RICERCA OPERATIVA

Written assessment, June 29, 2018

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First and Last name ______________________________

Exercise 1 (value 13)
A big tiles factory is reorganizing its production. The factory has n production lines which produce raw tiles with a basic uniform color. The raw tiles can go directly to one of the m ovens for the final cooking phase, or can go to a single painting department for artistic decoration, before going to one of the ovens. The orders of the next period consists of b batches of tiles. Each batch $j = 1, \ldots, b$, if produced, must be entirely processed on one production line and on one oven, and gives a revenue of r_j euros. The quality of a batch is either *normal* or *special*. Normal batches require only the production of the raw tiles and the cooking. The special batches must be also decorated. To produce a batch j are required: tp_j minutes for the production of the raw tile, td_j minutes for the decoration (only for the special ones), and tc_j minutes for cooking. In the period considered for this plan, the production lines work for TP minutes each, the painting department for TD minutes, and each oven for TC minutes.

Write a linear model to help the company to allocate the batches to the resources in order to maximise the total revenue.

Modify the above model adding the following constraint. Let $A \subset \{1, \ldots, b\}$, $B \subset \{1, \ldots, b\}$ such that $A \cap B = \emptyset$. For technological reasons if a batch of set A is produced on a line k, than no batch of set B can be produced on the same line.

Exercise 2 (value 8)
Consider the following PLC problem. Solve it with the dual simplex method, using the Bland's rule, than write the dual problem and compute the corresponding solution.

$$\begin{aligned} \min \quad & 2x_1 + 3x_2 + x_3 \\ & -x_1 + 3x_2 - 2x_3 \geq 8 \\ & x_2 - x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Write the dual problem using only non-negative variables. Draw the feasible region and identify on the region the vertices corresponding to the solutions obtained in each iteration of the simplex (remind that the primal and the dual solutions are related by the slackness conditions).

Exercise 3 (value 7)
Write a GLPK or XPRESS model corresponding to the following mathematical model

$$\min \ z = \sum_{i=1}^{n} \sum_{j \in F} c_{ij} x_{ij} - \sum_{i=1}^{n} \sum_{j \in F} r_j f_{ij} + (1 - \theta)$$

$$f_{ij} \leq x_{ij} Q \quad i = 1, \ldots, n, \ j \in F \tag{1}$$

$$\sum_{i=1}^{n} f_{ij} \geq q_j \quad j \in F \tag{2}$$

$$\sum_{j \in F} f_{ij} \leq C_i \quad i = 1, \ldots, n \tag{3}$$

$$x_{1j} + x_{2j} \leq \theta \quad j \in F \tag{4}$$

$$x_{ij} \in \{0, 1\} \quad i = 1, \ldots, n, \ j \in F \tag{5}$$

$$f_{ij} \geq 0 \quad i = 1, \ldots, n, \ j \in F \tag{6}$$

$$\theta \in \{0, 1\} \tag{7}$$

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Exercise 1

Variables

$x_{jk} = 1$ if batch j is assigned to line k for raw tiles production, 0 otherwise.
$y_{jh} = 1$ if batch j is assigned to oven h for cooking, 0 otherwise.
$z_j = 1$ if batch j is produced, 0 otherwise.

Data

$c_j = 1$ if batch j is *special*, 0 otherwise.

Model

$$\max z = \sum_{j=1}^{b} r_j z_j$$

$$\sum_{k=1}^{n} x_{jk} \le z_j \quad j = 1, \ldots, b$$

$$\sum_{k=1}^{m} y_{jh} \le z_j \quad j = 1, \ldots, b$$

$$\sum_{j=1}^{b} tp_j x_{jk} \le TP \quad k = 1, \ldots, n$$

$$\sum_{j=1}^{b} tc_j y_{jh} \le TC \quad h = 1, \ldots, m$$

$$\sum_{j=1}^{b} td_j c_j z_j \le TD$$

$$x_{jk} \in \{0, 1\} \quad j = 1, \ldots, b, k = 1, \ldots, n$$

$$y_{jh} \in \{0, 1\} \quad j = 1, \ldots, b, h = 1, \ldots, m$$

$$z_j \in \{0, 1\} \quad j = 1, \ldots, b$$

$$\sum_{j \in B} x_{jk} \le b(1 - x_{lk}) \quad k = 1, \ldots, n, l \in A$$

Exercise 2

$$\begin{array}{ll} \min & 2x_1 + 3x_2 + x_3 \\ & -x_1 + 3x_2 - 2x_3 \geq 8 \\ & x_2 - x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

