General aspects of the renormalization of standard model (SM)

Gauge: $[SU(3)_c \times] SU(2)_L \times U(1)_Y \xrightarrow[\langle H \rangle]{} U(1)_Q$

$g_c \quad g\, W_\mu^a \quad g'\, B_\mu$

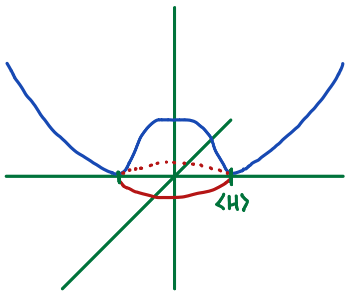
$$Q = T_3 + \frac{1}{2} Y$$

Matter:

$$L(1,2,-1) = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L \; ; \quad \ell_R(1,1,-2) \qquad \ell = e, \mu, \tau$$

$$Q(3,2,1/3) = \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L \; ; \quad q_{uR}(3,1,4/3);\; q_{dR}(3,1,-2/3)$$

$$q_u = u, c, t \quad ; \; q_d = d, s, b.$$



$H(1,2,1)$

$$V(H) = -\mu^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 \longrightarrow \langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$v^2 = 4\mu^2/\lambda$$

$|D\langle H \rangle|^2$:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm i W_\mu^2) \; ; \quad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} C_{\theta_W} & -S_{\theta_W} \\ S_{\theta_W} & C_{\theta_W} \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

$$e = g \sin(\theta_W) = g' \cos(\theta_W) \qquad \cos(\theta_W) = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$m_W^2 = \frac{1}{4} g^2 v^2 \longrightarrow \boxed{m_Z \cos(\theta_W) = m_W}$$

$$m_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2$$

$$D_\mu \rightarrow g W_\mu^a T_a + \frac{1}{2} Y g' B_\mu$$

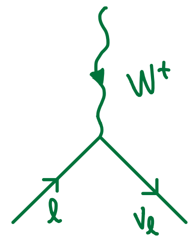
$$= \frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) + \frac{g}{\cos(\theta_W)} Z_\mu (T_3 - \sin^2(\theta_W) Q) + e Q A_\mu$$

$$T^\pm = T^1 \pm i T^2 \quad ; \quad T^i = 1/2\, \sigma^i$$

Current:

$$\mathcal{L}_{cc} = -\frac{g}{2\sqrt{2}} W^+_\mu J^\mu_{cc} + h.c.$$

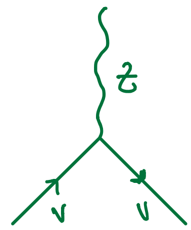
$$J^\mu_{cc} = \bar{\nu}_\ell \gamma^\mu (1-\gamma_5)\ell.$$



$$: -\frac{ig}{2\sqrt{2}} \gamma_\mu (1-\gamma_5)$$

$$\mathcal{L}_{CN} = -\frac{e}{\sin^2(\theta_W)} Z_\mu \left(J^\mu_{CN,\nu} + J^\mu_{CN,\ell}\right)$$

$$J^\mu_{CN,\nu} = \frac{1}{2}\left(\bar{\nu}_\ell \gamma^\mu (1-\gamma_5)\nu_\ell\right)$$

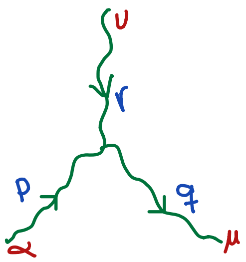


$$: -\frac{ig}{4C_{\theta_W}} \gamma^\mu (1-\gamma_5)$$

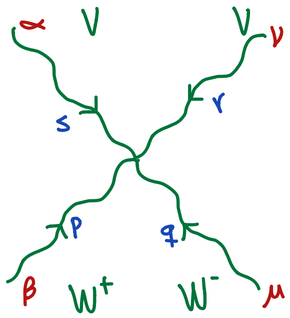
$$J^\mu_{CN,\ell} = \left(-\frac{1}{2} + \xi\right)\left(\bar{\ell}\gamma^\mu(1-\gamma_5)\ell\right) + \xi\left(\bar{\ell}\gamma^\mu(1+\gamma_5)\ell\right)$$

$$\xi = \sin^2(\theta_W)\ ;\ g_A = 1/2 - 2\xi\ ;\ g_V = 1/2.$$

F^2:



$$\begin{matrix} W^+W^-\gamma \\ W^+W^-Z \end{matrix} : \begin{bmatrix} ie \\ igC_{\theta_W} \end{bmatrix} \left[(r-q)_\alpha \eta_{\mu\nu} + (q-p)_\nu \eta_{\mu\alpha} + (p-r)_\mu \eta_{\alpha\nu}\right]$$

