$\lambda\varphi^4$: Feynman Rules

$$W[J] \propto \exp\left(-i\int dx\,\mathcal{L}_I\left(-i\frac{\delta}{\delta J(x)}\right)\right)W_0[J]$$

$$\mathcal{L}_I = -\frac{\lambda}{4}\varphi^4(x)$$

$\lambda \ll J \longrightarrow$ We give a perturbative approximation.

$$e^{-i\lambda I} = 1 - i\lambda I + \cdots$$

$$W[J] \propto W_0[J] - \frac{i\lambda}{4!}\int dx\,\frac{\delta^4 W_0[J]}{\delta J(x)^4} + \mathcal{O}(\lambda^2).$$

$$W_0[J] = \exp\left(-\frac{1}{2}\int dy\,dz\,J(y)\Delta_F(y-z)J(z)\right)$$

then,

$$\frac{\delta W_0}{\delta J(x)} = -W_0\int dy\,\Delta_F(x-y)\,J(y)$$

$$\frac{\delta^2 W_0}{\delta J(x)^2} = W_0\left[-\underbrace{\Delta_F(x-x)}_{\Delta_F(0)} + \left(\underbrace{\int dx\,\Delta(x-y)\,J(y)}_{=:\int\Delta_F J}\right)^2\right]$$

$$\frac{\delta^3 W_0}{\delta J(x)^3} = W_0\left[\Delta_F(0)\int\Delta_F J + 2\Delta(0)\int\Delta J - \left(\int\Delta_F J\right)^3\right]$$

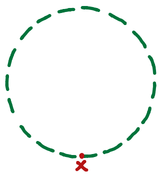
$$= W_0\left[3\Delta_F(0)\int\Delta_F J - \left(\int\Delta_F J\right)^3\right]$$

$$\frac{\delta^4 W_0}{\delta J(x)^4} = W_0\left[3\Delta_F^2(0) - 3\Delta_F(0)\left(\int\Delta_F J\right)^2 - 3\Delta_F(0)\left(\int\Delta_F J\right)^2 + \left(\int\Delta_F J\right)^4\right]$$

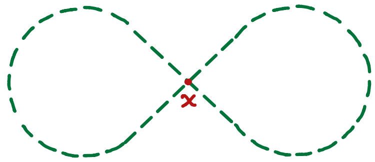
$$= W_0\left[3\Delta_F^2(0) - 6\Delta_F(0)\left(\int\Delta_F J\right)^2 + \left(\int\Delta_F J\right)^4\right]$$

Introduce:

$\boxed{\Delta_F(x-y):}$ 

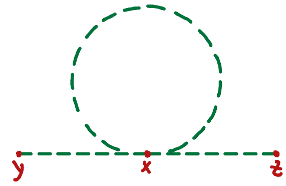
$\Delta_F(x-x):$ 

$\Delta_F(x-x)\Delta_F(x-x)$

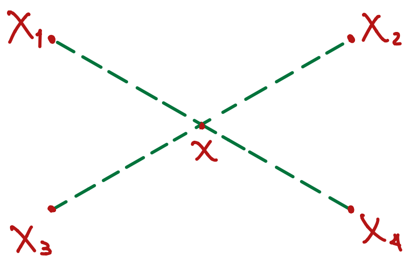


$\int dy\,dz$

$\Delta_F(y-x)\Delta_F(x-x)\Delta(x-z)$



$\Delta_F(x-x_1)\Delta_F(x-x_2)\Delta_F(x-x_3)\Delta_F(x-x_4)$



factors 3 and 6

Combinatorics.

Again,

$$W[J] \propto W_0[J] - \frac{i\lambda}{4!}\int dx\, \frac{\delta^4 W_0[J]}{\delta J(x)^4} + \mathcal{O}(\lambda^2).$$

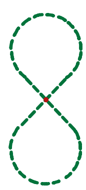
$$\frac{\delta^4 W_0}{\delta J(x)^4} = W_0\left[3\Delta^2(x-x) - 6\Delta(x-x)\left(\int \Delta_F J\right)^2 + \left(\int \Delta_F J\right)^4\right]$$

$$W_0 = \exp\left(-\frac{1}{2}\int dy\,dz\, J(y)\Delta_F(y-z)J(z)\right)$$

Then, $W[J]$ is cuadratic in J. $\longrightarrow$ $G^{(2n+1)} = 0$

but,

$$W[0] \propto 1 - \frac{i\lambda}{8}\int dx\, \Delta_F^2(x-x) + \mathcal{O}(\lambda^2) \neq 1.$$

$= 1 + \frac{1}{8}$  $+ \dots$ $\longleftarrow$ only vacuum diagrams.

$=: G^{(0)}$ $\longrightarrow$ Divergent normalization factor.

$$\Delta(x-y) = \int \frac{d^4p}{(2\pi)^4}\, \frac{i}{p^2 - m^2 + i\epsilon}\, e^{ip(x-y)}$$

Now, if we get

$$\Delta(0) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} \sim \int d^3p \sim \Lambda^2 !!$$

then,

$$W[J] = \frac{e^{i\int dx \mathcal{L}_{INT}\left(-i\frac{\delta}{\delta J(x)}\right)} W_0[J]}{e^{i\int dx \mathcal{L}_{INT}\left(-i\frac{\delta}{\delta J(x)}\right)} W_0[J]|_{J=0}} \longrightarrow \frac{\infty}{\infty}$$

Finite part.

