Gauge fields

$$W \propto \int DA_\mu D\psi D\bar{\psi} D\eta^* D\eta \, e^{i\int dx (\mathcal{L} + J^\mu_a A^a_\mu + \bar{\sigma}\psi + \bar{\psi}\sigma)}$$

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_D - \frac{1}{2\xi}(\partial\cdot A)^2 + \mathcal{L}_{FP} \longrightarrow \text{is not gauge invariant.}$$

$$\left.\begin{aligned} \mathcal{L}_{YM} &= -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \\ \mathcal{L}_D &= \bar{\psi}(i\not{\partial} - m)\psi \end{aligned}\right\} \text{Gauge invariant.}$$

$$\mathcal{L}_{FP} = \partial_\mu \eta^*_a D^\mu_{ab} \eta_b \; ; \quad D^\mu_{ab} = \delta_{ab}\partial^\mu + g f_{abc} A^\mu_c$$

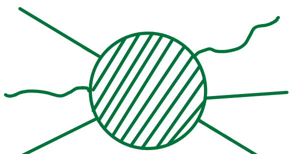
Gauge symmetry $\longleftrightarrow$ S must be invariant.

Noether theorem:

$$S[\varphi] = \int d^4x \, \mathcal{L}(\varphi)$$

$$\text{Symmetry: } \varphi \longrightarrow \varphi' = \varphi + \delta\varphi \,/ = S[\varphi]$$

$$\text{then, } \quad \partial_\mu J^\mu = 0$$

$S = \sum$  $\longleftarrow \Gamma \longleftarrow X \longleftarrow W = \langle 0|0\rangle^J$

$\Gamma^{(n)}$

therefore, W must be gauge invariant.

$$\rightarrow \Theta\left(\frac{\delta}{\delta J}, \frac{\delta}{\delta \bar{\sigma}}, \frac{\delta}{\delta \bar{\sigma}}\right) W = 0.$$

QED:

$$W \propto \int DA_\mu D\psi D\bar{\psi} D\eta^* D\eta \, e^{i\int dx (\mathcal{L}_{QED} - \frac{1}{2\xi}(\partial\cdot A)^2 + JA + \bar{\sigma}\psi + \bar{\psi}\sigma)}$$

Under,

$$A_\mu \longrightarrow A_\mu + \partial_\mu \Lambda(x) \quad ; \quad \psi \longrightarrow e^{-ie\Lambda(x)} = \psi - ie\Lambda(x)$$

The measure: $\int DA_\mu D\psi D\bar{\psi}$ is invariant.

The integrand changes in

$$\exp\left\{ i \int dx \left[-\frac{1}{\xi}(\partial\cdot A)\Box\Lambda + J^\mu \partial_\mu \Lambda - ie\Lambda(\bar{\sigma}\psi - \bar{\psi}\sigma)\right]\right\}$$

$$= 1 + i\int dx\left[-\frac{1}{\xi}\Box(\partial\cdot A) - \partial_\mu J^\mu - ie(\bar{\sigma}\psi - \bar{\psi}\sigma)\right]\Lambda(x)$$

consider $\delta W = 0;\quad \psi \to -i\frac{\delta}{\delta\bar{\sigma}};\quad \bar{\psi} \to i\frac{\delta}{\delta\sigma};\quad A_\mu \to -i\frac{\delta}{\delta J^\mu}$

So, we get

$$\left[\frac{i}{\xi}\Box\partial^\mu \frac{\delta}{\delta J^\mu} - \partial_\mu J^\mu - e\left(\bar{\sigma}\frac{\delta}{\delta\bar{\sigma}} - \sigma\frac{\delta}{\delta\sigma}\right)\right]W = 0,$$

for $W = e^{iX}$:

$$-\frac{1}{\xi}\Box\partial^\mu\frac{\delta X}{\delta J^\mu} - i\partial_\mu J^\mu - ie\left(\bar{\sigma}\frac{\delta X}{\delta\bar{\sigma}} - \sigma\frac{\delta X}{\delta\sigma}\right) = 0$$

