RICERCA OPERATIVA

Written assessment, June 11, 2018 A

First and Last name _____

Exercise 1 (value 13)
A Social Media Manager (SMM) needs to plan the social campaign for the company he works for. The campaign is for only one day: the company's birthday. The day is discretized in a set $T = \{1, \ldots, p\}$ of time periods. The SMM has a set $P = \{1, \ldots, m\}$ of prepared posts that can be published on the used social media. Each post can be published at most twice, but not all posts need to be published necessarily. One post cannot be published twice in the same period and, moreover, the same post cannot be published in two consecutive periods. Not all periods must be covered by posts, but in every period of the following time-slots (=*fasce orarie*) there should be at least one post: in the lunch time-slot $LT \subset T$ and in the dinner time-slot $DT \subset T$.

Publishing a post $i \in P$ in a period $t \in T$ has a return r_{it}. However, not using a time period produces a gain of g_t. Help the SMM to plan the social campaign by writing an integer programming model that maximizes the profit.

After reporting the first version of the model, Sales&Marketing required to include the following constraint. The company sells a set H of products. Each product $h \in H$ is associated with two posts: a *discount type* post $s(h) \in P$ and an *advertisement video* $a(h) \in P$. If one of the two posts of product $h \in H$ is published in one period than also the other must be published in the same period.

Exercise 2 (value 8)
Consider the following PLC problem. Solve it with the simplex method, using the Bland's rule, than write the dual problem and compute the corresponding solution.

$$\begin{aligned} \min \quad & -2x_1 + 3x_2 - x_3 \\ & x_1 - 3x_2 + 2x_3 \leq 8 \\ & 2x_2 - x_3 \leq 12 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Exercise 3 (value 7)
Consider a knapsack problem with 5 objects and a bin of capacity C=29. The object's profits and weights are, respectively, are $p = (19, 21, 16, 7, 10)$ $w = (14, 16, 13, 6, 9)$. Solve the problem with the branch-and-bound method.

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Exercise 1

Variables

$x_{it} = 1$ if the post $i \in P$ is published in period $t \in T$, 0 otherwise.
$y_t = 1$ if no post is published in period $t \in T$, 0 otherwise.

Model

$$
\begin{aligned}
\max z = & \sum_{i \in P} \sum_{t \in T} r_{it} x_{it} + \sum_{t \in T} g_t y_t \\
& \sum_{t \in T} x_{it} \leq 2 \quad i \in P \\
& \sum_{i \in P} x_{it} \geq 1 \quad t \in LT \cup DT \\
& x_{it} + x_{i,t+1} \leq 1 \quad i \in P, t = 1, \ldots, p-1 \\
& x_{s(h),t} = x_{a(h),t} \quad h \in H, t \in T \\
& \sum_{i \in P} x_{it} \leq |P|(1 - y_t) \quad t \in T \\
& x_{it} \in \{0,1\} \quad i \in P, t \in T \\
& y_t \in \{0,1\} \quad t \in T.
\end{aligned}
$$

Exercise 2

x_1	x_2	x_3	x_4	x_5		
-2	3	-1	0	0	0	$-z$
(1)	-3	2	1	0	8	x_4
0	2	-1	0	1	12	x_5

x_1	x_2	x_3	x_4	x_5		
0	-3	3	2	0	16	$-z$
1	-3	2	1	0	8	x_1
0	(2)	-1	0	1	12	x_5

x_1	x_2	x_3	x_4	x_5		
0	0	$\frac{3}{2}$	2	$\frac{3}{2}$	34	$-z$
1	0	$\frac{1}{2}$	1	$\frac{3}{2}$	26	x_1
0	1	$-\frac{1}{2}$	0	$\frac{1}{2}$	6	x_2

