

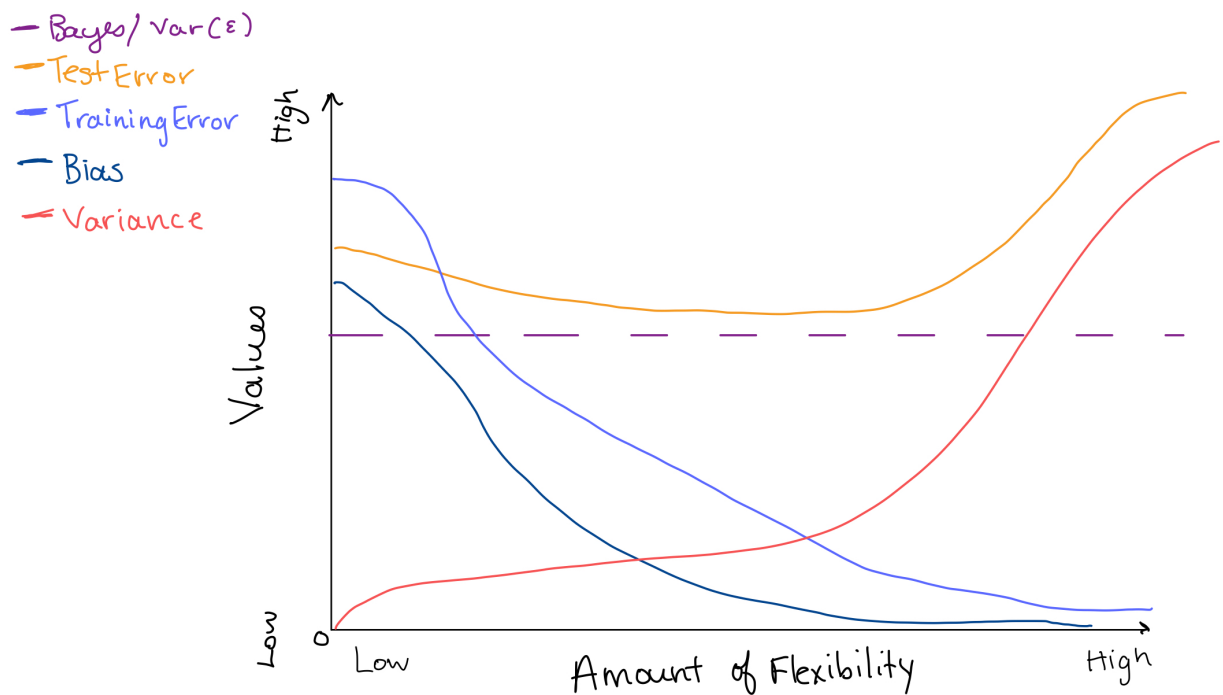
## ML Problem Set 1

I worked with Wasil Engel as a partner on this PSet. (UCID: 12231558)

All working code can be located via my [GitHub](#).

### Chapter 2 Problems

3.



**Bias:** Decreases as the model becomes more flexible.

**Variance:** Increases as we have a more flexible model. The variance refers to the amount by which our estimate of  $f$  would change if we estimated it using a different training data set. High variance means small changes in the training data can result in large changes in our estimate of  $\hat{f}$ .

**Training Error:** Training error rate consistently declines as flexibility increases. This is because as we have a more flexible model, it fits to our training data

**Test error:** Test error can never lie below our Bayes/irreducible error curves. At a point a model can be optimally flexible, at a point where it is not too biased or overfit, and is close to the Bayes/irreducible error.

**Bayes/irreducible error:** This is the error associated inherently with estimated models; it serves as a baseline of error in the chosen model.

5. What are the advantages and disadvantages of a very flexible (versus a less flexible) approach for regression or classification? Under what circumstances might a more flexible approach be preferred to a less flexible approach? When might a less flexible approach be preferred?

A flexible model excels when data is non-linear. If the model will need to make many estimates, a flexible model will also excel when the variance in the data is low. Less flexible approaches are preferred when we are conducting general tasks such as linear regression, because it can only generate linear functions. Whenever inference and interpretability are of importance, the simplicity of inflexible models is advantageous in accomplishing this.

