Applied Mathematical Modelling 38 (2014) 2476–2489

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

The coordinating contracts for a fuzzy supply chain with effort and price dependent demand

Jing Zhao [a], Jie Wei [b,*]

[a] School of Science, Tianjin Polytechnic University, Tianjin 300387, PR China
[b] General Courses Department, Military Transportation University, Tianjin 300161, PR China

ARTICLE INFO

Article history:
Received 22 August 2012
Received in revised form 22 September 2013
Accepted 11 October 2013
Available online 15 November 2013

Keywords:
Supply chain coordination
Fuzzy demand
Symmetric information
Asymmetric information
Sales effort

ABSTRACT

This paper investigates the coordination of a two-echelon supply chain with fuzzy demand that is dependent on both retail price and sales effort. In contrast with the centralized and decentralized decision models, two coordinating models based on symmetric information and asymmetric information about retailer's scale parameter are developed by game theory, and the corresponding analytical solutions are obtained. Theoretical analysis and numerical examples yield the maximal supply chain profits in two coordination situations are equal to that in the centralized situation and greater than that in the decentralized situation. Furthermore, under asymmetric information contract, the maximal expected profit obtained by the low-scale-level retailer is higher than that under symmetric information contract.

1. Introduction

The supply chain coordination has gained considerable attention lately from both practitioners and researchers. With supply chain coordination, the supplier offers a set of appropriate contract parameters to the retailer such that the retailer's self-profit maximizing objective when making decisions is aligned with the objective of the entire supply chain. Without the coordination, double marginalization [1] exists in a supply chain system because there are two profit margins from the up-stream and down-stream supply chain member, respectively and neither firm considers the entire supply chain profit when making a decision. A properly designed coordination contract can completely eliminate the problem of double marginalization [2]. Many contracts have been presented to improve the supply chain performance, such as buyback contracts, quantity discounts, quantity flexibility agreements and so on [3–8]. Tsay et al. [9], Cachon [10] and Yano and Gilbert [11] provided extensive reviews of coordinating contracts for decentralized supply chains.

As retail pricing is an important vehicle to enhance supply chain revenue, some works have been done that study channel coordination with stochastic and price dependent demand. Apart from the retail price, in most situations, retailer sales effort is also important in influencing demand. A retailer can spur a product's demand by merchandising, commercial advertising, providing attractive shelf space, and guiding consumer purchases with sales personnel, etc. In order to coordinate the supply chain when the sales effort influences the market demand, Taylor [12] designed a returns policy with channel rebates and achieved a win–win outcome, but the retail price was exogenous. He et al. [2] extended Taylor's works and investigated the issue of channel coordination for a supply chain facing stochastic demand that is sensitive to both sales effort and retail price. The foundation of the above-mentioned papers was the manufacturer and the retailer had complete information on each other's operations.

* Corresponding author.
E-mail address: weijie2288@163.com (J. Wei).

0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.10.063

In many scenarios, one party holds private information and the other party makes decisions with limited available information. There are some papers on the coordination problem of a supply chain with asymmetric information, for instance, Corbett et al. [13] studied the design of supply contracts in the presence of asymmetric information, Lau and Lau [14] and Lau et al. [15] modeled a manufacturer and a retailer in a supply chain as a noncooperative game with symmetric and asymmetric information where the market demand was unknown to both manufacturer and retailer and was a function of price only.

In the real world, sometimes probability distribution of market demand may not be available to the decision maker, due, at least in part, to lack of historical data. In this situation, the uncertainty parameters are able to be approximately estimated by managers' judgements, intuitions and experience, and can be characterized as fuzzy variables [16]. The fuzzy theory provided by Zadeh [17] can be an alternative approach to deal with this kind of uncertainty. There have been a few research studies in the area of supply chain modeling with fuzzy demand [18–24]. Wong and Lai [25] provided extensive reviews for the applications of the fuzzy set theory technique in production and operations management. Zhao et al. [26] analyzed the pricing problem of substitutable products with fuzzy manufacturing costs and fuzzy demands. Wei and Zhao [27] and Wei et al. [28] considered pricing decisions in a fuzzy closed-loop supply chain. Wei and Zhao [29] explored the decisions of reverse channel choice in a fuzzy closed-loop supply chain. However, in fuzzy environment, the supply chain coordination problem under asymmetric information has not been considered.

In this paper, we consider a two-echelon supply chain and mainly focus on the fuzziness aspect of effort and price dependent demand. The decision models are developed to study the coordination problem under symmetric and asymmetric information about retailer's scale parameter. We extend the franchise fee contract to coordinate the decentralized decision situation. Contract design in franchising has already been addressed by several researchers. Important theoretical contributions include [30–33]. The approaches indicated above do not address contract design in franchising when private information is an issue [34]. The method used in the paper is similar to that of Liu et al. [35]. Liu et al. [35] investigated online dual channel supply chain system and its joint decisions on production and pricing under information asymmetry, and two kinds of contracts are designed for the decentralized system to coordinate the channel system. The main difference between [35] and this paper is the research problem.

In this paper, (a) When the retailer's scale parameter is a common information for the manufacturer, the manufacturer provides complete knowledge about the manufacturing cost to the retailer, which is required to pay a commission fee. We find the commission fee decreases as the retailer's scale parameter increases. That is, the high-scale-level retailer will pay less commission fee than the low-scale-level retailer. (b) When the retailer's scale parameter is his private information, the low-scale-level retailer may lie to pay less commission fee. To prevent the low-scale-level retailer lying, the manufacturer has to design a contract to reveal the retailer's private information. We design a menu of contracts with a principle-agent method under asymmetric information. By contrasting the centralized and decentralized decision situations, we can see that the maximal supply chain profits in two coordination situations are equal to that in the centralized situation and greater than that in the decentralized situation. Furthermore, under asymmetric information contract, the maximal expected profit obtained by the low-scale-level retailer is higher than that under symmetric information contract.

The organization of the paper is as follows. In Section 2, we present model assumptions and notations and then analyze price and sales effort decisions in the centralized and decentralized settings. Two types of supply chain contracts under symmetric and asymmetric information about retailer's scale parameter to coordinate the decentralized supply chain are designed in Section 3. To demonstrate the performance of contracts, a numerical example is presented in Section 4. Section 5 concludes with summary insights. For clarity of presentation, preliminaries and proofs of all propositions are relegated to the appendices.

2. Decision models

2.1. Model descriptions and assumptions

Consider a two-echelon supply chain consisting of a manufacturer and a retailer. The manufacturer wholesales a product to the retailer, who in turn retails it to the customers. The retailer needs to make his pricing and sales effort decision in order to achieve maximal expected profit. The manufacturer needs to decide the product's wholesale price to achieve his maximal expected profit. The following notations are used to formulate the supply chain model discussed in this paper.

p: unit retail price, which is the retailer's decision variable, satisfying $p > 0$;
θ: sales effort level, which is the retailer's decision variable, satisfying $\theta > 0$;
g: retailer's sales effort cost, which is a function of θ;
b: retailer's scale parameter, $b > 0$;
c: unit manufacturing cost;
w: unit wholesale price, which is the manufacturer's decision variable, satisfying $p \geqslant w$;
D: consumer's demand, which is a function of p and θ;
π_m: manufacture's profit, which is a function of w, p and θ;
π_r: retailer's profit, which is a function of w, p and θ;
π_c: profit of the total supply chain system, which is a function of p and θ.

Throughout the paper, the following assumptions are made:

A1. The function of the customer demand is assumed to be

$$D(p,\theta) = a - \beta p + \gamma\theta, \tag{1}$$

where a represents the market base of this product, β and γ denote the measure of the responsiveness of the consumer demand to retail price and to sales effort level, respectively. Here a, β and γ are nonnegative fuzzy variables since they are usually estimated by the decision-makers' subjective judgements due to lack of historical data.

