RICERCA OPERATIVA

Written assessment, October 28, 2021

Last name, First name ____________________

Exercise 1 (value 13)
A mining company is deciding how to operate in a certain area for the next T years. In the area there are n mines, but at most $P < n$ can operate in the same year. Let U_{it} denote the upper limit on the tons of material that can be extracted from mine $i = 1, \dots, n$ in year $t = 1, \dots, T$. The revenue of one ton of material depends on the mine: Let g_i denote such revenue for mine i. To use a mine i in one year the company must pay royalties r_i to the mine owner. If a mine is not used in one year, but it will be used in a next year, the royalties must be paid although no material is extracted.
Write a linear MILP model that decides how to use the mines in the given period so as to maximize the profit of the company.

Exercise 2 (value 8)
Given the following LP problem.

$$
\begin{aligned}
\min \quad & 56x_1 - 60x_2 + 12x_3 \\
& 7x_1 + 2x_2 + 3x_3 \geq 3 \\
& 8x_1 + 5x_2 - 4x_3 \geq 1 \\
& x_1, x_3 \geq 0 \\
& x_2 \leq 0
\end{aligned}
$$

Write the corresponding dual problem and solve it with the simplex method implementing the Bland's rule. Obtain the optimal primal solution from the optimal dual one.

Exercise 3 (value 7)
Consider a 0-1 knapsack problem with a bin of capacity $c = 16$ and four items with profits $p_j = (21, 11, 16, 6)$ and weights $w_j = (7, 4, 6, 3)$. Solve it by applying the branch-and-bound method.

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Exercise 1

Constants

Variables

$x_{it} = 1$ if mine i is open in year t, 0 otherwise.
$u_{it} = 1$ if mine i is used in year t, 0 otherwise.
q_{it} = tons extracted from mine i in year t, 0 otherwise.

Model

$$\max z = \sum_{i=1}^{n} \sum_{t=1}^{T} (g_i q_{it} - r_i x_{it})$$

$$q_{it} \leq U_{it} u_{it} \qquad i = 1, \dots, n, t = 1, \dots, T$$

$$u_{it} \leq x_{it} \qquad i = 1, \dots, n, t = 1, \dots, T$$

$$\sum_{i=1}^{n} u_{it} \leq P \qquad t = 1, \dots, T$$

$$|T| x_{it} \geq \sum_{s=t+1}^{T} x_{is} \qquad i = 1, \dots, n, t = 1, \dots, T-1$$

alternatively...

$$x_{it} \geq x_{i,t+1} \qquad i = 1, \dots, n, t = 1, \dots, T-1$$

$$x_{it} \in \{0, 1\}, u_{it} \in \{0, 1\} \qquad i = 1, \dots, n, t = 1, \dots, T$$

$$q_{it} \geq 0 \qquad i = 1, \dots, n, t = 1, \dots, T$$

Exercise 2

$$
\begin{array}{llcll}
\min & 56x_1 - 60x_2 + 12x_3 & \qquad & \max & 3u_1 + u_2 \\
 & 7x_1 + 2x_2 + 3x_3 \geq 3 & & & 7u_1 + 8u_2 \leq 56 \\
 & 8x_1 + 5x_2 - 4x_3 \geq 1 & & & 2u_1 + 5u_2 \geq -60 \\
 & x_1, x_3 \geq 0 & & & 3u_1 - 4u_2 \leq 12 \\
 & x_2 \leq 0 & & & u_1, u_2 \geq 0
\end{array}
$$

u_1	u_2	u_3	u_4	u_5		
3	1	0	0	0	0	$-z$
7	8	1	0	0	56	u_3
-2	-5	0	1	0	60	u_4
3	-4	0	0	1	12	u_5

u_1	u_2	u_3	u_4	u_5		
0	5	0	0	-1	-12	$-z$
0	$\frac{52}{2}$	1	0	-$\frac{7}{3}$	28	u_3
0	- $\frac{23}{3}$	0	1	$\frac{2}{3}$	68	u_4
1	-$\frac{4}{3}$	0	0	$\frac{1}{4}$	4	u_5

