Laboratório de Programação Avançada 2018/19

Week 4 – Dynamic Programming

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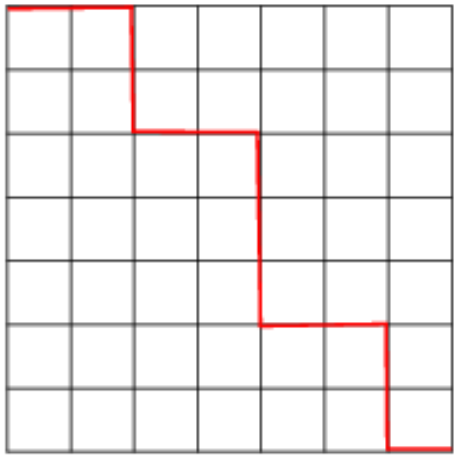
Number of monotonic paths

- Given a grid of size $n \times m$, count the number of monotonic paths.
- Monotonic means that you can only turn right or down



Number of monotonic paths

Recursive solution

Function $count(x, y)$
  if $x = 1$ **or** $y = 1$ **then** {base case}
    return 1
  $C1 = count(x - 1, y)$ {recursion}
  $C2 = count(x, y - 1)$
  return $C1 + C2$

Number of monotonic paths

Top-down dynamic programming

Function $count(x, y)$
  if $T[x, y]$ is cached **then** {if already visited}
    return $T[x, y]$
  if $x = 1$ **or** $y = 1$ **then** {base case}
    return 1
  $C1 = count(x - 1, y)$ {recursion}
  $C2 = count(x, y - 1)$
  $T[x, y] = C1 + C2$ {memoizing}
  return $T[x, y]$

Number of monotonic paths

Bottom-up dynamic programming

Function $count(n, m)$

  for $i = 1$ **to** n **do** {1st base case}

    $T[i, 1] = 1$

  for $j = 1$ **to** m **do** {2nd base case}

    $T[1, j] = 1$

  for $i = 2$ **to** n **do**

    for $j = 2$ **to** m **do**

      $T[i, j] = T[i - 1, j] + T[i, j - 1])$

  return $T[n, m]$

- Bottom-up approach in $O(mn)$ time.