Problem Set 2

To be solved before the class of November 4, 2021

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Numbered references to equations and sections refer to the textbook Macroeconomic Theory, by M. Wickens (2^{nd} edition).

Question 1

Consider the centralized economy of section (2.4) and its optimal solution. Show that the steady state level of consumption for this economy is smaller than the golden rule level of consumption.

Question 2

Consider the centralized economy of section (2.4). Suppose that, in a modified version of that model, capital only depreciates every other year, i.e., in periods $\bar{s}$=1,3,5,...

2. a) Set up the Lagrangian for this economy.

2. b) Calculate the derivative of the Lagrangian with respect to k_{t+318}.

Question 3

Consider the example in section (2.4.5.1) with the utility function $U(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ and the production function $y_t = \bar{A}k_t^{\alpha}$. Choose the following parameter values for σ, α, $\bar{A}$, β and δ:
$\sigma = 2$, $\alpha = \frac{1}{2}$, $\bar{A} = 1$, $\beta = 0.95$ and $\delta = 0.1$.

3. a) Calculate the steady state of this model (using the parameters given above).

3. b) Evaluate the following derivatives for this model at their respective steady state levels: $U'(c^*)$, $U''(c^*)$ and $F''(k^*)$, i.e., provide numerical derivatives given the parameter values.

3. c) Suppose the Matrix A in the system

$$\begin{bmatrix} c_{t+1} - c^* \\ k_{t+1} - k^* \end{bmatrix} = A \begin{bmatrix} c_t - c^* \\ k_t - k^* \end{bmatrix}$$

consists of the following numerical elements: $A = \begin{bmatrix} 1.0073 & -0.0078 \\ -1 & 1.0526 \end{bmatrix}$

Suppose that in period 17 consumption is one unit above its steady state level and the capital stock is two units above its steady state level. Calculate the deviations of c_{19} and k_{19} from their respective steady state values.

Comment: In the book the symbol A is used for both the coefficient of total factor productivity in the production function and the the matrix which describes the evolution of deviations from the long-run equilibrium. To avoid confusion we changed the parameter in the production function to $\bar{A}$ for this problem set.

Question 4

Consider the dynamics of the optimal solution as specified by the equations in slide 2 of part 3. Once more, choose the following parameter values: $\sigma = 2$, $\alpha = \frac{1}{2}$, $\bar{A} = 1$, $\beta = 0.95$ and $\delta = 0.1$.

4. a) Express c_{t+1} and k_{t+1} as functions of c_t and k_t.

4. b) Using your smartphone or a pocket calculator and the dynamic relation from (a) calculate 5 periods of the path of consumption and capital for a starting value of $k_1 = 1$ and $c_1 = 1$.

4. c) Using your smartphone or a pocket calculator and the dynamic relation from (a) calculate 3 periods of the path of consumption and capital for a starting value of $k_1 = 1$ and $c_1 = 0.627582223701021$.

4. d) Using MATLAB and the dynamic relation from (a) calculate 30 periods of the path of consumption and capital for a starting value of $k_1 = 1$ and $c_1 = 0.627582223701021$.