CSC236 Assignment 5

1.)

a)

(1 + 01)*1

b)

((0 + 1)(0 + 1)(0 + 1))*

2.)

a)

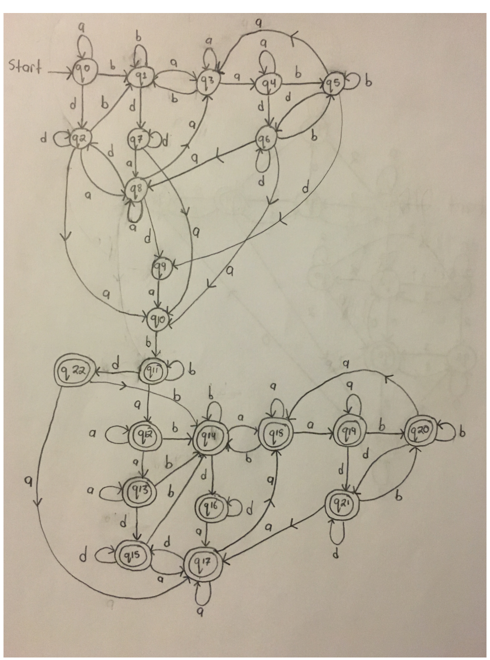
L = {x∈{0, 1}* : x contains an odd number or 1's}.

b)

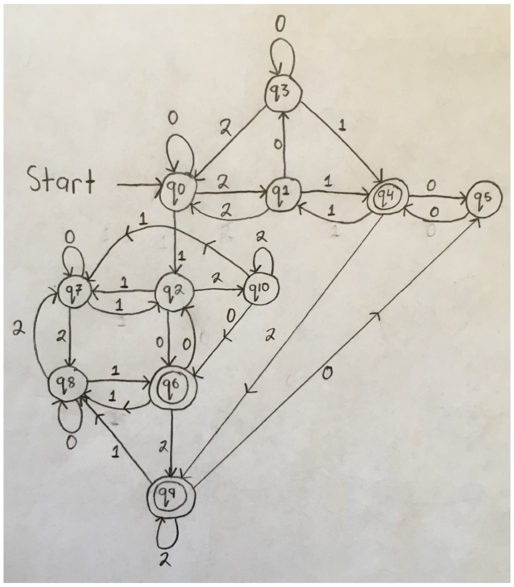
L = {x∈{0, 1}* : x contains no occurence of the substring "0011"}.

3.)

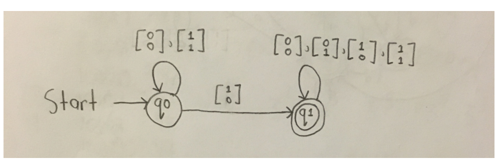
a)



b)



4.)



$\delta^*(q_0, x)$ = q_0 if and only if x is empty or the bottom row of x is equal to the top row of x.

= q_1 if and only if the bottom row of x is smaller than the top row of x.

The initial state is q_0. The only accepting state is q_1.

5)

Suppose $L \subseteq \Sigma^*$ is regular. Then there is a DFA $A = \langle Q_A, \Sigma_A, \delta_A, q_0, F_A\rangle$ such that $L = \mathcal{L}(A)$. Let

$B = \langle Q_B, \Sigma_B, \delta_B, q_0, F_B\rangle$ be a DFA where,

- $Q_B = Q_A \cup \{q_B\}$.
- $\Sigma_B = \Sigma_A$.
- $F_B = \{q_B\}$.
- for all $q \in Q_A$, and $c \in \Sigma_A$, $\delta_B(q, c) = \delta_A(q, c)$.
- for all $w \in \Sigma_A^*$, if $a \in \Sigma_A$ and $\delta_B^*(q_0, wa) \in F_A$, $\delta_B^*(\delta_B^*(q_0, wa), w) = q_B$.

