General aspects of renormalization of the SM

MS ($\overline{MS}$) are complex schemes but realizable.

- There is no agreement on which scheme is better.

In the present discussion we will use the scheme "on-shell", also called the Sirling scheme.

$$\varphi_B = Z_\varphi^{1/2}\varphi \longrightarrow \overline{Z_\varphi := 1}.$$

Postulates an outline where the divergences are removed only by the renormalization of the parameters.

Imply the rescaling of the Green Functions to ensure that propagators keep a unitary waste, in its contribution to S (exterior lines).

On-shell scheme, moreover, suggest renormalised parameters as physical parameters (experimentally measured):

$$e, m_W^2, m_Z^2, m_H^2, m_\ell \; (\ell = e, \mu, \tau)$$

$\hookrightarrow$ 5 constants of renormalization independents.

Counter terms definition:

Boson masses:

$$\mathcal{L}_m^B = m_{WB}^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_{ZB}^2 Z_\mu Z^\mu$$

from SSB: $\quad m_{WB}^2 = \frac{1}{4} g_B^2 V_B^2 \;;\quad m_{ZB}^2 = \frac{m_{WB}^2}{\cos(\theta_{WB})} \;;\quad \tan(\theta_{WB}) = \frac{g_B'}{g_B}$

$$\left.\begin{array}{l} m_{WB}^2 = m_W^2(1+K_W) \\ m_{ZB}^2 = m_{ZB}^2(1+K_Z) \end{array}\right. \longrightarrow \Delta\mathcal{L}_m = K_W m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} K_Z m_Z^2 Z_\mu Z^\mu$$

Identify (on-shell) $\quad m_W = 80.379 \pm 0.012$ GeV

$m_Z = 91.1876 \pm 0.0021$ GeV

Goldstone's: $\quad \left.\begin{array}{l} m_B^2(h^+) = \xi\, m_{WB}^2 \\ m_B^2(h^0) = \xi\, m_Z^2 \end{array}\right. : \; Z_\xi = 1$

then, $- - - \times - - - : -iK_{W,Z}\,\xi\, m^2_{W,Z}$

Ghost: $\quad \eta_\pm : \xi\, m^2_{WB} \quad - - - \times - - - : -iK_W\,\xi\, m_W^2$

't Hooft gauge $\quad \eta_Z : \xi\, m^2_{ZB} \quad - - - \times - - - : -iK_Z\,\xi\, m_Z^2$

Charge e:

$$\mathcal{L}_B(em) : -e_B\, j_\mu A^\mu \qquad j_\mu = \bar{\psi}\gamma_\mu\psi$$

$$\downarrow$$

$$e_B = e(1+K_e) \qquad e := \sqrt{4\pi\alpha} = 0.3028221$$

→ then,

$$\Delta\mathcal{L} : -K_e\, e\, j_\mu A^\mu$$

Given $e_B, m_{WB,ZB} \longrightarrow g_B, g'_B \ldots$ in the vertices of SM.

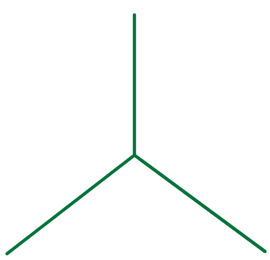
Couplings:

Definition: $g = e\, m_Z (m_Z^2 - m_W^2)^{-1/2}$; $\quad g' = e\,\frac{m_Z}{m_W} = g\tan(\theta_W)$

thereafter,

$$\tan(\theta_W) := \frac{g'}{g} \longrightarrow \sin^2(\theta_W) = \frac{m_Z^2 - m_W^2}{m_Z^2} = 0.22343 \pm 0.00007$$

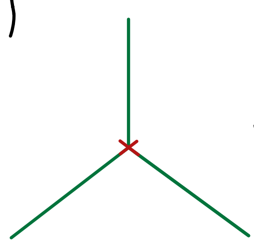
All vertex in SM will generate a counter term with similar structure:



$$(f_{ver})_B = (f_{ver})(1+K_{fV})$$

$$\downarrow$$

$$:(f_{ver})\times \cdots \longrightarrow$$

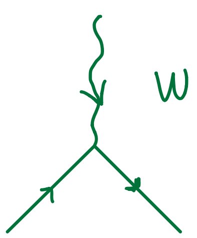


$: \quad K_{fV}(f_{ver})\times \cdots$

$$K_{fV} = K_{fV}(K_W, K_Z, K_e, K_H, K_\ell)$$

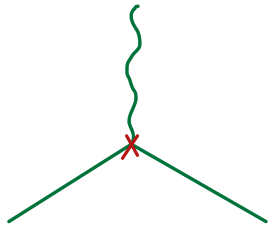
$m^2_{HB} = m_H^2(1+K_H) \qquad m_H^2 = 125.10 \pm 0.14$ GeV

