Written assessment, January 29, 2020 M

Last name, First name ______________________________

Exercise 1 (value 13)
Red-Fir is a forestry company that owns and manages a large forest in the Alps. They have to plan a set of m tree cutting activities in the next W weeks. Each activity v requires $a_v \leq W$ consecutive work weeks. The company uses teams of woodcutters (boscaioli) hired in the near villages. Each team may execute any activity of any duration $w(1 \leq w \leq W)$. The cost of a team to work for w weeks is c_w euros. When two teams are hired for the same duration w, the woodcutters recognize to the company a discount of d_w euros, so, sometimes, it is convenient to hire a team for a period longer than necessary, to gain the discount. Write a linear mathematical programming model to help the Red-Fir to choose the set of hiring periods which allow to complete all the activities and minimizes the total cost. Now consider that each activity v has a given starting week s_v ($1 \leq s_v \leq W$). In this case the discount d_w is applied only if two teams are hired both for a_v weeks, and the two activities start on the same week. Reformulate the above model by considering this new requirement.

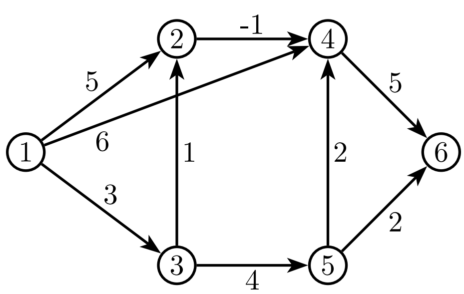
Exercise 2 (value 9)
Consider the following LP problem.

$$\begin{aligned} \min \quad & 4x_1 + 8x_2 + 2x_3 \\ & -2x_1 + 3x_2 \geq 3 \\ & -3x_1 + 2x_2 + x_3 \leq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Perform the following tasks:
(i) solve the problem using the symplex method;
(ii) write the dual problem, draw the feasible region and determine its solution.

Exercise 3 (value 6)

Consider the following digraph and find the shortest path from 1 to all other vertices using the Bellman-Ford algorithm reporting all iteration in the table in the rear.



iter	$f^k(j)$ 1	2	3	4	5	6	$pred_j$ 1	2	3	4	5	6
0												
1												
2												
3												
4												
5												

Written assessment, January 29, 2020 M

Last name, First name ____________________

Exercise 1

Variables

$x_{vw} = 1$ if activity v is executed by a team hired for w weeks, 0 otherwise.
z_w = number of pairs of teams hired for w weeks

Model

$$\min \sum_{v=1}^{m} \sum_{w=1}^{W} c_w x_{wv} - \sum_{w=1}^{W} d_w z_w \tag{25}$$

$$\sum_{w=1}^{W} x_{wv} = 1 \quad v = 1, \dots, m \tag{26}$$

$$\sum_{w=1}^{W} w x_{wv} \geq a_v \quad v = 1, \dots, m \tag{27}$$

$$z_w \leq \sum_{v=1}^{m} x_{wv}/2 \quad w = 1, \dots, W \tag{28}$$

$$x_{wv} \in \{0, 1\} \quad w = 1, \dots, W, v = 1, \dots, m \tag{29}$$

$$z_w \geq 0, \text{integer} \quad w = 1, \dots, m \tag{30}$$

Second Model

Constants

A_s = set of all activities $v \in \{\dots, m\}$ starting in s (i.e., $s_v = s$).

Variables

$x_{wv} = 1$ if activity v is executed by a team hired for w weeks (starting in week s_v), 0 otherwise.
z_{ws} = number of pairs of teams hired for w weeks, starting at week s

Model

$$\min \sum_{v=1}^{m} \sum_{w=1}^{W} c_w x_{wv} - \sum_{w=1}^{W} \sum_{s=1}^{W-w+1} d_w z_{ws} \tag{31}$$

$$\sum_{w=1}^{W} x_{wv} = 1 \quad v = 1, \dots, m \tag{32}$$

$$\sum_{w=1}^{W} w x_{wv} \geq a_v \quad v = 1, \dots, m \tag{33}$$

$$z_{ws} \leq \sum_{v \in A_s} x_{wv}/2 \quad w = 1, \dots, W, s = 1, \dots, W - w + 1 \tag{34}$$

$$x_{wv} \in \{0, 1\} \quad w = 1, \dots, W, v = 1, \dots, m \tag{35}$$

$$z_{ws} \geq 0, \text{integer} \quad w = 1, \dots, m, s = 1, \dots, W - w + 1 \tag{36}$$

