SANS2021-22-HW1: Review of probability and Linear Algebra

jorge.garcia.vidal

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1 Discrete random variables

Generate $M = 10000$ samples of discrete random variables with the following distributions:

- Bernoulli with $p = p\{X = 1\} = 3/4$,
- Binomial with $n = 10$ and $p = 3/4$,
- Geometric with $p = 3/4$ (X taking values in $\{0, 1, ...\}$),
- Poisson with $\lambda = 1$,

For the all cases plot the graphic $X(m)$ for $m = 1...1000$ and the histogram.

2 Continuous random variables

Generate = 10000 samples of continuous random variables with the following distributions:

- Uniform in $[0, 1]$,
- Exponential with $\lambda = 1$,
- Gaussian with $\mu = 1$, $\sigma^2 = 1$
- Gaussian with $\mu = 1$, $\sigma^2 = 5$

For the three cases plot the graphic X(m) for m=1...1000 and the histogram.

3 Law Large Numbers, LLN

Generate $N = 100$ sets of $M = 1000$ samples of random variables following Bernoulli($p = pX = 1 = 3/4$), Uniform in $[0, 1]$, Exponential with $\lambda = 1$, and Gaussian with $\mu = 1$, $\sigma^2 = 1$ distributions.

For every distribution, use the different generated sets for generate M=1000 samples of the random variable $X^* = \frac{\sum_{k=1}^{N} X_k}{N}$. For the all cases, plot the graphic $X(m)$ for $m = 1...1000$ and the histogram.

4 Central Limit Theorem, CLT

Generate $N = 100$ sets of $M = 1000$ samples of random variables following Bernoulli($p = pX = 1 = 3/4$), Uniform in $[0, 1]$, Exponential with $\lambda = 1$, and Gaussian with $\mu = 1$, $\sigma^2 = 1$ distributions.

For each distribution, generate now $N = 100$ sets of $M = 1000$ samples of the normalized random variables $Z = \frac{X - \mathbb{E}(X)}{\sigma}$.

For every distribution, use the different generated sets for generate $M = 1000$ samples of the random variable $X^{**} = \frac{\sum_{k=1}^{N} Z_k}{\sqrt{N}}$. For the all cases, plot the graphic $Z(m)$ for $m = 1...1000$ and the histograms.

Check that the CLT holds.

5 Multivariate Gaussian

Generate $M = 10000$ samples from a multivariate gaussian distribution with $\mu = (3, 3)$ and $\Sigma = \begin{bmatrix} \frac{5}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$. Plot the scatterplot for (x_1, x_2). Find the eigenvalues and eigenvectors of Σ and relate them with the axis of the ellipse that appears in the plot. Use a python numpy.linalg.eig() that returns a tuple (eigvals,eigvecs) where eigvals is a 1D NumPy array of complex numbers giving the eigenvalues of , and eigvecs is a 2D NumPy array with the corresponding eigenvectors in the columns.

6 Subspaces, eigenvalues and eigenvectors

Find the subspaces *Col(A), Ker(A), Col(A^t), Ker(A^t)* for the matrices: $A = \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 \end{bmatrix}$, $D = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. Plot these subspaces.

For matrices A and B, find the eigenvalues and eigenvectors using numpy.linalg.eig(). Plot the scatter plot of the vectors in the unit circle $x(n) = (cos(n\frac{2\pi}{50}), sin(n\frac{2\pi}{50}))$

for $n = 0..49$. Plot also the scatter plot for the vectors $Ax(n)$ and $Bx(n)$. Relate the obtained plots with the eigenvectors and eigenvalues of the matrices.

7 Orthogonal, symmetric and positive definite matrices

Using numpy.linalg.eig() find the eigenvectors and eigenvalues of: $A = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 \\ 5 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$.