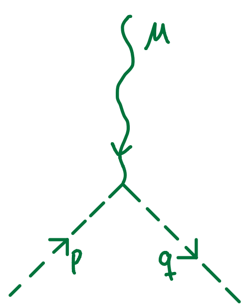


$$W^+W^- \left\{\begin{matrix} W^+W^- \\ \gamma\gamma \\ ZZ \\ Z\gamma \end{matrix}\right\} : \begin{bmatrix} ig^2 \\ -ie^2 \\ -ig^2 C^2_{\theta_W} \\ -iegC_{\theta_W} \end{bmatrix} \left[2\eta_{\beta\nu}\eta_{\alpha\mu} - \eta_{\mu\nu}\eta_{\alpha\beta} + \eta_{\alpha\nu}\eta_{\beta\mu}\right]$$

Gauge - Scalar:

$$(D_\mu H)(D^\mu H) - V(H);\qquad V(H) = -\mu^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2$$

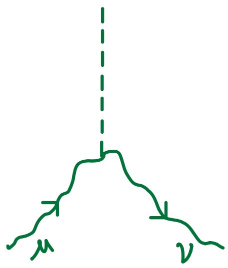
$$H = \tilde{\varphi} + \langle H \rangle := \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix}$$



$$\begin{matrix} \gamma G^+ G^+ \\ Z G^+ G^+ \end{matrix} : \begin{pmatrix} -ie \\ -ig\frac{\cos(2\theta_W)}{2\cos(\theta_W)} \end{pmatrix} \cdot (p+q)_\mu$$

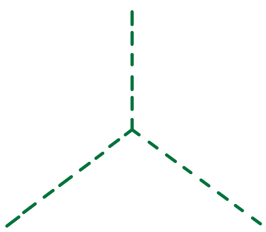
$W^+G^0G^+$	$W^-G^+G^0$	W^+hG^+	W^-G^+h	ZhG^0	
$\frac{1}{2}g$	$-\frac{1}{2}g$	$-\frac{i}{2}g$	$-\frac{i}{2}g$	$\frac{1}{2}\frac{g}{2C_{\theta_W}}$	$\times(p+q)_\mu$

Although in the unitary gauge the Goldstone's



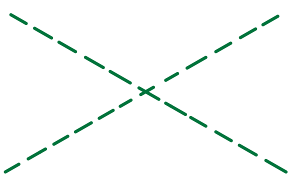
$G^-W^+\gamma$	G^-W^+Z	hW^+W^+	hZZ	
ieM_W	$-igM_Z\sin^2(\theta_W)$	igM_W	$ig\frac{M_Z^2}{M_W}$	$\times\eta_{\mu\nu}$

Scalars:



$hG^\pm G^\pm$; hG^0G^0 $\qquad$ hhh

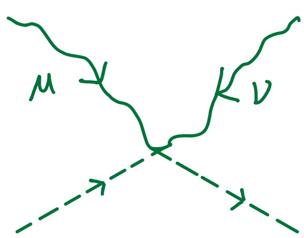
$-\frac{1}{2}ig\frac{M_H^2}{M_W}$ $\qquad$ $-\frac{3}{2}ig\frac{M_H^2}{M_W}$



$$-\frac{i}{4}\lambda = -\frac{1}{8}ig^2\frac{M_H^2}{M_W^2}$$

$G^+G^-G^+G^-$; $G^0G^0G^+G^-$; hhG^+G^-

$hhhh$; $G^0G^0G^0G^0$; hhG^0G^0



$$\left.\begin{matrix} hh \\ G^0G^0 \\ G^+G^- \end{matrix}\right\} W^+W^- : i\frac{1}{2}g^2\eta_{\mu\nu}$$