10. A) How many rows are in this data set?

There are 506 rows and 14 columns This data comes from a study where Boston housing market data is used to generate quantitative estimates of the willingness to pay for air quality improvements, where each observation contains different neighborhoods

variables/indicators in Boston. It is necessary to isolate independent influence of air pollution to reduce bias in drawn conclusions.

B) My findings show that throughout Boston there persists a high volume of nitric oxide concentrations. The higher proportion of Blacks neighborhoods that are subject to higher levels of NOX gives evidence to the notion of environmental oppression, as well as the narrative of having higher crime rates. [See [GitHub](#) for graphs]

C) Nitric oxide levels, neighborhoods with higher representation of Blacks in a town, the median value of owner-occupied homes, and a larger share of being in the lower percentage status in comparison to the population.

D) When we look at the range of some of the predictors for neighborhoods 380, 418, 405, 410, and 414, we can see that there are large disparities between crime, tax rate, and pupil-teacher ratio by town. Where the tax rate is highest, crime is relatively low. Where crime is low, the pupil-teacher is usually pretty high. In the range of these three predictors, it is evident that some neighborhoods are subject to higher rates of unequal opportunity to prosper.

E) We see there are 35 suburbs in this data set that bound the Charles River.

F) We see that the median is 19.05 for pupil-teacher ratio.

G) Suburbs '398' and '405' have the lowest median value of owner-occupied homes, and the other predictors are just as alarming. Crime is high, all units were built prior to 1940, hold a high proportion of blacks in the town, and are at a higher percentage of being the lower status in the Boston population. The 5 suburbs that have the lowest median value of owner-occupied homes also do have high levels of nitric oxides concentration.

H) 64 average more than seven rooms per dwelling. 13 suburbs average more than 8 rooms per their dwelling. My understanding of the suburbs that average more than eight rooms per dwelling are either due to high rent driving households to increase the amount dwelling in their spaces. I assume that these still could be suburbs that suffer from high crime rates, higher levels of NOX, and yet some surprising predictor values (low-vs-high median value of owner-occupied homes, fluctuating tax rates, and varying pupil-teacher ratio).

### Chapter 3 Problems

3A. Which answer is correct, and why?

- For a fixed value of IQ and GPA, males earn more on average than females provided that the GPA is high enough. Therefore, **option iii** is correct.
  - o Males earn more on average than females provided that the GPA is high enough. This is due to the predictors  $X_3$ ,  $X_4$ , and  $X_5$  all interacting with each other in one way. For example, we have a male and female with the exact same GPA and IQ. Men will have less interacting predictors if we fit the model to the estimated values of beta, where  $\hat{\beta}_5$  is -10, then by multiplication, males will earn more on average than females, provided that their GPA is high enough.

3B. Predict the salary of a female with IQ of 110 and a GPA of 4.0.

- $y = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3 + \hat{\beta}_4 X_4 + \hat{\beta}_5 X_5$
- $y = 50 + 20 * 4.0 + 0.07 * 110 + 35 * 1 + 0.01 * 4.0 * 110 - 10 * 4.0$
- $\hat{y} = 137.1$

3C. True or false: Since the coefficient for the GPA/IQ interaction term is very small, there is very little evidence of an interaction effect. Justify your answer.

**False.** The coefficient estimate does not provide any statistical insight to conclude that any evidence exists between any of the interactions. There is no criteria for us to check for hypothesis testing, or to compare any values against t-statistics, p-values, or confidence intervals.

15. This problem involves the Boston data set, which we saw in the lab for this chapter. We will now try to predict per capita crime rate using the other variables in this data set. In other words, per capita crime rate is the response, and the other variables are the predictors.

A) For each predictor, fit a simple linear regression model to predict the response. Describe your results. In which of the models is there a statistically significant association between the predictor and the response? Create some plots to back up your assertions.

It appears that all are statistically significant except CHAS, an indicator/dummy variable. While some predictors have some outliers (NOX, AGE, TAX, PTRATIO, B, LSTAT), they are still statistically significantly.

