20210119/Exercises v1.1

1. **MM-value=7**

   A family is used to go shopping to the same supermarket every week. Because of that, at the end of the year they have earned P points to be used to select some prizes from the catalogue. Each object $i \in O$ can be obtained in two ways: (i) by using p_i points; or (ii) by using $\pi_i < p_i$ points and a monetary contribute m_i. Each object can be selected at most once. After a family meeting, each of the O objects (prizes) is assigned a preference l_i. Write a MILP formulation to minimize the total monetary contribution that the family must spend, while satisfying a level of total preference (sum of the preference of the selected objects) of at least L.

   Notes: (not included in XML)

   - Solution:
     - $x_i = 1$ if the object $i \in O$ is obtained only with the use of points; 0 otherwise.
     - $y_i = 1$ if the object $i \in O$ is obtained with the use of points and money; 0 otherwise.

   $$\min z = \sum_{i \in O} m_i y_i$$
   $$x_i + y_i \leq 1 \quad i \in O$$
   $$\sum_{i \in O} (p_i x_i + \pi_i y_i) \leq P$$
   $$\sum_{i \in O} l_i (x_i + y_i) \geq L$$
   $$x_i \in \{0, 1\} \quad i \in O$$
   $$y_i \in \{0, 1\} \quad i \in O$$

2. **PLI-value=6**

   Given the following inequality

   $$x_1 + 3/4 x_2 - 4/3 x_3 = 5/2$$

- Write the correspondent Gomory's cut.
- Transform the obtained inequality in the standard form you use to include it into a tableau during the optimization. Motivate the answer.

Notes: (not included in XML)

- Solution: The corresponding Gomory cuts is the following:

$$3/4x_2 + 2/3x_3 \geq 1/2$$

It's standard form ready to be included in the tableau can be written in the following forms:

$$\begin{aligned} 0x_1 + 3/4x_2 + 2/3x_3 - s_1 &= 1/2 \\ 0x_1 - 3/4x_2 - 2/3x_3 + s_1 &= -1/2 \end{aligned}$$

3. **PLC-value=9**

Consider the following linear programming problem.

$$\begin{array}{lll} \min & z = & 3x_1 + 4x_2 + 3x_3 \\ & & 5x_1 + 3x_2 + 4x_3 \geq 5 \\ & & 6x_1 - 3x_2 + 3x_3 \geq 3 \\ & & 7x_1 - 7x_2 + 3x_3 \leq 4 \\ & & x_1, x_2, x_3 \geq 0 \end{array}$$

- Perform the first iteration of the dual simplex algorithm to solve the problem by applying the Bland's rule. Write the tableau before and after the iteration.
- Is the obtained solution optimal? Motivate your answer.

Notes: (not included in XML)

- Solution:
First of all we write the standard form:

$$\begin{aligned} \min \quad z = & 3x_1 + 4x_2 + 3x_3 \\ & 5x_1 + 3x_2 + 4x_3 - s_1 = 5 \\ & 6x_1 - 3x_2 + 3x_3 - s_2 = 3 \\ & 7x_1 - 7x_2 + 3x_3 + s_3 = 4 \\ & x_1, x_2, x_3, s_1, s_2, s_3 \geq 0 \end{aligned}$$

Basis	x_1 3	x_2 4	x_3 3	s_1 0	s_2 0	s_3 0	$-z$ 0
s_1	$\mathbf{-5}$	-3	-4	1	0	0	-5
s_2	-6	3	-3	0	1	0	-3
s_3	7	-7	3	0	0	1	4

Two rows has a negative rhs: rows 1 and 2. We follow the Bland's rule and select the row in base with the smallest index: s_1 is the variable that will leave the basis. The entering variable will be the one that provides the minimum among $|-5|/3; |-3|/4; |-4|/3$ which is x_1. Thus -5 is the pivot.

Basis	x_1 0	x_2 11/5	x_3 3/5	s_1 3/5	s_2 0	s_3 0	$-z$ -3
x_1	1	3/5	4/5	$-1/5$	0	0	1
s_2	0	33/5	9/5	$-6/5$	1	0	3
s_3	0	$-56/5$	$-13/5$	7/5	0	1	-3

The current solution is not optimal because it's not primal feasible.

