20210630

1. **MM-value=11/time=20**

ESSAY | 11 points | 0.10 penalty | editor

The public authority of a rural province wants to optimize the water supply to a set of cultivation fields F. Each field $v \in F$ requires q_v^t m^3 at period $t \in T$, being T the time horizon. The water can be collected from a set of basins B. Each basin $w \in B$ has h_w m^3 available in total for the entire time horizon. Each $v \in B$ is connected to each $w \in F$ by a pipe for the distribution of water. Each pipe from v to w has a cost l_{vw} per m^3 transiting in it and a maximum capacity of p_{vw} for each period $t \in T$.

Write a mixed integer programming model to obtain a feasible water flow for each period while respecting the constraints. No water pressure, speed, or turbulence is considered.

Include the following constraint into the model: if basin 1 is used in period τ, then basin 2 cannot be used in *either* period $\tau + 1$ *or* $\tau + 2$ (i.e., basin 2 can be used at most in one of the two periods).

Notes for grader:

- Solution:
  - f_{vw}^t = m^3 of water carried on arc $(v, w) \in A$ in period $t \in T$;
  - $\gamma = 1$ if basin 2 is not used in period $\tau + 1$; 0 otherwise;
  - $\delta = 1$ if basin 2 is not used in period $\tau + 2$; 0 otherwise;
  - M is an arbitrary large constant.

$$
\begin{aligned}
\min z = \sum_{(v,w)\in A} l_{vw} \sum_{t\in T} f^t_{vw}& \\
\sum_{t\in T}\sum_{w\in F} f^t_{vw} \le h_v \quad & v \in B \\
\sum_{v\in B} f^t_{vw} \ge q^t_w \quad & w \in F, t \in T \\
f^t_{vw} \le p_{vw} \quad & w \in B, w \in F, t \in T \\
\sum_{w\in F} f^\tau_{1w} \le M(\gamma+\delta)& \\
\sum_{w\in F} f^{\tau+1}_{2w} \le M(1-\gamma)& \\
\sum_{w\in F} f^{\tau+2}_{2w} \le M(1-\delta)& \\
f^t_{vw} \ge 0 \quad & v \in B, w \in F, t \in T \\
\delta \in \{0,1\}&
\end{aligned}
$$

2. **PLC-value=9/time=15**

ESSAY 9 points 0.10 penalty editor

Consider the following LP problem:

$$
\begin{aligned}
\min \quad & 2x_1 - 3x_2 + 11x_3 \\
& 4x_1 - 12x_2 + 2x_3 \ge 3 \\
& -x_1 + 3x_2 + 9x_3 \ge 8 \\
& x_1, x_2 \ge 0.
\end{aligned}
$$

Write the corresponding dual problem and the complementary slackness. Given the dual solution $\overline{u} = (10/19, 21/19)$, verify if it is the optimal solution by using the complementary slackness. Explain your rationale.

Notes for grader:

- Solution:

$$
\begin{aligned}
\min \quad & 3u_1 + 8u_2 \\
& 4u_1 - u_2 \leq 2 \\
& -12u_1 + 3u_2 \leq -3 \\
& 2u_1 + 9u_2 = 11 \\
& u_1, u_2 \geq 0.
\end{aligned}
$$

$$
\begin{aligned}
&(4x_1 - 12x_2 + 2x_3 - 3)u_1 = 0 \\
&(-x_1 + 3x_2 + 9x_3 - 8)u_2 = 0 \\
&(4u_1 - u_2 - 2)x_1 = 0 \\
&(-12u_1 + 3u_2 + 3)x_2 = 0 \\
&(2u_1 + 9u_2 - 11)x_3 = 0
\end{aligned}
$$

$$
\begin{aligned}
u_1 &= 10/19 \\
u_2 &= 21/19 \\
x_1 &= 0 \\
x_2 &= -11/114 \\
x_3 &= 35/38
\end{aligned}
$$

The dual solution is not the optimal solution because one of the corresponding primal variables does not respect its non negativity constraint for variable x_2.

3. **DP-value=8/time=15 (FRO 6CFU)**

ESSAY | 8 points | 0.10 penalty | editor

Given a knapsack of capacity $c = 7$, and 3 objects of weights w_i=(2,3,4) and profits p_i=(2,3,3), solve the corresponding knapsack problem with dynamic programming using the profits as phases. We also know that an upper bound for the maximum profit is 6 (this information can be used to reduce the number of stages considered).

Notes for grader:

- Solution:

	0	1	2	3	4	5	6
f_0	0	M	M	M	M	M	M
f_1	0	M	2	M	M	M	M
f_2	0	M	2	3	M	5	M
f_3	0	M	2	3	M	5	7

	0	1	2	3	4	5	6
J_0	E	E	E	E	E	E	E
J_1	E	E	$\{1\}$	E	E	E	E
J_2	E	E	$\{1\}$	$\{2\}$	E	$\{1,2\}$	E
J_3	E	E	$\{1\}$	$\{2\}$	E	$\{1,2\}$	$\{2,3\}$

The optimal solution $x = \{2, 3\}$ has value 6.

Total of marks: 28