OLS results for predictor: ZN

OLS Regression Results						
Dep. Variable:	CRIM	R-squared:	0.040			
Model:	OLS	Adj. R-squared:	0.038			
Method:	Least Squares	F-statistic:	20.88			
Date:	Mon, 25 Jan 2021	Prob (F-statistic):	6.15e-06			
Time:	20:33:12	Log-Likelihood:	-1795.8			
No. Observations:	506	AIC:	3596.			
Df Residuals:	504	BIC:	3604.			
Df Model:	1					
Covariance Type:	nonrobust					
	coef	std err	t	P> t	[0.025	0.975]
Intercept	4.4292	0.417	10.620	0.000	3.610	5.249
boston[col]	-0.0735	0.016	-4.570	0.000	-0.105	-0.042
Omnibus:	568.366	Durbin-Watson:	0.862			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	32952.356			
Skew:	5.270	Prob(JB):	0.00			
Kurtosis:	41.103	Cond. No.	28.8			

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS results for predictor:INDUS

#### OLS Regression Results

```
=====
Dep. Variable:          CRIM      R-squared:                0.164
Model:                  OLS      Adj. R-squared:             0.162
Method:                 Least Squares      F-statistic:         98.58
Date:                  Mon, 25 Jan 2021    Prob (F-statistic):      2.44e-21
Time:                  20:33:12           Log-Likelihood:        -1760.9
No. Observations:      506             AIC:                  3526.
Df Residuals:          504             BIC:                  3534.
Df Model:              1
Covariance Type:       nonrobust
=====
```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	-2.0509	0.668	-3.072	0.002	-3.362	-0.739
boston[col]	0.5068	0.051	9.929	0.000	0.407	0.607

```
=====
Omnibus:                585.528      Durbin-Watson:          0.990
Prob(Omnibus):          0.000        Jarque-Bera (JB):       41469.710
Skew:                   5.456        Prob(JB):               0.00
Kurtosis:               45.987       Cond. No.               25.1
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS results for predictor:CHAS

#### OLS Regression Results

```
=====
Dep. Variable:          CRIM      R-squared:                0.003
Model:                  OLS      Adj. R-squared:             0.001
Method:                 Least Squares      F-statistic:         1.546
Date:                  Mon, 25 Jan 2021    Prob (F-statistic):      0.214
Time:                  20:33:12           Log-Likelihood:        -1805.3
No. Observations:      506             AIC:                  3615.
Df Residuals:          504             BIC:                  3623.
Df Model:              1
Covariance Type:       nonrobust
=====
```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	3.7232	0.396	9.404	0.000	2.945	4.501
boston[col]	-1.8715	1.505	-1.243	0.214	-4.829	1.086

```
=====
Omnibus:                562.698      Durbin-Watson:          0.822
Prob(Omnibus):          0.000        Jarque-Bera (JB):       30864.755
Skew:                   5.205        Prob(JB):               0.00
Kurtosis:               39.818       Cond. No.               3.96
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS results for predictor:NOX

```

=====
                        OLS Regression Results
=====
Dep. Variable:          CRIM      R-squared:                0.174
Model:                  OLS      Adj. R-squared:            0.173
Method:                 Least Squares      F-statistic:          106.4
Date:                   Mon, 25 Jan 2021    Prob (F-statistic):      9.16e-23
Time:                   20:33:12    Log-Likelihood:         -1757.6
No. Observations:      506      AIC:                    3519.
Df Residuals:          504      BIC:                    3528.
Df Model:               1
Covariance Type:       nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	-13.5881	1.702	-7.986	0.000	-16.931	-10.245
boston[col]	30.9753	3.003	10.315	0.000	25.076	36.875

```

=====
Omnibus:                591.496    Durbin-Watson:          0.994
Prob(Omnibus):           0.000    Jarque-Bera (JB):        42994.381
Skew:                    5.544    Prob(JB):                0.00
Kurtosis:                46.776    Cond. No.                11.3
=====

```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS results for predictor:RM