Now, let's consider

$$\Gamma[\psi,\bar{\psi},A_\mu] = X[\sigma,\bar{\sigma},J] - \int dx\left(J\cdot A + \bar{\sigma}\psi + \bar{\psi}\sigma\right)$$

then,

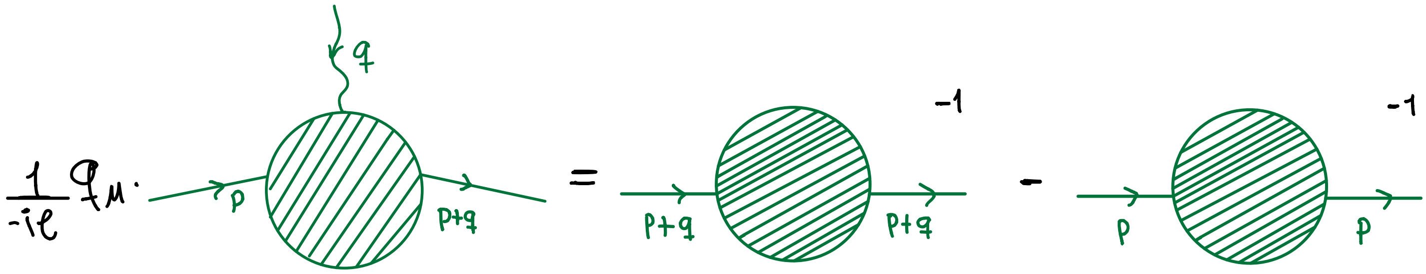
$$\frac{\delta\Gamma}{\delta A^\mu} = -J^\mu \qquad \frac{\delta\Gamma}{\delta\psi} = \bar{\sigma} \qquad \frac{\delta\Gamma}{\delta\bar{\psi}} = -\sigma$$

$$\frac{\delta X}{\delta J^\mu} = A^\mu \qquad \frac{\delta X}{\delta\sigma} = -\bar{\psi} \qquad \frac{\delta X}{\delta\bar{\sigma}} = \psi$$

$$\frac{\delta}{\delta\bar{\psi}(x_1)}\frac{\delta}{\delta\psi(y_1)}\left[\cancelto{0}{-\frac{1}{\xi}\Box\partial^\mu A_\mu} + i\partial_\mu\frac{\delta\Gamma}{\delta A_\mu} + ie\psi\frac{\delta\Gamma}{\delta\psi} - ie\bar{\psi}\frac{\delta\Gamma}{\delta\bar{\psi}}\right]_{A=0=\psi=\bar{\psi}} = 0$$

$$i\partial_{x\mu}\frac{\delta^3\Gamma}{\delta\bar{\psi}(x_1)\delta\psi(y_1)\delta A_\mu(x)} + ie\delta(x-y_1)\frac{\delta^2\Gamma}{\delta\bar{\psi}(x_1)\delta\psi(y_1)} - ie\delta(x-x_1)\frac{\delta^2\Gamma}{\delta\bar{\psi}(x_1)\delta\psi(y_1)} = 0$$

$$q_\mu \tilde{\Gamma}^{(3)}(p,q,p+q) = [\tilde{S}_F'(p+q)]^{-1} - [\tilde{S}_F'(p)]^{-1}$$ Ward-Takahashi identity



lower order:

$$\not p + \not q - m - (\not p - m) = \not q = q_\mu \gamma^\mu$$

$$\tilde{\Gamma}^{(3)} = (-ie)\gamma^\mu$$

$q_\mu \to 0$:

non-zero photon insertion.

$$\frac{\partial \tilde{S}_F'^{-1}}{\partial p^\mu} = \Gamma_\mu(p,0,p)$$ Ward Identity

→ Vertex function.

Yang-Mills:

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_0 + \mathcal{L}_{GF} + \mathcal{L}_{FP}$$

- Invariant
- No invariant
- ?

$$\mathcal{L}_{FP} = \partial^\mu \eta_a^* D_\mu^{ab} \eta_b = -\eta_a^* \partial^\mu D_\mu^{ab} \eta_b + \partial K$$

$$D_\mu^{ab} = \delta^{ab}\partial_\mu + g f^{abc} A_\mu^c$$

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}(\partial A)^2$$

Under gauge transformations:

$$\delta A_\mu^a = \partial_\mu \Lambda^a(x) + g f^{abc} A_\mu^b \Lambda^c = (D_\mu \Lambda)^a$$

