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THE BENEFITS OF DECENTRALIZED DECISION-MAKING IN SUPPLY CHAINS

ABSTRACT. The inefficiency of decentralized decision-making is one of the most influential findings of the supply chain coordination literature. This paper shows that with the possibility of continuing trade, decentralization can be *beneficial* in improving supply chain performance. In a supply chain with decentralized decision-making and continuing trade, it is easier to incentivize players to coordinate on efficient actions. There are more gains to be shared from coordination, and by virtue of each player being a smaller influence on the system, any individual player's opportunism is less of a threat to coordination. These stronger incentives to coordinate manifest themselves in higher profits of supply chains with decentralized decision-making and additional terms of contracting acceptable to all players. Our analysis demonstrates that the widely accepted inefficiency of decentralized decision-making is an artifact of the simplifying assumption of one-off trade, and identifies conditions for departures from this result with continuing trade. The newly identified phenomena provide a possible explanation for the paradoxically good performance of very decentralized supply chains seen in emerging market cooperatives, urban logistics, micro-retailing, and other settings.

1. INTRODUCTION

The study of incentives in supply chains has grown to be one of the most important fields of research in supply chain management. An influential finding of this literature is the inefficiency that arises from the delegation of decision-making from a central planner to individual tiers in a supply chain. Decentralization of decision-making has been shown to reduce supply chain performance for a variety of different supply chain decisions including inventory levels, capacity investments, information sharing and quality efforts (cf. Perakis and Roels (2007) and the references therein).

Our analysis demonstrates that in situations with continuing (or repeated) trade, *in contrast with the literature, more decentralized decision-making can in fact strictly improve the performance of a supply chain.* Specifically, the impact of decentralization depends on the discount factor, which can be interpreted as a composite of the firm's time value of money and the probability of continuing trade. For firms that place a low probability on continuing trade or those that have high costs of capital, that is for *low levels* of the discount factor, as one-off trade literature predicts, more decentralized

decision-making is harmful. More interestingly, when supply chain partners have *intermediate levels* of the discount factor, supply chains with more decentralized decision-making can strictly outperform supply chains with more centralized decision making, whereas for sufficiently *high levels*, both more centralized and decentralized supply chains perform equally well.

We build on the classic one-off trade models that illustrate the inefficiency of decentralization: an N-tier version of the price-only contracts model of Lariviere and Porteus (2001) (Section 3) and a more general, generic supply chain interaction model (Section 4). We revisit these while considering the possibility of continuing trade. We compare two supply chains that are the same in all respects save an elemental difference in the degree of decentralization of decision-making: one has N, while the other has $N + 1$ independent decision-makers. Common in the supply chain coordination literature, comparison of the two under one-off trade suggests that the performance of the supply chain with $N + 1$ decision-makers is worse. Our analysis, on the other hand, shows that with the possibility of continuing trade, the more decentralized $N + 1$ decision-maker supply chain can earn higher profits.

With a positive probability of continuing trade, the interactions between supply chain partners are captured as a game of uncertain horizon. In such a setup, inter-temporal incentives may be used to eliminate any incentive misalignments between tiers (cf. Taylor and Plambeck (2007b)). Specifically, players can be incentivized to act in the interest of the entire supply chain rather than in their own self-interest with the offer of a future reward. The reward is typically the continued behavior by all other members in the supply chain's interest, and a share in the benefits from such decision-making. All incentive conflicts in the supply chain are eliminated if and only if for each player, this future reward, is higher than her immediate profits foregone from behaving in an opportunistic way. Our analysis shows that decentralization makes it easier to meet this condition.

Specifically, with more decentralized decision-making, the lack of supply chain optimal behavior hurts the supply chain more. Put differently, the gains realized from supply chain optimal behavior are higher. Consequently there are more gains available to distribute amongst individual partners as the future reward that incentivizes them to behave in a supply chain optimal fashion. With an appropriate distribution of the gains realized, each player gains more by behaving in a supply chain optimal fashion in a chain with more decentralized decision making– there *is a higher value of relationships*. Further, with more decentralized decision-making, individual decision-makers have a comparatively lower influence on the supply chain, and they have less to gain by behaving in an opportunistic way, or alternately less immediate profits to forgo by acting in the interest of the supply chain rather than

in their opportunistic self-interest, i.e. there is *reduced opportunism.* Taken together, the higher value of relationships and the reduced opportunism that arise out of more decentralized decision-making can improve supply chain performance.

This improvement manifests itself in two ways. First, even for discount factors, when more centralized supply chains cannot use inter-temporal incentives to eliminate incentive conflicts, more decentralized supply chains can do so and consequently achieve higher supply chain profits. This requires that the influence of the higher value of relationships and the reduced opportunism overcome the additional opportunism that may arise on the account of more decision-makers. Second, more decentralized decision-making improves supply chain performance by allowing for additional allocations of profits that are acceptable to all supply chain partners, while earning the same profits as a more centralized supply chain. This greater flexibility could help achieve supply chain management objectives beyond profit maximization, such as equity (Loch and Wu (2008)) and financial health (Swinney and Netessine (2009)) among others.

Section 3 provides our analysis for the most commonly studied N-tier serial, push supply chain with price-only contracts. Necessary and sufficient conditions that identify when decentralization has a detrimental or beneficial effect on the supply chain are provided. Section 4 generalizes these to any uncoordinated supply chain with general supply chain structures (serial, assembly, etc.), actions by the tiers (capacity/quality investments, forecast sharing, promotion efforts, etc.), governance/contract forms, and profit functions.

Our results also speak to the paradoxically good performance of highly decentralized supply chains often seen in emerging economies. The industry leading performance of supply chains with a large number of independent decision-makers has been documented in dairy cooperatives (Goldberg et al. (1998)), in micro-retailing (Pierson and van Ryzin (2010)), in low-cost urban logistics (Menor and Ramasastry (2004)), in labor-intensive manufacturing (Prahalad (2010)), etc. While theories on the effects of decentralization from the existing supply chain literature would predict poor performance of supply chains or the use of complex coordinating contracts, there is no evidence of either. Instead, these supply chains often outperform vertically integrated supply chains and employ no formal, written contracts, relying on relationships between players. Our theory on the benefits of decentralization in maintaining relationships and the consequent superior performance explains both the unexpectedly good performance of these supply chains and the absence of formal coordinating contracts. Finally, our results also line up with the increasing incidence of hyper-specialization (Malone et al. (2011)).

This study makes three important contributions to the theory and practice of supply chain management. First, we advance supply chain management theory by demonstrating that the widely accepted and studied decentralization inefficiencies are an artifact of the one-off trade assumption in the literature. In our analysis, with the possibility of continuing trade, we isolate conditions where decentralization can strictly improve supply chain profits, and we demonstrate the increased flexibility available with decentralized decision-making. Second, from a managerial point of view, our analysis suggests that supply chain designers and managers need not be unnecessarily worried about the hyper-specialization and outsourcing of value-adding activities and in fact could improve relationships and supply chain performance while increasing number of tiers. In continuing trade, there are limited, if any, detrimental effects to having more independent decision-makers. Managers planning for continuing sourcing, should reap the benefits of specialization without fear of decentralization inefficiencies. Finally, our analysis highlights the importance of considering repeated trade and inter-temporal incentives in the economic analysis of supply chains. Even if the products provisioned by the supply chain are perishable and there is no physical linkage through inventory between different time periods, in multi-player decision-making, multiple periods may be linked by the strategic memory of different decisions makers. This strategic memory and the consequent inter-temporal incentives can drastically alter the results from the analysis of one-off trade.

2. Literature Review

Our work is related to two streams of supply chain management literature, studying the effect of *decentralized decision-making* and *continuing trade* on supply chain performance.

Decentralized Decision-Making. Decentralized decisions are driven by the objectives of individual players, which are different from those of the supply chain. This leads to actions that are not in the interest of the supply chain, thereby deteriorating its performance. This effect, first documented as double marginalization in the industrial organization literature, is now a frequent theme in the supply chain economics literature. The findings of the decentralization literature are best summarized in Majumder and Srinivasan (2006): "It has always been known that shorter chains [with less independent self-interested tiers] are better, even in the early research in two stage supply chains".

Different sources of inefficiency are considered in the literature. In Lariviere and Porteus (2001), the use of price-only contract between two tiers of a supply chain leads to suboptimal performance and inefficient inventory levels. In Cachon and Zipkin (1999) and Parker and Kapuscinski (2011), the

use of independent, inventory-cost minimizing objectives by two tiers of the supply chain results in inefficient base-stock levels. Information asymmetry may also lead to decentralization inefficiencies. Cachon and Lariviere (2001) consider forecast-sharing by a manufacturer in a setup where optimal supply chain performance requires truthful revelation of the forecast, but it is in manufacturer's best interest to inflate their forecasts. Baiman et al. (2001) considers asymmetric information about the returns from a supplier's design/production investments, Yang et al. (2009) considers asymmetric information about supply disruptions. Bernstein and DeCroix (2004) consider a modular assembly system with an assembler, sub-assemblers and suppliers. Decentralized assembly results in lower capacity and supply chain profits.

The literature suggests remedial measures for decentralization inefficiencies, often involving more complicated contracting forms: advance-purchase discounts, shared-savings, revenue-sharing, buy-back and two-part tariff contracts (see Cachon (2003) for an overview). While these complicated contract forms are appealing, the literature increasingly shows that they are rarely used in practice, or are unlikely to work in supply chains more complex than those studied in the literature (see for example Krishnan et al. (2004)).

While the vast majority of the operations literature has highlighted the disadvantages of decentralized decision making, Su and Zhang (2008) is a notable exception. Like this paper, it shows that decentralization of decision making can improve supply chain performance. While Su and Zhang (2008) consider the effects of strategic customer behavior on decentralization, this paper examines the role of continuing trade.

Our paper continues in this broad stream of literature studying the effect of decentralized decision-making on supply chain performance. The model of Section 3 is an N-tier generalization of the price-only contracts model by Lariviere and Porteus (2001), and the generalized model of Section 4 can cover many other operational settings including those discussed above. In contrast with the extant literature, we consider the possibility of supply chain partners trading more than once. This reverses the key insights of prior literature that considers one-off interactions.

Continuing Trade. A growing body of literature highlights the use of informal agreements and inter-temporal incentives as a remedy to the inefficiencies brought by decentralization. Taylor and Plambeck (2007a,b) study settings where price and capacity are non-contractible, while Debo and Sun (2004) study a setup with non-contractible inventory levels. Plambeck and Taylor (2006) study joint production with unobservable utility-relevant actions. Ren et al. (2010) consider forecast-sharing by a

buyer in a setup where he has an incentive to inflate the forecasts. Tunca and Zenios (2006) consider non-contractible promotion and quality efforts with multiple buyers and sellers. Belavina and Girotra (2012) consider a general class of non-contractible actions in a supply chain with two buyers, two suppliers and an intermediary. Li and Debo (2009) compare the capacity investment incentives in short and long term relationships. Chen et al. (2012) study the effect of long-term relationships between managers and firms in a supply chain. In each of these studies, use of long-term relational strategies mitigates some of the decentralization inefficiencies.

In this paper, in line with this literature, we also consider the possibility of continuing trade. Rather than modeling any of the specific non-contractible actions studied in this literature, we consider a generic game that captures the key elements of each of the above settings. More importantly, in our paper we do not show how repeated interactions can mitigate the decentralization inefficiencies; we show how more decentralized decision-making can improve supply chain performance.

