Two levels - 2

$$\varepsilon_1 = 0 \qquad \varepsilon_2 = \varepsilon > 0$$

$$t_j = \begin{cases} 0 & \text{if } \varepsilon_j = 0 \\ 1 & \text{if } \varepsilon_j = \varepsilon \end{cases} \qquad E\{t_j\} = \sum_{j=1}^{N} \varepsilon t_j$$

$$Z = \sum_{\{t_j\}} \exp(-\beta E\{t_j\}) = \sum_{t_1,\dots,t_N} \exp\left(-\sum_{j=1}^{N} \beta \varepsilon t_j\right)$$

$$Z = \left[\sum_{t} \exp(-\beta\varepsilon t)\right]^N = Z_1^N$$

$$Z_1 = \exp(-\beta\varepsilon 0) + \exp(-\beta\varepsilon)$$

$$= 1 + \exp(-\beta\varepsilon)$$

In the other hand

$$Z = \sum_{E} \Omega(E,N) \exp(-\beta E)$$

$$= \sum_{N_1,N_2}^{N} \frac{N!}{N_1!\,N_2!} \exp(0) \exp(-\beta\varepsilon N_2)$$

$$= \left[1 + \exp(-\beta\varepsilon)\right]^N$$

$$f = f(t) = -\frac{1}{\beta} \lim_{N\to\infty} \frac{1}{N} \ln(Z) = -k_B T \ln\left(1 + \exp\left(-\frac{\varepsilon}{K_B T}\right)\right)$$

$$s = -\frac{\partial f}{\partial T} = K_B \ln\left(1 + \exp\left(-\frac{\varepsilon}{K_B T}\right)\right) + \frac{\varepsilon}{T} \frac{\exp\left(-\frac{\varepsilon}{K_B T}\right)}{\left(1 + \exp\left(-\frac{\varepsilon}{K_B T}\right)\right)}$$

$$c = T\frac{\partial s}{\partial T} = K_B \left(\frac{\varepsilon}{K_B T}\right)^2 \frac{\exp\left(-\frac{\varepsilon}{K_B T}\right)}{\left(1 + \exp\left(-\frac{\varepsilon}{K_B T}\right)\right)^2}$$

In terms of probability

$$p_1 = \frac{1}{Z_1} \qquad p_2 = \frac{\exp(-\beta\varepsilon)}{Z_1}$$

$Z_1 = 1 + \exp(-\beta\epsilon)$

If $\varepsilon > 0$ and $T > 0$, then $p_1 > p_2$

If $T \to 0$ then $p_1 \to 1/2$ and $p_2 \to 0$

If $T \to \infty$ then $p_1 \to 1/2$ and $p_2 \to 1/2$

The energy per particle is $\frac{\varepsilon}{2}$

$$s = s(u) = -K_B\left(1-\frac{u}{\varepsilon}\right)\ln\left(1-\frac{u}{\varepsilon}\right) - K_B\left(\frac{u}{\varepsilon}\right)\ln\left(\frac{u}{\varepsilon}\right).$$