20200608/Exercises

1. **E01-value=6**

   A company has to define to produce exactly one among two products. If product 1 is made, than exactly $k1$ units have to be realized. If, instead, product 2 is made, than exactly $k2$ units have to be made. Write linear constraints to model the above condition.

   Notes: (not included in XML)

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   $$x_i = \text{units of product } i, (i = 1, 2)$$
   $$\delta = \begin{cases} 1 \text{ if } k1 \text{ units of product 1 are made;} \\ 0 \text{ if } k2 \text{ units of product 2 are made} \end{cases}$$

   $$x_1 = k1\delta$$
   $$x_2 = k2(1 - \delta)$$
   $$x_i \geq 0, i = 1, 2$$
   $$\delta \in \{0, 1\}$$

2. **E02-value=8**

   A warehouse has to store n boxes in a rack with m shelves. The first $m1$ shelves have length $L1$, while the remaining have length $L2$. Each box $j = 1, \ldots, n$ has length ℓ_j and a frequency of usage f_j. Write a linear programming model that helps the warehouse to decide how to store the boxes, so that the sum of the frequencies of the boxes stored in the first $m1$ shelves is maximized. (N.B. It is not known if all the boxes can be stored.)

   Notes: (not included in XML)

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   $$x_{ij} = 1 \text{ if box } j \text{ is stored in self } i; 0 \text{ otherwise}$$

$$
\begin{aligned}
&\max \sum_{i=1}^{m1} \sum_{j=1}^{n} x_{ij} f_j \\
&\sum_{i=1}^{m} x_{ij} \leq 1, j = 1, \ldots, n \\
&\sum_{j=1}^{n} \ell_j x_{ij} \leq L1, i = 1, \ldots, m1 \\
&\sum_{j=1}^{n} \ell_j x_{ij} \leq L2, i = m1+1, \ldots, m \\
&x_{ij} \in \{0, 1\} \quad i = 1, \ldots, m, j = 1, \ldots, n
\end{aligned}
$$

3. **E03-value=7**

   Consider a 0-1 knapsack problem with 5 items with profits $p_j = (10, 15, 8, 12, 11)$, weights $w_j = (8, 15, 10, 17, 20)$ and a knapsack of size 35. Consider the branch-and-bound used to solve the problem and the first subproblem generated by a branching of the type $x_j = 0$. Write the upper bound of this subproblem, and say if the exploration must proceed to lower levels: justify your choice.

   Notes: (not included in XML)

   - The subproblem is the fifth one (including the root node) and corresponds to $x_3 = 0$. The upper bound is $UB = 10 + 15 + 12\lfloor\frac{12}{17}\rfloor = 33$. The search stops for this subproblem, because we already have a feasible solution of value 33 obtained in the fourth node with $x = (1, 1, 1, 0, 0)$, $z = 33$.

4. **E04-value=7**

Given the following linear model write the corresponding dual problem.

$$
\begin{aligned}
\max\ & 3x_1 - 2x_2 + x_4 \\
& x_1 - x_2 + 3x_3 \geq 8 \\
& 4x_2 + 5x_4 \leq 12 \\
& 3x_1 - 4x_3 + x_4 \geq 15 \\
& x_i \geq 0 \quad i = 1, \ldots, 4
\end{aligned}
$$

Notes: (not included in XML)

- $$
\begin{aligned}
\min\ & -8u_1 + 12u_2 - 15u_3 \\
& -u_1 - 3u_3 \geq 3 \\
& u_1 + 4u_2 \geq -2 \\
& -3u_1 + 4u_3 \geq 0 \\
& 5u_2 - u_3 \geq 1 \\
& u_i \geq 0 \quad i = 1, \ldots, 3
\end{aligned}
$$