RICERCA OPERATIVA

Written assessment, January 9, 2019 A

First and Last name ____________________

Exercise 1 (value 8)
Consider the following PLC problem and solve it with the symplex method using the Blend rule

$$
\begin{aligned}
\min \quad & -x_2 + x_3 \\
& -2x_1 + 2x_2 \leq 1 \\
& x_1 + x_2 \leq 4 \\
& x_2 - x_3 \leq 1 \\
& x_1 \leq 3 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Write the dual and solve the dual using the complementary slackness conditions.

Exercise 2 (value 14)
The manager of a large company wants to optimize the supply chain. The company produces two goods, called A and B in the following. There are n suppliers, each with an available quantity of a_i items ($i = 1, \ldots, n$) that can be used to produce both A and B. The use of an item from supplier i has a cost c_i. The items must be transformed into a semifinished product (of type A or B) and next to be transformed into a final product. (Note that the type of the product is defined by the first transformation.) There are m first level plants that can perform both the first and the second transformation. There are p second level plans that can perform only the second transformation. Each plant can perform the first transformation for both product types, but the second transformation for a unique good type (A or B). Each first level plant j can make f_j first-transformations and s_j transformations from semifinished product to final goods. Moreover, there is a limit on the total number of transformations, namely t_j. The cost of the first transformation for an item is $c1_j$, while the cost of a second transformation is $c2_j$. Each second level plants k can transform at most l_k semifinished products into final goods at a cost $c3_k$ for each good. There is also a cost for the transport of items and semifinished products, namely t_{ij} is the cost to transport an item from supplier i to first level plant j, and r_{jk} is the cost to transport a semifinished product from first level plan j to second level plant k.
Write a linear model to help the company to decide how to produce F_A (resp. F_B) final products of type A (resp. B) at minimum cost.

Exercise 3 (value 6).
Consider the following ILP and write the corresponding model in GLPK or Mosel (XPRESS) language.

$$\min z = \sum_{t \in T} (a_t x_t - \sum_{j \in J} b_{tj} y_{tj}) + \sum_{\substack{t \in T \\ t \neq 0}} c_t z_t \tag{1}$$

$$\text{s.t. } x_t + \sum_{j \in J} y_{tj} \geq \alpha z_t \qquad t \in T, t \neq 0 \tag{2}$$

$$x_t + x_{t+1} + x_{t+2} \leq B \qquad t \in T, t \leq |T| - 2 \tag{3}$$

$$\sum_{j \in J} y_{tj} \leq 1 \qquad t \in T \tag{4}$$

$$2y_{tj} + y_{t,\beta} \geq 0 \qquad t \in T, j \in J, j \neq \beta \tag{5}$$

$$x_t, z_t \geq 0 \qquad t \in T \tag{6}$$

$$y_{tj} \geq 0 \text{ integer} \qquad t \in T, j \in J. \tag{7}$$

Exercise 1

x_1	x_2	x_3	x_4	x_5	x_6	x_7		
0	-1	1	0	0	0	0	0	$-z$
-2	2	0	1	0	0	0	1	x_4
1	1	0	0	1	0	0	4	x_5
0	1	-1	0	0	1	0	1	x_6
1	0	0	0	0	0	1	3	x_7

x_1	x_2	x_3	x_4	x_5	x_6	x_7		
-1	0	1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$-z$
-1	1	0	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	x_2
2	0	0	$-\frac{1}{2}$	1	0	0	$\frac{7}{2}$	x_5
1	0	-1	$-\frac{1}{2}$	0	1	0	$\frac{1}{2}$	x_6
1	0	0	0	0	0	1	3	x_7

x_1	x_2	x_3	x_4	x_5	x_6	x_7		
0	0	0	0	0	1	0	1	$-z$
0	1	-1	0	0	1	0	1	x_2
0	0	2	$\frac{1}{2}$	1	-2	0	$\frac{5}{2}$	x_5
1	0	-1	$-\frac{1}{2}$	0	1	0	$\frac{1}{2}$	x_1
0	0	1	$\frac{1}{2}$	0	-1	1	$\frac{5}{2}$	x_7

$x = (\frac{1}{2}, 1, 0, 0, \frac{5}{2}, 0, \frac{5}{2})$ $z_P = -1$

$$\begin{array}{ll} -\max & x_2 - x_3 \\ & -2x_1 + 2x_2 \leq 1 \\ & x_1 + x_2 \leq 4 \\ & x_2 - x_3 \leq 1 \\ & x_1 \leq 3 \\ & x_1, x_2, x_3 \geq 0 \end{array} \qquad \begin{array}{ll} -\min & u_1 + 4u_2 + u_3 + 3u_4 \\ & -2u_1 + u_2 + u_4 \geq 0 \\ & 2u_1 + u_2 + u_3 \geq 1 \\ & -u_3 \geq -1 \\ & u_1, \ldots, u_4 \geq 0 \end{array}$$

$$\left\{ \begin{array}{ll} (-2x_1 + 2x_2 - 1)u_1 = 0 & \\ (x_1 + x_2 - 4)u_2 = 0 & \Rightarrow \ u_2 = 0 \\ (x_2 - x_3 - 1)u_3 = 0 & \\ (x_1 - 3)u_4 = 0 & \Rightarrow \ u_4 = 0 \end{array} \right. \left\{ \begin{array}{l} (-2u_1 + u_2 + u_4)x_1 = 0 \\ (2u_1 + u_2 + u_3 - 1)x_2 = 0 \\ (u_3 - 1)x_3 = 0 \end{array} \right. \qquad \left\{ \begin{array}{l} -2u_1 = 0 \\ 2u_1 + u_3 = 1 \end{array} \right.$$

