RICERCA OPERATIVA

Written assessment, July 23, 2019 M

Last name, First name ______________________________

Exercise 1 (value 13)

A fishing company needs to organize its operations. The company has been given a see surface where to fish, that has been partitioned into a set S of sectors. Each sector $s \in S$ has an expected amount of production q_s that the company want to fish entirely. The fishing is made by *fishing ships* each of which must be associated with a *mother ship* used for fish preparation and packaging (no fishing activity can be done by the mother ship). Each mother ship provides support to a maximum of 5 fishing ships, but each fishing ship is associated with only one mother ship. The company can use up to m mother ships that can be located in one of the sectors $s \in S$. At most one mother ship per sector is allowed. Let K denote the set of available fishing ships. Each ship $k \in K$ has a fishing capacity c^k. A fishing ship operates in only one sector, which can be different from that of its mother ship. In each sector can operate several fishing ships. The cost of using a mother ship is b, while the cost of assigning the fishing ship $k \in K$ to a mother ship located in sector $s \in S$ to fish in sector $i \in S$ is a^k_{si}.

Write a MILP model including all the given specifications and minimizing the total cost. Clearly explain the used variables. *Suggestion: mother ships can be identified using only the index of the sector they are assigned.*

Sector 1 is very relevant and if no mother ship is located in sector 1 then a cost D must be paid: change the objective function to address this new cost.

For contract reasons, if fishing ship α is assigned to fish in a sector $i \in S$ then it must be the one and only fishing ship operating in that sector. Note that fishing ship α has a sufficient capacity for doing that.

Exercise 2 (value 9)
Consider the following ILP problem.

$$
\begin{aligned}
P = \min \quad & -5x_1 - 3x_2 \\
& 2x_1 + x_2 \le 14 \\
& x_1 + 8x_2 \le 32 \\
& x_1, x_2 \ge 0, integer
\end{aligned}
$$

Perform the following tasks:
(i) Represent graphically problem P.
(ii) Compute the optimal solution of the continuous relaxation by using the symplex method, than define all the Gomory's cuts associated with the optimal tableau.
(iii) Draw the Gomory's cuts on the graphical representation and say if they are faces of the convex hull (explain why)
(iv) Insert the first cut in the tableau and select the pivot (without computing it). Explain your choice.

Exercise 3 (value 6)
Write a GLPK or XPRESS model corresponding to the following mathematical model

$$\max \ z = \sum_{i \in N} \sum_{k=1}^{m} b_{ik} x_{ik} - \sum_{i \in N} w_i - \sum_{k=1}^{m} l_k y_k$$

$$\sum_{k=1}^{m} x_{ik} \leq w_i \qquad i \in N \tag{1}$$

$$\sum_{i \in N \setminus \{0\}} x_{ik} \leq M y_k, \qquad k = 1, \ldots, m \tag{2}$$

$$w_i = \sum_{j=0}^{4} j \gamma_{ij}, \qquad i \in N \tag{3}$$

$$\sum_{j=0}^{4} \gamma_{ij} = 1, \qquad i \in N \tag{4}$$

$$x_{ik} \in \{0, 1\} \qquad i \in N, \ k = 1, \ldots, m \tag{5}$$

$$y_k \in \{0, 1\} \qquad k = 1, \ldots, m \tag{6}$$

$$\gamma_{ij} \in \{0, 1\} \qquad i \in N \ j = 0, \ldots, 4 \tag{7}$$

$$w_i \geq 0 \qquad i \in N \tag{8}$$

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Exercise 1

Variables

$y_s = 1$ if a mother ship is located in sector $s \in S$, 0 otherwise.
$x_{si}^k = 1$ if fishing ship $k \in K$ operates in sector $i \in S$ from mother ship located in sector $s \in S$, 0 otherwise.