$G^+G^-\gamma\gamma$	$(hh; G^0G^0)ZZ$	G^+G^-ZZ	
$i2e^2$	$i\frac{g^2}{2\cos^2(\theta_W)}$	$ig^2\frac{\cos^2(2\theta_W)}{\cos^2(\theta_W)}$	$\times\eta_{\mu\nu}$

$hG^+\gamma W^+$ $hG^-\gamma W^-$	$G^0G^\pm\gamma W^\pm$	$G^+G^-\gamma Z$	$hG^\pm ZW^\pm$	
$i\frac{1}{2}eg$	$\mp\frac{1}{2}eg$	$ieg\frac{\cos(2\theta_w)}{\cos(\theta_w)}$	$\pm ig^2\frac{(1/2\cos(2\theta_w)-1)}{\cos(\theta_w)}$	$\times\eta_{\mu\nu}$

$$G^0G^\pm ZW^\pm : \mp\frac{1}{2}g^2\frac{(1/2\cos(2\theta_w)-1)}{\cos(\theta_w)}\eta_{\mu\nu}$$

Gauge Propagators

$$SSB: \langle H\rangle \neq 0 \longrightarrow H=\tilde{\varphi}+\langle H\rangle \longrightarrow m_{W,Z}$$

but the R_ξ gauge does not consider the mass.
Is better to consider use the gauge of 't Hooft

$$\mathcal{L}_{GF}=-\frac{1}{2\xi}\left[\partial^\mu W^a_\mu-2i\xi g\tilde{\varphi}^+T^a\langle H\rangle\right]^2-\frac{1}{2\xi}\left[\partial_\mu B^\mu-i\xi g'\tilde{\varphi}^+\langle H\rangle\right]^2$$

$$D_\mu H=\left(\partial_\mu+igT_aW^a_\mu+\frac{1}{2}g'B_\mu\right)H$$

Cancelation

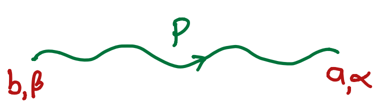
then,
$$(D_\mu H)^+(D^\mu H)=(D_\mu\tilde{\varphi})^+(D^\mu\tilde{\varphi})+(D_\mu\langle H\rangle)^+(D^\mu H)$$
$$+\left[i(\partial_\mu\tilde{\varphi})^\mu\left(gT_aW^a_\mu+\frac{1}{2}g'B_\mu\right)\langle H\rangle+h.c.\right]$$

$$SU(2)_L: \; F^2_a(x)=\partial_\mu W^\mu_a-2i\xi g\tilde{\varphi}^+T^a\langle H\rangle-f^2(x)=0$$
$$U(1)_Y: \; F^1_a(x)=\partial_\mu B^\mu-i\xi g'\tilde{\varphi}^+\langle H\rangle-f^1(x)=0$$

Generical Quadratic terms:

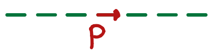
$$\mathcal{L}_{A^2}=-\frac{1}{2}(\partial_\mu A^a_\nu)(\partial^\mu A^\nu_a-\partial^\nu A^\mu_a)-\frac{1}{2\xi}(\partial A)^2+A^a_\mu(M^2)_{ab}A^\mu_b$$

$$\tilde{\Delta}^{\alpha\beta}_F(p^2,m_a^2)=\frac{-\eta^{\alpha\beta}+(1-\xi)p^\alpha p^\beta(p^2-\xi m_a^2)^{-1}}{p^2-m_a^2+i\epsilon}$$



$: i\delta^{ab}\tilde{\Delta}^{\alpha\beta}_F(p^2,m_a^2)$ $\quad Z,W$

$$\mathcal{L}_{\phi^2}: \frac{1}{2}(\partial h)^2 - \frac{1}{2} m_H h^2 + \frac{1}{2}(\partial G^0)^2 + (\partial G^+)\cdot(\partial G^-) - \frac{1}{2}\xi m_Z^2 (G^0)^2 - \xi m_W^2 G^+ G^-$$



$G^\pm$	G^0	h
$\frac{i}{p^2 - \xi m_W^2 + i\epsilon}$	$\frac{i}{p^2 - \xi m_Z^2 + i\epsilon}$	$\frac{i}{p^2 - \xi m_H^2 + i\epsilon}$

