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20160164

Lab No. 4

PROBLEMA 1
Metodo de Sustitucion

Demuestren que la solucion dada para cada recurrencia es la correcta utilizando el metodo de sustitucion.

1. $T(n) = T(n-1) + n$ || Resp: $O(n^2)$

Handwritten solution for Problem 1:

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Problema 1

Tarea 4
 $T(n) = T(n-1) + n$
Resp: $O(n^2)$

$T(n-1) + n \leq cn^2 \quad c < 0$

$T(\frac{n}{2}-1) + \frac{n}{2} \leq c(\frac{n^2}{4})$

$T(\frac{n}{2}-1) \leq c(\frac{n^2}{4}) - \frac{n}{2}$

$T(n) \leq 2[c(\frac{n^2}{4}) - \frac{n}{2} + 1]$

$T(n) \leq 2(\frac{cn^2}{4} - \frac{2n}{2} + 2)$

$T(n) \leq \frac{cn^2}{2} - n + 2$

$\leq O(n^2)$

2. $T(n) = T(n/2) + 1$ || Resp: $O(\lg n)$

Handwritten solution for Problem 2:

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Problema 2: $T(n) = T(\frac{n}{2}) + 1$
Resp: $O(\lg n)$

$T(\frac{n}{2}) + 1 \leq c \lg(\frac{n}{2})$

$T(\frac{n}{4}) + 1 \leq c(\lg(\frac{n}{2}))$

$T(n) \leq 4[c(\lg(\frac{n}{2})) - 1]$

$T(n) \leq 4c \lg(n) - 4c \lg(2) - 4$

$T(n) \leq 4c \lg(n) - 4$

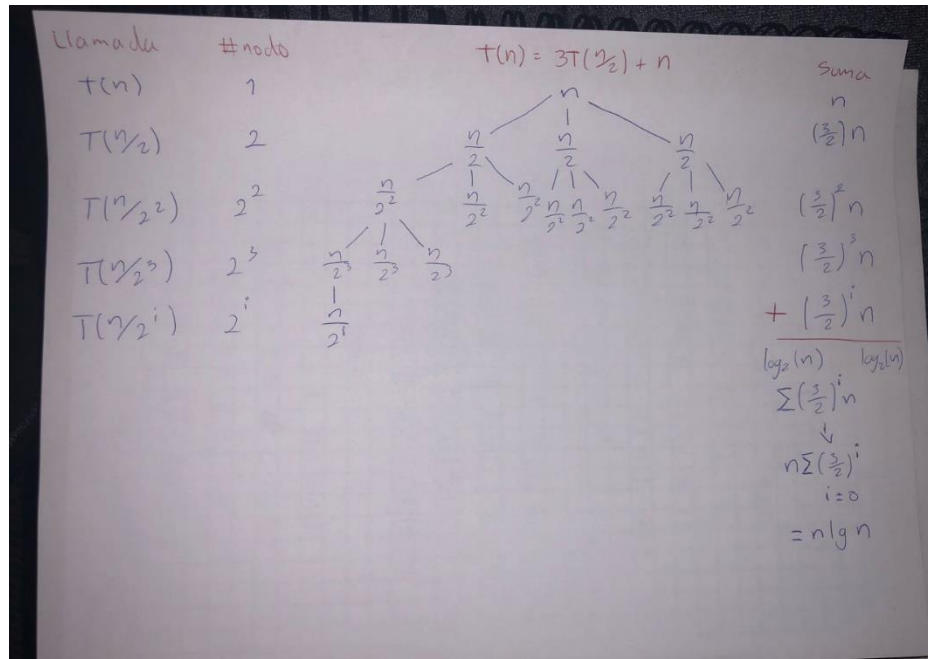
$\leq O(\lg(n))$

PROBLEMA 2

Metodo de Arbol recursivo

Utilicen el metodo de arbol recursivo para encontrar un limite asintotico. Utilicen el metodo de sustitucion para comprobar.

1. $T(n) = 3T(n/2) + n$



Comprobacion problema 2

Comprobacion P#2:

$$\begin{aligned} T(n) &\leq 3\left[\frac{n}{2} \cdot \lg\left(\frac{n}{2}\right)\right] + n \\ &\leq \frac{3}{2}n \cdot \lg\left(\frac{n}{2}\right) + n \\ &\leq \frac{3}{2}n \cdot \lg n - \lg 2 + n \\ &\leq \frac{3n}{2} \cdot \lg(n) + n \\ &\leq \boxed{c n \lg n} \end{aligned}$$

PROBLEMA 3

Metodo Maestro

Encuentren un limite asymptotico para cada problema utilizando el metodo maestro.

1. $T(n) = 2T(n/4) + 1$

Problema 3

1) $T(n) = \underbrace{2}_{a} T(\underbrace{n/4}_{b}) + \underbrace{1}_{f(n)}$

$n^{\log_4 2} = \sqrt{n}$

$n^{\log_4 2} = \sqrt{n} > 1$

$\boxed{N/A}$

2. $T(n) = 2T(n/4) + \sqrt{n}$

2) $T(n) = \underbrace{2}_{a} T(\underbrace{n/4}_{b}) + \underbrace{\sqrt{n}}_{f(n)}$

$n^{\log_4 2} = \sqrt{n}$

$\sqrt{n} = \sqrt{n}$

$f(n) = O(\sqrt{n})$

$f(n) = (n^{\log_4 2} \cdot \log(n))$

$= O(\sqrt{n} \cdot \log(n))$

$\boxed{\text{Caso 2: El running time esta distribuido igualmente a travez del arbol}}$

3. $T(n) = 2T(n/4) + n$

3) $T(n) = \underbrace{2}_{a} T(\underbrace{n/4}_{b}) + \underbrace{n}_{f(n)}$

$n^{\log_4 2} = \sqrt{n}$

$n^{1/2} < n$

$a \cdot (\frac{a}{b}) \leq c f(n)$

$2 \cdot \sqrt{\frac{n}{4}} \leq cn$

$\sqrt{n} \leq cn$

$f(n) = O(n^{\log_4 2})$

$T(n) = O(f(n)) \Rightarrow O(n)$

$\boxed{\text{Caso 3: El running time es dominado por el costo en la raíz.}}$

4. $T(n) = 2T(n/4) + n^2$

$$4) T(n) = \underbrace{2}_a T\left(\underbrace{\frac{n}{4}}_b\right) + \underbrace{n^2}_{f(n)}$$

$$n^{\log_4 2} = \sqrt{n}$$

$$\sqrt{n} = n^2$$

Caso 3: Running time
dominado por el costo
de la raíz

$$a \cdot f\left(\frac{n}{b}\right) \leq c f(n) \quad c < 1$$

$$2 \cdot \sqrt{\frac{n}{4}} \leq c n^2$$

$$\sqrt{n} \leq c n^2$$

$$f(n) = \Omega(n^{\log_4 2})$$

$$f(n) = O(f(n)) = O(n^2)$$