$m_{\ell B} = m_\ell(1+K_\ell) \qquad m_\ell = m_{\ell\, phys}$



$: \quad g_B = g(1+K) \longrightarrow \Delta\mathcal{L}_{CC} = \frac{Kg}{\sqrt{2}} W^+_\mu \bar{\nu}_{\ell L}\gamma^\mu \ell_L + h.c.$

$\rightarrow \dfrac{-ig}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)$



$: \dfrac{-iK}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)$

but,

$$g=\frac{em_z}{(m_z^2-m_w^2)^{1/2}} \quad \text{and} \quad \sin^2(\theta_w)=\frac{(m_z^2-m_w^2)}{m_z^2}$$

$$\rightarrow (1+K)^2=(1+K_e)^2(1+K_z)\sin^2(\theta_w)\left(\sin^2(\theta_w)+K_z-K_w\cos^2(\theta_w)\right)^{-1}$$

For the others vertices we define:

$Z\nu\nu$: $\dfrac{g_B}{\cos(\theta_{wB})}=(1+K_3)\dfrac{g}{\cos(\theta_{wB})}$

Definition: In this way that $\Delta\mathcal{L}$ not contains K's in denominators.

$Z\ell\ell$: $\dfrac{g_B\sin^2(\theta_{wB})}{\cos(\theta_{wB})}=(1+K_4)\dfrac{g\sin^2(\theta_w)}{\cos(\theta_w)}$

ZWW : $g_B\cos(\theta_{wB})=(1+K_5)g\cos(\theta_w)$

ZG^+G^+: $\dfrac{g_B\cos(2\theta_{wB})}{\cos(\theta_{wB})}=(1+K_6)g\dfrac{\cos(2\theta_w)}{\cos(\theta_w)}$

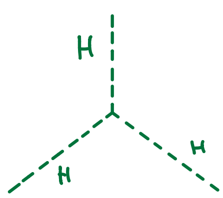
$GW\gamma$: $e_B m_{wB}=(1+K_7)em_w$

GWZ: $g_B m_{zB}\sin^2(\theta_w)=(1+K_8)gm_z\sin^2(\theta_w)$

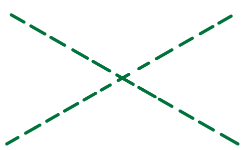
HWW : $g_B m_{wB}=(1+K_9)gm_w$

HZZ : $g_B m_{zB}=(1+K_{10})gm_z$

Scalars:



$: -\dfrac{1}{2}i\lambda v$ $\qquad \lambda_B v_B=\lambda v(1+K_{11})$



$: -\dfrac{i}{4}\lambda$ $\qquad \lambda_B=\lambda(1+K_\lambda)$

but, $\quad m_{HB}^2=\dfrac{1}{2}\lambda_B v_B^2$ $\longrightarrow$

Definition:

