Discrete random variables

• Bernoulli p=3/4, E(x)=p=3/4, var(x)=p(1-p)
• Binomial with n = 10 and p = 3=4, E(x)=n*p, , var(x)= np(1-p)
• Geometric with p = 3=4, E(x)=1/p, Var(x)=(1-p)/(p^2)
• Poisson with lambda= 1, E(x)= lambda, Var(x)= lambda

Continuous random variables

• Uniform in [0; 1], , E(x)=(b+a)/2, var(x)=(b-a)^2 /12
• Exponential with lamba= 1, , E(x)=1/ lamba, var(x)=1/ (lamba^2) memoryless
• Gaussian with lamba = 1, σ^2 = 1
• Gaussian with lamba = 1, σ ^2= 5

LAW LARGE NUMBERS

In probability theory, the law of large numbers (LLN) is a theorem that describes the result of performing the same experiment a large number of times. According to the law, the average of the results obtained from a large number of trials should be close to the expected value and will tend to become closer to the expected value as more trials are performed.

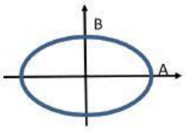
CENTRAL LIMIT THEOREM

In probability theory, the central limit theorem (CLT) establishes that, in many situations, when independent random variables are summed up, their properly normalized sum tends toward a normal distribution (informally a bell curve) even if the original variables themselves are not normally distributed.

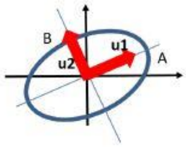
MULTIVARIATE GAUSSIAN

In probability theory and statistics, the multivariate normal distribution, multivariate Gaussian distribution, is a generalization of the one-dimensional (univariate) normal distribution to higher dimensions.

Ellipse $(x_1/A)^2+(x_2/B)^2=1$



$$(x_1, x_2)\begin{pmatrix} 1/A^2 & 0 \\ 0 & 1/B^2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=1$$



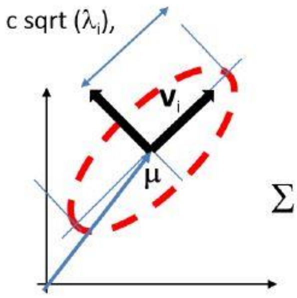
$U = [u_1 u_2]$ *u1 and u2 are column vectors...*

$$(x_1, x_2)U\begin{pmatrix} 1/A^2 & 0 \\ 0 & 1/B^2 \end{pmatrix}U^t\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=1$$

If we plot the geometrical locus of the points that fulfill the equation:

$$(\boldsymbol{x}-\boldsymbol{\mu})^t\Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu}) = c^2$$

we would obtain an n dimensional ellipsoid, centered at the point $\boldsymbol{\mu}$, with axis aligned with the (orthonormal) eigenvectors $\boldsymbol{q_i}$ of the matrix Σ, and with semi-axis length in the axis pointed by $\boldsymbol{q_i}$ equal to $c\sqrt{\lambda_i}$, where λ_i is the eigenvalue associated with $\boldsymbol{q_i}$:



$$\Sigma\, \mathbf{v}_i = \lambda_i\, \mathbf{v}_i$$

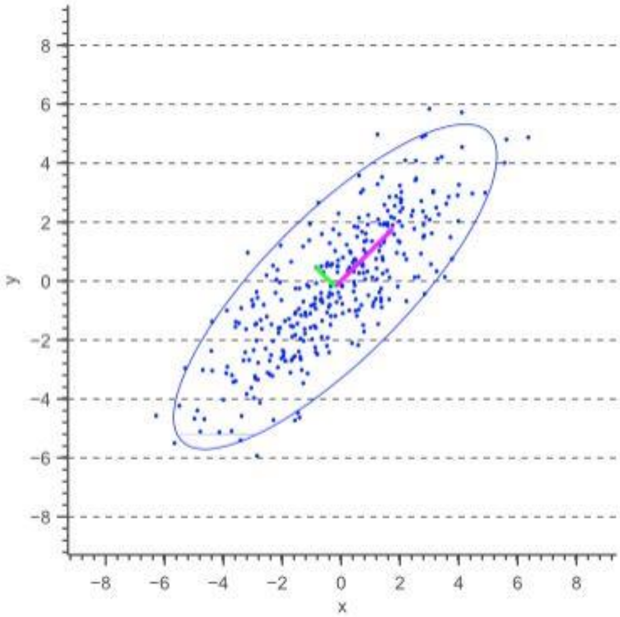


Figure 1. 2D confidence ellipse for normally distributed data

These directions are actually the directions in which the data varies the most, and are defined by the covariance matrix. The covariance matrix can be considered as a matrix that linearly transformed some original data to obtain the currently observed data. In a previous article about eigenvectors and eigenvalues we showed that the direction vectors along such a linear transformation are the eigenvectors of the transformation matrix. Indeed, the vectors shown by pink and green arrows in figure 1, are the eigenvectors of the covariance matrix of the data, whereas the length of the vectors corresponds to the eigenvalues.

Subspaces, eigenvalues and eigenvectors

- Fundamental theorem of linear algebra:

In mathematics, the **fundamental theorem of linear algebra** is a collection of statements regarding vector spaces and linear algebra, popularized by Gilbert Strang. The naming of these results is not universally accepted.

More precisely, let f be a linear map between two finite-dimensional vector spaces, represented by a $m \times n$ matrix M of rank r, then:

- r is the dimension of the column space of M, which represents the image of f;
- $n - r$ is the dimension of the null space of M, which represents the kernel of f;
- $m - r$ is the dimension of the cokernel of f.

The transpose M^T of M is the matrix of the dual f^* of f. It follows that one has also:

- r is the dimension of the row space of M, which represents the image of f^*;
- $m - r$ is the dimension of the left null space of M, which represents the kernel of f^*;
- $n - r$ is the dimension of the cokernel of f^*.

The two first assertions are also called the rank–nullity theorem.

Col(A)=1

Ker(At)=1

Col(At)=1

Ker(A)=1

Col(B)=2

Ker(Bt)=0

Col(Bt)=2

Ker(B)=0

Col(C)=R

Ker(Ct)=0

Col(Ct)=1

Ker(C)=1

Col(D)=1

Ker(Dt)=1

Col(Dt)=R

Ker(D)=0