RICERCA OPERATIVA

June 27,2016

A

First and Last name ______________________

Exercise 1

A large software house has a new building in which wants to allocate some hundreds of its employees. The building has three floors and each floor i has n_i rooms. A room j at floor i has a size of s_{ij} square meters. The employees have been divided into n small groups. Each group k consists of g_k persons. It is known that each person has to be allocated γ square meters. The first question that the software house wants to answer is if in the building it is possible to allocate all the n groups of employees. To do that it is necessary to write a linear mathematical model (without objective function) (value 5).

As a second step, they want to add an objective function aiming to maximize the square meters of the rooms that remain completely empty after the allocation of all employees (value 2).

For the next step the model has to be improved with the following requirement. A set S of groups of employees is defined and the requirement is to allocate all the groups of S in the same floor (value 2).

Another refinement identifies two sets of groups, namely set A and set B. If at least 50% of the groups of set A are allocated to the same floor, then no group of the set B can be allocated to this floor (value 4).

Exercise 2

Consider the following PLC problem and solve it using the primal simplex with two phases method and Bland rule.(value 4)

$$\begin{aligned}
\min \quad & 2x_1 + 3x_2 + x_3 + 4x_4 \\
& x_1 + x_2 - 2x_3 + 2x_4 \geq 1 \\
& 2x_1 + 3x_2 + 2x_3 \geq 3 \\
& x_1, \ldots, x_4 \geq 0
\end{aligned}$$

Write the dual problem, add the integrality constraint to the dual and solve it using the standard branch-and-bound method (solve the relaxed problems with a graphical method). For the first branch select variable u_1.(value 4)

Exercise 3 (value 6)

Write a GLPK or XPRESS model corresponding to the following mathematical model

$$\max\ z = \sum_{i=n_1}^{n-2} \sum_{j=1}^{m} a_{ij}(1 - x_{ij})$$

$$\sum_{i=n_1}^{n-2} p_j x_{ij} \geq b_j \quad j = 1, \ldots, m \tag{1}$$

$$\sum_{i=n_1+3}^{n} x_{ij} \leq K \quad j = 1, \ldots, m \tag{2}$$

$$\sum_{i=n_1+3}^{n} x_{ij} + \alpha y_j \leq r_j \quad j = 1, \ldots, m \tag{3}$$

$$x_{ij} \in \{0, 1\} \quad i = n_1, \ldots, n,\ j = 1, \ldots, m \tag{4}$$

$$y_j \in \{0, 1\} \quad j = 1, \ldots, m \tag{5}$$

$$\tag{6}$$

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Exercise 1

$$
\begin{aligned}
x_{ijk} &= 1 \text{ if group } k \text{ is allocated to floor } i\text{, room } j\text{; 0 otherwise} \\
y_{ij} &= 1 \text{ if room } j \text{ at floor } i \text{ is used; 0 otherwise} \\
z_i &= 1\text{if all the groups of } S \text{ are on the same floor; 0 otherwise} \\
\delta_i &= 1 \text{ if at least 50\% of groups of } A \text{ are allocated to floor } i\text{; 0 otherwise}
\end{aligned}
$$

