RICERCA OPERATIVA

July 19, 2016 **First and Last name** A

Exercise 1 (value 13)
The project manager of a company has to plan the activities for the next T weeks. She has a list of n activities to be executed. Each activity $i = 1, \dots, n$ has a starting week $s(i)$, an ending week $e(i)$ and can be executed by using different personnel and equipments. There are m different modes for executing each activity. The project manager must select exactly one mode for each activity. If we use the mode $j = 1, \dots, m$ we have an overall cost c_{ij}, we must use q_{ij} units of personnel. There are r resources available to perform an activity (equipments of different kinds). Mode j of activity i uses r_{ij} units of each resource ($r_{ij} = 0$ when the resource is not needed). The company has Q units of personnel. The total budget for all the activities is B. Each resource $k = \dots, r$ is available in R_k units for week. For each activity i there is a preferred mode $p(i)$ to execute it.

Write a linear program to help the project manager to find the solution that satisfies all constraints while maximizing the number of activities executed in the preferred mode (value 10). Modify the above model adding the following constraint: if activity a is executed in mode $m(a)$, then activities b and c must be executed in mode $m(b)$ and $m(c)$, respectively (value 3).

Exercise 2 (value 7)
Consider the following PLI problem. Solve the continuous relaxation using the simplex method with the Bland rule. (value 4)

$$
\begin{aligned}
\min \quad & -2x_1 - 3x_2 + x_3 \\
& 2x_1 + 3x_2 + 2x_3 \le 12 \\
& 4x_1 + 2x_2 \le 14 \\
& x_1, \dots, x_3 \ge 0 \text{ integer}
\end{aligned}
$$

Determine a Gomory cut, using the first possible row (the highest in the tableau), add the cut to the model and compute the new continuous solution (value 3)

Exercise 3 (value 5)
Write a GLPK or XPRESS model corresponding to the following mathematical model

$$
\max\ z = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{ij} x_{ij}
$$

$$
\sum_{i=1}^{n} q_i x_{ij} \le c \quad j = 1, \dots, m \tag{1}
$$

$$
\sum_{i \in S} \sum_{j=1}^{m} x_{ij} \le nm\, y_S \tag{2}
$$

$$
\sum_{i \notin S} \sum_{j=1}^{m} x_{ij} \le nm\, y_S \tag{3}
$$

$$
x_{ij} \in \{0, 1\} \quad i = 1, \dots, n,\ j = 1, \dots, m \tag{4}
$$

$$
y_S \in \{0, 1\} \tag{5}
$$

$$
\tag{6}
$$

Exercise 1

$$x_{ij} = 1 \text{ if activity } i \text{ is executed in mode } j;\ 0 \text{ otherwise}$$

$$\max\ z = \sum_{i=1}^{n} x_{ip(i)}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \quad i = 1, \ldots, n \tag{1}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \leq B \tag{2}$$

$$\sum_{\substack{i=1,\ldots,n \\ s(i) \leq t \leq e(i)}} \sum_{j=1}^{m} q_{ij} x_{ij} \leq Q \quad t = 1, \ldots, T \tag{3}$$

$$\sum_{\substack{i=1,\ldots,n \\ s(i) \leq t \leq e(i)}} \sum_{j=1}^{m} r_{ij} x_{ij} \leq R_k \quad k = 1, \ldots, r; t = 1, \ldots, T \tag{4}$$

$$2x_{a\,m(a)} \leq x_{b\,m(b)} + x_{c\,m(c)} \tag{5}$$

$$x_{ij} \in \{0, 1\} \quad i = 1, \ldots, n; j = 1, \ldots, m \tag{6}$$

Exercise 2

x_1	x_2	x_3	x_4	x_5		
-2	-3	1	0	0	0	$-z$
2	3	2	1	0	12	x_4
4	2	0	0	1	14	x_5

x_1	x_2	x_3	x_4	x_5		
0	-2	1	0	$\frac{1}{2}$	7	$-z$
0	2	2	1	$-\frac{1}{2}$	5	x_4
1	$\frac{1}{2}$	0	0	$\frac{1}{4}$	$\frac{7}{2}$	x_1

x_1	x_2	x_3	x_4	x_5		
0	0	3	1	0	12	$-z$
0	1	1	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{5}{2}$	x_2
1	0	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{3}{8}$	$\frac{9}{4}$	x_1

Gomory cut: $\frac{1}{2}x_4 + \frac{3}{4}x_5 \geq \frac{1}{2}$

x_1	x_2	x_3	x_4	x_5	x_6		
0	0	3	1	0	0	12	$-z$
0	1	1	$\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{5}{2}$	x_2
1	0	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{3}{8}$	0	$\frac{9}{4}$	x_1
0	0	0	$-\frac{1}{2}$	$-\frac{3}{4}$	1	$-\frac{1}{2}$	x_6

x_1	x_2	x_3	x_4	x_5	x_6		
0	0	3	1	0	0	12	$-z$
0	1	1	$\frac{2}{3}$	0	$-\frac{1}{3}$	$\frac{8}{3}$	x_2
1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	2	x_1
0	0	0	$\frac{2}{3}$	1	$-\frac{4}{3}$	$\frac{2}{3}$	x_5

Exercise 3

```
/* Exercise 3, 2016 07 19 */

param n, integer, > 0;
param m, integer, > 0;
set I := 1..n;
set J := 1..m;
set S;
param c, integer, > 0;

param p{i in I, j in J}, >= 0;
param q{i in I}, >= 0;

var  x{i in I, j in J},  binary;
var  ys,  binary;

maximize z: sum{i in I, j in J} p[i,j]*x[i,j];

s.t.  cap{j in J}:  sum{i in I} q[i]*x[i,j] <=   c;
  yS1: sum{i in S, j in J} x[i,j] <= n*m*ys;
  yS2: sum{i  in I, j in J: i not in S} x[i,j] <= n*m* ys;
solve;

printf "\n";
for{i in I} {
   printf "\n%1d)",i;
   printf{j in J} "%5d ", x[i,j];
}
printf "\n\n-----z = %g\n\n",z;
printf "\n\n  ";
printf "yS =  %5d ", ys;

data;

param n := 6;
param m := 3;
param p :  1  2 3  :=
1  5  2 3
2  6  2 4
3  2  5 1
4  2  3 1
5  4  2 9
6  3  8 4;
param q := [1] 20 [2] 12 [3] 15 [4] 11 [5] 9 [6] 13;
param c := 40;
set S := 1 3;
end;
```