

Exercise 1 Consider the two following problems:

- Sorting problem: Given a set $S = \{s_1, \dots, s_n\}$ of n non-negative integer values, order them in non-decreasing order.
 - Median problem: Given a set $R = \{r_1, \dots, r_m\}$ of m non-negative integer values, find its median.
- a) Reduce the median problem to the sorting problem. What is the time complexity of this transformation?

Exercise 2 Consider the following problem:

- Graph coloring: Given a graph $G = (V, E)$, and a positive integer k , color the nodes with at most k colors such that no two adjacent nodes have the same color.
- a) Sketch an algorithm that, given a graph that is arbitrarily coloured with k colors, and a function $color(u)$ that returns the color of node u , it outputs the number of pairs of nodes that violate the constraint above. Discuss its time complexity.
- b) Assume that you have an algorithm \mathcal{A} to solve the Graph Coloring problem as stated above. Sketch an algorithm to solve the optimization version of this problem (to minimize the number of colors) that uses \mathcal{A} . Give the number of calls to \mathcal{A} .

Exercise 3 Consider the two following problems:

- 3-Sum: Given a set of n distinct integers, find whether it contains three elements that sum to 0.
 - 3-Collinear: Given a set of m distinct points in the plane, find whether it contains three collinear points.
- a) A set of points is collinear if the slopes between any pair of points of that set match. Explore this property to solve the 3-Collinear problem. Discuss its time complexity.
- b) Show that the 3-Sum problem can be reduced to the 3-Collinear problem. Hint: Consider the property above.
- c) A widely known conjecture claims that it is not possible to solve the 3-Sum problem in less than $O(n^2)$ amount of time. If this conjecture is true, what can you state about the time complexity of solving the 3-Collinear problem?
- d) Read the problem *A - Party Icebreaker* in EA2020_ProblemasAula in Mooshak. Relate it with the 3-Sum problem and solve it. Hint: An $O(n^3)$ -time algorithm may not be enough to solve all test cases.