x_1	x_2	x_3	x_4	x_5		
2	3	1	0	0	0	$-z$
1	-3	2	1	0	-8	x_4
0	1	-1	0	1	2	x_5

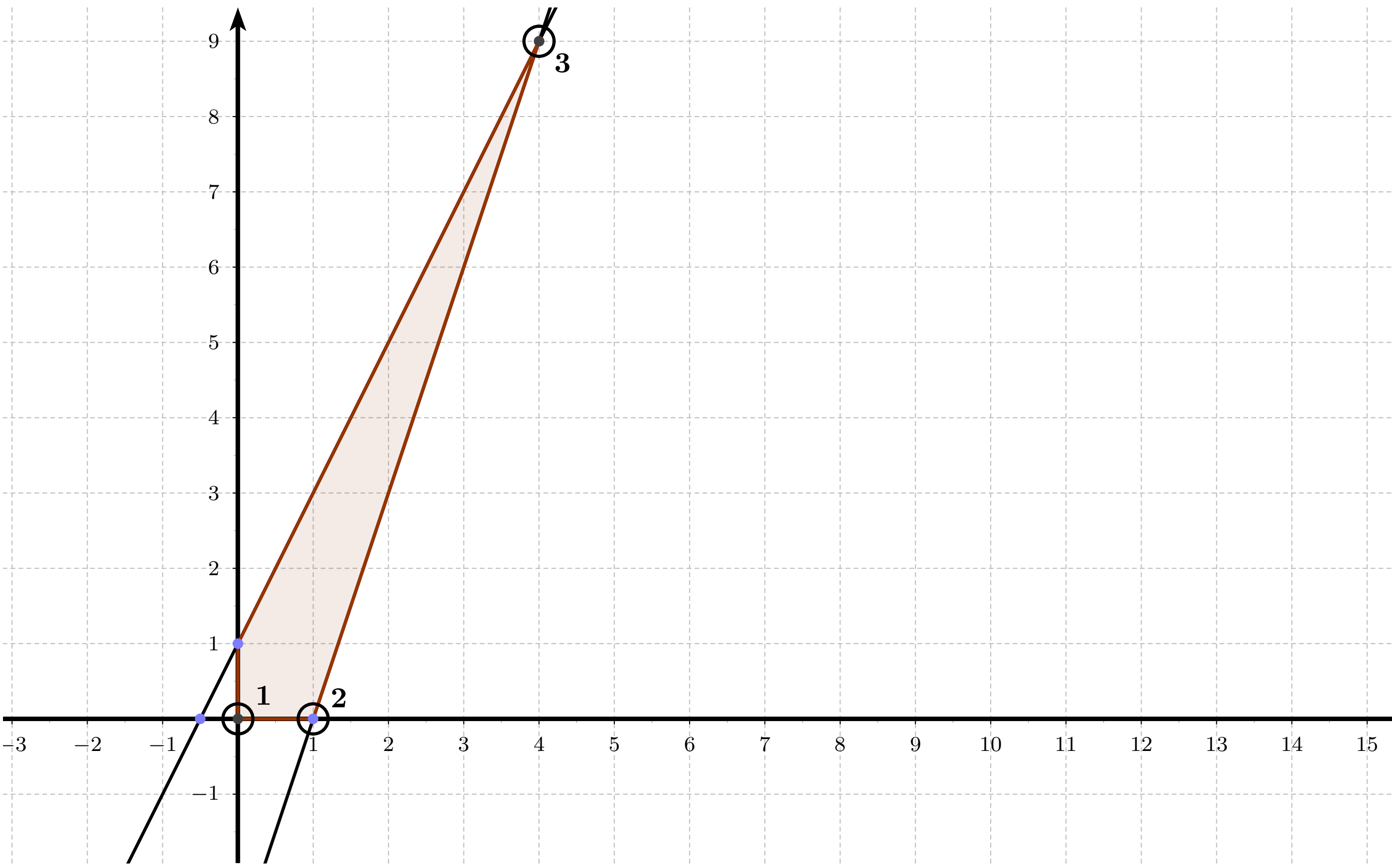
x_1	x_2	x_3	x_4	x_5		
3	0	3	1	0	-8	$-z$
$-\frac{1}{3}$	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{8}{3}$	x_2
$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	1	$-\frac{2}{3}$	x_5

