Einstein Solid

$$E_{n_1,n_2,\dots,n_N} = M\hbar\omega + \frac{1}{2}N\hbar\omega$$

$$\Omega(E,N) = \frac{\left(\frac{E}{\hbar\omega} + \frac{N}{2} - 1\right)!}{\left(\frac{E}{\hbar\omega} - \frac{N}{2}\right)!\,(N-1)!}$$

$$\ln(\Omega(E,N)) = \left(\frac{E}{\hbar\omega} - \frac{N}{2} - 1\right)\ln\left(\frac{E}{\hbar\omega} + \frac{N}{2} - 1\right)$$

$$-\left(\frac{E}{\hbar\omega} + \frac{N}{2} - 1\right) - \left(\frac{E}{\hbar\omega} - \frac{N}{2}\right)\ln\left(\frac{E}{\hbar\omega} - \frac{N}{2}\right)$$

$$+\left(\frac{E}{\hbar\omega} - \frac{N}{2}\right) - (N-1)\ln(N-1) + (N-1)$$

$$= -N\ln(N-1) + \ln(N-1) + \left(\frac{E}{\hbar\omega} + \frac{N}{2} - 1\right)\ln\left(\frac{E}{\hbar\omega} + \frac{N}{2} - 1\right)$$

$$-\left(\frac{E}{\hbar\omega} - \frac{N}{2}\right)\ln\left(\frac{E}{\hbar\omega} - \frac{N}{2}\right)$$

$$\frac{1}{N}\ln(\Omega(E,N)) = -\ln(N) + \left(\frac{u}{\hbar\omega} + \frac{1}{2} - \frac{1}{N}\right)\left[\ln(N) + \ln\left(\frac{u}{\hbar\omega} + \frac{1}{2} - \frac{1}{N}\right)\right]$$

$$-\left(\frac{u}{\hbar\omega} - \frac{1}{2}\right)\left[\ln(N) + \ln\left(\frac{u}{\hbar\omega} - \frac{1}{2}\right)\right]$$

$$= \left(\frac{u}{\hbar\omega} + \frac{1}{2}\right)\ln\left(\frac{u}{\hbar\omega} + \frac{1}{2}\right) - \left(\frac{u}{\hbar\omega} - \frac{1}{2}\right)\ln\left(\frac{u}{\hbar\omega} - \frac{1}{2}\right)$$

$$s(u) = \lim_{\substack{E,N\to\infty \\ \frac{E}{N}=u}} \frac{1}{N} k_B \ln(\Omega(E,N)).$$

$$= k_B\left(\frac{u}{\hbar\omega} + \frac{1}{2}\right)\ln\left(\frac{u}{\hbar\omega} + \frac{1}{2}\right)$$

$$- k_B\left(\frac{u}{\hbar\omega} - \frac{1}{2}\right)\ln\left(\frac{u}{\hbar\omega} - \frac{1}{2}\right)$$

$$\frac{1}{T} = \frac{\partial s}{\partial u} = \frac{k_B}{\hbar\omega}\left[\ln\left(\frac{u}{\hbar\omega} + \frac{1}{2}\right) - \ln\left(\frac{u}{\hbar\omega} - \frac{1}{2}\right)\right]$$

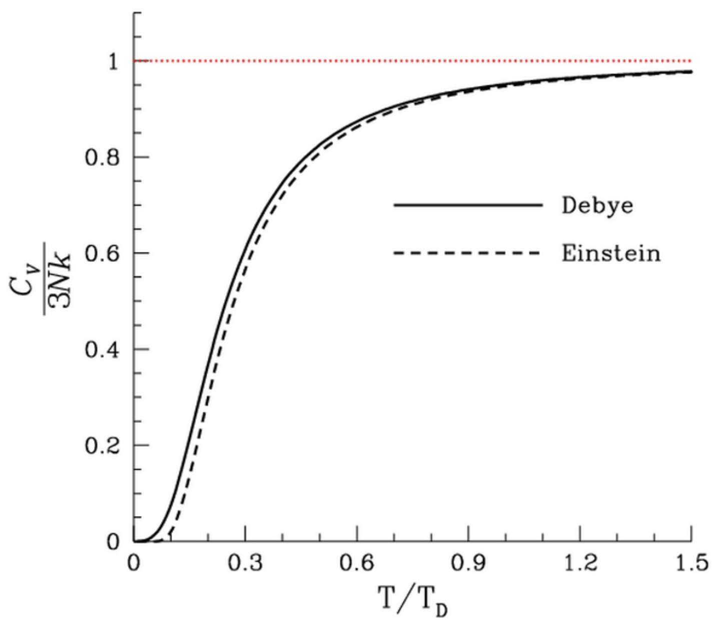
$$\exp\left(\frac{\hbar\omega}{k_B T}\right) = \frac{\frac{u}{\hbar\omega} + \frac{1}{2}}{\frac{u}{\hbar\omega} - \frac{1}{2}} \rightarrow \left(\frac{u}{\hbar\omega} - \frac{1}{2}\right)\exp\left(\frac{\hbar\omega}{k_B T}\right) = \frac{u}{\hbar\omega} + \frac{1}{2}$$

$$\rightarrow u\left(\frac{\exp\left(\frac{\hbar\omega}{k_B T}\right)}{\hbar\omega} - \frac{1}{\hbar\omega}\right) = \frac{\exp\left(\frac{\hbar\omega}{k_B T}\right)}{2} + \frac{1}{2}$$

$$\rightarrow u = \frac{\hbar\omega}{2}\,\frac{1+\exp\left(\frac{\hbar\omega}{k_B T}\right)}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1} = \frac{\frac{\hbar\omega}{2}\exp\left(\frac{\hbar\omega}{k_B T}\right) - \frac{\hbar\omega}{2} + \hbar\omega}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1}$$

$$= \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1}$$

$$c = \frac{\partial u}{\partial T} = k_B\left(\frac{\hbar\omega}{k_B T}\right)^2 \frac{\exp\left(\frac{\hbar\omega}{k_B T}\right)}{\left[\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1\right]^2}$$



Dolong-petit (law)

Debye model