RICERCA OPERATIVA

Written assesment, January 26, 2023

Last name, First name _____

Exercise 1 (value 9, time 20)

The FancyTile company needs to define the production plan for the next period. The company must produce a set O of orders. Each order $j \in O$ is defined by a product class τ_j and the total duration, p_j, for the production of the entire order. Let C be the set of all the possible product classes. The production of one order, when started, cannot be interrupted until the order is completed. Due to the contract with the workforce and technical constraints, the production must be done in *batches*, where a batch is a group of orders to be executed one after the other without interruption. If a batch contains products belonging to at least 2 different classes, the batch duration must be increased by s time units. Note that there are at most $|O|$ batches. The total duration of each batch must be at most t. Moreover, orders $\alpha \in O$ and $\beta \in O$ cannot be assigned to the same batch.

Help FancyTile to define an optimal production plan, by writing a linear mathematical model which minimizes the number of batches used, while satisfying all the constraints.

Exercise 2 (value 9, time 25)

Consider the following LP

$$\begin{array}{ll} \min & x_1 + 4x_2 + x_3 \\ & -x_1 - 4x_2 + 5x_3 = -4 \\ & -x_1 - 2x_2 + 2x_3 = 0 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

Write the dual problem, solve it and obtain the primal solution with the complementary slackness conditions.

Exercise 3 (value 9, time 30)

Consider a 0-1 knapsack problem with 6 items with profits $p_j = (10, 5, 12, 20, 8, 14)$, weights $w_j = (4, 2, 5, 10, 5, 9)$, and a knapsack capacity $c = 17$. Solve it with the branch-and-bound method, drawing the bracnh decision tree. Report the optimal profit and the items inserted in the knapsack in the optimal solution.

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Solution sketch

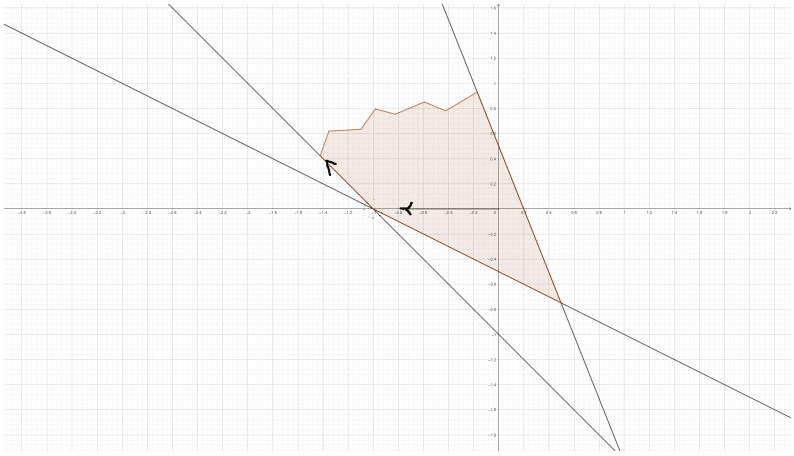
Exercise 1

$x_{ij} = 1$ if order j is produced in the i-th batch; 0 otherwise$j \in O, i \in O$
$z_i = 1$ if batch i contains some order; 0 if it is empty
$\delta_i = 1$ if batch i contains orders from at least two different classes; 0 otherwise

$$
\begin{aligned}
\min \quad & \sum_{i \in O} z_i & & \\
& \sum_{i \in O} x_{ij} & = 1 & \quad j \in O \\
& \sum_{j \in O} p_j x_{ij} + s\delta_i & \leq t z_i & \quad i \in O \\
& x_{ih} + x_{ij} & \leq 1 + \delta_i & \quad i \in O, h, j \in O, \tau_h \neq \tau_j \\
& x_{i\alpha} + x_{i\beta} & \leq 1 & \quad i \in O \\
& x_{ij} & \in \{0,1\} & \quad i, j \in O \\
& z_i & \in \{0,1\} & \quad i \in O \\
& \delta_i & \in \{0,1\} & \quad i \in O
\end{aligned}
$$

Exercise 2

$$
\begin{array}{lll}
\min & x_1 + 4x_2 + x_3 & \\
& -x_1 - 4x_2 + 5x_3 = -4 & \\
& -x_1 - 2x_2 + 2x_3 = 0 & \\
& x_1, x_2, x_3 \geq 0. &
\end{array}
\qquad
\begin{array}{lll}
\max & -4u_1 & \\
& -u_1 - u_2 \leq 1 & \\
& -4u_1 - 2u_2 \leq 4 & \\
& 5u_1 + 2u_2 \leq 1 &
\end{array}
$$



