The Floor is LAVA

Consider an individual version of the TV game "The Floor is LAVA", where an extremely large number of players attempt to escape the lava filled room. The room has the following structure:

Players start from the ENTRANCE. 70 % of the players will head to crate C_1 [yellow path], and the other 30 % will go directly to crate C_2 [light blue path]. Considering the players choosing the yellow path, 20 % will fall after an exponentially distributed amount of time, with rate $\lambda = 0.5\ min^{-1}$; the other 80 % will reach crate C_1 after an Erlang distributed amount of time, with $k = 4$, and rate $\lambda = 1.5\ min^{-1}$. Only 30 % of the players choosing the light blue path will reach crate C_2, after a time uniformly distributed between $a = 3\ min$ and $b = 6\ min$. Instead, 70 % of the players choosing the light blue path, will fail after an exponentially distributed amount of time, with rate $\lambda = 0.25\ min^{-1}$.

Let us now focus on players that have successfully reached crate C_1. 50% will continue to crate C_2 on the yellow path, while the other half will try to go there along the white path. The yellow path between C_1 and C_2 can be successful only 25 % of the times: failure will occur after an exponentially distributed amount of time with rate $\lambda = 0.4\ min^{-1}$, while success will lead to crate C_2 in an Erlang distributed amount of time with parameters $k = 3$, and rate $\lambda = 2\ min^{-1}$. The white path between C_1 and C_2 is instead successful 60 % of the times: failure occurs after an exponentially distributed amount of time with rate $\lambda = 0.2\ min^{-1}$, and success occurs after an exponentially distributed rate $\lambda = 0.15\ min^{-1}$.

The final step across the green path from crate C_2 to the EXIT, is successful 60 % of the times. Both success or failure will occur after an Erlang distributed amount of time with parameters $k = 5$, and rate $\lambda = 4\ min^{-1}$.

Whenever a player either wins or falls in the lava, the room requires a deterministic amount of time, with T = *5 min*, to prepare for the next player to try.

Analyze the considered game:

1. Draw a state machine of the system.
2. Compute the winning probability.
3. Compute the average duration of a game, either from the ENTRANCE to the EXIT, or from the ENTRANCE to the fall in the LAVA.
4. Compute the throughput of the system: how many games per hour the room can accommodate.