Layout Optimization Model for Multi-Recipe and Multi-Route Problems with Application to the Design of a Steel Factory

Box placement MILP model (with or without simulated annealing) for the layout of the scrap yard, given a left / right split, to explore the space of possible loading sequences for each grade

In [1]:
```
from itertools import permutations, product
import random
import math
```

In [2]:
```
from datetime import datetime
```

In [3]:
```
import numpy as np
import pandas as pd
```

In [4]:
```
from IPython.core.magic import register_cell_magic

@register_cell_magic
def skip(line, cell):
    return
```

DATA

Scrapyard data

Width of the 40m deep box for each scrap type

In [5]:
```
box_width_dict = {0: 11.4, 1: 10.0, 2: 11.7, 3: 10.0, 4: 10.0, 5: 10.0, 6: 23.4,
                  7: 10.0, 8: 34.4, 9: 12.4, 10: 10.0, 11: 10.0, 12: 38.3,
                  13: 26.5, 14: 56.9, 15: 43.6, 16: 27.5, 17: 14.3, 18: 69.3,
                  19: 27.2, 20: 56.1, 21: 38.4, 22: 26.0, 23: 10.0, 24: 33.6,
                  25: 31.3, 26: 14.4, 27: 30.9, 28: 16.0, 29: 10.0, 30: 14.3,
                  31: 15.5}
```

Grades data

Each grade contains a given quantity of certain scrap types

In [6]:
```
grade_ingredients_dict = {'G01': [6, 8, 4, 5, 0, 1, 2, 7, 9, 16, 20],
 'G02': [6, 8, 4, 5, 0, 1, 3, 7, 16, 20],
 'G03': [6, 8, 4, 0, 3, 7, 9, 16, 20],
 'G04': [6, 8, 18, 4, 5, 7, 9, 11, 17, 30, 16, 19, 20, 27],
 'G05': [8, 5, 0, 1, 7, 17, 16, 20],
 'G06': [6, 8, 0, 1, 3, 9, 16, 20],
 'G07': [6, 8, 18, 4, 5, 0, 1, 3, 16, 20],
 'G08': [8, 28, 22, 15, 17, 14, 16, 21, 19, 24, 27],
 'G09': [12, 13, 15, 29, 14, 19, 24],
 'G10': [6, 8, 7, 15, 14, 19, 24],
 'G11': [22, 23, 15, 21, 19, 20, 24],
 'G12': [12, 13, 18, 28, 0, 1, 2, 3, 29, 30, 16, 31, 20, 24],
 'G13': [12, 13, 15, 29, 19, 24, 25],
 'G14': [12, 15, 29, 14, 19, 24],
 'G15': [6, 8, 28, 5, 16, 19, 20],
 'G16': [6, 8, 5, 7, 11, 15, 29, 14, 20, 24],
 'G17': [12, 13, 15, 29, 14, 19, 24, 25],
 'G18': [6, 8, 12, 13, 0, 9, 15, 29, 14, 19, 24, 25],
 'G19': [12, 15, 20, 24],
 'G20': [8, 12, 13, 15, 29, 19, 24, 25],
 'G21': [6, 8, 10, 4, 5, 9, 11, 15, 20, 24],
 'G22': [8, 10, 5, 7, 15, 29, 14, 20, 24],
 'G23': [8, 12, 15, 20],
 'G24': [6, 8, 12, 5, 7, 9, 15, 14, 20, 24],
 'G25': [6, 8, 4, 5, 7, 11, 15, 14, 20, 24],
 'G26': [12, 13, 15, 20, 24, 25],
 'G27': [6, 8, 10, 12, 4, 5, 7, 9, 15, 20, 24],
 'G28': [6, 8, 5, 0, 3, 9, 17, 16, 20, 27],
 'G29': [12, 23, 15, 21, 19, 24],
 'G30': [12, 15, 29, 14, 21, 19, 24],
 'G31': [12, 13, 23, 15, 29, 21, 19, 24, 25],
 'G32': [12, 22, 15, 21, 19, 24, 25],
 'G33': [12, 13, 22, 23, 15, 29, 14, 21, 19, 24, 25],
 'G34': [6, 8, 5, 0, 1, 2, 3, 17, 16, 20],
 'G35': [8, 10, 28, 5, 16, 31, 20, 27],
 'G36': [8, 10, 28, 4, 11, 15, 17, 16, 20, 27],
 'G37': [6, 8, 18, 4, 5, 15, 30, 16, 20, 27],
 'G38': [12, 15, 29, 14, 19, 24, 25],
 'G39': [12, 22, 23, 15, 29, 21, 19, 24, 25],
 'G40': [6, 8, 18, 28, 4, 7, 9, 15, 17, 16, 20, 27],
 'G41': [8, 12, 13, 22, 15, 29, 14, 21, 19, 24, 25],
```

```
'G42': [12, 23, 15, 29, 14, 21, 19, 24],
'G43': [12, 13, 15, 21, 19, 24, 25],
'G44': [12, 15, 24, 25],
'G45': [23, 15, 21, 19, 20, 24],
'G46': [12, 13, 28, 15, 17, 29, 14, 16, 19, 24, 25],
'G47': [12, 15, 24],
'G48': [12, 0, 15, 29, 14, 24],
'G49': [12, 15, 29, 14, 24],
'G50': [8, 18, 28, 0, 2, 23, 17, 30, 16, 21, 20, 27],
'G51': [18, 26, 28, 16, 19],
'G52': [6, 8, 10, 12, 18, 28, 0, 2, 9, 16, 20, 25],
'G53': [12, 15, 14, 24, 25],
'G54': [12, 13, 15, 29, 14, 24],
'G55': [12, 15, 24, 25],
'G56': [6, 8, 18, 28, 0, 3, 9, 16, 20],
'G57': [12, 26, 19, 25],
'G58': [6, 8, 10, 28, 11, 16, 19, 24],
'G59': [6, 8, 4, 9, 11, 30, 16, 19, 24],
'G60': [8, 12, 13, 28, 16, 19, 20],
'G61': [8, 10, 18, 4, 5, 11, 17, 16, 19, 24],
'G62': [12, 15, 29, 14, 24],
'G63': [15, 14, 20, 24],
'G64': [12, 13, 15, 14, 19, 24],
'G65': [12, 13, 15, 29, 14, 19, 24, 25],
'G66': [12, 13, 15, 14, 19, 24, 25],
'G67': [15, 29, 21, 24],
'G68': [6, 8, 12, 5, 15, 19, 24],
'G69': [12, 13, 21, 19, 24, 25],
'G70': [12, 22, 21, 20, 24],
'G71': [12, 15, 16, 24, 25],
'G72': [0, 3, 15, 14, 16, 24],
'G73': [12, 18, 0, 1, 15, 14, 16, 24, 27],
'G74': [12, 13, 15, 16, 24],
'G75': [8, 4, 5, 0, 1, 2, 7, 9, 16, 20],
'G76': [6, 8, 0, 3, 16, 20],
'G77': [8, 4, 0, 1, 3, 9, 16, 20],
'G78': [6, 8, 4, 0, 3, 7, 9, 16, 20],
'G79': [6, 8, 18, 28, 0, 1, 16, 20],
'G80': [18, 0, 2, 17, 30, 16, 31, 20, 27],
'G81': [18, 28, 0, 17, 30, 16, 31, 20, 27],
'G82': [0, 3, 30, 16, 20, 27],
'G83': [18, 0, 17, 30, 16, 20]}
```

