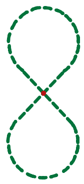
$\lambda\varphi^4$

$$W[J] \propto \exp\left(-i\int dx\, \mathcal{L}_I\left(-i\frac{\delta}{\delta J(x)}\right)\right) W_0[J]$$

$\mathcal{L}_I(\varphi(x))$:

$\varphi(x) \longrightarrow -i\frac{\delta}{\delta J(x)}$

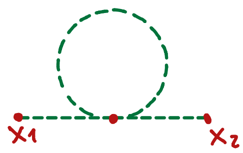
Vacuum diagonal ($J=0$):

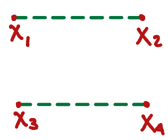
$1 + \frac{1}{8}$  $+ \cdots$

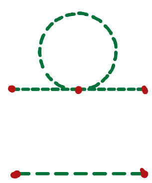
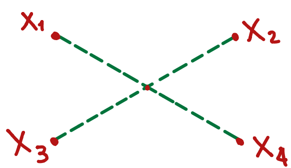
$$W[J] = \frac{e^{i\int dx\, \mathcal{L}_{INT}\left(-i\frac{\delta}{\delta J(x)}\right)} W_0[J]}{e^{i\int dx\, \mathcal{L}_{INT}\left(-i\frac{\delta}{\delta J(x)}\right)} W_0[J]\big|_{J=0}}$$

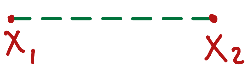
for 2,4-point Green function

$G^{(2)}(x_1, x_2) =$ 

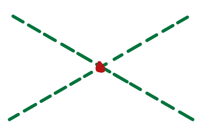
$+ \frac{1}{2}$ 

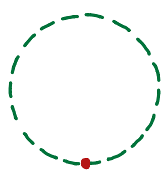
$G^{(4)}(x_1, \ldots, x_4) = \sum_{\text{Perm}}$ 

$+ \sum_{\text{Perm}} \frac{1}{2}$  $+$ 



: $\Delta_F(x_1 - x_2)$

 : $(-i\lambda)\int dx\; \Delta_F(x - x_i)\cdots$

 : $\Delta_F(x - x)$

Connected diagrams:

$W[J] = \{\exp\} \cdot W_0$

$iX[J] = \ln(W[J])$

$iX[J] = \underbrace{\ln(\{\exp\}}_{iX_{INT}} + \underbrace{\ln(W_0)}_{iX_0}$

$\mathcal{O}(\lambda)$:

$$iX_{INT} = \ln\left[1 + \frac{i\lambda}{4!}\int dx\left[6\Delta_F(0)\left(\int\Delta J\right)^2 - \left(\int \Delta J\right)^4\right]\right]$$

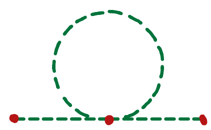
for $\ln(1+x) = x$

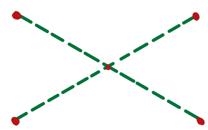
$$iX_{INT} = \frac{(-i\lambda)}{4!}\left[-6\int dx\, dx_1\, dx_2\, \Delta_F(x_1-x)\Delta_F(x-x)\Delta_F(x-x_2)J(x_1)J(x_2)\right.$$

$$\left. + \int dx\, dx_1 \cdots dx_4\, \Delta_F(x_1-x)\cdots\Delta_F(x_4-x)\cdot J(x_1)J(x_2)J(x_3)J(x_4)\right]$$

Then, the connected diagrams:

$$i^n g^{(n)} = \frac{i\,\delta^n X[J]}{\delta J(x_n)\cdots\delta J(x_1)}$$

$g^{(2)}(x_1,x_2) =$ $+\frac{1}{2}$ 

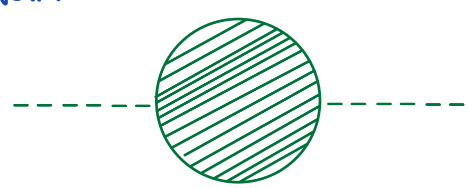
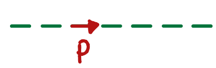
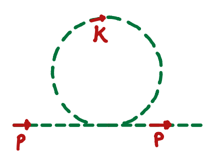
$g^{(4)}(x_1,\ldots,x_4) =$ 

In the moment space.