A2. The unit manufacturing cost c is assumed to be a nonnegative fuzzy variable. It is difficult to get the exact value of the unit manufacturing cost c because of the change of circumstances such as the raw material shortage, the new tax or tariff policy, machine breakdown, and so on. So, this assumption is reasonable.

A3. The makeup $p - c$ and $w - c$, and the consumer's demand $a - \beta p + \gamma\theta$ are non- negative to ensure that the manufacturer and the retailer are profitable. Thus $\mathrm{Pos}(\{p - c < 0\}) = 0,\ \mathrm{Pos}(\{w - c < 0\}) = 0,\ \mathrm{Pos}(\{a - \beta p + \gamma\theta < 0\}) = 0$.

A4. The fuzzy parameters β, γ, a, c are all independent.

A5. The function of sales effort cost $g(\theta) = \frac{b}{2}\theta^2$ is convex, it is obvious that $g(0) = 0,\ g'(\theta) = b\theta > 0,\ g''(\theta) = b > 0$.

A6. The retailer's scale parameter b satisfies the condition $b > \frac{E^2[\gamma]}{2E[\beta]}$, which means b has a lower boundary.

The manufacturer's profit can be expressed as follows

$$\pi_m(w) = (w - c)(a - \beta p + \gamma\theta). \tag{2}$$

The retailer's profit is

$$\pi_r(p,\theta) = (p - w)(a - \beta p + \gamma\theta) - \frac{b}{2}\theta^2. \tag{3}$$

Thus, the profit of the total supply chain system can be written as follows

$$\pi_c(p,\theta) = \pi_m(w) + \pi_r(p,\theta) = (p - c)(a - \beta p + \gamma\theta) - \frac{b}{2}\theta^2. \tag{4}$$

2.2. Centralized decision (CD) model

To establish a performance benchmark, consider a supply chain operated by an integrated-firm which can also be regarded as the manufacturer and the retailer making cooperation. In this case, the wholesale price w, is regarded as inner transfer price, which influences the profit of each participant, but not affect the profit of the total supply chain system. The integrated-firm tries to maximize its expected profit, denoted as $E[\pi_c(p,\theta)]$ (about the expect operator $E[\cdot]$, see Definition 7 in Appendix A), so the expected value model can be formulated as

$$\textbf{CD model}: \max_{(p,\theta)} E[\pi_c(p,\theta)]. \tag{5}$$

Solving the CD model, we can obtain the optimal retail price, the optimal sales effort level, and the maximal expected profit of the integrated-firm. The following proposition gives the results.

Proposition 1. *The expected profit $E[\pi_c(p,\theta)]$ is jointly concave in (p,θ). The optimal retail price and the optimal sales effort level, denoted as p_c^* and θ_c^* respectively, are given as follows*

$$p_c^* = \frac{b(E[a] + E[c\beta]) - \frac{E[\gamma]}{2}\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)d\alpha}{2bE[\beta] - E^2[\gamma]}, \tag{6}$$

$$\theta_c^* = \frac{E[\gamma](E[a] + E[c\beta]) - E[\beta]\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)d\alpha}{2bE[\beta] - E^2[\gamma]}. \tag{7}$$

The maximal expected profit of the integrated-firm, denoted as π_c^*, is

$$\begin{aligned}\pi_c^* &= E[\pi_c(p_c^*,\theta_c^*)]\\ &= -E[\beta](p_c^*)^2 + (E[a] + E[c\beta])p_c^* + E[\gamma]p_c^*\theta_c^* - \frac{b}{2}(\theta_c^*)^2 - \frac{\theta_c^*}{2}\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)d\alpha - \frac{1}{2}\int_0^1 (c_\alpha^L a_\alpha^U + c_\alpha^U a_\alpha^L)d\alpha.\end{aligned} \tag{8}$$

2.3. Decentralized decision (DD) model

In this scenario, the manufacturer first announces his wholesale price, after observing the wholesale price w, the retailer then decides the retail price and the sales effort level. The optimal retail price and the optimal sales effort level, are denoted as $p^*(w)$ and $\theta^*(w)$ respectively. The objective of each participant is to maximize his own expected profit. The following DD model can be formulated as

$$\textbf{DD model}: \begin{cases} \max\limits_{w} E[\pi_m(w, p^*(w), \theta^*(w))] \\ (p^*(w), \theta^*(w)) \text{ solves the problem} \\ \max\limits_{(p,\theta)} E[\pi_r(p, \theta)]. \end{cases} \tag{9}$$

We first derive the retailer's reaction function as follows.

Proposition 2. *In the DD model, given earlier decision w made by the manufacturer, the retailer's optimal retail price and the optimal sales effort level are*

$$p^*(w) = \frac{(bE[\beta] - E^2[\gamma])w + bE[a]}{2bE[\beta] - E^2[\gamma]}, \tag{10}$$

$$\theta^*(w) = \frac{-E[\beta]E[\gamma]w + E[\gamma]E[a]}{2bE[\beta] - E^2[\gamma]}. \tag{11}$$

Then, after knowing the retailer's reaction function, the manufacturer sets the optimal wholesale price to maximize his expected profit $E[\pi_m(w, p^*(w), \theta^*(w))]$.

Proposition 3. *In the DD model, the manufacturer's optimal wholesale price, denoted as w_d^*, is given as*

$$w_d^* = \frac{E[\beta](bE[a] + \frac{E[\gamma]}{2}\int_0^1 (c_\alpha^L \gamma_\alpha^U + c_\alpha^U \gamma_\alpha^L)\,\mathrm{d}\alpha) + E[c\beta](bE[\beta] - E^2[\gamma])}{2bE^2[\beta]}. \tag{12}$$

By substituting Eq. (12) into Eqs. (10) and (11), the retailer's optimal retail price and optimal sales effort level can be obtained. Further, the maximal expected profits of the manufacturer and retailer can be obtained. The following proposition gives the results.

Proposition 4. *In the DD model, the retailer's optimal retail price and optimal sales effort level, given the manufacturer' earlier strategies w_d^*, are*

$$p_d^* = \frac{(bE[\beta] - E^2[\gamma])w_d^* + bE[a]}{2bE[\beta] - E^2[\gamma]}, \tag{13}$$

$$\theta_d^* = \frac{-E[\beta]E[\gamma]w_d^* + E[\gamma]E[a]}{2bE[\beta] - E^2[\gamma]}. \tag{14}$$

The maximal expected profits of the manufacturer and retailer, denoted as π_{md}^* and π_{rd}^* respectively, are

$$\begin{aligned} \pi_{md}^* &= E[(w_d^* - c)(a - \beta p_d^* + \gamma\theta_d^*)] \\ &= (E[a] - E[\beta]p_d^* + E[\gamma]\theta_d^*)w_d^* + E[c\beta]p_d^* - \frac{1}{2}\int_0^1 (c_\alpha^L a_\alpha^U + c_\alpha^U a_\alpha^L)d\alpha - \frac{\theta_d^*}{2}\int_0^1 (c_\alpha^L \gamma_\alpha^U + c_\alpha^U \gamma_\alpha^L)\,d\alpha, \end{aligned} \tag{15}$$

$$\pi_{rd}^* = E\left[(p_d^* - w_d^*)(a - \beta p_d^* + \gamma\theta_d^*) - \frac{b}{2}\theta_d^{*2}\right] = (p_d^* - w_d^*)(E[a] - E[\beta]p_d^* + E[\gamma]\theta_d^{ast}) - \frac{b}{2}\theta_d^{*2}. \tag{16}$$

3. Coordinating contracts

Due to the "double marginalization", $\pi_{md}^* + \pi_{rd}^* < \pi_c^*$. So, in this section, we will design the supply chain coordinating contracts that provide incentives to the manufacturer and the retailer so that the decentralized supply chain behaves exactly the same as the centralized one.