Taken together, our paper builds on the supply chain coordination literature, takes the classic models from the literature and examines them with the possibility of continuing trade. We demonstrate a potential advantage of decentralized decision-making with the possibility of continuing trade. To the best of our knowledge, this is the first study that demonstrates this departure from the widely held conventional wisdom in supply chain theory on the detrimental effects of decentralized decision-making.

3. Price Only Contracts

We first establish our key results in a supply chain with price-only contracts, one of the simplest and most common mechanisms governing transactions in supply chains. It is well known that in such a setup, the supply chain is not coordinated (Cachon (2003); Perakis and Roels (2007); Lariviere and Porteus (2001); Cachon and Lariviere (2001)). We first replicate this result in our model of an N-tier, serial, push supply chain, where in line with the supply chain coordination literature, the tiers have only one opportunity to trade. Next, we consider the possibility of continuing trade in the same supply chain. We establish our key result, the superior performance of decentralized decision-making, and identify and characterize the contingencies and key effects driving the result.

3.1. **An N-Tier, Serial, Push Supply Chain.** Consider an N-tier serial supply chain (Figure 3.1) for a perishable/seasonal product. Demand for the product in the selling season, D, is stochastic, with a strictly increasing continuous cumulative distribution function $F(x)$, density, $f(x)$, and
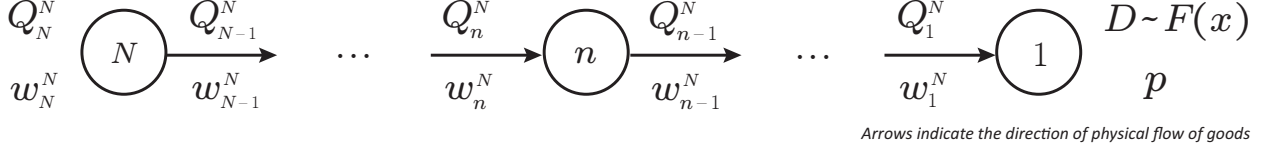
FIGURE 3.1. The N-tier Supply Chain

survival function, $\overline{F}(x) \equiv 1 - F(x)$. During the selling season, the product sells for p monetary units. We normalize its post-season price, the salvage value, to zero. The retailer, called tier 1, faces a newsvendor problem: it has a single order opportunity when it builds its inventory, Q_1^N.[1] Consistent with the literature (Cachon (2004); Cachon and Lariviere (2001); Lariviere and Porteus (2001); Perakis and Roels (2007)), we model the strategic interactions in the supply chain as a sequential move game. The originating tier, tier N, moves first, followed by tier $N-1$, $N-2$, and so on. The retailer, tier 1, moves last. Specifically,

Originating tier N, produces Q_N^N units at a cost, w_N^N, $w_N^N \equiv c$.

Originating tier N, offers tier $N-1$ a per-unit price w_{N-1}^N. Tier $N-1$ orders Q_{N-1}^N units.

$\vdots$

Tier n, $n \in \{2, 3, ..., N\}$, offers tier $n-1$ a per-unit price w_{n-1}^N. Tier $n-1$ orders Q_{n-1}^N units.

$\vdots$

Tier 2 offers the retailer, tier 1, a per-unit price w_1^N. The retailer orders Q_1^N units.

Demand D is realized, fulfilled from the retailer's inventory, Q_1^N. Excess demand is lost.

Each unit of satisfied demand generates a revenue, p, $p > c > 0$. The prices set and quantities ordered are observable to other players. Specifically, when tier n, $n \in \{1, 2, ..., N\}$, decides on the quantity to order from tier $n+1$ and the price to offer to tier $n-1$, it knows the price set and the quantity ordered by the $N-n$ tiers that already acted. The above described setup corresponds to an N-tier, serial, push supply chain as per the nomenclature used by Cachon (2004) and Perakis and Roels (2007).

3.2. **Supply Chains with One-off Trade.** Consistent with the literature, consider one-off trade in the above described supply chain: the supply chain is disbanded after one trade opportunity or the players do not factor in the possibility of any future trade in their decision-making. The actions of independently acting self-interested tiers are now governed solely by their immediate strategic tradeoffs.

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[1]Throughout the paper, we use the superscript to denote the number of self-interested decision-makers in the supply chain, and the subscript to identify the decision-maker relevant to the statistic.

Consider demand distributions such that $\forall n$, $n \in \{2, 3, ..., N\}$ and $\forall x$ in the support of F, $\varphi_n(x)$ is a decreasing concave function of x, where $\varphi_n(x) \equiv \frac{\partial}{\partial x} x \varphi_{n-1}(x)$ and $\varphi_1(x) \equiv \overline{F}(x)$. In the two-tier setting, these conditions on the demand distribution boil down to the commonly assumed IGFR (increasing generalized failure rate) property of the demand distribution (Lariviere and Porteus (2001); Lariviere (2006)). Many probability distributions satisfy this property: for instance, the Uniform and the Exponential distributions (cf. Chod and Rudi (2005); Perakis and Roels (2007) for more examples and details on the ranges of acceptable parameters). For supply chains with more than two tiers, this property always holds when demand is distributed according to the Uniform distribution, and also for a variety of other common distributions such as the Exponential, the mix of Power distributions, etc., with mild restrictions on distribution parameters.

The above restriction ensures that there is a unique equilibrium to the game with one-off trade. For ease of exposition, we present the results in this section when this is the case. Nevertheless, the results from this section on price-only contracts can be extended to a wider class of distributions: for example, those provided in Chen (2012), where there are potential multiple equilibria in subgames that arise from lower-tier analysis. In Section 4, we go even further and extend our main result to generic supply chain interactions.

The ensuing subgame perfect equilibrium of this game is computed as solution to a multi-level optimization program. Lemma 5 (Appendix, Page 29) characterizes the equilibrium actions and outcomes. In equilibrium, all tiers order the same quantity: $\hat{Q}_1^N = ... = \hat{Q}_n^N = ... = \hat{Q}_N^N$. We denote this common quantity by $\hat{Q}^N$, omitting the subscript for the tier. The equilibrium incoming transfer price to tier n is denoted by $\hat{w}_n^N$. The equilibrium profits of tier n are denoted by $\hat{\Pi}_n^N$, and the total profits of the N-tier supply chain are denoted by $\hat{\Pi}^N$. If there was only one independently acting tier in the supply chain, the supply chain would achieve the first-best solution: the retailer would order the traditional newsvendor quantity, $Q^{FB} \equiv \overline{F}^{-1}\left(\frac{c}{p}\right)$. We denote the associated supply chain profit as Π^{FB}.

We compare the above described N-tier supply chain with one that is identical to it in all respects, but is elementally more decentralized in its decision-making. Specifically, both supply chains produce and provision products with identical economics, but the value-adding activities of *one* of the tiers in the centralized supply chain are instead done by *two independently acting tiers* in the decentralized supply chain. Consequently, the latter supply chain has $N + 1$ independent tiers. Like all the other tiers, these two independently acting tiers also choose a quantity and a transfer price to earn a margin. Even though the two supply chains add the same value to the product, or do the same activities, the

$N + 1$-tier has more independent decision-makers, or is a more decentralized version of the N-tier supply chain. In all subsequent discussion, for brevity, we will refer to the supply chain with N tiers as the *centralized* supply chain and the one with $N + 1$ tiers as the *decentralized* supply chain, rather than using more/less centralized/decentralized supply chain. The classic comparison of an integrated supply chain and a decentralized supply chain consisting of an independently acting wholesaler and a retailer (see for example Lariviere and Porteus (2001)) corresponds to a special case of our setup with $N = 1$.

The following lemma echoes a classic result from the literature: supply chain profits decrease with decentralization in the supply chain, reflecting the impact of double marginalization that manifests itself in the quantity ordered to meet uncertain demand. A classic version of this result, with $N = 1$ is presented in Lariviere and Porteus (2001).

Lemma 1. *1) The centralized supply chain provisions a higher quantity and earns higher total profits. Formally, $\forall N \in \{1, 2, ...\}$, $\hat{Q}^N > \hat{Q}^{N+1}$ and $\hat{\Pi}^N > \hat{\Pi}^{N+1}$.*

2) The profits of individual tiers are higher in the centralized supply chain than those earned by their counterparts in the decentralized supply chain. Formally, $\forall n \in \{1, 2, ..., N\}$, $\hat{\Pi}_n^N > \hat{\Pi}_n^{N+1}$.

Detailed proofs for this and all subsequent results are provided in the Appendix.

Overall supply chain profits and performance are determined by the quantity ordered by tier 1, the retailer, the only tier in the supply chain selling to the customer. This quantity depends on the margin that the retailer can make from selling the product, $p - w_1$. All other tiers order the same quantity. With more independently acting tiers in the decentralized supply chain, each of which moves before the retailer, there are more players that appropriate a part of the total product margin, $p - c$, and a smaller margin is left for the retailer, $p - \hat{w}_1^{N+1} < p - \hat{w}_1^N$, which decreases the order quantity. The lower order quantity reduces supply chain profits. The lower quantity ordered and the lower margins accruing to each tier together reduce the profits of individual tiers in the decentralized supply chain as compared to their counterparts, or tiers at their equivalent positions in the centralized supply chain.

The model developed above follows in the tradition of the literature on supply chain coordination in considering only one trading opportunity in the supply chain, implicitly not allowing for the possibility of continuing trade. On the other hand, almost all real-world supply chains have the possibility of trade and of strategic interactions beyond one selling cycle. The firms involved may continue to trade

with each other to sell the same product, new versions of the product, or entirely new products. Rarely are supply chains disbanded after one trade cycle. Interestingly, the canonical example used to motivate these models, the stocking of an edition of a newspaper, illustrates the incompleteness of the existing models. The implicit one-trading-opportunity assumption in the literature suggests that the newsvendor and her supplier, publish, distribute and sell only one edition of the newspaper. However, entities involved in the newspaper supply chain do not change on a day-to-day basis; on the contrary, the players in such a supply chain typically interact repeatedly over a time horizon that involves many editions of a newspaper. Thus, from the standpoint of accurately capturing the key elements of a supply chain, it is essential that supply chain coordination models allow for the possibility of continuing trade between supply chain partners. More interestingly, this consideration can significantly change our understanding of the strategic effects of decentralization.

The products considered in supply chain coordination models are typically perishable and are not inventoried from one period to another, so there is no physical linkage between successive time periods. In a single decision-maker analysis, it is thus sufficient to consider a model with one-off trade. However, in multi-player decision analysis or game-theoretic modeling of supply chain interactions, even if the physical products are perishable, players may have a *strategic memory* that can link the strategic decisions in different periods. In fact, it is well known in the study of repeated games and reputations that this strategic memory allows for players to engage in inter-temporal tradeoffs that can drastically alter the nature of equilibria observed in the game (Mailath and Samuelson (2006)).

Taken together, the need for accurately modeling supply chains and the potential for arriving at different insights, both suggest that the existing supply chain coordination models, which allow only for one-off trade, may be incomplete in the understanding they provide of supply chain economics and the subsequent managerial prescriptions. We next extend the above base model to allow for the possibility of continuing trade in the supply chain.