$u = (0, 0, 1, 0)$ $z_D = -1$

Exercise 2

Variables

$$
\begin{aligned}
x_{ij} &= \text{amount of items from supply } i \text{ to plant } j\\
y_j^A &= \text{amount of final goods of type A produced by plant } j\\
y_j^B &= \text{amount of final goods of type B produced by plant } j\\
w_{jk}^A &= \text{semifinished products of type A transported from } j \text{ to } k\\
w_{jk}^B &= \text{semifinished products of type B transported from } j \text{ to } k\\
\delta_j &= \text{1/0 if first level plant } j \text{ produces goods of type A/B}\\
\gamma_k &= \text{1/0 if second level plant } k \text{ produces goods of type A/B}
\end{aligned}
$$

Constants

$$M = \text{a big number, e.g. } M = F_A + F_B$$

$$\min z = \sum_{i=1}^{n} c_i \sum_{j=1}^{m} x_{ij} + \sum_{j=1}^{m} c1_j \sum_{i=1}^{n} x_{ij} + \sum_{j=1}^{m} c2_j (y_j^A + y_j^B) \tag{8}$$

$$+ \sum_{k=1}^{p} c3_k \sum_{j=1}^{m} w_{jk} + \sum_{i=1}^{n}\sum_{j=1}^{m} t_{ij} x_{ij} + \sum_{j=1}^{m}\sum_{k=1}^{p} r_{jk}(w_{jk}^A + w_{jk}^B) \tag{9}$$

$$\text{s.t.} \quad \sum_{j=1}^{m} x_{ij} \le a_i \qquad i = 1, \ldots, n \tag{10}$$

$$\sum_{i=1}^{n} x_{ij} \le \min(f_j, t_j) \qquad j = 1, \ldots, m \tag{11}$$

$$y_j^A + y_j^B \le s_j \qquad j = 1, \ldots, m \tag{12}$$

$$\sum_{i=1}^{n} x_{ij} \le y_j^A + y_j^B \qquad j = 1, \ldots, m \tag{13}$$

$$\sum_{i=1}^{n} x_{ij} - (y_j^A + y_j^B) = \sum_{k=1}^{p} (w_{jk}^A + w_{jk}^B) \qquad j = 1, \ldots, m \tag{14}$$

$$\sum_{j=1}^{m} (w_{jk}^A + w_{jk}^B) \le l_k \qquad k = 1, \ldots, p \tag{15}$$

$$\sum_{j=1}^{m} (y_j^A + \sum_{k=1}^{p} w_{jk}^A) \ge F_A \tag{16}$$

$$\sum_{j=1}^{m} (y_j^B + \sum_{k=1}^{p} w_{jk}^B) \ge F_B \tag{17}$$

$$y_j^A \le M\delta_j \qquad j = 1, \ldots, m \tag{18}$$

$$y_j^B \le M(1 - \delta_j) \qquad j = 1, \ldots, m \tag{19}$$

$$w_{jk}^A \le M\delta_j \qquad j = 1, \ldots, m, k = 1, \ldots, p \tag{20}$$

$$w_{jk}^B \le M(1 - \delta_j) \qquad j = 1, \ldots, m, k = 1, \ldots, p \tag{21}$$

$$x_{ij} \ge 0 \text{ integer} \qquad i = 1, \ldots, n, j = 1, \ldots, m \tag{22}$$

$$w_{jk}^A, w_{jk}^B \ge 0 \text{ integer} \qquad j = 1, \ldots, m, k = 1, \ldots, p \tag{23}$$

$$y_j^A, y_j^B \ge 0 \text{ integer} \qquad j = 1, \ldots, m \tag{24}$$

$$\delta_j \in \{0, 1\} \text{ integer} \qquad j = 1, \ldots, m \tag{25}$$

Exercise 3

```
param n integer > 0 ;
param m integer > 0 ;
set T:=0..n;
set J:=0..m;

param B integer > 0 ;
param alpha integer > 0 ;
param beta integer > 0 ;
param a{t in T};
param b{t in T, j in J};
param c{t in T};

/* variables */
var x {t in T } >= 0 ;
var y {t in T, j in J } >= 0, integer ;
var z {t in T} >= 0;

/* objective function  */
maximize zz :   sum{t in T} (a[t] * x[t]+ sum{j in J} b[t,j]*y[t,j])
                    + sum{t in T: t> 0} c[t]*z[t];

/* constraints */
s.t.
one{t in T: t>0} : x[t] +sum{j in J} y[t,j] >= alpha*z[t];
two{t in T: t<= n-2} : x[t]+x[t+1]+x[t+2]<= B ;
three{t in T} : sum{j in J} y[t,j] <=  1;
four{t in T, j in J: j <> beta} : 2*y[t,j] + y[t,beta] >= 0;

solve ;

end;
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