# CSC236H, Winter 2016
# Assignment 1
# Due January 31th, 10:00 p.m.

- You may work in groups of no more than **two** students, and you should produce a single solution in a PDF file named `a1.pdf`, submitted to MarkUs. Submissions must be **typed**.
- Please refer to the course information sheet for the **late submission policy**.

1. Let $f_1, f_2, ...$ be a sequence of natural numbers which is defined as follows:

$$\begin{aligned} f_1 =& 1, \\ f_2 =& 1, \\ f_n =& f_{n-1} + f_{n-2}, \qquad n \geq 3. \end{aligned}$$

   Use induction to prove that for any $n \geq 1$, $\gcd(f_n, f_{n+1}) = 1$.

   Reminder: for two natural numbers $a$ and $b$, $\gcd(a, b)$ denotes the greatest common divisor of. That is, the larget natural number that divides both $a$ and $b$. A well-known theorem about gcd that can be useful for this problem states that for natural numbers $a, b$, if $a > b$, then $\gcd(a, b) = \gcd(b, a - b)$.

2. Consider the sequence $f_1, f_2, ...$ defined in the previous question, and the following sequence of natural numbers:

$$\begin{aligned} a_1 &= 1, \\ a_2 &= 1, \\ a_n &= a_{n-1} + a_{n-2} + 1, \quad \text{for } n \geq 3. \end{aligned}$$

   Prove that for all $n \geq 1$, $a_n = 2f_n - 1$.

3. Use the well-ordering principle to prove that every natural number greater than 1 is divisible by a prime number.

4. Suppose that $h_0, h_1, h_2, ...$ is a sequence defined as follows:

$$\begin{aligned} h_0 &= 1, \\ h_1 &= 2, \\ h_2 &= 3, \\ h_k &= h_{k-1} + h_{k-2} + h_{k-3}, \quad \text{for } k \geq 3. \end{aligned}$$

   Suppose that $s$ is any real number such that $s^3 \geq s^2 + s + 1$.

   Prove that for all $n \geq 2$, $h_n \leq s^n$.

   Note that $s^3 \geq s^2 + s + 1$ implies that $s > 1.83$.