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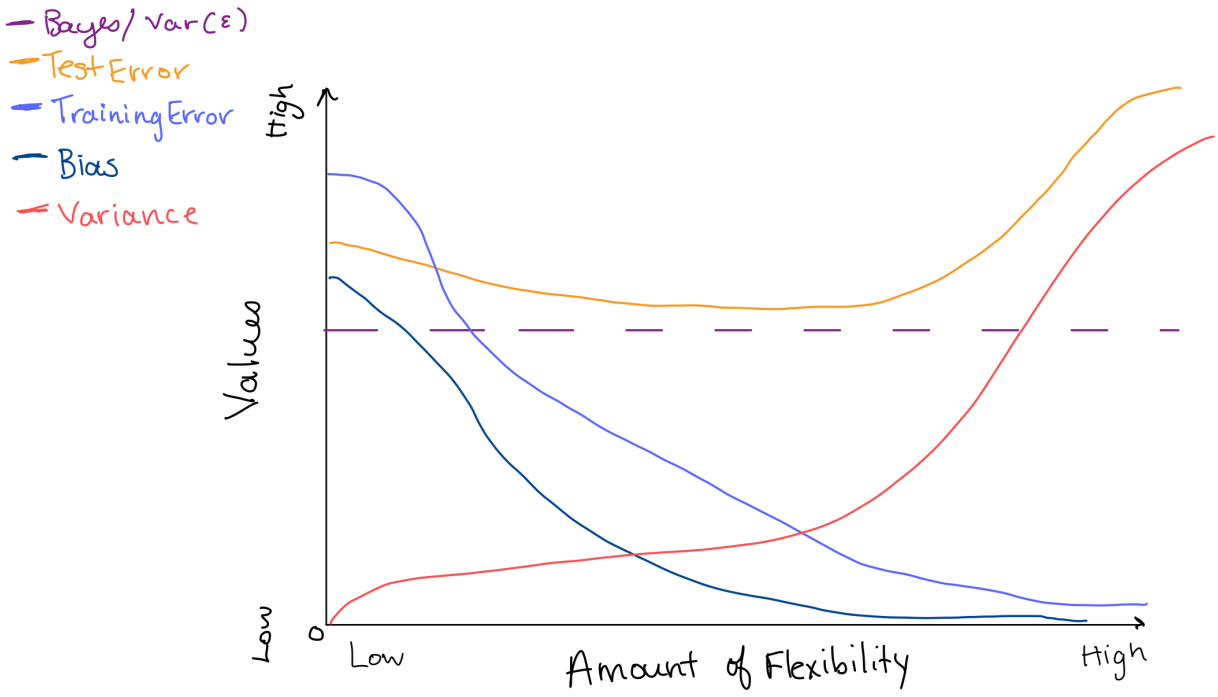
ML Problem Set 1

I worked with Wasil Engel as a partner on this PSet. (UCID: 12231558)

All working code can be located via my GitHub.

Chapter 2 Problems

3.



Bias: Decreases as the model becomes more flexible.

Variance: Increases as we have a more flexible model. The variance refers to the amount by which our estimate of f would change if we estimated it using a different training data set. High variance means small changes in the training data can result in large changes in our estimate of $\hat{f}$.

Training Error: Training error rate consistently declines as flexibility increases. This is because as we have a more flexible model, it fits to our training data

Test error: Test error can never lie below our Bayes/irreducible error curves. At a point a model can be optimally flexible, at a point where it is not too biased or overfit, and is close to the Bayes/irreducible error.

Bayes/irreducible error: This is the error associated inherently with estimated models; it serves as a baseline of error in the chosen model.

5. What are the advantages and disadvantages of a very flexible (versus a less flexible) approach for regression or classification? Under what circumstances might a more flexible approach be preferred to a less flexible approach? When might a less flexible approach be preferred?

   A flexible model excels when data is non-linear. If the model will need to make many estimates, a flexible model will also excel when the variance in the data is low. Less flexible approaches are preferred when we are conducting general tasks such as linear regression, because it can only generate linear functions. Whenever inference and interpretability are of importance, the simplicity of inflexible models is advantageous in accomplishing this.

10. A) How many rows are in this data set?

    There are 506 rows and 14 columns This data comes from a study where Boston housing market data is used to generate quantitative estimates of the willingness to pay for air quality improvements, where each observation contains different neighborhoods

variables/indicators in Boston. It is necessary to isolate independent influence of air pollution to reduce bias in drawn conclusions.

