20200910/Exercises

1. **Value=7**

   Write a set of constraints in which variable x_1 and x_2 are non-negative quantities having that:

   - x_2 cannot exceed 5;
   - either x_1 is at most x_2 or the sum of the two variables is less than or equal to 5.

   Notes: (not included in XML)

   - M is a large enough number

   $$\begin{aligned}
   & x_1 - x_2 \leq 0 + M(1-y) \\
   & x_1 + x_2 \leq 5 + My \\
   & x_2 \leq 5 \\
   & x_1, x_2 \geq 0 \\
   & y \in \{0, 1\}
   \end{aligned}$$

2. **Value=7**

   Consider the following LP problem.

   $$\begin{aligned}
   \min \quad & 3x_1 + 5x_2 + 4x_3 \\
   & 3x_1 + 2x_2 + 4x_3 \geq 1 \\
   & 2x_1 + 3x_2 + 2x_3 \geq 3 \\
   & x_1, x_2, x_3 \geq 0
   \end{aligned}$$

   - Check if the vector $\bar{x} = (0, 1, 0)$ is a feasible solution for the LP problem. Motivate your answer.
   - Write the corresponding dual problem and the complementary slackness conditions.
   - Write the dual solution corresponding to the primal solution $\bar{x}$.
   - Is it possible to conclude that vector $\bar{x}$ is the optimal solution of the primal problem? Motivate your answer.

Notes: (not included in XML)

- Dual:

$$
\begin{aligned}
\max \quad & u_1 + 3u_2 \\
& 3u_1 + 2u_2 \leq 3 \\
& 2u_1 + 3u_2 \leq 5 \\
& 4u_1 + 2u_2 \leq 4 \\
& u_1, u_2, u_3 \geq 0
\end{aligned}
$$

  Complementary slackness:

$$
\begin{aligned}
& x_1(3u_1 + 2u_2 - 3) = 0 \\
& x_2(2u_1 + 3u_2 - 5) = 0 \\
& x_3(4u_1 + 2u_2 - 4) = 0 \\
& u_1(3x_1 + 2x_2 + 4x_3 - 1) = 0 \\
& u_2(2x_1 + 3x_2 + 2x_3 - 3) = 0
\end{aligned}
$$

  Corresponding dual solution:

$$
\begin{aligned}
& \bar{u}_2 = 5/3 \\
& \bar{u}_1 = 0
\end{aligned}
$$

  Both solution have the same value of 5 but the first dual constraint is violated by $\bar{u}$, so the vector $\bar{x}$ is not the optimal solution.

3. **Value=6**

   Given a set of 4 objects: A, B, C, D, write a model in which those objects can be selected with the following requests:

   - maximise the profit, knowing that selecting object C provides the double of the profit of objects A, B, and D.
   - If objects A and B are both selected then C cannot be selected.
   - Exactly one between A and D must be selected.

Notes: (not included in XML)

- $$\begin{aligned} \max x_A + x_B + 2x_C + x_D \\ x_C \leq 2 - x_A - x_B \\ x_A + x_D = 1 \\ x_A, x_B, x_C, x_D \in 0, 1 \end{aligned}$$

4. **Value=8**

   Mr. Smith must decide how to invest 9 thousands euro. He is considering 4 possible investments, respectively with cost (3, 5, 2, 4) and profit (7, 9, 3, 5) (everything is expressed in thousands euro).

   - Find the best investment policy with a B&B algorithm knowing that if an investment is made, then it must be acquired completely.
   - Write the upper bound computed at the root node (UB_0) and write all the upper bounds having a different value with respect to UB_0.
   - Write the optimal solution and the optimal solution value.

   Notes: (not included in XML)

   - $UB_0 = 17$. Other upper bounds are $UB = 15$ when branching the variable x_2 and $UB = 14$ when branching the variable x_1. The optimal solution is $x = (1, 1, 0, 0)$ and has value 16.