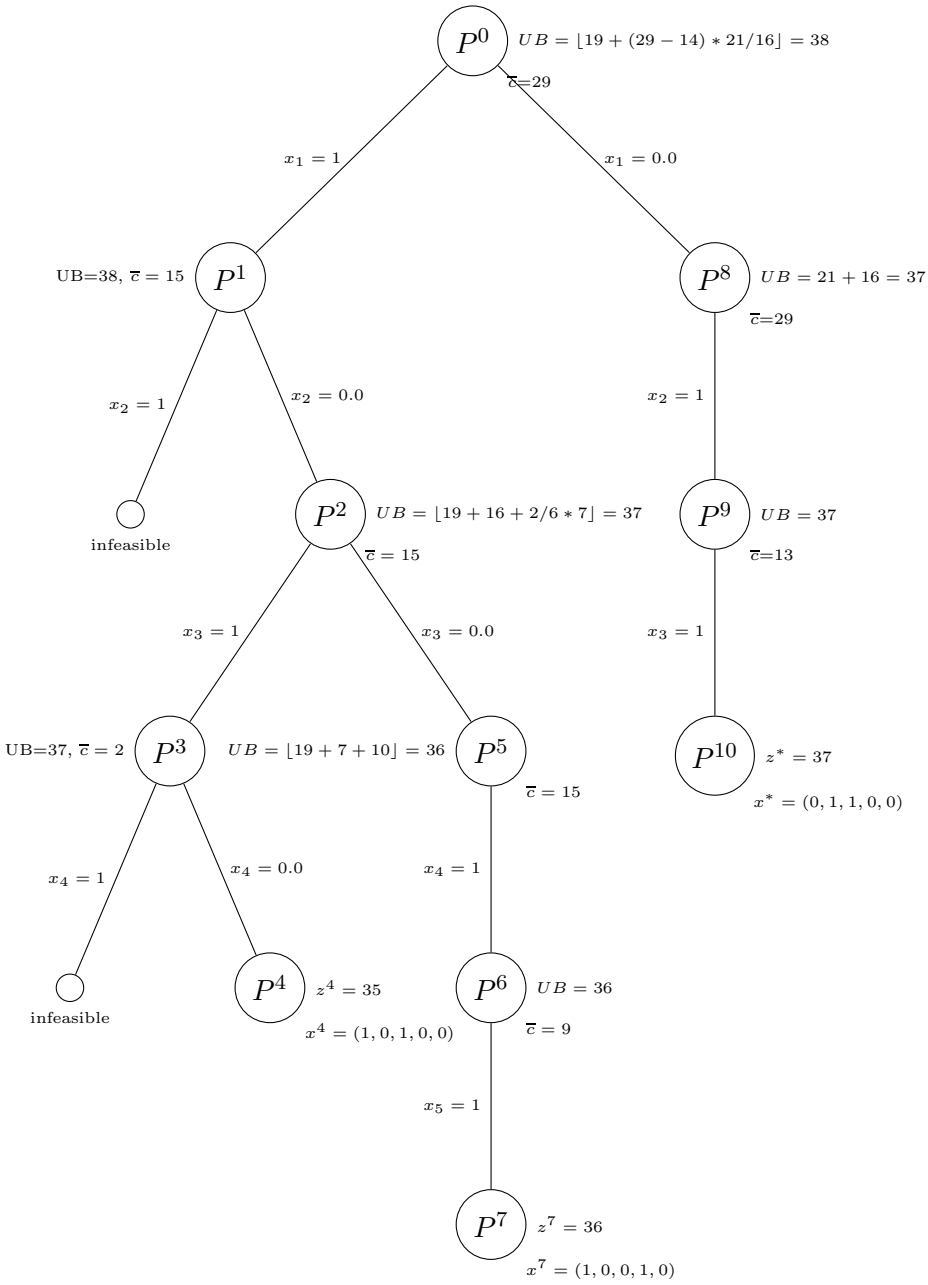
$x = (26, 6, 0, 0, 0),\ z_P = -34$

$$\begin{array}{ll} \min & -2x_1 + 3x_2 - x_3 \\ & -x_1 + 3x_2 - 2x_3 \geq -8 \\ & -2x_2 + x_3 \geq -12 \\ & x_1, x_2, x_3 \geq 0 \end{array} \qquad \begin{array}{rcl} \max 8u_1 + 12u_2 & & \\ -u_1 & \leq & -2 \\ 3u_1 - 2u_2 & \leq & 3 \\ -2u_1 + u_3 & \leq & -1 \\ u_1, u_2 & \leq & 0 \end{array}$$

$$\begin{cases} (-x_1 + 3x_2 - 2x_3 + 8)u_1 = 0 \\ (-2x_2 + x_3 + 12)u_2 = 0 \\ (-u_1 + 2)x_1 = 0 \\ (3u_1 - 2u_2 - 3)x_2 = 0 \\ (-2u_1 + u_3 + 1)x_3 = 0 \end{cases} \quad \begin{cases} (0)u_1 = 0 \\ (0)u_2 = 0 \\ (u_1 - 2)26 = 0 \\ (3u_1 - 2u_2 - 3)6 = 0 \\ (-2u_1 + u_2 + 1)0 = 0 \end{cases} \quad \begin{cases} -- \\ -- \\ u_1 = 2 \\ u_2 = 3/2 \end{cases}$$

$u = (2, 3/2),\ z_D = 34$

Exercise 3


P^0
$UB = \lfloor 19 + (29 - 14) * 21/16 \rfloor = 38$
$\overline{c}$=29
$x_1 = 1$
$x_1 = 0.0$
UB=38, $\overline{c} = 15$
P^1
P^8
$UB = 21 + 16 = 37$
$\overline{c}$=29
$x_2 = 1$
$x_2 = 0.0$
$x_2 = 1$
infeasible
P^2
$UB = \lfloor 19 + 16 + 2/6 * 7 \rfloor = 37$
$\overline{c} = 15$
P^9
$UB = 37$
$\overline{c}$=13
$x_3 = 1$
$x_3 = 0.0$
$x_3 = 1$
UB=37, $\overline{c} = 2$
P^3
$UB = \lfloor 19 + 7 + 10 \rfloor = 36$
P^5
$\overline{c} = 15$
P^{10}
$z^* = 37$
$x^* = (0, 1, 1, 0, 0)$
$x_4 = 1$
$x_4 = 0.0$
$x_4 = 1$
infeasible
P^4
$z^4 = 35$
$x^4 = (1, 0, 1, 0, 0)$
P^6
$UB = 36$
$\overline{c} = 9$
$x_5 = 1$
P^7
$z^7 = 36$
$x^7 = (1, 0, 0, 1, 0)$