Good morning, everyone! Welcome back to Math 201. I hope you're all ready to dive into our first official lecture of the semester. As you know from our last class, today we're going to start exploring some of the foundational concepts that will be critical as we move forward: **Graphs of Equations** and **Lines**.

Before we jump in, let me briefly remind you why these topics are so important. Graphs and lines are everywhere—in the graphs you see in the news, in the patterns you notice in data, and even in the way you plot out trends in your daily life, like tracking expenses or grades. Understanding how to graph equations and work with lines isn't just a mathematical exercise; it's a skill that will help you interpret and communicate information effectively.

Today, we'll start by reviewing the coordinate plane, which is the stage where all our graphs will take place. We'll learn how to plot points, how these points relate to equations, and how those equations can form the lines we'll be working with. By the end of the class, you'll not only understand how to graph these equations but also how to interpret what those graphs are telling you.

Now, let's get started. First, we'll take a look at the Cartesian coordinate system, which is simply a fancy way of saying the grid that helps us map out our equations visually. Ready? Let's dive in!

Alright, let's begin by exploring the **Coordinate Plane**, which is the foundation of everything we'll be discussing today.

Imagine a flat grid, like a piece of graph paper. This grid is what we call the **Cartesian coordinate plane**. It's named after the mathematician René Descartes, who developed this system that allows us to visually represent equations.

The coordinate plane is made up of two perpendicular lines, or **axes**: the **x-axis** and the **y-axis**. The x-axis runs horizontally, from left to right, while the y-axis runs vertically, from bottom to top. These two axes intersect at a point called the **origin**, which is where both xxx and yyy are zero (0,0)(0,0)(0,0).

Now, let's talk about how we use these axes. The plane is divided into four sections, or **quadrants**, by the x and y axes. Each quadrant represents a different combination of positive and negative values for xxx and yyy. Starting in the top right and moving counterclockwise:

- **Quadrant I**: Both xxx and yyy are positive.
- **Quadrant II**: xxx is negative, but yyy is positive.
- **Quadrant III**: Both xxx and yyy are negative.
- **Quadrant IV**: xxx is positive, but yyy is negative.

Understanding these quadrants is important because it helps us determine the signs of the coordinates of any point we plot.

Let's take a moment to practice visualizing this. Picture a point somewhere on this grid. To specify exactly where it is, we use a pair of numbers called **coordinates**, written as (x,y)(x, y)(x,y). The first number, xxx, tells us how far to move left or right from the origin, and the second number, yyy, tells us how far to move up or down.

For example, if I give you the point (3,2)(3, 2)(3,2), you would start at the origin, move three units to the right (because x=3x = 3x=3 is positive), and then two units up (because y=2y = 2y=2 is positive). This places our point in **Quadrant I**.

On the other hand, if I give you the point (−4,−1)(-4, -1)(−4,−1), you'd move four units to the left (because x=−4x = -4x=−4 is negative) and one unit down (because y=−1y = -1y=−1 is negative). This point would be in **Quadrant III**.

Understanding how to plot these points is the first step toward graphing equations, which we'll get into shortly. But before we move on, let's practice a bit. I'll give you a few coordinates, and I want you to visualize where they would be on the plane, or better yet, sketch them out on some graph paper if you have it handy.

Ready? Let's start with (5,−3)(5, -3)(5,−3), then (−2,4)(-2, 4)(−2,4), and finally (0,−6)(0, -6)(0,−6). Take a moment to think about where each of these points would be located.

Great! By getting comfortable with plotting points, you're laying the groundwork for everything else we'll do today. Once we're confident with this, we'll see how these points come together to form graphs of equations. But first, let's ensure everyone is clear on how to navigate the coordinate plane.

Any questions before we move on to plotting points?

Now that we've got a good grasp of the coordinate plane and how it's structured, let's move on to something just as important: **Plotting Points on the Coordinate Plane**.

Think of the coordinate plane as a map, and the points as specific locations on that map. Each point is defined by a pair of coordinates (x,y)(x, y)(x,y), where the first number tells us how far to move left or right, and the second number tells us how far to move up or down.

Let's start with a basic example. Imagine I give you the point (2,3)(2, 3)(2,3). How do we plot it? Here's what you do:

1. **Start at the Origin**: Always begin at the origin (0,0)(0, 0)(0,0). This is your reference point.
2. **Move Horizontally**: From the origin, look at the first number, x=2x = 2x=2. Since it's positive, you'll move 2 units to the right along the x-axis.
3. **Move Vertically**: Next, look at the second number, y=3y = 3y=3. This is also positive, so you'll move 3 units up along the y-axis.
4. **Plot the Point**: Where these two movements intersect is where you plot your point. So, (2,3)(2, 3)(2,3) is 2 units to the right and 3 units up from the origin.

