Written assessment, February 21, 2022

Last name, First name ______________________________

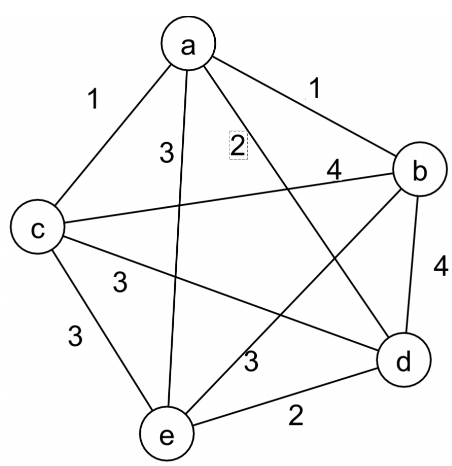
Exercise 1 (value 13)
In a certain period a set K of refrigerator trucks arrives at the unloading dock of a logistic company. Each truck $i \in K$ arrives at time a_i and requires m_i time units to be unloaded. The dock has one unloading team that can operate only one truck at the time. If the team is unloading another truck, then the newly arrived one must wait until the team is free. The trucks carry perishable goods and thus the trucks must keep refrigerating the goods while waiting to be unloaded. This costs c_i per each minute waited by truck $i \in K$. When the unloading of one truck starts, it must be completed without interruptions. Write a MILP to minimize the costs for unloading all the trucks, while respecting the given constraints.

Exercise 2 (value 8)
Consider the following ILP problem.

$$\begin{array}{lrrcrl}
\max & 2x_1 & +5x_2 & & & \\
 & x_1 & -2x_2 & \geq & -4 & \\
 & 2x_1 & -3x_2 & \leq & 5 & \\
 & 4x_1 & +x_2 & \leq & 8 & \\
 & x_1, & x_2 & \geq & 0 & \textit{integer}
\end{array}$$

- Solve the linear relaxation of the problem with the simplex algorithm using the Bland rule (show tableaus) and report its optimal solution and the value of the optimal solution.
- If the obtained solution is not integer consider to apply a branch and bound algorithm and answer to the following questions. 1) Is it correct to branch first on x_2 ? 2) what inequalities define the branching on this variable in the two newly generated subproblems?

Exercise 3 (value 7) Given the following graph, find the Shortest Path Problem from node **b** to node **d** using Dijkstra's algorithm. Show the procedure by reporting, for each iteration, labels and predecessors. Write the final path and its cost.



RICERCA OPERATIVA

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Exercise 1

- t_i non-negative variable that defines the time in which truck $i \in K$ starts unloading.
- $y_{ij} = 1$ if the truck $j \in K$ is unloaded after truck $i \in K$; 0 otherwise.
- s_i is the time the truck spends waiting with the refrigerator on.

T is a large enough number

$$\begin{aligned}
&\min z = \textstyle\sum_{i\in K} c_i s_i \\
&s_i = t_i - a_i \quad i \in K \\
&t_j \geq t_i + m_i - T(1 - y_{ij}) \quad i, j \in K \\
&y_{ij} + y_{ji} = 1 \quad i, j \in K \\
&y_{ij} \in \{0, 1\} \quad i, j \in K \\
&t_i, s_i \geq 0, i \in K.
\end{aligned}$$

Exercise 2

x_1	x_2	x_3	x_4	x_5		
-2	-5	0	0	0	0	$-z$
-1	2	1	0	0	4	x_3
2	-3	0	1	0	5	x_4
4	1	0	0	1	8	x_5

x_1	x_2	x_3	x_4	x_5		
0	9/2	0	0	$-1/2$	-4	$-z$
0	**9/4**	1	0	1/4	6	x_3
0	$-7/2$	0	1	$-1/2$	1	x_4
1	1/4	0	0	1/4	2	x_1

x_1	x_2	x_3	x_4	x_5		
0	0	-20	0	-1	-16	$-z$
0	1	4/9	0	1/9	8/3	x_2
0	0	14/9	1	$-1/9$	31/3	x_4
1	0	$-1/9$	0	2/9	4/3	x_1

The optimal solution of the linear relaxation is $x_{LR} = (4/3, 8/3)$ with value -16.

The optimal solution is not integer. YES: both x_1 and x_2 are fractional, so it is possible to branch on x_2.
The two tree branches include inequalities $x_2 \leq 2$ and $x_2 \geq 3$, respectively.

Exercise 3

L_j S	**a**	**b**	**c**	**d**	**e**
$\{b\}$	**1**	–	4	4	3
$\{a, b\}$	1	–	**2**	3	3
$\{a, b, c\}$	1	–	2	**3**	3

$Pred_j$ S	**a**	**b**	**c**	**d**	**e**
$\{b\}$	b	–	b	b	b
$\{a, b\}$	b	–	a	a	b
$\{a, b, c\}$	b	–	a	a	b

The shortest path from b to d is $\{b, a, d\}$ and costs 3.