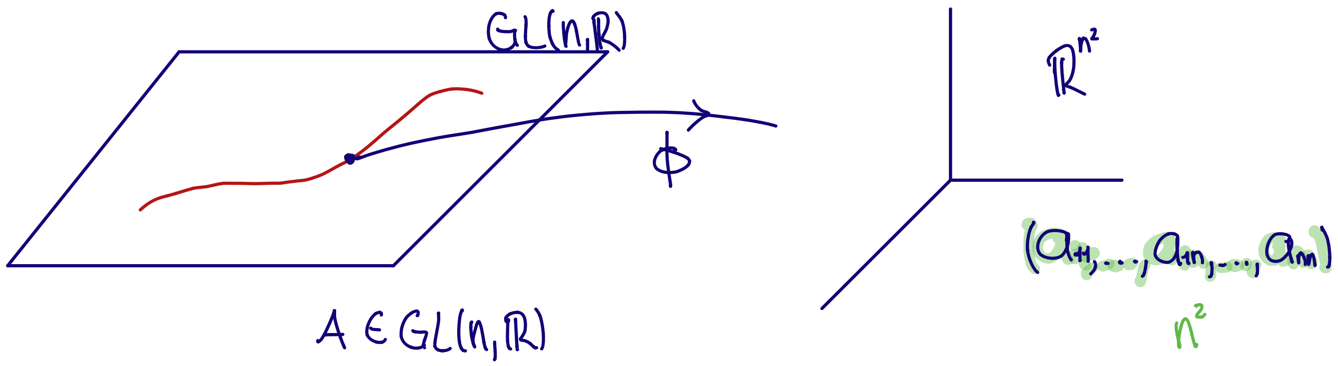
Homework: Show

$$* : \Omega^p(M) \longrightarrow \Omega^{n-p}(M)$$
$$*^2 = (-1)^{p(n-p)+s}$$

This G groups results being manifolds, i.e., the multiplication and inverse are smooth mapping.



$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & & a_{nn} \end{pmatrix}$$

We say that G is a Lie group if it is a manifold and the operations:

- $\cdot : G \times G \longrightarrow G$
- ${}^{-1} : G \longrightarrow G$

are smooth maps. (Sophius Lie 1880).

As same that linearit in a vector space, in a group we have:

Given two groups G and H, we say that a function $\rho : G \longrightarrow H$ is an homomorphism.

$$\rho(gh) = \rho(g)\rho(h).$$

If ρ is one-to-one and onto, is and isomorphism.

Homework: Prove that, if $\rho: G \longrightarrow H$, an homomorphism

$$\rho(1) = 1$$
$$\rho(g^{-1}) = \rho(g)^{-1}$$

Homework: let $U(1)=\{e^{i\theta}, \theta\in\mathbb{R}\}$. Show that $U(1)$ is an isomorphism to $SO(2)$

$$\rho(e^{i\theta})=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$

We say that a group G acts over a vector space V, if there is a map ρ of G to the linear transformations of the vector space V, such that.

$$\rho(gh)v=\rho(g)\rho(h)v, \quad \forall\, v\in V.$$

i.e., ρ is a representation of G in V which is an homomorphism.

If we define $GL(V)$ as the linear invertible transformation group in V, a representation of G in V, is an homomorphism.

$$\rho: G\longrightarrow GL(V).$$

If G is a Lie group, $\rho: G\longrightarrow GL(V)$; V of finite dimension ρ is smooth, g is labeled as the Gauge group.

Given two groups G, H, let $G\times H$ the set form by the pairs (g,h) with $g\in G$ and $h\in H$ $G\times H$ is a group with product

$$(g,h)(g',h')=(gg',hh')$$

$$1=(1,1) \qquad \text{identity.}$$

$$(g,h)^{-1}=(g^{-1},h^{-1}) \qquad \text{inverse.}$$

We say that two representations.

$$\rho: G\longrightarrow GL(V) \quad \text{and} \quad \rho': G\longrightarrow GL(V),$$

are equivalent if there exists a map one-to-one and onto $T: V\rightarrow V$!

$$\rho(g)T=T\rho'(g), \quad \forall g\in G.$$