Stochastic Processes

Assignment One

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1 Problem 1

1.1 Question a

We can model the different channels as states in a Markov chain, with two absorbing states representing the conversion or non-conversion of a client. This assumes however that the channel each user explores is only decided by the last channel he or she clicked on, which might not be completely realistic.

1.2 Question b

The state space is all of the unique values of the chain existing in our sample. This is easily computed with R, and it appears our state space consists of:

- **Three channels** labeled as `Ch 1`, `Ch 2` and `Ch 3`.
- **Two absorbing states** representing the `Conversion` and `Non-conversion` states.

We found the estimate of the transition probabilities using maximum likelihood, as we have from our notes:

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_{j=1}^{k}} j, i \in S$$

Where n_{ij} is the total number of transitions from state i to state j and the denominator is the total amount of transitions *into* state j. The estimate found for our sample is the following transition matrix:

$$P = \begin{bmatrix} 0.3042 & 0.0792 & 0.1518 & 0.3132 & 0.1515 \\ 0.1660 & 0.1612 & 0.3365 & 0.1601 & 0.1761 \\ 0.1358 & 0.0756 & 0.1459 & 0.2202 & 0.4225 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix},$$

where we have encoded our states as numbers using the following mapping: `Ch 1` is row (and column) 1, `Ch 2` is number 2, `Ch 3` is number 3, `Conversion` is number 4 and `Non-Conversion` is number 5.

1.3 Question c

Our first consumer journey is `Ch 3 -> Ch 2 -> Conversion`, which is equivalent in our numerical mapping to $X_1 = 3, X2 = 2, X3 = 4$. The probability we then have to calculate is $P(X_3 = 4|X_2 = 2, X_1 = 3)$. This can be done as follows:

$$\begin{aligned} &P(X_3 = 4|X_2 = 2, X_1 = 3) = P(X_3 = 4|\ X_2 = 2) * P(X_2 = 2|X_1 = 3) * P(X_1 = 3) \\ &= P_{2,4} * P_{3,2} * (\boldsymbol{\alpha} P)_5 \end{aligned}$$

Assuming our initial distribution $\boldsymbol{\alpha} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0)$[1] the answer is 0.0026.

[1] We are assuming the client cannot start in one of the conversion or non-conversion states and that it is uniform on the rest, which seems reasonable.

1.4 Question d

We can find the limiting distribution for the chain by using the result from equation 3.11 in Dobrow. This allows us to compute the limiting submatrix as $\boldsymbol{(I-Q)^{-1}R}$ for the absorbing states, which is:

$$\boldsymbol{(I-Q)^{-1}R} = \begin{bmatrix} 0.5887 & 0.4113 \\ 0.4650 & 0.5350 \\ 0.3925 & 0.6075 \end{bmatrix}$$

Where the first column represents the limiting distribution of the conversion state and the second column represents the limiting distribution of the non-conversion state. We can then compute the total conversion ratio as the ratio between the sum of column 1 (which is the total proportion of realizations that end in a conversion) divided by the sum of the matrix.

$$\text{Conversion Ratio } = 0.4821.$$

1.5 Question e