CSC236H, Winter 2016
Assignment 2
Due February 14th, 10:00 p.m.

- You may work in groups of no more than **two** students, and you should produce a single solution in a PDF file named `a2.pdf`, submitted to MarkUs. Submissions must be **typed**.
- Please refer to the course information sheet for the **late submission policy**.

1. Let T be a set of rooted trees such that for each $t \in T$:
   - nodes are labeled with positive integers. That is, each node v is labeled with an integer a_v;
   - if w is a child of v, then $a_w < a_v$.

   Prove that for all $t \in T$, the root of t has the largest label of all nodes in the tree.

2. Let $M \subseteq \mathbb{Z}^2$ be a set defined as follows:
   - $(3, 2) \in M$;
   - for all $(x, y) \in M$, $(3x - 2y, x) \in M$;
   - nothing else belongs to M.

   Use structural induction to prove that for all $(x, y) \in M$, there exists $k \in \mathbb{N}$, such that

$$(x, y) = (2^{k+1} + 1, 2^k + 1).$$

3. Let G be a set defined as follows:
   - if x is a propositional variable, then $x \in G$;
   - if $f_1, f_2 \in G$, then $\neg f_1 \in G$, and $(f_1 \wedge f_2) \in G$;
   - nothing else belongs to G.

   For a formula $f \in G$, let $c_{not}(f)$ be the number of occurrences of $\neg$ in f, and $c_{and}(f)$ be the number of occurrences of $\wedge$ in f. Let $H = \{f \in G : c_{not}(f) = c_{and}(f)\}$. That is, H is the set of formulas in G with equal number of $\neg$'s and $\wedge$'s.
   Prove that for any formula $f \in G$, there is a formula f' such that $f' \in H$ and f' and f are logically equivalent.