3.3. **Supply Chains with the Possibility of Continuing Trade.** We continue to consider exactly the same supply chain settings as before, we depart in only way: we now allow for the possibility of continuing trade. In particular, the above described sequential move game is repeated with some probability. Formally, we model this as a game with uncertain horizon or an infinitely repeated game where the sequence of the events outlined in Section 3.1 is repeated in every period t, $t \in \{0, 1, 2, ...\}$. All supply chain partners discount future profits with a per-period discount factor $\delta \in (0, 1)$, which captures the time value of money and the probability of continuing trade in the supply chain. We

use the subgame perfect equilibrium concept to identify equilibrium outcomes. As before, profits in one-off trade are denoted by Π, with the superscripts denoting the number of independent actors and the subscript denoting the individual actor. The corresponding profits in repeated trade are denoted by π. In repeated trade, supply chain profits are not just a function of the number of independently acting tiers in the supply chain, but also of the discount factor. We denote the highest equilibrium profit achievable in an N-tier supply chain with repeated trading by $\pi^N(\delta)$.

As opposed to a supply chain with one-off trade, tiers in the supply chain now consider the immediate and long-term consequences of their actions. This facilitates the use of price-only contracts to establish inter-temporal trade-offs that lead to different equilibrium outcomes than those in supply chains with one-off trade. Typically, such "relational" strategies are characterized by a set of *continuation actions* that are played in equilibrium and a set of *punishment actions* that enforce them. Players continue to behave according to the continuation actions as long as the continuation actions were observed in the previous periods. On the other hand, if any player deviates from the prescribed continuation actions, the players revert to punishing each other with a set of punishment actions.

In our context, a relational strategy prescribes a set of transfer prices, $\boldsymbol{w}^N \equiv \left(w_1^N, w_2^N, ..., w_N^N\right)$, and an order quantity, Q^N. In following this relational strategy, in its upstream role, tier n proposes the transfer price w_n^N specified by $\boldsymbol{w}^N$. In its downstream role, tier n responds to the contract by ordering quantity Q^N. If and only if all preceding actions were these continuation actions, the tier continues playing the continuation actions, else it plays the punishment actions: the myopically optimal actions described in Lemma 5, earning per-period profits of Π_n^N. The prescribed order quantity Q^N determines the total supply chain profit, while the vector of transfer prices, $\boldsymbol{w}^N$, determines only its *distribution* among the tiers of the supply chain. The following Lemma provides the conditions necessary to sustain the relational strategy as an equilibrium.

To describe our results intuitively, we use the following short-form notation: $v_n(Q)$ denotes the myopically optimal transfer price that tier $n+1$ sets if it anticipates that the downstream tiers will order a quantity Q. Specifically, $v_1(Q) = p\overline{F}(Q)$, $v_n(Q) = \frac{\partial}{\partial Q}Q \cdot v_{n-1}(Q)$, $n \in \{2, 3, ..., N\}$; $v_0(Q) = \frac{1}{Q}p\int_0^Q \overline{F}(x)\,dx$ and $w_0^N = \frac{1}{Q^N}p\int_0^{Q^N}\overline{F}(x)\,dx$. Further, denote the solution to $v_n(Q) = y$ as $q_n(y)$, $n \in \{1, 2, ..., N\}$. Now, $q_n(y)$ represents the myopically optimal order quantity of the supply chain with n tiers and a production cost y. Specifically, $q_N(c) = \hat{Q}^N$.

Lemma 2. *A relational strategy $\left(\boldsymbol{w}^N, Q^N\right)$ is a subgame perfect equilibrium iff*

$$(3.1) \qquad \phi\left(\boldsymbol{w}^N, Q^N, \delta\right) \equiv \boldsymbol{C}^N\left(\boldsymbol{w}^N, Q^N\right) - (1-\delta)\, \boldsymbol{D}^N\left(\boldsymbol{w}^N\right) - \delta\hat{\boldsymbol{\Pi}}^N \geq \boldsymbol{0},$$

where the vectors $\boldsymbol{C}^N\left(\boldsymbol{w}^N, Q^N\right)$, $\boldsymbol{D}^N\left(\boldsymbol{w}^N\right)$ and $\hat{\boldsymbol{\Pi}}^N$ are defined below.

$\boldsymbol{C}^N\left(\boldsymbol{w}^N, Q^N\right)$	$\boldsymbol{D}^N\left(\boldsymbol{w}^N\right)$	$\hat{\boldsymbol{\Pi}}^N$
$Q^N \cdot \begin{bmatrix} w_0^N - w_1^N \\ \vdots \\ w_{n-1}^N - w_n^N \\ \vdots \\ w_{N-1}^N - w_N^N \end{bmatrix}$	$\begin{bmatrix} q_1\left(w_1^N\right)\cdot\left(v_0\left(q_1\left(w_1^N\right)\right) - w_1^N\right) \\ \vdots \\ q_n\left(w_n^N\right)\cdot\left(v_{n-1}\left(q_n\left(w_n^N\right)\right) - w_n^N\right) \\ \vdots \\ q_N\left(w_N^N\right)\cdot\left(v_{N-1}\left(q_N\left(w_N^N\right)\right) - w_N^N\right) \end{bmatrix}$	$\hat{Q}^N \cdot \begin{bmatrix} v_0\left(\hat{Q}^N\right) - v_1\left(\hat{Q}^N\right) \\ \vdots \\ v_{n-1}\left(\hat{Q}^N\right) - v_n\left(\hat{Q}^N\right) \\ \vdots \\ v_{N-1}\left(\hat{Q}^N\right) - v_N\left(\hat{Q}^N\right) \end{bmatrix}.$

A relational strategy is a subgame perfect equilibrium iff the expected discounted profit earned by each tier n of the N-tier supply chain following the relational strategy exceeds the best profit that tier n can secure by deviating in any given period and facing the resulting one-off trade profits in the future. The vector ϕ is composed of the slack in these equilibrium constraints. Vector $\boldsymbol{C}^N$ captures the value of continuing to play the continuation actions prescribed by the relational strategy; it is the margin earned, $\left(w_{n-1}^N - w_n^N\right)$, times the quantity, Q^N, prescribed by the relational strategy. Vector $\boldsymbol{D}^N$ is composed of the immediate profits that a player may realize from the best deviation from the continuation actions. A deviation by player n is immediately detected by tier $n-1$ and all subsequent tiers from $n-1$ to 1, which now act as per the punishment (the actions in one-off trade) in this very period. For tier n, the best deviation profits are $D_n^N\left(w_n^N\right) \equiv q_n\left(w_n^N\right)\cdot\left(v_{n-1}\left(q_n\left(w_n^N\right)\right) - w_n^N\right)$; $q_n\left(w_n^N\right)$ is the order quantity that all subsequent tiers will order. Anticipating this order, tier n sets a price $v_{n-1}\left(q_n\left(w_n^N\right)\right)$, receiving the margin $v_{n-1}\left(q_n\left(w_n^N\right)\right) - w_n^N$. Finally, vector $\hat{\boldsymbol{\Pi}}^N$ captures the payoff in all periods subsequent to the deviation period. All tiers play in their short-term interest as per the myopic outcome described in Lemma 5 in the Appendix, earning utility $\hat{\Pi}_n^N \equiv \hat{Q}^N \cdot \left(v_{n-1}\left(\hat{Q}^N\right) - v_n\left(\hat{Q}^N\right)\right)$.

With the use of the above described equilibrium relational strategies, it is possible to achieve the first-best supply chain profits, Π^{FB}. As it is typical in repeated games, to understand the equilibrium outcomes, it is crucial to characterize the lowest discount factor at which the first-best profit can be achieved. Our first Theorem characterizes this threshold discount factor and the "marginal" relational strategy, the relational strategy that achieves the first-best profit at this discount factor.

Theorem 1. *The lowest discount factor at which an N-tier supply chain with the possibility of continuing trade can achieve first-best profits is*

$$\bar{\delta}^N = \left(\frac{\mathbf{1} \cdot \boldsymbol{D^N}\left(\boldsymbol{\bar{w}}^N\right) - \Pi^{FB}}{\Pi^{FB} - \hat{\Pi}^N} + 1 \right)^{-1}.$$

Further, the transfer prices in the "marginal" relational strategy, $\boldsymbol{\bar{w}}^N$, are given by the solution to $\phi\left(\boldsymbol{\bar{w}}^N, Q^{FB}, \bar{\delta}^N\right) = \mathbf{0}$.

The above theorem shows that the lowest discount factor at which a relational strategy can be enforced is such that *each* player's normalized profit earned from the relational strategy is *exactly equal* to the sum of the deviation profit and the subsequent normalized profit from the punishment path. A careful examination of the equilibrium constraints from Lemma 2 (Equation 3.1) reveals that the constraints of all tiers are interlinked. If for a given discount factor, the constraint of one or several tiers is binding, but there is a slack in the constraint for some other tier, that is, there is a tier that has a higher margin than necessary to enforce the strategy, part of this tier's extra profits can be redistributed to all other tiers to ensure that all tiers also receive more than minimum profits on the continuation path. Now, this strategy can be enforced at a lower discount factor. Thus, as stated in the second part of Theorem 1, the marginal relational strategy must be such that all tiers receive exactly as much profits as necessary to ensure that they prefer not to deviate. This property of the marginal relational strategy allows us to characterize the threshold discount factor as a function of two intuitive metrics of the relational strategy:

$$\bar{\delta}^N = \left(\frac{\mathbf{1} \cdot \boldsymbol{D^N}\left(\boldsymbol{\bar{w}}^N\right) - \Pi^{FB}}{\Pi^{FB} - \hat{\Pi}^N} + 1 \right)^{-1} = \left(\frac{Total\,Deviation\,Gain}{Value\,of\,Relationships} + 1 \right)^{-1}.$$

The first, is the difference between the first-best supply chain profit (achieved by following the relational strategy) and the profits in the supply chain with one-off trade, $\Pi^{FB} - \hat{\Pi}^N$. This can be interpreted as the *value of relationship*. Keeping all else fixed, the higher is the value of relationships, the lower is the threshold discount factor. Second, the threshold discount factor also depends on the *total deviation gain*, $\mathbf{1} \cdot \boldsymbol{D^N}\left(\boldsymbol{\bar{w}}^N\right) - \Pi^{FB}$, which is the sum of the extra profits that the players in the N-tier supply chain can earn by unilaterally deviating from the continuation terms prescribed by the relational strategy. The effect of the total deviation gain is opposite to the effect of the value of relationships. The higher is the total deviation gain, the higher is the threshold discount factor required for achieving the first-best outcomes.

Taken together, the threshold discount factor depends on the competing effects of the value of relationship and the total deviation gain. In the next section, we examine and characterize the fine balance between the changes in the value of relationships and the total deviation gains that determine changes in the performance of the supply chain in response to decentralization of decision-making in the supply chain.

3.4. **Supply Chain Performance: Effect of Decentralization.** As in our analysis of the supply chain with one-off trade, to study the effects of decentralization, we compare the performance of a supply chain with centralized and decentralized decision-making (N vs. $N + 1$ independently acting tiers). The conventional wisdom suggests that the supply chain with centralized decision-making always outperforms one with decentralized decision-making (Lemma 1).

We first define $\bar{D}_{\mathcal{N}}^N(\delta)$ to be the *minimal total deviation profit* of players in set $\mathcal{N}$, $\mathcal{N} \subset \{1, ...N\}$ that can be achieved with any equilibrium relational strategy that achieves the first-best profits, when the discount factor is δ.