The quantity of the scrap type required determines how many grabs the crane has to do

In [7]:
```
total_grabs_dict = {'G01': 25, 'G02': 25, 'G03': 28, 'G04': 36, 'G05': 32, 'G06': 26, 'G07': 32, 'G08': 46, 'G09': 33, 'G10
                    'G11': 27, 'G12': 47, 'G13': 30, 'G14': 30, 'G15': 31, 'G16': 40, 'G17': 32, 'G18': 44, 'G19': 26, 'G20
                    'G21': 38, 'G22': 37, 'G23': 28, 'G24': 37, 'G25': 37, 'G26': 30, 'G27': 36, 'G28': 34, 'G29': 41, 'G30
                    'G31': 42, 'G32': 39, 'G33': 38, 'G34': 40, 'G35': 43, 'G36': 46, 'G37': 45, 'G38': 31, 'G39': 40, 'G40
                    'G41': 37, 'G42': 32, 'G43': 34, 'G44': 22, 'G45': 24, 'G46': 34, 'G47': 22, 'G48': 23, 'G49': 21, 'G50
                    'G51': 41, 'G52': 43, 'G53': 23, 'G54': 25, 'G55': 22, 'G56': 31, 'G57': 22, 'G58': 42, 'G59': 39, 'G60
                    'G61': 38, 'G62': 21, 'G63': 16, 'G64': 30, 'G65': 32, 'G66': 34, 'G67': 27, 'G68': 24, 'G69': 38, 'G70
                    'G71': 22, 'G72': 26, 'G73': 31, 'G74': 23, 'G75': 26, 'G76': 25, 'G77': 27, 'G78': 28, 'G79': 28, 'G80
                    'G81': 32, 'G82': 33, 'G83': 32}
```

Based on the real production plan, there is a maximum time available for the loading of each grade

In [8]:
```
grades_max_time_dict = {'G07': 2821000.0, 'G08': 3600000.0, 'G09': 4464000.0, 'G11': 2581000.0,
                        'G13': 3541000.0, 'G28': 5076000.0, 'G40': 4260000.0, 'G44': 3541000.0,
                        'G46': 4146331.0, 'G47': 3421000.0, 'G64': 3990000.0, 'G65': 3601000.0,
                        'G67': 3298718.0, 'G78': 2581000.0}
```

Scrap types data

The layer of each scrap type

In [9]:
```
scrap_layer_dict = {6: 1.0, 8: 1.0, 10: 1.0, 12: 1.0, 13: 1.0, 18: 1.0, 26: 1.0, 28: 2.0, 4: 3.0, 5: 3.0,
                    0: 4.0, 1: 4.0, 2: 4.0, 3: 4.0, 7: 4.0, 9: 4.0, 11: 4.0, 22: 4.0, 23: 4.0, 15: 5.0,
                    17: 5.0, 29: 5.0, 30: 5.0, 14: 6.0, 16: 6.0, 21: 6.0, 31: 6.0, 19: 7.0, 20: 7.0, 24: 7.0,
                    25: 7.0, 27: 7.0}
```

Production plan data

(Real) Production plan data

In [10]:
```
jobs_eaf1_list_RPP = ['G11', 'G11', 'G11', 'G40', 'G40', 'G40', 'G11', 'G11', 'G11', 'G28', 'G28', 'G28', 'G09', 'G09', 'G0
                      'G11', 'G11', 'G11', 'G07', 'G07', 'G07', 'G28', 'G28', 'G28', 'G46', 'G46', 'G46', 'G46', 'G46', 'G0
                      'G07', 'G07', 'G47', 'G47', 'G47', 'G47', 'G47', 'G47', 'G67', 'G67', 'G67', 'G67', 'G67', 'G78', 'G7
                      'G78', 'G13', 'G13', 'G13', 'G13', 'G13', 'G13', 'G64', 'G64', 'G64', 'G64', 'G64', 'G65', 'G65', 'G6
                      'G65', 'G65', 'G65', 'G44', 'G44', 'G44', 'G44', 'G44', 'G67', 'G67', 'G67', 'G67', 'G67', 'G47', 'G4
                      'G47', 'G47', 'G47', 'G11', 'G11', 'G11', 'G11', 'G11', 'G65', 'G65', 'G65']

jobs_eaf2_list_RPP = ['G11', 'G11', 'G11', 'G40', 'G40', 'G40', 'G11', 'G11', 'G11', 'G28', 'G28', 'G28', 'G09', 'G09', 'G0
                      'G11', 'G11', 'G11', 'G07', 'G07', 'G07', 'G28', 'G28', 'G28', 'G07', 'G07', 'G78', 'G78', 'G78', 'G0
```

```
                         'G08', 'G08', 'G47', 'G47', 'G78', 'G78', 'G78', 'G11', 'G11', 'G65', 'G65', 'G65']
```

In [11]:
```
# Grades that appear in the real production plan
grades_list_RPP = ['G07', 'G08', 'G09', 'G11', 'G13', 'G28', 'G40', 'G44', 'G46', 'G47', 'G64', 'G65', 'G67', 'G78']
```

Leveled Production plan data (LPP)

In [12]:
```
jobs_eaf1_list_LPP = ['G40', 'G12', 'G08', 'G37', 'G35', 'G31', 'G29', 'G51',
                      'G59', 'G32', 'G33', 'G24', 'G25', 'G22', 'G46', 'G28',
                      'G66', 'G60', 'G82', 'G80', 'G42', 'G17', 'G07', 'G73',
                      'G15', 'G14', 'G26', 'G78', 'G79', 'G77', 'G11', 'G75',
                      'G72', 'G76', 'G54', 'G68', 'G74', 'G48', 'G55', 'G44',
                      'G49']

jobs_eaf2_list_LPP = ['G36', 'G18', 'G52', 'G58', 'G30', 'G10', 'G16', 'G39',
                      'G34', 'G21', 'G61', 'G69', 'G41', 'G04', 'G27', 'G43',
                      'G20', 'G09', 'G65', 'G70', 'G81', 'G83', 'G05', 'G56',
                      'G38', 'G13', 'G64', 'G23', 'G50', 'G03', 'G67', 'G19',
                      'G06', 'G01', 'G02', 'G45', 'G53', 'G71', 'G57', 'G47',
                      'G62', 'G63']
```

