June 07, 2016 A

First and Last name ____________________

Exercise 1 (value 12)
The owner of a well known supermarket wants to build a new store in the city. Before proceeding with the detailed design some preliminary decisions must be taken. First of all the total area, in square meters (m^2) has to be fixed. The location chosen for the store is compatible with a total area between 2000 m^2 and 3000 m^2. The whole surface must be allocated to the following uses. There must be between n_1 and n_2 shelves for generic goods each having a total size of A_g m^2. The area allocated to these shelves must be between 20% and 45% of the total area. For each shelf there must be an aisle and the total area used for the aisles must be between 1.2 and 1.4 times the area of the shelves. The area dedicated to fruit and vegetables is between 5% and 15% of the total area, while the area for delicatessen is not less that 50% of the area dedicated to fruit and vegetables and not more that 300 m^2. There are m generic goods that must be allocated on the shelves following a set of rules. Let A, B and C be three distinct sets of products. A product $j \in A$ cannot be assigned at the same shelf with a product $j \in B$. Moreover if two or more products of set A are on the same shelf then at least one product $j \in C$ must be assigned to that shelf.

The expected revenue for one square meter allocated to generic goods, fruits and vegetables, and delicatessen, is, respectively, r^g, r^{fv} and r^d. Write a Linear Programming model to help the manager to define the total areas to be allocated to each kind of good and the total area of the store, maximizing the expected revenue.

Exercise 2 (value 8)
Consider the following PLC problem and solve it using the primal simplex with Bland rule.

$$\begin{aligned} \max \quad & 2x_1 + x_2 - x_3 \\ & 2x_1 + 4x_2 - 2x_3 \le 5 \\ & -x_1 + 3x_2 + 2x_3 \le 6 \\ & x_1, \dots, x_3 \ge 0 \end{aligned}$$

Write the dual problem, drawn the region of the feasible solutions and identify the points of the plane u_1, u_2 associated with the bases visited by the primal simplex algorithm (remind that $u^T = c_B^T B^{-1}$).

Exercise 3 (value 7)

Write a GLPK or XPRESS model corresponding to the following mathematical model

$$\max \ z = \sum_{i=1}^{n} \sum_{j=1}^{m} r_{ij} x_{ij} - \sum_{j=1}^{m} c_j y_j$$

$$\sum_{i=1}^{n} x_{ij} \le 2 \quad j = 1, \dots, m \tag{1}$$

$$\sum_{i=1}^{n} x_{ij} \le n y_j \quad j = 1, \dots, m \tag{2}$$

$$\sum_{i=1}^{n} x_{ij} + m\delta_j \ge 2 \quad j \in A \tag{3}$$

$$\delta_j \le y_j \quad j \in A \tag{4}$$

$$x_{ij} \in \{0, 1\} \quad i = 1, \dots, n, \ j = 1, \dots, m \tag{5}$$

$$y_j \in \{0, 1\} \quad j = 1, \dots, m \tag{6}$$

$$\delta_j \in \{0, 1\} \quad j \in A \tag{7}$$

(8)

June 7, 2016 **A**

First and Last name ______________________

Exercise 1

$$
\begin{aligned}
x_{tot} &= \text{area of the supermarket}\\
x_i^g &= \text{1 if shelf } i \text{ is used; 0 otherwise}\\
x^a &= \text{area dedicated to aisles}\\
x^{fv} &= \text{area dedicated to fruit and vegetables}\\
x^d &= \text{area dedicated to delicatessen}\\
y_{ij} &= \text{1 if product } j \text{ is assigned to shelf } i\text{; 0 otherwise}\\
\delta_i^A &= \text{1 if there are at least two products of type } A \text{ on shelf } i\text{; 0 otherwise}
\end{aligned}
$$

$$\max\ z = r^g A_g \sum_{i=1}^{n_2} x_i^g + r^{fv} x^{fv} + r^d x^d$$

$$2000 \leq x_{tot} \leq 3000 \tag{1}$$

$$A_g \sum_{i=1}^{n_2} x_i^g + x^a + x^{fv} + x^d = x_{tot} \tag{2}$$

$$\sum_{i=1}^{n_2} x_i^g \geq n_1 \tag{3}$$

$$0.2x_{tot} \leq A_g \sum_{i=1}^{n_2} x_i^g \leq 0.45x_{tot} \tag{4}$$

$$1.2A_g \sum_{i=1}^{n_2} x_i^g \leq x^a \leq 1.4A_g \sum_{i=1}^{n_2} x_i^g \tag{5}$$

$$0.05x_{tot} \leq x^{fv} \leq 0.15x_{tot} \tag{6}$$

$$0.5x^{fv} \leq x^d \leq 300 \tag{7}$$

$$\sum_{i=1}^{n_2} y_{ij} = 1 \quad j = 1, \dots, m \tag{8}$$

$$m\, x_i^g \leq \sum_{j=1}^{m} y_{ij} \quad i = 1, \dots, n_2 \tag{9}$$

$$y_{ij} + y_{ik} \leq 1 \quad j \in A, k \in B \tag{10}$$

$$\sum_{j \in A} y_{ij} - 1 \leq |A|\delta_i^A \quad i = 1, \dots, n_2 \tag{11}$$

$$\delta_i^A \leq \sum_{j \in C} y_{ij} \quad i = 1, \dots, n_2 \tag{12}$$

$$x_{tot}, x^a, x^{fv}, x^d \geq 0 \tag{13}$$

$$x_i^g \in \{0, 1\} \quad i = 1, \dots, n_2 \tag{14}$$

$$y_{ij} \in \{0, 1\} \quad i = 1, \dots, n_2, j = 1, \dots, m \tag{15}$$

$$\delta_i^A \in \{0, 1\} \quad i = 1, \dots, n_2 \tag{16}$$

Exercise 2

$$\begin{array}{ll} \max & 2x_1 + x_2 - x_3 \\ & 2x_1 + 4x_2 - 2x_3 \le 5 \\ & -x_1 + 3x_2 + 2x_3 \le 6 \\ & x_1, \ldots, x_3 \ge 0 \end{array}$$