Definition. For a given discount factor δ, define $\bar{D}_{\mathcal{N}}^N(\delta)$ as,

$$(3.2) \qquad \bar{D}_{\mathcal{N}}^N(\delta) = \min_{\boldsymbol{w}^N} \sum_{n \in \mathcal{N}} D_n^N\left(w_n^N\right)$$

$$s.t. \quad \forall n \in \mathcal{N}$$

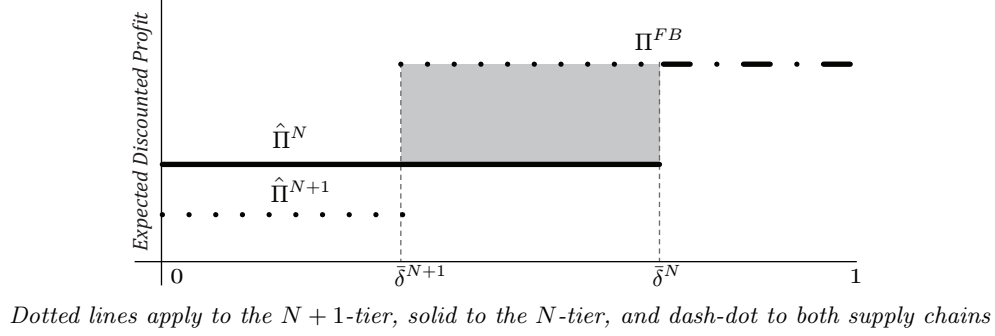$$\phi_n\left(\boldsymbol{w}^N, Q^{FB}, \delta\right) \geq 0,$$

where $\phi_n(\cdot)$ are the constituent elements of vector $\boldsymbol{\phi}(\cdot)$ defined in Equation 3.1.

Theorem 2 provides necessary and sufficient conditions for a supply chain with decentralized decision making to strictly outperform one with centralized decision-making for a non-empty range of discount factors. This is in contrast with the literature on supply chain coordination and contracting, which shows that the decentralized supply chains always perform *worse* than the centralized ones.

Theorem 2. *Supply Chains with Repeated Trade: The Effect of Decentralization*

*The profit of the decentralized $N + 1$-tier supply chain are **strictly higher** than the profit of the centralized N-tier supply chain, $\pi^{N+1}(\delta) > \pi^N(\delta)$, for $\delta \in \left(\bar{\delta}^{N+1}, \bar{\delta}^N\right)$, iff*

$$(3.3) \qquad \bar{\delta}^N\left(\hat{\Pi}^N - \hat{\Pi}^{N+1}\right) > \left(1 - \bar{\delta}^N\right)\left(\bar{D}_{\{1,2,...,N\}}^{N+1}\left(\bar{\delta}^N\right) - \bar{D}_{\{1,2,...,N\}}^N\left(\bar{\delta}^N\right) + \hat{\Pi}_{N+1}^{N+1}\right).$$

Dotted lines apply to the $N+1$-tier, solid to the N-tier, and dash-dot to both supply chains

FIGURE 3.2. The Profit in Supply Chain with N vs $N+1$ tiers

The superior performance of the decentralized supply chain is driven by the fact that the threshold discount factor at which the first-best outcome can be achieved is lower for the decentralized supply chain than for the centralized one, $\bar{\delta}^{N+1} < \bar{\delta}^N$. For $\delta \in \left(\bar{\delta}^{N+1}, \bar{\delta}^N\right)$, the decentralized supply chain can achieve first-best outcomes, whereas the centralized supply chain can not (Figure 3.2). In Theorem 1, we showed that this threshold discount factor depends on two key properties of the supply chain interactions: the value of relationship and the total deviation gain. Thus, to understand the drivers of the superior performance, we examine how these two properties change with decentralization.

In repeated trading with the benefit of relational strategies, both the more centralized N-tier and the decentralized $N+1$-tier supply chains, achieve the first-best supply chain profit, Π^{FB}. In the absence of relationships, one-off outcomes ensue in which the decentralized supply chain has lower profits (Lemma 1). Thus, the relationship is worth more in a decentralized supply chain, $\Pi^{FB} - \hat{\Pi}^{N+1} > \Pi^{FB} - \hat{\Pi}^N$. We call the difference, $\Pi^{FB} - \hat{\Pi}^{N+1} - \left(\Pi^{FB} - \hat{\Pi}^N\right)$, *the increased value of relationship,* which is associated with decentralization.

The difference in total deviation gain between the centralized and decentralized supply chain can be split into two parts: the difference in the deviation gains of the N decision-makers at the same position (tier 1 to N) in the two supply chains, $\bar{D}^N_{\{1,2,...,N\}}\left(\bar{\delta}^N\right)$ and $\bar{D}^{N+1}_{\{1,2,...,N\}}\left(\bar{\delta}^N\right)$, and the deviation gain of tier $N+1$ in the decentralized supply chain, $\bar{D}^{N+1}_{\{N+1\}}\left(\bar{\delta}^N\right)$. First, we consider the total deviation gains of the N "common" tiers.

Consider transfer prices for the $N+1$-tier supply chain, $\boldsymbol{w}^{N+1}$, where the first N elements are set to the transfer prices that minimize the deviation gains in the N-tier supply chain while sustaining first-best cooperation in equilibrium. Formally, $w_n^{N+1} = \ddot{w}_n^N$, $n \in \{1,..N\}$, where $\ddot{\boldsymbol{w}}^N$ is a solution to equation 3.2 at $\delta = \bar{\delta}^N$, $\mathcal{N} = \{1,2,...,N\}$. At these transfer prices, $\boldsymbol{w}^{N+1}$, there is slack in the first N equilibrium constraints for the $N+1$-tier supply chain: $\phi_n^{N+1}\left(\left\{\ddot{\boldsymbol{w}}^N, w_{N+1}^{N+1}\right\}, Q^{FB}, \bar{\delta}^N\right) > 0$,

$n \in \{1, ..N\}$. This implies that the first N transfer prices at which the constraints for the $N + 1$-tier supply chain are binding will be higher than these prices from the N-tier supply chain.

Put differently, the transfer prices to the N common tiers, which minimize the deviation gains and sustain cooperation, are higher in the $N + 1$-tier supply chain. Further, each player's deviation profit, $D_n^N \left(w_n^N \right)$, is a decreasing function of the transfer price, w_n^N (Lemma 6 in the Appendix). Taken together, this implies that the common tiers have less to gain by deviation in the decentralized supply chain on account of higher transfer prices. We call this *Reduced opportunism* in the supply chain on account of decentralization. Formally, $\bar{D}_{\{1,2,...,N\}}^{N+1} \left(\bar{\delta}^N \right) < \bar{D}_{\{1,2,...,N\}}^N \left(\bar{\delta}^N \right)$.

Finally, consider tier $N + 1$, which also has an opportunity to deviate in the decentralized supply chain. The deviation profit of this tier is exactly its one-off profit, $\bar{D}_{\{N+1\}}^{N+1} \left(\bar{\delta}^N \right) = \hat{\Pi}_{N+1}^{N+1}$, since this tier moves first and any deviation by this tier is immediately observed and leads to the traditional one-off outcome. Deviations by tier $N + 1$ increase potential opportunism in the supply chain, which we call this *Additional opportunism.*

To summarize, there are three key differences between the centralized N-tier and the decentralized $N + 1$-tier supply chains.

(1) The decentralized supply chain values relationships more highly than the centralized supply chain (*Increased value of relationship*).

(2) The minimal total deviation profit of the N common tiers is lower in an $N + 1$-tier supply chain (*Reduced opportunism*).

(3) The presence of the independently acting tier $N + 1$ adds to the total deviation profit in the decentralized supply chain (*Additional opportunism*).

The third effect is competing with the first two effects. The left-hand side of inequality 3.3 captures increased value of relationship. The right-hand side of inequality 3.3 captures the net impact on total deviation gain, the combined force of reduced and additional opportunism effects.

Put together, a supply chain with $N + 1$ tiers can outperform a supply chain with N tiers, iff the increase in the value of relationships on account of decentralization surpasses the increase in opportunism due to decentralization. Inequality 3.3 formalizes this condition. When this inequality holds, the threshold discount factor at which a relational strategy achieves first-best outcomes in a decentralized supply chain, $\bar{\delta}^{N+1}$, is lower than the equivalent threshold for the centralized N-tier supply chain, $\bar{\delta}^N$.

The above results demonstrate a departure from the conventional wisdom when the discount factor is in the interval $\left(\bar{\delta}^{N+1}, \bar{\delta}^{N}\right)$. However, our analysis illustrates that the advantages of decentralization are present across the board, and are not limited to this range of discount factors. In particular, these benefits arise from the additional set of relational strategies that can be enforced in equilibrium. The next theorem highlights this.

Theorem 3. *Whenever relational strategies are enforceable in a supply chain with decentralized decision-making, there exists a set of transfer prices that are enforceable in equilibrium in the decentralized supply chain, but not enforceable in the centralized supply chain. Formally, for all $\delta > \bar{\delta}_{N+1}$ there exists relational strategy $\left(w_1^{N+1}, w_2^{N+1}, ..., w_N^{N+1}, c, Q\right)$, which is an equilibrium of the $N + 1$-tier supply chain, while a relational strategy with the same transfer prices to tiers $\{N - 1, ..., 1\}$, $\left(w_1^{N+1}, w_2^{N+1}, ..., w_{N-1}^{N+1}, c, Q\right)$ is not an equilibrium of the N-tier supply chain.*

Theorem 3 highlights that the advantages of decentralization are present in a wider range of discount factors and even when condition 3.3 does not hold. It shows that the set of first-best achieving equilibrium transfer prices, and consequently profit shares in decentralized supply chains, includes transfer prices and profit shares that are not acceptable in the centralized supply chain. The additional transfer prices available with decentralization, while achieving the same profits, may help firms address concerns beyond profit maximization. For example, some of this additional freedom may be used by supply chain designers to account for considerations of equity between different tiers, the long-term health and bankruptcy of suppliers, the need for capacity investments, anti-competitive regulations, intellectual property and consequent royalties, etc. (Loch and Wu (2008), Swinney and Netessine (2009)).

A key drivers of the increase in supply chain profits on account of decentralization is the decentralization induced lower profit of individual tiers in one-off trade. This leads to higher value of relationships and ability to set higher transfer prices, resulting in the reduced opportunism of common tiers. While high transfer prices are beneficial for reducing the opportunism by the supply chain tiers, they might also lead to lower margins accruing to individual tiers. Hence, even when with decentralization the overall supply chain profit increases, the profits of the common N tiers may increase or decrease. Our analysis also sheds light on this.

If condition of theorem 2 holds, there exists the shaded area in Figure 3.2 where the decentralized supply chain performs strictly better than supply chain with N tiers. In particular the decentralized

supply chain can achieve the best possible profit, Π^{FB}, while the more centralized supply chain only achieves the one-off profit of $\hat{\Pi}^N$, with each individual tier earning $\hat{\Pi}_n^N$. The total profit left for allocation to the N "common" players in the decentralized system is at most $\Pi^{FB} - \hat{\Pi}_{N+1}^{N+1}$, as from the total pie, Π^{FB}, tier $N + 1$ must at a minimum be given his one-off payoff, $\hat{\Pi}_{N+1}^{N+1}$. Thus, only if this remaining profit is higher than what the "common" players earned in the centralized system, $\hat{\Pi}^N$, can all players be better off: only if $\Pi^{FB} - \hat{\Pi}^N > \hat{\Pi}_{N+1}^{N+1}$, can individual players be also better off with decentralization. Put differently, if the extent of improvement that the first-best provides over one-off profits in the centralized supply chain is high enough, all players can be better off with decentralization of decision-making.

Our analysis above considers the smallest possible increase in decentralization. We showed that for a supply chain with a generic number of independent decision-makers, an elemental increase in decentralization can improve supply chain performance, both in terms of profits and flexibility of profit shares. This would suggest that with further increase in the degree of decentralization, the supply chain should continue to improve its performance, which would imply that a supply chain with infinitely many (small) decision-makers would perform best! Our analysis shows that this is not always the case.

