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Innovative Applications of O.R.

Price competition with integrated and decentralized supply chains

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ABSTRACT

We consider price competition with a linear demand function and compare two cases. In the first case each distribution channel is vertically integrated, while in the second, decentralised, case the manufacturers and retailers act independently. We explore the effect of varying the level of price competition on the profits of the industry participants and demonstrate the important role played by the spread of underlying market shares. The coefficient of variation of these market shares determines whether decentralised supply chains can outperform integrated supply chains with an appropriate level of competition.

1. Introduction

In this paper, we investigate the equilibrium behavior of competing supply chains, in which similar products are sold into the same market and compete on price. We study the influence of supply chain structure on the equilibrium solution and hence on industry performance. We are able to give conditions under which decentralized supply chains are more profitable than integrated supply chains, and we investigate the impact of changes in the level of competition.

A key idea in supply chains is that of double marginalization, which describes the way that supply chain members, each taking decisions independently in order to maximize their own profit, will not, in general, operate in a way that maximizes the entire supply chain profit. We can be more specific: they will make decisions on inventory or pricing which end up with a lower volume of sales through the supply chain as a whole than would be optimal if they were cooperating. For example in the decentralized case the retailer will choose a price which is higher than the optimal price for the supply chain as a whole. There has been much attention given to the structure of contracts (e.g. buyback or profit sharing) that eliminate the negative effects of double marginalization.

We are interested in competition between supply chain participants at the same point in their supply chains. We call this horizontal competition to distinguish it from vertical competition between different stages in the same supply chain. Horizontal competition will reduce profits and, in many circumstances, as competition increases supply chain participants tend to set lower prices than they would in order to maximize supply chain profits (and similarly hold more stock than they should). Thus competition has the opposite effect to double marginalization. A number of studies have shown how horizontal competition can benefit the decentralized supply chain by compensating for the effect of double marginalization. When price is exogenous and stocking decisions are being made, Netessine and Zhang (2005) show that the retailer tends to overstock products under horizontal competition and is compensated for understocking due to double marginalization. As we discuss in more detail below, this is paralleled by research that shows that horizontal competition can make a decentralized channel perform better than an integrated channel when pricing decisions are being made.

Research on horizontal supply chain competition has considered two different ways in which competition may occur: in-chain competition and chain-to-chain competition. The former deals with competition between different parties in the same echelon of a single supply chain, e.g. competition between retailers who buy product from the same manufacturer. On the other hand chain-to-chain competition is concerned with competition at the downstream end of two entirely separate supply chains.

Most authors who have considered in-chain competition have focused on inventory theory, and considered competition in the context of product availability. This work is based on single echelon inventory systems studied by Parlar (1988), Lippman and McCardle (1997) and Mahajan and van Ryzin (2001). They model the competition between manufacturers at the same echelon level and assume the pricing decision is exogenous; thus a firm may obtain an advantage by judicious choice of inventory level through being able to meet customer demand. On the other hand, a number of authors have discussed in-chain competition from a multi-echelon viewpoint, e.g. Anupindi and Bassok (1999) consider inventory competition between two retailers who procure from the same manufacturer. Cachon (2003) and Tsay et al. (2004) summarize the literature in this area.

In this paper we will focus on chain-to-chain competition in which different manufacturers sell through exclusive retailers that

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compete for end customers. Thus there is direct competition between the retailers, while the competition between manufacturers is indirect. Our work extends the paper by McGuire and Staelin (1983) (MS) who study a model with two competitors. They point out that this kind of model fits several industries such as soft drink, gasoline and cars, where exclusive dealership is not uncommon. Within each region, each bottler produces only one brand of cola, different car dealers carry only one brand of car (or cars from a single manufacturer), and petrol stations sells only one brand of petrol.

We will concentrate on competition through price; thus we are concerned with a type of competitive substitutability, in which supply chain *i*s retail price influences supply chain *j*s demand. This is often modeled through the cross-price parameter in a demand curve. Generally, we suppose a supply chain's demand is decreasing with the supply chain's own price and increasing with the competitors' price. Milgrom and Roberts (1990) argue that, in addition, the logarithm of the demand function needs to have increasing differences.[1] They show that a linear demand function, $d_i(p) = \alpha_i - \beta_i p_i + \sum_{j\neq i}\beta_{ij}p_j$, with$\alpha_i > 0$, $\beta_i, \beta_{ij} \geqslant 0 \forall i, j\neq i$, will satisfy these properties (as will Logit, Cobb-Douglas and Constant Elasticity of Substitution). The linear demand function is commonly used because it is tractable and enables closed-form solutions. For example Dixit (1979) use a linear demand function to analyze the problem of entry barriers in a duopoly setting. Also Yano (2005) uses the linear demand function to study the impact of capacity constraints in a two-supplier one-retailer supply chain.

MS use the linear demand function and examine various supply chain structures in the context of two competing manufacturers, each selling his products through an exclusive retailer who determines the retail price. They show that when the level of competitive substitutability is high, the total channel profits (or industry profits) are greater when manufacturers use decentralized distribution systems (i.e. a decentralized supply chain). Following MS, a number of other papers have explored the circumstances in which these results hold such as Coughlan (1985) and Moorthy (1988); while Choi (1991) has considered an extensions of this model with one retailer selling both products in which the retailer may exercise some power in the relationship with the manufacturer.

