## 20200716/Exercises

1. **MM01-value=8**

   A company that distributes medical products wants to open a warehouse in one or more of $L$ possible locations, to serve the set K of customers. Let $d_{ik}$ represent the distance from location $i$ to customer $k \in K$.
   We want: a) to decide which warehouse(s) to open, and b) to assign each customer to exactly one warehouse.
   Write a MILP problem to minimize the maximum distance from a customer to the assigned warehouse.

   Clearly define the variables used.

   Notes: (not included in XML)

   - $$y_{ik} = 1 \text{ if the customer } k \in K \text{ is assigned to warehouse } i \in L; 0 \text{ otherwise}$$
     $$z_i = 1 \text{ if warehouse in location } i \in L \text{ is open}; 0 \text{ otherwise}$$
     $$\alpha \text{ maximum distance.}$$

   $$\begin{aligned}
   &\min \alpha \\
   &\alpha \geq d_{ik} y_{ik} \quad i \in L; k \in K \\
   &\sum_{k \in K} y_{ik} \leq |K| z_i \quad i \in L \\
   &\sum_{i \in L} y_{ik} = 1 \quad k \in K \\
   &y_{ik} \in \{0, 1\} \quad i \in L; k \in K \\
   &z_i \in \{0, 1\} \quad i \in L \\
   &\alpha \geq 0
   \end{aligned}$$

2. **MM02-value=5**

Write the dual problem of the following LP problem.

$$
\begin{aligned}
\max \quad & 4x_1 + 5x_2 + 3x_3 \\
& 3x_1 + 5x_2 + 4x_3 \geq 1 \\
& 2x_1 - 3x_2 + x_3 \leq 3 \\
& x_1, x_3 \geq 0 \\
& x_2 \text{ free}
\end{aligned}
$$

Notes: (not included in XML)

- $$
\begin{aligned}
\min \quad & -u_1 + 3u_2 \\
& -3u_1 + 2u_2 \geq 4 \\
& -5u_1 - 3u_2 = 5 \\
& -4u_1 + u_2 \geq 3 \\
& u_1, u_2 \geq 0
\end{aligned}
$$

3. **MM03-value=5**

Consider the following maximization LP in tableau form:

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $-z$ |
|---|---|---|---|---|
| 2 | -3 | 0 | 0 | -3 |
| 5 | -2 | 0 | 1 | 4 |
| 3 | -1 | 1 | 0 | 2 |

- Write the current basic solution. Is it optimal? Motivate the answer. If not, apply one iteration of the simplex method and write the resulting solution. Is it optimal ?

Notes: (not included in XML)

- Initial solution: $z = 3$; $(x_1 = 0, x_2 = 0, x_3 = 2, x_4 = 4)$. It is not optimal because there is still a positive reduced cost.
Tableau after one iteration:

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $-z$ |
|---|---|---|---|---|
| 0 | -7/3 | -2/3 | 0 | -13/3 |
| 0 | -1/3 | -5/3 | 1 | 2/3 |
| 1 | -1/3 | 1/3 | 0 | 2/3 |

Final solution: $z = 13/3$; $(x_1 = 2/3, x_2 = 0, x_3 = 0, x_4 = 2/3)$. Optimal.

4. **PL02-value=5**

   Consider this following tableau corresponding to a minimization problem, and perform a single pivot. Report: a) the resulting tableau; b) the corresponding solution and value; c) discuss the optimality (or non optimality) of the solution.

| | | | | | |
|---|---|---|---|---|---|
| 3 | 0 | 2 | 0 | 1 | 12 |
| 1 | 1 | 1 | 0 | 3 | 4 |
| 2 | 0 | 2 | $-1$ | 0 | 2 |

   Notes: (not included in XML)

   -
| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 10 |
| 0 | 1 | 0 | 1/2 | 3 | 3 |
| 1 | 0 | 1 | $-1/2$ | 0 | 1 |

     $x = (0, 3, 1, 0, 0)$ $z = -10$
     The solution is optimal since we have a feasible solution and non negative reduced costs

5. **GLPK01-value=5**

   Write the following minimization problem in GLPK or XPRESS Mosel.

$$\min\ z = \sum_{i \in A} x_i + \sum_{i \in A} \sum_{j \in B} y_{ij}$$
$$\sum_{j \in B \setminus \{0\}} y_{ij} \leq x_i W \quad i \in A$$
$$\sum_{i \in A} x_i \geq L$$
$$x_i \in \{0, 1\}$$
$$y_{ij} \in \{0, 1\}.$$

Notes: (not included in XML)

- ```
  param L integer > 0;
  set A;
  set B;
  var x { i in A } >= 0, binary ;
  var y { i in A, j in B } >= 0, binary ;
  minimize z :  sum {i in A} x[i]+ sum {i in A, j in B} y[i,j];
  C1{i in A} :  sum {j in B : j<>0} y[i,j] <= x[i]W;
  C2 :  sum{i in A} x[i] >= L;
  solve ;
  end;
  ```
  ```