x_1	x_2	x_3	x_4	x_5		
6	0	0	4	9	-14	$-z$
-1	1	0	-1	-2	4	x_2
-1	0	1	-1	-3	2	x_3

$x = (0, 4, 2, 0, 0),\ z_P = 14$

$$\begin{array}{ll} \min & 2x_1 + 3x_2 + x_3 \\ & -x_1 + 3x_2 - 2x_3 \geq 8 \\ & x_2 - x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

$$\begin{array}{rcl} \max 8u_1 - 2u_2 & & \\ -u_1 & \leq & 2 \\ 3u_1 - u_2 & \leq & 3 \\ -2u_1 + u_2 & \leq & 1 \\ u_1, u_2 & \geq & 0 \end{array}$$



$$\begin{cases} (-x_1 + 3x_2 - 2x_3 - 8)u_1 = 0 \\ (-x_2 + x_3 + 2)u_2 = 0 \\ (-u_1 - 2)x_1 = 0 \\ (3u_1 - u_2 - 3)x_2 = 0 \\ (-2u_1 + u_2 - 1)x_3 = 0 \end{cases}$$

Tableau 1: $x = (0, 0, 0, -8, 2),\ u = (0, 0)$
Tableau 2: $x = (0, 8/3, 0, 0, -2/3),\ u = (1, 0)$
Tableau 3: $x = (0, 4, 2, 0, 0),\ u = (4, 9)$

Exercise 3

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/* Exercise 3, 2016 09 12 */

param n, integer, > 0;
set I := 1..n;
set F;

param c{i in I, j in F}, >= 0;
param r{j in F}, >= 0;
param Q, >= 0;
param q{j in F}, >= 0;
param C{i in I}, >= 0;

var  x{i in I, j in J},  binary;
var  f{i in I, j in J},  >=0;
var  theta,  binary;

minimize z: sum{i in I, j in F} c[i,j]*x[i,j] - sum{i in I, j in F} r[j]*f[i,j]
        + (1-theta);

s.t.  one{i in I, j in F}:  f[i,j] <=   x[i,j]*Q;
   two{j in F}: sum{i in I} f[i,j] >= q[j];
   three{i in I}: sum{j in F} f[i,j] <= C[i];
        four{j in F}: x[1,j] + x[2,j] <= theta;
solve;

printf "\n";
for{i in I} {
   printf "\n%1d)",i;
   printf{j in F} "%5d ", x[i,j];
}
printf "\n";
for{i in I} {
   printf "\n%1d)",i;
   printf{j in F} "%5d ", f[i,j];
}
printf "%5d ", theta;

printf "\n\n-----z = %g\n\n",z;
printf "\n\n ";

data;
param n := 5;
set F := 1 2 3 4 5 6;
param c :  1  2 3 4 5 6 :=
1  5  2 3 3 2 4
2  6  2 4 1 1 1
3  2  5 1 2 3 1
4  2  3 1 4 5 1
5  4  2 9 2 1 2;
param r:= [1] 2 [2] 3 [3] 1 [4] 3 [5] 2 [6] 1;
param Q := 20;
param q:= [1] 10 [2] 12 [3] 15 [4] 11 [5] 3 [6] 8;
param C:= [1] 10 [2] 10 [3] 20 [4] 30 [5] 15;

end;
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