RICERCA OPERATIVA

Written assessment, September 29, 2017 A

First and Last name _____

Exercise 1 (value 8)
Consider the following PLC problem and solve it using the Simplex algorithm with the Bland rule.

$$\begin{aligned} \min \quad & -2x_1 + 4x_3 \\ & -x_1 - 3x_2 + x_3 \leq 6 \\ & 2x_1 + x_2 + 2x_3 = 9 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Exercise 2 (value 6)
Consider the final solution of the previous exercise. Add the integrality constraints to the model and apply the cutting plane method with Gomory cuts, performing one iteration by selecting the first (highest) row of the tableau.

Exercise 3 (value 10).
The logistic company INTERTRANS must deliver n parcels, each with a weight $w_i, i = 1, \ldots, n$. They will use m independent drivers, each owning a single truck with total capacity (regarding weight) $V_j (j = 1, \ldots, m)$. INTERTRANS aim is to deliver all the parcels using a fair approach with the drivers, i.e., they want to give each driver used a "similar" filling of the truck. To do this they want to maximize the minimum percentage load of the trucks used *(percentage load = ratio between the weight of the parcels transported by the truck over the capacity of the truck).* Help the company to find an optimal solution by writing a Linear Program.

RICERCA OPERATIVA

Answers A

First and Last name _____

Exercise 1

$$\begin{aligned} \min \quad & -2x_1 + 4x_3 \\ & -x_1 - 3x_2 + x_3 \leq 6 \\ & 2x_1 + x_2 + 2x_3 = 9 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

PHASE I

x_1	x_2	x_3	x_4	x_5		
-2	-1	-2	0	0	0	$-z$
-1	-3	1	1	0	6	x_4
(2)	1	2	0	1	9	x_5

x_1	x_2	x_3	x_4	x_5		
0	0	0	0	1	9	$-z$
0	$-\frac{5}{2}$	2	1	$\frac{1}{2}$	$\frac{21}{2}$	x_4
1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{9}{2}$	x_1

PHASE II

x_1	x_2	x_3	x_4			
0	1	6	0	9	$-z$	$x = (\frac{9}{2}, 0, 0, \frac{21}{2}),\ z = -9$
0	$-\frac{5}{2}$	2	1	$\frac{21}{2}$	x_4	
1	$\frac{1}{2}$	1	0	$\frac{9}{2}$	x_1	

Exercise 2

Gomory cut from first row:

$$\frac{1}{2}x_2 \geq \frac{1}{2}$$

x_1	x_2	x_3	x_4	x_5			
0	1	6	0	0	9	$-z$	
0	$-\frac{5}{2}$	2	1	0	$\frac{21}{2}$	x_4	$x = (3, 1, 0, 13, 0),\ z = -8$
1	$\frac{3}{2}$	1	0	0	$\frac{9}{2}$	x_1	
0	(-$\frac{1}{2}$)	0	0	1	$-\frac{1}{2}$	x_5	

x_1	x_2	x_3	x_4	x_5		
0	0	6	0	2	8	$-z$
0	0	2	1	-5	13	x_4
1	0	1	0	3	3	x_1
0	1	0	0	-2	1	x_2

Exercise 3
Variables

$$\begin{aligned} x_{ij} &= \text{1 if parcel } i \text{ is transported by truck } j\text{; 0 otherwise} \\ z_j &= \text{1 if truck } j \text{ is used; 0 otherwise} \\ L &= \text{minimum percentage load of an used truck} \end{aligned}$$

$$\max L \tag{1}$$

$$\sum_{j=1}^{m} x_{ij} = 1 \quad i = 1, \ldots, n \tag{2}$$

$$\sum_{i=1}^{n} w_i x_{ij} \leq V_j z_j \quad j = 1, \ldots, m \tag{3}$$

$$\sum_{i=1}^{n} w_i x_{ij} / V_j \geq L - M(1 - z_j) \quad j = 1, \ldots, m \tag{4}$$

$$x_{ij} \in \{0, 1\} \quad i = 1 \ldots, n; j = 1 \ldots, m \tag{5}$$

$$z_i \in \{0, 1\} \quad i = 1 \ldots, n \tag{6}$$

$$L \geq 0 \tag{7}$$

$M =$ constant representing a large number