



Homework 3. Partially Observable Markov Decision Problems

1.

(a)

$X = \{A\clubsuit, A\diamondsuit\}$, the first state means the opponent is holding Ace of Clubs and the second one means he's holding Ace of Diamonds.

$A = \{GA\clubsuit, GA\diamondsuit, \text{peek}\}$, the first and second actions are Guess Ace of Clubs and Ace of Diamonds respectively and the third one is peek.

$Z = \{OA\clubsuit, OA\diamondsuit\}$, the possible observations are Observe Ace of Clubs and Observe Ace of Diamonds respectively.

(b)

$$O \approx \left\{ \begin{array}{l} O_{\text{Guess Ace of Clubs}} = O_{\text{Guess Ace of Diamonds}} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \quad O_{\text{peek}} = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix} \end{array} \right\}$$

$$P \approx \left\{ \begin{array}{l} P_{\text{Guess Ace of Clubs}} = P_{\text{Guess Ace of Diamonds}} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \quad P_{\text{peek}} = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} \end{array} \right\}$$

$$C \approx \begin{bmatrix} 1.0 & 0.0 & 0.5 \\ 0.0 & 1.0 & 0.5 \end{bmatrix}$$

(c)

The initial belief is that the opponent has an ace of clubs with probability 0.7 which means that the distribution for that belief is $[0.7, 0.3]$.

The agent decides to peek and observes an ace of diamonds, which means that the agent performs the action "peek" and receives the observation $OA\diamondsuit$.

Using the forward algorithm:

$$\mu_0 = [0.7 \quad 0.3]$$

$$Z_{1:1} = \{OA\diamondsuit\}$$

$$\alpha_0 = \mu_0^T$$

$$\alpha_1 = \text{diag}(O_1) * P^T * \alpha_0 = \begin{bmatrix} 0.1 & 0.0 \\ 0.0 & 0.9 \end{bmatrix} * \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} * \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 0.07 \\ 0.27 \end{bmatrix}$$

$$\text{Normalizing: } \alpha_1 = \begin{bmatrix} 0.206 \\ 0.794 \end{bmatrix}$$

Which means the the belief of the agent after action peek and observation $OA\diamondsuit$ is the distribution:

$$\text{Belief} = [0.206, 0.794]$$