta_{ij}$$

$$C_3 = \frac{d_a}{d_r} C_r$$

$$= \hat{g}^2 M^{\varepsilon} C_3 \delta_{ij} \int \frac{d^{2\omega}K}{(2\pi)^{2\omega}} (\gamma_\nu \tilde{S}_F(p+K) \gamma_\mu) \tilde{D}_F^{\mu\nu}(K)$$

using $\gamma_\mu \gamma^\mu = 2\omega$; $\gamma^\mu \gamma_\rho \gamma^\mu = 2(1-\omega)\gamma_\rho$

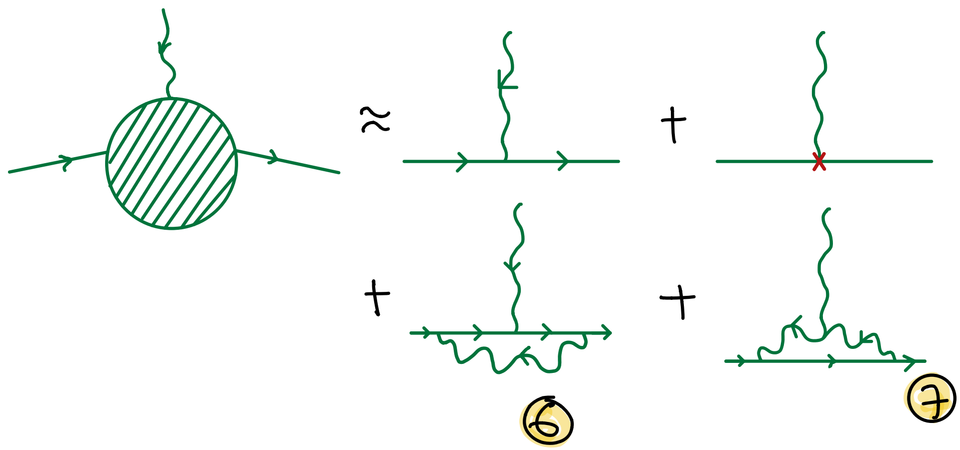
$$⑤ = \frac{2i\hat{g}^2 C_3 \delta_{ij}}{16\pi^2} \frac{1}{\varepsilon} (\xi \not{p} - (3+\xi) m) + (\text{finite})$$

therefore,

$$\Delta Z_\psi = \frac{-2\hat{g}^2 C_3 \xi}{16\pi^2} \frac{1}{\varepsilon} \quad ; \quad K_m = -\frac{2\hat{g}^2 C_3}{16\pi^2} \frac{1}{\varepsilon} (3+\xi)$$

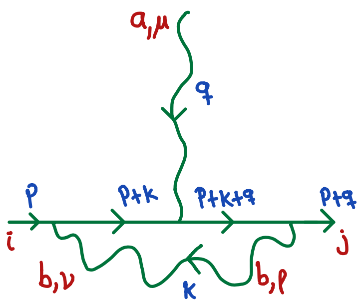
$N_B = 1\ ;\ N_F = 2 \longrightarrow D = 0$

$\tilde{\Gamma}^{(3)}_\mu$:



$ig K_2 (T_a)_{ij}\gamma_\mu$

⑥ =



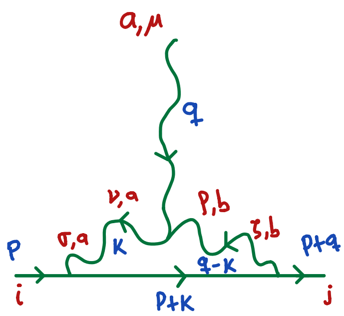
$$= g\hat{g}^2 M^\varepsilon (T_b T_a T_b)_{ji}\int \frac{d^{2\omega}k}{(2\pi)^{2\omega}}\, \tilde{D}^{\rho\nu}(k)\left[\gamma_\rho \tilde{S}_F(p+k+q)\gamma_\mu \tilde{S}_F(p+k)\gamma_\nu\right]$$

$$T_bT_aT_b = \left(C_3 - \frac{1}{3}C_1\right)T_a$$

$$\gamma_\rho\gamma_\alpha\gamma_\mu\gamma_\beta\gamma^\rho = -2\gamma_\beta\gamma_\mu\gamma_\alpha + \varepsilon\gamma_\alpha\gamma_\mu\gamma_\beta$$

$$⑥ = \frac{-2ig\hat{g}^2(C_3 - 1/3\, C_1)\xi}{16\pi^2\varepsilon}(T_a)_{ji}\gamma_\mu + (\text{finite})$$

