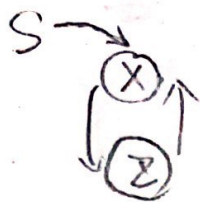


Problem 2

a) In Perez-Carrasco's paper, AC-DC circuit model, from eqn (1).

the system is



$$\frac{d\bar{x}}{dt} = \frac{\bar{\alpha}_x + \bar{\beta}_x S}{1 + S + \left(\frac{\bar{z}}{\bar{x}}\right)^{n_{zx}}} - \bar{\delta}_x \bar{x}$$

$$\frac{d\bar{z}}{dt} = \frac{\bar{\alpha}_z}{1 + \left(\frac{\bar{x}}{\bar{x}_0}\right)^{n_{xz}}} - \bar{\delta}_z \bar{z}$$

b) Non-dimensionalize the system:

$t = \bar{t} \delta_x$ $\bar{t} = \frac{t}{\delta_x}$ Mistake happens here, right $\bar{t} = \bar{t} \delta_x$ $\bar{t} = \frac{t}{\delta_x}$
and all. \bar{z} should be \bar{z}

$$x = \frac{\bar{x} \delta_x}{\bar{\alpha}_z}, \quad \frac{\bar{z} \delta_x}{\bar{\alpha}_z} = \bar{z}$$

$$\rightarrow \bar{x} = \frac{x \bar{\alpha}_z}{\delta_x}, \quad \bar{z} = \frac{\alpha_z \bar{z}}{\delta_x}$$

Substitute all terms into equation.

$$\frac{d\bar{x}}{d\left(\frac{t}{\delta_x}\right)} = \frac{\bar{\alpha}_x + \bar{\beta}_x S}{1 + S + \left(\frac{\bar{z}}{\bar{x}}\right)^{n_{zx}}} - \bar{\delta}_x \bar{x} \quad ; \quad \frac{d\bar{z}}{d\left(\frac{t}{\delta_x}\right)} = \frac{\bar{\alpha}_z}{1 + \left(\frac{\bar{x}}{\bar{x}_0}\right)^{n_{xz}}} - \bar{\delta}_z \bar{z}$$

$$\rightarrow \frac{\bar{\alpha}_z}{\delta_x} \frac{d\bar{x}}{dt} = \frac{1}{\delta_x} \left(\frac{\bar{\alpha}_x + \bar{\beta}_x S}{1 + S + \left(\frac{\bar{z}}{\bar{x}}\right)^{n_{zx}}} \right) - \frac{x \bar{\alpha}_z}{\bar{x}} \quad ; \quad \frac{\bar{\alpha}_z}{\delta_x} \frac{d\bar{z}}{dt} = \frac{1}{\delta_x} \left(\frac{\bar{\alpha}_z}{1 + \left(\frac{\bar{x}}{\bar{x}_0}\right)^{n_{xz}}} \right) - \frac{\delta_z \bar{\alpha}_z}{\delta_x}$$

$$\alpha_x = \frac{\bar{\alpha}_x}{\bar{\alpha}_z}, \quad \frac{\bar{\beta}_x}{\bar{\alpha}_z} = \beta_x$$

$$\therefore \frac{dx}{dt} = \frac{\alpha_x + \beta_x S}{1 + S + \left(\frac{\bar{z}}{\bar{x}}\right)^{n_{zx}}} - x$$

$$\frac{dz}{dt} = \frac{1}{\left(1 + \frac{\bar{x}}{\bar{x}_0}\right)^{n_{xz}}} - \delta_z z$$