The use of a linear demand function is not without problems: Ingene and Pappy (2004) have criticised this assumption on the grounds that it makes an increase in competitive substitutability lead to an increase in aggregate demand. We look again at the circumstances when this form of demand function may be appropriate, and formulate a model which is subtly different to that of MS. Most of the existing literature on chain-to-chain competition assumes that there are only two chains; we extend these studies and model the competition between an arbitrary number of supply chains.

Our main result addresses the question of when decentralised supply chains outperform integrated chains. We are considering a situation in which there are no additional production or transaction costs that arise as a result of the decentralised environment. The question of when firms will choose to vertically integrate is a complex one and has been addressed from a number of perspectives. A key insight of Williamson (1991) is that the degree of asset specificity is central to this decision, with purchase in a market place being preferred when asset specificity is low. Other issues that may influence a company towards vertical integration are the difficulties arising from incomplete contracts, and the possibility of leakage of intellectual property in dealing with a separate company. There is a large literature in this area of transaction cost economics; see for example the books Williamson (1975), Williamson (1985) and the survey by Grover and Malhotra (2003). Our analysis focusses on coordination and competition issues and will not treat these wider concerns directly.

The rest of the paper is organized as follows: In Section 2, we will formulate the supply chain competition model. An example of competition between two supply chains is discussed in Section 3 to give a general idea of the problem. Section 4 presents the results where *n* supply chains are competing.

2. The model

We assume there are *n* supply chains competing in the market with substitutable products. The supply chains are either integrated, i.e. manufacturers sell their own product (perhaps through an in-house outlet), or they are decentralized where manufacturers sell their products to the market through retailers. In this paper, we look at the situation in which the competing supply chains all have the same structure. That is, we consider the case in which integrated chains only compete with integrated chains, and we also consider the case in which decentralized chains compete with decentralized chains.

Our model applies to a mature industry in which the total potential number of customers is constant, and there is a given split between the products in the market. Moreover, no manufacturer has a technology advantage, and so the manufacturing cost *c* is the same for every manufacturer. We will investigate the effect of price competition at the retail level.

We will assume that the demand for product *i* as a function of the retail prices $p_i, i = 1, 2, \dots, n$, is given by

$$q_i = \alpha k_i - \delta_i p_i + \gamma \sum_{j=1, j\neq i}^{n} (p_j - p_i), \quad i = 1, 2, \dots, n, \tag{1}$$

where $\sum_{i=1}^{n} k_i = 1$. In this model, we can think of α as a parameter giving the total potential market size (if prices were all 0) and k_i can be thought of as product *i*'s underlying market share; thus αk_i is the demand for product *i* if all prices are 0. Since we do not have negative demand we assume

$$\alpha k_i - \delta_i p_i > \gamma \sum_{j=1, j\neq i}^{n} (p_i - p_j).$$

When a supply chain reduces the price for its product, demand will be increased. We suppose that there are two types of customers who can be gained: switching customers and marginal customers. A switching customer will definitely buy one of the competing products, but is price sensitive and will look at the combination of product attributes and price to make a purchase decision. Marginal customers, on the other hand, will buy one of the competing products only if the price is below a certain level. Both types of customers are included in demand model (1). In the model, the term in γ represents a "leakage" of the demand from one supply chain to the other, corresponding to switching customers. Clearly, the total demand is not affected by γ since $\sum_{i=1}^{n} q_i = \alpha - \sum_{i=1}^{n} p_i$. The term in δ_i corresponds to the marginal customers who can be attracted by a reduced price. In considering product *i*s demand sensitivity to its own price, p_i, note that a unit of price reduction increases demand by $(\delta_i + \gamma)$ corresponding to the total of marginal and switching customers.

The fact that there are only a fixed number of switching customers makes it likely that the model we give will be a poor representation of customer demand for extreme price values. For example, consider a large value of γ which corresponds to switching customers who are very sensitive to differences between

[1] A function $f(x_1, \dots, x_n)$ has increasing differences in (x_i, x_j) if $f(x_1, \dots, x_i^1, \dots, x_n) - f(x_1, \dots, x_i^2, \dots, x_n)$ is increasing in x_j, for all $x_i^2 < x_i^1$. If the function *f* is twice differentiable, the property is equivalent to $\partial^2 f / \partial x_i \partial x_j \geqslant 0$.

prices. In this case the demand function is likely to be a poor model for a low-priced product for which almost all the potential switching customers have already been won. However this drawback may not be severe. Our attention will be focused on equilibrium solutions in a stable market and it is the behavior near the equilibrium which is important. We can think of taking a linearization of a non-linear demand function near the equilibrium point.

In this model, we will use a single parameter γ to capture competitive effects rather than a different γ_{ij} for each product pair (i,j) (with $\gamma_{ij} = \gamma_{ji}$). This is for convenience since we will be considering changes in overall competition levels with many different products.

To focus on the effects of competition, we suppose that all model parameters are fixed and common knowledge. The order in which decisions are taken is as follows:

- For the integrated supply chain, manufacturers simultaneously decide the retail prices that maximize their own profits.
- For the decentralized supply chain, manufacturers act as Stackelberg leaders; they decide the wholesale price w_i that maximizes their profits, given the responses of retailers. For any given wholesale price, retailers, acting as followers, choose retail prices to maximize their profits.