Considering the function $F_a(x)$ in the norm of 't Hooft

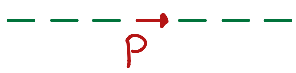
$$F_a(x) = \partial_\mu A_a^\mu - i\xi g \tilde{\varphi}^+ T^a \langle H \rangle - f_a(x) = 0$$

we will get that

$$\frac{\delta F_a(x')}{\delta \Lambda_b(x)} = \partial_{\mu x'} \left\{ \left[\partial_{x'}^\mu \delta^{ab} + g f^{abc} A^{c\mu}(x') \right] \delta(x'-x) \right\}$$
$$+ \xi g^2 (\langle H \rangle^+ + \tilde{\varphi}^+(x')) T^b T^a \langle H \rangle \delta(x-x')$$

then,

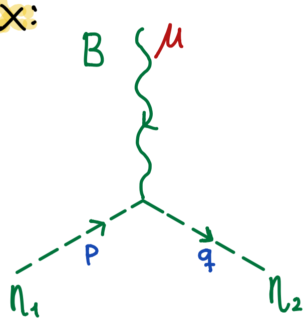
$$\mathcal{L}_{FP} = \partial^\mu \eta_a^* \cdot D_\mu^{ab} \eta_b - \xi \eta_a^* (M^2)^{ab} \eta_b - \xi g^2 \eta_a^* \eta_b \tilde{\varphi}^+ T^a T^b \langle H \rangle$$



$: i\delta^{ab}\tilde{\Delta}_F(p, \xi m_a^2)$

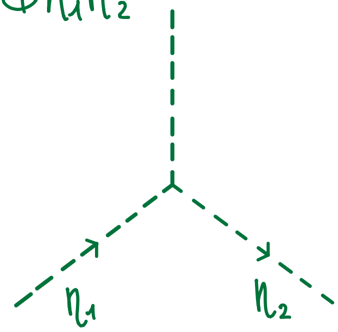
S.M.	η_γ	$\eta_\pm$	η_Z
	$i(p^2+i\epsilon)^{-1}$	$i(p^2-\xi m_W^2+i\epsilon)$	$i(p^2-\xi m_Z^2+i\epsilon)$

Vertex:



$$: \pm ieq_\mu \begin{cases} \gamma \eta_\mp \eta_\mp \\ W^\mp \eta_\pm \eta_\gamma \\ W^\pm \eta_\gamma \eta_\mp \end{cases} ; \quad \begin{array}{l} \gamma \to Z \\ \eta_\gamma \to \eta_Z \\ e \to g\cos(\theta_W) \end{array}$$

$\Phi \eta_1 \eta_2$



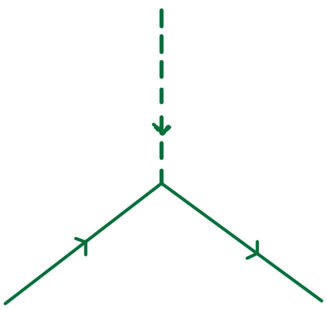
$h\eta_- \eta_-$	$G^0 \eta_\mp \eta_\mp$	$G^\pm \eta_\gamma \eta_\pm$	$G^\pm \eta_\mp \eta_Z$
$-\frac{1}{2} i g \xi m_W$	$\mp \frac{1}{2} g \xi m_W$	$-i e \xi m_W$	$\frac{1}{2} i g \xi m_Z$

$$G^\pm \eta_Z \eta_\pm : -i \frac{1}{2} g \frac{\cos(2\theta_W)}{\cos(\theta_W)} \xi m_W$$

Landau gauge $\rightarrow$ All null

$(\xi \rightarrow 0)$

Yukawa Interaction



$$y_\ell \bar{L}_\ell H \ell_R \rightarrow m_\ell = \frac{y_\ell v}{\sqrt{2}}$$

$$G^+ \ell \nu \longrightarrow y_\ell = \frac{g m_\ell}{\sqrt{2} m_W}$$

$$-i \frac{g}{\sqrt{2}} \frac{m_\ell}{M_W} \frac{1}{2} (1-\gamma_5)$$

$G^+ \ell \nu$	$h \ell \ell$	$G^0 \ell \ell$
$-i \frac{g}{\sqrt{2}} \frac{m_\ell}{M_W} \frac{1}{2} (1-\gamma_5)$	$-\frac{1}{2} i g \frac{m_\ell}{M_W}$	$\frac{1}{2} g \frac{m_\ell}{M_W} \gamma_5$