RICERCA OPERATIVA

Written assessment, February 15, 2018

A

Last name______________________________ **First name**________________________

WRITE THE REQUIRED INFORMATION ONLY IN THE SPACES AVAILABLE

Exercise 1 (value 8)

Consider the following PLC problem and solve it using the Simplex algorithm with the Bland rule. Than, write the dual of the problem and find its optimal solution using the complementary slackness conditions.

$$
\begin{aligned}
\min \quad & 2x_1 + 4x_2 - 3x_3 \\
& -x_1 + 3x_2 + x_3 \leq 5 \\
& -2x_1 - 2x_2 + x_3 \geq -10 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

ANSWER: Simplex iterations (tableau)

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WRITE THE REQUIRED INFORMATION ONLY IN THE SPACES AVAILABLE

ANSWER: Dual problem

ANSWER: Complementary slackness and optimal dual solution

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Exercise 2 Step 1, value 14.
An airline company is looking for the best way to store the passenger's luggage in the aircraft. For each fly it is necessary to load on the aircraft a set $J = \{1, \ldots, n\}$ of luggage, each associated with a volume v_j ($j \in J$). Moreover, there is a set $I = \{1, \ldots, m\}$ of identical containers of capacity V, which can be loaded on the aircraft after storing some luggage inside them. Let $P = \{1, \ldots, p\}$ denote the set of passengers. Each passenger $k \in P$ is associated with the luggage of the set $S_k \subset J$, with $S_k \cap S_h = \emptyset$ for $k, h \in P$, $k \neq h$. The luggage of the same passenger must be stored into the same container $i \in I$, or in two contiguous containers (say i and $i+1$). Write a linear program aimed to define a loading of all the luggage in the containers, while minimizing the number of containers using.

Step 2, value 4. Modify the above model as follows. Assume that all the containers can be used (i.e., we do not want any more to minimize the number of containers). Let w_j be the weight of luggage $j \in J$ and let θ be a positive integer. We want to find a 'balanced' loading of the luggage such that the absolute difference of the total weight of the first θ containers and that of the last θ containers is minimized.

ANSWER: Variables

ANSWER: Objective function step 1

ANSWER: Objective function step 2

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WRITE THE REQUIRED INFORMATION ONLY IN THE SPACES AVAILABLE

ANSWER: Constraints

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Exercise 3 (value 6).
Consider a 0-1 knapsack problem with 8 items, with weights $w = (5, 5, 10, 4, 12, 7, 7)$, profits $p = (130, 115, 190, 75, 220, 125, 120)$ and bin capacity 25. Solve the problem with the branch and bound method.

ANSWER: Branch-and-bound tree.

Answers

A

First and Last name ______________________________

Exercise 1

$$\begin{aligned} \min \quad & 2x_1 + 4x_2 - 3x_3 \\ & -x_1 + 3x_2 + x_3 \leq 5 \\ & -2x_1 - 2x_2 + x_3 \geq -10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

FASE II

x_1	x_2	x_3	x_4	x_5		
2	4	-3	0	0	0	$-z$
-1	3	1	1	0	5	x_4
2	2	-1	0	1	10	x_5

x_1	x_2	x_3	x_4	x_5		
-1	13	0	3	0	15	$-z$
-1	3	1	1	0	5	x_3
1	5	0	1	1	15	x_5

x_1	x_2	x_3	x_4	x_5		
0	18	0	4	1	30	$-z$
0	8	1	2	1	20	x_3
1	5	0	1	1	15	x_1

$x = (15, 0, 20, 0, 0)$ $z_P = -30$

$$\begin{aligned} \min \quad & 2x_1 + 4x_2 - 3x_3 \\ & x_1 - 3x_2 - x_3 \geq -5 \\ & -2x_1 - 2x_2 + x_3 \geq -10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

$$\begin{aligned} \max & -5u_1 - 10u_2 \\ u_1 - 2u_2 &\leq 2 \\ -3u_1 - 2u_2 &\leq 4 \\ -u_1 + u_2 &\leq -3 \\ u_1, u_2 &\geq 0 \end{aligned}$$

$$\begin{cases} (x_1 - 3x_2 - x_3 + 5)u_1 = 0 \\ (-2x_1 - 2x_2 + x_3 + 10)u_2 = 0 \\ (u_1 - 2u_2 - 2)x_1 = 0 \\ (-3u_1 - 2u_2 - 4)x_2 = 0 \\ (-u_1 + u_2 + 3)x_3 = 0 \end{cases} \qquad \begin{cases} (0)u_1 = 0 \\ (0)u_2 = 0 \\ u_1 - 2u_2 = 2 \\ (-3u_1 - 2u_2 - 2)\ 0 = 0 \\ -u_1 + u_2 = -3 \end{cases}$$

