20210616

1. **MM-value=12/time=25**

ESSAY | 12 points | 0.10 penalty | editor

An electric company of a city is considering to change some of the installed meters (=contatori) with new smart meters that provide data to the company from remote. Let C denote the set of customers, and suppose C is partitioned into subsets A and B. The use of a smart meter provides a cost save estimated in g_i for customer $i \in C$. The company wants to guarantee to the customers a total saving $G > 0$, while minimizing the number of smart meters installed. Moreover, the public authority asked to the electric company to impose that customers in A and B have the same amount of new smart meters installed.

Write a MILP model to help the electric company to decide where to install smart meters, while respecting all the constraints.

Modify the model by adding the following constraint. Let $L \subset C$ and $S \subset C$, denote, respectively customers located at a large distance from the headquarter of the company, and customers at a short distance from the headquarter. If at least K customers that are located at a large distance have their meter changed, then at most H customers that are located at a small distance can have their meter changed.

Notes for grader:

- Solution:
  - $x_i = 1$ f the meter of customer $i \in C$ is changed; 0 otherwise
  - $\delta = 1$ if at least K large distance customers have their meter changed; 0 otherwise.
  - M is a large number

$$
\begin{aligned}
&\min z = \sum_{i \in C} x_i \\
&\sum_{i \in C} g_i x_i \geq G \\
&\sum_{i \in A} x_i = \sum_{i \in B} x_i \\
&\sum_{i \in L} x_i \leq K + M\delta \\
&\sum_{i \in S} x_i \leq H + M(1-\delta) \\
&x_i \in \{0,1\},\ i \in C \\
&\delta \in \{0,1\}
\end{aligned}
$$

2. **PLI-value=7/time=15**

ESSAY | 7 points | 0.10 penalty | editor

Consider the following LP problem:

$$
\begin{array}{ll}
\max & 4x_1 + x_2 + x_3 \\
& x_1 + 4x_2 - 2x_3 = 4 \\
& -x_1 - 2x_2 + 5x_3 = 7 \\
& x_1, x_2, x_3 \geq 0.
\end{array}
$$

Write the first tableau obtained when starting the first phase of the Two Phase Method, perform the first iteration using the Bland rule, and show the resulting tableau. Did you obtain an initial feasible basic solution? Motivate your answer.

Notes for grader:

- Solution:

$$
\begin{array}{ll}
\min & y_1 + y_2 \\
& x_1 + 4x_2 - 2x_3 + y_1 = 4 \\
& -x_1 - 2x_2 + 5x_3 + y_2 = 7 \\
& x_1, x_2, x_3, y_1, y_2 \geq 0.
\end{array}
$$

	x_1	x_2	x_3	y_1	y_2	$-z$
	0	-2	-3	0	0	-11
y_1	1	4	-2	1	0	4
y_2	-1	-2	5	0	1	7

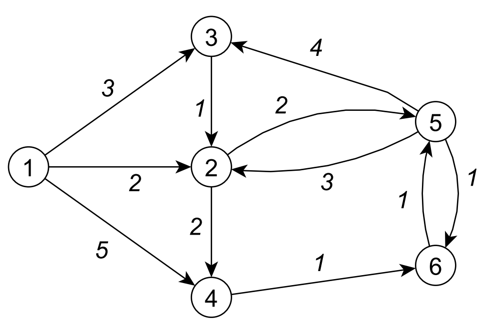
	x_1	x_2	x_3	y_1	y_2	$-z$
	$1/2$	0	-4	$1/2$	0	-9
x_2	$1/4$	1	$-1/2$	$1/4$	0	1
y_2	$-1/2$	0	4	$1/2$	1	9

The obtained solution is not a feasible solution for the original problem because an auxiliary variable in still in the basis.

3. **SPP-value=9/time=15**

ESSAY | 9 points | 0.10 penalty | editor

Consider the graph $G = (V, A)$ in figure. Find the shortest path from node 1 to node 5 by using the Dijkstra algorithm. Show the tables of costs and predecessors, and write the optimal path with the associated cost.



Notes for grader:

- Solution:

S	L_j				
nodes	**2**	**3**	**4**	**5**	**6**
$\{1\}$	**2**	3	5	M	M
$\{1,2\}$	2	**3**	4	4	M
$\{1,2,3\}$	2	3	**4**	4	M
$\{1,2,3,4\}$	2	3	4	**4**	5
$\{1,2,3,4,5\}$	2	3	4	4	5

S	$Pred_j$					
nodes	**2**	**3**	**4**	**5**	**6**	
$\{1\}$	**1**	1	1	1	1	
$\{1,2\}$	1	**1**	2	2	1	
$\{1,2,3\}$	1	1	**2**	2	1	
$\{1,2,3,4\}$	1	1	2	**2**	4	
$\{1,2,3,4,5\}$	1	1	2	2	4	5

Total of marks: 28