RICERCA OPERATIVA

Assignment

First and Last name ______________________________

Exercise 1 (value 13)
A logistic company is planning the loading of a cargo ship. One of the main issues when loading a ship, is to balance the weight, in order to avoid that the boat recline on a side. The company must deliver n items, each having a weight w_j ($j = 1, \ldots, n$). In a first phase the boat deck is divided into m zones and the loading of the items in the zones must guarantee that the difference of weight between the zone with maximum weight and the zone with the minimum weight does not exceed a threshold δ.
Once the assignment of the items to the zones has been defined, the packing of the items inside each zone is considered. Usually the total size of the items assigned to a zone exceed the area of the zone, so multiple layers have to be used. Each zone i is divided into a grid having $\hat{r}$ rectangles. Each rectangle in the grid allows to pack one item for each layer. At most three layers are allowed (i.e., three items can be packed one over the other in a rectangle). The set $F \subseteq \{1, \ldots, n\}$ denotes fragile items that cannot have other items packed over them.
The objective function asks to minimize the number of items packed at the third level.

Exercise 2 (value 8)
Consider the following PLC problem. Solve it with the simplex method, than write the dual problem and compute the corresponding optimal solution using the complementary slackness conditions.

$$
\begin{aligned}
\min 2x_1 + 4x_2 + 3x_3 & \\
2x_1 - 3x_2 + 2x_3 &\leq 6 \\
4x_1 + 2x_2 &\leq 10 \\
-2x_1 + x_3 &\geq 4 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}
$$

Exercise 3 (value 7)
Write a GLPK or XPRESS model corresponding to the following mathematical model

$$
\max\ z = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij} x_{ij} - \sum_{j=1}^{m} w_j y_j
$$

$$
\sum_{i=1}^{n} c_i x_{ij} \leq C y_j \quad j = 1, \ldots, m \tag{1}
$$

$$
\sum_{j=1}^{m} x_{ij} = 1, \quad i = 1, \ldots, m \tag{2}
$$

$$
\sum_{j \in F_i} x_{ij} = 0, \quad i = 1, \ldots, m \tag{3}
$$

$$
x_{ij} \in \{0, 1\} \quad i = 1, \ldots, n,\ j = 1, \ldots, m \tag{4}
$$

$$
y_j \in \{0, 1\} \quad j = 1, \ldots, m \tag{5}
$$

$n = 6, m = 3$ $F = \{\{1\}, \{2\}, \{1, 3\}, \{3\}, \{3\}, \{2, 3\}\}$,

$C = 50$, $c = (10, 22, 5, 14, 9, 11)$ $w = (20, 30, 15)$ $p := \begin{bmatrix} 5 & 2 & 3 \\ 6 & 2 & 4 \\ 2 & 5 & 1 \\ 2 & 3 & 1 \\ 4 & 2 & 9 \\ 3 & 8 & 4 \end{bmatrix}$

RICERCA OPERATIVA

Assignment **A**

First and Last name ______________________________

Exercise 1

$$
\begin{aligned}
x_{ij} &= \text{1 if item } j \text{ is loaded in zone } i\text{; 0 otherwise}\\
WM &= \text{maximum weight of a zone}\\
wm &= \text{minimum weight of a zone}\\
y_{ijrl} &= \text{1 if item } j \text{ is packed in rectangle } r \text{ of zone } i \text{ at layer } l\text{; 0 otherwise}
\end{aligned}
$$

$$\min \ z = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{r=1}^{\hat{r}} y_{ijr3}$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad j = 1,\ldots,n; \tag{6}$$

$$WM - \sum_{j=1}^{n} w_j x_{ij} \geq 0 \quad i = 1,\ldots,m; \tag{7}$$

$$wm - \sum_{j=1}^{n} w_j x_{ij} \leq 0 \quad i = 1,\ldots,m; \tag{8}$$

$$WM - wm \leq \delta; \tag{9}$$

$$\sum_{j=1}^{n} y_{ijrl} \leq 1 \quad i = 1,\ldots,m; r = 1,\ldots; l = 1,2,3; \tag{10}$$

$$y_{ijrl} \geq \sum_{\substack{h=1\\h\neq j}}^{n} y_{ihr(l+1)} \quad i = 1,\ldots,m; j = 1\ldots,n; r = 1,\ldots,\hat{r}; l = 1,2; \tag{11}$$

$$1 - y_{ijrl} \geq \sum_{\substack{h=1\\h\neq j}}^{n} y_{ihr(l+1)} \quad i = \ \ldots,m; j \in F; r = 1,\ldots,\hat{r}; l = 1,2; \tag{12}$$

$$\sum_{r=1}^{\hat{r}}\sum_{l=1}^{3} y_{ijrl} = x_{ij} \quad i = 1,\ldots,m; j = 1\ldots,n; \tag{13}$$

$$x_{ij} \in \{0,1\} \quad i = 1,\ldots,m; j = 1,\ldots,n \tag{14}$$

$$y_{ijrl} \in \{0,1\} \quad i = 1,\ldots,m; j = 1,\ldots,n; r = 1,\ldots,\hat{r}; l = 1,2,3; \tag{15}$$