Decentralized supply chains can earn higher profits if condition 3.3 holds. But as the degree of decentralization in a supply chain, or the number of tiers, increases, this condition becomes "harder" to satisfy. The left-hand side of inequality 3.3 is decreasing in N: the increase in the value of relationships is diminishing with more decentralization. If the right-hand side, $\bar{D}_{\{1,2,...,N\}}^{N+1}\left(\bar{\delta}^N\right) - \bar{D}_{\{1,2,...,N\}}^{N}\left(\bar{\delta}^N\right) + \hat{\Pi}_{N+1}^{N+1}$, is increasing in N and is greater than zero for some N, as is often the case with most realistic distributions, then the supply chain that achieves first-best profits for the widest range of discount factors has *finitely* many independently acting tiers.

4. A GENERIC MODEL OF SUPPLY CHAIN INTERACTIONS

The above analysis demonstrated the benefits of decentralized decision-making in an N-tier, serial, push supply chain governed by price-only contracts. In this section, we extend our analysis to a generic uncoordinated supply chain, that is a supply chain where the conventional wisdom on supply chain coordination holds: decentralized decision-making is harmful in one-off trades. We consider uncoordinated supply chains that can have a generic supply chain structure (serial, assembly, etc.), actions by the tiers (capacity investments, quality investments, forecast-sharing, promotion efforts,

etc.), a governance/contract form and profit functions. In the preceding section, governance by price-only contracts led to a decentralization inefficiency that manifested itself as inefficient inventory levels in the face of uncertain demand. The analysis in this section allows for a generic incentive misalignment that leads to inefficiency. This includes the lack of coordination arising out of insufficient forecast-sharing (Cachon and Lariviere (2001)), low capacity investments (Taylor and Plambeck (2007b)), less than requisite quality efforts (Tunca and Zenios (2006)), promotion efforts (Krishnan et al. (2004)) in supply chains governed by price-only contracts or due to other forms of contractual incompleteness (Aghion and Holden (2011)). In many of these cases, there exist no practically viable solutions to eliminating the incentive misalignment.
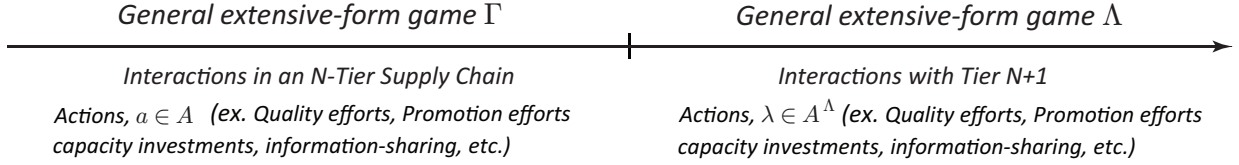
4.1. **A Supply Chain with N Independent Decision-Makers.** We model the strategic interactions between the N independently acting tiers of the supply chain as a generic, finite, N-player extensive form game, denoted by Γ. In game Γ, an action for a player specifies a move for the player at each information set owned by that player. The set of all feasible actions for player n, $n \in \{1, 2, ..., N\}$ is denoted as $A_n \subset \mathbb{R}^k$. The set of feasible action profiles is then given as $A \equiv A_1 \times A_2 \times ... \times A_N$. Each element of set A, a, describes a feasible action profile, that is a set of actions taken by all N players in this game. On completion of game Γ, the action profile a is perfectly and publicly observable.[2] The profit of each player n is given by a general profit function $\Pi_n^N \colon A \to \mathbb{R}$.

Let Ξ be the collection of initial nodes of all subgames of game Γ. The subgame of Γ with initial node $\xi \in \Xi$ is denoted by Γ_ξ. $\Gamma_{\xi^0} = \Gamma$, ξ^0 is the initial node of the game, Γ. The set Ξ is partially ordered by a precedence relation $\prec$, where $\xi \prec \xi'$, iff ξ' is a node in the subgame Γ_ξ (node ξ' appears "chronologically after" node ξ in the game). Given a node $\xi \in \Xi$, $\Pi_n^N(a|\xi)$ is player n's payoff from Γ_ξ, given the moves in Γ_ξ implied by a. The set of all terminal nodes of the game Γ is denoted by Y, with typical element y. A unique terminal node is reached under a path of play implied by actions a. At the end of the game, the players observe terminal node y reached as a result of play. The terminal node reached by a conditional on being at sub-game node ξ is denoted by $y(a|\xi)$. We denote the Nash equilibrium of game Γ as $\hat{a} \in A$, and we assume that it is unique.[3]

As before, we model the continuing trade between N tiers of the supply chain as a game with uncertain horizon. The game Γ is repeated in every period t, $t \in \{0, 1, 2, ...\}$. All parties discount future profits with a discount factor $\delta \in (0, 1)$, which captures the time value of money and the

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[2]In Section 5, we discuss the effect of imperfect observability of actions.
[3]Our results extend to games with multiple equilibria as long as the equilibrium with the highest total profits is inefficient.

General extensive-form game Γ | *General extensive-form game* Λ

Interactions in an N-Tier Supply Chain

Actions, $a \in A$ *(ex. Quality efforts, Promotion efforts capacity investments, information-sharing, etc.)*

Interactions with Tier N+1

Actions, $\lambda \in A^{\Lambda}$ *(ex. Quality efforts, Promotion efforts capacity investments, information-sharing, etc.)*

FIGURE 4.1. The Stage Game for the Supply Chain with $N+1$ Tiers

probability of termination of trade. Let $\sigma(a)$ be a relational strategy in this game that prescribes playing an action profile a in every instance of game Γ, iff terminal node $y(a|\xi^0)$ was observed in all preceding instances, and playing action $\hat{a}$ (the stage-game equilibrium) otherwise. The following Lemma provides conditions that characterize the equilibrium outcome of this repeated game.

Lemma 3. *Equilibrium with N Independent Tiers:* *The relational strategy* $\sigma(a)$ *is a subgame-perfect equilibrium iff for all* $n \in \{1, 2, ..., N\}$:

$$\Pi_n^N(a) \geq (1 - \delta) D_n^N(a) + \delta \Pi_n^N(\hat{a}),$$

where the immediate profits from the best one-shot deviation for player n are represented by $D_n^N(a) \equiv \max\limits_{a_n', \xi} \Pi_n^N(a_n', a_{-n}|\xi)$ *and* a_{-n} *are the actions prescribed by strategy a for all other players.*

As in our analysis of the supply chain with price-only contracts, we consider a supply chain and a version of it where some of the activities are decentralized. Specifically, we compare the above described supply chain of N independently acting decision-makers with a more decentralized supply chain, one with $N+1$ independent decision-makers.

4.2. **A Supply Chain With $N+1$ Independent Decision-Makers.** The interactions between the original N tiers remain the same as before, the game Γ. We model the interaction of the original N decision-makers with the $N+1^{th}$ decision-maker by a distinct, completely general finite extensive-form game, Λ. This game captures how the $N+1^{th}$ decision-maker interacts with the original N players. In game Λ, the set of feasible action profiles is then given by A^{Λ}, with typical element λ. The sourcing game is illustrated in Figure 4.1. The profit of tier n in the supply chain with $N+1$ decision-makers is given as:

$$\Pi_n^{N+1}(a, \lambda) = \Pi_n^N(a) - \chi_n(a, \lambda), \ n \in \{1, 2, ..., N\};$$

$$\Pi_{N+1}^{N+1}(a, \lambda) = \chi_{N+1}(a, \lambda),$$

where $\chi_n(a, \lambda) \geq 0$, $n \in \{1, 2, ..., N\}$ for all (a, λ) and $\chi_{N+1}(a, \lambda) \leq \sum_1^N \chi_n(a, \lambda)$, i.e. individual players can only be hurt by the interaction with the new decision-maker and the new decision-maker does not increase the supply chain profits or value added.[4] We denote the Nash equilibrium of the augmented game $(\Gamma + \Lambda)$ as $\hat{\alpha} = (\hat{a}, \hat{\lambda}) \in \{A \times A^\Lambda\}$; we assume that it is unique. We further assume that Nash behavior in this interaction *strictly* decreases the sourcing profit as compared to the base model, $\chi_{N+1}(\hat{\alpha}) < \sum_1^N \chi_n(\hat{\alpha})$; that is, there is a significant decentralization inefficiency in one-off trade. Further, we require that there exists a continuing action, an action profile $\lambda^C \in A^\Lambda$ such that $\chi_n(a, \lambda^C) < \chi_n(a, \hat{\lambda})$, for all $n \in \{1, 2, ..., N\}$, and $\chi_{N+1}(a, \lambda^C) > \chi_{N+1}(a, \hat{\lambda})$. The continuing action, λ^C, improves all players' profits compared to the Nash action, though it can not be enforced as equilibrium in one-off trade. Further, when this action is played there is no loss in efficiency; that is, the supply chain earns the same profits as it would if there was no $N+1^{th}$ independent decision-maker, $\chi_{N+1}(a, \lambda^C) = \sum_1^N \chi_n(a, \lambda^C)$. For all other actions besides action λ^C, the additional transaction reduces the supply chain profits as compared to the outcome of the original game Γ.

The restrictions on $\boldsymbol{\chi}$ described above allow us to capture the conventional wisdom on the effect of decentralization, i.e. inefficiency is increasing with decentralization of decision-making. In essence, in our setup, self-interested interactions with tier $N + 1$ reduce the total supply chain profits in a Nash equilibrium, even though there exist continuing actions that could make everyone better off. This is the decentralization inefficiency that is widely demonstrated by the literature on one-off trade and is the only restriction on the supply chain that we impose in our model. Next, we consider repeated trade in this supply chain.

We model the continuing trade between the $N + 1$ independent decision-makers in the supply chain as a game of uncertain horizon, with discount factor $\delta \in (0, 1)$ capturing the time value of money and the probability of termination of trade. The augmented stage game, $(\Gamma + \Lambda)$, is repeated in every period $t \in \{0, 1, 2, ...\}$. Let $\sigma(a, \lambda)$ be a relational strategy in the augmented game that prescribes playing an action profile (a, λ) in an instance of the augmented game, iff terminal node $y(a, \lambda|\xi^0)$ was observed in all preceding interactions, otherwise playing $\hat{\alpha}$, the Nash equilibrium outcome in one-off trading.

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[4]Note here that there may also be benefits of decentralization that come from specialization, economies of scale, pooling benefits, etc., but to focus on the role of decentralization and its widely accepted detrimental effect on supply chain profits, we do not consider any of these effects in our model. This biases our results to underestimating the advantages of decentralization. Our subsequent results would be even stronger if we considered any of these effects.

Lemma 4. *Equilibrium with N+1 independent tiers:* *The relational strategy* $\sigma(a, \lambda)$ *is a subgame-perfect equilibrium iff for all* $n \in \{1, 2, ..., N+1\}$,

$$\Pi_n^{N+1}(a, \lambda) \geq (1 - \delta) D_n^{N+1}(a, \lambda) + \delta \Pi_n^{N+1}(\hat{\alpha}),$$

where $D_n^{N+1}(a, \lambda) = \max\left\{D_n^{\Gamma, N+1}(a, \lambda), D_n^{\Lambda, N+1}(a, \lambda)\right\}$, $D_n^{\Gamma, N+1}(a, \lambda) = \max\limits_{a_n', \xi}\left(\Pi_n^N(a_n', a_{-n}|\xi) - \chi_n\left(a_n', a_{-n}, \hat{\lambda}\right)\right)$, $D_n^{\Lambda, N+1}(a, \lambda) = \max\limits_{\lambda_n', \xi}\left(\Pi_n^N(a) - \chi_n\left(a, \lambda_n', \lambda_{-n}|\xi\right)\right)$, $\forall n \in \{1, 2, ..., N\}$; $D_{N+1}^{N+1}(a, \lambda) = \max\limits_{\lambda_{N+1}', \xi}\chi_{N+1}\left(a, \lambda_{N+1}', \lambda_{-(N+1)}|\xi\right)$.

