Written assessment, July 1, 2019 A

Last name, First name ______________________________

Exercise 1 (value 13)
A company is deciding how to invest B Euro, in particular, is evaluating to rent parcels of land where to set wind mills and solar panels to produce energy. The set of parcels to possibly rent is L and renting the parcel $\ell \in L$ costs c_ℓ Euro. Each parcel has a size d_ℓ in m^2 and, if rented, must be used completely; however its use can be diversified. Do to exposition to wind and sunlight, each parcel $\ell \in L$ can produce, for each m^2, e_ℓ^W or e_ℓ^S MW wind and solar power, respectively. Each MW produced by wind and solar installations has a net revenue of r^W and r^S Euro, respectively. The parcels can also be used for agricultural purpose, in such a case each m^2 of $\ell \in L$ produces a net gain of r_ℓ^A Euro. To reduce the risk the company decided to have at most 50% revenue from each type (wind, solar or agricultural). All values are considered for the entire evaluated period, and thus there is no need to include time in the model.

Due to its particular topography, if parcel 1 is rented, then all of it must be dedicated to solar panels.

The owner of parcels $1, 2, 3$ and 4 is willing to rent parcel 4 only if both parcels 1 and 2 are rented.

a) Write a linear MILP model that decides how to use the budget by maximizing the total revenue. Clearly describe the decision variables in your proposed formulation.

b) Consider now parcel 5. Being 5 very large, if it is rented then the sum of the areas of the remaining parcels rented must be at most 60% of d_5.

Exercise 2 (value 8)
Given the following LP problem.

$$
\begin{aligned}
\min \quad & 56x_1 + 60x_2 - 12x_3 \\
& 7x_1 + 12x_2 + 3x_3 \geq 3 \\
& 8x_1 + 5x_2 - 4x_3 \geq 1 \\
& x_1, x_2 \geq 0 \\
& x_3 \leq 0
\end{aligned}
$$

Write the corresponding dual problem and solve it with the simplex method implementing the Bland's rule. Obtain the optimal primal solution from the optimal dual one.

Exercise 3 (value 7)
A company has 15 M€ to invest in 4 projects. The four projects have an expected revenue and a cost as in the following table. The company wants to define the best mix of projects to invest in without exceeding its budget. Note that each project must be financed entirely, if selected.

Project	1	2	3	4
Revenue	21	11	16	6
Cost	7	4	6	3

Solve the problem by applying the branch-and-bound for Knapsack 0-1.

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Exercise 1

Constants

Variables

$y_l = 1$ if parcel $l \in L$ is rented, 0 otherwise.
$x_\ell^W, x_\ell^S, x_\ell^A$: m^2 of parcel $\ell \in L$ dedicated to wind energy production, solar energy production or agriculture.
$R =$ total revenue

Model

$$
\begin{aligned}
\max z = \ & R - \sum_{\ell \in L} c_\ell y_\ell \\
& R = \sum_{\ell \in L} (r^W e_\ell^W x_\ell^w + r^S e_\ell^S x_\ell^S + r_\ell^A x_\ell^A) \\
& x_\ell^W + x_\ell^S + x_\ell^A = d_\ell y_\ell \quad l \in L \\
& \sum_{\ell \in L} c_\ell y_\ell \leq B \\
& \sum_{\ell \in L} r^W e_\ell^W x_\ell^w \leq 50\% R \\
& \sum_{\ell \in L} r^S e_\ell^S x_\ell^S \leq 50\% R \\
& \sum_{\ell \in L} r_\ell^A x_\ell^A \leq 50\% R \\
& x_1^S = d_1 y_1 \\
& y_4 + 1 \leq y_1 + y_2 \\
& \sum_{\ell \in L \setminus \{5\}} d_\ell y_\ell \leq 0.6 d_5 + M(1 - y_5) \\
& x_\ell^W, x_\ell^S, x_\ell^A \geq 0 \text{ integer} \quad \ell \in L \\
& y_\ell \in \{0, 1\} \quad l \in L
\end{aligned}
$$

Exercise 2

$$\begin{array}{llcll}
\min & 56x_1 + 60x_2 - 12x_3 & \qquad & \max & 3u_1 + u_2 \\
 & 7x_1 + 12x_2 + 3x_3 \geq 3 & & & 7u_1 + 8u_2 \leq 56 \\
 & 8x_1 + 5x_2 - 4x_3 \geq 1 & & & 12u_1 + 5u_2 \leq 60 \\
 & x_1, x_2 \geq 0 & & & 3u_1 - 4u_2 \geq -12 \\
 & x_3 \leq 0 & & & u_1, u_2 \geq 0
\end{array}$$

	u_1	u_2	s_1	s_2	s_3	
	3	1	0	0	0	0
s_1	7	8	1	0	0	56
s_2	**12**	5	0	1	0	60
s_3	-3	4	0	0	1	12

