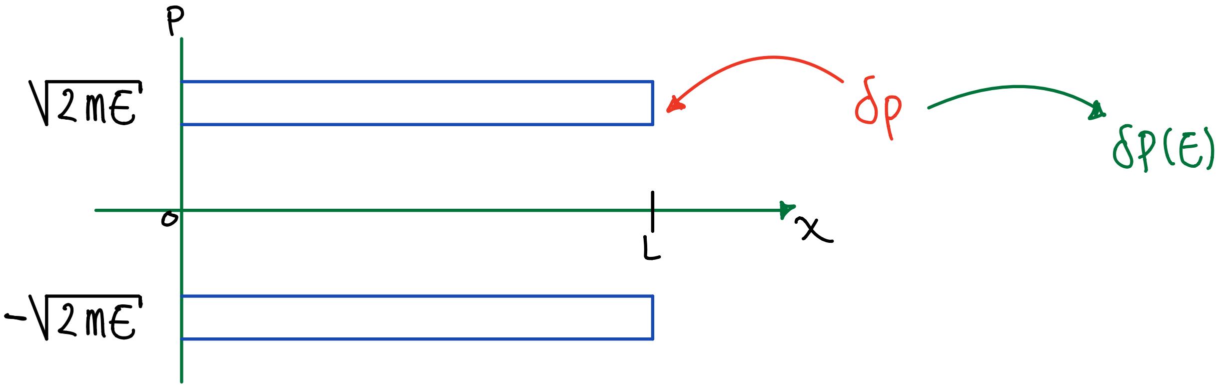
Classic systems

Particle in a box of lenth L:

$$E = \frac{p^2}{2m} \longrightarrow p = \pm\sqrt{2mE} \longrightarrow \delta p = \sqrt{\frac{m}{2E}}\,\delta E$$

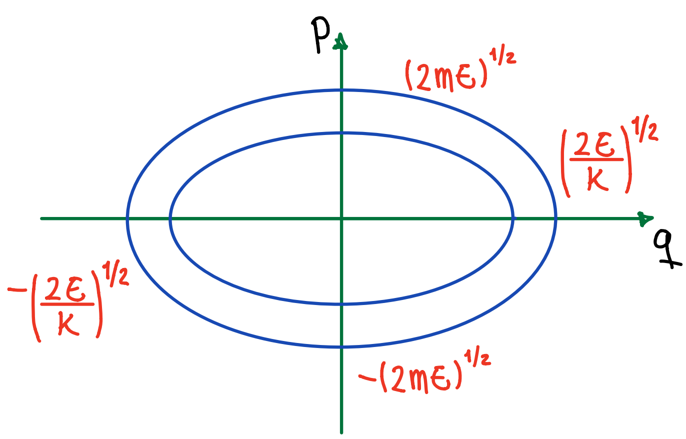


$$\Omega(E, L; \delta E) = 2L\,\delta p = \sqrt{\frac{2m}{L}}\,L\,\delta E$$

Harmonic oscillator 1D:

$$\mathcal{H} = \frac{p^2}{2m} + \frac{1}{2}k^2 q^2 \quad ; \quad k > 0.$$

$$\longrightarrow \frac{p^2}{2mE} + \frac{q^2}{2E/k} = 1$$



Area of an ellipse $(\pi R r)$

$$\longrightarrow \pi\left[(2mE)^{1/2}\left(\frac{2E}{k}\right)^{1/2}\right] = 2\pi\sqrt{\frac{m}{k}}\,E$$

and

$$\pi\left\{\left[2m(E+\delta E)\right]^{1/2}\left[\frac{2(E+\delta E)}{k}\right]^{1/2}\right\}$$

$$= 2\pi\sqrt{\frac{m}{k}}\,(E+\delta E)$$

Substracting,

$$\longrightarrow 2\pi\left(\frac{m}{k}\right)^{1/2}\delta E$$

$$\Omega(E;\delta E) = 2\pi\left(\frac{m}{k}\right)^{1/2}\delta E \neq f(E)$$

Ideal monoatomic gas of N molecules

$$\mathcal{H} = \sum_{j=1}^{N} \frac{1}{2m}\vec{p}_j^{\,2}$$

$$\Omega = \int_V \cdots \int d^3\vec{r}_1 \cdots d^3\vec{r}_N \quad \int \cdots \int_{2mE \leq \vec{p}_1^{\,2} + \cdots + \vec{p}_N^{\,2} \leq 2^2 m(E+\delta E)} d^3\vec{p}_1 \cdots d^3\vec{p}_N$$

$$= V^N \int \cdots \int_{2mE \leq \vec{p}_1^{\,2} + \cdots + \vec{p}_N^{\,2} \leq 2^2 m(E+\delta E)} d^3\vec{p}_1 \cdots d^3\vec{p}_N$$

$$\Omega^N(R;\delta R) = C_n R^{n-1}\delta R$$

hyperspheric shell volume of radius R and width δR

$n = 3N$ and $R = (2mE)^{1/2}$

$$\Omega(E,V,N,\delta E) = \left(\frac{m}{2}\right)^{1/2} C_{3N}(2m)^{3N/2 - 1/2} V^N E^{3N/2 - 1}\delta E$$