

Universidade de Coimbra

Faculty of Science and Technology Department of Informatics Engineering

Laboratório de Programação Avançada First Written Test – April 19 2017

Student ID: _____

Name: ______

4 grade points in total, 1 hour and 30 m	ninutes, closed books.
the number of elements in list A an	plexity of the following recursive algorithm with respect to adjustify your answer with the Master Theorem. Assume first index of A is 1, and each arithmetic operation takes a
Function $product(A, n)$	Master Theorem (general version):
if $n = 1$ then	Let $a \ge 1$, $b > 1$, $d \ge 0$.
return	
for $i = 1$ to $n/2$ do	$T(n) = \begin{cases} aT(n/b) + n^c & \text{if } n > 1 \end{cases} \Rightarrow$
$A[i] = A[i] \times A[i + (n+1)/2]$	$\int d \qquad \text{if } n = 1$
product(A,(n+1)/2)	$T(n) = \begin{cases} aT(n/b) + n^c & \text{if } n > 1 \\ d & \text{if } n = 1 \end{cases} \Rightarrow$ $T(n) = \begin{cases} \Theta(n^c) & \text{if } \log_b a < c \\ \Theta(n^c \log n) & \text{if } \log_b a = c \\ \Theta(n^{\log_b a}) & \text{if } \log_b a > c \end{cases}$
	$T(n) = \begin{cases} \Theta(n^c \log n) & \text{if } \log_b a = c \end{cases}$
	$\Theta(n^{\log_b a})$ if $\log_b a > c$

2. Consider the following recursive algorithm to compute the arithmetic mean of n > 0 elements in a list L. Assume that the first index of list L is 1 and its elements are nonnegative reals.

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\begin{aligned} & \textbf{Function} \ mean(L,n) \\ & \textbf{if} \ n=1 \ \textbf{then} \\ & \textbf{return} \ \ L[n] \\ & \textbf{else} \\ & \textbf{return} \ \ L[n]/n + mean(L,n-1) \times (n-1)/n \end{aligned}
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Show by induction that the algorithm is correct, using the mathematical definition of arithmetic mean. Explicitly state the base case, the inductive hypothesis and the inductive step. (1 g.p.)

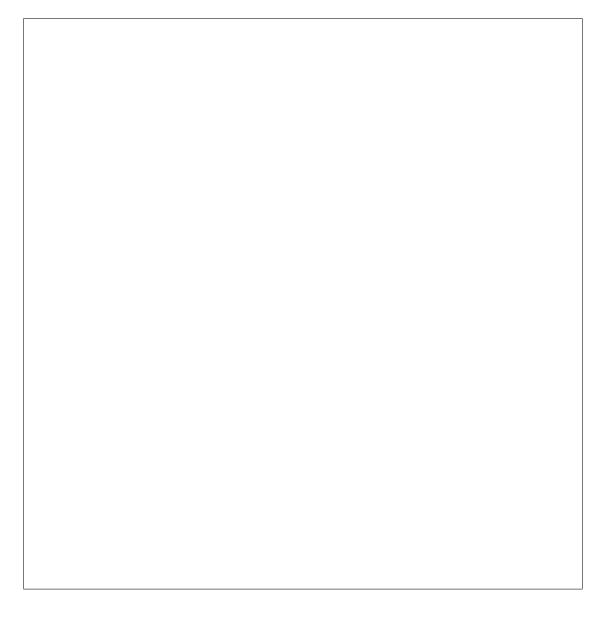
3.	3. Consider the following problem: Given a sequence	e of $n > 0$ integers, compute a contiguous
	subsequence that has the largest sum. For instance, f	for the sequence

$$(-2,1,-3,4,-1,2,1,-5,4)$$

a contiguous subsequence with the largest sum is (4, -1, 2, 1) with a value of 6. The following dynamic programming algorithm solves the problem for a sequence A of n elements by reporting only the largest sum.

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\begin{aligned} & \textbf{Function} \ msum(A) \\ & DP[1] = A[1] \\ & \textbf{for} \ i = 2 \ \textbf{to} \ n \ \textbf{do} \\ & DP[i] = \max(A[i], DP[i-1] + A[i]) \\ & \textbf{return} \ \max(DP[1], \dots, DP[n]) \end{aligned}
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(a)	Show that the	problem has o	otimal	substructure as ex	plored b	y the algorithm abov	e. (1	g.1	p.`
()						,		0.1	



array <i>DF</i>	algorithm to returned by t	he algorithm	above. (1 g.p	p.)	