$$\begin{array}{ll} \min & 5u_1 + 6u_2 \\ & 2u_1 - u_2 \ge 2 \\ & 4u_1 + 3u_2 \ge 1 \\ & 2u_1 - 2u_2 \le 1 \\ & u_1, u_2 \ge 0 \end{array}$$

x_1	x_2	x_3	x_4	x_5		
2	1	-1	0	0	0	$-z$
2	4	-2	1	0	5	x_4
-1	3	2	0	1	6	x_5

x_1	x_2	x_3	x_4	x_5		
0	-3	1	-1	0	-5	$-z$
1	2	-1	$\frac{1}{2}$	0	$\frac{5}{2}$	x_1
0	5	1	$\frac{1}{2}$	1	$\frac{17}{2}$	x_5

x_1	x_2	x_3	x_4	x_5		
0	-8	0	$-\frac{3}{2}$	-1	$-\frac{27}{2}$	$-z$
1	7	0	1	1	11	x_1
0	5	1	$\frac{1}{2}$	1	$\frac{17}{2}$	x_3

$z_P = 27/2, x = (11, 0, -17/2, 0, 0)$

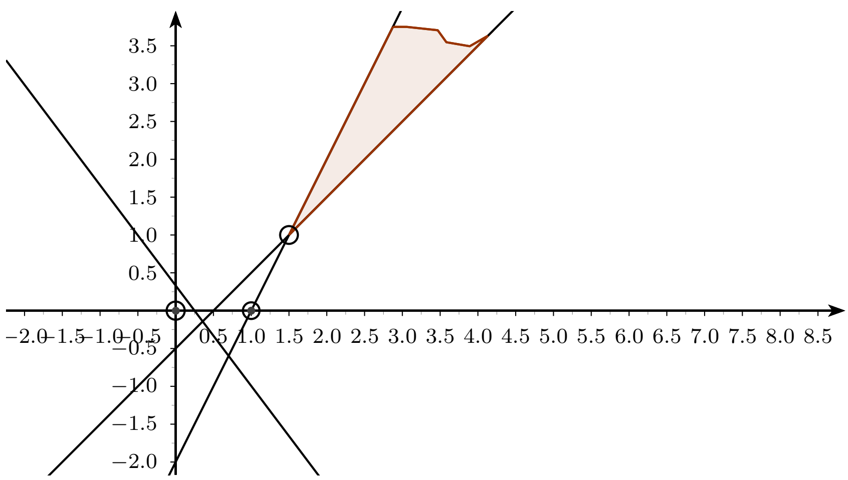
Initial base is x_4, x_5, $c_B^T = [0, 0]$, so $u_1 = u_2 = 0$

The next base is x_1, x_5, so $B = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$, $c_B^T = [2, 0]$ and $u_1 = 1, u_2 = 0$

The optimal base gives

$$\begin{cases} (2x_1 + 4x_2 - 2x_3 - 5)u_1 = 0 \\ (-x_1 + 3x_2 + 2x_3 - 6)u_2 = 0 \\ (2u_1 - u_2 - 2)x_1 = 0 \\ (4u_1 + 3u_2 - 1)x_2 = 0 \\ (2u_1 - 2u_2 - 1)x_3 = 0 \end{cases} \qquad \begin{cases} (0)u_1 = 0 \\ (0)u_2 = 0 \\ 2u_1 - u_2 - 2 = 0 \\ (4u_1 + 3u_2 - 1)0 = 0 \\ 2u_1 - 2u_2 - 1 = 0 \end{cases} \qquad \begin{cases} 2u_1 - u_2 = 2 \\ 2u_1 - 2u_2 = 1 \end{cases}$$

$u = (3/2, 1), z_D = 27/2$



Exercise 3

```
/* Exercise 3, 2016 06 07G */

param n, integer, > 0;
param m, integer, > 0;
set I := 1..n;
set J := 1..m;
set A;

param r{i in I, j in J}, >= 0;
param c{j in J}, >= 0;

var  x{i in I, j in J},  binary;
var  y{j in J},  binary;
var  deltaA{i in I},  binary;

maximize z: sum{i in I, j in J} r[i,j]*x[i,j] - sum{j in J} c[j]*y[j];

s.t.  all{j in J}:  sum{i in I} x[i,j] <=  2;
  usej{j in J}: sum{i in I} x[i,j] <= n *y[j];
  two{j in A}: sum{i in I} x[i,j] + m*deltaA[j] >= 2;
  delta_y{j in A}: deltaA[j] <= y[j];
solve;

printf "\n";
for{i in I} {
   printf "\n%1d)",i;
   printf{j in J} "%5d ", x[i,j];
}
printf "\n\n-----z = %g\n\n",z;
printf "\n\n  ";
printf{j in J} "%5d ", y[j];


/* the data section is not required in the exam */
data;

param n := 5;
param m := 10;
param r :  1  2 3 4 5 6 7 8 9 10 :=
1  2  2 1 3 4 2 2 5 6  8
2  2  2 4 3 4 2 4 5 6  1
3  2  2 1 9 4 2 8 5 3  2
4  2  2 1 3 7 6 4 7 3  3
5  2  2 9 3 4 2 4 4 6  5;
param c := [1]3  [2]4 [3]6 [4]8 [5]11 [6]6 [7]9 [8]12 [9]13 [10]11;
set A := 3 4;

end;
```