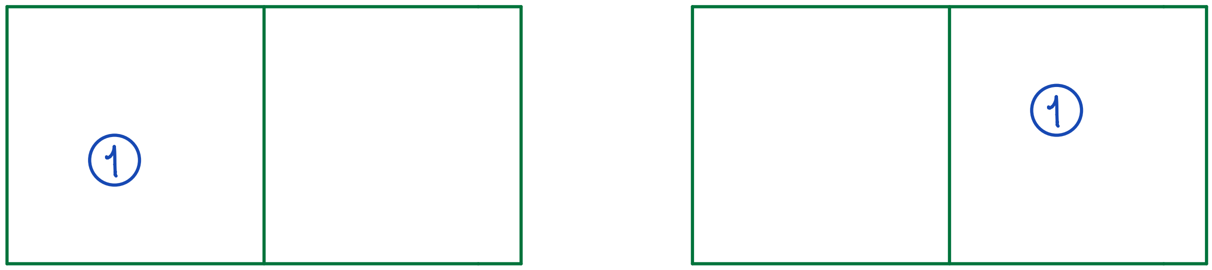
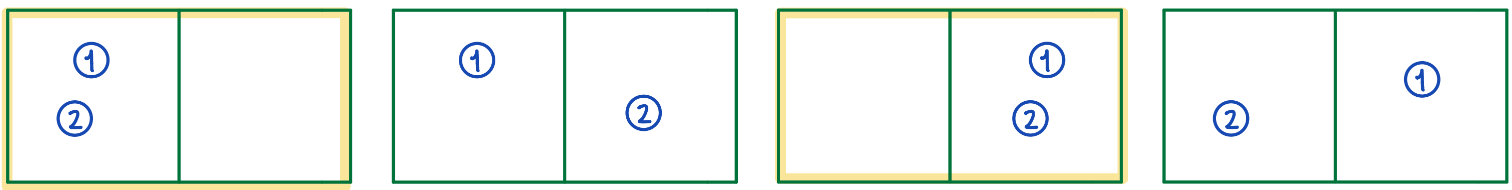
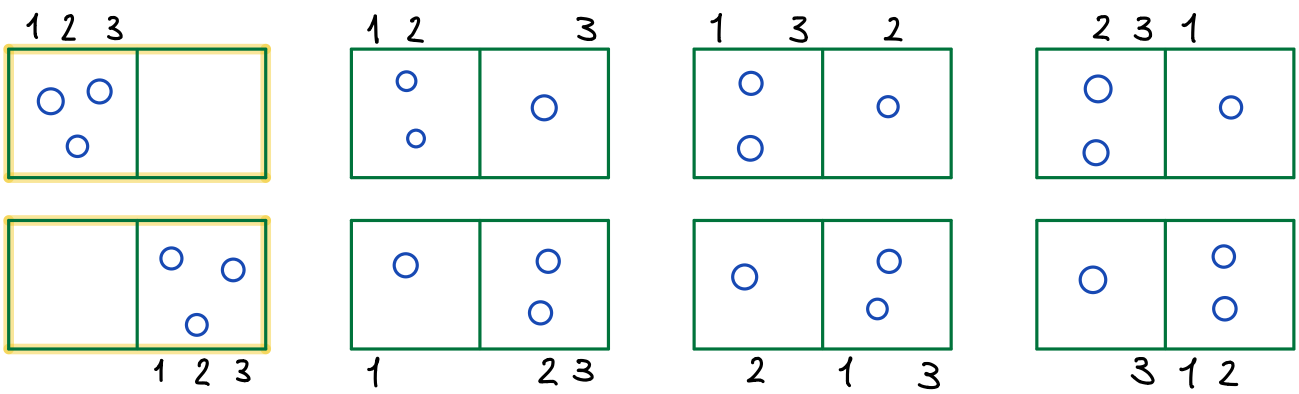
Introduction

Can all the particles be on one side of the box?



Yes





$$2^1 = 2 \;,\; 2^2 = 4 \;,\; 2^3 = 8 \;,\; \dots \;,\; 2^{100} \approx 1.2 \times 10^{30}$$

$$\frac{2}{2} = 1 \;,\; \frac{2}{4} = \frac{1}{2} \;,\; \frac{2}{8} = \frac{1}{4} \;,\; \dots \;,\; \frac{2}{2^{100}} = 1.6 \times 10^{-30}$$

Probability

$$P_A = \frac{N_A}{N} = \frac{\text{\# of configurations that show A.}}{\text{total \# of systems.}}$$

if $N_A = N \longrightarrow P_A = 1 \longrightarrow$ 100% of probability.

$P(A \text{ y } B) = P(A \cap B) \rightarrow$ Combined probability

$P(A \cap B) = \dfrac{N_{AB}}{N}$ if A and B are independent events, then $N_{AB} = N_A P_B$

From the events of N_A where is obtained A, there is a fraction P_B that gives the B event.

Therefore,

$$P(A \cap B) = \frac{N_A P_B}{N} = P_A P_B$$ Indepent events.

If

$$P(A \cap B) = 0 \longrightarrow$$ exclusive events

A and B may happens, but never simultaneosly.

$$P(A \text{ or } B) = P(A \cup B) = P_A + P_B - (P_A \cap P_B)$$

if A and B are exclusive events, then

$$P(A \cup B) = P_A + P_B.$$

Let $r = 1, 2, \dots, \alpha$ be exclusive events where

$$N_1 + N_2 + \cdots + N_\alpha = N \longrightarrow \sum_{r=1}^{\alpha} \frac{N_r}{N} = 1$$

then

$$\sum_{r=1}^{\alpha} P_r = 1 \longrightarrow \text{Normalization}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \longrightarrow \text{Conditional probability.}$$

Probability of A given B

Example

4 balls, 2 blue, 2 red.

Which is the probability of take out the second red?

Answer: There are two possible probability paths:

I. If we take out the first blue ball, we get:

$$P(2^{nd} \text{ red} / 1^{st} \text{ blue}) = \frac{\#\text{ red balls}}{\#\text{ total balls}} = \frac{2}{3}$$

II. If we take out the first red balls, we get:

$$P(2^{nd} \text{ red} / 1^{st} \text{ red}) = \frac{\#\text{ red balls}}{\#\text{ total balls}} = \frac{1}{3}$$

$\blacksquare$