In [13]:
```
# Complete list of grades
grades_list_LPP = ['G01', 'G02', 'G03', 'G04', 'G05', 'G06', 'G07', 'G08', 'G09', 'G10',
                   'G11', 'G12', 'G13', 'G14', 'G15', 'G16', 'G17', 'G18', 'G19', 'G20',
                   'G21', 'G22', 'G23', 'G24', 'G25', 'G26', 'G27', 'G28', 'G29', 'G30',
                   'G31', 'G32', 'G33', 'G34', 'G35', 'G36', 'G37', 'G38', 'G39', 'G40',
                   'G41', 'G42', 'G43', 'G44', 'G45', 'G46', 'G47', 'G48', 'G49', 'G50',
                   'G51', 'G52', 'G53', 'G54', 'G55', 'G56', 'G57', 'G58', 'G59', 'G60',
                   'G61', 'G62', 'G63', 'G64', 'G65', 'G66', 'G67', 'G68', 'G69', 'G70',
                   'G71', 'G72', 'G73', 'G74', 'G75', 'G76', 'G77', 'G78', 'G79', 'G80',
                   'G81', 'G82', 'G83']
```

In [13]:

SPLITS

Split (0) - left / right split of boxes from the Layout 0 (for Layout 0 star)

In [14]:
```
split0star_left_scraps = sorted([26, 18, 13, 4, 1, 23, 22, 3, 15, 14, 31, 24, 19, 25])
split0star_right_scraps = sorted([10, 6, 8, 12, 28, 5, 2, 11, 7, 9, 0, 30, 17, 29, 16, 21, 20, 27])
```

Split (1) - optimized left/right split of scrap types based on the perfect balance between the total length of both sides of the yard

In [15]:
```
split1_left_scraps = [0, 2, 8, 9, 14, 17, 18, 20, 22, 24, 27, 30, 31]
split1_right_scraps = [1, 3, 4, 5, 6, 7, 10, 11, 12, 13, 15, 16, 19, 21, 23, 25, 26, 28, 29]
```

Split (2a) - optimized left/right split of scrap types based on:

1. balance between the total length of both sides of the yard;
2. balanced split for scraps of the layers in A.

In [16]:
```
split2a_left_scraps = [0, 3, 4, 5, 6, 7, 15, 16, 18, 19, 21, 22, 25, 26, 27]
split2a_right_scraps = [1, 2, 8, 9, 10, 11, 12, 13, 14, 17, 20, 23, 24, 28, 29, 30, 31]
```

Split (2b) - optimized left/right split of scrap types based on:

1. balanced split for scraps of the layers in A;
2. balance between the total length of both sides of the yard.

In [17]:
```
split2b_left_scraps = [0, 2, 4, 5, 7, 9, 11, 13, 16, 17, 18, 20, 21, 24, 26, 29, 30]
split2b_right_scraps = [1, 3, 6, 8, 10, 12, 14, 15, 19, 22, 23, 25, 27, 28, 31]
```

Split (3) - optimized left/right split of scrap types based on the balance between the total length of both sides of the yard with weight 0.5 and the balance between the split for scraps of layer 1, 4, 5, 6 and 7 with weight 0.5.

In [18]:
```
# alpha = 1.0

split3_10_left_scraps = [0, 2, 8, 9, 14, 17, 18, 20, 22, 24, 27, 30, 31]
split3_10_right_scraps = [1, 3, 4, 5, 6, 7, 10, 11, 12, 13, 15, 16, 19, 21, 23, 25, 26, 28, 29]
```

In [19]:
```
# alpha = 0.9

split3_09_left_scraps = [2, 7, 8, 9, 10, 11, 12, 13, 14, 17, 20, 23, 24, 28, 29, 30, 31]
split3_09_right_scraps = [0, 1, 3, 4, 5, 6, 15, 16, 18, 19, 21, 22, 25, 26, 27]
```

In [20]:
```
# alpha = 0.8

split3_08_left_scraps = [4, 5, 7, 11, 13, 16, 17, 18, 19, 21, 22, 23, 25, 26, 27, 29, 30]
split3_08_right_scraps = [0, 1, 2, 3, 6, 8, 9, 10, 12, 14, 15, 20, 24, 28, 31]
```

In [21]:
```
# alpha = 0.7

split3_07_left_scraps = [4, 5, 7, 11, 13, 16, 17, 18, 19, 21, 22, 23, 25, 26, 27, 29, 30]
split3_07_right_scraps = [0, 1, 2, 3, 6, 8, 9, 10, 12, 14, 15, 20, 24, 28, 31]
```

In [22]:
```
# alpha = 0.6

split3_06_left_scraps = [0, 2, 4, 5, 9, 11, 13, 16, 17, 18, 20, 21, 23, 24, 26, 29, 30]
split3_06_right_scraps = [1, 3, 6, 7, 8, 10, 12, 14, 15, 19, 22, 25, 27, 28, 31]
```

In [23]:
```
# alpha = 0.5

split3_05_left_scraps = [0, 2, 6, 8, 9, 10, 11, 12, 14, 15, 20, 23, 24, 28, 31]
split3_05_right_scraps = [1, 3, 4, 5, 7, 13, 16, 17, 18, 19, 21, 22, 25, 26, 27, 29, 30]
```

In [24]:
```
# alpha = 0.4

split3_04_left_scraps = [6, 7, 8, 10, 11, 12, 14, 15, 19, 22, 23, 25, 27, 28, 31]
split3_04_right_scraps = [0, 1, 2, 3, 4, 5, 9, 13, 16, 17, 18, 20, 21, 24, 26, 29, 30]
```

In [25]:
```
# alpha = 0.3

split3_03_left_scraps = [4, 5, 7, 11, 13, 16, 17, 18, 19, 21, 22, 23, 25, 26, 27, 29, 30]
split3_03_right_scraps = [0, 1, 2, 3, 6, 8, 9, 10, 12, 14, 15, 20, 24, 28, 31]
```

In [26]:
```
# alpha = 0.2

split3_02_left_scraps = [6, 7, 8, 10, 11, 12, 14, 15, 19, 22, 23, 25, 27, 28, 31]
split3_02_right_scraps = [0, 1, 2, 3, 4, 5, 9, 13, 16, 17, 18, 20, 21, 24, 26, 29, 30]
```

In [27]:
```
# alpha = 0.1

split3_01_left_scraps = [0, 1, 2, 6, 8, 9, 10, 12, 14, 15, 20, 23, 24, 28, 31]
split3_01_right_scraps = [3, 4, 5, 7, 11, 13, 16, 17, 18, 19, 21, 22, 25, 26, 27, 29, 30]
```

In [28]:
```
# alpha = 0.0

split3_00_left_scraps = [0, 2, 4, 5, 9, 11, 13, 15, 16, 18, 20, 21, 23, 24, 26, 28]
split3_00_right_scraps = [1, 3, 6, 7, 8, 10, 12, 14, 17, 19, 22, 25, 27, 29, 30, 31]
```