3.1. Coordinating contract under symmetric information

Assume that the retailer's scale parameter b is a common information for the manufacturer and the retailer. The manufacturer provides complete knowledge about the manufacturing cost c to the retailer, which is required to pay a commission fee (denoted as F). To determine optimal contract parameters, we formulate the following model

$$\begin{cases} \max\limits_{F} \pi_m = F \\ subject\ to: \\ \quad E[\pi_r(p^*, \theta^*)] \geqslant \pi^*_{rd} \\ (p^*, \theta^*) \text{ solves the problem} \\ \qquad \max\limits_{(p,\theta)} E[\pi_r(p, \theta)] = E[(p-c)(a-\beta p+\gamma\theta) - \frac{b}{2}\theta^2] - F \end{cases} \tag{17}$$

Solving the model (17), we can obtain the following optimal solutions.

Proposition 5. *The optimal solutions of the model (17), denoted as p^*, θ^*, F^*, are*

$$p^* = p^*_c, \ \ \theta^* = \theta^*_c$$

$$F^* = -E[\beta](p^*_c)^2 + (E[a] + E[c\beta])p^*_c + E[\gamma]p^*_c\theta^*_c - \frac{b}{2}\theta^{*2}_c - \frac{\theta^*_c}{2}\int_0^1 (c^L_\alpha \gamma^U_\alpha + c^U_\alpha \gamma^L_\alpha) d\alpha$$
$$-\frac{1}{2}\int_0^1 (c^L_\alpha a^U_\alpha + c^U_\alpha a^L_\alpha)\ d\alpha - (p^*_d - w^*_d)(E[a] - E[\beta]p^*_d + E[\gamma]\theta^*_d) + \frac{b}{2}\theta^{*2}_d. \tag{18}$$

By analyzing and comparing the results, the following insights can be obtained:

(i) Under the coordinating contract with symmetrical information, the retailer gets the same maximal expected profit as under decentralized decision case, the manufacturer obtains more maximal expected profit than under the decentralized decision case.
Therefore, in order to attract the retailer in agreeing to enter this coordinating contract, the manufacturer should adjust F such that the retailer gets a profit which is larger than that under the decentralized decision.
(ii) The retailer's optimal effort level in coordination contract case $\theta^* = \theta^*_c$ is more than that in the decentralized decision case θ^*_d by comparing Eq. (7) with Eq. (14).
(iii) It follows from Eq. (18) that F^* decreases as the retailer's scale parameter b increases. That is, the high-scale-level retailer will pay less commission fee than the low-scale-level retailer.

3.2. Coordinating contract under asymmetric information

Consider the following situation: the manufacturer provides complete knowledge about the manufacturing cost c to the retailer, yet the retailer has his private information, the scale parameter b. For simplicity, we assume the scale parameter b has only two levels: the high level b_H and the low level b_L. In order to decrease the commission fee F^* and obtain more profit, the low-scale-level retailer may lie to the manufacturer that his scale level is high. To prevent the low-scale-level retailer lying, the manufacturer has to design a contract to reveal the retailer's private information.

We summary the notations used in this subsection as follows.

F_H: the commission fee paid by the high-scale-level retailer to the manufacturer;
F_L: the commission fee paid by the low-scale-level retailer to the manufacturer;
$E[\pi_{r_H}(p_H, \theta_H)]$: the expected profit of the high-scale-level retailer;
$E[\pi_{r_L}(p_L, \theta_L)]$: the expected profit of the low-scale-level retailer;
$E[\pi_{r_H}(p_L, \theta_L)]$: the expected profit of the high-scale-level retailer lied to the manufacturer that his scale level is low;
$E[\pi_{r_L}(p_H, \theta_H)]$: the expected profit of the low-scale-level retailer lied to the manufacturer that his scale level is high;
π^*_{rdH}: the expected profit of the high-scale-level retailer under decentralized decision scenario;
π^*_{rdL}: the expected profit of the low-scale-level retailer under decentralized decision scenario.
To obtain the contract parameters, we establish the following model

$$\begin{cases} \max\limits_{F_H, F_L} \pi_m = F_H + F_L \\ \text{subject to}: \\ \quad E[\pi_{r_H}(p^*_H, \theta^*_H)] \geqslant \pi^*_{rdH} \\ \quad E[\pi_{r_L}(p^*_L, \theta^*_L)] \geqslant \pi^*_{rdL} \\ \quad E[\pi_{r_H}(p^*_H, \theta^*_H)] \geqslant E[\pi_{r_H}(p^*_L, \theta^*_L)] \\ \quad E[\pi_{r_L}(p^*_L, \theta^*_L)] \geqslant E[\pi_{r_L}(p^*_H, \theta^*_H)] \\ (p^*_H, \theta^*_H) \text{ solves the problem} \\ \qquad \max\limits_{(p_H,\theta_H)} E[\pi_{r_H}(p_H, \theta_H)] = E[(p_H - c)(a - \beta p_H + \gamma\theta_H) - \frac{b_H}{2}\theta^2_H] - F_H. \\ (p^*_L, \theta^*_L) \text{ solves the problem} \\ \qquad \max\limits_{(p_L,\theta_L)} E[\pi_{r_L}(p_L, \theta_L)] = E\Big[(p_L - c)(a - \beta p_L + \gamma\theta_L) - \frac{b_L}{2}\theta^2_L\Big] - F_L. \end{cases} \tag{19}$$

Solving the model (19), we obtain the optimal solution as follows.

Proposition 6. *The optimal solution of the model* (19) *is denoted as* $(p_L^*, \theta_L^*, F_L^*, p_H^*, \theta_H^*, F_H^*)$, *where*

$$
\begin{aligned}
p_L^* &= \frac{b_L(E[a]+E[c\beta]) - \frac{E[\gamma]}{2}\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)\,\mathrm{d}\alpha}{2b_LE[\beta]-E^2[\gamma]},\\
\theta_L^* &= \frac{E[\gamma](E[a]+E[c\beta]) - E[\beta]\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)\,\mathrm{d}\alpha}{2b_LE[\beta]-E^2[\gamma]},\\
F_L^* &= -E[\beta]p_L^{*2} + (E[a]+E[c\beta])p_L^* + E[\gamma]p_L^*\theta_L^* - \frac{b_L}{2}\theta_L^{*2} - \frac{1}{2}\int_0^1 (c_\alpha^L a_\alpha^U + c_\alpha^U a_\alpha^L)\,\mathrm{d}\alpha\\
&\quad - \frac{\theta_L^*}{2}\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)\,\mathrm{d}\,\alpha - (p_{dH}^* - w_{dH}^*)(E[a]-E[\beta]p_{dH}^* + E[\gamma]\theta_{dH}^*) + \frac{(b_L-b_H)}{2}\theta_H^{*2} + \frac{b_H}{2}(\theta_{dH}^*)^2,\\
p_H^* &= \frac{b_H(E[a]+E[c\beta]) - \frac{E[\gamma]}{2}\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)\,\mathrm{d}\alpha}{2b_HE[\beta]-E^2[\gamma]},\\
\theta_H^* &= \frac{E[\gamma](E[a]+E[c\beta]) - E[\beta]\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)\,\mathrm{d}\alpha}{2b_HE[\beta]-E^2[\gamma]},\\
F_H^* &= -E[\beta]p_H^{*2} + (E[a]+E[c\beta])p_H^* + E[\gamma]p_H^*\theta_H^* - \frac{b_H}{2}\theta_H^{*2} - \frac{\theta_H^*}{2}\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)\,\mathrm{d}\alpha\\
&\quad - \frac{1}{2}\int_0^1 (c_\alpha^L a_\alpha^U + c_\alpha^U a_\alpha^L)\,\mathrm{d}\alpha - (p_{dH}^* - w_{dH}^*)(E[a]-E[\beta]p_{dH}^* + E[\gamma]\theta_{dH}^*) + \frac{b_H}{2}(\theta_{dH}^*)^2.
\end{aligned}
\tag{20}
$$

4. Numerical examples

In this section, we give numerical examples in order to illustrate the proposed models. The data adopted in the numerical example are estimated from clothing retail industry in China (www.chinairn.com and www.ceicdata.com/Chinese.htm). These data have been appropriately manipulated, for example, standardization and non-dimensionalization, before being employed, which satisfies or closely complies with certain assumptions of this research. We think that these data can represent the real-world condition as closely as possible due to the difficulty of accessing the actual industry data. The relationship between linguistic expressions and triangular fuzzy variables for the manufacturing cost, market base, price elasticity and the sales effort elasticity are often determined by experts' experiences, as shown in Table 1.