⑦ =



$$= ig\hat{g}^2 M^\varepsilon f_{abc}(T_cT_b)_{ji}\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}\left[\gamma_\xi \tilde{S}_F(p+k)\gamma_\sigma\right]\tilde{D}_F^{\xi\rho}(q-k)$$

$$\times \tilde{D}_F^{\nu\sigma}(k)\left[(2k-q)_\mu\eta_{\nu\rho} - (q+k)_\rho\eta_{\mu\nu} + (2q-k)_\nu\eta_{\rho\mu}\right]$$

Group factor: $f_{abc}T_cT_b = -\frac{i}{2}C_1T_a$

$$⑦ = \frac{3 i g \hat{g}^2 C_1 (1-\xi)}{32\pi^2 \varepsilon} (T_a)_{ji} \gamma_\mu + (\text{finite})$$

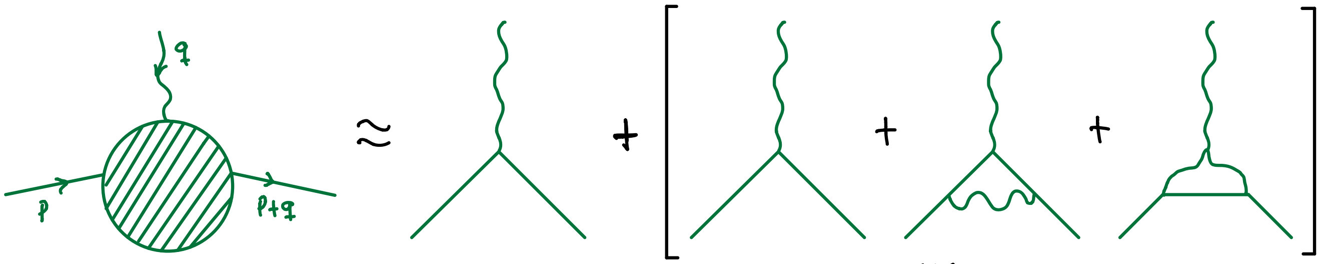
Therefore,

$$K_2 = \frac{-\hat{g}^2}{32\pi^2} \frac{1}{\varepsilon} \left[3C_1 + (C_1 + 4C_3)\xi \right]$$

Anomalous magnetic moment

e, γ

$\tilde{\Gamma}_\mu =$



$-ie\gamma_\mu$ $\qquad$ $-ie\Lambda_\mu^{MS}$

finite part

Effective interaction:

I. $\bar{u}(p')\gamma_\mu u(p)$: Usual interaction $\quad g=2$

II. $\bar{u}(p')\Lambda_\mu u(p)$: Quantum correction.

QED: (See Chang)

$$\bar{u}(p')\Lambda_\mu u(p) = \bar{u}(p') \left[\gamma_\mu (F_1(q^2) - 1) + \frac{i \sigma_{\mu\nu} q^\nu}{2m} F_2(q^2) \right]$$

$$\bar{u}(p')\tilde{\Gamma}_\mu u(p) = \bar{u}(p') \left[\gamma_\mu F(q^2) + \frac{i \sigma_{\mu\nu} q^\nu}{2m} F_2(q^2) \right]$$

Electric charge: $\longleftarrow$ electric form factor

$$Q = eF_1(0) =: e_{phys}$$

magnetic form factor $\longrightarrow$ $F_2(0)$: magnetic moment: $\qquad \alpha = \frac{e^2}{4\pi}$

$$\mu_e = 1 + \frac{\alpha}{2\pi} \qquad [e^2/2m]$$

$\longrightarrow$ Schinger correction

$$\mu_{AMM} = a_e \cdot \frac{e^2}{2m} \simeq \frac{\alpha}{2\pi} e^2/2m$$

$$(a_e)_{Th} = 0.001\,159\,652\,181\,643(764) \leftarrow \text{4-loops}$$

v.s.

$$(a_e)_{exp} = 0.001\,159\,652\,1\underline{80\,73(28)} \qquad \text{Precision: } \sim 10^{-9}$$