In [29]:
```
# Function to set left_scraps and right_scraps according to the split
```

```python
def get_scraps(split):
    left_var_name = f"{split}_left_scraps"
    right_var_name = f"{split}_right_scraps"

    # Fallback if split is not recognized
    if left_var_name not in globals() or right_var_name not in globals():
        left_var_name = split3_05_left_scraps
        right_var_name = split3_05_right_scraps
        print('Error! Using Split 3 0.5 as default')

    left_scraps = globals()[left_var_name]
    right_scraps = globals()[right_var_name]

    return left_scraps, right_scraps
```

In [29]:

```python

```

LAYER PERMUTATION

In [30]:

```python
layer_1 = [key for key in scrap_layer_dict.keys() if scrap_layer_dict[key] == 1.0]
layer_2 = [key for key in scrap_layer_dict.keys() if scrap_layer_dict[key] == 2.0]
layer_3 = [key for key in scrap_layer_dict.keys() if scrap_layer_dict[key] == 3.0]
layer_4 = [key for key in scrap_layer_dict.keys() if scrap_layer_dict[key] == 4.0]
layer_5 = [key for key in scrap_layer_dict.keys() if scrap_layer_dict[key] == 5.0]
layer_6 = [key for key in scrap_layer_dict.keys() if scrap_layer_dict[key] == 6.0]
layer_7 = [key for key in scrap_layer_dict.keys() if scrap_layer_dict[key] == 7.0]
#print(layer_1)
#print(layer_2)
#print(layer_3)
#print(layer_4)
#print(layer_5)
#print(layer_6)
#print(layer_7)
```

In [31]:

```python
layer_1_perms = list(permutations(layer_1))
layer_2_perms = list(permutations(layer_2))
layer_3_perms = list(permutations(layer_3))
layer_4_perms = list(permutations(layer_4))
layer_5_perms = list(permutations(layer_5))
layer_6_perms = list(permutations(layer_6))
layer_7_perms = list(permutations(layer_7))
```

```
#layer_1_perms
```

In [32]:
```
perms = [layer_1_perms, layer_2_perms, layer_3_perms, layer_4_perms, layer_5_perms, layer_6_perms, layer_7_perms]
```

Function to get recipe out of a permutation for all layers

In [33]:
```
def get_recipe(grade, sigma):
    # Input: a grade (eg. 'G07') and a sequence of all scraps in each layer
    # Output: the recipe for the grade using the sequences

    ingredients = grade_ingredients_dict[grade]
    recipe = [scrap for scrap in sigma[0] if scrap in ingredients]
    recipe += [scrap for scrap in sigma[1] if scrap in ingredients]
    recipe += [scrap for scrap in sigma[2] if scrap in ingredients]
    recipe += [scrap for scrap in sigma[3] if scrap in ingredients]
    recipe += [scrap for scrap in sigma[4] if scrap in ingredients]
    recipe += [scrap for scrap in sigma[5] if scrap in ingredients]
    recipe += [scrap for scrap in sigma[6] if scrap in ingredients]

    return recipe
```

GET INFO FROM OUTPUT

Layout

In [34]:
```
def get_precedences(scraps):
    precedences = []

    for i in scraps:
      for j in scraps:
        if x[i,j].X == 1:
          precedences.append(f'[{i},{j}]')

    return precedences
```

In [35]:
```
def print_row(scraps, precedences):
    aux = []
    for i in sorted(scraps):
      for j in sorted(scraps):
        if f'[{i},{j}]' in precedences:
          aux.append(i)
```

```python
    row = []
    for k in range(len(scraps)):
      for i in sorted(scraps):
        if aux.count(i) == k:
          row.append(i)

    return row
```

Yard total length

In [36]:
```python
def compute_side_length(side):
    length = 20*2 + 1.7
    for scrap in side:
      length += box_width_dict[scrap] + 1.7

    return length
```

In [37]:
```python
def compute_yard_length(L, R):

    return max(compute_side_length(L), compute_side_length(R))
```

Loading time for the production plan

In [38]:
```python
def compute_midpoints(L, R):
    midpoints = {}

    L_aux = L.copy()
    L_aux.reverse()
    midpoint = 20 + 1.7 + box_width_dict[L_aux[0]]/2
    midpoints[L_aux[0]] = midpoint
    for i in range(1,len(L_aux)):
      midpoint += box_width_dict[L_aux[i-1]]/2 + 1.7 + box_width_dict[L_aux[i]]/2
      midpoints[L_aux[i]] = midpoint

    R_aux = R.copy()
    R_aux.reverse()
    midpoint = 20 + 1.7 + box_width_dict[R_aux[0]]/2
    midpoints[R_aux[0]] = midpoint
    for i in range(1,len(R_aux)):
      midpoint += box_width_dict[R_aux[i-1]]/2 + 1.7 + box_width_dict[R_aux[i]]/2
      midpoints[R_aux[i]] = midpoint
```

```
    return midpoints
```

In [39]:
```
def get_grade_loading_time(grade, sigma, L, R):

  recipe = get_recipe(grade, sigma)
  total_grabs = total_grabs_dict[grade]

  midpoint = compute_midpoints(L,R)

  time = 6.2 + 2*1.47 + 0.67*total_grabs + (midpoint[recipe[0]] + midpoint[recipe[-1]])/60
  for i in range(len(recipe)-1):
    time += abs(midpoint[recipe[i]]-midpoint[recipe[i+1]])/40

  return time
```

In [40]:
```
def get_eaf_loading_time(prodplan, sigma, L, R):

    time = 0
    for grade in prodplan:
      time += get_grade_loading_time(grade, sigma, L, R)

    return time
```

In [41]:
```
def get_prod_plan_loading_time(prodplan1, prodplan2, sigma, L, R):

    return max(get_eaf_loading_time(prodplan1, sigma, L, R), get_eaf_loading_time(prodplan2, sigma, L, R))
```

In [41]:

MODEL FORMULATION

This is the model to be run in each iteration of the simulated annealing. For each grade, there is a single recipe to consider.

Sets

S is the set of scrap types

L is the set of scraps on the left side of the scrap yard

R is the set of scraps on the right side of the scrap yard

G is the set of grades

P is the sequence of jobs in EAF1

Q is the sequence of jobs in EAF2

Parameters

- For each scrap type $s \in S$ we are given the width of the box containing the scrap, w_s.
- For each grade $g \in G$ we are given a recipe, z_g.
- For each grade $g \in G$ we are given the total number of grabs required to load the necessary quantities, n_g.
- (ONLY FOR REAL PROD PLAN) For each grade $g \in G$ we are given the maximum time allowed for the loading of the bucket, r_g (in ms).
- For each job $p \in P$ in EAF1 we are given the corresponding grade g_p.
- For each job $q \in Q$ in EAF2 we are given the corresponding grade h_q.

Variables

For each scrap type $s \in S$:

- m_s is a non-negative continuous variable describing the midpoint of the box containing s, starting at the entry of the scrap yard closer to the EAFs.