As for how to construct triangular fuzzy numbers, we adopt the method presented in [36]. Specifically, the managers or experts may give a set of scores on manufacturing costs, market bases, price elastic coefficients, service elastic coefficients and service cost coefficients. The set of collected scores is viewed as a sample from the possibility distribution of the grading process, and is used to estimate the parameters of the fuzzy number. When estimating the mode and spreads of a fuzzy number, a weight determination technique to associate a weight with each score is employed. Please see [36] for more details.

Firstly, we investigate the change of the optimal decisions and expected profits with the parameter b in fuzzy environments.

Consider the case where the manufacturing cost c is medium (about 14), the market bases a is small (about 150), price elasticity β is sensitive (about 0.5), and sales effort elasticity γ is sensitive (about 0.3). Using Table 1, the fuzzy parameters $c = (9, 14, 18)$, $a = (100, 150, 220)$, $\beta = (0.3, 0.5, 1)$, $\gamma = (0.1, 0.3, 0.6)$. Using the Definition 7 in Appendix A, similar to Example 2, the expected values are

$$E[c] = 13.73,\quad E[a] = 155,\quad E[\beta] = 0.575,\quad E[\gamma] = 0.325.$$

Table 1
Relation between linguistic expression and triangular fuzzy variable.

	Linguistic expression	Triangular fuzzy variable
Manufacturing cost c	High (about 30)	(20,30,35)
	Medium (about 14)	(9,14,18)
	Low (about 6)	(5,6,9)
Market base a	Large (about 300)	(230,300,350)
	Small (about 150)	(100,150,220)
Price elasticity β	Very sensitive (about 1.5)	(1,1.5,2)
	Sensitive (about 0.5)	(0.3,0.5,1)
Sales effort elasticity γ	Very sensitive (about 1)	(0.6,1,1.5)
	Sensitive (about 0.3)	(0.1,0.3,0.6)

According to the Assumption A6., we know the parameter b should satisfy $b > 0.09$.

Using the Definition 5 in Appendix A, similar to Example 1, the α-optimistic value and α-pessimistic values of c, a, β and γ are

$$c_{1\alpha}^L = 9 + 5\alpha, \quad a_{1\alpha}^L = 100 + 50\alpha, \quad \beta_\alpha^L = 0.3 + 0.2\alpha, \quad \gamma_\alpha^L = 0.1 + 0.2\alpha,$$

$$c_{1\alpha}^U = 18 - 4\alpha, \quad a_{1\alpha}^U = 220 - 70\alpha, \quad \beta_\alpha^U = 1 - 0.5\alpha, \quad \gamma_\alpha^U = 0.6 - 0.3\alpha.$$

Tables 2,3 and Figs. 1,2 show that the change of the optimal decisions and expected profits with the parameter b under centralized and decentralized decision cases in fuzzy environments.

Through the analysis, we gain the following intuitive insights:

(1.1) From Fig. 1, the maximal expected profit of the total system under centralized decision case is higher than that under decentralized decision case and both maximal expected profits decrease as the retailer's scale parameter increases.
(1.2) From Fig. 2, both the optimal retail price and effort level will decrease as the retailer's scale parameter increases, no matter in centralized decision case or in decentralized decision case. Whereas the optimal wholesales price increases slightly as the retailer's scale parameter increases.
(1.3) From Fig. 2, the retail price is lower under centralized decision case than that under decentralized decision case, and yet effort level higher.

Table 2
Optimal decisions and expected profits under centralized decision case.

b	p_c^*	θ_c^*	π_c^*
0.3	199.26	202.23	13827
0.5	171.25	103.13	11742
0.7	161.67	69.216	11028
0.9	156.83	52.086	10668

Table 3
Optimal decisions and expected profits under decentralized decision case.

b	p_d^*	θ_d^*	w_d^*	π_{md}^*	π_{rd}^*	π_{cd}^*
0.3	233.78	99.99	141.48	7028.5	3399.1	10428
0.5	220.03	50.89	141.73	5985.8	2877.8	8863.6
0.7	215.35	34.13	141.84	5629.0	2699.4	8328.3
0.9	212.99	25.67	141.89	5448.8	2609.2	8058

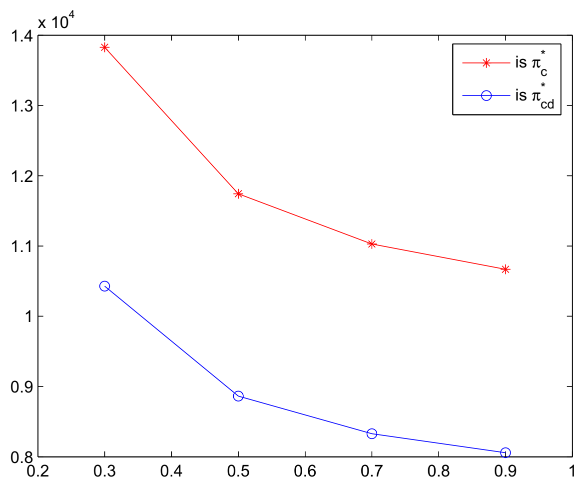

Fig. 1. The change of π_c^* and π_{cd}^* with the parameter b.

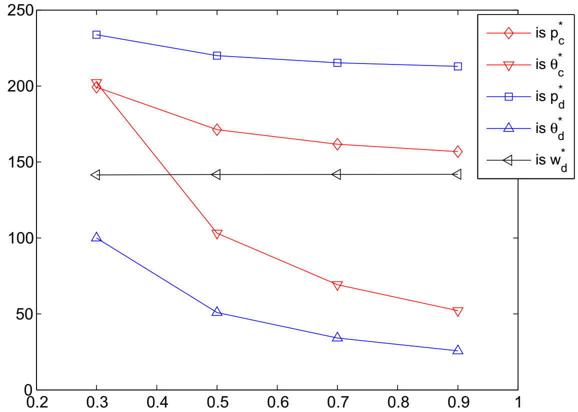

Fig. 2. The change of p, w and θ with the parameter b.

Table 4
Optimal results under symmetric information contract.

b	p^*	θ^*	$\pi_m^*(F^*)$	π_r^*	$\pi_c^* = \pi_m^* + \pi_r^*$
0.3	199.26	202.23	10427.9	3399.1	13827
0.5	171.25	103.13	8864.2	2877.8	11742
0.7	161.67	69.216	8328.6	2699.4	11028
0.9	156.83	52.086	8058.8	2609.2	10668

Table 5
Optimal results of the low scale retailer under asymmetric information contract.

b_H	p_L^*	θ_L^*	$\pi_{mL}^*(F_L^*)$	π_{rL}^*	π_{cL}^*
0.5	199.26	202.23	9886	3941	13827
0.7	199.26	202.23	10170	3647	13827
0.9	199.26	202.23	10404	3423	13827

($b_L = 0.3$).

Table 6
Optimal results of the high scale retailer under asymmetric information contract.

b_H	p_H^*	θ_H^*	$\pi_{mH}^*\ (F_H^*)$	π_{rH}^*	π_{cH}^*
0.5	171.25	103.13	8864.2	2877.8	11742
0.7	161.67	69.216	8328.6	2699.4	11028
0.9	156.83	52.086	8058.8	2609.2	10668

($b_L = 0.3$).

Then, we illustrate the effectiveness of the proposed coordination mechanisms for symmetric and asymmetric information by using the numerical example. The parameters are the same values as before. The results obtained are given in Tables 4–6 and Figs. 3,4.