B) My findings show that throughout Boston there persists a high volume of nitric oxide concentrations. The higher proportion of Blacks neighborhoods that are subject to higher levels of NOX gives evidence to the notion of environmental oppression, as well as the narrative of having higher crime rates. [See GitHub for graphs]

C) Nitric oxide levels, neighborhoods with higher representation of Blacks in a town, the median value of owner-occupied homes, and a larger share of being in the lower percentage status in comparison to the population.

D) When we look at the range of some of the predictors for neighborhoods 380, 418, 405, 410, and 414, we can see that there are large disparities between crime, tax rate, and pupil-teacher ratio by town. Where the tax rate is highest, crime is relatively low. Where crime is low, the pupil-teacher is usually pretty high. In the range of these three predictors, it is evident that some neighborhoods are subject to higher rates of unequal opportunity to prosper.

E) We see there are 35 suburbs in this data set that bound the Charles River.

F) We see that the median is 19.05 for pupil-teacher ratio.

G) Suburbs '398' and '405' have the lowest median value of owner-occupied homes, and the other predictors are just as alarming. Crime is high, all units were built prior to 1940, hold a high proportion of blacks in the town, and are at a higher percentage of being the lower status in the Boston population. The 5 suburbs that have the lowest median value of owner-occupied homes also do have high levels of nitric oxides concentration.

H) 64 average more than seven rooms per dwelling. 13 suburbs average more than 8 rooms per their dwelling. My understanding of the suburbs that average more than eight rooms per dwelling are either due to high rent driving households to increase the amount dwelling in their spaces. I assume that these still could be suburbs that suffer from high crime rates, higher levels of NOX, and yet some surprising predictor values (low-vs-high median value of owner-occupied homes, fluctuating tax rates, and varying pupil-teacher ratio).

Chapter 3 Problems

3A. Which answer is correct, and why?

- For a fixed value of IQ and GPA, males earn more on average than females provided that the GPA is high enough. Therefore, **option iii** is correct.
    - Males earn more on average than females provided that the GPA is high enough. This is due to the predictors X_3, X_4, and X_5 all interacting with each other in one way. For example, we have a male and female with the exact same GPA and IQ. Men will have less interacting predictors if we fit the model to the estimated values of beta, where $\hat{\beta}_5$ is -10, then by multiplication, males will earn more on average than females, provided that their GPA is high enough.

3B. Predict the salary of a female with IQ of 110 and a GPA of 4.0.

- $y = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3 + \hat{\beta}_4 X_4 + \hat{\beta}_5 X_5$
- $y = 50 + 20 * 4.0 + 0.07 * 110 + 35 * 1 + 0.01 * 4.0 * 110 - 10 * 4.0$
- $\hat{y} = 137.1$

3C. True or false: Since the coefficient for the GPA/IQ interaction term is very small, there is very little evidence of an interaction effect. Justify your answer.

False. The coefficient estimate does not provide any statistical insight to conclude that any evidence exists between any of the interactions. There is no criteria for us to check for hypothesis testing, or to compare any values against t-statistics, p-values, or confidence intervals.

15. This problem involves the Boston data set, which we saw in the lab for this chapter. We will now try to predict per capita crime rate using the other variables in this data set. In other words, per capita crime rate is the response, and the other variables are the predictors.

A) For each predictor, fit a simple linear regression model to predict the response. Describe your results. In which of the models is there a statistically significant association between the predictor and the response? Create some plots to back up your assertions.