Model

$$
\begin{aligned}
\min\ & b \sum_{s \in S} y_s + \sum_{k \in K} \sum_{i \in S} \sum_{i \in S} a_{si}^k x_{si}^k + D(1 - y_1) \\
& \sum_{s \in S} y_s \leq m \\
& \sum_{s \in S} \sum_{k \in K} c^k x_{si}^k \geq q_i \quad i \in S \\
& \sum_{i \in S} \sum_{k \in K} x_{si}^k \leq 5 y_s \quad s \in S \\
& \sum_{s \in S} \sum_{i \in S} x_{si}^k \leq 1 \quad k \in K \\
& \sum_{s \in S} \sum_{k \in K \setminus \{\alpha\}} x_{si}^k \leq |K|(1 - \sum_{s \in S} x_{si}^{\alpha}) \quad i \in S \\
& x_{si}^k \in \{0, 1\} \quad s, i \in S, k \in K \\
& y_s \in \{0, 1\} \quad s \in S.
\end{aligned}
$$

Exercise 2

Let's write the model in standard form:

$$\begin{aligned} \min \quad & -5x_1 - 3x_2 \\ & 2x_1 + x_2 + x_3 &= 14 \\ & x_1 + 8x_2 + x_4 &= 32 \\ & x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

x_1	x_2	x_3	x_4		
-5	-3	0	0	0	$-z$
2	1	1	0	14	x_3
1	8	0	1	32	x_4

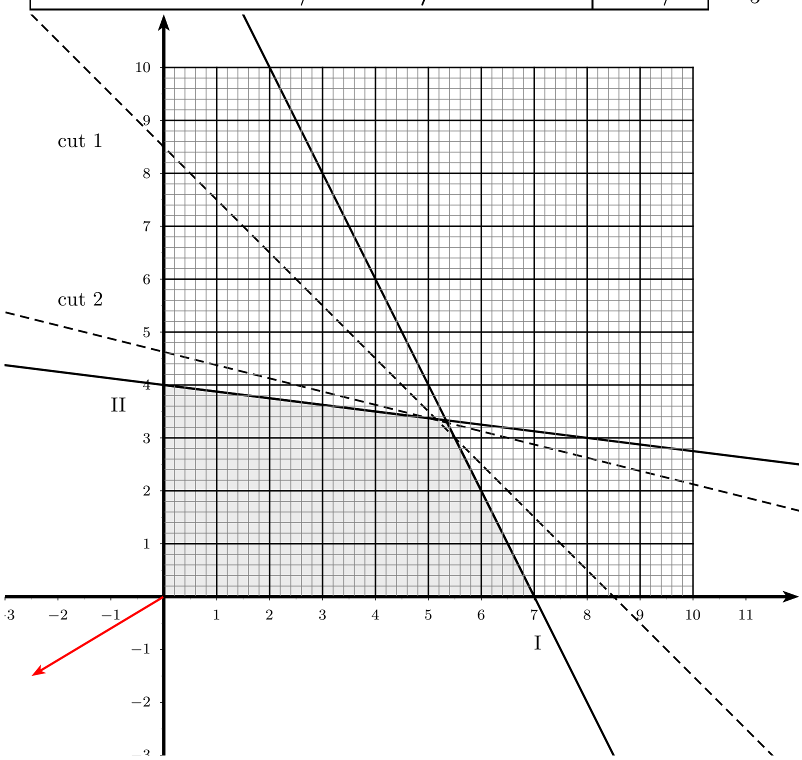
x_1	x_2	x_3	x_4		
0	-1/2	5/2	0	35	$-z$
1	1/2	1/2	0	7	x_1
0	**15/2**	-1/2	1	25	x_4

x_1	x_2	x_3	x_4		
0	0	37/15	1/15	110/3	$-z$
1	0	8/15	-1/15	16/3	x_1
0	1	-1/15	2/15	10/3	x_2

The Gomory's cuts built on both rows are:
$8/15x_3 + 14/15x_4 \geq 1/3 \quad \rightarrow \quad 8x_3 + 14x_4 \geq 5$
$14/15x_3 + 2/15x_4 \geq 1/3 \quad \rightarrow \quad 14x_3 + 2x_4 \geq 5$

