Good morning, everyone! Welcome back to Math 201. I'm glad to see you all here for our second lecture of the semester. Today, we're going to dive into some fascinating concepts that will expand on what we covered in our last class.

Just to briefly recap, in Lecture 1, we explored the foundations of graphing on the coordinate plane. We started by understanding the **coordinate plane** itself—how the x-axis and y-axis create a grid that allows us to plot points using coordinates (x,y)(x, y)(x,y). We practiced **plotting points** and then moved on to **graphing equations** by connecting those points to form straight lines, which represent the solutions to linear equations. Finally, we introduced the concept of **lines**, discussing their slopes and how to write their equations.

These concepts laid the groundwork for everything we'll be doing in this course because they allow us to visualize mathematical relationships. Today, we're going to build on that foundation by exploring the concept of **functions**. Functions are a crucial part of mathematics because they describe how one quantity depends on another. You can think of them as a kind of machine: you input a number, and the function gives you an output based on a specific rule.

We'll start by defining what a function is, learning how to recognize one, and getting comfortable with function notation. Then, we'll take a tour through what I like to call the **Library of Functions**, where we'll meet some of the most common types of functions you'll encounter—like linear, quadratic, and exponential functions, to name a few. Finally, we'll explore how we can transform these functions by shifting, stretching, or reflecting their graphs.

By the end of today's lecture, you'll not only understand what functions are but also how to manipulate and graph them, which is essential for tackling more complex mathematical problems in the future.

So, without further ado, let's jump into today's topic: **Functions**. We'll start with the basics—what is a function, and how do we identify one? Let's get started!

Great! Let's start by diving into the concept of **Functions**.

So, what exactly is a function? At its core, a function is like a machine that takes an input, processes it according to a specific rule, and then produces an output. In mathematical terms, a function is a relationship between two sets of numbers, where each input from one set is paired with exactly one output from the other set.

Let's break that down a bit further. Imagine you have a set of inputs, which we call the **domain**. These are all the possible values you can put into your function. On the other side, you have a set of outputs, which we call the **range**. These are all the possible values that come out of the function after processing the inputs.

A function essentially maps each input in the domain to a specific output in the range. For example, if you have a function f(x)=2x+3f(x) = 2x + 3f(x)=2x+3, and you input x=1x = 1x=1, the function processes it and gives you an output of f(1)=2(1)+3=5f(1) = 2(1) + 3 = 5f(1)=2(1)+3=5.

Now, let's talk about **function notation**. You'll often see functions written like this: f(x)=some expressionf(x) = \text{some expression}f(x)=some expression. The fff stands for the function, and xxx is the variable that you're inputting. When you see f(2)f(2)f(2), it means 'apply the function to 2.' Using our earlier example, f(2)=2(2)+3=7f(2) = 2(2) + 3 = 7f(2)=2(2)+3=7.

One important thing to remember is that for something to be a function, each input must correspond to exactly one output. If you put in an input and the function gives you back two different outputs, then it's not a function.

Let's visualize this with a simple example. Imagine you have a function that squares its input: f(x)=x2f(x) = x^2f(x)=x2. If you input x=2x = 2x=2, you get f(2)=4f(2) = 4f(2)=4. If you input x=−2x = -2x=−2, you still get f(−2)=4f(-2) = 4f(−2)=4. Notice that even though different inputs can produce the same output, each input still produces only one output. That's a key characteristic of a function.

Now, to make sure a relationship is truly a function, we can use something called the **Vertical Line Test** when looking at its graph. If you can draw a vertical line anywhere on the graph, and it crosses the graph at more than one point, then what you're looking at is not a function. We'll get into that more in a moment.

For now, let's focus on understanding the basic idea: a function is a rule that assigns each input exactly one output. You'll see this concept over and over as we explore different types of functions today.