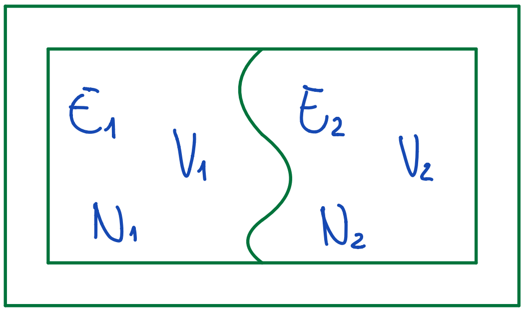
Microcanonical ensemble

$$P(\{x_i\}) \sim \Omega(E, V, N; \{x_i\})$$

\# of microscopic states

Probability



$$\Omega = \Omega_1(E_1, V_1, N_1)\, \Omega_2(E_2, V_2, N_2)$$

$$E_1 + E_2 = E_0$$

V_1, V_2, N_1, N_2 constants.

$$\Omega(E_1; E_0) = \Omega_1(E_1)\, \Omega_2(E_0 - E_1)$$

$$\rightarrow P(E_1) = c\,\Omega(E_1; E_0) = c\,\Omega_1(E_1)\,\Omega_2(E_0 - E_1)$$

$$\frac{1}{c} = \sum_{E_1=0}^{E_0} \Omega_1(E_1)\,\Omega_2(E_0 - E_1)$$

- $\Omega(E)$ grows with E
- $\Omega_1(E_1)$ grows while $\Omega_2(E_0 - E_1)$ decreases
- $P(E_1)$ has a maximum.

$$f(E_1) = \ln(P(E_1)) = \ln(c) + \ln(\Omega_1(E_1)) + \ln(\Omega_2(E_0 - E_1))$$

in the maximum

$$\frac{\partial \ln(P(E_1))}{\partial E_1} = \frac{\partial \ln \Omega_1(E_1)}{\partial E_1} - \frac{\partial \ln(\Omega_2(E_0 - E_1))}{\partial E_1}$$

If $\quad S(E) = K_B \ln(\Omega(E))$ Entropy

$$\rightarrow \quad T_1 = T_2$$

Max probability $\longleftrightarrow$ Max Entropy

Example: 2 Classical gas.

$$\Omega(E,V,N;\delta E)=\left(\frac{m}{2}\right)^{1/2} C_{3N}(2m)^{(N-1)/2}\, V^N E^{N/2-1}\,\delta E$$

for $N \gg 1$, $\Omega(E)=cE^{3/2N}$

$$P(E_1)=c\,C_1E_1^{3/2N}C_2E_2^{3/2N}=\text{constant}\cdot E_1^{3/2N}E_2^{3/2N}$$

$$\rightarrow \ln(p(E_1))=\text{constant}+\frac{3}{2}N_1\ln(E_1)+\frac{3}{2}N_2\ln(E_2).$$

$$E_2:=E_0-E_1$$

$$\frac{\partial \ln(P(E_1))}{\partial E_1}=\frac{3}{2}\frac{N_1}{E_1}-\frac{3}{2}\frac{N_2}{E_2}=0 \longrightarrow \frac{\tilde{E}_1}{N_1}=\frac{\tilde{E}_2}{N_2}$$

As

$$T=\left(\frac{\partial U}{\partial S}\right)_{V,N} \longrightarrow \frac{1}{T}=\left(\frac{\partial S}{\partial U}\right)_{V,N}=\left(\frac{\partial K_B\ln(\Omega)}{\partial U}\right)_{V,N}$$

$$\frac{1}{K_BT}=\left(\frac{\partial \ln(\Omega)}{\partial U}\right)_{V,N}$$

let $U=\tilde{E}$, then

$$\frac{1}{K_BT_1}=\frac{3}{2}\frac{N_1}{U_1}=\frac{3}{2}\frac{N_2}{U_2}=\frac{1}{K_BT_2}$$

Also

$$T_1=T_2 \quad \text{and} \quad U=\frac{3}{2}NK_BT.$$

$$\frac{\partial^2\ln(P(E_1))}{\partial E_1^2}=-\frac{3}{2}\frac{N_1}{E_1^2}-\frac{3}{2}\frac{N_2}{E_1^2}$$

if $E_1=U_1=\frac{3}{2}N_1K_BT$ and $E_2=U_2=\frac{3}{2}N_2K_BT$, then

$$\left(\frac{\partial^2\ln(P(E_1))}{\partial E_1^2}\right)_{max}=-\frac{3}{2}\left(\frac{2}{3K_BT}\right)^2\left(\frac{N_1}{N_1^2}+\frac{N_2}{N_2^2}\right)$$

$$=-\frac{2}{3}\left(\frac{N_1+N_2}{K_B^2T^2N_1N_2}\right)<0$$

Using Taylor expansion arround the max

$$\ln(p(E_1)) = \text{constant} - \frac{1}{3}\frac{N_1+N_2}{K_B^2T^2N_1N_2}\left(E - \frac{3}{2}N_1K_BT\right)^2 + \dots$$

$$P_G = A\exp\left[-\frac{1}{3}\frac{N_1+N_2}{K_B^2T^2N_1N_2}\left(E - \frac{3}{2}N_1K_BT\right)^2\right]$$

Therefore

$$\langle E_1\rangle_G = \frac{3}{2}N_1K_BT$$

$$\langle(\Delta E_1)^2\rangle_G = \frac{3}{2}\frac{K_B^2T^2N_1N_2}{(N_1+N_2)}$$

$$\frac{\sqrt{\langle(\Delta E_1)^2\rangle_G}}{\langle E_1\rangle_G} = \left(\frac{2}{3}\frac{N_2}{N_1(N_1+N_2)}\right)^{1/2} \longrightarrow 0.$$