```

=====
                        OLS Regression Results
=====
Dep. Variable:          CRIM      R-squared:                0.048
Model:                  OLS      Adj. R-squared:            0.046
Method:                 Least Squares      F-statistic:          25.62
Date:                   Mon, 25 Jan 2021    Prob (F-statistic):      5.84e-07
Time:                   20:33:12    Log-Likelihood:         -1793.5
No. Observations:      506      AIC:                    3591.
Df Residuals:          504      BIC:                    3600.
Df Model:               1
Covariance Type:       nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	20.5060	3.362	6.099	0.000	13.901	27.111
boston[col]	-2.6910	0.532	-5.062	0.000	-3.736	-1.646

```

=====
Omnibus:                576.890    Durbin-Watson:          0.883
Prob(Omnibus):           0.000    Jarque-Bera (JB):        36966.825
Skew:                    5.361    Prob(JB):                0.00
Kurtosis:                43.477    Cond. No.                58.4
=====

```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS results for predictor:AGE

```

=====
                        OLS Regression Results
=====
Dep. Variable:          CRIM      R-squared:                0.123
Model:                  OLS      Adj. R-squared:            0.121

```

```

Method:          Least Squares      F-statistic:          70.72
Date:            Mon, 25 Jan 2021    Prob (F-statistic):    4.26e-16
Time:            20:33:12            Log-Likelihood:        -1772.9
No. Observations: 506                AIC:                   3550.
Df Residuals:    504                BIC:                   3558.
Df Model:        1
Covariance Type: nonrobust

```

```

=====
              coef      std err          t      P>|t|      [0.025      0.975]
-----
Intercept    -3.7527      0.944     -3.974      0.000     -5.608     -1.898
boston[col]   0.1071      0.013      8.409      0.000      0.082      0.132
=====
Omnibus:            575.090    Durbin-Watson:           0.960
Prob(Omnibus):      0.000    Jarque-Bera (JB):        36851.412
Skew:               5.331    Prob(JB):                0.00
Kurtosis:          43.426    Cond. No.:               195.
=====

```

#### Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

#### OLS results for predictor:DIS

```

              OLS Regression Results
=====
Dep. Variable:          CRIM      R-squared:            0.143
Model:                  OLS      Adj. R-squared:       0.141
Method:                 Least Squares      F-statistic:         83.97
Date:                   Mon, 25 Jan 2021    Prob (F-statistic):   1.27e-18
Time:                   20:33:12            Log-Likelihood:       -1767.1
No. Observations:       506                AIC:                 3538.
Df Residuals:           504                BIC:                 3547.
Df Model:               1
Covariance Type:        nonrobust
=====
              coef      std err          t      P>|t|      [0.025      0.975]
-----
Intercept      9.4489      0.731     12.934      0.000      8.014     10.884
boston[col]    -1.5428      0.168     -9.163      0.000     -1.874     -1.212
=====
Omnibus:            577.090    Durbin-Watson:           0.957
Prob(Omnibus):      0.000    Jarque-Bera (JB):        37542.100
Skew:               5.357    Prob(JB):                0.00
Kurtosis:          43.815    Cond. No.:               9.32
=====

```

#### Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

#### OLS results for predictor:RAD

```

              OLS Regression Results
=====
Dep. Variable:          CRIM      R-squared:            0.387
Model:                  OLS      Adj. R-squared:       0.386
Method:                 Least Squares      F-statistic:         318.1
Date:                   Mon, 25 Jan 2021    Prob (F-statistic):   1.62e-55
Time:                   20:33:12            Log-Likelihood:       -1682.3
No. Observations:       506                AIC:                 3369.
Df Residuals:           504                BIC:                 3377.
Df Model:               1

```



```

Covariance Type: nonrobust
=====
              coef      std err          t      P>|t|      [0.025      0.975]
-----
Intercept      -2.2709      0.445      -5.105      0.000      -3.145      -1.397
boston[col]      0.6141      0.034      17.835      0.000      0.546      0.682
=====
Omnibus:                654.232      Durbin-Watson:                1.336
Prob(Omnibus):           0.000      Jarque-Bera (JB):            74327.568
Skew:                    6.441      Prob(JB):                    0.00
Kurtosis:                60.961      Cond. No.                    19.2
=====