First we need to show that for all $w \in \Sigma_A^*$, $P(w)$ holds, where $P(w)$ denotes the assertion that

$$\delta_B^*(q_0, w) = \delta_A^*(q_0, w).$$

Base Case: Let $w = \varepsilon$.

Since $\delta_B^*(q_0, \varepsilon) = q_0$ and $\delta_B^*(q_0, \varepsilon) = q_0$, so $P(w)$ holds.

Induction Step: Let $w = yc$, where $y \in \Sigma_A^*$ and $c \in \Sigma_A$. Suppose $P(y)$ holds. [IH]

Then $\delta_B^*(q_0, w) = \delta_B^*(q_0, yc)$ # $w = yc$

$\delta_B^*(q_0, w) = \delta_B(\delta_B^*(q_0, y), c)$ # by definition of δ_B^*

$\delta_B^*(q_0, w) = \delta_B(\delta_A^*(q_0, y), c)$ # by IH

$\delta_B^*(q_0, w) = \delta_A(\delta_A^*(q_0, y), c)$ # by definition of δ_B since $\delta_A^*(q_0, y) \in Q_A$ and $c \in \Sigma_A$

$\delta_B^*(q_0, w) = \delta_A^*(q_0, yc)$ # by definition of δ_A^*

$\delta_B^*(q_0, w) = \delta_A^*(q_0, w)$ # since $w = yc$

Therefore P(w) holds.

Now suppose $x = wa$ where $a \in \Sigma_A, w \in \Sigma_A^*$ and $x \in L(A)$.

Then $\delta_B^*(q_0, x) = \delta_B^*(q_0, wa)$

$\delta_B^*(q_0, x) = q_A$ **(A)** # where $q_A \in F_A$, since $wa \in L(A)$

Then $\delta_B^*(\delta_B^*(q_0, x), w) = \delta_B^*(\delta_B^*(q_0, wa), w)$ # $x = wa$

$\delta_B^*(\delta_B^*(q_0, x), w) = \delta_B^*(q_A, w)$ # by **(A)** $\delta_B^*(q_0, wa) = q_A$

$\delta_B^*(\delta_B^*(q_0, x), w) = q_B$ # by definition of δ_B^* and since $q_A \in F_A$

Therefore, we have $w \in \text{Quot}(L, a) \Rightarrow$ exists $a \in \Sigma_A$ such that $wa \in L(A)$.

$\Rightarrow \delta_B^*(q_0, w) = q_B$

$\Rightarrow w \in L(B)$

Therefore, $\text{Quot}(L, a) \subseteq L(B)$. **(1)**

We also need to show that $L(B) \subseteq \text{Quot}(L, a)$.

For a contradiction, suppose $L(B)$ is not a subset of Quot(L, a).

Then there exists $w \in L(B)$ such that $w \notin \text{Quot}(L, a)$.

That is $w \in \Sigma_B^*$, $a \in \Sigma_B$ such that $x = wa$ and $\delta_B^*(q_0, w) = q_B$, but $x \notin L(A)$.

If $x \notin L(A)$, then $\delta_B^*(q_0, x) \notin F_A$, and by definition of δ_B, $\delta_B^*(q_0, x)$ is a dead state which means

$\delta_B^*(q_0, w) = \delta_B^*(\delta_B^*(q_0, x), w)$ is also a dead state, which contradicts the assumption that

$\delta_B{}^*(q_0, w) = q_B$. Therefore, $L(B) \subseteq$ Quot(L, a). **(2)**

By **(1)** and **(2)** we can conclude $L(B) =$ Quot(L, a).

6.)

Let L' = $\{0^i1^j : i, j \geq 0, i \neq j\}$. Assume L' is regular. Regular languages are closed under complementation so the complementation of L' is regular. The complementation of L' is the set $\{0^i1^j : i, j \geq 0, i = j\}$. Regular languages are closed under intersection so the intersection of $\{0^i1^j : i, j \geq 0, i = j\}$and the regular expression 0*1* is the set L = $\{0^n1^n : n \geq 0\}$ which we know is not regular. Therefore, L' is not regular.