20210714

1. **MM-value=13/time=25**

   ESSAY | 13 points | 0.10 penalty | editor

   A supermarket provides home delivery service to customers by using a fleet of autonomous robots, each with capacity Q, expressed in number of products. The company needs to optimize the deliveries for the next day from its stores S to the customers C. Each product $p \in P$ have been ordered by a customer $j \in C$ in quantity q_{pj} (note that $q_{pj} = 0$ if the customer $j \in C$ does not select that product), while n_{pi} is the number of available products of type $p \in P$ at a store $i \in S$.

   Every customer can be served by more than one robot, depending on the amount of products requested, and not necessarily from the same store. Each robot performs a single trip.

   Write a linear model in order to serve all the customers, which respects the capacity of the robots, and minimizes the number of robots used. Among the solutions with the same minimal number of robots, we want to minimize the total distance traveled by the robots, knowing that d_{ij} is the distance between store $i \in S$ and customer $j \in C$ and that each robot only makes missions of type store/customer/store (i.e. each mission serves only one customer).

   The store α required to apply the following constraint to its deliveries: only one robot can be sent to each customer, and only if at least 80% of the robot capacity is used for that delivery.

   Clearly define all the used variables.

   Notes for grader:

   - – x_{ij}: number of robots servicing customer $j \in C$ from store $i \in S$.;
     – y_{ij}^p: number of products $p \in P$ delivered from store $i \in S$ to customer $j \in C$;
     – K is an arbitrary large constant, $K \geq \sum_{i \in S} \sum_{j \in C} 2d_{ij}$.

$$
\begin{aligned}
\min z =& K\left(\sum_{i\in S}\sum_{j\in C} x_{ij}\right) + \sum_{i\in S}\sum_{j\in C} 2d_{ij}x_{ij} \\
& \sum_{i\in S} y^p_{ij} = q_{pj} \quad p \in P, j \in C \\
& \sum_{j\in C} y^p_{ij} \le n_{pi} \quad p \in P, i \in S \\
& \sum_{p\in P} y^p_{ij} \le Qx_{ij} \quad i \in S, j \in C \\
& x_{ij} \ge 0, integer \quad i \in S, j \in C \\
& y^p_{ij} \ge 0, integer \quad i \in S, j \in C, p \in P
\end{aligned}
$$

$$
\begin{aligned}
& \sum_{p\in P} y^p_{\alpha j} \ge 0.8Qx_{\alpha j} \quad j \in C \\
& x_{\alpha j} \le 1 \quad j \in C
\end{aligned}
$$

2. **PLC-value=7/time=15**

ESSAY | 7 points | 0.10 penalty | editor

Consider the following LP problem:

$$
\begin{aligned}
\min z = \quad & 2x_1 + 5x_2 - 3x_3 + 10x_4 \\
& 3x_1 - 2x_2 + 8x_3 - x_4 = 110 \\
& 4x_1 - 10x_2 - x_3 + 5x_4 = 80 \\
& x_1, x_2, x_3, x_4 \ge 0.
\end{aligned}
$$

The optimal solution is $x^* = (150/7, 0, 40/7, 0)$ with value $z^* = 180/7$ and the optimal basis is $B = \begin{bmatrix} 3 & 8 \\ 4 & -1 \end{bmatrix}$ and with $B^{-1} = \begin{bmatrix} 1/35 & 8/35 \\ 4/35 & -3/35 \end{bmatrix}$.

Perform sensitivity analysis in order to understand how much each of the two right hand side values can change without altering the current optimal basis. Explain the procedure.

Notes for grader:

- We need to compute $B^{-1}(b + \Delta b) \geq 0$:

$$\begin{bmatrix} 1/35 & 8/35 \\ 4/35 & -3/35 \end{bmatrix} \begin{bmatrix} 110 + \Delta b_1 \\ 80 + \Delta b_2 \end{bmatrix} \geq 0$$

$$\begin{cases} 750 + \Delta b_1 + 8\Delta b_2 \geq 0 \\ 200 + 4\Delta b_1 - 3\Delta b_2 \geq 0 \end{cases}$$

Setting $\Delta b_2 = 0$ we have:

$$\begin{cases} \Delta b_1 \geq -750 \\ \Delta b_1 \geq -50 \end{cases}$$

Hence the basis does not change for $-50 \leq \Delta b_1 \leq \infty$

Setting $\Delta b_2 = 0$ we have:

$$\begin{cases} \Delta b_2 \geq -750/8 \\ \Delta b_2 \leq 200/3 \end{cases}$$

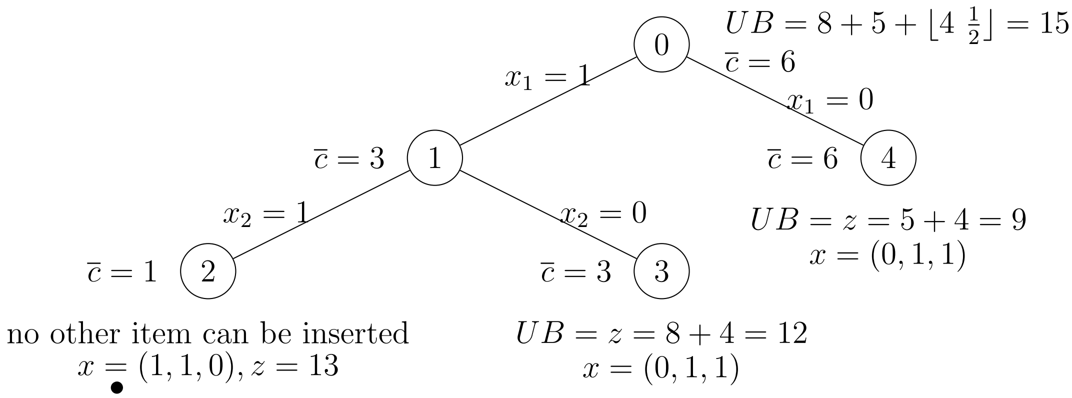
Hence the basis does not change for $-750/8 \leq \Delta b_2 \leq 200/3$

3. **BB-value=8/time=15**

ESSAY | 8 points | 0.10 penalty | editor

Given a knapsack of capacity $C = 6$, and 3 objects of weights w_i=(3,2,2) and profits p_i=(8,5,4), solve the corresponding knapsack problem with branch-and-bound. Number each subproblem in the order in which is visited in the enumeration tree and, for each of them, show the upper bound value, the residual capacity, and an explanation in case the subproblem is solved.

Notes for grader:



- The optimal solution is $x^* = (1, 1, 0)$ with value $z^* = 13$ found in node 2.

Total of marks: 28