Comparing the results presented in the above Tables, we have the following insights:

(2.1) Both contracts can achieve supply chain coordination, in which the independent retailer makes the same decisions as the centralized decision case.
(2.2) Under both contracts, the manufacturer obtains higher maximal expected profit than under the decentralized decision.
(2.3) Under symmetric information contract, the maximal expected profit of the retailer is equal to that under the decentralized decision.
(2.4) Under asymmetric information contract, the maximal expected profit of the high-scale-level retailer is equal to that under the decentralized decision, the maximal expected profit of the low-scale-level retailer is higher than that under the decentralized decision.

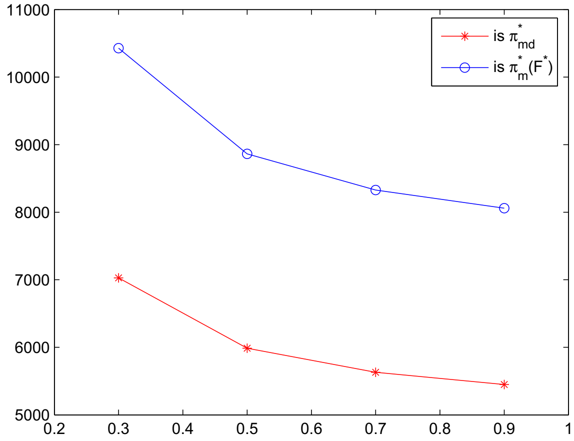

Fig. 3. The change of π^*_{md} and $\pi^*_m(F^*)$ with the parameter b.

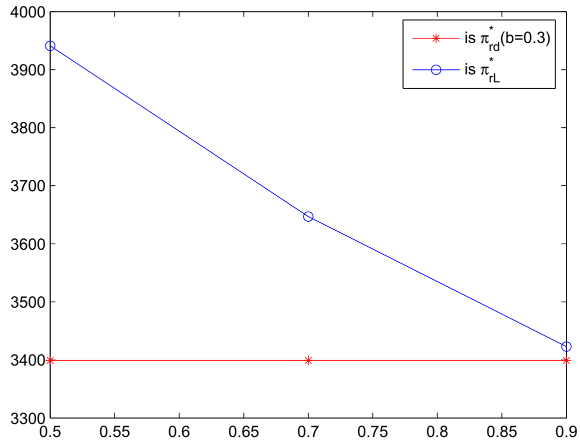

Fig. 4. π^*_{rd} $(b = 0.3)$ and b_L=0.3 the change of π^*_{rL} and π^*_{cd} with the parameter b_H.

5. Conclusions

Different from the conventional studies, this paper considers the case in which the demand is fuzzy rather than stochastic or determinate and dependent on retail price and sales effort. We develop the decision models to determine the optimal price and effort level for centralized and decentralized supply chains. Based on the analytical solutions, the corresponding maximal expected profit can be obtained. To coordinate the decentralized supply chain, we design two coordinating contracts under symmetric and asymmetric information about the retailer's scale parameter. By analyzing numerical examples, we find that the maximal supply chain profits in two coordination situations are equal to that in the centralized situation and higher than that in the decentralized situation.

Compared to the traditional approach, the proposed approach does not need plenty of data to model the uncertain demand and the uncertain manufacturing cost, which can make use of the subjective estimation based on decision maker's judgment, experience and intuitions. It is appropriate when the situation is ambiguous and lacks historical data.

Our results, however, are based upon simplistic assumptions about the demand function. Thus, there are possible extensions to improve our model. Such as, different or more general forms of the demand function can be used to analyze the problem, the supply chain with many manufacturers and many retailers and the model over multiple periods can also be considered in the future.

Acknowledgments

This work was supported by the National Science Foundation of China Grant Nos. 71001106, 71301116, 71371186.

Appendix A. Preliminaries

A possibility space is defined as a triplet $(\Theta, \mathcal{P}(\Theta), \text{ Pos})$, where Θ is a nonempty set, $\mathcal{P}(\Theta)$ the power set of Θ, and Pos a possibility measure. Each element in $\mathcal{P}(\Theta)$ is called a fuzzy event. For each event A, $\text{Pos}(A)$ indicates the possibility that A will occur. Nahmias [37] and Liu [38] gave the following four axioms.

Axiom 1. $\text{Pos}(\Theta) = 1$.
Axiom 2. $\text{Pos}(\phi) = 0$, where ϕ denotes the empty set.
Axiom 3. $\text{Pos}(\bigcup_{i=1}^{m} A_i) = \sup_{1\leqslant i\leqslant m} \text{Pos}(A_i)$ for any collection A_i in $\mathcal{P}(\Theta)$.
Axiom 4. Let Θ_i be nonempty sets, on which Pos_i is possibility measure satisfying the first three axioms, $i = 1, 2, \ldots, n$, and $\Theta = \prod_{i=1}^{n} \Theta_i$. Then

$$\text{Pos}(A) = \sup_{(\theta_1,\theta_2,\ldots,\theta_n)\in A} \text{Pos}_1(\theta_1) \wedge \text{Pos}_2(\theta_2) \wedge \cdots \wedge \text{Pos}_n(\theta_n),$$

for each $A \in \mathcal{P}(\Theta)$. In that case we write $\text{Pos} = \wedge_{i=1}^{n} \text{Pos}_i$.

Lemma 1 [38]. *Suppose that $(\Theta_i, \mathcal{P}(\Theta_i), \text{ Pos}_i)$ is a possibility space, $i = 1, 2, \ldots, n$. By Axiom 4, $(\prod_{i=1}^{n} \Theta_i, \mathcal{P}(\prod_{i=1}^{n} \Theta_i), \wedge_{i=1}^{n} \text{ Pos}_i)$ is also a possibility space, which is called the product possibility space.*

Definition 1 [37]. A fuzzy variable is defined as a function from the possibility space $(\Theta, \mathcal{P}(\Theta), \text{ Pos})$ to the set of real numbers and its membership function is derived from the possibility by

$$\mu_\xi(x) = \text{Pos}(\{\theta \in \Theta | \xi(\theta) = x\}), \forall x \in R.$$

Definition 2 [38]. A fuzzy variable ξ is said to be nonnegative (or positive) if $\text{Pos}(\{\xi < 0\}) = 0$ (or $\text{Pos}(\{\xi \leq 0\}) = 0$).

Definition 3 [38]. Let $f : R^n \to R$ be a function, and ξ_i a fuzzy variable defined on the possibility space $(\Theta_i, \mathcal{P}(\Theta_i), \text{ Pos}_i), i = 1, 2, \ldots, n$, respectively. Then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ is a fuzzy variable defined on the product possibility space $(\prod_{i=1}^{n} \Theta_i, \mathcal{P}(\prod_{i=1}^{n} \Theta_i), \wedge_{i=1}^{n} \text{ Pos}_i)$.

The independence of fuzzy variables was discussed by several researchers, such as [38,37,39].

Definition 4 [38]. The fuzzy variables $\xi_1, \xi_2, \ldots, \xi_n$ are independent if for any sets $\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_n$ of R,

$$\text{Pos}(\{\xi_i \in \mathcal{B}_i, i = 1, 2, \ldots, n\}) = \min_{1\leqslant i\leqslant n} \text{Pos}(\{\xi_i \in \mathcal{B}_i\}).$$

Lemma 2 [40]. *Let ξ_i be independent fuzzy variable, and $f_i : R \to R$ function, $i = 1, 2, \ldots, m$. Then $f_1(\xi_1), f_2(\xi_2), \ldots, f_m(\xi_m)$ are independent fuzzy variables.*

Definition 5 [38]. Let ξ be a fuzzy variable on the possibility space $(\Theta, \mathcal{P}(\Theta), \text{ Pos})$, and $\alpha \in (0, 1]$. Then

$$\xi_\alpha^L = \inf\{r | \text{Pos}(\{\xi \leqslant r\}) \geqslant \alpha\} \text{ and } \xi_\alpha^U = \sup\{r | \text{Pos}(\{\xi \geqslant r\}) \geqslant \alpha\}$$

are called the α-pessimistic value and the α-optimistic value of ξ, respectively.