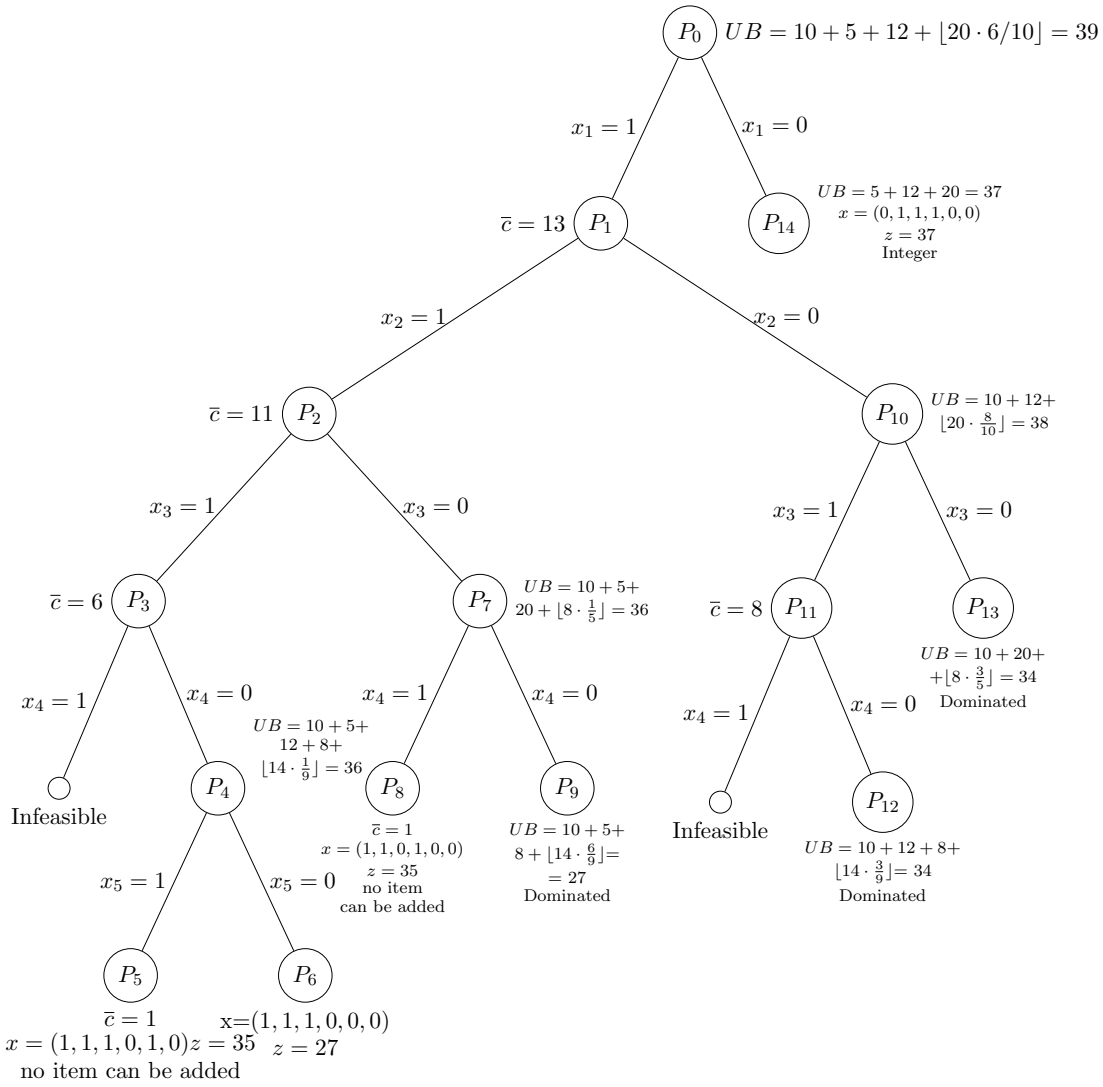
Optimal solution: Unbounded. The primal problem is therefore empty.
The complementary slackness conditions are:

$$
\begin{cases}
(-x_1 - 4x_2 + 5x_3 + 4)u_1 = 0 \\
(-x_1 - 2x_2 + 2x_3)u_2 = 0 \\
(-u_1 - u_2 - 1)x_1 = 0 \\
(-4u_1 - 2u_2 - 4)x_2 = 0 \\
(5u_1 + 2u_2 - 1)x_3 = 0
\end{cases}
$$

Exercise 3

$p_j = (10, 5, 12, 20, 8, 14)$ $w_j = (4, 2, 5, 10, 5, 9)$, $c = 17$

The objects are sorted correctly.



Optimal solution $z = 37$, items in the knapsack = 2,3,4.