Written assessment, July uno, 2023

Last name, First name ______________________________

Exercise 1 (value 8)

The Daily New needs to redesign its *distribution network*. The newspapers are printed in an single *print house*, than are transported to some *distribution centers* (DCs), and later are transported from the distribution centers to the *kiosks*. Let us denote with 0 the print house, with $C = \{\dots, n\}$ the set of DCs, and with $S = \{1, \dots, m\}$ the set of kiosks. Each kiosk $j \in S$ requires d_j newspapers. Each DC $i \in C$ has a maximum storage capacity of u_i newspapers. The transport times from the print house to the DCs and from the these to the kiosks are given and independent of the quantity of newspapers transported. Let $t_i, i \in C$ denote the transport time from the print shop and DC i, and let $\hat{t}_{ij}$ denote the transport time from DC $i \in C$ to kiosk $j \in S$. Each kiosk must be served by a unique DC. It is given a maximum time T to serve each kiosk, i.e., supposing that the newspapers are printed at time zero, each kiosk must receive its newspapers within T time units. Help the company to find an optimal solution where the number of used DCs is minimized, by writing an Integer Linear Program.

Exercise 2 (value 11)

Consider the following LP problem.

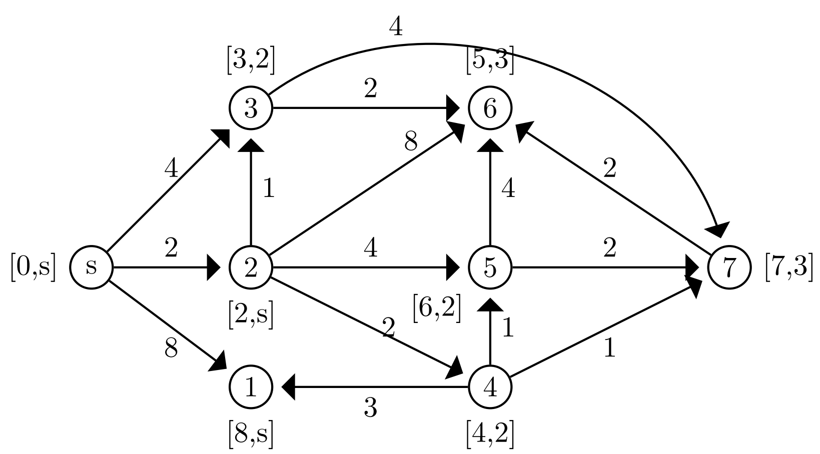
$$
\begin{aligned}
z = \min \quad & x_1 + x_2 + 2x_3 \\
& x_1 + 2x_2 = 10 \\
& 4x_1 + x_2 - x_3 = 4 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

- Write the dual problem, the complementary slackness and calculate the dual solution associated with a primal basic solution with x_1 and x_2 in the basis.
- Is the dual solution optimal ? (justify)

Exercise 3 (value 8)

Consider the following graph $G = (V, A)$ and the labels associated to the vertices after some iterations of the Dijkstra algorithm, starting from vertex s (the labels are [value, predecessor]). Answer to the following demands (justifying your assertions)

- What vertices have permanent labels (i.e., a label with a value corresponding to the shortest path from s)?
- Has the shortest path from s to 1 value 8 ?
- If we are looking for the shortest path from s to all other vertices, how many iterations we have still to perform ?



RICERCA OPERATIVA

Written assessment, June 19, 2023

Solution sketch

Exercise 1

Variables

$f_i =$ number of newspapers from print house to DC i
$\hat{f}_{ij} =$ number of newspapers from DC i to kiosk j
$x_i = 1$ if DC i receives newspapers; 0 otherwise
$\hat{x}_{ij} = 1$ if kiosk j receives newspapers from DC i; 0 otherwise
$\tau_j =$ time in which kiosk j receives the newspapers

$$
\begin{array}{lrll}
\min & \sum_{i \in C} x_i & & \\
& f_i - \sum_{j \in S} \hat{f}_{ij} & = 0 & i \in C \\
& f_i & \leq u_i x_i & i \in C \\
& \hat{f}_{ij} & = d_j \hat{x}_{ij} & i \in C, j \in S \\
& (t_i + \hat{t}_{ij})\hat{x}_{ij} & \leq \tau_j & i \in C, j \in S \\
& \tau_j & \leq T & j \in S \\
& f_i & \geq 0 & i \in C \\
& \hat{f}_{ij} & \geq 0 & i \in C, j \in S \\
& x_i & \in \{0,1\} & i \in C \\
& \hat{x}_{ij} & \in \{0,1\} & i \in C, j \in S
\end{array}
$$

Exercise 2

$$
\begin{array}{rl}
z = \min & x_1 + x_2 + 2x_3 \\
& x_1 + 2x_2 = 10 \\
& 4x_1 + x_2 - x_3 = 4 \\
& x_1, x_2, x_3 \geq 0
\end{array}
\qquad\qquad
\begin{array}{rl}
z = \max & 10u_1 + 4u_2 \\
& u_1 + 4u_2 \leq 1 \\
& 2u_1 + u_2 \leq 1 \\
& \quad - u_2 \leq 2 \\
& u_1, u_2 \text{ free}
\end{array}
$$

Complementary slackness

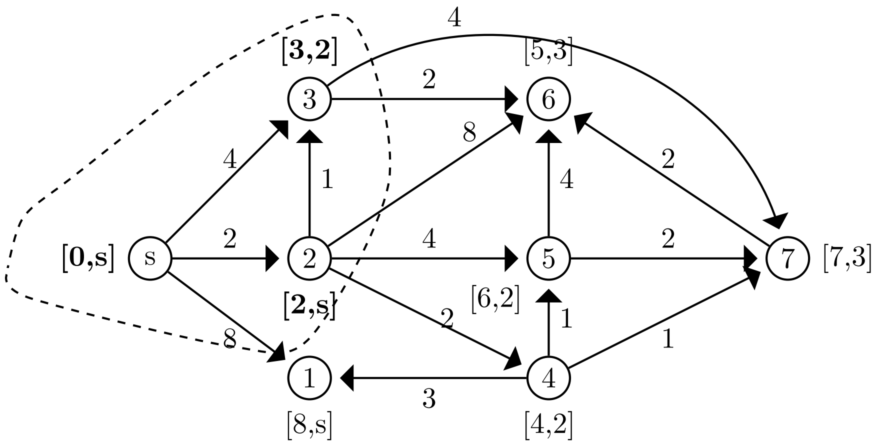
$$
\begin{cases}
(u_1 + 4u_2 - 1)x_1 = 0 \\
(2u_1 + u_2 - 1)x_2 = 0 \\
(-u_2 - 2)x_3 = 0
\end{cases}
\Rightarrow
\begin{cases}
(u_1 + 4u_2) = 1 \\
2u_1 + u_2 = 1 \\
--
\end{cases}
\Rightarrow (u_1 = 3/7, u_2 = 1/7) \; z_D = 34/7
$$

The dual solution is feasible
The primal solution is

$$
\begin{cases}
x_1 + 2x_2 = 10 \\
4x_1 + x_2 = 4
\end{cases}
\Rightarrow (x_1 = -2/7 x_2 = 36/7)
$$

unfeasible, therefore the dual solution is not optimal.

Exercise 3



- The algorithm has performed three iterations. Permanent labels have been given to vertices s, 2 and 3.
- The label of vertex 1 is not yet permanent. If we continue the algorithm the next vertex to be selected is vertex 4 which will relabel vertex 1 with a path of length 7.
- Since there are 5 vertices with a non permanent label, to compute the shortest path to each vertex we need other 4 iterations.