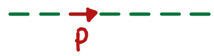
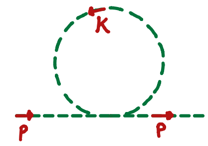
Renormalization

$\lambda \varphi^4$:

$$G^{(2)}(p,-p) = $$  $$+ \frac{1}{2}$$  $$+ \mathcal{O}(\lambda^2)$$

Loop: $$-i\Sigma(p) := \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \sim \lambda^2 :$$ divergent (ultraviolet divergence)

Notice that:

$$\tilde{G}^{(2)}(p,-p) \simeq \tilde{\Delta}_F(p)\left(1 + \frac{(-i\lambda)}{2}\left(-i\Sigma(p)\right)\tilde{\Delta}_F(p)\right)$$

$$\simeq \tilde{\Delta}_F(p)\left(1 + \frac{i\lambda}{2}\left(-i\Sigma(p)\right)\tilde{\Delta}_F(p)\right)^{-1}$$

$$\tilde{G}^{(2)}(p,-p) \simeq \left[\tilde{\Delta}_F^{-1}(p) + \frac{i\lambda}{2}\left(-i\Sigma(p)\right)\right]^{-1} := \frac{i}{p^2 - m_R^2 + i\epsilon}$$

where $m_R^2 := m^2 - \frac{i\lambda}{2}\Sigma(p)$ $\rightarrow$ Renormalized mass.

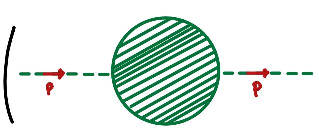
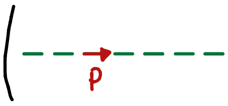
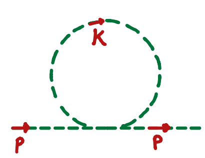
- m: Bare mass $\longrightarrow m_B$
- $\delta m^2 = -\frac{i\lambda}{2}\Sigma(p)$: Quantum correction

Renormalization

The physical (measurable) quantity is not the same as the parameter in the Lagrangian in the presence of interactions.

Definition: (Renormalizable theory) If it can be made finite by renormalizing only the parameters and fields in $\mathcal{L}_B$.

Remark: The redefinition of m is clearer if we consider

$$i\Gamma^{(2)}_{(p)} := -\left(\right.$$  $$\left.\right)^{-1} = -\left(\right.$$  $$\left.\right)^{-1} + \frac{1}{2}$$  $$+ \cdots$$

$$= i(p^2 - m^2) + \frac{1}{2}(-i\lambda)\left(-i\Sigma(p)\right) + \cdots$$

$$=: i(p^2 - m_R^2)$$

Superficial degree of divergence: Dimensional count.

Scalar theory:

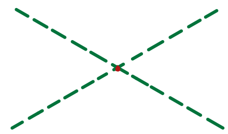
($\lambda\varphi^4$)

- each loop: $\int d^4K$: $4L$
- Interior lines: $\frac{1}{p^2-m^2}$: $-2I$

$$D = 4L - 2I$$

D=0: Logarithmic divergence $\sim \int \frac{dK}{K}$

D=2: Cuadratic divergence



$L \leq I$ ($\sum p = 0$ in vertices)

In a connected diagram:

$$L = I - V + 1$$

V: vertices

then,

$$D = 2I - 4V + 4$$

but, number of exterior lines: $E = 4V - 2I$

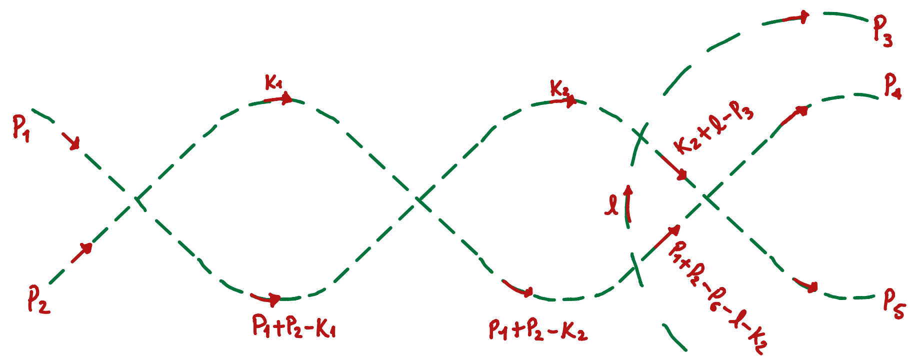
then,

$$D = 4 - E$$

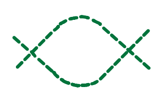
E is even. $\longrightarrow$ $D \geq 0$: $\tilde{\Gamma}^{(0)}$; $\tilde{\Gamma}^{(2)}$; $\tilde{\Gamma}^{(4)}$

Does not mean that diagrams with $E > 4$ are not divergent:

$\Gamma^{(6)}$:



$D = -2$

- $\int d^4K_1$; $\int d^4K_2$ are independent
- each one is of the associated form to $\tilde{\Gamma}^{(4)}$: 

Weinberg theorem: If $D<0$ for a diagram, and for each sub-graph the degree of divergence is also negative, then the diagram is convergent.

(Condition of sufficiency)

Is the theory is renormalizable, the unique divergences that must appear are those associated to m, λ and the renormalization of the fields.

QM: $H = H_0 + \lambda V$,

$$|n\rangle \approx \frac{1}{Z}\left[|n\rangle_0 + \lambda \sum_{k}^{N} \frac{{}_0\langle k|V|n\rangle_0}{E_k - E_N} |k\rangle_0 \right]$$

$N \to \infty$; $\Delta E \to 0$; $Z \to \infty$

Strategy to remove divergences

I. Divergence isolation: Regularization.

II. Removal the divergences: Renormalization.

Regularization Methods:

- Cut-off:

$$\int d^4K \longrightarrow \int_0^{\Lambda} \Omega_K \, K^3 dK$$

- Pauli-Villars:

$$\frac{1}{K^2} \longrightarrow \frac{1}{K^2} - \frac{1}{K^2 - \Lambda^2} = -\frac{\Lambda^2}{K^2(K^2-\Lambda^2)}$$

Renormalization:

$$\varphi_R(x) = \frac{1}{Z^{1/2}} \varphi_B(x)$$

Z: Renormalization constant

φ_R: Renormalized field — finite theory

φ_B: Bare field — Theory with divergences