Parametrize: $\Lambda^a = \lambda \eta^a(x)$ λ = constant → Grassmann

then, $$\delta A_\mu^a \approx -D_\mu^{ab} \eta_b(x) \lambda$$

$$\to \delta \mathcal{L}_{GF} \approx \frac{1}{\xi}(\partial A^a) \cdot \partial^\alpha D_\alpha^{ab} \eta_b \lambda$$

and, $\delta\mathcal{L}_{FP} \simeq -\delta\eta^*_a \partial^\alpha D^{ab}_\alpha \eta_b - \eta^*_a \partial^\mu(\delta D^{ab}_\mu \eta_b)$

then,

Becchi
Rouet
Stora
(BRS)

$$\begin{cases} \delta\eta^*_a = -\frac{1}{\xi}(\partial A_a)\lambda \\ \delta\eta_a = -\frac{1}{2} g f_{abc}\eta_b\eta_c\lambda \longrightarrow \delta D^{ab}_\mu \eta_b = 0 \\ \delta A^a_\mu = -D^{ab}_\mu \eta_b \lambda \\ \delta\psi = -igT_a\eta_a\lambda\psi \end{cases}$$

$\mathcal{L}$ invariant.

moreover, $\int DA_\mu D\psi D\bar\psi D\eta^* D\eta$ is BRS invariant.

then

$$W \propto \int DA_\mu D\psi D\bar\psi D\eta^* D\eta\, e^{i(S+\int dx(\cancel{JA+\bar\sigma\psi+\bar\psi\sigma}))}$$

must be invariant

not invariant terms.

consider

$$I = \int DA_\mu D\eta^* D\eta\, \eta^*_a(x) e^{i(S+\int dx J\cdot A)} = 0$$

BRS $\quad \delta\eta^*_a = -\frac{1}{\xi}(\partial\cdot A_a)\lambda\, ; \quad \delta A^a_\mu = -D^{ab}_\mu \eta_b\lambda$

$$0 = \delta I = \int DA_\mu D\eta^* D\eta \left[\frac{1}{\xi}(\partial\cdot A_a) + iJ^\mu_a \eta^*_a D^{ab}_\mu \eta_b\right] e^{i(S+\int dx J\cdot A)}$$

Slanov-Taylor Identity.

$\hookrightarrow$ Generalization of Ward-Takashi.

We will use this identity for studing $\xi \to \xi + d\xi$ in.

$$W_\xi \propto \int DA_\mu D\eta^* D\eta\, e^{i\int dx(\mathcal{L}_{YM} - 1/2\xi(\partial A)^2 + \mathcal{L}_{FP} + JA)}$$

then, $\Delta W_\xi \equiv W_{\xi+d\xi} - W_\xi$

$$\propto \int DA_\mu Dn^* Dn \int dx \left(\frac{i d\xi}{2\xi^2} (\partial A)^2 \right) e^{i(S+\int dx J\cdot A)}$$

while:

$$\int dx dy \delta(x-y) \partial_{y\mu} \frac{\delta}{\delta J^a_\mu(y)} [\delta I] = 0$$

$$= \int DA_\mu Dn^* Dn \int dx (\partial\cdot A^a) \left[\frac{1}{\xi} (\partial A_a) + i J^\mu_b n^*_a D^{bc}_\mu n_c \right] e^{i(S+\int dx J\cdot A)}$$

then

$$W_{\xi+d\xi} \approx \int DA_\mu Dn^* Dn \left[1 + \frac{d\xi}{2\xi} \int dx (\partial A^a) J^\mu_b n^*_a D^{bc}_\mu n_c \right] e^{i(S+\int dx J\cdot A)}$$

$$W_{\xi+d\xi} \propto \int DA_\mu Dn^* Dn \exp\left[iS + \int dx J^\mu_b \left(A^a_\mu - i \frac{d\xi}{2\xi} (\partial A_a) n^*_a D^{ab}_\mu n_c \right) \right]$$

$\xi \longrightarrow \xi + d\xi$: Just change the coupled field to J.

W Keeps its shape.

- In general, $G^{(n)}$ are modified.
- S_{fi}: Are obtained rescaling the $G^{(n)}$ such that the residue in the poles is unitary.

(S$_{fi}$ cuts the extend propagators)

- Change the coupled field to J, changes the residue in the pole.

Rescaling different when we have to cut the propagators.

$\rightarrow$ S is invariant under $\xi \rightarrow \xi + d\xi$.