We assume $p_i \geqslant w_i \geqslant c \geqslant 0$ for $i = 1, \dots n$, to avoid trivial problems. When prices are set at c (as low as they can be if firms are profitable), we will assume that demand for all products is positive: hence we assume that $\alpha k_i > \delta_i c$. Summing gives

$$\alpha > c \sum_i \delta_i. \tag{2}$$

3. Two competing supply chains

In this section, we discuss the simplest case where there are only two competing products. With only two competitors, the demand function (1) can be simplified to

$$q_i = \alpha k_i - \delta_i p_i + \gamma(p_j - p_i). \tag{3}$$

With just two products we can explore the behavior of an example in detail. We will show that with sufficiently intense competition, decentralized supply chains perform better than integrated supply chains. We will defer the proofs of all results to the next section, in which we deal with an arbitrary number of supply chains.

We will discuss the equilibrium solutions for integrated supply chains and decentralized supply chains separately in the following sections.

3.1. Two competing integrated supply chains

For an integrated supply chain, the manufacturer produces the product with a unit cost c and sells it at retail price p_i. Thus the profit for manufacturer i is

$$\Pi_i^I = (p_i - c)(\alpha k_i - \delta_i p_i + \gamma(p_j - p_i)).$$

Differentiating this and solving for equilibrium prices for both products simultaneously, we have

$$\hat{p}_i^I = \frac{(2\delta_j + \gamma)\alpha k_i + \alpha\gamma - (2\delta_0 - \gamma\delta_j)c}{4\delta_0 + 3\gamma^2} + c,$$

where

$$\delta_0 = \delta_i\delta_j + \gamma(\delta_i + \delta_j). \tag{4}$$

With the equilibrium prices, we can obtain the equilibrium demand $\hat{q}_i^I$, and profits $\hat{\Pi}_i^I$:

$$\hat{q}_i^I = \frac{(2\delta_j + \gamma)\alpha k_i + \alpha\gamma - (2\delta_0 - \gamma\delta_j)c}{4\delta_0 + 3\gamma^2}(\delta_i + \gamma),$$

$$\hat{\Pi}_i^I = \left[\frac{(2\delta_j + \gamma)\alpha k_i + \alpha\gamma - (2\delta_0 - \gamma\delta_j)c}{4\delta_0 + 3\gamma^2}\right]^2(\delta_i + \gamma).$$

We write $\hat{\Pi}^I$ for the total industry profits, $\hat{\Pi}^I = \hat{\Pi}_1^I + \hat{\Pi}_2^I$.

3.2. Two competing decentralized supply chains

In a decentralized supply chain, manufacturer i who has a unit production cost c, sells the product to his exclusive retailer i at wholesale price w_i. Given the wholesale price, the retailer then decides a retail price which maximizes its profits. We look for an equilibrium choice of retail prices. The manufacturer can predict the retailers' pricing decisions for any given wholesale price, and thus can optimize its own profits.

With the demand function given by (3), the manufacturers' and retailers' profits Π_i^M, and Π_i^R are

$$\Pi_i^M = (w_i - c)q_i,$$
$$\Pi_i^R = (p_i - w_i)q_i.$$

Solving for the equilibrium we can get the optimal solution for the retail price $\hat{p}_i^D$, wholesale price $\hat{w}_i$, demand $\hat{q}_i^D$, manufacturer's profits $\hat{\Pi}_i^M$, retailer's profits $\hat{\Pi}_i^R$:

$$\hat{w}_i = R_i(4\delta_0 + 3\gamma^2) + c,$$
$$\hat{p}_i^D = R_i(6\delta_0 + 4\gamma^2) + c,$$
$$\hat{q}_i^D = R_i(2\delta_0 + \gamma^2)(\delta_i + \gamma),$$
$$\hat{\Pi}_i^M = R_i^2(4\delta_0 + 3\gamma^2)(2\delta_0 + \gamma^2)(\delta_i + \gamma),$$
$$\hat{\Pi}_i^R = R_i^2(2\delta_0 + \gamma^2)^2(\delta_i + \gamma),$$

where

$$R_i = \frac{\Delta_{2j}\alpha k_i + \Delta_3\alpha - \Delta_{4i}c}{\Delta_1}, \tag{5a}$$
$$\Delta_1 = (16\delta_0^2 + 15\delta_0\gamma^2 + 3\gamma^4)(4\delta_0 + 3\gamma^2), \tag{5b}$$
$$\Delta_{2i} = 8\delta_i\delta_0 + 2\delta_0\gamma + 5\delta_i\gamma^2 + \gamma^3, \tag{5c}$$
$$\Delta_3 = 6\delta_0\gamma + 4\gamma^3, \tag{5d}$$
$$\Delta_{4i} = 8(\delta_i\delta_j)^2 + 2\delta_i\delta_j(8\delta_i + 7\delta_j)\gamma + (\delta_i + 2\delta_j)(8\delta_i + 3\delta_j)\gamma^2 + (5\delta_i + 4\delta_j)\gamma^3 \tag{5e}$$

and δ_0 is defined in (4).

We can also calculate $\hat{\Pi}_i^D$, the combined profit of manufacturer and retailer in a single supply chain

$$\hat{\Pi}_i^D = R_i^2(6\delta_0 + 4\gamma^2)(2\delta_0 + \gamma^2)(\delta_i + \gamma).$$

Again we write $\hat{\Pi}^D = \hat{\Pi}_1^D + \hat{\Pi}_2^D$ for the total industry profits.

3.2.1. Numerical analysis

In this section, we will explore a numerical example. We choose the following parameter values to illustrate the equilibrium solutions' sensitivity to competition intensity γ:

$$a = 250, \quad k_1 = 0.45, \quad \delta_1 = 1,$$
$$c = 10, \quad k_2 = 0.55, \quad \delta_2 = 1.$$

With these numerical values, we can calculate the supply chain profits and industry profits as a function of competition intensity γ.