For each pair of scrap types $s_1,s_2 \in S$:

- x_{s_1,s_2} is a binary variable that equals 1 iff s_1 and s_2 are both on the left or both on the right side of the scrap yard and scrap type s_1 is closer to the EAFs that s_2;
- d_{s_1,s_2} is a non-negative continuous variable describing the distance between m_{s_1} and m_{s_2} (regardless of their being or not in the same side of the scrap yard).

For each grade $g \in G$:

- t_g is a non-negative continuous variable describing the time it takes to have the bucket loaded with z_g ready at the EAF, including the unloading of the previous mixture.

And also:

- U is a non-negative continuous variable describing the total time taken to load all grades bound to EAF1;

- V is a non-negative continuous variable describing the total time taken to load all grades bound to EAF2;
- W is a non-negative continuous variable describing the total time taken to load the whole production plan.

Constraints

- Each midpoint m_s is given by (in meters): $$m_s = 20 + 1.7 + \Big(\sum_{s^\ast=0}^{31} (w_{s^\ast} + 1.7)x_{s^\ast,s} \Big) + \frac{w_s}{2} \, , \quad \forall \, s\in S$$
- Given any two scrap types on the same side of the scrap yard, exactly one sits closer to the EAFs: $$x_{s_1,s_2}+x_{s_2,s_1} = 1 \, , \quad \forall \, s_1,s_2 \in L$$ $$x_{s_1,s_2}+x_{s_2,s_1} = 1 \, , \quad \forall \, s_1,s_2 \in R$$
- No scrap type preceeds itself: $$x_{s,s} = 0 \, , \quad \forall \, s \in S$$
- Given any two scrap types on different sides of the scrap yard, no one preceeds the other: $$x_{s_1,s_2} = 0 \, , \quad \forall \, s_1 \in L, s_2 \in R$$ $$x_{s_1,s_2} = 0 \, , \quad \forall \, s_1 \in R, s_2 \in L$$
- Variables x are transitive: $$x_{s_1,s_2} = x_{s_2,s_3} = 1 ⇒ x_{s_1,s_3} = 1 \, , \quad \forall \, s_1,s_2,s_3 \in L$$ $$x_{s_1,s_2} = x_{s_2,s_3} = 1 ⇒ x_{s_1,s_3} = 1 \, , \quad \forall \, s_1,s_2,s_3 \in R$$
- The distance between any two scrap types: $$d_{s_1,s_2} = |m_{s_1}-m_{s_2}| \, , \quad \forall \, s_1,s_2 \in S$$
- The time necessary to load a grade $g \in G$ into the bucket (incluinding the unloading of the previous grade) is given by (in minutes): $$t_g = 6.2 + 1.47\times 2 + 0.67n_g + \frac{m_{z_g[0]} + m_{z_g[-1]}}{60} + \sum_{i=0}^{\text{len}(z_g)-2} \frac{d_{z_g[i],z_g[i+1]}}{40} \, , \quad \forall \, g\in G$$
- (ONLY FOR REAL PROD PLAN) The time necessary to load a grade $g \in G$ into the bucket cannot exceed the maximum time allowed: $$t_g \leq \frac{r_g}{1000} \, , \quad \forall \, g\in G$$
- The total time it takes to load all grades in the production plan: $$U = \sum_{p \in P} t_{g_p}$$ $$V = \sum_{q \in Q} t_{h_q}$$
- The variable to minimize: $$W \geq U$$ $$W \geq V$$

Objective function

The objective is to minimize the total time taken to produce all the grades in the production plan. This can be stated as

$$\min W$$

In [41]:

```

```

GUROBI STUFF

In [42]:

```
!pip install gurobipy  # install gurobipy, if not already installed
```

```
import gurobipy as gp  # import the installed package
```

```
Requirement already satisfied: gurobipy in /usr/local/lib/python3.11/dist-packages (12.0.2)
```

In [43]:
```
from gurobipy import *
from gurobipy import GRB # explicitly import GRB from gurobipy
```

In [44]:
```
# Create an environment with your WLS license
params = {
"WLSACCESSID": '********-****-****-****-************',
"WLSSECRET": '********-****-****-****-************',
"LICENSEID": *******,
}
env = gp.Env(params=params)

# Create the model within the Gurobi environment
model = gp.Model(env=env)
```

```
Set parameter WLSAccessID
Set parameter WLSSecret
Set parameter LicenseID to value 2619152
Academic license 2619152 - for non-commercial use only - registered to fi___@isel.pt
```

In [44]:

MODEL IMPLEMENTATION & SIMULATED ANNEALING

In [45]:
```
k = random.randint(1, 5000)
random.seed(k)

#print(f'Seed in use: {k}')
```

Chosing the neighbour

Neighbour functions considered:

v1 Chooses 1 layer in σ, chooses 2 scraps in that layer and swaps their boxes

v2 Chooses 1 layer in σ and replaces the entire layer permutation for another

v3 Chooses 2 layers in σ, chooses 2 scraps in each and swaps their boxes

v4 Chooses 2 layers in σ and swaps their entire permutation for another

v5 Chooses n layers in σ, chooses 2 scraps in each and swaps their boxes

v6 Chooses n layers in σ and swaps their entire permutation for another **v7** Chooses $n\geq 2$ layers in σ and swaps their entire permutation for another

In [46]:
```
# Function to choose 2 scraps (within same layer) and swap them in sigma

def choose_and_swap(layer):

    # Make copy as a list to allow modification
    new_layer_perm = list(sigma[layer-1])

    # Pick two different scraps in the layer
    scrap1 = random.choice(sigma[layer-1])
    scrap2 = random.choice(sigma[layer-1])
    while scrap2 == scrap1:
        scrap2 = random.choice(sigma[layer-1])
    # Swap them
    new_layer_perm = [scrap2 if x == scrap1 else scrap1 if x == scrap2 else x for x in new_layer_perm] # Use the modifiable

    return new_layer_perm
```

In [47]:
```
# Function to choose new permutation of a given layer

def choose_new_perm(layer):

    # Pick random permutation in layer
    new_perm = random.choice(perms[layer-1])
    while new_perm == sigma[layer-1]:
        new_perm = random.choice(perms[layer-1])
    # Return the new permutation
    return new_perm
```

In [48]:
```
# v1 Chooses 1 layer and swaps 2 scraps in that layer

def get_neighbour_v1(sigma):

    # Make copy
    new_sigma = sigma.copy()
```

```
    # Pick random layer
    i = random.choice([1,3,4,5,6,7])
    # Pick two different scraps in the chosen layer
    scrap1 = random.choice(sigma[i-1])
    scrap2 = random.choice(sigma[i-1])
    while scrap2 == scrap1:
        scrap2 = random.choice(sigma[i-1])
    # Swap them
    new_sigma[i-1] = [scrap2 if x == scrap1 else scrap1 if x == scrap2 else x for x in sigma[i-1]]

    return new_sigma
```