Example 1. The triangular fuzzy variable $\xi = (a_1, a_2, a_3)$ has its α-pessimistic value and α-optimistic value

$$\xi_\alpha^L = a_2\alpha + a_1(1 - \alpha) \text{ and } \xi_\alpha^U = a_2\alpha + a_3(1 - \alpha).$$

Lemma 3 [41]. *Let ξ_i be independent fuzzy variables defined on the possibility spaces $(\Theta_i, \mathcal{P}(\Theta_i), \text{Pos}_i)$ with continuous membership function, $i = 1, 2, \ldots, n$, and $f : X \subset \mathcal{R}^n \to \mathcal{R}$ a measurable function. If $f(x_1, x_2, \ldots, x_n)$ is monotonic with respect to x_i, respectively, then*

(a) $f_\alpha^U(\xi) = f(\xi_{1\alpha}^V, \xi_{2\alpha}^V, \ldots, \xi_{n\alpha}^V)$, *where* $\xi_{i\alpha}^V = \xi_{i\alpha}^U$, *if* $f(x_1, x_2, \ldots, x_n)$ *is nondecreasing with respect to* x_i; $\xi_{i\alpha}^V = \xi_{i\alpha}^L$, *otherwise,*
(b) $f_\alpha^L(\xi) = f(\xi_{1\alpha}^{\overline{V}}, \xi_{2\alpha}^{\overline{V}}, \ldots, \xi_{n\alpha}^{\overline{V}})$, *where* $\xi_{i\alpha}^{\overline{V}} = \xi_{i\alpha}^L$, *if* $f(x_1, x_2, \ldots, x_n)$ *is nondecreasing with respect to* x_i; $\xi_{i\alpha}^{\overline{V}} = \xi_{i\alpha}^U$, *otherwise,*

where $f_\alpha^U(\xi)$ and $f_\alpha^L(\xi)$ denote the α-optimistic value and the α-pessimistic value of the fuzzy variable $f(\xi)$, respectively.

Definition 6 [42]. Let $(\Theta, \mathcal{P}(\Theta), \text{ Pos})$ be a possibility space and A a set in $\mathcal{P}(\Theta)$. The credibility measure of A is defined as

$$\text{Cr}(A) = \frac{1}{2}(1 + \text{Pos}(A) - \text{Pos}(A^c)),$$

where A^c denotes the complement of A.

Definition 7 [42]. Let ξ be a fuzzy variable. The expected value of ξ is defined as

$$E[\xi] = \int_0^{+\infty} \text{Cr}\ (\{\xi \geqslant x\})\ \mathrm{d}x - \int_{-\infty}^{0} \text{Cr}(\{\xi \leqslant x\})\ \mathrm{d}x$$

provided that at least one of the two integrals is finite.

Example 2. The triangular fuzzy variable $\xi = (a_1, a_2, a_3)$ has an expected value

$$E[\xi] = \frac{a_1 + 2a_2 + a_3}{4}.$$

Definition 8 [42]. Let f be a function on $R \to R$ and ξ be a fuzzy variable. Then the expected value $E[f(\xi)]$ is defined as

$$E[f(\xi)] = \int_0^{+\infty} \text{Cr}(\{f(\xi) \geqslant x\})\ \mathrm{d}x - \int_{-\infty}^{0} \text{Cr}(\{f(\xi) \leqslant x\})\ \mathrm{d}x$$

provided that at least one of the two integrals is finite.

Lemma 4 [43]. *Let ξ be a fuzzy variable with finite expected value. Then*

$$E[\xi] = \frac{1}{2}\int_0^1 (\xi_\alpha^L + \xi_\alpha^U)\ \mathrm{d}\alpha.$$

Lemma 5 [43]. *Let ξ and η be independent fuzzy variables with finite expected values. Then for any numbers a and b,*

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta].$$

Appendix B. Proofs of propositions

Proof of Proposition 1. Noting that the fuzzy variables a, c, β and γ are all independent and nonnegative, the expected profit $E[\pi_c(p,\theta)]$ can be expressed as

$$\begin{aligned} E[\pi_c(p,\theta)] &= E\left[(p-c)(a-\beta p+\gamma\theta) - \frac{b}{2}\theta^2\right] \\ &= -E[\beta]p^2 + (E[a] + E[c\beta])p + E[\gamma]p\theta - \frac{b}{2}\theta^2 - \frac{\theta}{2}\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)\ \mathrm{d}\alpha - \frac{1}{2}\int_0^1 (c_\alpha^L a_\alpha^U + c_\alpha^U a_\alpha^L)\ \mathrm{d}\alpha, \end{aligned} \tag{21}$$

where $c_\alpha^L, c_\alpha^U, \gamma_\alpha^L, \gamma_\alpha^U, a_\alpha^L, a_\alpha^U$ are the α-pessimistic and α-optimistic values of c, γ, a, respectively (see Definition 5 in Appendix A).

The first-order and second-order partial derivatives of $E[\pi_c(p,\theta)]$ to p and θ are

$$\frac{\partial E[\pi_c(p,\theta)]}{\partial p} = -2E[\beta]p + E[\gamma]\theta + E[a] + E[c\beta], \tag{22}$$

$$\frac{\partial E[\pi_c(p,\theta)]}{\partial \theta} = -b\theta + E[\gamma]p - \frac{1}{2}\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)\ \mathrm{d}\alpha, \tag{23}$$

$$\frac{\partial^2 E[\pi_c(p,\theta)]}{\partial p^2} = -2E[\beta], \tag{24}$$

$$\frac{\partial^2 E[\pi_c(p,\theta)]}{\partial \theta^2} = -b, \tag{25}$$

$$\frac{\partial^2 E[\pi_c(p,\theta)]}{\partial p \partial \theta} = \frac{\partial^2 E[\pi_c(p,\theta)]}{\partial \theta \partial p} = E[\gamma]. \tag{26}$$

By Eqs. (24)–(26), Hessian matrix

$$H_1 = \begin{bmatrix} \frac{\partial^2 E[\pi_c(p,\theta)]}{\partial p^2} & \frac{\partial^2 E[\pi_c(p,\theta)]}{\partial p \partial \theta} \\ \frac{\partial^2 E[\pi_c(p,\theta)]}{\partial \theta \partial p} & \frac{\partial^2 E[\pi_c(p,\theta)]}{\partial \theta^2} \end{bmatrix} = \begin{bmatrix} -2E[\beta] & E[\gamma] \\ E[\gamma] & -b \end{bmatrix}. \tag{27}$$

According to assumption A5., $b > \frac{E^2[\gamma]}{2E[\beta]}$, the Hessian matrix H_1 is negative definite, so $E[\pi_c(p,\theta)]$ is jointly concave with respect to (p,θ).

By setting Eqs. (22) and (23) to zero and solving them with respect to p and θ simultaneously, the results (6) and (7) can be obtained.

