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Exercise 1 (value 8)
Consider the following PLC problem. Solve it with the simplex method, using the Bland's rule, and solve the same problem graphically.

$$\begin{array}{ll} \min & -x_1 - x_2 \\ & -x_1 + x_2 \leq 5 \\ & x_1 + x_2 \leq 8 \\ & 4x_1 - x_2 \leq 24 \\ & x_1, x_2 \geq 0 \end{array}$$

(a) Is the optimal solution unique ?
(b) A slack variable has a reduced cost that equals zero. Does this relate to the number of optimal solutions ? Provide a brief explanation.

Exercise 2 (value 14)
A distribution company has started a drone delivery service from a set of supermarkets S to a set of customers C. All customers represent one request and each of them can be satisfied by exactly one drone flight. A subset of customers $H \subseteq C$ can be served by the drone or by a normal delivery (while all the customers in $C \setminus H$ **must** be served by the drone). If a customer $j \in H$ is not served by the drone a price p_j must be paid for serving him by other means. There is a second subset of customers $I \subseteq C \setminus H$: the customers of set H and those of set I cannot be served by the same supermarket. Drone can be launched by all supermarkets to each of the customers, the only limit is the battery endurance E. Therefore, the flying and service times of all the deliveries assigned to a drone must have a total duration smaller or equal to E. A drone service requires the time $2 * t_{ij}$ for flying from supermarket $i \in S$ to customer $j \in C$ both ways, plus service time s. If supermarket $i \in S$ is used for drone launching, the launching area has to be activated at a cost of a_i. When activated for drone delivery, each supermarket can serve at most L_i customers. Each drone used has a cost of D euro.

Write a linear program that helps the company to define the drone use, while minimizing the total costs.

Exercise 3 (value 6).
Consider a 0-1 knapsack problem with capacity $C = 40$, $n = 5$ items with profits $p_j = (8, 6, 5, 6, 5)$ and weights $w_j = (10, 12, 14, 18, 25)$. Find the optimal solution using the branch-and-bound method.

Exercise 1

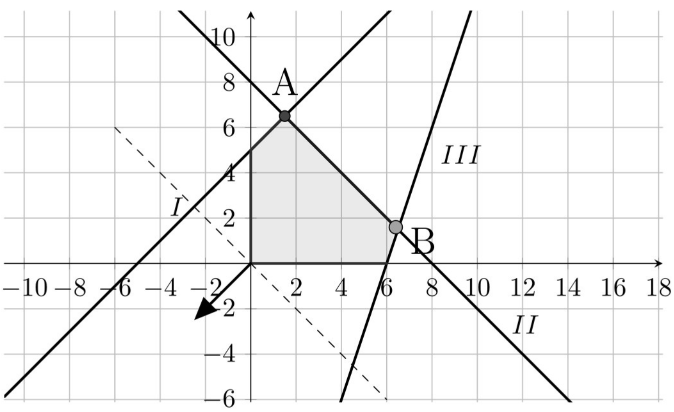
$$
\begin{array}{lll}
\min & -x_1 - x_2 & \\
 & -x_1 + x_2 + x_3 & = 5 \\
 & x_1 + x_2 + x_4 & = 8 \\
 & 4x_1 - x_2 + x_5 & = 24 \\
 & x_1, x_2, x_3, x_4, x_5 & \geq 0
\end{array}
$$

x_1	x_2	x_3	x_4	x_5		
-1	-1	0	0	0	0	$-z$
-1	1	1	0	0	5	x_3
1	1	0	1	0	8	x_4
4	-1	0	0	1	24	x_5

x_1	x_2	x_3	x_4	x_5		
0	-5/4	0	0	1/4	6	$-z$
0	3/4	1	0	1/4	11	x_3
0	**5/4**	0	1	-1/4	2	x_4
1	-1/4	0	0	1/4	6	x_1

x_1	x_2	x_3	x_4	x_5		
0	0	0	1	0	8	$-z$
0	0	1	-3/5	2/5	49/5	x_3
0	1	0	4/5	-1/5	8/5	x_2
1	0	0	1/5	1/5	32/5	x_1

$x^* = (32/5, 8/5, 49/5, 0, 0)$, $z^* = -8$



(a) The gradient is perpendicular to a facet of the polyhedron, given by constraint II, so s a infinite set of optimal solutions with the same optimal value of 8 exists along the segment A-B.
(b) Slack variable x_5, out of the basis, has reduced cost of value zero. A pivot can be performed on column x_5 to include x_5 in the basis, so switching from point A to point B, without changing the objective function value.