```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS results for predictor:TAX

```

              OLS Regression Results
=====
Dep. Variable:          CRIM      R-squared:                0.336
Model:                  OLS      Adj. R-squared:           0.335
Method:                 Least Squares      F-statistic:          254.9
Date:                  Mon, 25 Jan 2021      Prob (F-statistic):    9.76e-47
Time:                  20:33:12      Log-Likelihood:       -1702.5
No. Observations:      506      AIC:                  3409.
Df Residuals:          504      BIC:                  3418.
Df Model:              1
Covariance Type:       nonrobust
=====
              coef      std err          t      P>|t|      [0.025      0.975]
-----
Intercept      -8.4748      0.818     -10.365      0.000     -10.081      -6.868
boston[col]      0.0296      0.002     15.966      0.000      0.026      0.033
=====
Omnibus:                634.003      Durbin-Watson:                1.252
Prob(Omnibus):           0.000      Jarque-Bera (JB):            63141.063
Skew:                    6.134      Prob(JB):                    0.00
Kurtosis:                56.332      Cond. No.                    1.16e+03
=====

```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 1.16e+03. This might indicate that there are strong multicollinearity or other numerical problems.

OLS results for predictor:PTRATIO

```

              OLS Regression Results
=====
Dep. Variable:          CRIM      R-squared:                0.083
Model:                  OLS      Adj. R-squared:           0.081
Method:                 Least Squares      F-statistic:          45.67
Date:                  Mon, 25 Jan 2021      Prob (F-statistic):    3.88e-11
Time:                  20:33:12      Log-Likelihood:       -1784.1
No. Observations:      506      AIC:                  3572.
Df Residuals:          504      BIC:                  3581.
Df Model:              1
Covariance Type:       nonrobust
=====
              coef      std err          t      P>|t|      [0.025      0.975]
-----

```

Intercept	-17.5307	3.147	-5.570	0.000	-23.714	-11.347
boston[col]	1.1446	0.169	6.758	0.000	0.812	1.477

```
=====
Omnibus:                    568.808    Durbin-Watson:                0.909
Prob(Omnibus):              0.000    Jarque-Bera (JB):            34373.378
Skew:                      5.256    Prob(JB):                    0.00
Kurtosis:                  41.985    Cond. No.                    160.
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

OLS results for predictor:B

#### OLS Regression Results

```
=====
Dep. Variable:              CRIM    R-squared:                0.142
Model:                    OLS      Adj. R-squared:           0.141
Method:                  Least Squares    F-statistic:             83.69
Date:                    Mon, 25 Jan 2021    Prob (F-statistic):      1.43e-18
Time:                    20:33:12    Log-Likelihood:         -1767.2
No. Observations:        506    AIC:                    3538.
Df Residuals:            504    BIC:                    3547.
Df Model:                1
Covariance Type:         nonrobust
=====
```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	16.2680	1.430	11.376	0.000	13.458	19.078
boston[col]	-0.0355	0.004	-9.148	0.000	-0.043	-0.028

```
=====
Omnibus:                    591.626    Durbin-Watson:                1.001
Prob(Omnibus):              0.000    Jarque-Bera (JB):            43282.465
Skew:                      5.543    Prob(JB):                    0.00
Kurtosis:                  46.932    Cond. No.                    1.49e+03
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 1.49e+03. This might indicate that there are strong multicollinearity or other numerical problems.