$$WM, wm \geq 0. \tag{16}$$

Exercise 2

x_1	x_2	x_3	x_4	x_5	x_6		
2	4	3	0	0	0	0	$-z$
2	-3	2	1	0	0	6	x_4
4	2	0	0	1	0	10	x_5
-2	0	1	0	0	-1	4	x_6

x_1	x_2	x_3	x_4	x_5	x_6		
2	4	3	0	0	0	0	$-z$
2	-3	2	1	0	0	6	x_4
4	2	0	0	1	0	10	x_5
2	0	-1	0	0	1	-4	x_6

x_1	x_2	x_3	x_4	x_5	x_6		
8	4	0	0	0	3	-12	$-z$
6	-3	0	1	0	2	-2	x_4
4	2	0	0	1	0	10	x_5
-2	0	1	0	0	-1	4	x_3

x_1	x_2	x_3	x_4	x_5	x_6		
16	0	0	$\frac{4}{3}$	0	$\frac{17}{3}$	$-\frac{44}{3}$	$-z$
-2	1	0	$-\frac{1}{3}$	0	$-\frac{2}{3}$	$\frac{2}{3}$	x_2
8	0	0	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{26}{3}$	x_5
-2	0	1	0	0	-1	4	x_3

$x = (0, \frac{2}{3}, 4, 0, \frac{26}{3}, 0, 0),\ z_P = \frac{44}{3}$

$$\begin{aligned} \min\ & 2x_1 + 4x_2 + 3x_3 \\ 2x_1 - 3x_2 + 2x_3 &\leq 6 \\ 4x_1 + 2x_2 &\leq 10 \\ -2x_1 + x_3 &\geq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

$$\begin{aligned} \max\ & 6u_1 + 10u_2 + 4u_3 \\ 2u_1 + 4u_2 - 2u_3 &\leq 2 \\ -3u_1 + 2u_2 &\leq 4 \\ 2u_1 + u_3 &\leq 3 \\ u_1, u_2 &\leq 0 \\ u_3 &\geq 0 \end{aligned}$$

$$\left\{\begin{array}{lcl} (2x_1 - 3x_2 + 2x_3 - 6)u_1 & = & 0 \\ (4x_1 + 2x_2 - 10)u_2 & = & 0 \\ (-2x_1 + x_3 - 4)u_3 & = & 0 \\ (2u_1 + 4u_2 - 2u_3 - 2)x_1 & = & 0 \\ (-3u_1 + 2u_2 - 4)x_2 & = & 0 \\ (2u_1 + u_3 - 3)x_3 & = & 0 \end{array}\right. \quad \left\{\begin{array}{lcl} (0)u_1 & = & 0 \\ (-26/3)u_2 & = & 0 \\ (0)u_3 & = & 0 \\ (2u_1 + 4u_2 - 2u_3 - 2)0 & = & 0 \\ (-3u_1 + 2u_2 - 4)2/3 & = & 0 \\ (2u_1 + u_3 - 3)4 & = & 0 \end{array}\right. \quad \left\{\begin{array}{lcl} -- & & \\ u_2 & = & 0 \\ -- & & \\ -- & & \\ -3u_1 & = & 4 \\ 2u_1 + u_3 & = & 3 \end{array}\right.$$

$u = (-4/3, 0, 17/3),\ z_D = 44/3$

Exercise 3

```
/* Exercise 3, 2016 09 12 */

param n, integer, > 0;
param m, integer, > 0;
set I := 1..n;
set J := 1..m;
set F{i in I};

param p{i in I, j in J}, >= 0;
param c{i in I}, >= 0;
param w{j in J}, >= 0;
param C, >= 0;

var  x{i in I, j in J},  binary;
var  y{j in J},  binary;

maximize z: sum{i in I, j in J} p[i,j]*x[i,j]-sum{j in J} w[j]*y[j];

s.t.  cap{j in J}:  sum{i in I} c[i]*x[i,j] <=   C*y[j];
   one{i in I}: sum{j in J} x[i,j] = 1;
   forbidden{i in I}: sum{j in F[i]} x[i,j] = 0;
solve;

printf "\n";
for{i in I} {
   printf "\n%1d)",i;
   printf{j in J} "%5d ", x[i,j];
}
printf "\n\n-----z = %g\n\n",z;
printf "\n\n  ";

data;

param n := 6;
param m := 3;

param C := 50;
param c:= [1] 10 [2] 22 [3] 5 [4] 14 [5] 9 [6] 11;
param w:= [1] 20 [2] 30 [3] 15 ;
param p :  1  2 3  :=
1  5  2 3
2  6  2 4
3  2  5 1
4  2  3 1
5  4  2 9
6  3  8 4;
set F[1] := 1 ;
set F[2] := 2;
set F[3] := 1 3;
set F[4] := 3;
set F[5] := 3;
set F[6] := 2 3;
end;
```