Written assessment, February 16, 2023

Last name, First name ______________________________

Exercise 1 (value 13)

The **Alpine Rescue Service** has been asked to find and help a trekker lost in a mountain region. The service sent to that region n rescue operators. The region has been partitioned into m zones and the operators must be assigned to the zones in such a way that all the zones are explored. Given the morphological characteristics of each region $j \in \{1, \dots, m\}$ and its size, we must assign to zone j a minimum of $a_j \geq 2$ and a maximum of $b_j \geq a_j$ operators. The set of operators assigned to a same zone is called a *team*. The operators have different experience and competencies. Let E be the subset of operators with highest experience, and F the subset of operators with competencies in first-aid (note that one operator may belong to both E and F). We must assign to each zone a team with at least one operator from E and one from F and they must be different persons (i.e., if one operator belongs to both E and F, we still need to assign another operator from E or F). Preferences have been expressed about the composition of the teams: let P be a $n \times n$ matrix where p_{hk} gives the preference score associated with having operators h and k working together in a same team.

Write a linear mathematical model to help the coordinator of the service to assign the operators to the zones, by satisfying the above constraint and maximizing the overall preferences for the composition of the teams.

Exercise 2 (value 7)

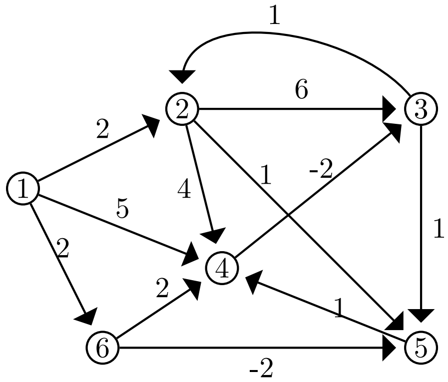
Consider the following LP problem.

$$
\begin{array}{lrrrcl}
\min & x_1 & +2x_2 & -x_3 & & \\
 & x_1 & +2x_2 & -x_3 & \leq & 8 \\
 & 3x_1 & +x_2 & +x_3 & \leq & 12 \\
 & x_1, & x_2, & x_3 \geq & & 0
\end{array}
$$

Write the dual problem, solve its graphically, and compute the optimal primal solution using the complementary slackness conditions. Show the procedure.

Exercise 3 (value 7)

Consider the following graph $G = (V, A)$ and find the shortest path from 1 to 2 using an appropriate algorithm. Report in a table all the steps of the algorithm.



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Solutions

Exercise 1

x_{ij} $= 1$ if operator $i = 1, \ldots, n$ is assigned to zone $j = 1, \ldots, m$, 0 otherwise

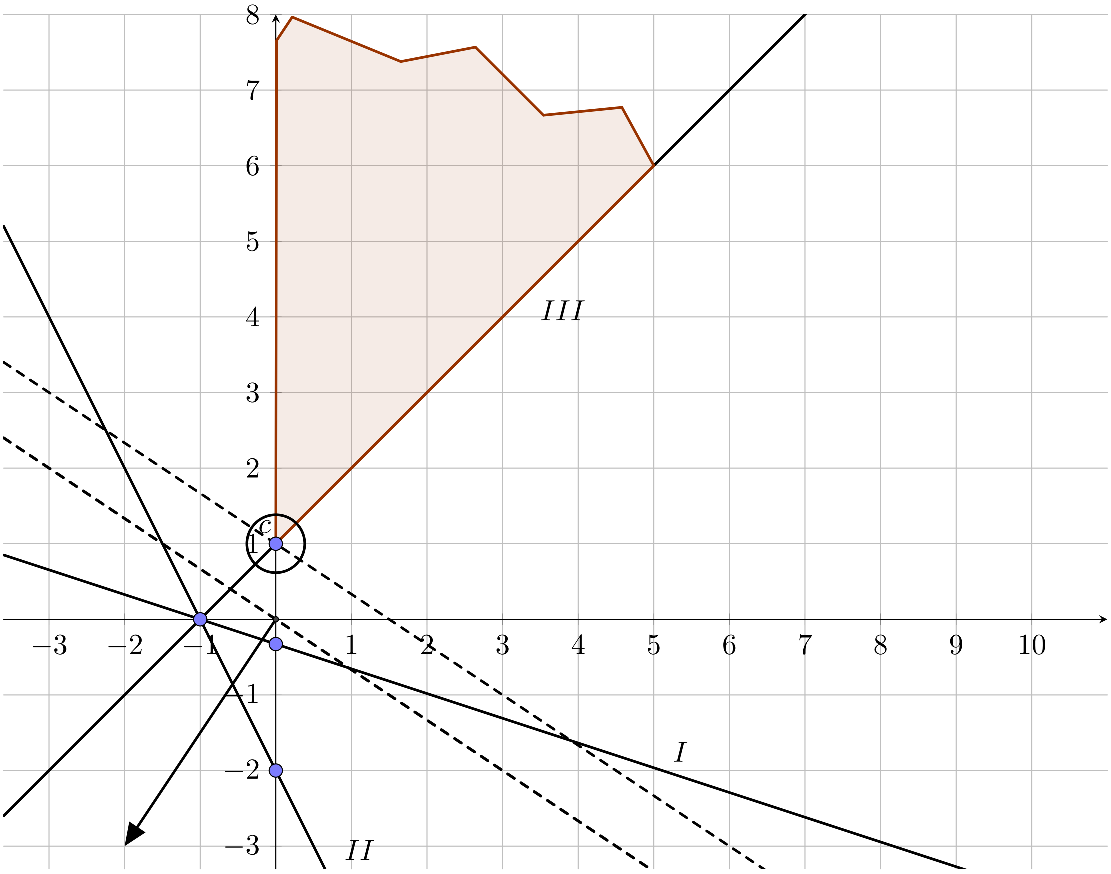
y_{hkj} $= 1$ if both operators $h, k \in \{1, \ldots, n\}$ are assigned to zone $j = 1, \ldots, m$, 0 otherwise

