Quantum Systems

$$\Psi = \sum_n C_n \phi_n \qquad \hat{H}\phi_n = E_n \phi_n$$

Located particle of spin $\frac{1}{2}$

$$\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad ; \quad \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathcal{H} = -\vec{\mu}\cdot\vec{H} = -\mu_z H = \begin{cases} -\mu_0 H & \text{for } \uparrow \\ +\mu_0 H & \text{for } \downarrow \end{cases}$$

$$\mu \sim -S \longrightarrow \mu_z = -\mu_0 \quad \text{or} \quad \mu_z = +\mu_0$$

3 located particle none interacting of spin $\frac{1}{2}$

$$\mathcal{H} = -\vec{\mu}_1\cdot\vec{H} - \vec{\mu}_2\cdot\vec{H} - \vec{\mu}_3\cdot\vec{H}$$

↑↑↑			$-3\mu_0 H$
↑↑↓	↑↓↑	↓↑↑	$-\mu_0 H$
↑↓↓	↓↑↓	↓↓↑	$+\mu_0 H$
↓↓↓			$+3\mu_0 H$

N located particle none interacting of spin $\frac{1}{2}$

$$\mathcal{H} = -\sum_{j=1}^{N} \vec{\mu}_j \vec{H} = -\mu_0 H \sum_{j=1}^{N} \sigma_j \qquad \{\sigma_j \,/\, j = 1,\dots,N\} \text{ and } \sigma_j = \pm 1.$$

$$E = -\mu_0 H N_1 + \mu_0 H (N - N_1) \quad \text{with} \quad N_1 := \#\uparrow \text{ and } N_2 := \#\downarrow$$

$$\longrightarrow N_1 = \frac{1}{2}\, \frac{E - \mu_0 H N}{(\mu_0 H)} = \frac{1}{2}\left(N - \frac{E}{\mu_0 H}\right)$$

$$N_2 = N - N_1 = \frac{1}{2}\left(N + \frac{E}{\mu_0 H}\right)$$

$$\Omega(E,N) = \frac{N!}{N_1! \cdot N_2!} = \frac{N!}{\left[\frac{1}{2}\left(N - \frac{E}{\mu_0 H}\right)\right]! \left[\frac{1}{2}\left(N + \frac{E}{\mu_0 H}\right)\right]!}$$

One dimensional harmonic oscillator

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \; ; \qquad n = 0, 1, 2, \ldots$$

Two oscillators

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 \longrightarrow \phi = \phi_1 \phi_2 \quad \text{and} \quad E = E_1 + E_2$$

$$E_{n_1,n_2} = \left(n_1 + \frac{1}{2}\right)\hbar\omega + \left(n_2 + \frac{1}{2}\right)\hbar\omega = (n_1 + n_2 + 1)\hbar\omega$$

$$\begin{aligned}
(n_1, n_2) &\longrightarrow \text{Energy} \\
(0,0) &\longrightarrow \hbar\omega \\
(0,1) &\longrightarrow 2\hbar\omega \\
(1,0) &\longrightarrow 2\hbar\omega \\
(0,2) &\longrightarrow 3\hbar\omega \\
(2,0) &\longrightarrow 3\hbar\omega \\
(1,1) &\longrightarrow 3\hbar\omega
\end{aligned}$$

N none interacting harmonic oscillators

$$E_{n_1,n_2,n_3,\ldots,n_N} = \left(n_1 + \frac{1}{2}\right)\hbar\omega + \left(n_2 + \frac{1}{2}\right)\hbar\omega + \ldots + \left(n_N + \frac{1}{2}\right)\hbar\omega$$

$$= \left(n_1 + n_2 + \ldots + n_N + \frac{N}{2}\right)\hbar\omega = M\hbar\omega + \frac{N}{2}\hbar\omega$$

$$M = \sum_{i=1}^{N} n_i = \frac{E}{\hbar\omega} - \frac{N}{2}$$

How many configurations can m and n give me?

$\bullet\ \bullet \mid \bullet\ \bullet\ \bullet \mid \bullet \mid \cdots \mid$ M balls in N boxes!

N-1 divisions.

$$\Omega(E,N) = \frac{(M+N-1)!}{M!\,(N-1)!} = \frac{\left(\frac{E}{\hbar\omega} + \frac{N}{2} - 1\right)!}{\left(\frac{E}{\hbar\omega} - \frac{N}{2}\right)!\,(N-1)!}$$

configurations for E and N given.

Particle in a box.

$$\mathcal{H} = \frac{p_x^2}{2m} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} \longrightarrow -\frac{\hbar^2}{2m}\frac{d^2\phi(x)}{dx^2} = E\phi(x)$$

$$\longrightarrow \phi(x) = A\sin(kx) + B\cos(kx) \quad \text{and} \quad E = \frac{\hbar k^2}{2m}$$

As $\phi(0) = \phi(L) = 0$, then

$$\phi_n(x) = A\sin(K_n x) \quad \text{and} \quad E_n = \frac{\hbar^2 K_n^2}{2n} \ ; \quad K_n = \frac{n\pi}{L} \quad n = 1, 2, 3, \dots$$

or

$$K_n = 0, \pm\frac{2\pi}{L}, \pm 2\,\frac{2\pi}{L}, \dots \qquad \phi_x(x) = c\exp(iK_n x)$$

for N particles

$$\mathcal{H} = \frac{1}{2m}\sum_{j=1}^{N} p_j^2 = -\frac{\hbar^2}{2m}\sum_{j=1}^{N}\frac{d^2}{dx_j^2}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Phi(x_1, x_2, \dots, x_N)}{dx^2} = E\,\Phi(x_1, \dots, x_N)$$

$$\Phi(x_1, \dots, x_N) = \Phi_{K_1}(x_1)\,\Phi_{K_2}(x_2)\cdots\Phi_{K_N}(x_N)$$

$$E = E_{K_1, K_2, \dots, K_N} = \frac{\hbar^2}{2m}(K_1^2 + K_2^2 + \dots + K_N^2)$$

$$K_1 = n_1\frac{2\pi}{L}, \dots, K_N = n_N\frac{2\pi}{L}$$

$$n_1, \dots, n_N = 0, \pm 1, \pm 2, \dots$$