Exercise 2 Let's write the model in standard form:

$$
\begin{array}{ll}
\min & 4x_1 + 8x_2 + 2x_3 \\
 & 2x_1 - 3x_2 + x_4 = -3 \\
 & -3x_1 + 2x_2 + x_3 + x_5 = 1 \\
 & x_1, \dots, x_5 \geq 0
\end{array}
$$

x_1	x_2	x_3	x_4	x_5		
4	8	2	0	0	0	$-z$
2	-3	0	1	0	-3	x_4
-3	2	1	0	1	1	x_5

x_1	x_2	x_3	x_4	x_5		
$\frac{28}{3}$	0	2	$\frac{8}{3}$	0	-8	$-z$
$-\frac{2}{3}$	1	0	$-\frac{1}{3}$	0	1	x_2
$-\frac{5}{3}$	0	1	$\frac{2}{3}$	1	-1	x_5

x_1	x_2	x_3	x_4	x_5		
0	0	$\frac{38}{5}$	$\frac{32}{5}$	$\frac{28}{5}$	$-\frac{68}{5}$	$-z$
0	1	$-\frac{2}{5}$	$-\frac{3}{5}$	$-\frac{2}{5}$	$\frac{7}{5}$	x_2
1	0	$-\frac{3}{5}$	$-\frac{2}{5}$	$-\frac{3}{5}$	$\frac{3}{5}$	x_1

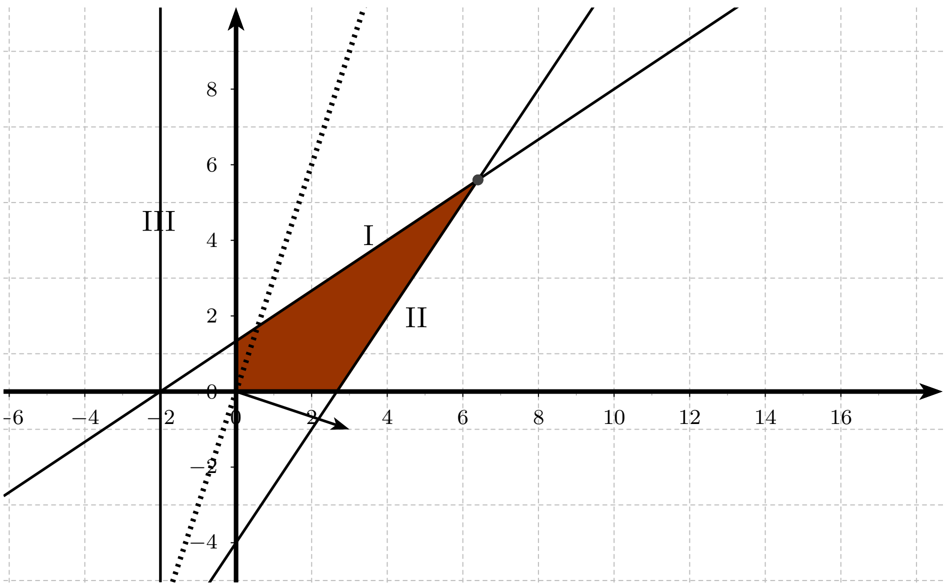
$x = (3/5, 7/5, 0, 0, 0), z_P = 68/5$

PRIMAL

$$
\begin{array}{ll}
\min & 4x_1 + 8x_2 + 2x_3 \\
 & -2x_1 + 3x_2 \geq 3 \\
 & 3x_1 - 2x_2 - x_3 \geq -1 \\
 & x_1, x_2, x_3 \geq 0
\end{array}
$$

DUAL

$$
\begin{array}{ll}
\max & 3u_1 - u_2 \\
 & -2u_1 + 3u_2 \leq 4 \\
 & 3u_1 - 2u_2 \leq 8 \\
 & -u_2 \leq 2 \\
 & u_1, u_2 \geq 0
\end{array}
$$



$u = (32/5, 28/5), z_D = 68/5$

Exercise 3

iter	$f^k(j)$ 1	2	3	4	5	6	$pred_j$ 1	2	3	4	5	6
0	0	∞	∞	∞	∞	∞	1	-	-	-	-	-
1	0	5	3	6	∞	∞	1	1	1	1	-	-
2	0	4	3	4	7	∞	1	3	1	2	3	-
3	0	4	3	3	7	9	1	3	1	2	3	5
4	0	4	3	3	7	8	1	3	1	2	3	4
5	0	4	3	3	7	8	1	3	1	2	3	4