Substituting p_c^* and θ_c^* into Eq. (21) yields Eq. (8). □

Proof of Proposition 2. It follows from Eq. (3) that

$$E[\pi_r(p,\theta)] = E[(p-w)(a-\beta p+\gamma\theta) - \frac{b}{2}\theta^2] = (p-w)(E[a] - E[\beta]p + E[\gamma]\theta) - \frac{b}{2}\theta^2. \tag{28}$$

The first-order and second-order partial derivatives of $E[\pi_r(p,\theta)]$ to p and θ can be shown as

$$\frac{\partial E[\pi_r(p,\theta)]}{\partial p} = -2E[\beta]p + E[\gamma]\theta + E[a] + E[\beta]w, \tag{29}$$

$$\frac{\partial E[\pi_r(p,\theta)]}{\partial \theta} = -b\theta + E[\gamma](p-w), \tag{30}$$

$$\frac{\partial^2 E[\pi_r(p,\theta)]}{\partial p^2} = -2E[\beta] < 0, \tag{31}$$

$$\frac{\partial^2 E[\pi_r(p,\theta)]}{\partial \theta^2} = -b < 0, \tag{32}$$

$$\frac{\partial^2 E[\pi_r(p,\theta)]}{\partial p \partial \theta} = \frac{\partial^2 E[\pi_r(p,\theta)]}{\partial \theta \partial p} = E[\gamma]. \tag{33}$$

By Eqs. (31)–(33), Hessian matrix

$$H_2 = \begin{bmatrix} \frac{\partial^2 E[\pi_r(p,\theta)]}{\partial p^2} & \frac{\partial^2 E[\pi_r(p,\theta)]}{\partial p \partial \theta} \\ \frac{\partial^2 E[\pi_r(p,\theta)]}{\partial \theta \partial p} & \frac{\partial^2 E[\pi_r(p,\theta)]}{\partial \theta^2} \end{bmatrix} = \begin{bmatrix} -2E[\beta] & E[\gamma] \\ E[\gamma] & -b \end{bmatrix} \tag{34}$$

is negative definite when $b > \frac{E^2[\gamma]}{2E[\beta]}$, so $E[\pi_r(p,\theta)]$ is jointly concave with respect to (p,θ).

By setting Eqs. (29) and (30) to zero and solving them simultaneously with respect to p and θ, the retailer's optimal retail price and the optimal sales effort level can be established as in Eqs. (10) and (11). □

Proof of Proposition 3. Substituting Eqs. (10) and (11) into Eq. (2), we have

$$\begin{aligned} E[\pi_m(w,p^*(w),\theta^*(w))] &= E[(w-c)(a-\beta p^*(w)+\gamma\theta^*(w))] \\ &= E[a]w - E[\beta]wp^*(w) + E[\gamma]w\theta^*(w)_+E[c\beta]p^*(w) - \frac{\theta^*(w)}{2}\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)\,\mathrm{d}\alpha - \frac{1}{2}\int_0^1 (c_\alpha^L a_\alpha^U \\ &\quad + c_\alpha^U a_\alpha^L)\,\mathrm{d}\alpha. \end{aligned} \tag{35}$$

The first-order and second-order partial derivative of $E[\pi_m(w,p^*(w),\theta^*(w))]$ with respect to w are

$$\frac{\partial E[\pi_m(w,p^*(w),\theta^*(w))}{\partial w} = E[\gamma]\theta^*(w) - E[\beta]p^*(w) + w\left[E[\gamma]\frac{\partial\theta^*(w)}{\partial w} - E[\beta]\frac{\partial p^*(w)}{\partial w}\right], \tag{36}$$

$$\frac{\partial^2 E[\pi_m(w,p^*(w),\theta^*(w))]}{\partial w^2} = \frac{-bE^2[\beta]}{2bE[\beta] - E^2[\gamma]}, \tag{37}$$

where

$$\frac{\partial p^*(w)}{\partial w} = \frac{bE[\beta] - E^2[\gamma]}{2bE[\beta] - E^2[\gamma]}, \quad \frac{\partial \theta^*(w)}{\partial w} = \frac{-E[\beta]E[\gamma]}{2bE[\beta] - E^2[\gamma]}.$$

It follows from Eq. (37) and $b > \frac{E^2[\gamma]}{2E[\beta]}$ that the expected profit $E[\pi_m(w, p^*(w), \theta^*(w))]$ is concave with respect to w. By setting Eq. (36) to zero, the first order condition can be obtained as

$$E[\gamma]\theta^*(w) - E[\beta]p^*(w) + w\left[E[\gamma]\frac{\partial \theta^*(w)}{\partial w} - E[\beta]\frac{\partial p^*(w)}{\partial w}\right] = 0. \tag{38}$$

Solving Eq. (38) yields Eq. (12). □

Proof of Proposition 5. Noting that the first-order and second-order partial derivatives of $E[\pi_r(p, \theta)]$ to p and θ are equal to the first-order and second-order partial derivatives of $E[\pi_c(p, \theta)]$ to p and θ, respectively. So, similar to the Proposition 1, the optimal retail price and the optimal sales effort level, *i.e.* p^* and θ^*, respectively, are equal to p_c^* and θ_c^*.

For each given (p^*, θ^*), the value of F should be selected as big as possible in order to satisfy the condition $E[\pi_r(p^*, \theta^*)] \geqslant \pi_{rd}^*$ and to maximize the objective function π_m. Therefore, the optimal F^* is given at the point where $E[\pi_r(p^*, \theta^*)] \geqslant \pi_{rd}^*$ is satisfied with equality. Therefore,

$$\begin{aligned} F^* &= E[\pi_c(p_c^*, \theta_c^*)] - \pi_{rd}^* = E[(p_c^* - c)(a - \beta p_c^* + \gamma\theta_c^*) - \frac{b}{2}\theta_c^{*2}] - E[(p_d^* - w_d^*)(a - \beta p_d^* + \gamma\theta_d^*) - \frac{b}{2}\theta_d^{*2}] \\ &= -E[\beta](p_c^*)^2 + (E[a] + E[c\beta])p_c^* + E[\gamma]p_c^*\theta_c^* - \frac{b}{2}\theta_c^{*2} - \frac{\theta_c^*}{2}\int_0^1 (c_\alpha^L\gamma_\alpha^U + c_\alpha^U\gamma_\alpha^L)\,\mathrm{d}\alpha - \frac{1}{2}\int_0^1 (c_\alpha^L a_\alpha^U + c_\alpha^U a_\alpha^L)\,\mathrm{d}\alpha - (p_d^* \\ &\quad - w_d^*)(E[a] - E[\beta]p_d^* + E[\gamma]\theta_d^*) + \frac{b}{2}\theta_d^{*2}. \end{aligned} \tag{39}$$

Thus, Proposition 5 is proven. □

Proof of Proposition 6. Similar to the Proposition 5, p_L^*, θ_L^* and p_H^*, θ_H^* can be obtained. For each given $p_L^*, \theta_L^*, p_H^*, \theta_H^*$, values of F_H and F_L should be selected by the manufacturer as big as possible in order to satisfy four constraint conditions, *i.e.* $E[\pi_{r_H}(p_H^*, \theta_H^*)] \geqslant \pi_{rdH}^*, E[\pi_{r_L}(p_L^*, \theta_L^*)] \geqslant \pi_{rdL}^*, E[\pi_{r_H}(p_H^*, \theta_H^*)] \geqslant E[\pi_{r_H}(p_L^*, \theta_L^*)], E[\pi_{r_L}(p_L^*, \theta_L^*)] \geqslant E[\pi_{r_L}(p_H^*, \theta_H^*)]$, and to maximize the objective function π_m. Substituting p_L^*, θ_L^* and p_H^*, θ_H^* into four constraint conditions, we have

$$\begin{aligned} F_H &\leqslant E[\pi_{r_H}(p_H^*, \theta_H^*)] - \pi_{rdH}^* \\ &= E\left[(p_H^* - c)(a - \beta p_H^* + \gamma\theta_H^*) - \frac{b_H}{2}(\theta_H^*)^2\right] - \pi_{rdH}^*, \end{aligned} \tag{40}$$

$$\begin{aligned} F_L &\leqslant E[\pi_{r_L}(p_L^*, \theta_L^*)] - \pi_{rdL}^* \\ &= E\left[(p_L^* - c)(a - \beta p_L^* + \gamma\theta_L^*) - \frac{b_L}{2}(\theta_L^*)^2\right] - \pi_{rdL}^*, \end{aligned} \tag{41}$$

$$\begin{aligned} F_H \geqslant F_L &+ E\left[(p_H^* - c)(a - \beta p_H^* + \gamma\theta_H^*) - \frac{b_L}{2}(\theta_H^*)^2\right] \\ &- E\left[(p_L^* - c)(a - \beta p_L^* + \gamma\theta_L^*) - \frac{b_L}{2}(\theta_L^*)^2\right], \end{aligned} \tag{42}$$

$$\begin{aligned} F_L \geqslant F_H &+ E\left[(p_L^* - c)(a - \beta p_L^* + \gamma\theta_L^*) - \frac{b_H}{2}(\theta_L^*)^2\right] \\ &- E\left[(p_H^* - c)(a - \beta p_H^* + \gamma\theta_H^*) - \frac{b_H}{2}(\theta_H^*)^2\right]. \end{aligned} \tag{43}$$

It follows from (40)–(43) that F_H^* and F_L^* in Eq. (20) can be obtained. Hence, Proposition 6 is proven.

