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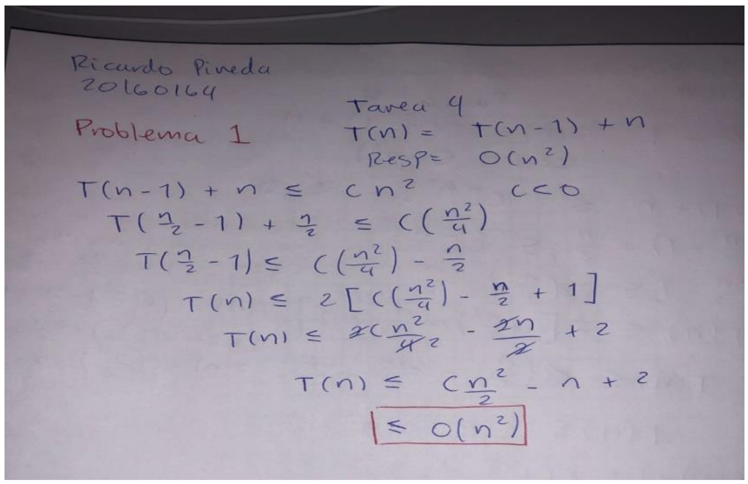
20160164

Tarea # 4

PROBLEMA 1
Metodo de Sustitucion

Demuestren que la solucion dada para cada recurrencia es la correcta utilizando el metodo de sustitutcion.

1. T(n) = T(n-1) + n || Resp: `O(n^2)`



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Tarea 4

Problema 1 $T(n) = T(n-1) + n$

Resp= $O(n^2)$

$T(n-1) + n \leq cn^2 \quad c<0$

$T(\frac{n}{2} - 1) + \frac{1}{2} \leq c(\frac{n^2}{4})$

$T(\frac{n}{2} - 1) \leq c(\frac{n^2}{4}) - \frac{n}{2}$

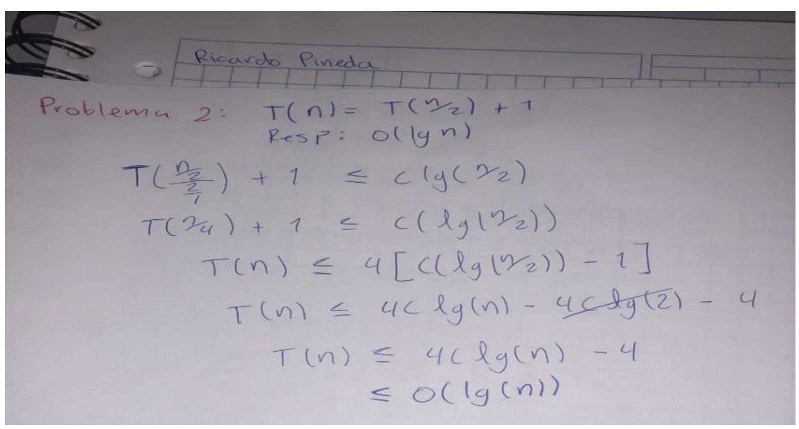
$T(n) \leq 2[c(\frac{n^2}{4}) - \frac{n}{2} + 1]$

$T(n) \leq \cancel{2}c\frac{n^2}{\cancel{4}2} - \frac{\cancel{2}n}{\cancel{2}} + 2$

$T(n) \leq c\frac{n^2}{2} - n + 2$

$\boxed{\leq O(n^2)}$

2. T(n) = T(n/2) + 1 || Resp: `O(lg n)`



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Problema 2: $T(n) = T(n/2) + 1$