First we consider the integrated supply chains and Fig. 1 shows how the profits of the two chains change as competition intensity increases from 0 to 25. The solid line represents the profit of the large supply chain ($k_2 = 0.55$) and the dashed line is the small chain's profit. We observe that the large supply chain always has a higher profit than the small chain, but the difference decreases as competition intensity increases, with both profits tending to 0

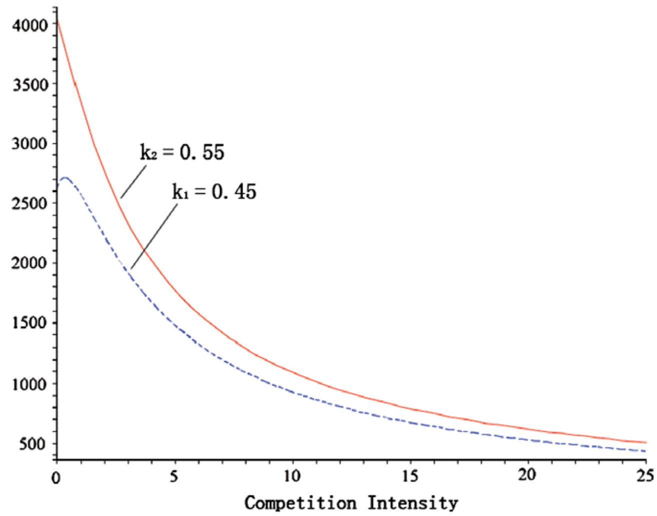

Fig. 1. Profits of integrated chains.

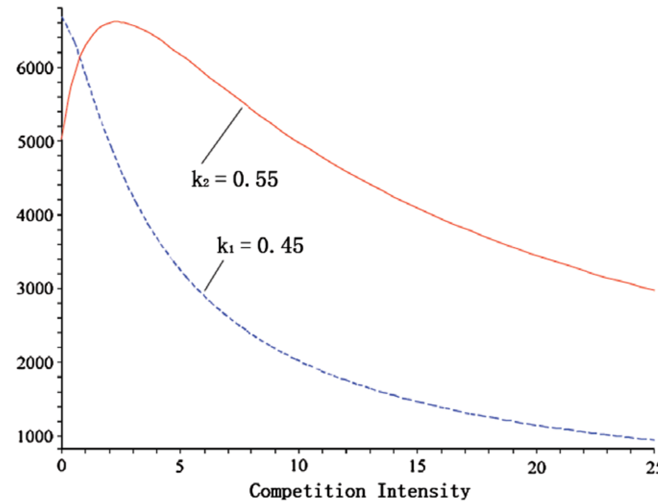

Fig. 3. Industry profits.

as competition intensity becomes large. Notice that the maximum profit for the smaller chain does not occur at $\gamma = 0$, so that this supply chain makes more profit with limited competition than it does without any competition. The introduction of a small amount of horizontal competition increases the smaller chain's profits, though the large chain profit decreases.

For decentralized supply chains, we can plot a similar figure to observe what happens as competition intensity increases. The result is shown in Fig. 2 where the solid line represents the large chain and the dashed line is the small chain. In the figure, manufacturers' profits are shown in thin lines and bold lines represent supply chain profits. As in the integrated case the large chain earns more profit than the small chain and both chain's profits tend to 0 as horizontal competition intensity gets large. Furthermore, from Fig. 2, we see that the introduction of a limited amount of horizontal competition increases both chains' profits. For decentralized supply chains, an appropriate level of horizontal competition can benefit everyone. In each supply chain, a manufacturer's profit shows the same trend as the supply chain profit does.

Fig. 3 compares the industry profits with integrated and decentralized supply chains as competition intensity increases. The solid and dashed lines show the industry profits for the decentralized and integrated supply chains, respectively. The industry profit with integrated supply chains is strictly decreasing as competition intensity increases; so when two integrated supply chains compete, industry profit is maximized if there is no horizontal competition. On the other hand with decentralized supply chains the industry profit starts by increasing as competition emerges; but then at a certain level of competition intensity the industry profit begins to decrease. In this example we can compare industry profits for the integrated and decentralized supply chains, and observe that decentralized supply chains give an advantage when competition is intense. This observation is consistent with the conclusion drawn by McGuire and Staelin (1983) that manufacturers are better off by using a decentralized distribution system when products are highly substitutable.

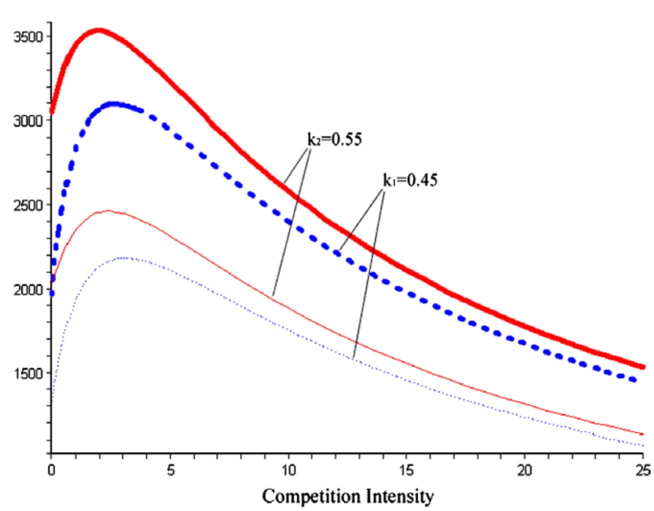

Fig. 2. Profits of decentralized chains.