In [49]:

```
# v2 Chooses 1 layer and replaces the layer's permutation for another

def get_neighbour_v2(sigma):

    # Make copy
    new_sigma = sigma.copy()

    # Pick random layer
    i = random.choice([1,3,4,5,6,7])
    # Pick random permutation in layer i
    new_layer_perm = random.choice(perms[i-1])
    while new_layer_perm == sigma[i-1]:
        new_layer_perm = random.choice(perms[i-1])
    # Replace old layer i perm by new layer i perm in sigma
    new_sigma[i-1] = new_layer_perm

    return new_sigma
```

In [50]:

```
# v3 Chooses 2 layers and swaps 2 scraps in each

def get_neighbour_v3(sigma):

    # Make copy
    new_sigma = sigma.copy()

    # Pick two different random layers
    i = random.choice([1,3,4,5,6,7])
    j = random.choice([1,3,4,5,6,7])
    while j == i:
        j = random.choice([1,3,4,5,6,7])
```

```python
    # Swap scraps in chosen layers
    new_sigma[i-1] = tuple(choose_and_swap(i))
    new_sigma[j-1] = tuple(choose_and_swap(j))

    return new_sigma
```

In [51]:

```python
# v4 Chooses 2 layers and swaps its permutation for another

def get_neighbour_v4(sigma):

    # Make copy
    new_sigma = sigma.copy()

    # Pick two different random layers
    i = random.choice([1,3,4,5,6,7])
    j = random.choice([1,3,4,5,6,7])
    while j == i:
        j = random.choice([1,3,4,5,6,7])
    # Swap scraps in chosen layers
    new_sigma[i-1] = tuple(choose_new_perm(i))
    new_sigma[j-1] = tuple(choose_new_perm(j))

    return new_sigma
```

In [52]:
```
# v5 Chooses n layers to be modified and swaps 2 scraps in each

def get_neighbour_v5(sigma):

    # Make copy
    new_sigma = sigma.copy()

    # Pick the number of layers which will be changed
    n = random.randint(1, 6)
    print(n)

    # Pick n different random layers
    layers = random.sample([1,3,4,5,6,7], n)
    print(layers)
    # Swap scraps in chosen layers
    for i in layers:
        new_sigma[i-1] = tuple(choose_and_swap(i))

    return new_sigma
```

In [53]:
```
# v6 Chooses n layers to be modified and swaps their permutation for another

def get_neighbour_v6(sigma):

    # Make copy
    new_sigma = sigma.copy()

    # Pick the number of layers which will be changed
    n = random.randint(1, 6)
    print(n)

    # Pick n different random layers
    layers = random.sample([1,3,4,5,6,7], n)
    print(layers)
    # Swap scraps in chosen layers
    for i in layers:
        new_sigma[i-1] = tuple(choose_new_perm(i))

    return new_sigma
```

In [54]:
```
# v7 Chooses n»2 layers to be modified and swaps their permutation for another
```

```python
def get_neighbour_v7(sigma):

    # Make copy
    new_sigma = sigma.copy()

    # Pick the number (at least 2) of layers which will be changed
    n = random.randint(2, 6)
    print(n)

    # Pick n different random layers
    layers = random.sample([1,3,4,5,6,7], n)
    print(layers)
    # Swap scraps in chosen layers
    for i in layers:
        new_sigma[i-1] = tuple(choose_new_perm(i))

    return new_sigma
```

Implementing & runing the model

In [55]:
```python
def run_model(sigma, time_limit, left_scraps, right_scraps, grades_list, jobs_eaf1_list, jobs_eaf2_list):

    global x

    model.reset()
    # OPTIONAL: Limit each iteration to time_limit seconds
    #model.setParam("TimeLimit", time_limit)

    # Sets
    L = left_scraps # Use the passed left_scraps argument
    R = right_scraps # Use the passed right_scraps argument
    S = sorted(left_scraps + right_scraps)
    G = grades_list
    P = list(range(len(jobs_eaf1_list)))
    Q = list(range(len(jobs_eaf2_list)))

    # Parameters
    box_width = {s:box_width_dict[s] for s in S}
    recipes = {g:get_recipe(g, sigma) for g in G}
    total_grabs = {g:total_grabs_dict[g] for g in G}
    # Comment for LPP:
    max_time = {g:grades_max_time_dict[g]/1000 for g in G}
    jobs_eaf1 = dict(zip(list(range(len(jobs_eaf1_list))), jobs_eaf1_list))
```

```
jobs_eaf2 = dict(zip(list(range(len(jobs_eaf2_list))), jobs_eaf2_list))

# Variables
midpoint = model.addVars(S, name="midpoint")
x = model.addVars(S, S, vtype=GRB.BINARY, name="x")
aux = model.addVars(S, S, lb=-gp.GRB.INFINITY, name="aux")
dist = model.addVars(S, S, name="dist")
timetoready = model.addVars(G, name="timetoready")
U = model.addVar(name="U")
V = model.addVar(name="V")
W = model.addVar(name="W")

# Constrains
model.addConstrs((x[i,j] + x[j,i] == 1 for i in L for j in L if j != i), name="sides1a")
model.addConstrs((x[i,j] + x[j,i] == 1 for i in R for j in R if j != i), name="sides1b")
model.addConstr((sum(x[i,i] for i in S) == 0), name="sides2")
model.addConstrs((x[i,j] == 0 for i in L for j in R), name="sides3a")
model.addConstrs((x[i,j] == 0 for i in R for j in L), name="sides3b")
model.addConstrs((x[i,k] >= 1 - 10*(1-x[i,j]) - 10*(1-x[j,k]) for i in L for j in L for k in L), name="sides4a")
model.addConstrs((x[i,k] >= 1 - 10*(1-x[i,j]) - 10*(1-x[j,k]) for i in R for j in R for k in R), name="sides4b")
model.addConstrs((midpoint[s] == 20 + 1.7 + sum( (box_width[y] + 1.7)*x[y,s] for y in S) + box_width[s]/2
    for s in S), name="midpoints")
for i in S:
    for j in S:
        model.addConstr(aux[i, j] == midpoint[i] - midpoint[j], name=f'distanceaux_{i}_{j}')
        model.addConstr(dist[i,j] == abs_(aux[i,j]), name=f'distance_{i}_{j}')
# Comment for LPP:
model.addConstrs((timetoready[g] <= max_time[g] for g in G), name="maxtime")
model.addConstrs((timetoready[g] ==
    (6.2 + 2*1.47 + 0.67*total_grabs[g] + (midpoint[recipes[g][0]] + midpoint[recipes[g][-1]])/60 + sum( dist[recipes[g
    for i in range(len(recipes[g])-1))/40) for g in G), name="time")
model.addConstr((U == sum( timetoready[jobs_eaf1[p]] for p in P)), name="totaltime1")
model.addConstr((V == sum( timetoready[jobs_eaf2[q]] for q in Q)), name="totaltime2")
model.addConstr((U <= W), name = "totaltimeW1")
model.addConstr((V <= W), name = "totaltimeW2");

# Objective function
model.setObjective(W, GRB.MINIMIZE)

# Run model
model.optimize()

# do IIS if the model is infeasible
```

```python
    if model.Status == GRB.INFEASIBLE:
        model.computeIIS()

    # Output results
    new_t = W.X
    new_output = model.getVars()

    print(f'Sigma: {sigma}')
    print(f'Time: {new_t}')

    left_precedences = get_precedences(left_scraps)
    new_left_row = print_row(left_scraps, left_precedences)
    #print(f'Left row: {new_left_row}')
    right_precedences = get_precedences(right_scraps)
    new_right_row = print_row(right_scraps, right_precedences)
    #print(f'Right row: {new_right_row}')

    return new_t, new_output, new_left_row, new_right_row
```