Resp: $O(\lg n)$

$T(\frac{n/2}{2}) + 1 \leq c\lg(n/2)$

$T(n/4) + 1 \leq c(\lg(n/2))$

$T(n) \leq 4[c(\lg(n/2)) - 1]$

$T(n) \leq 4c\lg(n) - \cancel{4c\lg(2)} - 4$

$T(n) \leq 4c\lg(n) - 4$

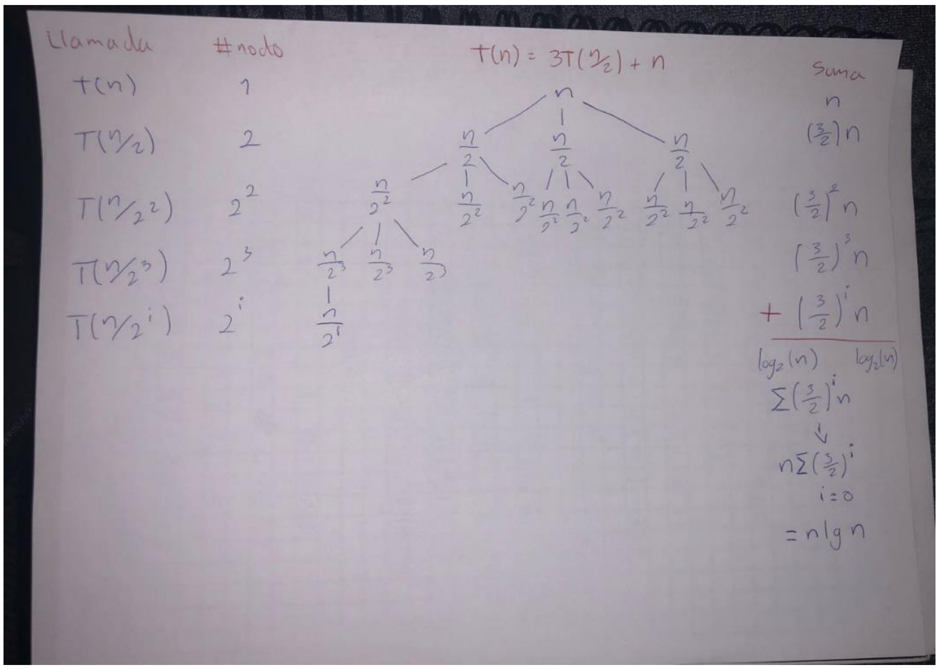
$\leq O(\lg(n))$

PROBLEMA 2

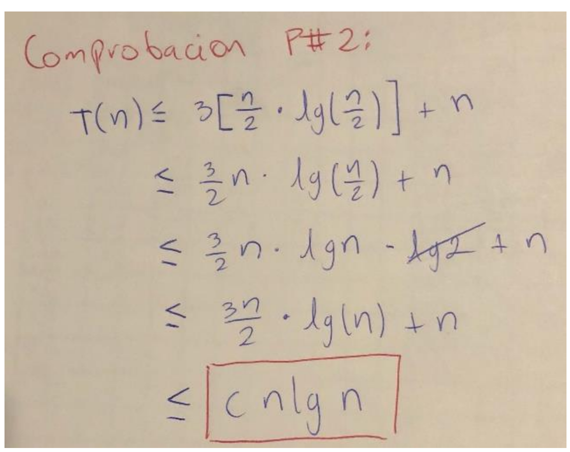
Metodo de Arbol recursivo

Utilicen el metodo de arbol recursivo para encontrar un limite asintotico. Utilicen el metodo de sustitucion para comprobar.

1. T(n) = 3T(n/2) + n



Comprobacion problema 2

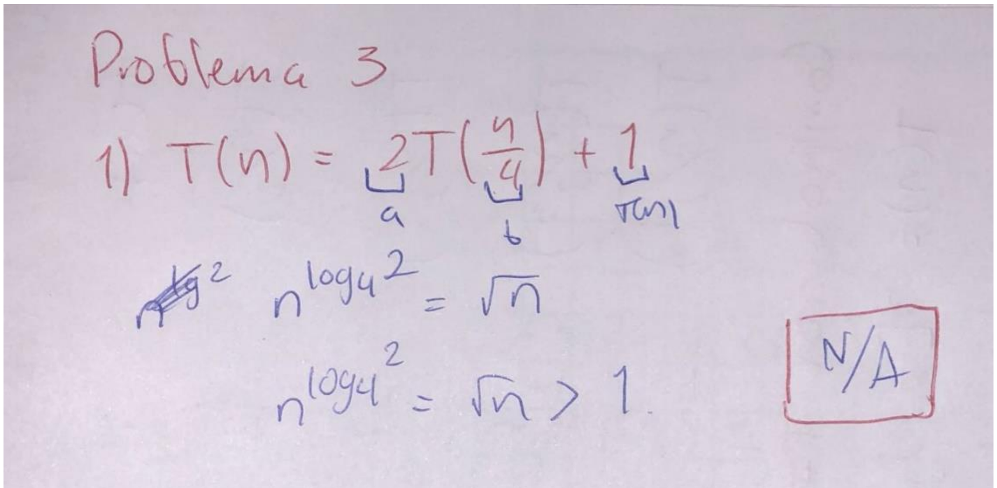


PROBLEMA 3

Metodo Maestro

Encuentren un limite asintotico para cada problema utilizando el metodo maestro.

