Assignment July 8, 2022 **A**

Last name, First name ____________________

Exercise 1 (value 12)

In the **Set Covering Problem (SCP)** we are given a set I of n elements and a set $S = \{s_1, \ldots, s_m\}$ of subsets of I. The basic problem asks to find the minimum number of subsets from S such that each element of I belongs to at least one of the selected subsets. We extend the SCP by considering, for each $s_j \in S$, a *weight* w_{s_j} and a *class* c_{s_j} (let $\mathcal{C}$ denote the set of all classes). We want to select some subset from S so that : a) the sum of the weights of the subsets selected is at least W; b) the number selected subsets is minimized; c) each element $i \in I$ belongs to al least one selected subset; d) no more than two subset from the same class are selected; e) if subset s_2 is selected, then subsets s_3 and s_6 are not selected.

Write a linear mathematical model to solve the above problem.

Answer:

$$x_{s_j} = 1 \text{ if subset } s_j \in S \text{ is selected, 0 otherwise}$$

$$
\begin{array}{llll}
\min & \sum_{j=1}^{m} x_{s_j} & & \\
& \sum_{j=1}^{m} w_{s_j} x_{s_j} & \geq W & \\
& \sum_{\substack{j=1 \\ i \in s_j}}^{m} x_{s_j} & \geq 1 & i \in I \\
& \sum_{\substack{j=1 \\ c_{s_j}=c}}^{m} x_{s_j} & \leq 2 & c \in \mathcal{C} \\
& x_{s_3} + x_{s_6} & \leq 2(1 - x_{s_2}) & \\
& x_{s_j} & \in \{0,1\} & j = 1, \ldots, m
\end{array}
$$

Exercise 2 (value 9)

Consider the following ILP problem.

$$
\begin{array}{rrrrcl}
\min & & -x_2 & +x_3 & & \\
& x_1 & +3x_2 & +3x_3 & \leq & 2 \\
& 2x_1 & +x_2 & -x_3 & \leq & 5 \\
& x_1, & x_2, & x_3 \geq & 0 & \textit{integer}
\end{array}
$$

- Solve with the standard Branch and Bound method for Integer Programming.
  - Solve the relaxed problems with the simplex method and show all the tableaus.
  - Apply Bland's rule.
- Show the search tree.
- Report the final solution and its value.

Answer:

Node P_0:

x_1	x_2	x_3	s_1	s_2	$-z$	
0	-1	1	0	0	0	
1	**3**	3	1	0	2	s_1
2	1	-1	0	1	5	s_2

x_1	x_2	x_3	s_1	s_2	$-z$
$\frac{1}{3}$	0	2	$\frac{1}{3}$	0	$\frac{2}{3}$
$\frac{1}{3}$	1	1	$\frac{1}{3}$	0	$\frac{2}{3}$
$\frac{5}{3}$	0	-2	$-\frac{1}{3}$	1	$\frac{13}{3}$

The optimal fractional solution is $x = (0, \frac{2}{3}, 0, 0, \frac{13}{3})$ with value $-\frac{2}{3}$. Since the coefficients of the objective function of ILP are integers we have $LB = \lceil -\frac{2}{3} \rceil = 0$.

We branch on x_2 imposing $x_2 \leq 0$ in Node P_1 and $x_2 \geq 1$ in Node P_2.

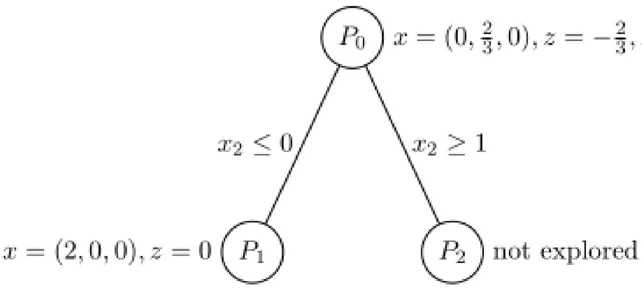
Node P_1 $(x_2 \leq 0)$:

x_1	x_2	x_3	s_1	s_2	s_3	$-z$
$\frac{1}{3}$	0	2	$\frac{1}{3}$	0	0	$\frac{2}{3}$
$\frac{1}{3}$	1	1	$\frac{1}{3}$	0	0	$\frac{2}{3}$
$\frac{5}{3}$	0	-2	$-\frac{1}{3}$	1	0	$\frac{13}{3}$
0	1	0	0	0	1	0

x_1	x_2	x_3	s_1	s_2	s_3	$-z$
$\frac{1}{3}$	0	2	$\frac{1}{3}$	0	0	$-\frac{2}{3}$
$\frac{1}{3}$	1	1	$\frac{1}{3}$	0	0	$\frac{2}{3}$
$\frac{5}{3}$	0	-2	$-\frac{1}{3}$	1	0	$\frac{13}{3}$
$-\boldsymbol{\frac{1}{3}}$	0	-1	$-\frac{1}{3}$	0	1	$-\frac{2}{3}$

x_1	x_2	x_3	s_1	s_2	$-z$	
0	0	3	0	0	0	0
0	1	0	0	0	1	0
0	0	-7	-2	1	5	1
1	0	3	1	0	-3	2

The integer solution $x = (2, 0, 0, 0, 1, 0)$ with value 0 has been retrieved. Since it has the same value of the bound found at father Node P_0, there is no need to evaluate node P_2.



Exercise 3 (value 6)

Consider the graph $G = (V, A)$ described by the following adjacency matrix with costs:

	1	2	3	4	5	6
1	–	3	8	5	6	4
2	3	–	–	2	2	–
3	6	1	–	–	–	–
4	5	–	–	–	–	1
5	2	3	4	–	–	1
6	1	2	12	–	1	–

Find the shortest path from node 1 to node 5 by using Dijkstra's method. Show the tables of labeles and predecessors, and write the optimal path with the associated cost.

Answer:

S	L_j					$pred_j$				
	2	**3**	**4**	**5**	**6**	**2**	**3**	**4**	**5**	**6**
$\{1\}$	**3**	8	5	6	4	1	1	1	1	1
$\{1,2\}$	3	8	5	5	**4**	1	1	1	2	1
$\{1,2,6\}$	3	8	5	**5**	4	1	1	1	2	1

The shortest path is $\{1, 2, 5\}$ with cost 5.