Problem 2

a) In PerezCarrasco's paper, AC-DC circuit model, from eqn(1).

$\because$ the system is

$$S \rightarrow \textcircled{X} \rightleftarrows \textcircled{Z}$$

$$\therefore \frac{d\bar{x}}{d\bar{t}} = \frac{\tilde{\alpha}_x + \tilde{\beta}_x S}{1+S+\left(\frac{\bar{z}}{\bar{z}_x}\right)^{n_{zx}}} - \bar{\delta}_x \bar{x}$$

$$\frac{d\bar{z}}{d\bar{t}} = \frac{\tilde{\alpha}_z}{1+\left(\frac{\bar{x}}{\bar{x}_z}\right)^{n_{xz}}} - \bar{\delta}_z \bar{z}$$

b) Non-dimensionalize the system:

$t = \bar{t}\delta_x \quad \bar{t} = \frac{t}{\delta_x}$ Mistake happens here. right: $t = \bar{t}\bar{\delta}_x \quad \bar{t} = \frac{t}{\bar{\delta}_x}$

and all. z should be $\bar{z}$

$$x = \frac{\bar{x}\bar{\delta}_x}{\bar{\alpha}_z}, \quad \frac{\bar{z}\,\bar{\delta}_x}{\tilde{\alpha}_z} = z.$$

$$\rightarrow \bar{x} = \frac{x\bar{\alpha}_z}{\delta_x} \quad \bar{z} = \frac{\bar{\alpha}_z \delta_z}{\delta_x}$$

Subsitute all terms into equation.

$$\frac{d\bar{x}}{d\left(\frac{t}{\delta_x}\right)} = \frac{\bar{\alpha}_x + \bar{\beta}_x S}{1+S+\left(\frac{\bar{z}}{\bar{z}_x}\right)^{n_{zx}}} - \bar{\delta}_x \bar{x} \quad ; \quad \frac{d\bar{z}}{d\left(\frac{x}{\delta_x}\right)} = \frac{\bar{\alpha}_z}{1+\left(\frac{x}{\bar{x}_z}\right)^{n_{xz}}} - \bar{\delta}_z \bar{z}$$

$$\rightarrow \frac{\bar{\alpha}_z}{\delta_x}\frac{dx}{dt} = \frac{1}{\delta_x}\left(\frac{\bar{\alpha}_x + \bar{\beta}_x S}{1+S+\left(\frac{\bar{z}}{\bar{z}_x}\right)^{n_{zx}}}\right) - \frac{x\bar{\alpha}_z}{\delta_x} \quad ; \quad \frac{\bar{\alpha}_z}{\delta_x}\frac{dz}{dt} = \frac{1}{\delta_x}\left(\frac{\bar{\alpha}_z}{1+\left(\frac{\bar{x}}{\bar{x}_z}\right)^{n_{xz}}}\right) - \frac{\delta_z \bar{\alpha}_z}{\delta_x}$$

$$\alpha_x = \frac{\bar{\alpha}_x}{\bar{\alpha}_z} \quad \frac{\bar{\beta}_x}{\bar{\alpha}_z} = \beta_x$$

$$\therefore \underline{\frac{dx}{dt} = \frac{\alpha_x + \beta_x S}{1+S+\left(\frac{z}{z_x}\right)^{n_{zx}}} - x} \qquad \underline{\frac{dz}{dt} = \frac{1}{\left(1+\frac{x}{x_z}\right)^{n_{xz}}} - \delta_z z}$$