CSC236H, Winter 2016
Assignment 3
Due March 06th, 10:00 p.m.

- You may work in groups of no more than **two** students, and you should produce a single solution in a PDF file named `a3.pdf`, submitted to MarkUs. Submissions must be **typed**.
- Please refer to the course information sheet for the **late submission policy**.

1. Let $T(n)$ denote the worst-case running time of the algorithm below on inputs of size n.

```
# A is a list of integers.
mystery(A):
1.      if len(A) <= 1:
2.          return 0
3.      m = len(A)//4 # Integer division
4.      s = mystery(A[0..m - 1])
5.      for i = m to 3 * m - 1:
6.          s = s + A[i]
7.      s = s + mystery(A[3 * m..len(A) - 1])
8.      return s
```

   (a) Write a recurrence relation satisfied by T. You may assume that $len(A)$ is a power of 4. Make sure to define n precisely (as a function of the algorithm's parameters) and justify that your recurrence is correct (by referring to the algorithm to describe how you obtained each term in your answer).

   (b) Give an asymptotic upper-bound for the worst-case running time of the algorithm.

2. Let $a, b \in \mathbb{N}$. Consider the following function $f : \mathbb{N} \to \mathbb{N}$.

$$f(n) = \begin{cases} a, & n = 0 \\ b, & n = 1 \\ 2f(n-1) - f(n-2) + 1, & n \geq 2 \end{cases}$$

   Use the method of repeated substitution to find a closed-form expression for $f(n)$.
   You don't need to prove the correctness of the closed-form expression you obtained.

3. A person saving for retirement makes an initial deposit of $1,000 to a bank account earning interest at a rate of 3% per year compounded monthly, and at the end of each month she adds an additional $200 to the account.
   For each non-negative integer n, let $f(n)$ be the amount in the account at the end of n months.

   (a) Find a recurrence function relating $f(n)$ to $f(n-1)$. Provide explanations to justify your recurrence.

   (b) Use the method of repeated substitution to find a closed-form expression for $f(n)$.

   (c) Use mathematical induction to prove the correctness of the formula you obtained.