It appears that all are statistically significant except CHAS, an indicator/dummy variable. While some predictors have some outliers (NOX, AGE, TAX, PTRATIO, B, LSTAT), they are still statistically significantly.

```
OLS results for predictor:ZN

                            OLS Regression Results
==============================================================================
Dep. Variable:                   CRIM   R-squared:                       0.040
Model:                            OLS   Adj. R-squared:                  0.038
Method:                 Least Squares   F-statistic:                     20.88
Date:                Mon, 25 Jan 2021   Prob (F-statistic):           6.15e-06
Time:                        20:33:12   Log-Likelihood:                -1795.8
No. Observations:                 506   AIC:                             3596.
Df Residuals:                     504   BIC:                             3604.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      4.4292      0.417     10.620      0.000       3.610       5.249
boston[col]   -0.0735      0.016     -4.570      0.000      -0.105      -0.042
==============================================================================
Omnibus:                      568.366   Durbin-Watson:                   0.862
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            32952.356
Skew:                           5.270   Prob(JB):                         0.00
Kurtosis:                      41.103   Cond. No.                         28.8
```

```
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

OLS results for predictor:INDUS

                            OLS Regression Results
==============================================================================
Dep. Variable:                   CRIM   R-squared:                       0.164
Model:                            OLS   Adj. R-squared:                  0.162
Method:                 Least Squares   F-statistic:                     98.58
Date:                Mon, 25 Jan 2021   Prob (F-statistic):           2.44e-21
Time:                        20:33:12   Log-Likelihood:                -1760.9
No. Observations:                 506   AIC:                             3526.
Df Residuals:                     504   BIC:                             3534.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      -2.0509      0.668     -3.072      0.002      -3.362      -0.739
boston[col]     0.5068      0.051      9.929      0.000       0.407       0.607
==============================================================================
Omnibus:                      585.528   Durbin-Watson:                   0.990
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            41469.710
Skew:                           5.456   Prob(JB):                         0.00
Kurtosis:                      45.987   Cond. No.                         25.1
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

OLS results for predictor:CHAS

                            OLS Regression Results
==============================================================================
Dep. Variable:                   CRIM   R-squared:                       0.003
Model:                            OLS   Adj. R-squared:                  0.001
Method:                 Least Squares   F-statistic:                     1.546
Date:                Mon, 25 Jan 2021   Prob (F-statistic):              0.214
Time:                        20:33:12   Log-Likelihood:                -1805.3
No. Observations:                 506   AIC:                             3615.
Df Residuals:                     504   BIC:                             3623.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept       3.7232      0.396      9.404      0.000       2.945       4.501
boston[col]    -1.8715      1.505     -1.243      0.214      -4.829       1.086
==============================================================================
Omnibus:                      562.698   Durbin-Watson:                   0.822
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            30864.755
Skew:                           5.205   Prob(JB):                         0.00
Kurtosis:                      39.818   Cond. No.                         3.96
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

```
OLS results for predictor:NOX

                            OLS Regression Results
==============================================================================
Dep. Variable:                   CRIM   R-squared:                       0.174
Model:                            OLS   Adj. R-squared:                  0.173
Method:                 Least Squares   F-statistic:                     106.4
Date:                Mon, 25 Jan 2021   Prob (F-statistic):           9.16e-23
Time:                        20:33:12   Log-Likelihood:                -1757.6
No. Observations:                 506   AIC:                             3519.
Df Residuals:                     504   BIC:                             3528.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     -13.5881      1.702     -7.986      0.000     -16.931     -10.245
boston[col]    30.9753      3.003     10.315      0.000      25.076      36.875
==============================================================================
Omnibus:                      591.496   Durbin-Watson:                   0.994
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            42994.381
Skew:                           5.544   Prob(JB):                         0.00
Kurtosis:                      46.776   Cond. No.                         11.3
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

OLS results for predictor:RM

                            OLS Regression Results
==============================================================================
Dep. Variable:                   CRIM   R-squared:                       0.048
Model:                            OLS   Adj. R-squared:                  0.046
Method:                 Least Squares   F-statistic:                     25.62
Date:                Mon, 25 Jan 2021   Prob (F-statistic):           5.84e-07
Time:                        20:33:12   Log-Likelihood:                -1793.5
No. Observations:                 506   AIC:                             3591.
Df Residuals:                     504   BIC:                             3600.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      20.5060      3.362      6.099      0.000      13.901      27.111
boston[col]    -2.6910      0.532     -5.062      0.000      -3.736      -1.646
==============================================================================
Omnibus:                      576.890   Durbin-Watson:                   0.883
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            36966.825
Skew:                           5.361   Prob(JB):                         0.00
Kurtosis:                      43.477   Cond. No.                         58.4
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

OLS results for predictor:AGE

                            OLS Regression Results
==============================================================================
Dep. Variable:                   CRIM   R-squared:                       0.123
Model:                            OLS   Adj. R-squared:                  0.121
```

```
Method:                 Least Squares   F-statistic:                     70.72
Date:                Mon, 25 Jan 2021   Prob (F-statistic):           4.26e-16
Time:                        20:33:12   Log-Likelihood:                -1772.9
No. Observations:                 506   AIC:                             3550.
Df Residuals:                     504   BIC:                             3558.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      -3.7527      0.944     -3.974      0.000      -5.608      -1.898
boston[col]     0.1071      0.013      8.409      0.000       0.082       0.132
==============================================================================
Omnibus:                      575.090   Durbin-Watson:                   0.960
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            36851.412
Skew:                           5.331   Prob(JB):                         0.00
Kurtosis:                      43.426   Cond. No.                         195.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

