Integração Newton-Cotes

Grau 4

$$g(s) = \sum_{k=0}^{4} \binom{s}{k} \Delta^k f_0$$

$k=0$: $\binom{s}{0} = \dfrac{s!}{0!(s-0)!} = 1$; $\Delta^0 f_0 = f_0$

$k=1$: $\binom{s}{1} = \dfrac{s!}{1!(s-1)!} = s$; $\Delta^1 f_0 = f_1 - f_0$

$k=2$: $\binom{s}{2} = \dfrac{s!}{2!(s-2)!} = \dfrac{1}{2!} s(s-1)$; $\Delta^2 f_0 = f_2 - 2f_1 + f_0$

$k=3$: $\binom{s}{3} = \dfrac{s!}{3!(s-3)!} = \dfrac{1}{3!} s(s-1)(s-2)$; $\Delta^3 f_0 = f_3 - 3f_2 + 3f_1 - f_0$

$k=4$: $\binom{s}{4} = \dfrac{s!}{4!(s-4)!} = \dfrac{1}{4!} s(s-1)(s-2)(s-3)$;

$$\Delta^4 f_0 = f_4 - 4f_3 + 6f_2 - 4f_1 + f_0$$

INTEGRAÇÃO NEWTON-COTES

FILOSOFIA FECHADA - GRAU 4

$$h = \frac{\Delta x}{4}$$

$$h\int_0^4 g(s)\,ds$$

$$g(s) = f_0 + s(f_1 - f_0) + \frac{1}{2!}s(s-1)(f_2 - 2f_1 + f_0) + \frac{1}{3!}s(s-1)(s-2)(f_3 - 3f_2 + 3f_1 - f_0) + \frac{1}{4!}s(s-1)(s-2)(s-3)(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)$$

$$\begin{aligned} g(s) = {} & f_0\left(1 - s + \frac{1}{2!}s(s-1) - \frac{1}{3!}s(s-1)(s-2) + \frac{1}{4!}s(s-1)(s-2)(s-3)\right) \\ & + f_1\left(s - \frac{2}{2!}s(s-1) + \frac{3}{3!}s(s-1)(s-2) - \frac{4}{4!}s(s-1)(s-2)(s-3)\right) \\ & + f_2\left(\frac{1}{2!}s(s-1) - \frac{3}{3!}s(s-1)(s-2) + \frac{6}{4!}s(s-1)(s-2)(s-3)\right) \\ & + f_3\left(\frac{1}{3!}s(s-1)(s-2) - \frac{4}{4!}s(s-1)(s-2)(s-3)\right) \\ & + f_4\left(\frac{1}{4!}s(s-1)(s-2)(s-3)\right) \end{aligned}$$

$$\int_{x_I}^{x_F} f(x)\,dx \approx$$

$$\begin{aligned} h\int_0^4 g(s)\,ds = h\Bigg(& f_0\int_0^4\left(1 - s + \frac{1}{2}s(s-1) - \frac{1}{6}s(s-1)(s-2) + \frac{1}{24}s(s-1)(s-2)(s-3)\right)ds \\ & + f_1\int_0^4\left(s - \frac{2}{2}s(s-1) + \frac{3}{6}s(s-1)(s-2) - \frac{4}{24}s(s-1)(s-2)(s-3)\right)ds \\ & + f_2\int_0^4\left(\frac{1}{2}s(s-1) - \frac{3}{6}s(s-1)(s-2) + \frac{6}{24}s(s-1)(s-2)(s-3)\right)ds \\ & + f_3\int_0^4\left(\frac{1}{6}s(s-1)(s-2) - \frac{4}{24}s(s-1)(s-2)(s-3)\right)ds \\ & + f_4\int_0^4\left(\frac{1}{24}s(s-1)(s-2)(s-3)\right)ds\Bigg) \end{aligned}$$

$$= h\left(\frac{14}{45}f_0 + \frac{64}{45}f_1 + \frac{8}{15}f_2 + \frac{64}{45}f_3 + \frac{14}{45}f_4\right)$$

$$\frac{24}{45}$$

$$= \frac{2}{45}h\left(7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4\right)$$

$$\boxed{\int_{x_i}^{x_f} f(x)\,dx \approx \frac{2}{45}h\left(7f(x_i) + 32f(x_i+h) + 12f(x_i+2h) + 32f(x_i+3h) + 7f(x_f)\right)}$$

INTEGRACIÓN NEWTON-COTES

FILOSOFIA ABIERTA - GRADO 4

$$\int_{x_i}^{x_f} f(x)\,dx \approx h\int_{-1}^{5} g(s)\,ds =$$

$$= h\Bigg(f_0\int_{-1}^{5}\left(1 - s + \frac{1}{2}s(s-1) - \frac{1}{6}s(s-1)(s-2) + \frac{1}{24}s(s-1)(s-2)(s-3)\right)ds$$

$$+ f_1\int_{-1}^{5}\left(s - \frac{2}{2}s(s-1) + \frac{3}{6}s(s-1)(s-2) - \frac{4}{24}s(s-1)(s-2)(s-3)\right)ds$$

$$+ f_2\int_{-1}^{5}\left(\frac{1}{2}s(s-1) - \frac{3}{6}s(s-1)(s-2) + \frac{6}{24}s(s-1)(s-2)(s-3)\right)ds$$

$$+ f_3\int_{-1}^{5}\left(\frac{1}{6}s(s-1)(s-2) - \frac{4}{24}s(s-1)(s-2)(s-3)\right)ds$$

$$+ f_4\int_{-1}^{5}\left(\frac{1}{24}s(s-1)(s-2)(s-3)\right)ds\Bigg)$$

$$= h\left(\frac{33}{10}f_0 - \frac{21}{5}f_1 + \frac{39}{5}f_2 - \frac{21}{5}f_3 + \frac{33}{10}f_4\right)$$

$$= \frac{3}{10}h\left(11f_0 - 14f_1 + 26f_2 - 14f_3 + 11f_4\right)$$

$$\boxed{\int_{x_i}^{x_f} f(x)\,dx \approx \frac{3}{10}h\Big(11f(x_i+h) - 14f(x_i+2h) + 26f(x_i+3h) - 14f(x_i+4h) + 11f(x_i+5h)\Big)}$$