$$
\begin{array}{llll}
\max & \displaystyle\sum_{h=1}^{n} \sum_{k=1, k\neq h}^{n} \sum_{j=1}^{m} p_{hk}\, y_{hkj} & & \\
& \displaystyle\sum_{j=1}^{m} x_{ij} & \leq 1 & i = 1, \ldots, n \\
& \displaystyle\sum_{i=1}^{n} x_{ij} & \geq a_j & j = 1, \ldots, m \\
& \displaystyle\sum_{i=1}^{n} x_{ij} & \leq b_j & j = 1, \ldots, m \\
& \displaystyle\sum_{i\in E} x_{ij} & \geq 1 & j = 1, \ldots, m \\
& \displaystyle\sum_{i\in F} x_{ij} & \geq 1 & j = 1, \ldots, m \\
& \displaystyle\sum_{i\in E\cup F} x_{ij} & \geq 2 & j = 1, \ldots, m \\
& x_{hj} + x_{kj} & \geq 2y_{hkj} & h, k = 1 \ldots, n,\ j = 1, \ldots, m \\
& x_{ij} & \in \{0, 1\} & i = 1, \ldots, n,\ j = 1, \ldots, m \\
& y_{hkj} & \in \{0, 1\} & h, k = 1 \ldots, n,\ j = 1, \ldots, m
\end{array}
$$

Exercise 2

$$
\begin{array}{lrrrcl}
\min & x_1 & +2x_2 & -x_3 & & \\
& x_1 & +2x_2 & -x_3 & \leq & 8 \\
& 3x_1 & +x_2 & +x_3 & \leq & 12 \\
& x_1, & x_2, & x_3 \geq & & 0
\end{array}
$$

$$
\begin{array}{lrrcl}
\max & -8u_1 & -12u_2 & & \\
& -u_1 & -3u_2 & \leq & 1 \\
& -2u_1 & -u_2 & \leq & 2 \\
& -u_1 & +u_2 & \geq & 1 \\
& u_1, & u_2, \geq & & 0
\end{array}
$$



Optimal dual solution: $u = (0, 1)$, $z_D = -12$

$$\begin{cases} (x_1 + 2x_2 - x_3 - 8)u_1 & = 0 \\ (3x_1 + x_2 + x_3 - 12)u_2 & = 0 \\ (u_1 + 3u_2 + 1)x_1 & = 0 \\ (2u_1 + u_2 + 2)x_2 & = 0 \\ (u_1 - u_2 + 1)x_3 & = 0 \end{cases} \quad \begin{cases} (x_1 + 2x_2 - x_3 - 8)(0) & = 0 \\ 3x_1 + x_2 + x_3 - 12 & = 0 \\ 4x_1 & = 0 \\ 3x_2 & = 0 \\ (0)x_3 & = 0 \end{cases} \quad \begin{cases} 0 & = 0 \\ x_3 & = 12 \\ x_1 & = 0 \\ x_2 & = 0 \\ 0 & = 0 \end{cases}$$

Optimal primal solution: $x = (0, 0, 12)$, $z_P = -12$

Exercise 3

We use the Bellman-Ford algorithm, having some negative costs.

Source node = 1, destination node = 2.

Iter	$f(j)$						pred					
	1	2	3	4	5	6	1	2	3	4	5	6
0	0	-	-	-	-	-	1	-	-	-	-	-
1	0	2	-	5	-	2	1	1	-	1	-	1
2	0	2	3	4	0	2	1	1	4	6	6	1
3	0	2	2	1	0	2	1	1	4	5	6	1
4	0	2	-1	1	0	2	1	1	4	5	6	1
5	0	0	-1	1	0	2	1	3	4	5	6	1

Optimal path $1- > 2$, cost $= 0$, path $= (1, 6, 5, 4, 3, 2)$.