4. *n* Competing supply chains

Up to this point we have just made observations on the basis of a single numerical example. Our aim in this section is to establish the circumstances in which these observations hold in the more general context of n competing supply chains. We will show how supply chain profits respond to the introduction of competition.

We will consider the competition between similar substitutable products, and it is convenient to assume

$$\delta_i = \delta, \quad i = 1, 2, \ldots, n,$$

so that each product has the same sensitivity to its own price. This will be helpful in working through the algebra we require. Thus, after normalizing units of demand we can use the following simple demand model:

$$q_i = \alpha k_i - p_i + \gamma \left(\sum_{j \neq i} p_j - (n-1) p_i \right). \tag{6}$$

Now suppose there are n supply chains competing in the market, each with a market share k_i for $1 \leqslant i \leqslant n$, with $\sum_{i=1}^{n} k_i = 1$. As in Section 3, we consider two cases: either all the supply chains are integrated or all the supply chains are decentralized. We will first give the equilibrium solutions for integrated and decentralized competing supply chains separately, and then compare the industry profits (the sum of all the competing supply chains' profits). All proofs are provided in the Appendix.

First we consider competition between n integrated supply chains, in which each manufacturer distributes his product direct to the market. This is a game between n players who simultaneously decide the retail price that maximizes their profits given their competitors' retail prices. Lemma 4.1 gives the equilibrium solution.

Lemma 4.1. *The equilibrium solution for n competing integrated supply chains is given by*

$$\hat{p}_i^I = \frac{\alpha k_i B_1 + \alpha\gamma - B_2 c}{B_1 B_2} + c, \tag{7}$$

$$\hat{q}_i^I = \frac{(\alpha k_i B_1 + \alpha\gamma - B_2 c)}{B_1 B_2}(B_1 - 1), \tag{8}$$

$$\hat{\Pi}_i^I = \left(\frac{\alpha k_i B_1 + \alpha\gamma - B_2 c}{B_1 B_2}\right)^2 (B_1 - 1), \quad i = 1, 2, \ldots, n, \tag{9}$$

$$\hat{\Pi}^I = \left(\frac{B_1 - 1}{B_1^2 B_2^2}\right)\left(\alpha^2 B_1^2 \frac{V^2 + 1}{n} + 2\alpha B_1(\alpha\gamma - B_2 c) + n(\alpha\gamma - B_2 c)^2\right), \tag{10}$$

where

$$B_1 = 2 + (n-1)\gamma, \tag{11}$$
$$B_2 = 2 + (2n-1)\gamma \tag{12}$$

and $V = (n\sum k_i^2 - 1)^{1/2}$ *is the coefficient of variation for* $\{k_1, k_2, \cdots k_n\}$

We will continue to assume that $\alpha k_i \geqslant c$, and hence that $\alpha \geqslant nc$; this is enough to show that the demand is positive for each i. Similar conclusions can be drawn for the decentralized case.

From the equilibrium solution we can observe that the profit of supply chain depends on its market share k_i as well as the number of competitors. It is interesting that the industry profits are increasing with V, the coefficient of variation of the underlying market shares for the competing products. An imbalance between market shares gives larger industry profits.

Lemma 4.2 establishes what happens to profits when competition increases. Introducing a low level of competition will benefit the products with a smaller than average underlying proportion of demand, but as competition increases further all the supply chains begin to do worse, and in the limit all profits are driven to zero. Moreover the initial benefits for products with small market share are outweighed by losses for products with larger market share, so even a small amount of competition does not benefit the industry as a whole.

Lemma 4.2. *With integrated supply chains:*

1. *The introduction of horizontal competition will increase the profit of product i,* $i = 1, 2, \ldots, n$*, if and only if*
$$k_i < \frac{1}{n}. \tag{13}$$
2. *The introduction of horizontal competition will decrease the industry profits.*
3. *When horizontal competition intensity approaches infinity, the profits for each supply chain (and hence the industry as a whole) will decrease to zero.*

Now we turn to the case of competition between n decentralized supply chains. Each manufacturer first decides the wholesale price, and then retailers, when they know the wholesale price, make decisions about the retail prices that maximize their profits. Thus there are a total of $2n$ decision makers in the system. The equilibrium solution for this model is given in the following lemma.

Lemma 4.3. *The equilibrium solutions for n competing decentralized supply chains is*

$$\hat{w}_i^D = A_i B_1 B_2 + c, \tag{14}$$
$$\hat{p}_i^D = A_i B_0 + c, \tag{15}$$
$$\hat{q}_i^D = A_i(B_1 - 1)(B_3 - B_2), \tag{16}$$
$$\hat{\Pi}_i^M = (A_i)^2 B_1 B_2 (B_1 - 1)(B_3 - B_2), \tag{17}$$
$$\hat{\Pi}_i^R = (A_i)^2 (B_3 - B_2)^2 (B_1 - 1), \tag{18}$$
$$\hat{\Pi}_i^D = (A_i)^2 B_0 (B_3 - B_2)(B_1 - 1), \quad i = 1, 2, \ldots, n, \tag{19}$$
$$\hat{\Pi}^D = B_0(B_3 - B_2)(B_1 - 1)\sum_{i=1}^{n}(A_i)^2 \tag{20}$$

with

$$A_i = \frac{\alpha k_i B_1 B_3 + \alpha\gamma B_0 - c B_2 B_4}{B_1 B_2 B_3 B_4}, \tag{21}$$
$$B_0 = (3n-2)(n-1)\gamma^2 + 3(3n-2)\gamma + 6, \tag{22}$$
$$B_3 = (n-1)^2\gamma^2 + (5n-3)\gamma + 4, \tag{23}$$
$$B_4 = (2n-1)(n-1)\gamma^2 + 3(2n-1)\gamma + 4 \tag{24}$$

and B_1, B_2 *are defined in (11) and (12).*

The following lemma gives some properties of the equilibrium solution for decentralized chain competition.

