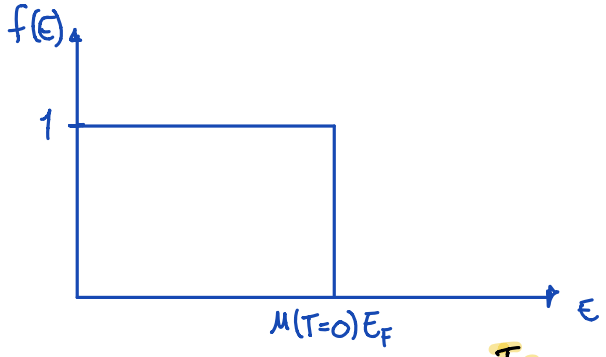
Completely degenerate Fermi gas



Fermi energy

$$N = \gamma V \int_0^{E_f} D(E)\,dE = \gamma V \int_0^{E_f} C\epsilon^{1/2}\,d\epsilon = \frac{2}{3}\gamma V C \epsilon^{3/2}\Big|_0^{E_f}$$

$$= \frac{2}{3}\gamma V C E_f^{3/2} = \frac{3}{2}\gamma V E_f D(E_f)$$

Then

$$E_f = \frac{\hbar^2}{2m}\left(\frac{6\pi^2}{\gamma}\right)^{3/2}\left(\frac{N}{V}\right)^{2/3}$$

$$U = \gamma V \int_0^{\infty} \epsilon\, C\epsilon^{1/2} f(\epsilon)\,d\epsilon = \gamma V \int_0^{E_f} \epsilon^{3/2} C\,d\epsilon$$

$$= \frac{2}{5}\gamma V \epsilon_f^{5/2} C = \frac{2}{5}\gamma V D(E_f) E_f^2$$

but $U = \frac{3}{2}pV$, so

$$p = \frac{2}{5}\frac{N}{V}E_f = \frac{\hbar^2}{5m}\left(\frac{6\pi^2}{\gamma}\right)^{2/3}\left(\frac{N}{V}\right)^{5/3}$$

pressure at $T=0$.

$$T_f = \frac{E_F}{k_B} = \frac{\hbar^2}{2mk_B}\left(\frac{6\pi^2}{\gamma}\right)^{2/3}\left(\frac{N}{V}\right)^{2/3}$$

T_f for metal $\sim 10^4 K$