Monoatomic ideal gas

$$\mathcal{H}=\sum_{i=1}^{N}\frac{1}{2m}\vec{p}_i^{\,2}+\sum_{i<j}V(|\vec{r}_i-\vec{r}_j|)$$

for $V=0$

$$\Omega(E,V,N;\delta E)=\left(\frac{m}{2}\right)^{1/2}C_{3N}(2m)^{3N/2-1}V^{N}E^{3/2N-1}\delta E$$

$$\frac{1}{N}\ln(\Omega(E,V,N;\delta E))=\left(\frac{3}{2}-\frac{1}{N}\right)\ln(u)+\ln(v)+\left(\frac{3}{2}-\frac{1}{N}\right)\ln(2m)$$
$$+\frac{1}{N}\ln(C_{3N})+\ln(N)+\left(\frac{3}{2}-\frac{1}{N}\right)\ln(N)+\frac{1}{2N}\ln\left(\frac{m}{2}\right)$$
$$+\frac{1}{N}\ln(\delta E)$$

$$\lim_{N\to\infty}\frac{1}{N}\ln(\delta E)=0$$

$$s(u,v)=\lim\frac{1}{N}K_B\ln(\Omega(E,V,N;\delta E))$$
$$=\frac{3}{2}K_B\ln(u)+K_B\ln(v)+s_0$$

$$s_0=\frac{3}{2}\ln(2m)+\frac{1}{N}\ln(C_{3N})+\ln(N)+\frac{3}{2}\ln(N).$$

$$\frac{1}{T}=\frac{\partial s}{\partial u}=\frac{3K_B}{2u}\longrightarrow u=\frac{3}{2}K_BT$$

$$\frac{p}{T}=\frac{\partial s}{\partial v}=\frac{K_B}{v}\longrightarrow pv=K_BT$$

$$-\frac{\mu}{T}=\frac{\partial s}{\partial N}$$

$$S(E,V,N;\delta E)=K_B\ln(\Omega(E,V,N;\delta E))$$
$$=\frac{3}{2}K_BN\ln(E)+K_BN\ln(V)+f(N,\delta E)$$

$$\longrightarrow\frac{1}{T}=\frac{\delta S}{\delta E}=\frac{3}{2}\frac{NK_B}{E}\longrightarrow E=\frac{3}{2}K_BTN$$

$$\frac{p}{T}=\frac{\partial S}{\partial V}=\frac{K_BN}{V}\longrightarrow pV=K_BTN$$

state equation

As $C_{3N} = \frac{2\pi^{3N/2}}{\Gamma\left(\frac{3N}{2}\right)} \sim N^{-3/2 N} b^{N}$

$$\longrightarrow \frac{1}{N}\ln(C_{3N}) = -\frac{3}{2}\ln(N) + \ln(b)$$

$$\longrightarrow s_0 = \frac{3}{2}\ln(2m) - \frac{3}{2}\ln(N) + \ln(b) + \ln(N) + \frac{3}{2}\ln(N)$$

$$= \frac{3}{2}\ln(2m) + \ln(b) + \ln(N)$$

$$\begin{matrix} s_0 \to \infty \\ N \to \infty \end{matrix} \quad !$$

what about if

$$C_{3N} \longrightarrow \frac{C_{3N}}{N!} \sim \frac{N^{-3/2N}\, b^{N}}{\sqrt{2\pi N}\, N^{N} e^{-N}} = N^{-5/2N} a^{N}$$

then,

$$s_0 = \frac{3}{2}\ln(2m) + \ln(b) \longrightarrow s_0 \xrightarrow[N \to \infty]{} s_0 \neq s_0(N)$$

finally

$$\Omega \longrightarrow \frac{1}{N!}\Omega$$ Boltzmann factor.