$$\lambda=\frac{e^2m_H^2m_z^2}{2m_w^2(m_z^2-m_w^2)}$$

and, $\quad m_w^2=\dfrac{1}{4}g_B v_B^2$

$$\lambda v=\frac{em_H^2m_z}{m_w(m_z^2-m_w^2)^{1/2}}$$

Evaluation of renormalization constanto at 1-loop

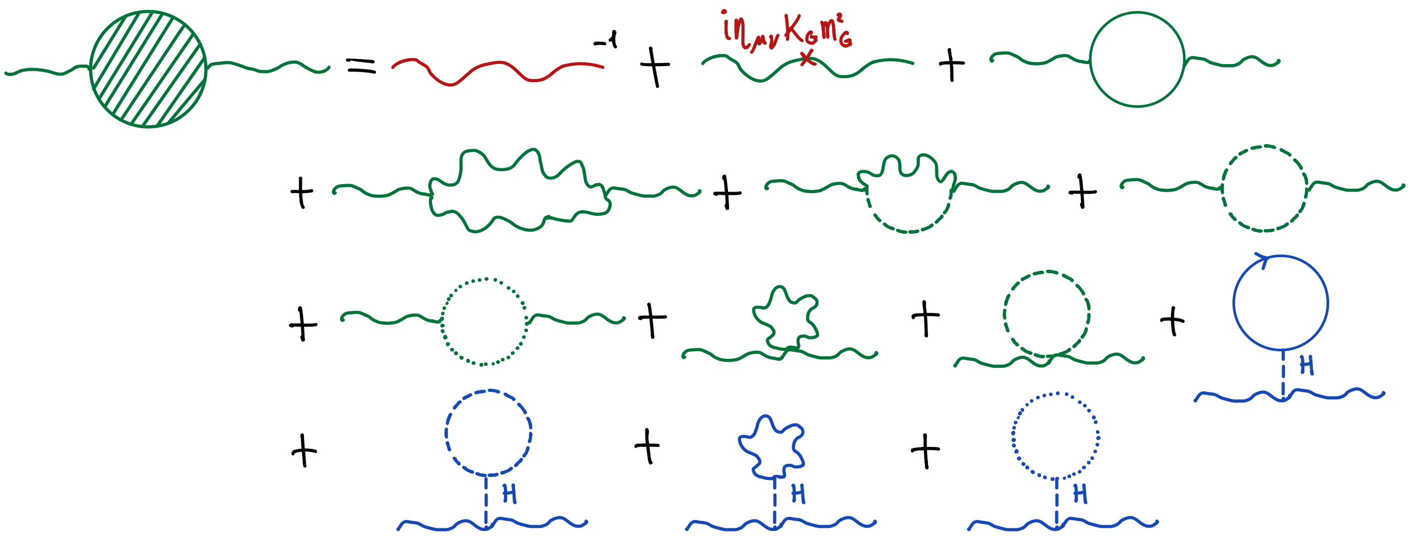
In the "Feynman norm":

$$\xi = 1$$

To be determined : $K_{W,Z}$; K_H ; K_ℓ ; K_e

Propagators   interaction γ-e

Gauge: $G \to G$.



but $m = m_{phys.}$

: Total contribution to S_{fi}

then,
$$\sum_2^{13} S_{fi}(\text{diag}) = 0$$

self-energy: $\sum S_{fi}(\text{diag}) := \varepsilon^\mu \Pi_{\mu\nu} \varepsilon^\nu$

Tadpoles: $\sum S_{fi}(\text{diag}) := \varepsilon^\mu T_{\mu\nu} \varepsilon^\nu$

$\Pi_{\mu\nu}$ is a covariant function of p_α

Expresions more general:

$$\Pi^G_{\mu\nu} = A_G(p^2)\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + B_G(p^2)\,\frac{p_\mu p_\nu}{p^2}$$

while, $T_{\mu\nu} = T_G \eta_{\mu\nu}$ $\to$ Is because the vertex VVH does not depend on p.

In a mass layer, $p^2=m^2$ and $\varepsilon\cdot p=0$:

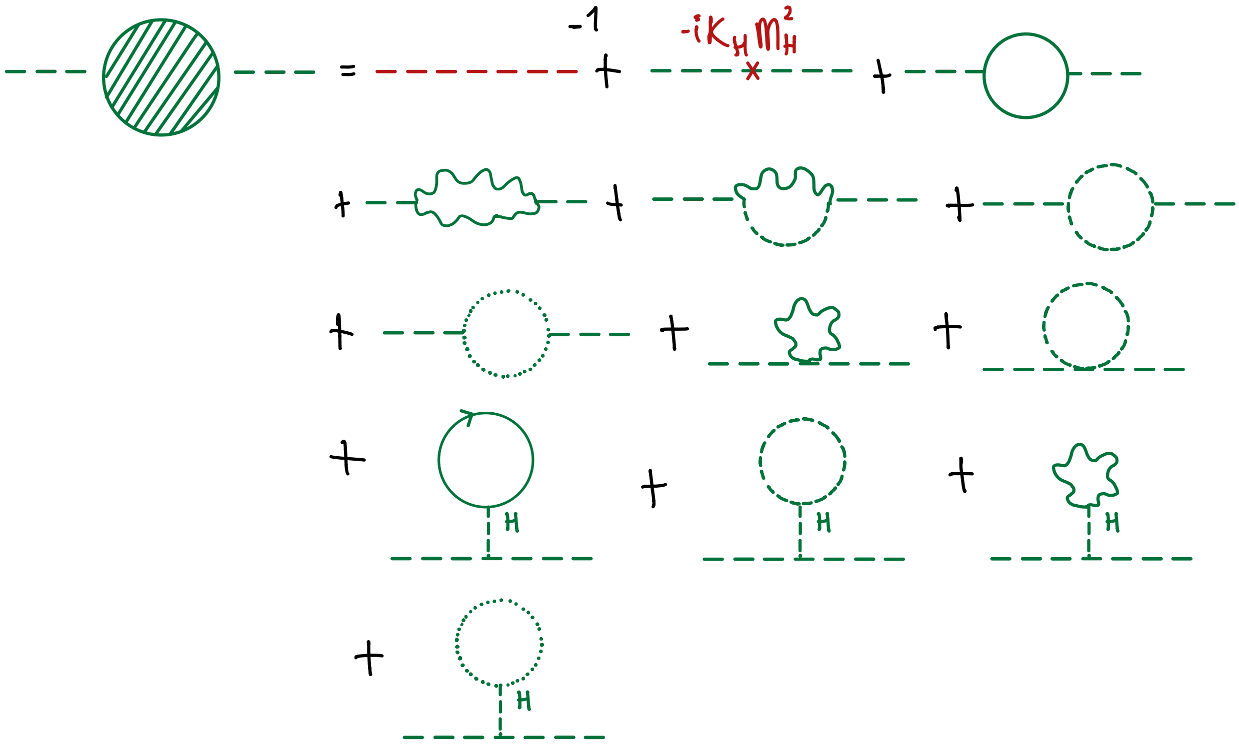
$$iK_G m_G^2 + A_G(m_G^2) + T_G = 0$$

therefore

$$K_G = i\, m_G^{-2}\left[A_G(m_G^2) + T_G\right]_{G=W,Z}$$

The compute of A and T: K. Aoki, et al, Supp. Prog. Theor. Phys. 73 (1982) 106.

Higgs: $H \longrightarrow H$



Again,

S_{fi}: 

Now all the integrals are scalar functions, then

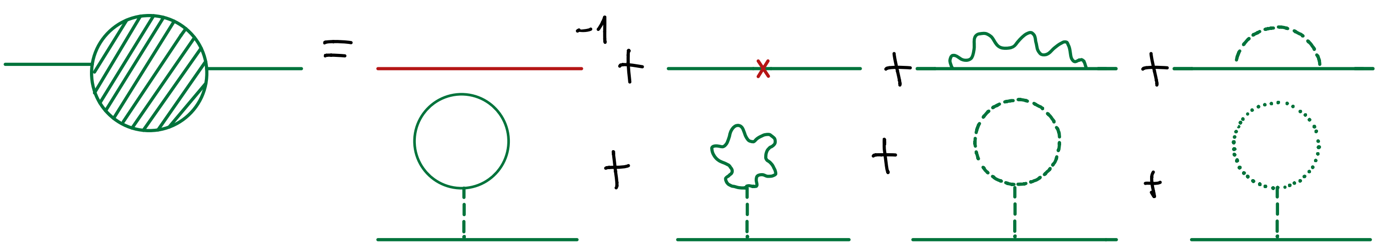
$$\sum S_{fi}(\text{Self-energy}) = A_H(p^2)$$

$$\sum S_{fi}(\text{Tadpoles}) = T_H$$

therefore,

$$K_H = -i m_H^{2}\left[A_H(m_H^2) + T_H\right]$$

Fermion: $l \longrightarrow l$.



$$\sum S_{fi}(\text{Self-energy}) = \bar{u}_2(p)\,\Sigma\, u_1(p)$$

$$\sum S_{fi}(\text{Tadpoles}) = \bar{u}_2(p)\,T\, u_1(p)$$

In general form

$$\Sigma(p) = \Sigma_s(p^2)\,\mathbb{1} + \Sigma_5(p^2)\,\gamma_5 + \Sigma_v(p^2)\,\not{p} + \Sigma_A(p^2)\,\not{p}\,\gamma_5$$