1. T(n) = 2T(n/4) + 1



Problema 3

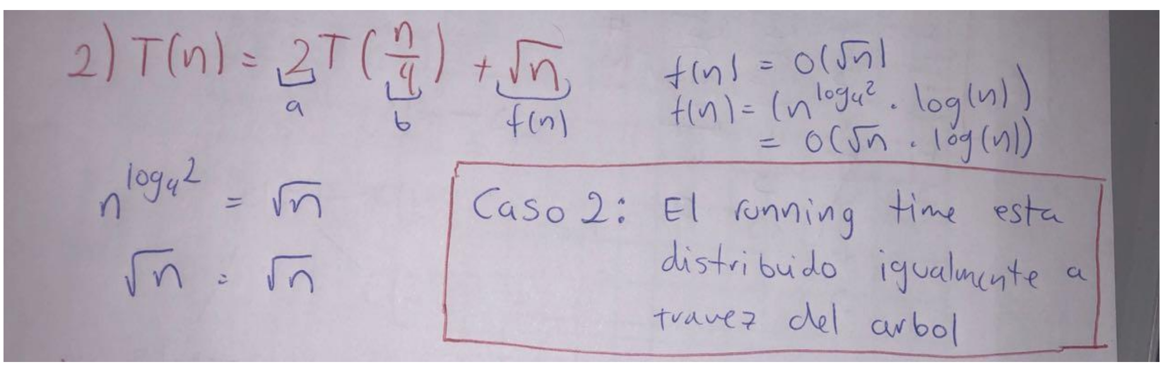
1) $T(n) = \underbrace{2}_{a}T\left(\frac{n}{\underbrace{4}_{b}}\right) + \underbrace{1}_{f(n)}$

$\cancel{n^{\log}}{}^{2}$ $n^{\log_4 2} = \sqrt{n}$

$n^{\log_4 2} = \sqrt{n} > 1$

$\boxed{N/A}$

2. T(n) = 2T(n/4) + sqrt(n)



2) $T(n) = \underbrace{2}_{a}T\left(\frac{n}{\underbrace{4}_{b}}\right) + \underbrace{\sqrt{n}}_{f(n)}$

$f(n) = O(\sqrt{n})$

$f(n) = (n^{\log_4 2} \cdot \log(n))$

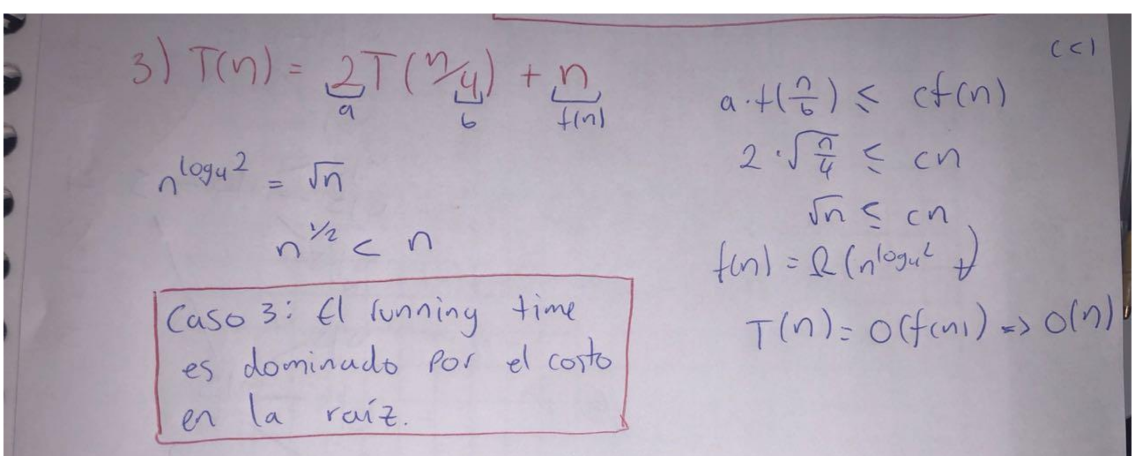
$= O(\sqrt{n} \cdot \log(n))$

$n^{\log_4 2} = \sqrt{n}$

$\sqrt{n} = \sqrt{n}$

Caso 2: El running time esta distribuido igualmente a travez del arbol

3. T(n) = 2T(n/4) + n



3) $T(n) = \underbrace{2}_{a}T(n/\underbrace{4}_{b}) + \underbrace{n}_{f(n)}$

$n^{\log_4 2} = \sqrt{n}$

$n^{1/2} < n$

Caso 3: El running time es dominado por el costo en la raiz.

$(c<1)$

$a \cdot f\left(\frac{n}{b}\right) \le cf(n)$

$2 \cdot \sqrt{\frac{n}{4}} \le cn$

$\sqrt{n} \le cn$

$f(n) = \Omega(n^{\log_4 2} + \epsilon)$

$T(n) = O(f(n)) \Rightarrow O(n)$

4. T(n) = 2T(n/4) + n^2

4) $T(n) = \underbrace{2}_{a}T\left(\underbrace{\frac{n}{4}}_{b}\right) + \underbrace{n^2}_{f(n)}$

$n^{\log_4 2} = \sqrt{n}$

$\sqrt{n} = n^2$

Caso 3: Running time dominado por el costo de la raíz

$a \cdot f\left(\frac{n}{b}\right) \leq cf(n) \qquad c<1$

$2 \cdot \sqrt{\frac{n}{4}} \leq cn^2$

$\sqrt{n} \leq cn^2$

$f(n) = \Omega(n^{\log_4 2})$

$t(n) = o(f(n)) = O(n^2)$