OLS results for predictor:LSTAT

#### OLS Regression Results

```
=====
Dep. Variable:              CRIM    R-squared:                0.205
Model:                    OLS      Adj. R-squared:           0.203
Method:                  Least Squares    F-statistic:             129.6
Date:                    Mon, 25 Jan 2021    Prob (F-statistic):      7.12e-27
Time:                    20:33:12    Log-Likelihood:         -1748.2
No. Observations:        506    AIC:                    3500.
Df Residuals:            504    BIC:                    3509.
Df Model:                1
Covariance Type:         nonrobust
=====
```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	-3.2946	0.695	-4.742	0.000	-4.660	-1.930
boston[col]	0.5444	0.048	11.383	0.000	0.450	0.638

```
=====
Omnibus:                    600.766    Durbin-Watson:                1.184
```

```

Prob(Omnibus):      0.000    Jarque-Bera (JB):      49637.173
Skew:               5.638    Prob(JB):           0.00
Kurtosis:           50.193    Cond. No.           29.7
=====

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

OLS results for predictor:MDEV

                        OLS Regression Results
=====
Dep. Variable:        CRIM    R-squared:                0.149
Model:                OLS    Adj. R-squared:           0.147
Method:               Least Squares    F-statistic:         88.15
Date:                Mon, 25 Jan 2021    Prob (F-statistic):   2.08e-19
Time:                20:33:12    Log-Likelihood:      -1765.3
No. Observations:    506    AIC:                 3535.
Df Residuals:        504    BIC:                 3543.
Df Model:             1
Covariance Type:     nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	11.7202	0.935	12.539	0.000	9.884	13.557
boston[col]	-0.3606	0.038	-9.389	0.000	-0.436	-0.285

```

=====
Omnibus:              559.282    Durbin-Watson:        1.000
Prob(Omnibus):        0.000    Jarque-Bera (JB):     32809.507
Skew:                 5.114    Prob(JB):             0.00
Kurtosis:             41.099    Cond. No.             64.5
=====

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

```

B) Fit a multiple regression model to predict the response using all the predictors. Describe your results. For which predictors can we reject the null hypothesis  $H_0: \beta_j = 0$ ?

# OLS Regression Results