Making the decision

In [56]:
```python
def get_decision(current_t, current_sigma, current_output, current_left_row, current_right_row,
                 new_t, new_sigma, new_output, new_left_row, new_right_row, beta, step):

    if new_t < current_t:
        current_t = new_t
        current_sigma = new_sigma
        current_output = new_output
        current_left_row = new_left_row
        current_right_row = new_right_row
        print('New best solution found!')
        print(f'New left row: {current_left_row}')
        print(f'New right row: {current_right_row}')
    else:
        p = random.random()
        if p < math.exp((current_t - new_t)/5*(1+beta*step)):
            current_t = new_t
            current_sigma = new_sigma
            current_output = new_output
            current_left_row = new_left_row
            current_right_row = new_right_row
            print('p update!')
            print(f'New left row: {current_left_row}')
```

```
            print(f'New right row: {current_right_row}')
        else:
            print('No update!')
            #pass

    return current_t, current_sigma, current_output, current_left_row, current_right_row
```

Simulated Annealing algorithm

In [57]:
```
currentWs = []
newWs = []
```

In [58]:
```
def simulated_annealing(prod_plan, split, n_iterations, time_limit, neighbour_fn, beta, sigma):

    # Prod plan
    if prod_plan == 'RPP':
        grades_list = grades_list_RPP
        jobs_eaf1_list = jobs_eaf1_list_RPP
        jobs_eaf2_list = jobs_eaf2_list_RPP
    else:
        grades_list = grades_list_LPP
        jobs_eaf1_list = jobs_eaf1_list_LPP
        jobs_eaf2_list = jobs_eaf2_list_LPP

    # Split
    left_scraps, right_scraps = get_scraps(split)

    # Initial sigma
    current_sigma = sigma

    # Intial solution
    current_t, current_output, current_left_row, current_right_row = run_model(current_sigma, time_limit, left_scraps, righ
                                                                          grades_list, jobs_eaf1_list, jobs_eaf2_list)
    currentWs.append(current_t)
    newWs.append(current_t)
    print(f'Left row: {current_left_row}')
    print(f'Right row: {current_right_row}')

    for i in range(n_iterations):

        print(f'\nIteration {i}')
```

```
        # Neighbout fn
        if neighbour_fn == 'v1':
          new_sigma = get_neighbour_v1(current_sigma)
        elif neighbour_fn == 'v2':
          new_sigma = get_neighbour_v2(current_sigma)
        elif neighbour_fn == 'v3':
          new_sigma = get_neighbour_v3(current_sigma)
        elif neighbour_fn == 'v4':
          new_sigma = get_neighbour_v4(current_sigma)
        elif neighbour_fn == 'v5':
          new_sigma = get_neighbour_v5(current_sigma)
        elif neighbour_fn == 'v6':
          new_sigma = get_neighbour_v6(current_sigma)
        else:
          new_sigma = get_neighbour_v7(current_sigma)

        new_t, new_output, new_left_row, new_right_row = run_model(new_sigma, time_limit, left_scraps, right_scraps,
                                                                  grades_list, jobs_eaf1_list, jobs_eaf2_list)

        print(f'Current W: {current_t}')
        print(f'New W: {new_t}')
        print(f'Current sigma: {current_sigma}')
        print(f'New sigma: {new_sigma}')
        print(f'Current left row: {current_left_row}')
        print(f'New left row: {new_left_row}')
        print(f'Current right row: {current_right_row}')
        print(f'New right row: {new_right_row}')

        current_t, current_sigma, current_output, current_left_row, current_right_row = get_decision(current_t, current_sig
                                                                                              current_left_row, curr
                                                                                              new_t, new_sigma, new_
                                                                                              new_left_row, new_righ
        currentWs.append(current_t)
        newWs.append(new_t)
        print(f'Current W: {current_t}')
        print(f'Current sigma: {current_sigma}')
        print(f'Current left row: {current_left_row}')
        print(f'Current right row: {current_right_row}')

    return current_t, current_sigma, current_output, current_left_row, current_right_row
```

In [58]:

SA parameters

In [59]:
```
prod_plan = 'RPP'                                                               # RPP / LPP
split = 'split3_05'                                                             # split0 / split1 / split2a / split2b / spl
n_iterations = 0                                                                # nonzero for SA
time_limit = None                                                               # OPTIONAL; in seconds
beta = 0.05
neighbour_fn = 'v1'                                                             # v1 / v2 / v3 / v4 / v5 / v6 / v7


# Layer permutations in increasing order
sigma0 = [perms[i][0] for i in range(7)]

# Random inital sigma
sigma = [random.choice(perms[i]) for i in range(7)]
```

