



### Homework 3. Partially Observable Markov Decision Problems

1.

(a)

$\mathcal{X} = \{A\clubsuit, A\diamond\}$ , the first state means the opponent is holding Ace of Clubs and the second one means he's holding Ace of Diamonds.

$\mathcal{A} = \{GA\clubsuit, GA\diamond, \text{peek}\}$ , the first and second actions are Guess Ace of Clubs and Ace of Diamonds respectively and the third one is peek.

$\mathcal{Z} = \{OA\clubsuit, OA\diamond, \text{nothing}\}$ , the possible observations are Observe Ace of Clubs and Observe Ace of Diamonds respectively.

(b)

$$\mathcal{O} = \{ O_{\text{Guess Ace of Clubs}} = O_{\text{Guess Ace of Diamonds}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad O_{\text{peek}} = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.9 & 0 \end{bmatrix} \}$$

$$\mathcal{P} = \{ P_{\text{Guess Ace of Clubs}} = P_{\text{Guess Ace of Diamonds}} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \quad P_{\text{peek}} = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} \}$$

$$\mathcal{C} = \begin{bmatrix} 0 & 1 & 0.1 \\ 1 & 0 & 0.1 \end{bmatrix}$$

(c)

The initial belief is that the opponent has an ace of clubs with probability 0.7 which means that the distribution for that belief is  $[0.7, 0.3]$ .

The agent decides to peek and observes an ace of diamonds, which means that the agent performs the action "peek" and receives the observation  $OA\diamond$ .

Using the forward algorithm:

$$\mu_0 = [0.7 \ 0.3]$$

$$Z_{1:1} = \{ OA\diamond \}$$

$$\alpha_0 = \mu_0^T$$

$$\alpha_1 = \text{diag}(O_1) * P^T * \alpha_0 = \begin{bmatrix} 0.1 & 0.0 \\ 0.0 & 0.9 \end{bmatrix} * \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} * \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 0.07 \\ 0.27 \end{bmatrix}$$

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$$\text{Normalizing: } \alpha_1 = \frac{0.206}{0.794}$$

Which means the the belief of the agent after action peek and observation OA ♦ is the distribution:

$$\text{Belief} = [0.206, 0.794]$$