References

[1] J. Spengler, Vertical restraints and antitrust policy, J. Political Economy 58 (1950) 347–352.
[2] Y. He, X. Zhao, L. Zhao, J. He, Coordinating a supply chain with effort and price dependent stochastic demand, Appl. Math. Model. 33 (2009) 2777–2790.
[3] B.A. Pasternack, Optimal pricing and return policies for perishable commodities, Manag. Sci. 4 (2005) 166–176.
[4] E. Kandel, The right to return, J. Law Econ. 39 (1996) 329–356.
[5] A. Jeuland, S. Shugan, Managing channel profits, Mark. Sci. 2 (1983) 239–372.
[6] R. Anupindi, Y. Bassok, Centralization of stocks: retailers vs. manufacturer, Manage. Sci. 45 (1999) 78–91.
[7] A. Arkan, S.R. Hejazi, Coordinating orders in a two echelon supply chain with controllable lead time and ordering cost using the credit period, Comput. Ind. Eng. 62 (2012) 56–69.
[8] R. Xu, X. Zhai, Analysis of supply chain coordination under fuzzy demand in a two-stage supply chain, Appl. Math. Model. 34 (2010) 129–139.

[9] A. Tsay, S. Nahmias, N. Agrawal, Modeling supply chain contracts: a review, in: S. Tayur, R. Ganeshan, M. Magazine (Eds.), Quantitative Models for Supply Chain Management, Kluwer Academic Publishers, Norwell, MA, 1999.
[10] G. Cachon, Supply chain coordination with contracts, in: A.G. de Kok, S.C. Graves (Eds.), Handbooks in Operations Research and Management Science-Supply Chain Management: Design Coordination and Operation, Elsevier, Amsterdam, The Netherlands, 2003.
[11] C. Yano, S. Gilbert, Coordinated pricing and production/procurement decisions: a review, in: A. Chakravarty, J. Eliashberg (Eds.), Managing Business Interfaces: Marketing, Engineering and Manufacturing Perspectives, Kluwer Academic Publishers, 2005.
[12] T.A. Taylor, Supply chain coordination under channel rebates with sales effort effects, Manage. Sci. 48 (2002) 992–1007.
[13] C.J. Corbett, D. Zhou, C.S. Tang, Designing supply contracts: contract type and information asymmetry, Manage. Sci. 50 (2004) 550–559.
[14] A. Lau, H. Lau, Some two-echelon supply-chain games: improving from deterministic-symmetric-information to stochastic-asymmetric-information models, Eur. J. Oper. Res. 161 (2005) 203–223.
[15] A. Lau, H. Lau, Y. Zhou, A stochastic and asymmetric information framework for a dominant manufacturer supply chain, Eur. J. Oper. Res. 176 (2007) 295–316.
[16] H. Zimmermann, An application-oriented view of modelling uncertainty, Eur. J. Oper. Res. 122 (2000) 190–198.
[17] L. Zadeh, Fuzzy sets, Inf. Control 8 (1965) 338–353.
[18] D. Petrovic, R. Petrovic, M. Vujoievib, Fuzzy models for the newsboy problem, Int. J. Prod. Econ. 45 (1996) 435–441.
[19] D. Petrovic, R. Roy, R. Petrovic, Supply chain modelling using fuzzy sets, Int. J. Prod. Econ. 59 (1999) 443–453.
[20] I. Giannaoccaro, P. Pontrandolfo, B. Scozzi, A fuzzy echelon approach for inventory management, Eur. J. Oper. Res. 149 (2003) 185–196.
[21] J. Wang, Y. Shu, Fuzzy decision modeling for supply chain management, Fuzzy Sets Syst. 150 (2005) 107–127.
[22] R. Aliev, B. Fazlollahi, B. Guirimov, R. Aliev, Fuzzy-genetic approach to aggregate production distribution planning in supply chain management, Inf. Sci. 177 (2007) 4241–4255.
[23] S. Chen, Y. Ho, Analysis of the newsboy problem with fuzzy demands and incremental discounts, Int. J. Prod. Econ. 129 (2011) 169–177.
[24] X. Wang, W. Tang, Optimal production run length in deteriorating production processes with fuzzy elapsed time, Comput. Ind. Eng. 56 (2009) 1627–1632.
[25] B. Wong, V. Lai, A survey of the application of fuzzy set theory in production and operations management: 1998–2009, Int. J. Prod. Econ. 129 (2011) 157–168.
[26] J. Zhao, W. Tang, J. Wei, Pricing decision for substitutable products with retailer competition in a fuzzy environment, Int. J. Prod. Econ. 135 (2012) 144–153.
[27] J. Wei, J. Zhao, Pricing decisions with retail competition in a fuzzy closed loop supply chain, Expert Syst. Appl. 38 (2011) 11209–11216.
[28] J. Wei, J. Zhao, Y. Li, Pricing decisions for a closed-loop supply chain in a fuzzy environment, Asia-Pacic J. Oper. Res. 29 (2012) 1–30.
[29] J. Wei, J. Zhao, Reverse channel decisions for a fuzzy closed-loop supply chain, Appl. Math. Model. 37 (2013) 1502–1513.
[30] F. Mathewson, R. Winter, The economics of franchise contracts, J. Law Econ. 28 (1985) 503–526.
[31] R. Lal, Improving channel coordination through franchising, Mark. Sci. 9 (1990) 299–318.
[32] B. Katz, J. Owen, On the existence of franchise contracts and some of their implications, Int. J. Ind. Organiz. 10 (1992) 567–593.
[33] S. Bhattacharyya, F. Lafontaine, Double-sided moral hazard and the nature of share contracts, Rand J. Econ. 26 (1995) 761–781.
[34] B. Hempelmann, Optimal franchise contracts with private cost information, Int. J. Ind. Organiz. 24 (2006) 449–465.
[35] B. Liu, R. Zhang, M. Xiao, Joint decision on production and pricing for online dual channel supply chain system, Appl. Math. Model. 34 (2010) 4208–4218.
[36] C. Cheng, Group opinion aggregation based on a grading process: a method for constructing triangular fuzzy numbers, Comput. Math. Appl. 48 (2004) 1619–1632.
[37] S. Nahmias, Fuzzy variables, Fuzzy Sets Syst. 1 (1978) 97–110.
[38] B. Liu, Theory and Practice of Uncertain Programming, Physica-Verlag, Heidelberg, 2002.
[39] L. Zadeh, Fuzzy sets as a basis for a theory of possibility, Fuzzy Sets Syst. 1 (1978) 3–28.
[40] B. Liu, Uncertainty Theory: An Introduction to its Axiomatic Foundations, Springer-Verlag, Berlin, 2004.
[41] C. Wang, W. Tang, R. Zhao, On the continuity and convexity analysis of the expected value function of a fuzzy mapping, J. Uncertain Syst. 1 (2007) 148–160.
[42] B. Liu, Y. Liu, Excepted value of fuzzy variable and fuzzy expected value models, IEEE Trans. Fuzzy Syst. 10 (2002) 445–450.
[43] Y. Liu, B. Liu, Expected value operator of random fuzzy variable and random fuzzy expected value models, Int. J. Uncertainty Fuzziness Knowledge-Based Syst. 11 (2003) 195–215.