Lemma 4.4. *For decentralized supply chains:*

1. *The introduction of horizontal competition will increase the profit of manufacturer i and retailer i, if and only if*
$$k_i < \frac{3\alpha - (n-1)c}{\alpha(1+2n)}. \tag{25}$$
2. *The introduction of horizontal competition will increase the industry profits if and only if*
$$V^2 < \frac{(\alpha - cn)^2(n-1)}{\alpha^2(2n+1)}, \tag{26}$$
where V is the Coefficient of Variation for $\{k_1, k_2, \ldots k_n\}$.
3. *Each participant's profits will decrease to zero as horizontal competition intensity tends to infinity.*

Similar to Lemma 4.2, this result describes the change of profits when competition increases. Introducing a low level of competition benefits every member of the supply chain if that chain's underlying market share satisfies (25). Note that

$$\frac{3\alpha - (n-1)c}{\alpha(1+2n)} > \frac{3 - (n-1)/n}{1+2n} = \frac{1}{n},$$

using (2). Thus condition (25) is weaker than (13) and it is not necessary for a supply chain's market share to be below the average in order for there to be benefits from horizontal competition. For the case of two chain competition, condition (25) is equivalent to

$$k_i < \frac{3\alpha - c}{5\alpha}$$

and in the case of the numerical example given in Section 3 this takes the value 0.592. Since the market shares in the example are both less than 0.592, both supply chains' profits increase when γ starts to increase from 0.

The second part of Lemma 4.4 shows that when the market shares are concentrated around $1/n$, the introduction of horizontal competition will increase the industry profits of the decentralized supply chain system. For two-chain competition, the condition is that $V^2 < (1 - 2c/\alpha)^2/5$.

When horizontal competition gets intense and product substitutability is high, the demand is extremely sensitive to the difference of each chain's retail price. So a retailer has a strong incentive to set the retail price lower than its rivals. Thus the equilibrium retail price decreases with increasing competition. It turns out that the supplier will decrease the wholesale price at the same time. In the end, the entire supply chain's profits shrink as both the retail and wholesale prices approach the unit cost c, as reflected in part 3 of Lemma 4.4.

Up to this point, we have dealt separately with the equilibrium solutions for the two cases of integrated and decentralized supply chain competition. Our final result compares the industry profits between these two different supply chain structures. We establish that industry profits are improved with decentralized supply chains provided that the distribution of the underlying market shares, k_i, is not widely dispersed.

Theorem 1. *When horizontal competition is sufficiently intense, the industry profits with decentralized supply chains is larger than that with integrated chains if and only if*

$$V^2 < \frac{(2n-1)^5}{(n^2+n-1)(n-1)^3}\left(1-\frac{nc}{\alpha}\right)^2.$$

5. Discussion

This paper focusses on the influence of horizontal competition on the performance of supply chains. Our model is built upon the common assumption of a linear demand function, but we give an interpretation in terms of switching customers and marginal customers, which leads to a slightly different form than has been used by others. We look at the problem both from the perspective of a specific player in a supply chain or an individual supply chain, and also from the broader perspective of industry performance, i.e. the total profit of all the supply chains.

One contribution of this paper is that our analysis is able to deal with competition between an arbitrary number of supply chains. Previous research has mainly focused on the case with two competitors, and we demonstrate that results in this area can be generalized to the n-chain case.

Our analysis shows that the smaller supply chain will benefit from horizontal competition if the competition intensity is small, in both integrated and decentralized cases. We also show that, although intense horizontal competition reduces industry profits, an appropriate level of horizontal competition can lead to an increase in industry profits if the underlying market shares of the supply chains are not widely dispersed (as measured by the coefficient of variation of the market shares). The coefficient of variation of the underlying market shares also plays a role in determining the relative industry profitability of decentralised and integrated supply chains. With intense competition, a decentralized arrangement is preferable provided that market shares are similar – as measured by the coefficient of variation.

One limitation of our model is the assumption that the demand function is linear. Another limitation is our assumption that the competing supply chains all have the same structure (either integrated or decentralized). Further work is desirable to test whether our conclusions extend to other forms of demand function.

This work can be seen as addressing the issue of whether the benefits of coordination achieved by manufacturers buying their downstream retailers are greater than the losses incurred through entering into more direct competition with other manufacturers. The results suggest that there will be no advantage in manufacturers buying downstream retailers when the market shares of different products are similar. In this context it would be interesting to explore what happens when some, but not all, of the supply chains are integrated; in order to look at the incentives for an individual manufacturer to vertically integrate in an industry with mixed structure.

Appendix. Proofs

Proof of Lemma 4.1. For n integrated chains, the supply chain profit is $\Pi_i^I = (p_i - c)q_i$ where q_i is given by the expression (6). For the first order conditions, we differentiate this over price p_i^I: we have

$$\begin{aligned}\frac{\partial \Pi_i^I}{\partial p_i} &= q_i + (p_i - c)\frac{\partial q_i}{\partial p_i} \\ &= \alpha k_i - 2(1+\gamma n-\gamma)p_i + \gamma\sum_{j\neq i} p_j + c(1+\gamma n-\gamma).\end{aligned} \tag{27}$$

Notice that the second order derivatives are

$$\frac{\partial^2 \Pi_i^I}{\partial p_i^2} = -2(1+\gamma(n-1)) < 0$$

and we can conclude there is a unique optimal solution which we write as $\hat{p}_i$. Thus it will be enough to show that the solution given in the lemma statement satisfies the first order conditions.

