



# Homework 1. Markov chains

1. (a)

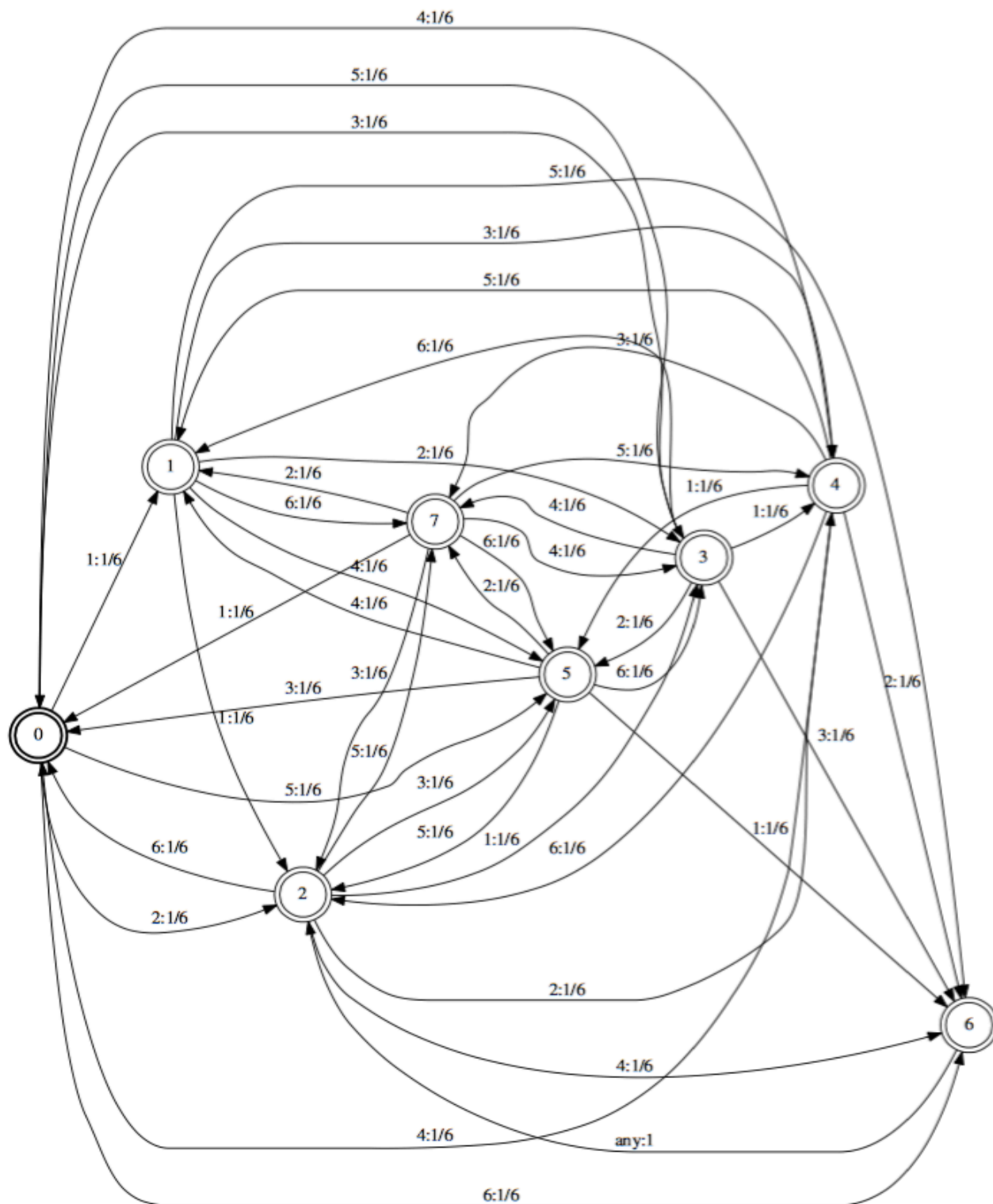


Figure 1 - transition diagram representing the motion of the player

The states from the board are represented by the numbers. The state and number translation is the following:

- Go = 0
- Vermont Avenue = 1
- Jail = 2
- Virginia Avenue = 3
- Free Parking = 4
- Marvin Gardens = 5
- Go to Jail = 6
- Pennsylvania Avenue = 7

The transitions are labeled with a  $\langle \text{number} \rangle : \langle \text{probability} \rangle$ . The  $\langle \text{number} \rangle$  on the label represents a possible number on the dice and  $\langle \text{probability} \rangle$  represents the probability of getting that number. The only exception in this syntax is in state 6 (Go to Jail) which only has a transition with the label “any:1” to the state 2 (Jail) which means that if a player ends in state 6 at some time  $T$ , in  $T+1$  the player will be in state 2.

(b)

$\chi = \{0, 1, 2, 3, 4, 5, 6, 7\}$  - the set of possible states

$$P = \begin{array}{ccccccc} & 0 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 0 \\ 0 & 0 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1/6 & 0 & 0 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1/6 & 1/6 & 0 & 0 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1/6 & 1/6 & 1/6 & 0 & 0 & 1/6 & 1/6 & 1/6 \\ 1/6 & 1/6 & 1/6 & 1/6 & 0 & 0 & 1/6 & 1/6 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 \end{array}$$

so the Markov chain model will be represented by:  $M(\chi, P)$

(c)

$$\mu_0 = [1, 0, 0, 0, 0, 0, 0, 0]$$

what's  $\mu_3$ ?

$$\mu_3 = \mu_0 P^3 = [0.11111111 \ 0.10648148 \ 0.10648148 \ 0.11574074 \ 0.12037037 \ 0.12037037, 0.11111111 \ 0.10185185]$$