$$
\max\ z = \sum_{i=1}^{3}\sum_{j=1}^{n_i} s_{ij}(1 - y_{ij})
$$

$$
\begin{aligned}
\sum_{i=1}^{3}\sum_{j=1}^{n_i} x_{ijk} &= 1 \quad k = 1,\dots,n && (1)\\
\sum_{k=1}^{n} \gamma g_k x_{ijk} &\le s_{ij} y_{ij} \quad i = 1,2,3; j = 1,\dots,n_i && (2)\\
\sum_{j=1}^{n_i}\sum_{k\in S} x_{ijk} &= |S| z_i \quad i = 1,2,3 && (3)\\
\sum_{j=1}^{n_i}\sum_{k\in A} x_{ijk} &\le \frac{1}{2}|A|(1+\delta_i) \quad i = 1,2,3 && (4)\\
\sum_{j=1}^{n_i}\sum_{k\in A} x_{ijk} &\ge \frac{1}{2}|A|\delta_i \quad i = 1,2,3 && (5)\\
\sum_{j=1}^{n_i}\sum_{k\in B} x_{ijk} &\le |B|(1-\delta_i) \quad i = 1,2,3 && (6)\\
x_{ijk} &\in \{0,1\} \quad i = 1,2,3; j = 1,\dots,n_i; k = 1,\dots,n && (7)\\
y_{ij} &\in \{0,1\} \quad i = 1,2,3; j = 1,\dots,n_i && (8)\\
\delta_i &\in \{0,1\} \quad i = 1,2,3 && (9)\\
& && (10)
\end{aligned}
$$

Exercise 2

$$\begin{array}{ll} \min & 2x_1 + 3x_2 + x_3 + 4x_4 \\ & x_1 + x_2 - 2x_3 + 2x_4 \geq 1 \\ & 2x_1 + 3x_2 + 2x_3 \geq 3 \\ & x_1, \dots, x_4 \geq 0 \end{array}$$

$$\begin{array}{lll} \max & u_1 + 3u_2 & \\ & u_1 + 2u_2 & \leq 2 \\ & u_1 + 3u_2 & \leq 3 \\ & -2u_1 + 2u_2 & \leq 1 \\ & 2u_1 & \leq 4 \\ & u_1, u_2 & \geq 0 \end{array}$$

PHASE I

x_1	x_2	x_3	x_4	x_5	x_6	x_1^a	x_2^a		
-3	-4	0	-2	1	1	0	0	-4	$-\xi$
1	1	-2	2	-1	0	1	0	1	x_1^a
2	3	2	0	0	-1	0	1	3	x_2^a

x_1	x_2	x_3	x_4	x_5	x_6	x_1^a	x_2^a		
0	-1	-6	4	-2	1	3	0	-1	$-\xi$
1	1	-2	2	-1	0	1	0	1	x_1
0	1	6	-4	2	-1	-2	1	1	x_2^a

x_1	x_2	x_3	x_4	x_5	x_6	x_1^a	x_2^a		
1	0	-8	6	-3	1	4	0	0	$-\xi$
1	1	-2	2	-1	0	1	0	1	x_2
-1	0	8	-6	3	-1	-3	1	0	x_2^a

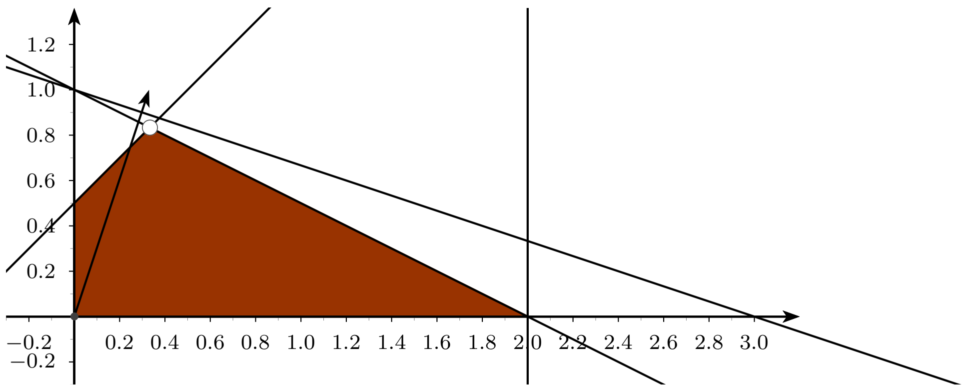
x_1	x_2	x_3	x_4	x_5	x_6	x_1^a	x_2^a		
0	0	0	0	0	0	1	1	0	$-\xi$
$\frac{3}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	1	x_2
$-\frac{1}{8}$	0	1	$-\frac{3}{4}$	$\frac{3}{8}$	$-\frac{1}{8}$	$-\frac{3}{8}$	$\frac{1}{8}$	0	x_3