Exercise 2

Variables

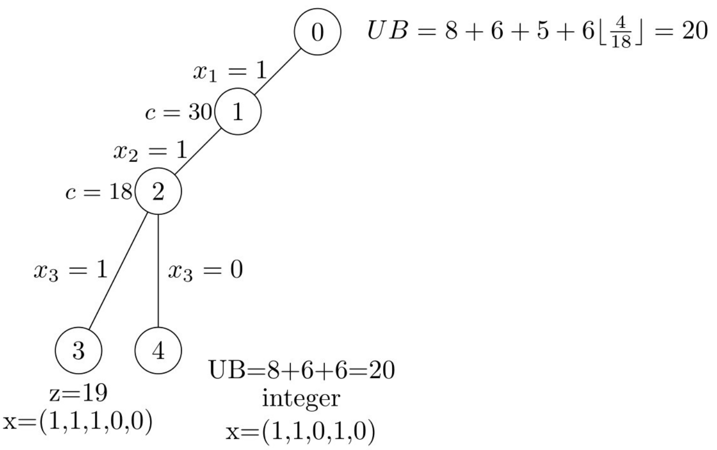
$$\begin{aligned}
x_k &= \text{1 if drone } k \text{ is used, 0 otherwise}\\
y_{ijk} &= \text{1 if drone } k \text{ serve customer } j \text{ starting from supermarket } i\text{, 0 otherwise}\\
\delta_i &= \text{1 if supermarket } i \text{ is used to launch drones, 0 otherwise}\\
h_i &= \text{1 if supermarket } i \text{ serves customers of set } H\text{, 0 otherwise}
\end{aligned}$$

Constant

$$K = \{1, \ldots, |C|\} \text{ set of all possible drones}$$

$$\begin{aligned}
\min\ z = &D \sum_{k \in K} x_k + \sum_{i \in S} a_i \delta_i + \sum_{j \in H} p_j \Big(1 - \sum_{i \in S} \sum_{k \in K} y_{ijk}\Big) && && (21)\\
\text{s.t. } &\sum_{i \in S} \sum_{k \in K} y_{ijk} = 1 && j \in C \setminus H && (22)\\
&\sum_{i \in S} \sum_{k \in K} y_{ijk} \leq 1 && j \in H && (23)\\
&\sum_{i \in S} \sum_{j \in C} (2t_{ij} + s) y_{ijk} \leq E x_k && k \in K && (24)\\
&\sum_{k \in K} \sum_{j \in C} y_{ijk} \leq L_i \delta_i && i \in S && (25)\\
&\sum_{k \in K} \sum_{j \in H} y_{ijk} \leq |C| h_i && i \in S && (26)\\
&\sum_{k \in K} \sum_{j \in I} y_{ijk} \leq |I|(1 - h_i) && i \in S && (27)\\
&x_k \in \{0, 1\} && k \in K && (28)\\
&\delta_i, h_i \in \{0, 1\} && i \in S && (29)\\
&y_{ijk} \in \{0, 1\} && i \in S, j \in C, k \in K && (30)
\end{aligned}$$

Exercise 3


0
$UB = 8 + 6 + 5 + 6\lfloor \frac{4}{18} \rfloor = 20$
$x_1 = 1$
$c = 30$
1
$x_2 = 1$
$c = 18$
2
$x_3 = 1$
$x_3 = 0$
3
4
z=19
x=(1,1,1,0,0)
UB=8+6+6=20
integer
x=(1,1,0,1,0)