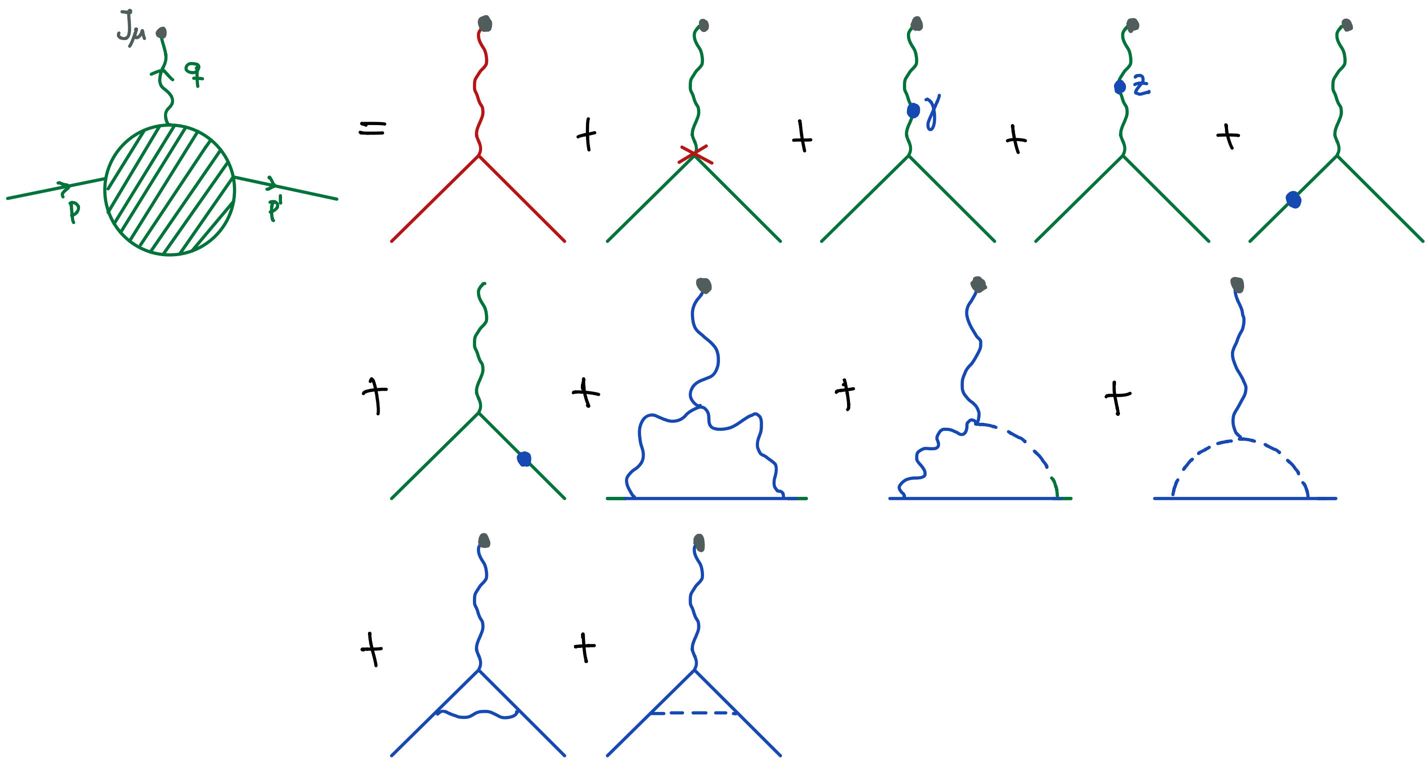
$$T = T_1\,\mathbb{1} + T_5\,\gamma_5$$

On-shell:

$$\not{p}\,u(p) = m\,u(p) \quad ; \quad \bar{u}_2(p)\,\gamma_5\, u_1(p) = 0$$

$$K_\ell = -i\,m_\ell^{-1}\left[\Sigma_s(m_\ell^2) + m_\ell\,\Sigma_v(m_\ell^2) + T_1\right]$$