PHASE II

x_1	x_2	x_3	x_4	x_5	x_6		
$-\frac{1}{8}$	0	0	$\frac{13}{4}$	$\frac{3}{8}$	$\frac{7}{8}$	-3	$-z$
$\frac{3}{4}$	1	0	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	1	x_2
$-\frac{1}{8}$	0	1	$-\frac{3}{4}$	$\frac{3}{8}$	$-\frac{1}{8}$	0	x_3

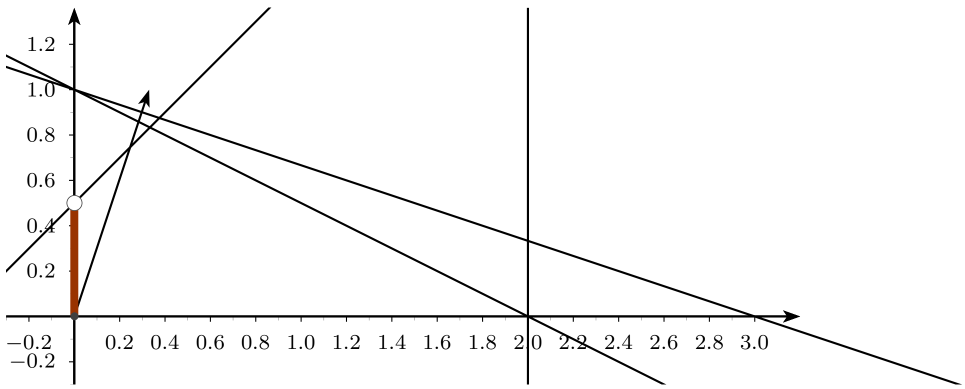
x_1	x_2	x_3	x_4	x_5	x_6		
0	$\frac{1}{6}$	0	$\frac{10}{3}$	$\frac{1}{3}$	$\frac{5}{6}$	$-\frac{17}{6}$	$-z$
1	$\frac{4}{3}$	0	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{4}{3}$	x_1
0	$\frac{1}{6}$	1	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{6}$	x_3

Problem P^0



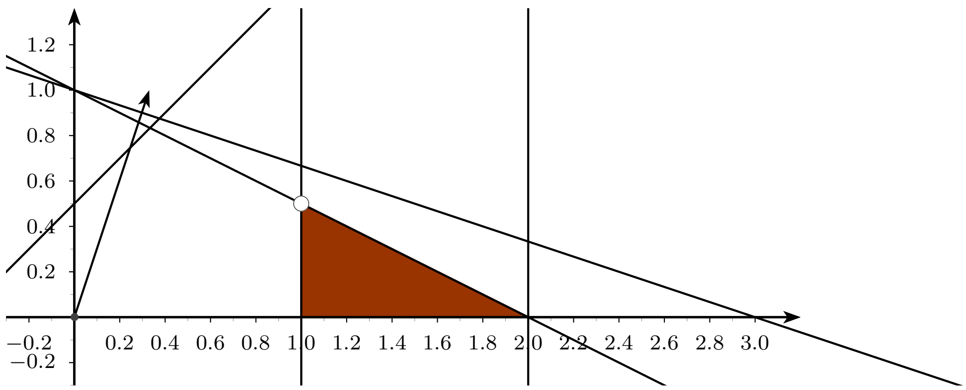
Optimal solution of the relaxed dual problem $u = (1/3, 5/6)$ $z_D = 17/6$