OLS results for predictor:DIS

                            OLS Regression Results
==============================================================================
Dep. Variable:                   CRIM   R-squared:                       0.143
Model:                            OLS   Adj. R-squared:                  0.141
Method:                 Least Squares   F-statistic:                     83.97
Date:                Mon, 25 Jan 2021   Prob (F-statistic):           1.27e-18
Time:                        20:33:12   Log-Likelihood:                -1767.1
No. Observations:                 506   AIC:                             3538.
Df Residuals:                     504   BIC:                             3547.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept       9.4489      0.731     12.934      0.000       8.014      10.884
boston[col]    -1.5428      0.168     -9.163      0.000      -1.874      -1.212
==============================================================================
Omnibus:                      577.090   Durbin-Watson:                   0.957
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            37542.100
Skew:                           5.357   Prob(JB):                         0.00
Kurtosis:                      43.815   Cond. No.                         9.32
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

OLS results for predictor:RAD

                            OLS Regression Results
==============================================================================
Dep. Variable:                   CRIM   R-squared:                       0.387
Model:                            OLS   Adj. R-squared:                  0.386
Method:                 Least Squares   F-statistic:                     318.1
Date:                Mon, 25 Jan 2021   Prob (F-statistic):           1.62e-55
Time:                        20:33:12   Log-Likelihood:                -1682.3
No. Observations:                 506   AIC:                             3369.
Df Residuals:                     504   BIC:                             3377.
Df Model:                           1
```

```
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      -2.2709      0.445     -5.105      0.000      -3.145      -1.397
boston[col]     0.6141      0.034     17.835      0.000       0.546       0.682
==============================================================================
Omnibus:                      654.232   Durbin-Watson:                   1.336
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            74327.568
Skew:                           6.441   Prob(JB):                         0.00
Kurtosis:                      60.961   Cond. No.                         19.2
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

OLS results for predictor:TAX

                            OLS Regression Results
==============================================================================
Dep. Variable:                   CRIM   R-squared:                       0.336
Model:                            OLS   Adj. R-squared:                  0.335
Method:                 Least Squares   F-statistic:                     254.9
Date:                Mon, 25 Jan 2021   Prob (F-statistic):           9.76e-47
Time:                        20:33:12   Log-Likelihood:                -1702.5
No. Observations:                 506   AIC:                             3409.
Df Residuals:                     504   BIC:                             3418.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      -8.4748      0.818    -10.365      0.000     -10.081      -6.868
boston[col]     0.0296      0.002     15.966      0.000       0.026       0.033
==============================================================================
Omnibus:                      634.003   Durbin-Watson:                   1.252
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            63141.063
Skew:                           6.134   Prob(JB):                         0.00
Kurtosis:                      56.332   Cond. No.                     1.16e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.16e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

OLS results for predictor:PTRATIO

                            OLS Regression Results
==============================================================================
Dep. Variable:                   CRIM   R-squared:                       0.083
Model:                            OLS   Adj. R-squared:                  0.081
Method:                 Least Squares   F-statistic:                     45.67
Date:                Mon, 25 Jan 2021   Prob (F-statistic):           3.88e-11
Time:                        20:33:12   Log-Likelihood:                -1784.1
No. Observations:                 506   AIC:                             3572.
Df Residuals:                     504   BIC:                             3581.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
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Intercept     -17.5307      3.147     -5.570      0.000     -23.714     -11.347
boston[col]     1.1446      0.169      6.758      0.000       0.812       1.477
==============================================================================
Omnibus:                      568.808   Durbin-Watson:                   0.909
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            34373.378
Skew:                           5.256   Prob(JB):                         0.00
Kurtosis:                      41.985   Cond. No.                         160.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

OLS results for predictor:B

                            OLS Regression Results
==============================================================================
Dep. Variable:                    CRIM   R-squared:                       0.142
Model:                             OLS   Adj. R-squared:                  0.141
Method:                  Least Squares   F-statistic:                     83.69
Date:                 Mon, 25 Jan 2021   Prob (F-statistic):           1.43e-18
Time:                         20:33:12   Log-Likelihood:                -1767.2
No. Observations:                 506   AIC:                             3538.
Df Residuals:                     504   BIC:                             3547.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      16.2680      1.430     11.376      0.000      13.458      19.078
boston[col]    -0.0355      0.004     -9.148      0.000      -0.043      -0.028
==============================================================================
Omnibus:                      591.626   Durbin-Watson:                   1.001
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            43282.465
Skew:                           5.543   Prob(JB):                         0.00
Kurtosis:                      46.932   Cond. No.                     1.49e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.49e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