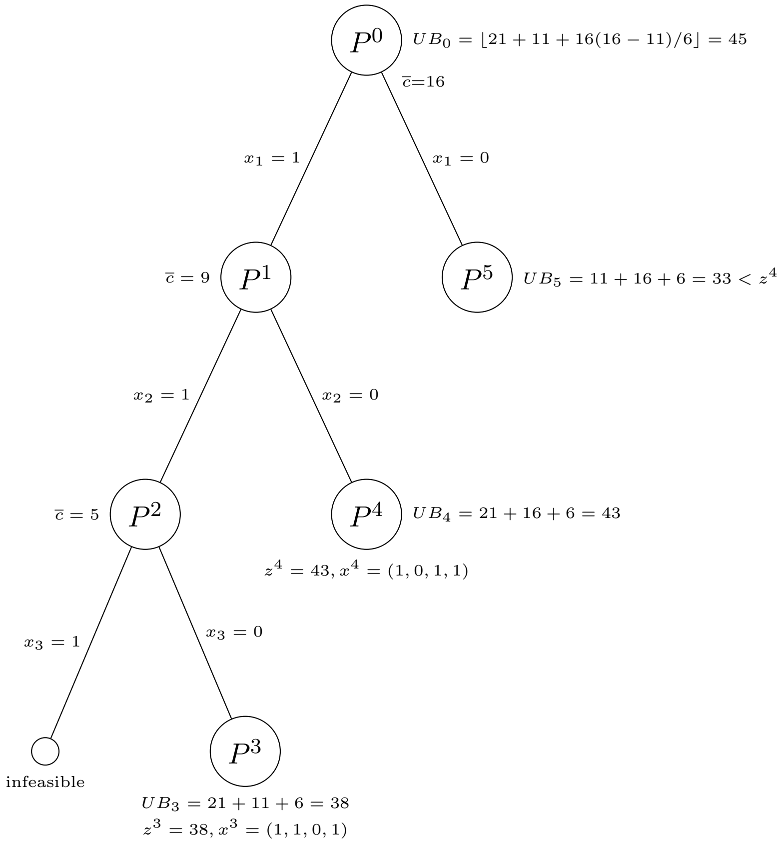
u_1	u_2	u_3	u_4	u_5		
0	0	- $\frac{15}{52}$	0	-$\frac{17}{52}$	-$\frac{261}{13}$	$-z$
0	1	$\frac{3}{52}$	0	-$\frac{7}{52}$	$\frac{21}{13}$	u_3
0	0	$\frac{23}{52}$	1	- $\frac{19}{52}$	$\frac{1045}{13}$	u_4
1	0	$\frac{1}{13}$	0	$\frac{2}{13}$	$\frac{80}{13}$	u_5

The dual solution is $u^* = (\frac{80}{13}, \frac{21}{13})$ with value $z_D^* = \frac{261}{13}$.

$$
\begin{cases}
(7x_1 + 2x_2 + 3x_3 - 3)u_1 = 0 \\
(8x_1 + 5x_2 - 4x_3 - 1)u_2 = 0 \\
(7u_1 + 8u_2 - 56)x_1 = 0 \\
(12u_1 + 5u_2 + 60)x_2 = 0 \\
(3u_1 - 4u_2 + 12)x_3 = 0
\end{cases}
\qquad
\begin{cases}
x_1 = \frac{15}{52} \\
x_2 = 0 \\
x_3 = \frac{17}{52}
\end{cases}
$$

$$x = (\tfrac{15}{52}, 0, \tfrac{17}{52}) \ z_P = \tfrac{261}{13}$$

Exercise 3



The optimal solution is $x^4 = (1, 0, 1, 1)$ with value $z^4 = 43$.