Problem $P^1 : u_1 \leq 0$



Optimal solution $u = (0, 1/2)$ $z_D = 3/2, UB = 1$

Problem $P^2 : u_1 \geq 1$



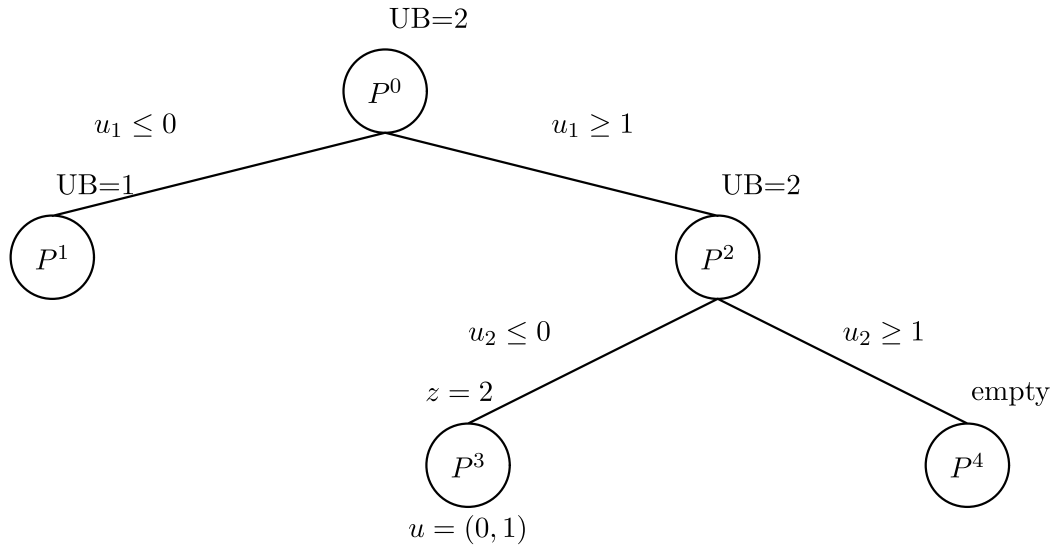
Optimal solution $u = (1, 1/2)$ $z_D = 5/2, UB = 2$

Problem $P^3 : u_2 \leq 0$

Optimal solution $u = (1, 0)$ $z_D = 2$ ** optimal

Problem $P^4 : u_2 \geq 1$

Un-feasible


UB=2
P^0
$u_1 \leq 0$
$u_1 \geq 1$
UB=1
P^1
UB=2
P^2
$u_2 \leq 0$
$u_2 \geq 1$
$z = 2$
empty
P^3
P^4
$u = (0, 1)$

Exercise 3

```
/* Exercise 3, 2016 06 27 */

param n, integer, > 0;
param n1, integer, > 0;
param m, integer, > 0;
set Ifirst := n1..n-2;
set Isecond := n1+3..n;
set Iall := n1..n;
set J := 1..m;

param a{i in Ifirst, j in J}, >= 0;
param p{j in J}, >= 0;
param b{j in J}, >= 0;
param K, >= 0;
param alfa, >= 0;
param r{j in J}, >= 0;

var  x{i in Iall, j in J},  binary;
var  y{j in J},  binary;

maximize z: sum{i in Ifirst, j in J} a[i,j]*(1-x[i,j]);

s.t.  all{j in J}:  sum{i in Ifirst} p[j]*x[i,j] >=   b[j];
  Kappa{j in J}: sum{i in Isecond} x[i,j] <= K;
  alpha{j in J}: sum{i in Isecond} x[i,j] + alfa*y[j] <= r[j];
solve;

printf "\n";
for{i in Iall} {
   printf "\n%1d)",i;
   printf{j in J} "%5d ", x[i,j];
}
printf "\n\n-----z = %g\n\n",z;
printf "\n\n  ";
printf{j in J} "%5d ", y[j];

data;

param n := 8;
param n1 := 2;
param m := 3;
param a :  1  2 3  :=
2  6  2 4
3  2  5 1
4  2  3 1
5  4  2 9
6  3  8 4;
param p := [1] 20 [2] 12 [3] 15;
param b := [1] 4 [2] 5 [3] 8;
param K := 11;
param alfa := 3;
param r := [1] 11 [2] 15 [3] 25;

end;
```