OLS results for predictor:LSTAT

                            OLS Regression Results
==============================================================================
Dep. Variable:                    CRIM   R-squared:                       0.205
Model:                             OLS   Adj. R-squared:                  0.203
Method:                  Least Squares   F-statistic:                     129.6
Date:                 Mon, 25 Jan 2021   Prob (F-statistic):           7.12e-27
Time:                         20:33:12   Log-Likelihood:                -1748.2
No. Observations:                 506   AIC:                             3500.
Df Residuals:                     504   BIC:                             3509.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      -3.2946      0.695     -4.742      0.000      -4.660      -1.930
boston[col]     0.5444      0.048     11.383      0.000       0.450       0.638
==============================================================================
Omnibus:                      600.766   Durbin-Watson:                   1.184
```

```
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            49637.173
Skew:                           5.638   Prob(JB):                        0.00
Kurtosis:                      50.193   Cond. No.                        29.7
================================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

OLS results for predictor:MDEV

                            OLS Regression Results
================================================================================
Dep. Variable:                    CRIM   R-squared:                       0.149
Model:                             OLS   Adj. R-squared:                  0.147
Method:                  Least Squares   F-statistic:                     88.15
Date:                 Mon, 25 Jan 2021   Prob (F-statistic):           2.08e-19
Time:                         20:33:12   Log-Likelihood:                -1765.3
No. Observations:                  506   AIC:                             3535.
Df Residuals:                      504   BIC:                             3543.
Df Model:                            1
Covariance Type:             nonrobust
================================================================================
                  coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
Intercept      11.7202      0.935     12.539      0.000       9.884      13.557
boston[col]    -0.3606      0.038     -9.389      0.000      -0.436      -0.285
==============================================================================
Omnibus:                      559.282   Durbin-Watson:                   1.000
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            32809.507
Skew:                           5.114   Prob(JB):                        0.00
Kurtosis:                      41.099   Cond. No.                        64.5
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

B) Fit a multiple regression model to predict the response using all the predictors. Describe your results. For which predictors can we reject the null hypothesis $H_0: \beta_j = 0$?

OLS Regression Results

Dep. Variable:	CRIM	R-squared:	0.448
Model:	OLS	Adj. R-squared:	0.434
Method:	Least Squares	F-statistic:	33.30
Date:	Mon, 25 Jan 2021	Prob (F-statistic):	4.17e-56
Time:	17:55:59	Log-Likelihood:	-1655.9
No. Observations:	506	AIC:	3338.
Df Residuals:	493	BIC:	3393.
Df Model:	12		
Covariance Type:	nonrobust		