That become:
$2x_1 + 8x_2 \leq 37$
$2x_1 + 2x_2 \leq 17$

x_1	x_2	x_3	x_4	x_5		
0	0	-37/15	-1/15	0	-110/3	$-z$
1	0	8/15	-1/15	0	16/3	x_1
0	1	-1/15	2/15	0	10/3	x_2
0	0	-8/15	**-14/15**	1	-1/3	x_5



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/* Exercise 3 */

param m, integer, > 0;
set N;
set J := 1..m;
set Q := 0,..,4;

param b{i in N, k in J}, >= 0;
param l{k in J}, >= 0;
param M, >= 0;

var  x{i in N, k in J},  binary;
var  y{k in J},  binary;
var  gamma{i in N, j in Q},  binary;
var  w{i in N},  >=0;

maximize z: sum{i in N, k in J} b[i,k]*x[i,k]-sum{i in N} w[i] - sum{k in J} l[k]*y[k];

s.t.  v1{i in N}:  sum{k in J} x[i,k] <=  w[i];
  v2{k in J}:  sum{i in N : i>0} x[i,k] <=  M*y[k];
  v3{i in N}:  w[i] = sum{j in Q} j*gamma[i,j];
  v4{i in N}:  sum{j in Q}gamma[i,j] = 1;

solve;
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Answer to the following questions. Each question may have zero, one or more correct answers. Each exact answer to a question has value +1, each wrong answer has value -1. No answer has zero value.

(Q1) Given the PLI model $\min\{c^T x : x \in P \cap Z^n\}$ with $P = \{Ax = b, x \geq 0\}$, and the optimal solution x^* of its continuous relaxation, a Gomory's cut is: (a) an inequality for P satisfying the Farkas' lemma; (b) an inequality $\alpha_F^T x_F \geq \alpha_0$ satisfied by any $x \in P$; (c) an inequality $\alpha_F^T x_F \geq \alpha_0$ satisfied by any $x \in P \cap Z^n$, but not by x^*; (d) an inequality $\alpha_F^T x_F \geq \alpha_0$ satisfied by any $x \in P$ but not by x^*; (e) none of the above answers.

a	b	c	d	e

(Q2) Consider the two problems, $P : \min\{c^T x : Ax = b, x \geq 0\}$, $D : \max\{u^T b : u^T A \leq c^T\}$, then (a) a feasible solution of P gives a lower bound on the optimal value of D; (b) a feasible solution of D gives a lower bound on the optimal value of P; (c) a pair of solutions x and u with x feasible for P and u feasible for D are optimal if $c^T - c_B^T B^{-1} A \geq 0$; (d) a pair of solutions x and u such that $(u^T A - c^T)x = 0$ are optimal if $x \geq 0$; (e) a pair of solutions x and u are optimal if $x \geq 0$ and $u \geq 0$.

a	b	c	d	e

(Q3) Given a PLC problem P in standard form with n variables and m constraints, consider the two phases method. The artificial problem used to solve the first phase : (a) has m variables and n constrains; (b) has, at most, $n + m$ variables and m constrains; (c) has an objective function in maximization form; (d) gives a feasible base for problem P, if its optimal solution has value zero; (e) gives a feasible and optimal solution for problem P if its optimal solution has a negative value;

a	b	c	d	e

(Q4) Consider a graph $G = (V, E)$, the Shortest Spanning Tree Problem and its optimal solution T^*. (a) the edge with minimum cost is always in T^*; (b) the edge with minimum cost is in T^* only if it does not close a cycle with other edges; (c) the edge with maximum cost is always in T^*; (d) the edge with second minimum cost is always in T^*; (e) the edge with second minimum cost is in T^* only if the edge with minimum cost is not in T^*.

a	b	c	d	e

(Q5) The reduced cost of a basic variable x_j, in a tableau in basic form: (a) is always null; (b) is strictly positive if the current solution is optimal and the problem is in minimization form; (c) has a non-negative value if the current solution is optimal and the problem is in minimization form; (d) has a non-positive value if the current solution is optimal and the problem is in minimization form; (e) represent the variation of the objective function when the r.h.s. of the j-th constraint increases of one unit;

a	b	c	d	e