Various variables

Let u and v random variables.

$P(u_j, v_k) \longrightarrow$ Joint probability distribution

$$0 \leq P(u_j, v_k) \leq 1 \quad \text{and} \quad \sum_{j,k} P(u_j, v_k) = 1.$$

$P_u(u_j) = \sum_k p(u_j, v_k) \longrightarrow$ Probability of $u = u_j$ no matter the value of v_k.

$$\sum_j P_u(u_j) = \sum_{j,k} P(u_j, v_k) = 1$$

If we have independent variables.

$$P(u_j, v_k) = P_u(u_j)\, P_v(v_j)$$

$$\langle u, v \rangle = \sum_{j,k} u_j v_k\, P(u_j, v_k) = \sum_{j,k} u_j v_k\, P_u(u_j)\, P_v(v_k)$$

$$= \sum_j u_j P_u(u_j) \sum_k v_k P_v(v_k)$$

$$= \langle u \rangle \langle v \rangle.$$

Continuous variables

Let $u \in \{ \mathbb{R} / a < u < b \}$ and $p(u)du \longrightarrow$ Probability of u being between u and $u + du$.

then $\displaystyle\int_a^b p(u)du = 1 \longrightarrow \langle f(u) \rangle = \int_a^b f(u)p(u) \rightarrow$ mean value