Charge: Electron dispersion by an external source ($M \to \infty$)



$$S_{fi}(1) = \bar{u}\, i e\,\gamma_\mu\, u(p)\,\frac{-i}{(p-p')^2}\, J^\mu (2\pi)^4\,\delta^4(p-p'-q)$$

definition of $-e$:

$$\lim_{q^2\to 0}\left[S_{fi}^{T} - S_{fi}(1)\right] = 0$$

then, $$\lim_{q^2\to 0}\sum_i S_{fi}(\text{diag}) = 0$$

$$S_{fi} := O_\mu\,\frac{-i}{q^2}\,J^\mu\,(2\pi^4)\,\delta\left(\sum p\right)$$

Insertion Off-Shell:  : $\Pi^{\gamma}_{\mu\nu}(q) \ni q^\mu q^\nu / q^2$

γ: $\quad O_\mu(\gamma_{in}) = \frac{1}{2}\bar{u}(p')\, ie\gamma_\mu u(p) \frac{-i}{q^2} A_\gamma(q^2)$

$$\bar{u}(p')\not{q}u(p) = \bar{u}(p')(\not{p}' - \not{p})u(p) = \bar{u}(p')(m-m)u(p) = 0$$

As

$$A_\gamma(0) = 0 \quad (m_\gamma = 0) \to A(q^2)/q^2 \xrightarrow[q^2\to 0]{} A'_\gamma(0)$$

$$S_{fi}(\gamma_{in}) \to \frac{-1}{2} i A'_\gamma(0) S_{fi}(1)_{\,q^2\to 0}$$

z: $\quad O_\mu(z_{in}) \to \bar{u}(p')ie\gamma_\mu \frac{m_z^2}{2M_W(m_z^2 - m_W^2)^{1/2}}(g_v - g_A\gamma_5)u(p)\cdot iA_{\gamma z}(0)m_z^{-2}$

e: $\quad O_\mu(e_{in}) = \bar{u}(p')ie\gamma_\mu \frac{i}{\not{p}-m}\left[-iK_e m_e + \Sigma(p) + T\right]u(p)$

$$+ \bar{u}(p')\left(\frac{i}{\not{p}-m} ie\gamma_\mu\right)[----]u(p)$$

After algebra,

$$O_\mu(e_{in}) = \bar{u}(p')ie\gamma_\mu i\left[\Sigma_A(m_e^2)\gamma_5 + 2m_e A_e(m_e^2) + \Sigma_v(m_e^2)\right]u(p)$$

where

$$A_e(m_e^2) := \lim_{p^2\to m_e^2}\left[\frac{\Sigma_B(p^2) - \Sigma_s(m_e^2)}{p^2 - m_e^2} + m_e\frac{\Sigma_v(p^2) - \Sigma_v(m_e^2)}{p^2 - m_e^2}\right]$$

$$= \Sigma'_s(m_e^2) + m_e\Sigma'_v(m_e^2)$$

diag Vertices: $\quad S_{fi} = ie\bar{u}(p')\Gamma_\mu(p,p')u(p)\frac{-i}{q}J_\mu(2\pi)^4\delta(p-p'-q)$

General form

$$\Gamma_\mu(p,p') = F_1(q^2)\gamma_\mu + F_2(q^2)i\sigma_{\mu\nu}q^\nu + F_3(q^2)q_\mu + G_1(q^2)\gamma_\mu\gamma_5$$
$$+ G_2(q^2)q_\mu\gamma_5 + G_3(q^2)i\sigma_{\mu\nu}q^\nu\gamma_5$$

In $q\to 0$, Just F_1 and G_1 are computable, then

$$\lim_{q\to 0}\sum_{2} S_{fi} = 0:$$

$$K_e + F_1(0) + G_1(0)\gamma_5 - i\frac{1}{2}A'_\gamma(0) + i\frac{1}{2}\frac{A_{\gamma Z}(0)}{m_W(m_Z^2 - m_W^2)^{1/2}}(g_V - g_A\gamma_5)$$
$$+ i\left[\Sigma_A(m_e^2)\gamma_5 + 2m_e A_e(m_e^2) + \Sigma_\gamma(m_e^2)\right] = 0$$

P-violating (γ_5) contributions must cancel apart.

$$G_1(0) + i\Sigma_A(m_e^2) - i\frac{A_{\gamma Z}(0)}{m_W(m_Z^2 - m_W^2)^{1/2}}g_A = 0$$

and

$$K_e = -F_1(0) + i\frac{1}{2}A'_\gamma(0) - i\Sigma_V(m_e^2) - \frac{A_{\gamma Z}(0)}{m_W(m_Z^2 - m_W^2)^{1/2}}g_V$$
$$- i2m_e\left[\Sigma'_1(m_e^2) + m_e\Sigma'_\gamma(m_e^2)\right]$$