RICERCA OPERATIVA

Written assessment, January 11, 2023

Last name, First name ______________________________

Exercise 1 (value 9)

A telecommunication company needs to install antennae for the new 6G technology. A set L of *locations* where antennae can be installed (maximum one antenna per location) has been identified. A set A of *areas* that the company aims to cover with the signal is also given. The signal intensity s_{ij}, namely the strength of the signal from an antenna installed in location $i \in L$ received in area $j \in A$, is known for each pair (i, j). An area j is classified as *covered* only when the sum of the strength of the signals received is at least T. Furthermore, due to interference issues, an area $j \in A$ is declared as *non-covered* if there is more than one signal with level greater than or equal to U (i.e., $s_{ij} \geq U$) in j. Finally, for technical reasons, an antenna can be placed in location α only if an antenna is installed in location β. Write an integer linear programming model to help the company to decide where to install antennae in order to maximize the number of *covered* areas.

Exercise 2 (value 9)

Solve the following LP problem with a simplex method of your choice:

$$
\begin{array}{ll}
\min & 4x_1 + x_2 + x_3 \\
 & x_1 + 4x_2 - 10x_3 \leq 4 \\
 & -x_1 - 2x_2 + 5x_3 = 5 \\
 & x_1, x_2, x_3 \geq 0.
\end{array}
$$

State the method you choose, show all the steps, and write the optimal solution together with its cost.

Exercise 3 (value 9)

Consider the graph $G = (V, A)$ described by the following adjacency matrix with costs:

	1	2	3	4	5	6
1	·	2	3	5	·	8
2	5	·	·	2	7	·
3	5	−2	·	·	·	·
4	5	·	·	·	·	1
5	6	3	4	·	·	1
6	1	12	14	·	1	·

Find the shortest path from node 1 to node 5 by using the Bellman-Ford algorithm. Show the procedure (show the tables of labels and predecessors) and write the optimal path with the associated cost.

Written assessment, January 11, 2023

Solution sketch

Exercise 1

$x_i = 1$ if an antenna is installed in location $i \in L$; 0 otherwise
$y_i = 1$ if the area $j \in A$ is covered; 0 otherwise

$$
\begin{array}{llll}
\max & \sum_{j \in A} y_j & & \\
& \sum_{i \in L} s_{ij} x_i & \geq T y_j & j \in A \\
& \sum_{i \in L : s_{ij} \geq U} x_i & \leq 1 + |L|(1 - y_j) & j \in A \\
& x_\alpha & \leq x_\beta & \\
& x_i & \in \{0,1\} & i \in L \\
& y_j & \in \{0,1\} & j \in A
\end{array}
$$

Exercise 2

Dual simplex method:

$$
\begin{array}{ll}
\min & 4x_1 + x_2 + x_3 \\
& x_1 + 4x_2 - 10x_3 + y_1 = 4 \\
& -x_1 - 2x_2 + 5x_3 + y_2 = 5 \\
& x_1 + 2x_2 - 5x_3 + y_3 = -5 \\
& x_1, x_2, x_3, y_1, y_2, y_3 \geq 0.
\end{array}
$$

x_1	x_2	x_3	y_1	y_2	y_3	$-z$	
4	1	1	0	0	0	0	
1	4	-10	1	0	0	4	y_1
-1	-2	5	0	1	0	5	y_2
1	2	$\mathbf{-5}$	0	0	1	-5	y_3

x_1	x_2	x_3	y_1	y_2	y_3	$-z$	
$21/5$	$7/5$	0	0	0	$1/5$	-1	
-1	0	0	1	0	-2	14	y_1
0	0	0	0	1	1	0	y_2
$-1/5$	$-2/5$	1	0	0	$-1/5$	1	x_3

Alternatively, two phases method:

x_1	x_2	x_3	y_1	a_1	$-z$	
0	0	0	0	1	0	
1	4	-10	1	0	4	y_1
-1	-2	5	0	1	5	a_1

x_1	x_2	x_3	y_1	a_1	$-z$	
1	2	-5	0	0	-5	
1	4	-10	1	0	4	y_1
-1	-2	$\mathbf{5}$	0	1	5	a_1

x_1	x_2	x_3	y_1	a_1	$-z$	
0	0	0	0	1	0	
-1	0	0	1	2	14	y_1
$-1/5$	$-2/5$	1	0	$1/5$	1	x_3

x_1	x_2	x_3	y_1	z	
$1/5$	$2/5$	0	0	-1	
-1	0	0	1	14	y_1
$-1/5$	$-2/5$	1	0	1	x_3

The optimal solution is $x = \{0, 0, 1\}$ with cost 1.

Exercise 3

	$f^k(j)$						$Pred_j$					
k	1	2	3	4	5	6	1	2	3	4	5	6
0	0	–	–	–	–	–	1	–	–	–	–	–
1	0	2	3	5	–	8	1	1	1	1	–	1
2	0	1	3	4	9	6	1	3	1	2	2	4
3	0	1	3	3	7	5	1	3	1	2	6	4
4	0	1	3	3	6	4	1	3	1	2	6	4
5	0	1	3	3	5	4	1	3	1	2	6	4

The optimal path is $\{1, 3, 2, 4, 6, 5\}$ and has cost 5.