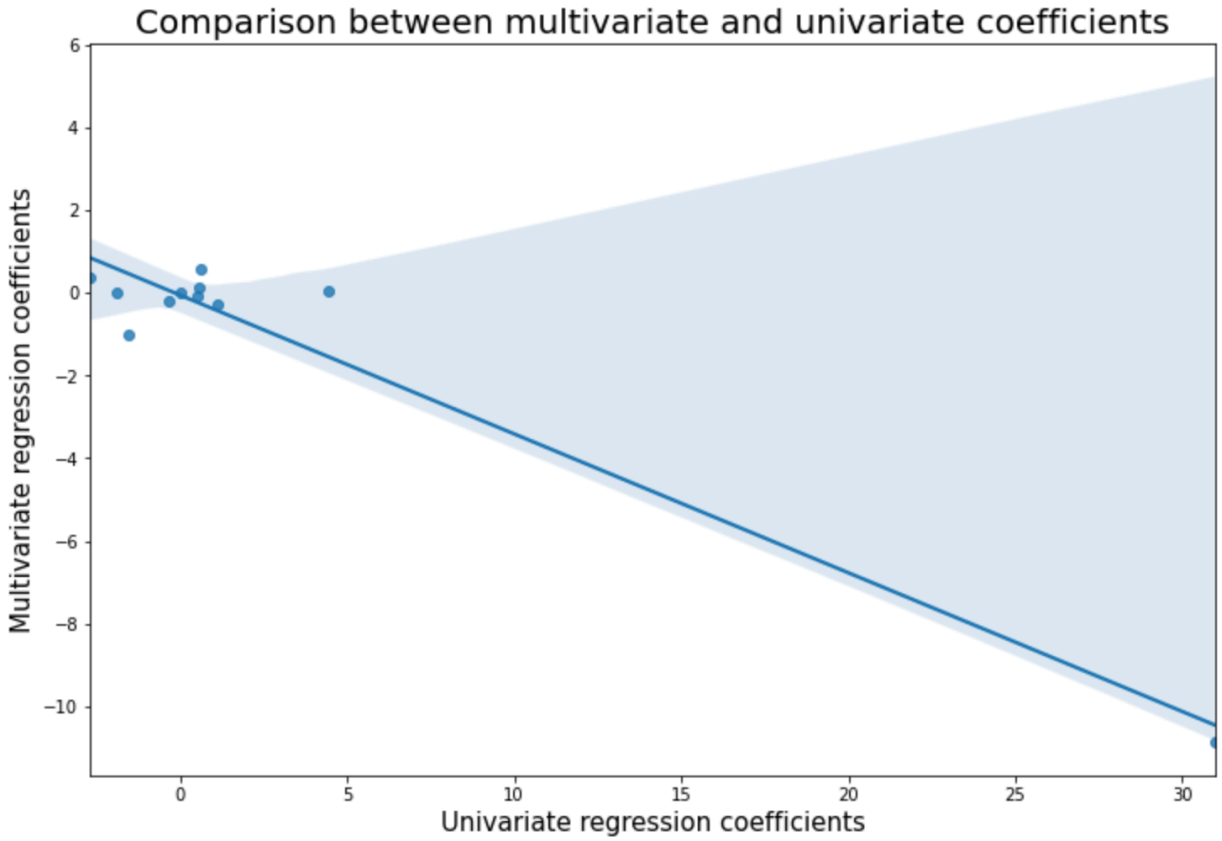
	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	17.5102	7.264	2.411	0.016	3.238	31.782
AGE	0.0014	0.018	0.081	0.936	-0.034	0.037
B	-0.0069	0.004	-1.879	0.061	-0.014	0.000
DIS	-1.0010	0.283	-3.539	0.000	-1.557	-0.445
INDUS	-0.0669	0.083	-0.802	0.423	-0.231	0.097
LSTAT	0.1211	0.076	1.593	0.112	-0.028	0.271
MDEV	-0.2047	0.060	-3.405	0.001	-0.323	-0.087
NOX	-10.8572	5.287	-2.054	0.041	-21.245	-0.470
PTRATIO	-0.2737	0.187	-1.463	0.144	-0.641	0.094
RAD	0.5849	0.088	6.632	0.000	0.412	0.758
RM	0.3908	0.615	0.635	0.526	-0.818	1.600
TAX	-0.0034	0.005	-0.666	0.506	-0.014	0.007
ZN	0.0450	0.019	2.394	0.017	0.008	0.082

Omnibus:	661.885	Durbin-Watson:	1.515
Prob(Omnibus):	0.000	Jarque-Bera (JB):	82471.479
Skew:	6.537	Prob(JB):	0.00
Kurtosis:	64.162	Cond. No.	1.58e+04

Using all of the predictors, we observe different results. Age, proportion of Blacks in the suburb, INDUS, LSTAT, PTRATIO, RM, and TAX were all predictors that were not significant, as we observe low t-statistics and p-values. All the others we can reject the null hypothesis. NOX is along the margin line in terms of t-statistics and its p-value; it is above a 2 t-statistic value, and comes close to being insignificant in terms of its p-value, hence it should be used with discretion.

C) How do your results from (a) compare to your results from (b)? Create a plot displaying the univariate regression coefficients from (a) on the x-axis, and the multiple regression coefficients from (b) on the y-axis. That is, each predictor is displayed as a single point in the plot. Its coefficient in a simple linear regression model is shown on the x-axis, and its coefficient estimate in the multiple linear regression model is shown on the y-axis.

The results from (a) provide us with context as to how each predictor influences the predicted value, that is per capita crime rate. As for (b), we observe the multicollinearity and interactions of using multiple predictors to predict our response.



When we plot the findings, we observe a downward trend, suggesting that independently, using predictors individually influence the response at greater magnitudes.

D) Is there evidence of non-linear association between any of the predictors and the response? To answer this question, for each predictor X, fit a model of the form:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + .$$