```

=====
Dep. Variable:          CRIM      R-squared:                0.448
Model:                  OLS       Adj. R-squared:           0.434
Method:                 Least Squares   F-statistic:             33.30
Date:                  Mon, 25 Jan 2021   Prob (F-statistic):      4.17e-56
Time:                  17:55:59    Log-Likelihood:          -1655.9
No. Observations:      506         AIC:                    3338.
Df Residuals:          493         BIC:                    3393.
Df Model:              12
Covariance Type:       nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
Intercept	17.5102	7.264	2.411	0.016	3.238	31.782
AGE	0.0014	0.018	0.081	0.936	-0.034	0.037
B	-0.0069	0.004	-1.879	0.061	-0.014	0.000
DIS	-1.0010	0.283	-3.539	0.000	-1.557	-0.445
INDUS	-0.0669	0.083	-0.802	0.423	-0.231	0.097
LSTAT	0.1211	0.076	1.593	0.112	-0.028	0.271
MDEV	-0.2047	0.060	-3.405	0.001	-0.323	-0.087
NOX	-10.8572	5.287	-2.054	0.041	-21.245	-0.470
PTRATIO	-0.2737	0.187	-1.463	0.144	-0.641	0.094
RAD	0.5849	0.088	6.632	0.000	0.412	0.758
RM	0.3908	0.615	0.635	0.526	-0.818	1.600
TAX	-0.0034	0.005	-0.666	0.506	-0.014	0.007
ZN	0.0450	0.019	2.394	0.017	0.008	0.082

```

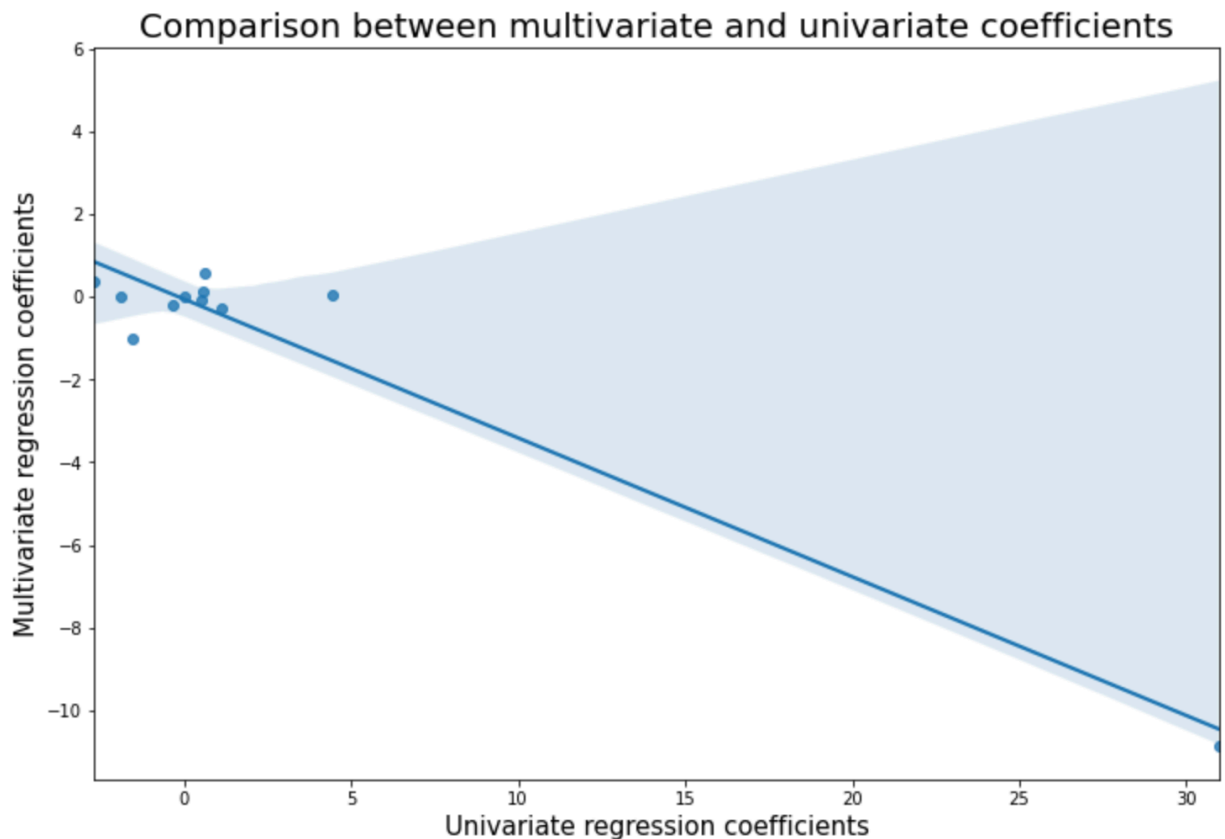
=====
Omnibus:                661.885    Durbin-Watson:           1.515
Prob(Omnibus):          0.000     Jarque-Bera (JB):        82471.479
Skew:                   6.537     Prob(JB):                 0.00
Kurtosis:               64.162    Cond. No.                 1.58e+04
=====

```

Using all of the predictors, we observe different results. Age, proportion of Blacks in the suburb, INDUS, LSTAT, PTRATIO, RM, and TAX were all predictors that were not significant, as we observe low t-statistics and p-values. All the others we can reject the null hypothesis. NOX is along the margin line in terms of t-statistics and its p-value; it is above a 2 t-statistic value, and comes close to being insignificant in terms of its p-value, hence it should be used with discretion.

- C) How do your results from (a) compare to your results from (b)? Create a plot displaying the univariate regression coefficients from (a) on the x-axis, and the multiple regression coefficients from (b) on the y-axis. That is, each predictor is displayed as a single point in the plot. Its coefficient in a simple linear regression model is shown on the x-axis, and its coefficient estimate in the multiple linear regression model is shown on the y-axis.

The results from (a) provide us with context as to how each predictor influences the predicted value, that is per capita crime rate. As for (b), we observe the multicollinearity and interactions of using multiple predictors to predict our response.



When we plot the findings, we observe a downward trend, suggesting that independently, using predictors individually influence the response at greater magnitudes.

- D) Is there evidence of non-linear association between any of the predictors and the response? To answer this question, for each predictor  $X$ , fit a model of the form:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3 + \text{error}$$

Yes, there is with MDEV, NOX, RAD, TAX, and the interaction terms. We see that with all predictors, the  $R^2$  is at a fairly significant level, showing that there is non-linear association between these predictors and predicting the per capita crime rate.