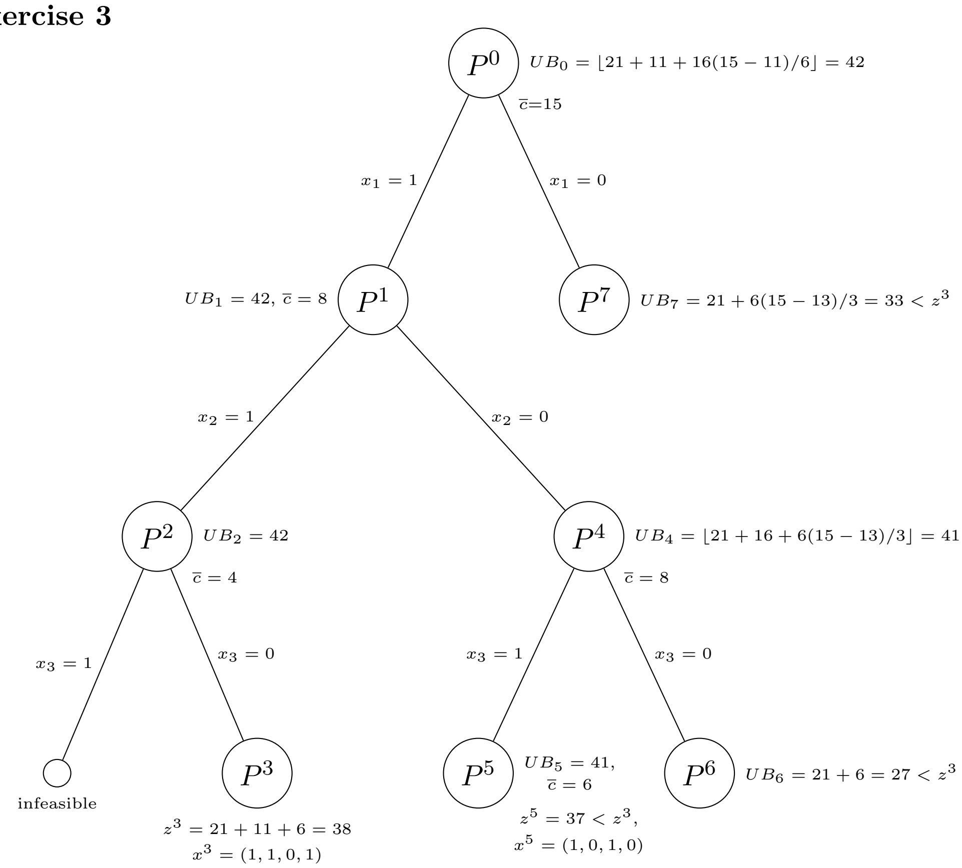
	u_1	u_2	s_1	s_2	s_3	
	0	-1/4	0	-1/4	0	-15
s_1	0	61/12	1	-7/12	0	21
u_1	1	5/12	0	1/12	0	5
s_3	0	21/4	0	1/4	1	27

The dual solution is $u^* = (5, 0)$ with value $z_D^* = 15$.

$$\left\{\begin{aligned} (7x_1 + 12x_2 + 3x_3 - 3)u_1 &= 0 \\ (8x_1 + 5x_2 - 4x_3 - 1)u_2 &= 0 \\ (7u_1 + 8u_2 - 56)x_1 &= 0 \\ (12u_1 + 5u_2 - 60)x_2 &= 0 \\ (3u_1 - 4u_2 + 12)x_3 &= 0 \end{aligned}\right. \qquad \left\{\begin{aligned} (7x_1 + 12x_2 + 3x_3 - 3)5 &= 0 \\ 0 &= 0 \\ (35 - 56)x_1 &= 0 \\ (60 - 60)x_2 &= 0 \\ (15 + 12)x_3 &= 0 \end{aligned}\right. \qquad \left\{\begin{aligned} x_2 &= 1/4 \\ 0 &= 0 \\ x_1 &= 0 \\ x_2 &\geq 0 \\ x_3 &= 0 \end{aligned}\right.$$

$x = (0, 1/4, 0)$ $z_P = 15$

Exercise 3



The optimal solution is $x^3 = (1, 1, 0, 1)$ with value $z^3 = 38$.