Part 30

Mauro Dell'Amico

DISMI, Universitá di Modena e Reggio Emilia
mauro.dellamico{at}unimore.it
www.or.unimore.it

Transportation problem

Are given

- *n sources* with maximum capacity $a_i, i = 1, \dots, n$
- *m sinks* with a request $b_j, j = 1, \dots, m$
- c_{ij} = transport cost of a unit from i to j

Problem: Find the set of transports giving to each sink the required amount, without exceeding the sources' capacity and minimizing the total cost.

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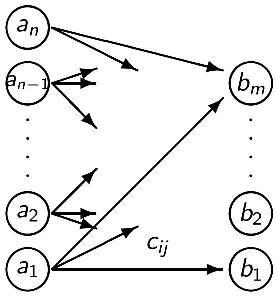

$$x_{ij} = \text{ quantity transported from } i \text{ to } j$$

$$\begin{aligned}
\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \\
\sum_{j=1}^{n} x_{ij} &\leq a_i, \quad i = 1, \ldots, n \\
\sum_{i=1}^{n} x_{ij} &\geq b_j, \quad j = 1, \ldots, n \\
x_{ij} &\geq 0 \quad \text{integer} \quad i, j = 1, \ldots, n
\end{aligned}$$

Linear assignment

Given an $n \times n$ square matrix $C = [c_{ij}]$, assign each row to a column, minimizing the sum of the selected elements and such that each column is assigned to a unique row.
(*given n workers, n works and the cost c_{ij} incurred if work j is done by worker i, assign each worker to one work, minimizing the cost.*)

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$$x_{ij} = \begin{cases} 1 & \text{if row } i \text{ is assigned to column } j \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} & & \\ \sum_{j=1}^{n} x_{ij} &= 1, \quad i = 1, \ldots, n \\ \sum_{i=1}^{n} x_{ij} &= 1, \quad j = 1, \ldots, n \\ x_{ij} &\in \{0, 1\} \quad i, j = 1, \ldots, n \end{aligned}$$

Knapsack

- n items
- Each item j has a profit $p_j$and a size w_j
- The knapsack has capacity c

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$$\begin{aligned} \max \sum_{j=1}^{n} p_j x_j & & \\ \sum_{j=1}^{n} w_j x_j & \le & c \\ x_j & \in & \{0,1\} \quad j = 1, \ldots, n \end{aligned}$$

Bin packing

- n items
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- Infinite number of bins of capacity c

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$$x_{ij} = \begin{cases} 1 & \text{if item } j \text{ is packed in bin } i \\ 0 & \text{otherwise} \end{cases} \qquad z_i = \begin{cases} 1 & \text{if bin } i \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

$$\min \sum_{i=1}^{n} z_i$$

$$\begin{aligned} \sum_{j=1}^{n} w_j x_{ij} &\leq c z_i, \quad i = 1, \ldots, n \\ \sum_{i=1}^{n} x_{ij} &= 1, \quad j = 1, \ldots, n \\ x_{ij} &\in \{0, 1\} \quad i, j = 1, \ldots, n \end{aligned}$$

Part 31

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Facility location

- $N = \{1, \dots, n\}$ potential facility locations
- $I = \{1, \dots, m\}$ customers' locations
- Site $j \in N$ has capacity u_j and activation cost c_j
- $b_i, i \in I$ is the request of customer i
- h_{ij} cost for serving customer i from site j
- Each customer must be served by a unique facility.

$$x_j = \begin{cases} 1 & \text{if a facility is set up in site } j \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if customer } i \text{ is served from site } j \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned}
\min \sum_{j \in N} c_j x_j \quad & + \quad \sum_{i \in I} \sum_{j \in N} h_{ij} y_{ij} \\
\sum_{j \in N} y_{ij} \quad & = \quad 1 \qquad i \in I && (1) \\
\sum_{i \in I} b_i y_{ij} \quad & \le \quad u_j x_j \quad j \in N && (2) \\
x_j \quad & \in \quad \{0,1\} \ \ j \in N && (3) \\
y_{ij} \quad & \in \quad \{0,1\} \ \ i \in I, j \in N
\end{aligned}$$

Fixed Charge

- n products
- $k_j > 0$ Set up cost *charge*) for j-th product
- c_j unitary cost for making one product of type j
- g_j revenue for one product unit of product j
- b_i quantity of resource $i = 1, \ldots, m$ available
- a_{ij} usage of resource i for one unit of product j

Problem: Find the optimal production mix.

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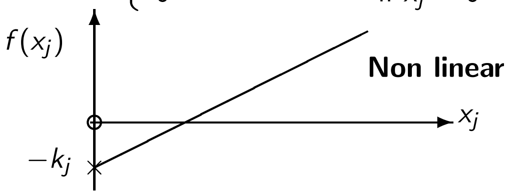
Problem: Find the optimal production mix.

$$x_j = \text{number of products of type } j$$

$$\max f(x_j) = \begin{cases} g_j x_j - k_j - c_j x_j & \text{if } x_j > 0 \\ 0 & \text{if } x_j = 0 \end{cases}$$



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$$\max \sum_{j=1}^{n} (g_j - c_j)x_j - \sum_{j=1}^{n} k_j y_j$$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \qquad i = 1, \dots, m \qquad (1)$$

$$x_j \le M y_j, \qquad j = 1, \dots, n \qquad (2)$$

$$y_j \in \{0, 1\}, \qquad j = 1, \dots, n \qquad (3)$$

$$x_j \ge 0, \qquad j = 1, \dots, n \qquad (4)$$