Now, what if we have a point like (−4,−2)(-4, -2)(−4,−2)? Let's break it down:

1. **Start at the Origin**: Again, begin at (0,0)(0, 0)(0,0).
2. **Move Horizontally**: For x=−4x = -4x=−4, you'll move 4 units to the left because the x-value is negative.
3. **Move Vertically**: For y=−2y = -2y=−2, you'll move 2 units down since the y-value is also negative.
4. **Plot the Point**: This point will be in Quadrant III, 4 units left and 2 units down from the origin.

Let's try a few more examples together. I'll call out a point, and I want you to imagine or, if you can, draw it out on your paper. Ready?

- How about (3,−5)(3, -5)(3,−5)? Start at the origin, move 3 units to the right, and then 5 units down.
- Now, try (−1,4)(-1, 4)(−1,4). Start at the origin, move 1 unit to the left, and then 4 units up.
- Finally, (0,−7)(0, -7)(0,−7). This one is special—since x=0x = 0x=0, you don't move left or right at all. Just move 7 units down.

The key to plotting points is practice. The more you do it, the more natural it becomes. And remember, each point on the plane represents a solution to some equation. As we move forward, you'll see how these points can connect to form the graphs of equations, which is what we'll be diving into next.

But before we do that, are there any questions or points of confusion about how to plot points on the coordinate plane? Let's make sure everyone's comfortable with this before we move on.

Great! Now that you're comfortable with plotting individual points on the coordinate plane, let's take things a step further and talk about how we can use those points to create **Graphs of Equations**.

So far, we've been plotting points that were simply given to us. But what if those points came from a relationship between two variables? That's exactly what an equation in two variables does—it defines a relationship between xxx and yyy, and every solution to that equation is a point on its graph.

Let's start with a basic example: the equation y=2x+1y = 2x + 1y=2x+1. This is an example of a **linear equation** because it represents a straight line when we graph it.

Here's how we can graph this equation:

1. **Choose Values for xxx**: To graph this equation, we need to find several points that lie on the line it represents. We start by choosing some values for xxx. Let's pick a few simple ones: −2-2−2, 000, and 222.
2. **Calculate Corresponding yyy Values**: For each xxx value we chose, we'll plug it into the equation to find the corresponding yyy value.
   - If x=−2x = -2x=−2: y=2(−2)+1=−4+1=−3y = 2(-2) + 1 = -4 + 1 = -3y=2(−2)+1=−4+1=−3 So, one point is (−2,−3)(-2, -3)(−2,−3).
   - If x=0x = 0x=0: y=2(0)+1=0+1=1y = 2(0) + 1 = 0 + 1 = 1y=2(0)+1=0+1=1 Another point is (0,1)(0, 1)(0,1).
   - If x=2x = 2x=2: y=2(2)+1=4+1=5y = 2(2) + 1 = 4 + 1 = 5y=2(2)+1=4+1=5 And another point is (2,5)(2, 5)(2,5).
3. **Plot the Points**: Now that we have our points—(−2,−3)(-2, -3)(−2,−3), (0,1)(0, 1)(0,1), and (2,5)(2, 5)(2,5)—we can plot them on the coordinate plane.
4. **Draw the Line**: Once the points are plotted, we can draw a line through them. Since this is a linear equation, the points will align perfectly on a straight line. This line is the graph of the equation y=2x+1y = 2x + 1y=2x+1.

Now, why does this happen? Because each point we plotted is a solution to the equation y=2x+1y = 2x + 1y=2x+1. For any xxx value you choose, if you calculate the corresponding yyy value using the equation, that point will fall on this line. This is what we mean when we say that the graph of an equation represents all its solutions.

Let's take a moment to see if you can connect this with what we've learned so far. If I give you the equation y=−x+4y = -x + 4y=−x+4, how would you start graphing it? That's right—first, choose some xxx values. Then, plug them into the equation to find the corresponding yyy values, and finally, plot those points on the plane.

For instance, if you pick x=−1x = -1x=−1, x=0x = 0x=0, and x=2x = 2x=2, and calculate the corresponding yyy values, you'll get points like (−1,5)(-1, 5)(−1,5), (0,4)(0, 4)(0,4), and (2,2)(2, 2)(2,2). Plot these, and then draw a line through them.

What's really powerful here is that this process works for any linear equation. The equation might look different—maybe the numbers will change, or the line will slope differently—but the core idea is the same: the equation tells you how to find the points that make up its graph.

In a bit, we'll look at more complex examples and see how changing the equation affects the graph. But first, let's make sure everyone's clear on how to go from an equation to a graph. Are there any questions before we practice this together?