$u = (4, 1)$ $z_D = -30$

Exercise 2
Variables

$$
\begin{aligned}
x_{ijk} &= 1 \text{ if luggage } j \text{ is stored in container } i\text{; 0 otherwise} \\
z_i &= 1 \text{ if container } i \text{ is user; 0 otherwise}
\end{aligned}
$$

$$\min z = \sum_{i \in I} z_i \tag{1}$$

$$\sum_{i \in I} x_{ij} = 1 \qquad j \in J \tag{2}$$

$$\sum_{j \in J} v_j x_{ij} \leq V z_i \qquad i \in I \tag{3}$$

$$n(1 - x_{ij}) \geq \sum_{h=i+2}^{m} \sum_{\ell \in S_p, \ell \neq j} x_{h\ell} \qquad p \in P, j \in S_p, i \in I \tag{4}$$

$$x_{ij} \in \{0, 1\} \qquad i \in I, j \in J \tag{5}$$

$$z_i \in \{0, 1\} \qquad i \in I \tag{6}$$

$$\min \Delta \tag{7}$$

$$\sum_{i \in I} x_{ij} = 1 \qquad j \in J \tag{8}$$

$$\sum_{j \in J} v_j x_{ij} \leq V \qquad i \in I \tag{9}$$

$$n(1 - x_{ij}) \geq \sum_{h=i+2}^{m} \sum_{\ell \in S_p, \ell \neq j} x_{h\ell} \qquad p \in P, j \in S_p, i \in I \tag{10}$$

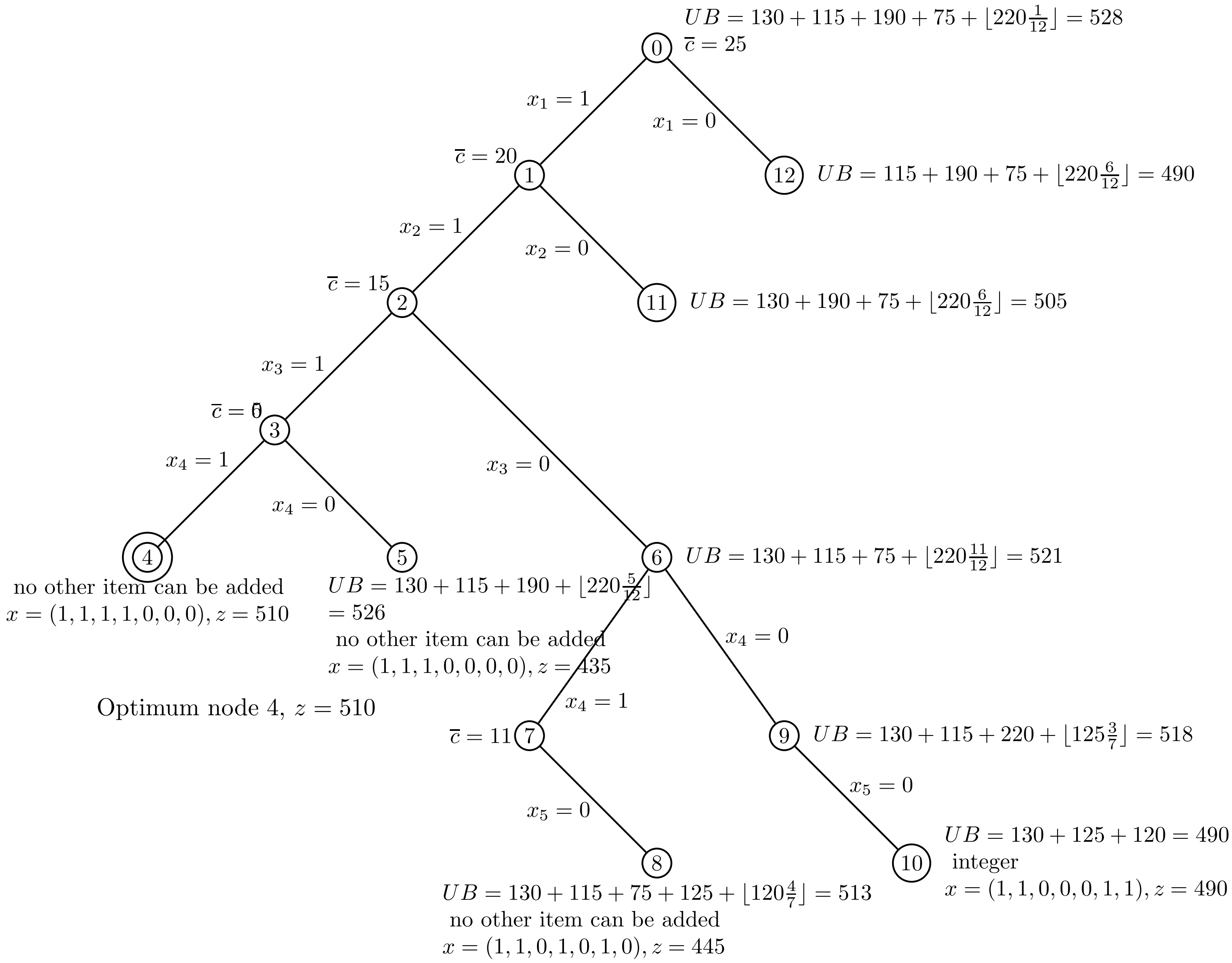
$$\sum_{i=1}^{\theta} w_j x_{ij} - \sum_{i=m-\theta+1}^{m} w_j x_{ij} \leq \Delta \tag{11}$$