4. **SP-value=6**

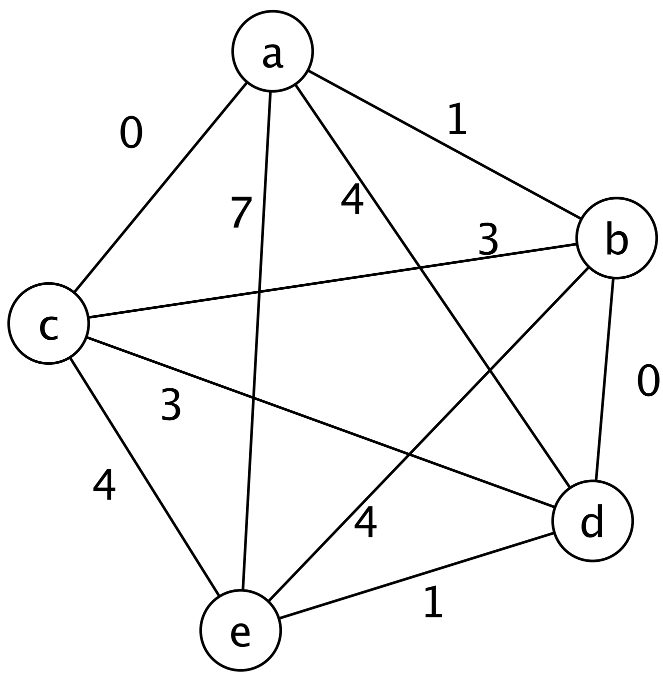
   Given the following graph $G = (V, E)$, with $V = \{a, b, c, d, e\}$ and the of edges as in the figure, each one with a cost associated:

   - Answer the following questions and provide motivations:
     - Is $G_1 = \{(a, d), (d, c), (c, b)\}$ a shortest path from a to b in G?
     - Is $G_2 = \{(a, c), (c, e), (a, e)\}$ a path from a to e in G?
   - Applying the Dijkstra's algorithm, find the shortest path from a to e in G. Report the sequence of nodes entering the set S, and the cost of the shortest path.

   Notes: (not included in XML)

   - Solution: G_1 is a path from a to b in G but is not a shortest one because, e.g., $\{(a, b)\}$ is shorter.
     G_2 is not a path because it contains a cycle.

     The unique shortest path from a to e in G is $P^* = \{(a, b), (b, d), (d, e)\}$ and has a cost of 2 units. The sequence of nodes entering S during the execution of Dijkstra's algorithm is $(a,)\ c, b, d, e$.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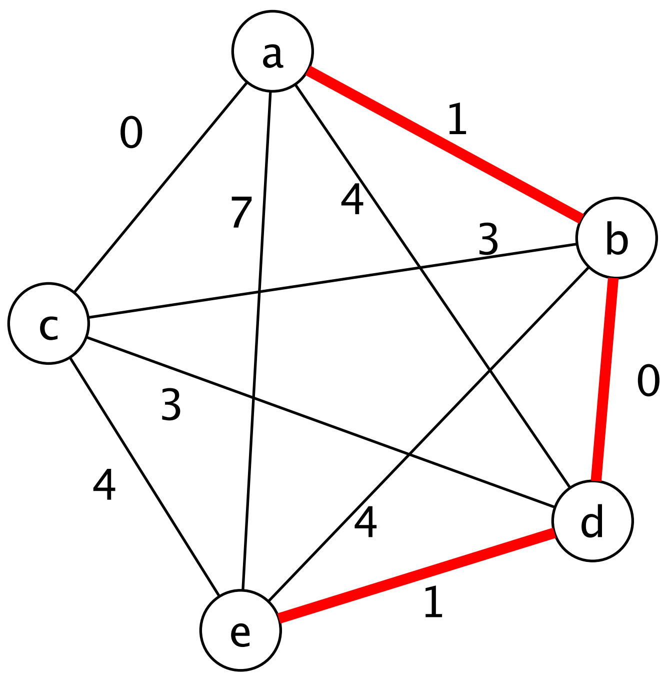
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