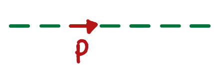
$$g^{(n)}(p_1,\ldots,p_n)(2\pi)^4\delta(p_1+\cdots+p_n) = \int dx_1\cdots dx_n\, g^{(n)}(x_1,\ldots,x_n)e^{-i\sum p_i x_i}$$

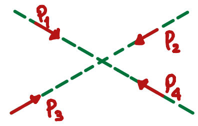
$$g^{(2)}(p,-p) = \tilde{\Delta}_F(p) + \frac{1}{2}(-i\lambda)\int\frac{d^4k}{(2\pi)^4}\tilde{\Delta}_F(p)\tilde{\Delta}_F(k)\tilde{\Delta}_F(p)$$

Feynman diagram:

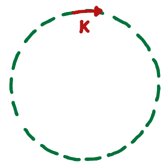
 $=$  $+\frac{1}{2}$ 

Feynman rules: $\lambda\phi^4$

1. line :  : $\frac{i}{p^2-m^2+i\epsilon} = \tilde{\Delta}_F(p)$
Propagator

III. Vertex : 

: $-i\lambda \qquad \left(\sum p_i = 0\right)$

IV. Loop : 

: $\displaystyle\int \frac{d^4K}{(2\pi)^4}$

OPI functions

$$\varphi_c(x) = \delta X[J] \quad ; \quad iX = iX_0 + iX_{INT}.$$

$$iX_0 = -\frac{1}{2}\int dx\,dy\, J(x)\Delta_F(x-y)J(y).$$

$$iX_{INV} = \frac{(-i\lambda)}{4!}\int dx\left[-6\Delta_F(0)\left(\int \Delta_F J\right)^2 + \left(\int \Delta_F J\right)^4\right] + \mathcal{O}(\lambda^2)$$

then

$$\varphi_c(x) = i\int dy\,\Delta_F(x-y)J(y) + \frac{\lambda}{2}\int dx_1\,dx_2\,\Delta_F(x-x_1)\Delta_F(x_1-x_1)\Delta_F(x_1-x_2)J(x_2)$$
$$-\frac{\lambda}{6}\int dx_1\left[\int dx_2\;\Delta_F(x_1-x_2)J(x_2)\right]^3 \Delta_F(x-x_1) + \dots$$

computing, $(\Box_x + m^2)\varphi_c(x)$ and using $(\Box_x+m^2)\Delta_F(x-y) = -i\delta(x-y)$, we may write

$$J(x) = (\Box + m^2)\varphi_c(x) + \frac{1}{2}\lambda\,\Delta_F(0)\varphi_c(x) + \frac{1}{6}\lambda\,(\varphi_c(x))^3 + \dots$$

self-interaction

Effective action:

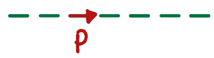
$$\Gamma[\varphi_c] = -\frac{1}{2}\int dx\,\varphi_c(\Box_x + m^2)\varphi_c - \frac{\lambda}{4}\int dx\,\Delta_F(x-x)(\varphi_c(x))^2$$

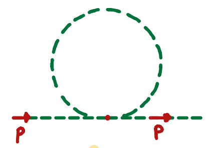
$$-\frac{1}{4!}\lambda\int dx\,(\varphi_c(x))^4 + \dots$$

$$\mathcal{F}[\varphi_c^2, \varphi_c^4] \to \Gamma^{(2)} ; \Gamma^{(4)} \qquad \text{non-null.}$$

then,

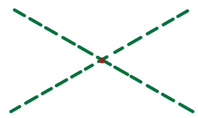
$$i\,\tilde{\Gamma}^{(2)}(p,-p) = i\,(p^2-\mu^2) + \frac{1}{2}(-i\lambda)\,\Delta_F(0)$$

$= \Big($ 

$\Big)^{-1} + \frac{1}{2}$ 

$$(-i\lambda)\int\frac{d^4k}{(2\pi)^4}\,\tilde{\Delta}_F(k)$$

$i\,\tilde{\Gamma}^{(4)}(p_1,\dots,p_4) = (-i\lambda)$:



Propagators of external legs "cutted".