The deviations, $D_n^{N+1}(a, \lambda)$, in the above Lemma arise considering all possible types of deviations: each player n, $n \in \{1, 2, ..., N\}$ can deviate in either her interactions with the other N players, that is in game Γ, or alternately in her interactions with player $N + 1$, that is in game Λ. The most profitable of these deviations are denoted by $D_n^{\Gamma, N+1}(a, \lambda)$ and $D_n^{\Lambda, N+1}(a, \lambda)$, respectively. The most profitable deviation from these two classes of deviations defines the best profit from deviation by player n, $D_n^{N+1}(a, \lambda)$. Player $N + 1$, on the other hand, can deviate only in game Λ, and as expected, her deviation, $D_{N+1}^{N+1}(a, \lambda)$, is her unilateral best response to other players' actions as per strategy (a, λ). Now, that we have established the basic analysis for both the centralized N-player game and the decentralized, $N + 1$-player game, we can proceed to comparing the outcomes and identifying the effects of decentralization, analogous to the analysis of Section 3.4.

4.3. **The Effects of Decentralization.** In Section 3.4, we saw that the effects of decentralization depend crucially on how decentralization changes the value of the relational strategy and the deviation gains. The same changes drive the effects of decentralization with the generic supply chain structure. Consider any relational strategy $\sigma(a)$ for the centralized game; now for each player $n \in \{1, 2, ..., N\}$ in the centralized decision game, we can define the *value of the relationship* as $V_n^N(a) = \Pi_n^N(a) - \Pi_n^N(\hat{a})$, the difference between the profits she earns by acting as per the norms of the relationship and the profits she would earn if there were no relationship, the Nash profits, $\Pi_n^N(\hat{a})$. Next, consider the profit equivalent counterpart of this strategy in the decentralized decision-making game, relational strategy, $\sigma(a, \lambda^C)$. As with the centralized game, for each player $n \in \{1, 2, ..., N + 1\}$ in the decentralized game, the value of the relationship can be computed as $V_n^{N+1}(a, \lambda^C) = \Pi_n^{N+1}(a, \lambda^C) - \Pi_n^{N+1}(\hat{a}, \hat{\lambda})$, the difference in the profits she can earn by acting as per the norms of the relationship and the profits outside of the relationship.

Theorem 4. *Consider any relational strategy* $\sigma(a)$ *for the centralized game and its profit equivalent counterpart* $\sigma(a, \lambda^C)$ *in the decentralized game:*

(1) *The value of the relationship for each player in the decentralized supply chain is higher than that for her counterpart in the centralized supply chain: $\forall n \in \{1, 2, ..., N\}$, $V_n^{N+1}\left(a, \lambda^C\right) > V_n^N(a)$. (Increased value of relationships)*

(2) *The highest immediate deviation profits of each player in the decentralized supply chain are lower than those for her counterpart in the centralized supply chain: $\forall n \in \{1, 2, ..., N\}$, $D_n^{N+1}\left(a, \lambda^C\right) < D_n^N(a)$. (Reduced opportunism)*

The first part of the theorem arises from the conventionally understood disadvantages of decentralization. With one-off trade, supply chain profits are lower in a decentralized supply chain. With continuing trade, relational strategies $\sigma(a)$ and $\sigma\left(a, \lambda^C\right)$ earn the same profits. Thus the value of relationships, defined as the difference between the profits from continuing trade and those from one-off trade, is higher for a decentralized supply chain. On the other hand, the deviation profits, the immediate benefits of the deviation, are lower in the decentralized chain. In the period of the deviation itself, deviations will be met by subsequent retaliatory actions in the remaining part of game Γ (as with the centralized system), but additionally by actions in the interactions with the tier $N + 1$ in game Λ. As a whole, this provides an extra degree of retaliation to a deviation, limiting its gain and resulting in *reduced opportunism.* Taken together, in the decentralized game, any deviations from a relational strategy hurt a deviator more in the long run on account of the loss of a more valuable relationship (part 1) and the gains in the immediate short run are also smaller (part 2).

The above theorem considers the N common decision-makers in the centralized and the decentralized system and highlights that with decentralization, relationships become more valuable and deviations from the relationships become less rewarding; thus, the incentives to get into and maintain a relationship are higher, or relational strategies are easier to enforce. Put differently, it is thus expected that enforcing an arbitrary action in equilibrium in the decentralized game is easier. With appropriately chosen actions that are prescribed by the relational strategy, this implies that a decentralized supply chain can outperform the centralized one. However, in addition to the above described effects, we must also take into account the role of deviations by the independent decision-maker $N + 1$ in the decentralized supply chain. Our next Theorem considers this decision-maker and illustrates the fine balance between the *increased value of relationships* and the *reduced opportunism* of the N decision-makers, and the *additional opportunism* on account of the decision-maker $N + 1$.

For all δ define $\pi^N(\delta) = \max_a \Pi^N(a)$, such that strategy $\sigma(a)$ is a sub-game perfect equilibrium of the centralized supply chain game for this δ (see the conditions in Lemma 3). Define $\pi^{N+1}(\delta) =$

$\max_a \Pi^{N+1}\left(a, \lambda^C\right)$, such that strategy $\sigma\left(a, \lambda^C\right)$ is an equilibrium of the decentralized supply chain game for this δ (see the conditions in Lemma 4). $\pi^N\left(\delta\right)$, $\pi^{N+1}\left(\delta\right)$ are the highest sourcing profits that are achievable as equilibria in respective supply chains, considering all possible actions a (and its profit equivalent counterpart, $\left(a, \lambda^C\right)$ in the decentralized supply chain) that can be sustained as equilibrium. Define $\bar{a} = \arg\max_a \Pi^N\left(a\right)$. Finally, for each action profile a, define the deviation gain of each player n, $n \in \{1, 2, ..., N\}$ in the centralized supply chain as $G_n^N\left(a\right) = D_n^N\left(a\right) - \Pi_n^N\left(a\right)$, and for each player n, $n \in \{1, 2, ..., N+1\}$ in the decentralized supply chain as $G_n^{N+1}\left(a, \lambda^C\right) = D_n^{N+1}\left(a, \lambda^C\right) - \Pi_n^{N+1}\left(a, \lambda^C\right)$. The deviation gain is the increase in its profits that each player can realize by deviating in the deviation period itself; it is the best deviation profit less the profit she would have earned by continuing to act as per the relational strategy.

Theorem 5. *There exists $\delta \in (0, 1)$ where the decentralized supply chain outperforms the more centralized supply chain, $\pi^{N+1}\left(\delta\right) > \pi^N\left(\delta\right)$, if for each player n, $n \in \{1, 2, ..., N\}$ the deviation gain to the value of the relational strategy ratio is lower in the decentralized supply chain:*

$$\frac{G_n^{N+1}\left(\bar{a}, \lambda^C\right)}{V_n^{N+1}\left(\bar{a}, \lambda^C\right)} < \frac{G_n^N\left(\bar{a}\right)}{V_n^N\left(\bar{a}\right)} \ , \ and$$

$$\frac{G_{N+1}^{N+1}\left(\bar{a}, \lambda^C\right)}{V_{N+1}^{N+1}\left(\bar{a}, \lambda^C\right)} \leq \max_n \frac{G_n^{N+1}\left(\bar{a}, \lambda^C\right)}{V_n^{N+1}\left(\bar{a}, \lambda^C\right)}.$$

This result is analogous to Theorem 2. However, unlike the price-only contracts supply chain, in a generic supply chain, we have to look at each player individually, since we don't have the exact form of how the constraints of different players are interlinked. Nevertheless, the central idea behind the result continues to hold in this much more generic setting. From Theorem 4, we know that each player values the relational strategy more in the decentralized supply chain (increased value of relationship). Now, as long as this increase in the value of relational strategy is higher than the increase (if any) in deviation gains, the decentralized supply chain will perform better. The first condition in the theorem ensures this is the case. Over and above this, the second condition ensures that the player $N + 1$ does not become a "bottle neck": the deviation gain to the value of the cooperation ratio is no higher for a new player as compared to other members of the supply chain.

Taken together, the results of this section demonstrate that even in a generic supply chain with generic structure, contracting form, source of incentive misalignment, etc., decentralization of decision-making in supply chains can improve supply chain profits. As with our analysis of price-only contracts, we find that in a supply chain with the possibility of continuing trade, an elemental increase in decentralization

increases the value of relationships for players, decreases the possibilities for opportunistic behavior for the original decision-makers, and creates another decision-maker who must be incentivized. The effect on supply chain performance is a fine balance of these effects. If the first two dominate the third effect, as a whole decentralization makes it easier for the supply chain partners to coordinate and provision inter-temporal trade-offs. This improves supply chain performance.

5. Discussion

The analysis of Section 3 considered the effects of an elemental increase in decentralization in an N-tier, serial, push, supply chain governed by price-only contracts. Section 4 extended the key insights from Section 3 to a generic uncoordinated supply chain. Nevertheless, in both sections we assumed that actions by decision-makers in the supply chain are observable by others. This may not always be the case. Players might instead get only an imperfect signal of the actions. We extended the model of Section 4 to consider such imperfect monitoring of actions. As one would expect, the set of possible relational strategies that can be enforced in equilibrium shrinks in both the more centralized N-tier and the more decentralized $N + 1$-tier supply chains, but as above, the impact of decentralization is driven by the fine balance of our three main effects: *Increased value of relationships*, *Reduced* and *Additional opportunism*. Again, a supply chain with more decentralized decision-making can strictly outperform one with more centralized decision-making.

Our results provide an alternate perspective on improving supply chain performance. Traditional operations literature has suggested a trade-off between the benefits of specialization on one side and the improved incentive alignment by vertical-integration or centralization of decision making, on the other. Our analysis suggests an alternate supply chain strategy of unbundling supply chain activities eliminates the trade-off between specialization and incentive alignment and improves supply chain relationships. Unbundling activities leads to an increased number of decision makers, which when accompanied by continuing trade or long-term relationship, helps mitigate incentive misalignments, while allowing the firms to reap the benefits of each individual firm's specialization and focus, all without the use of any complex coordinating contracts.

There is anecdotal evidence on the use and benefits of this supply chain management strategy in a variety of industries including emerging market cooperatives, urban logistics, and micro-retailing. For instance, consider the world-leading operational and financial performance of the Indian dairy cooperative, Amul, documented in Goldberg et al. (1998). At the heart of the cooperative is a

network of many independent processors or tiers of the supply chain. The independent processors repeatedly interact with each other, these interaction are not governed by any formal contracts but solely by an informal "relationship". While conventional supply chain wisdom would suggest significant inefficiencies in this supply chain or use of complex coordinating contracts, neither are observed; our analysis is in line with the observed excellent operational performance, the existence of many independent processors, the lack of complicated contracting mechanisms and focus on relationships and community. While this and other anecdotal evidence is compelling, a more rigorous large-sample analysis of the role of relationships and supply chain efficiency is needed to validate the practical implications of this study.

