Group 44:
Ricardo Rei nº 78047
Miguel Carvalho nº 78052

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Learning and Decision Making 2016-2017

Homework 4. Supervised learning

1.

(a)

Assuming $w^* = argmax_{\,w \in \mathbb{R}^{P+1}} l\,(D;\,\pi)$ then:

$$w^* = argmax_{\,w \in \mathbb{R}^{P+1}}\{\, \sum_{n=1}^{N}(\, a_n \log \pi(1|x_n) + (1 - \, a_n) \log(1 - \, \pi(1|x_n)))\}$$

$$= argmax_{\,w \in \mathbb{R}^{P+1}}\{\, \sum_{n=1}^{N}(\, a_n \log \pi(1|x_n) + (1 - \, a_n) \log(0|x_n)))\}$$

applying e^x:

$$= argmax_{\,w \in \mathbb{R}^{P+1}}\{\, \prod_{n=1}^{N}(\, \pi(1|x_n)^{a_n}\ \pi(0|x_n)^{1-a_n})\}$$

$$= argmax_{\,w \in \mathbb{R}^{P+1}}\{\, \prod_{n=1}^{N} P[a^n|\, x_n, w]\, \},$$

which is the same as minimizing the negative of the logarithm of the function:

$= argmin_{\,w \in \mathbb{R}^{P+1}}\{- \log\, (\prod_{n=1}^{N} P[a^n|\, x_n, w]\,)\} = w^*$, as we wanted to show.

(b)

applying the gradient to $l(D;\,\pi)$ with respect to **w** we have:

$$\boldsymbol{\nabla}_w\, l(D;\,\pi) = \sum_{\boldsymbol{n=1}}^{\boldsymbol{N}} \boldsymbol{\nabla}_w\, [a_n \log\,(\pi(1|x_n; w)) + (1 - \, a_n) \log\,(\, 1 - \, \pi(1|\, x_n; w))]$$

$$= \sum_{\boldsymbol{n=1}}^{\boldsymbol{N}} \boldsymbol{\nabla}_w\, [a_n \log\,(\pi(1|x_n; w)) + (1 - \, a_n) \log\,(\, \pi(0|\, x_n; w))]$$

$$= \sum_{\boldsymbol{n=1}}^{\boldsymbol{N}} \boldsymbol{\nabla}_w\, [\, \log\,(\pi(a_n|x_n; w))]$$

$$= \sum_{\boldsymbol{n=1}}^{\boldsymbol{N}} x_n\, (a_n - \pi(1|x_n; w)) \quad = \text{g},$$

thus the gradient of $l(D;\,\pi)$ with respect to **w** is equal to g as we wanted to show.

(c)

computing the Hessian of $l(D;\,\pi)$ with respect to **w**:

$$\boldsymbol{\nabla}_w\, \boldsymbol{\nabla}_{w'}\, l(D;\,\pi) = \sum_{\boldsymbol{n=1}}^{\boldsymbol{N}} \boldsymbol{\nabla}_w\, \boldsymbol{\nabla}_{w'} [a_n \log\,(\pi(1|x_n; w)) + (1 - \, a_n) \log\,(\, 1 - \, \pi(1|\, x_n; w))]$$

$$= - \sum_{\boldsymbol{n=1}}^{\boldsymbol{N}}\, [x_n\, \boldsymbol{\nabla}_{w'}\, \pi(a_n|x_n; w)]$$

$$= - \sum_{\boldsymbol{n=1}}^{\boldsymbol{N}}\, [x_n\ x_n^T\ \pi(1|x_n; w)\ \ (1 - \, \pi(1|x_n; w)] = \text{ H},$$

thus the Hessian of $l(D;\,\pi)$ with respect to **w** is equal to H as we wanted to show.