problem 1

a) $\langle n \rangle$ value : average mRNA number per cell.

$B = \langle M_c \rangle \hat{N}_c V$ $\quad OD = 0.1, \rightarrow 10^8$ cells/mL $\quad \langle M_c \rangle$: average mass per cell. gDW/cell

$\hat{N}_c$: cell number/mL.

$\therefore \hat{N}_c = 1 \times 10^8$ cells/mL $\qquad T_d \approx 40$ min.

given the bio number 103905, $\langle M_c \rangle = 2.8 \times 10^{-13}$ gDW/cell

$$B = \frac{gDW}{cell} \cdot \frac{cell}{mL} \cdot mL$$

$$= 2.8 \times 10^{-13} \cdot 10^8 = 2.8 \times 10^{-5}\ gDW$$

Converting: IPTG (mM) $\langle n \rangle$ mRNA/cell to Volume Basis (nmol/gDW)

$$\text{Volume Basis}: \frac{\langle n \rangle \times 10^9}{N_A} \cdot \frac{1}{B} = \frac{\langle n \rangle \times 10^9}{6.022 \times 10^{23} \times 2.8 \times 10^{-5}} \frac{nmol}{gDW}.$$

b) mRNA balances:

$$\dot{m}_i = r_{X,i} \bar{u}_i - (\mu + \theta_{m,i}) m_i$$

In pseudo steady state: $\dot{m}_i \to 0$

$$0 = \frac{dm_i}{dt} = r_{X,i} \bar{u}_i - (\mu + \theta_{m,i}) m_i^*$$

$$\rightarrow m_i^* = \frac{r_{X,i} \bar{u}_i}{\mu + \theta_{m,i}}$$

$$m_i^* = \underbrace{\left(\frac{r_{X,i}}{\mu + \theta_{m,i}}\right)}_{\text{gain function}} \underbrace{(\bar{u}_i)}_{\text{promoter function}}$$

$$= K_X(G, \theta)\, \bar{u}(I, K)$$

According to the Lecture notes from Transcription models - S20

$$r_{X,i} = k_{E,i} R_{X,T} \left(\frac{G_i}{\tau_{X,i} K_{X,i} + (\tau_{X,i} + 1) G_i}\right)$$

$$\bar{u} = \frac{k_1 + k_2 f_I}{1 + k_1 + k_2 f_I} \qquad f_I = \frac{I^n}{K^n + I^n}$$

$K_X = (G, \theta) \quad \bar{u} = (I, K)$

c) From equation in problem. b, we have some constant:

- doubleing time $\tau_d = 40$ min transcript length: $L = 1500$ nt.

  $m^* = K_{x_i} \cdot \bar{u}$ half life: 5 min.

  From previous PS 2. $K_{x_i} = 0.01364$ uM

For θ_i term. $\theta_{m,i} = \frac{\ln 2}{t_{1/2}} = 0.139$ min^{-1} dilution: $\mu = \frac{\ln(2)}{40\text{min}} = 0.0173$ min^{-1}

$m^* = \frac{r_{x_i}}{\mu + \theta_{m,i}} \cdot \bar{u}_i = \frac{r_{x_i}}{\mu + \theta_{m,i}} \left(\frac{1 + K_1 f_2}{1 + K_1 + K_2 f_2} \right)$

when IPTG = 0, $\therefore \bar{u} = \frac{K_1}{1 + K_1}$. solve K_1, $K_1 = 0.256$

$f_I = 0$ $\frac{1.127}{5.517} = \frac{K_1}{1 + K_1}$

all parameters are organized in Excel sheet.

d) The fitting data is pretty good. The plotting is in a great shape. The main factor to the fitting is the n value and the K_2 value comming from the regression analysis.