This study shows that with the possibility of continuing trade, decentralization of decision-making can be beneficial both for improving supply chain performance and providing additional profit-sharing terms. It is easier to maintain supply chain relationships to coordinate on efficient actions in decentralized supply chains. Each player values coordination in the system more and by virtue of having smaller influence on the system can gain less by damaging a coordinated system. Put differently, the higher anarchy resulting from a breakdown in coordination in a decentralized system, and the lower potential of any individual player to cause a breakdown in coordination, increase the incentives for all players to continue coordination. Further, this increased fear of anarchy and lower potential to cause anarchy allows for additional flexibility in the spectrum of relationships, or balance of power in relationships, that can be acceptable to all players. If these effects surpass the additional opportunism that may arise on account of more decision-makers, supply chain profits can strictly increase on account of more decentralized decision-making.

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Appendix A. Proofs for Section 3

Statements of the additional lemmas and proofs of lemmas and theorems of the main paper are given in their order of appearance.

A.1. **The equilibrium outcome in the N-tier supply chain with one-off trade.**

Lemma 5. *The vector of equilibrium transfer prices $\left(\hat{w}_1^N, \hat{w}_2^N, ..., \hat{w}_N^N\right)$ and the equilibrium order quantity $\hat{Q}^N$ is given by the solution to the following system of equations:*

$$w_1^N = p\bar{F}\left(Q^N\right), \quad w_N^N \equiv c,$$

$$w_n^N = w_{n-1}^N + Q^N \frac{\partial w_{n-1}^N}{\partial Q^N}, \quad n \in \{2, 3, ..., N\}.$$

Proof. The profit of each tier in the N-tier supply chain is given by:

$$\Pi_1^N = pE\left[\min\left\{Q_1^N, Q_2^N, ..., Q_N^N, D\right\}\right] - w_1^N \min\left\{Q_1^N, Q_2^N, ..., Q_N^N\right\}, \quad \text{for} \quad n = 1;$$

$$\Pi_n^N = w_{n-1}^N \min\left\{Q_{n-1}^N, Q_n^N, ..., Q_N^N\right\} - w_n^N \min\left\{Q_n^N, Q_{n+1}^N, ..., Q_N^N\right\}, \quad \text{for} \quad n \in \{2, 3, ..., N-1\};$$

$$\Pi_N^N = w_{N-1}^N \min\left\{Q_{N-1}^N, Q_N^N\right\} - w_N^N Q_N^N, \quad \text{for} \quad n = N.$$

When tier n chooses its order quantity, Q_n^N, and the transfer price, w_{n-1}^N, to maximize its profits (ensuring that it is non-negative), it takes into account how much it will be able to sell: anticipated order quantity Q_{n-1}^N. It can not acquire more inventory than what tier $n+1$ has in stock. By examining the profit functions, we can see that it will never be optimal for tier $n \in \{2, 3, ..., N\}$ to order/produce more than the anticipated order quantity of tier $n-1$. It also does not make sense to order less than the anticipated order quantity, as the tier can always receive a non-negative margin by appropriately setting the transfer price, i.e. $\hat{Q}_n^N = \hat{Q}_{n-1}^N$ for all $n \in \{2, 3, ..., N\}$. In other words, in equilibrium, all tiers order the same quantity: $Q_1^N = ... = Q_n^N = ... = Q_N^N$. We denote this common quantity by Q^N, omitting the subscript for the tier.

Applying the same logic as in Perakis and Roels (2007) p. 1252 and the fact that $\forall n, n \in \{1, 2, ..., N\}$ and $\forall x$ in the support of F, $\varphi_n(x)$ is a decreasing concave function of x; the optimal order quantity in the supply chain is either equal to the lowest value of the support of the demand distribution or uniquely determined by the following system of equations:

$$w_1^N = p\bar{F}\left(Q^N\right), \quad w_N^N \equiv c,$$

(A.1)
$$w_n^N = w_{n-1}^N + Q^N \frac{\partial w_{n-1}^N}{\partial Q^N}, \quad n \in \{2, 3, ..., N\}.$$

The degenerate case when the order quantity in the supply chain is equal to the lowest value of the support of the demand distribution is not of interest. For the rest of the paper, we will concentrate on situations where the solution is determined by the system of equations A.1. □

A.2. **Proof of Lemma 1.** 1a. We start out by showing that $\hat{Q}^N > \hat{Q}^{N+1}$. For a supply chain with N tiers, $\hat{Q}^N$ is determined from $w_N^N = w_{N-1}^N + Q^N \frac{\partial w_{N-1}^N}{\partial Q^N} = c$, while for supply chain with $N+1$ tiers, Q^{N+1} is determined from $w_{N+1}^{N+1} = w_N^{N+1} + Q^{N+1} \frac{\partial w_N^{N+1}}{\partial Q^{N+1}} = c, \implies w_N^{N+1} = c - Q^{N+1} \frac{\partial w_N^{N+1}}{\partial Q^{N+1}}$. We can rewrite this as $\hat{Q}^N$ is a solution to $p\varphi_N(x) = c$ and $\hat{Q}^{N+1}$ is a solution to $p\varphi_N(x) = c - x \frac{\partial \varphi_N(x)}{\partial x}$. We know (see section 3.2) that for all n, $\varphi_n(x)$ is decreasing in x, so we have $\frac{\partial \varphi_N(x)}{\partial x} < 0$ for any x. Since we are only interested in $x \geq 0$ (the order quantity must be non-negative to insure non-negativity of the profit of tier 1), $c - x \frac{\partial \varphi_N(x)}{\partial x} > c$. As φ_N is decreasing in x, $\hat{Q}^{N+1}$ must be lower than $\hat{Q}^N$.

1b. Next we show that $\hat{\Pi}^N > \hat{\Pi}^{N+1}$. The total supply chain profit does not depend on transfer prices (transfer prices only determine how profit is split among the supply chain partners) and is defined only by the order quantity: $\Pi^N(Q) \equiv \sum_1^N \Pi_n^N(Q) = p \int_0^Q \bar{F}(x)\, dx - cQ$. Taking derivative $\frac{\partial}{\partial Q} \Pi^N = p\bar{F}(Q) - c$, thus Π^N is increasing in Q for $Q \leq Q^{FB}$. In part 1, we showed that $Q^{FB} \geq \hat{Q}^N > \hat{Q}^{N+1}$, hence $\hat{\Pi}^N = \Pi^N\left(\hat{Q}^N\right) > \hat{\Pi}^{N+1} = \Pi^{N+1}\left(\hat{Q}^{N+1}\right)$.

2. Lastly we show that $\hat{\Pi}_n^N > \hat{\Pi}_n^{N+1}$ for all $n \in \{1, 2, ..., N\}$. In a supply chain with N tiers, the profit of tier $n \in \{2, 3, ..., N\}$, $\Pi_n^N = \left(w_{n-1}^N - w_n^N\right) Q^N$ and in a supply chain with $N+1$ tiers, $\Pi_n^{N+1} = \left(w_{n-1}^{N+1} - w_n^{N+1}\right) Q^{N+1}$. This can be expressed $\Pi_n^N = p\left(\varphi_{n-1}(Q^N) - \varphi_n(Q^N)\right) Q^N$ and $\Pi_n^{N+1} = p\left(\varphi_{n-1}(Q^{N+1}) - \varphi_n(Q^{N+1})\right) Q^{N+1}$. Using the definition of φ_n, we can rewrite it as:

$$\Pi_n^N = -p\left(Q^N\right)^2 \frac{\partial \varphi_{n-1}(x)}{\partial x}\Big|_{Q^N}, \qquad \Pi_n^{N+1} = -p\left(Q^{N+1}\right)^2 \frac{\partial \varphi_{n-1}(x)}{\partial x}\Big|_{Q^{N+1}}.$$

We know that $\frac{\partial \varphi_{n-1}(x)}{\partial x} < 0 \ \forall x$, hence iff $\left(\hat{Q}^N\right)^2 \left|\frac{\partial \varphi_{n-1}(x)}{\partial x}\Big|_{\hat{Q}^N}\right| > \left(\hat{Q}^{N+1}\right)^2 \left|\frac{\partial \varphi_{n-1}(x)}{\partial x}\Big|_{\hat{Q}^{N+1}}\right|$ is $\hat{\Pi}_n^N > \hat{\Pi}_n^{N+1}$. As $\varphi_{n-1}(x)$ is a decreasing concave function, it holds $\left|\frac{\partial \varphi_{n-1}(x)}{\partial x}\Big|_{\hat{Q}^N}\right| > \left|\frac{\partial \varphi_{n-1}(x)}{\partial x}\Big|_{\hat{Q}^{N+1}}\right|$ and thus $\hat{\Pi}_n^N > \hat{\Pi}_n^{N+1}$. Finally, $\hat{\Pi}_1^N > \hat{\Pi}_1^{N+1}$ as $\hat{Q}^N > \hat{Q}^{N+1}$.

A.3. **Proof of Lemma 2.** Follows directly from the definition of the subgame perfect equilibrium of the infinitely repeated game with discounting.

A.4. **Proof of Theorem 1.** The best profit the supply chain can make is Π^{FB}, which can only be achieved when tier 1 orders Q^{FB}. Next, we derive the range of discount factors where relational contracts of the form $\left(\boldsymbol{w}^N, Q^{FB}\right)$ can be maintained. From Lemma 2, an action profile $\left(\boldsymbol{w}^N, Q^{FB}\right)$ is an equilibrium of the repeated game iff:

$$\boldsymbol{C}^N\left(\boldsymbol{w}^N, Q^{FB}\right) - (1 - \delta)\, \boldsymbol{D}^N\left(\boldsymbol{w}^N\right) - \delta\hat{\boldsymbol{\Pi}}^N \geq \boldsymbol{0}.$$

For $\delta = 1$, there always exists a set of transfer prices $\boldsymbol{w}^N$ such that these inequalities hold for all $n \in \{1, 2, ...N\}$. For $\delta = 0$, the LHS of the inequality is negative or equal to zero at least for some n. Since, for any given δ, $C_n^N - (1 - \delta) D_n^N - \delta\hat{\Pi}_n^N$ is an increasing function of w_{n-1}^N and a decreasing function of w_n^N, $\forall n \in \{1, 2, ..., N\}$. It is a continuous function of δ, thus there exists a unique solution $\left(\bar{\boldsymbol{w}}^N, Q^{FB}, \bar{\delta}^N\right)$ to the following system of equations: $\boldsymbol{C}^N\left(\boldsymbol{w}^N, Q^{FB}\right) - (1 - \delta)\, \boldsymbol{D}^N\left(\boldsymbol{w}^N\right) - \delta\hat{\boldsymbol{\Pi}}^N = \boldsymbol{0}$. This defines the lowest discount factor at which any relational strategy $\left(\boldsymbol{w}^N, Q^{FB}\right)$ can be sustained in equilibrium.

A.5. **Proof of Theorem 2.** If at $\bar{\delta}^N$, the equilibrium conditions for a supply chain with $N+1$ tiers are satisfied with a slack, the threshold discount factor $\bar{\delta}^{N+1}$ will be lower than $\bar{\delta}^N$ (Theorem 1).