From (7) we can calculate

$$\sum_{j\neq i}\hat{p}_j^I = \frac{\alpha(1-k_i)B_1 + \alpha\gamma(n-1) - B_2(n-1)c}{B_1B_2} + (n-1)c,$$

where B_1, B_2 are defined in (11) and (12). Substituting $\hat{p}_i^I$, from (7), and $\sum_{j\neq i}\hat{p}_j^I$ into (27) we have

$$\left.\frac{\partial \Pi_i^I}{\partial p_i}\right|_{p_i=\hat{p}_i^I} = \frac{1}{B_1B_2}[\alpha k_iB_1B_2 - 2(1+\gamma m)W + \gamma X + c(1+\gamma m)B_1B_2],$$

where $W = \alpha k_iB_1 + \alpha\gamma + cB_2(B_1-1)$, $X = \alpha(1-k_i)B_1 + \alpha\gamma m + mcB_2(B_1-1)$ and $m = n-1$. After some algebra we can show that this is zero, as required. The other statements of the lemma follow immediately. □

Proof of Lemma 4.2. We can substitute for B_1 and B_2 in the integrated chain profit (9) to obtain

$$\hat{\Pi}_i^I = \frac{(m\gamma+1)(\alpha\gamma - c\gamma - 2c - 2cm\gamma + 2\alpha k_i + m\alpha\gamma k_i)^2}{(m\gamma+2)^2(\gamma+2m\gamma+2)^2}, \tag{28}$$

where again we write $m = n-1$. Then differentiating with respect to γ, and evaluating this at $\gamma = 0$, we can show that, for each i,

$$\left.\frac{\partial \hat{\Pi}_i^I}{\partial \gamma}\right|_{\gamma=0} = -\frac{1}{4}\alpha(\alpha k_i - c)(nk_i - 1). \tag{29}$$

Since $\alpha k_i - c > 0$, we can conclude that $\partial\hat{\Pi}_i^I/\partial\gamma|_{\gamma=0} > 0$ if and only if $k_i < 1/n$, giving the first part of the lemma.

For the second part, we add the expressions (29) to evaluate the derivative of industry profits at $\gamma = 0$. Using the coefficient of variation we can write this as

$$\left.\frac{\partial \hat{\Pi}^I}{\partial \gamma}\right|_{\gamma=0} = -\frac{1}{4}\alpha^2V^2.$$

Since the right hand side is less than 0, this means the introduction of horizontal competition always decreases the integrated chains' industry profits.

For the last part, it is straightforward from (28) to see that $\lim_{\gamma=\infty}\hat{\Pi}_i^I = 0$, $i = 1,2,\ldots,n$. From this it follows that $\lim_{\gamma=\infty}\hat{\Pi}^I = 0$. □

Proof of Lemma 4.3. For decentralized chains, the manufacturer and retailer's profit functions are

$$\begin{aligned}\Pi_i^M &= (w_i - c)q_i, \\ \Pi_i^R &= (p_i - w_i)q_i,\end{aligned} \tag{30}$$

where q_i is given by (6).

We begin with the retailer's problem. Retailer i needs to decide the optimal retail price for a given wholesale price w_i. For the first order conditions we have (from (30))

$$\frac{\partial \Pi_i^R}{\partial p_i} = q_i + (p_i - w_i)\frac{\partial q_i}{\partial p_i} = \alpha k_i - 2(1+\gamma n - \gamma)p_i + \gamma \sum_{j\neq i} p_j + w_i(1+\gamma n - \gamma) = 0 \quad (31)$$

Notice that

$$\frac{\partial^2 \Pi_i^R}{\partial p_i^2} = -2(1+\gamma n - \gamma) < 0,$$

so there is a unique optimal retail price for any given wholesale price w_i. Thus we can use the first order conditions above to find the optimal retail price as a function of w_i.

$$p_i^D(w_i) = \frac{\alpha k_i + \gamma \sum_{j\neq i} p_j^D(w_i)}{2(1+\gamma n - \gamma)} + \frac{w_i}{2}. \quad (32)$$

A simple manipulation of (32) gives

$$p_i^D(w_i) = \frac{\alpha k_i + \gamma \sum p_j^D(w_i) + (B_1 - 1)w_i}{B_2}. \quad (33)$$

Summing (33) over i, we and rearranging gives

$$\sum p_i^D(w_i) = \frac{1}{B_1}\left[\alpha + (B_1 - 1)\sum w_i\right]. \quad (34)$$

Substituting (34) into (32) gives the optimal price as a function of wholesale price w_i

$$p_i^D(w_i) = \frac{\alpha k_i B_1 + \alpha\gamma + (B_1 - 1)(\gamma \sum w_i + B_1 w_i)}{B_1 B_2}. \quad (35)$$

We combine (35) and (6) to get, after some algebra, the following expression for the demands $q_i^D(w_i)$

$$q_i^D(w_i) = (B_1 - 1)\left[\frac{\alpha k_i B_1 + \alpha\gamma - w_i(B_3 - B_2) + (B_1 - 1)\gamma \sum_{j\neq i} w_j}{B_1 B_2}\right],$$

where B_3 is given in the statement of the lemma.