In [60]:
```
start_time = datetime.now()
```

In [61]:
```
# run SA

final_t, final_sigma, final_output, final_left_row, final_right_row = simulated_annealing(prod_plan, split, n_iterations, t
                                                                                           neighbour_fn, beta, sigma0)
```

```
Discarded solution information
Gurobi Optimizer version 12.0.2 build v12.0.2rc0 (linux64 - "Ubuntu 22.04.4 LTS")

CPU model: Intel(R) Xeon(R) CPU @ 2.20GHz, instruction set [SSE2|AVX|AVX2]
Thread count: 1 physical cores, 2 logical processors, using up to 2 threads

Academic license 2619152 - for non-commercial use only - registered to fi___@isel.pt
Optimize a model with 10369 rows, 3121 columns and 29585 nonzeros
Model fingerprint: 0x55cc992d
Model has 1024 simple general constraints
  1024 ABS
Variable types: 2097 continuous, 1024 integer (1024 binary)
Coefficient statistics:
  Matrix range     [2e-02, 7e+01]
  Objective range  [1e+00, 1e+00]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 5e+03]
Presolve removed 7853 rows and 2634 columns
Presolve time: 0.12s
Presolved: 2516 rows, 487 columns, 7892 nonzeros
Variable types: 165 continuous, 322 integer (296 binary)
Found heuristic solution: objective 4281.2670833
Found heuristic solution: objective 4158.6937500
Extra simplex iterations after uncrush: 27

Root relaxation: objective 3.056452e+03, 637 iterations, 0.10 seconds (0.02 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 3056.45174    0  151 4158.69375 3056.45174  26.5%     -    0s
H    0     0                    4070.3820833 3056.45174  24.9%     -    0s
H    0     0                    4023.6495833 3056.45174  24.0%     -    0s
     0     0 3056.55160    0  166 4023.64958 3056.55160  24.0%     -    0s
     0     0 3056.57295    0  153 4023.64958 3056.57295  24.0%     -    0s
H    0     0                    3983.1837500 3056.57295  23.3%     -    0s
     0     0 3056.57295    0  146 3983.18375 3056.57295  23.3%     -    0s
     0     0 3056.57295    0  149 3983.18375 3056.57295  23.3%     -    0s
H    0     0                    3968.6745833 3056.57295  23.0%     -    0s
H    0     0                    3959.8995833 3056.57295  22.8%     -    0s
     0     0 3056.57295    0  149 3959.89958 3056.57295  22.8%     -    0s
     0     0 3056.60816    0  150 3959.89958 3056.60816  22.8%     -    1s
H    0     0                    3868.1712500 3056.60816  21.0%     -    1s
```

```
     0     0 3056.60816    0  149 3868.17125 3056.60816  21.0%     -    1s
     0     0 3056.60816    0  147 3868.17125 3056.60816  21.0%     -    1s
H    0     0                    3839.6095833 3056.60816  20.4%     -    1s
H    0     0                    3811.5595833 3056.60816  19.8%     -    1s
     0     0 3056.60816    0  147 3811.55958 3056.60816  19.8%     -    1s
     0     0 3056.60816    0  150 3811.55958 3056.60816  19.8%     -    1s
H    0     0                    3802.5595833 3056.60816  19.6%     -    1s
     0     0 3056.60816    0  147 3802.55958 3056.60816  19.6%     -    1s
     0     0 3056.60816    0  147 3802.55958 3056.60816  19.6%     -    1s
H    0     0                    3764.8504167 3056.60816  18.8%     -    1s
H    0     0                    3648.2479167 3056.60816  16.2%     -    1s
H    0     0                    3638.4412500 3056.60816  16.0%     -    1s
     0     0 3056.60816    0  147 3638.44125 3056.60816  16.0%     -    1s
     0     0 3056.60816    0  147 3638.44125 3056.60816  16.0%     -    1s
     0     0 3056.60816    0  147 3638.44125 3056.60816  16.0%     -    2s
     0     2 3056.60816    0  147 3638.44125 3056.60816  16.0%     -    2s
H   26    26                    3620.6937500 3073.09693  15.1%   334    3s
H   27    27                    3612.8612500 3073.09693  14.9%   325    3s
H   52    52                    3612.7712500 3073.09693  14.9%   228    3s
H   54    54                    3496.4787500 3073.09693  12.1%   221    3s
H   78    76                    3452.2604167 3073.09693  11.0%   190    4s
H   78    76                    3448.7679167 3073.09693  10.9%   190    4s
H   81    77                    3422.9904167 3073.09693  10.2%   186    4s
H  104    90                    3362.6679167 3073.09693  8.61%   157    4s
   120   104 3352.62488   13  165 3362.66792 3073.09693  8.61%   168    5s
H  130   100                    3352.3329167 3083.07553  8.03%   170    5s
H  156   124                    3348.6279167 3083.07553  7.93%   167    5s
H  156   122                    3346.5629167 3083.07553  7.87%   167    5s
H  186   128                    3344.8079167 3109.48842  7.04%   179    6s
H  243   141                    3343.4970833 3113.60055  6.88%   162    7s
H  276   150                    3342.1870833 3137.36629  6.13%   174    7s
   390   190 3208.42088    5  174 3342.18708 3151.28476  5.71%   169   10s
H  397   184                    3340.1245833 3151.28476  5.65%   168   10s
   680   216 3312.83241   17   90 3340.12458 3209.61629  3.91%   175   15s
  1002   225     cutoff   16      3340.12458 3251.35342  2.66%   178   20s
  1288   278 3320.96085   22  147 3340.12458 3273.61939  1.99%   171   25s
  1427   296 3326.85978   22  162 3340.12458 3273.61939  1.99%   178   30s
  1721   280 3327.88350   30  113 3340.12458 3282.08820  1.74%   172   35s
  2161   194 3336.51542   36   35 3340.12458 3298.10496  1.26%   157   40s
  2613   156 3339.58241   32   75 3340.12458 3313.33517  0.80%   149   45s

Cutting planes:
  Implied bound: 2
```

```
  MIR: 6
  Flow cover: 1
  Inf proof: 1

Explored 3174 nodes (425591 simplex iterations) in 49.27 seconds (30.95 work units)
Thread count was 2 (of 2 available processors)

Solution count 10: 3340.12 3342.19 3342.19 ... 3362.67

Optimal solution found (tolerance 1.00e-04)
Best objective 3.340124583333e+03, best bound 3.340096098626e+03, gap 0.0009%
Sigma: [(6, 8, 10, 12, 13, 18, 26), (28,), (4, 5), (0, 1, 2, 3, 7, 9, 11, 22, 23), (15, 17, 29, 30), (14, 16, 21, 31), (19,
20, 24, 25, 27)]
Time: 3340.124583333332
Left row: [2, 10, 11, 31, 8, 6, 0, 28, 9, 20, 14, 15, 24, 23, 12]
Right row: [26, 30, 18, 4, 5, 1, 3, 7, 27, 16, 17, 19, 29, 21, 25, 13, 22]
```

In [61]:

```
```

SA results

In [62]:

```
end_time = datetime.now()
runtime = (end_time - start_time).total_seconds()
print('Runtime (sec): {}'.format(runtime))
```

```
Runtime (sec): 50.259908
```

In [63]:

```
print(f'Prod plan: {prod_plan}')
print(f'Split: {split}')
print(f'Initial sigma: sigma0')                                          # sigma0 / random
if n_iterations != 0:
    print(f'Number of iterations: {n_iterations}')
    print(f'Time limit: {time_limit}')
    print(f'beta: {beta}')
    print(f'Neighbiour fn: {neighbour_fn}')
    print(f'Seed: {k}')
```

```
Prod plan: RPP
Split: split3_05
Initial sigma: sigma0
```

In [64]:

```
print(f'Final W: {final_t}')
print(f'Final sigma: {final_sigma}')
print(f'Final left row: {final_left_row}')
```

```
print(f'Final right row: {final_right_row}')
```

```
Final W: 3340.124583333332
Final sigma: [(6, 8, 10, 12, 13, 18, 26), (28,), (4, 5), (0, 1, 2, 3, 7, 9, 11, 22, 23), (15, 17, 29, 30), (14, 16, 21, 31), (19, 20, 24, 25, 27)]
Final left row: [2, 10, 11, 31, 8, 6, 0, 28, 9, 20, 14, 15, 24, 23, 12]
Final right row: [26, 30, 18, 4, 5, 1, 3, 7, 27, 16, 17, 19, 29, 21, 25, 13, 22]
```

In [65]:

```
print(f'currentWs = {currentWs}')
print(f'newWs = {newWs}')
```

```
currentWs = [3340.124583333332]
newWs = [3340.124583333332]
```

In [65]:

In [65]: