Quantum Gas

$\Psi = \Psi(q_1, \dots, q_N)$ wave function of N particles

$q_j :=$ spin and position

$\Psi(q_1, \dots, q_i, \dots, q_j, \dots, q_N) = \pm \Psi(q_1, \dots, q_j, \dots, q_i, \dots, q_N)$

Symmetric	Skew-Symmetric
Integer spin (Bosons)	Fractional spin (Fermions)
• Photons • Phonons • Magnons • ^4He	• Electrons • Protons • Neutrons • ^3He
Bose-Einstein	Fermi-Dirac

let us suppose 2 independent particles

$$\hat{H} = \hat{H}_1 + \hat{H}_2 \qquad \hat{H}\Psi = E\Psi$$

$$\hat{H}_j = \frac{\hat{\vec{p}}_j^{\,2}}{2m} + V(\hat{\vec{r}}_j) \qquad j = 1, 2.$$

$$\hat{H}_1 \varphi_{n_1}(\vec{r}_1) = E_{n_1} \varphi_{n_1}(\vec{r}_1)$$

$$\hat{H}_2 \varphi_{n_2}(\vec{r}_2) = E_{n_2} \varphi_{n_2}(\vec{r}_2)$$

$$E = E_{n_1} + E_{n_2}$$

$$\Psi_S(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} \left[\varphi_{n_1}(\vec{r}_1)\varphi_{n_2}(\vec{r}_2) + \varphi_{n_1}(\vec{r}_2)\varphi_{n_2}(\vec{r}_1) \right]$$

$$\Psi_A(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} \left[\varphi_{n_1}(\vec{r}_1)\varphi_{n_2}(\vec{r}_2) - \varphi_{n_1}(\vec{r}_2)\varphi_{n_2}(\vec{r}_1) \right]$$

If $n_1 = n_2 \rightarrow \Psi_A = 0 \longrightarrow$ Pauli exclusion principle.

Let us suppose 3 levels: 1 2 3

Bosons

AB	–	–	$\psi_1(\vec{r}_1)\psi_1(\vec{r}_2)$
–	AB	–	$\psi_2(\vec{r}_1)\psi_2(\vec{r}_2)$
–	–	AB	$\psi_3(\vec{r}_1)\psi_3(\vec{r}_2)$
A	B	–	$\frac{1}{\sqrt{2}}\left[\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)+\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\right]$
–	A	B	$\frac{1}{\sqrt{2}}\left[\psi_2(\vec{r}_1)\psi_3(\vec{r}_2)+\psi_2(\vec{r}_2)\psi_3(\vec{r}_1)\right]$
A	–	B	$\frac{1}{\sqrt{2}}\left[\psi_1(\vec{r}_1)\psi_3(\vec{r}_2)+\psi_1(\vec{r}_2)\psi_3(\vec{r}_1)\right]$

Fermions

A	B	–	$\frac{1}{\sqrt{2}}\left[\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)-\psi_1(\vec{r}_2)\psi_2(\vec{r}_1)\right]$
–	A	B	$\frac{1}{\sqrt{2}}\left[\psi_2(\vec{r}_1)\psi_3(\vec{r}_2)-\psi_2(\vec{r}_2)\psi_3(\vec{r}_1)\right]$
A	–	B	$\frac{1}{\sqrt{2}}\left[\psi_1(\vec{r}_1)\psi_3(\vec{r}_2)-\psi_1(\vec{r}_2)\psi_3(\vec{r}_1)\right]$

Maxwell-Boltzmann

AB	–	–	$\psi_1(\vec{r}_1)\psi_1(\vec{r}_2)$
–	AB	–	$\psi_2(\vec{r}_1)\psi_2(\vec{r}_2)$
–	–	AB	$\psi_3(\vec{r}_1)\psi_3(\vec{r}_2)$
A	B	–	$\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)$
B	A	–	$\psi_2(\vec{r}_1)\psi_1(\vec{r}_2)$
–	A	B	$\psi_2(\vec{r}_1)\psi_3(\vec{r}_2)$
–	B	A	$\psi_3(\vec{r}_1)\psi_2(\vec{r}_2)$
A	–	B	$\psi_1(\vec{r}_1)\psi_3(\vec{r}_2)$
B	–	A	$\psi_3(\vec{r}_1)\psi_1(\vec{r}_2)$