$\phi_{N+1}\left(\boldsymbol{w}^{N+1}, Q^{FB}, \delta\right) = C_{N+1}^{N+1}\left(\boldsymbol{w}^{N+1}, Q^{N+1}\right) - \hat{\Pi}_{N+1}^{N+1}$ as $D_{N+1}^{N+1}\left(\boldsymbol{w}^{N+1}\right) = \hat{\Pi}_{N+1}^{N+1}$ (if player $N+1$ deviates, his deviation will be immediately observed by all players in the supply chain and will result in myopic outcome). Thus, $\phi_{N+1}\left(\boldsymbol{w}^{N+1}, Q^{FB}, \delta\right)$ does not depend on the discount factor. Minimizing the deviation profits of players $1, 2, ...N$ will leave the highest possible slack to tier $N+1$. If the remaining portion of the first-best profit that has not been distributed to player $1, 2, ...N$ is strictly higher than $\hat{\Pi}_{N+1}^{N+1}$, there exists a relational strategy that can be enforced for $\delta < \bar{\delta}^N$ and thus $\bar{\delta}^{N+1} < \bar{\delta}^N$. For the marginal relational strategy, $C_n^N\left(\bar{\boldsymbol{w}}^N, Q^N\right) - \left(1 - \bar{\delta}^N\right) D_n^N\left(\bar{\boldsymbol{w}}^N\right) - \bar{\delta}^N \hat{\Pi}_n^N = 0$, summing up the equalities oven n, $n \in \{1, 2, ..., N\}$, we get:

$$(A.2) \qquad \Pi^{FB} = \left(1 - \bar{\delta}^N\right) \bar{D}_{\{1,2,...,N\}}^N\left(\bar{\delta}^N\right) + \bar{\delta}^N \hat{\Pi}^N.$$

Next, denote by $\ddot{\boldsymbol{w}}^M$, $M \geq N$, the solution to

$$\bar{D}_{\{1,2,...,N\}}^M\left(\bar{\delta}^N\right) \quad = \quad \min_{\boldsymbol{w}^M} \sum_{n \in \{1,2,...,N\}} D_n^M\left(w_n^M\right)$$
$$s.t. \quad \forall n \in \{1, 2, ..., N\}$$
$$\phi_n\left(\boldsymbol{w}^M, Q^{FB}, \bar{\delta}^N\right) \geq 0.$$

For a supply chain with $N + 1$ tiers, for $n \in \{1, 2, ..., N\}$:

$$(A.3) \qquad C_n^{N+1}\left(\ddot{\boldsymbol{w}}^{N+1}, Q^{N+1}\right) - \left(1 - \bar{\delta}^N\right) D_n^{N+1}\left(\ddot{\boldsymbol{w}}^{N+1}\right) - \bar{\delta}^N \hat{\Pi}_n^{N+1} \geq 0,$$

which satisfies the equilibrium conditions. Thus, we only need to make sure that for tier $N + 1$ equilibrium conditions also hold at $\ddot{\boldsymbol{w}}^{N+1}$:

$$(A.4) \qquad C_{N+1}^{N+1}\left(\ddot{\boldsymbol{w}}^{N+1}, Q^{N+1}\right) - \left(1 - \bar{\delta}^N\right) \hat{\Pi}_{N+1}^{N+1} - \bar{\delta}^N \hat{\Pi}_{N+1}^{N+1} > 0.$$

By adding inequalities A.3 to inequality A.4 and substituting the expression for Π^{FB} from A.2, we get the condition of the theorem.

A.6. Deviation gains.

Lemma 6. *(1) The best deviation profit of tier n in an N-tier supply chain, $D_n^N\left(y\right)$, is a decreasing function of the input price y. (2) The total deviation gain is higher in a supply chain with N tiers than in a supply chain with $N+1$ tiers: $\bar{D}_{\{1,2,...,N\}}^{N+1}\left(\bar{\delta}^N\right) < \bar{D}_{\{1,2,...,N\}}^N\left(\bar{\delta}^N\right)$.*

Proof. 1. We can rewrite $D_n^N\left(y\right) = p\left(\varphi_{n-1}\left(q_n\left(y\right)\right) - \varphi_n\left(q_n\left(y\right)\right)\right) q_n\left(y\right)$ or equivalently $D_n^N\left(y\right) = -pq_n^2\left(y\right) \frac{\partial \varphi_{n-1}(x)}{\partial x}\big|_{q_n(y)}$. Further, since $\varphi_{n-1}\left(x\right)$ is a decreasing concave function of x, if $y_1 > y_2$, $q_n\left(y_1\right) < q_n\left(y_2\right)$ and following along the lines of the proof of Part 3 of Lemma 1, we obtain $D_n^N\left(y\right)$ is a decreasing function of the input price y.

2. In both supply chains with N and $N + 1$ tiers, the deviation gain of tier n is given by the same function: $D_n^{N+1}\left(w_n\right) = D_n^N\left(w_n\right)$, which is a decreasing function of w_n as we have shown in part 1 of the Lemma. Since $\hat{\Pi}_n^{N+1} < \hat{\Pi}_n^N$ (see Lemma 1), $\ddot{w}_n^{N+1} > \ddot{w}_n^N$ (see the definition in the proof of theorem 2), for all $n \in \{1, 2, ..., N\}$. It follows that $\bar{D}_{\{1,2,...,N\}}^N\left(\bar{\delta}^N\right) < \bar{D}_{\{1,2,...,N\}}^N\left(\bar{\delta}^N\right)$. $\qquad\square$

A.7. **Proof of Theorem 3.** For each $\delta \geq \bar{\delta}_{N+1}$, we will construct relational strategy $\left(\dot{\boldsymbol{w}}^{N+1}, Q^{FB}\right)$ such that for $n \in \{1, 2, ..., N\}$

$$\phi_n\left(\dot{\boldsymbol{w}}^{N+1}, Q^{FB}, \delta\right) \equiv C_n^{N+1}\left(\dot{\boldsymbol{w}}^{N+1}, Q^{FB}\right) - (1-\delta) D_n^{N+1}\left(\dot{\boldsymbol{w}}^{N+1}\right) - \delta \hat{\Pi}_n^{N+1} = 0.$$

With such a relational strategy for $\delta \geq \bar{\delta}_{N+1}$, the constraint for tier $N+1$ is also satisfied:

$$C_{N+1}^{N+1}\left(\dot{\boldsymbol{w}}^{N+1}, Q^{FB}\right) - (1-\delta) D_{N+1}^{N+1}\left(\dot{\boldsymbol{w}}^{N+1}\right) - \delta \hat{\Pi}_{N+1}^{N+1} \geq 0.$$

This holds as $\dot{\boldsymbol{w}}_N^{N+1} < \bar{\boldsymbol{w}}_N^{N+1}$. Hence, the constructed relational strategy $\left(\dot{\boldsymbol{w}}^{N+1}, Q^{FB}\right)$ can be sustained in equilibrium in supply chain with $N+1$ tiers. But, relational strategy $\left(\dot{\boldsymbol{w}}^N, Q^{FB}\right)$ such that for $n \in \{1, 2, ..., N-1\}$ $\dot{\boldsymbol{w}}_n^N = \dot{\boldsymbol{w}}_n^{N+1}$ is not an equilibrium in a supply chain with N tiers. As,

$$C_1^N\left(\dot{\boldsymbol{w}}^N, Q^{FB}\right) - (1-\delta) D_1^N\left(\dot{\boldsymbol{w}}^N\right) = C_1^{N+1}\left(\dot{\boldsymbol{w}}^{N+1}, Q^{FB}\right) - (1-\delta) D_1^{N+1}\left(\dot{\boldsymbol{w}}^{N+1}\right),$$

and $\hat{\Pi}_1^N > \hat{\Pi}_1^{N+1}$, it follows that $\phi_1\left(\dot{\boldsymbol{w}}^N, Q^{FB}, \delta\right) < 0$, which contradicts the conditions of Lemma 2 that define the equilibrium conditions for a supply chain with N tiers.

Appendix B. Proofs for Section 4

B.1. **Proof of Lemma 3.** Follows directly from the definition of subgame perfect equilibrium of a repeated game.

B.2. **Proof of Lemma 4.** Follows directly from the definition of subgame perfect equilibrium of a repeated game and the fact that each player $n \in \{1, 2, ..., N\}$ can deviate both in game Γ and Λ, and tier $N+1$ in game Λ.

B.3. **Proof of Theorem 4.** For all $n \in \{1, 2, ..., N\}$,

1. $\Pi_n^N(a) - \Pi_n^N(\hat{a}) < \Pi_n^N\left(a, \lambda^C\right) - \Pi_n^N\left(\hat{a}, \hat{\lambda}\right)$ as $\chi_n\left(a, \lambda^C\right) < \chi_n\left(\hat{a}, \hat{\lambda}\right)$.

2. $D_n^N(a) \equiv \max\limits_{a_n', \xi} \Pi_n^N\left(a_n', a_{-n}|\xi\right)$.

2a. First, we consider the case where the highest of $D_n^{\Lambda, N+1}(a, \lambda)$ and $D_n^{\Gamma, N+1}(a, \lambda)$ is $D_n^{\Lambda, N+1}(a, \lambda)$:

$D_n^{N+1}(a, \lambda) = D_n^{\Lambda, N+1}(a, \lambda) = \max\limits_{\lambda_n', \xi}\left(\Pi_n^N(a) - \chi_n\left(a, \lambda_n', \lambda_{-n}\right)\right).$

$D_n^N(a) = \max\limits_{a_n', \xi} \Pi_n^N\left(a_n', a_{-n}|\xi\right) > \Pi_n^N(a) > \Pi_n^N(a) - \min\limits_{\lambda_n', \xi} \chi_n\left(a, \lambda_n', \lambda_{-n}\right) = D_n^{\Lambda, N+1}(a, \lambda).$

2b. Now, consider the case where the highest of $D_n^{\Lambda, N+1}(a, \lambda)$ and $D_n^{\Gamma, N+1}(a, \lambda)$ is $D_n^{\Gamma, N+1}(a, \lambda)$:

$D_n^{\Gamma, N+1}(a, \lambda) = \max\limits_{a_n', \xi}\left(\Pi_n^N\left(a_n', a_{-n}|\xi\right) - \chi_n\left(a_n', a_{-n}, \lambda^*\right)\right).$

$D_n^N(a) = \max\limits_{a_n', \xi} \Pi_n^N\left(a_n', a_{-n}|\xi\right) > \max\limits_{a_n', \xi} \Pi_n^N\left(a_n', a_{-n}|\xi\right) - \min\limits_{a_n', \xi} \chi_n\left(a_n', a_{-n}, \hat{\lambda}\right) \geq$

$\geq \max\limits_{a_n', \xi}\left(\Pi_n^N\left(a_n', a_{-n}|\xi\right) - \chi_n\left(a_n', a_{-n}, \hat{\lambda}\right)\right) = D_n^{\Gamma, N+1}(a, \lambda).$

B.4. **Proof of Theorem 5.** We can rewrite the conditions of Lemmas 3 and 4 as

$$\frac{\delta}{(1-\delta)} \geq \frac{\left(D_n^N(a) - \Pi_n^N(a)\right)}{\left(\Pi_n^N(a) - \Pi_n^N(\hat{a})\right)}, \qquad \text{for all } n \in \{1, 2, ..., N\};$$

$$\frac{\delta}{(1-\delta)} \geq \frac{\left(D_n^{N+1}(a, \lambda) - \Pi_n^{N+1}(a, \lambda)\right)}{\left(\Pi_n^{N+1}(a, \lambda) - \Pi_n^N(\hat{a}, \hat{\lambda})\right)}, \quad \text{for all } n \in \{1, 2, ..., N+1\}.$$

The conditions of the theorem follow.

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