Now we are able to rewrite the manufacturer's profit function as

$$\Pi_i^M = (w_i - c)q_i^D(w_i). \quad (36)$$

Since the wholesale price is chosen to maximize the manufacturer's profit, we differentiate (36) over the wholesale price w_i

$$\frac{\partial \Pi_i^M}{\partial w_i} = q_i^D(w_i) + (w_i - c)\frac{\partial q_i^D(w_i)}{\partial w_i}. \quad (37)$$

Now

$$\frac{\partial^2 \Pi_i^M}{\partial w_i^2} = 2\frac{\partial \hat{q}_i^D(w_i)}{\partial w_i} - \frac{\partial^2 \hat{q}_i^D(w_i)}{\partial w_i^2} = -\frac{2(B_3 - B_2)(B_1 - 1)}{B_1 B_2} < 0,$$

so the solution to the first order conditions is the unique optimal wholesale price for manufacturer i.

The next step is to show that the value $\hat{w}_i^D$ given in the Lemma statement do indeed satisfy the first order conditions. From (37), we have

$$\frac{\partial \Pi_i^M}{\partial w_i} = \frac{(B_1 - 1)}{B_1 B_2}\left[\alpha k_i B_1 + \alpha\gamma + (c - 2w_i)(B_3 - B_2) + (B_1 - 1)\gamma \sum_{j\neq i} w_j\right].$$

After straightforward but lengthy algebra we can deduce that this expression is zero at $\hat{w}_i^D$, and so this is the optimal choice of wholesale price. Next we calculate the optimal prices from (35), which gives a lengthy expression that can be simplified to (15). Finally we can obtain the quantities and profits shown in (16)–(20). □

Proof of Lemma 4.4. Differentiating the profit functions for the manufacturer i and retailer i over competition level γ separately, and setting $\gamma = 0$, we have

$$\left.\frac{\partial \hat{\Pi}_i^M}{\partial \gamma}\right|_{\gamma=0} = -\frac{1}{16}(\alpha k_i - c)(\alpha k_i(1+2n) - 3\alpha + (n-1)c),$$

$$\left.\frac{\partial \hat{\Pi}_i^R}{\partial \gamma}\right|_{\gamma=0} = -\frac{1}{32}(\alpha k_i - c)(\alpha k_i(1+2n) - 3\alpha + (n-1)c).$$

These slopes are positive, and hence the introduction of horizontal competition increases the manufacturer and retailer profits, if and only if

$$k_i < \frac{3\alpha - (n-1)c}{\alpha(1+2n)}$$

and clearly this is also the condition for the entire supply chain profits to increase.

Similarly, we can differentiate the industry profits for decentralized supply chains over competition level γ and set γ to 0,

$$\left.\frac{\partial \hat{\Pi}^D}{\partial \gamma}\right|_{\gamma=0} = -\frac{3}{32}\left[\frac{V^2+1}{n}\alpha^2(1+2n) - (3\alpha^2 - 2\alpha c(n-1) + n(n-1)c^2)\right].$$

So when

$$V^2 < \frac{(\alpha - cn)^2(n-1)}{\alpha^2(2n+1)}$$

the introduction of horizontal competition increases the decentralized supply chains' industry profits.

Notice that $\hat{\Pi}_i^M$ has in the denominator a polynomial with γ^{12} as the highest power of γ, and γ^{11} is the highest power in the numerator. The same is true for $\hat{\Pi}_i^R$ and hence both

$$\lim_{\gamma=\infty} \hat{\Pi}_i^M = 0 \quad \text{and} \lim_{\gamma=\infty} \hat{\Pi}_i^R = 0$$

establishing part 3 of the Lemma. □

Proof of Lemma 4.5. We need to compare industry profits between integrated chains and decentralized chains. We set $\Delta = \hat{\Pi}^D - \hat{\Pi}^I$. Moreover we can rewrite the expression (20) for $\hat{\Pi}^D$ as

$$\frac{B_0(B_3 - B_2)(B_1 - 1)}{(B_1 B_2 B_3 B_4)^2} \times \left[n(\alpha\gamma B_0 - cB_2 B_4)^2 + \alpha B_1 B_3(\alpha B_1 B_3 \frac{V^2+1}{n} + 2\alpha\gamma B_0 - 2cB_2 B_4)\right].$$

Thus, using (10), we have

$$\Delta = \frac{(B_1 - 1)c^2}{(B_1 B_2 B_3 B_4)^2} f(\gamma),$$

where $f(\gamma)$ is a polynomial of γ. The exact expression for $f(\gamma)$ is lengthy, but the highest power of γ is γ^{10}. Allowing γ to increase to infinity, we can see that Δ will eventually be positive as long as the coefficient of γ^{10} in $f(\gamma)$ is positive. Define the coefficient of γ^{10} as $\tilde{\alpha}_{10}$, then expanding the various terms we finally obtain

$$\tilde{\alpha}_{10} = (3n-2)(11n^3-20n^2+13n-3)(n-1)^5\beta^2 - (n^2+n-1)(n-1)^8\left(\frac{V^2+1}{n}\right)\beta^2 - (2n-1)^5(n-1)^5(2\beta-n),$$

where $\beta = \alpha/c > n$. The requirement that $\tilde{\alpha}_{10} > 0$ leads to the following condition for V^2:

$$V^2 < \frac{(2n-1)^5}{(n^2+n-1)(n-1)^3}\left(\frac{nc}{\alpha}-1\right)^2. \qquad \square$$

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