$$\lambda\phi^4: \frac{\left[1 - i\frac{\lambda}{4!}\int dx\left[3\Delta_F^2(0) - 6\Delta_F(0)\left(\int \Delta_F J\right)^2 + \left(\int \Delta_F J\right)^4\right]\right] W_0[J]}{1 - i\frac{\lambda}{4!}\int dx 3\Delta_F^2(0)}$$

$$= \left[1 - i\frac{\lambda}{4!}\int dx\left[3\Delta_F^2(0) - 6\Delta_F(0)\left(\int \Delta_F J\right)^2 + \left(\int \Delta_F J\right)^4\right]\right] W_0$$

$$\times \left(1 + i\frac{\lambda}{4!}\int dx\, 3\Delta_F^2(0)\right)$$

$$= \left[1 - i\frac{\lambda}{4!}\int dx 3\Delta_F^2(0) + i\frac{\lambda}{4!}\int dx\, 3\Delta_F^2(0) + \cdots\right] W_0$$

$$W[J] = \left[1 + i\frac{\lambda}{4!}\int dx\left[6\Delta_F(x-y)\left(\int \Delta J\right)^2 - \left(\int \Delta J\right)^4\right]\right] W_0[J]$$

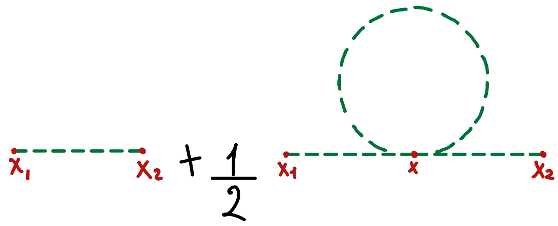
$$= \}\text{expr.}\{ \cdot W_0[J]$$

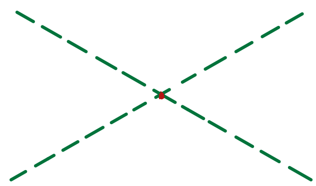
$$G^{(n)}(x_1, \ldots, x_n) = \frac{(-i)^n \delta^n W[J]}{\delta J(x_n) \cdots \delta J(x_1)}\Big|_{J=0}$$

$$= \sum_{\substack{\text{Perm} \\ \{P_i\}}} \sum_{k=0}^{n} (-i)^{n-k} \frac{\delta^{n-k} \}\text{expr}\{}{\delta J(x_{P_1}) \cdots \delta J(x_{n-k})}\Big|_{J=0} \cdot G_0^{(k)}(x_{P_{n-k+1}} \cdots x_{P_n})$$

1. $G^{(2)}(x_1, x_2) = G_0^{(2)}(x_1, x_2) + \frac{(-i\lambda)}{4!}\int dx\; 12\Delta_F(x-x)\Delta_F(x_1 - x)\Delta_F(x_2 - x)$

$$G^{(2)}(x_1,x_2) = G_0^{(2)}(x_1,x_2) + \frac{1}{2}(-i\lambda)\int dx\, \Delta_F(x_1-x)\Delta_F(x-x)\Delta_F(x_2-x)$$

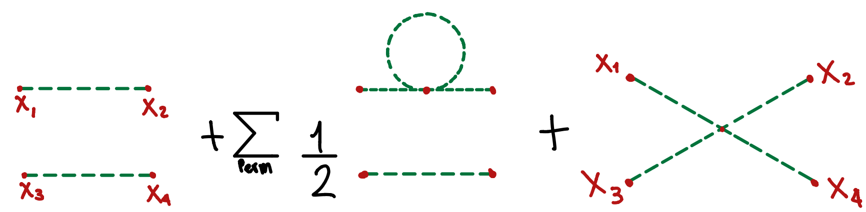
$=$ 

Conversion:  $: -i\lambda$ $\boxed{\text{Feynman Rule}}$

II. $G^{(4)}(x_1,\ldots,x_4) = G_0^{(4)}(x_1,\ldots,x_4)$

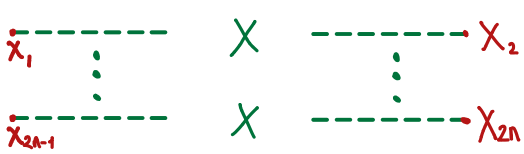
$$+\sum_{\text{Perm}} -\frac{i\lambda}{2}\int dx\, \Delta_F(x_1-x)\Delta_F(x-x)\Delta_F(x-x_2)\Delta_F(x_3-x_4)$$

$$+(-i\lambda)\int dx\, \Delta_F(x_1-x)\Delta_F(x_2-x)\Delta_F(x_3-x)\Delta_F(x_4-x)$$

$G^{(4)}(x_1,\ldots,x_4) = \sum_{\text{Perm}}$ 

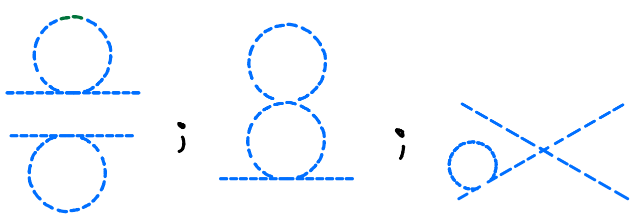
3 diag. 6 diag

III. $G^{(2n)}$; order $\lambda^N \longrightarrow N$-vertex.



Topology $\left\{ \right.$
- Join exterior lines (by pairs) to the vertex.
- Join the free legs in the vertex formed by loops
- Consider all posibilities

(Include expectators)

 ; ...