$$-\sum_{i=1}^{\theta} w_j x_{ij} + \sum_{i=m-\theta+1}^{m} w_j x_{ij} \leq \Delta \tag{12}$$

$$x_{ij} \in \{0, 1\} \qquad i \in I, j \in J \tag{13}$$

$$\tag{14}$$

0 **Exercise 3**


$UB = 130 + 115 + 190 + 75 + \lfloor 220\frac{1}{12} \rfloor = 528$
$\overline{c} = 25$
$x_1 = 1$
$x_1 = 0$
$\overline{c} = 20$
$UB = 115 + 190 + 75 + \lfloor 220\frac{6}{12} \rfloor = 490$
$x_2 = 1$
$x_2 = 0$
$\overline{c} = 15$
$UB = 130 + 190 + 75 + \lfloor 220\frac{6}{12} \rfloor = 505$
$x_3 = 1$
$\overline{c} = 0$
$x_4 = 1$
$x_4 = 0$
$x_3 = 0$
$UB = 130 + 115 + 75 + \lfloor 220\frac{11}{12} \rfloor = 521$
no other item can be added
$x = (1, 1, 1, 1, 0, 0, 0), z = 510$
$UB = 130 + 115 + 190 + \lfloor 220\frac{5}{12} \rfloor$
$= 526$
no other item can be added
$x = (1, 1, 1, 0, 0, 0, 0), z = 435$
$x_4 = 0$
Optimum node 4, $z = 510$
$x_4 = 1$
$\overline{c} = 11$
$UB = 130 + 115 + 220 + \lfloor 125\frac{3}{7} \rfloor = 518$
$x_5 = 0$
$x_5 = 0$
$UB = 130 + 125 + 120 = 490$
integer
$x = (1, 1, 0, 0, 0, 1, 1), z = 490$
$UB = 130 + 115 + 75 + 125 + \lfloor 120\frac{4}{7} \rfloor = 513$
no other item can be added
$x = (1, 1, 0, 1, 0, 1, 0), z = 445$

RICERCA OPERATIVA

Written assessment, February 15, 2018

B

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WRITE THE REQUIRED INFORMATION ONLY IN THE SPACES AVAILABLE

Exercise 1 (value 8)
Consider the following PLC problem and solve it using the Simplex algorithm with the Bland rule. Than, write the dual of the problem and find its optimal solution using the complementary slackness conditions.

$$\begin{aligned} \min \quad & -2x_1 + 4x_2 - 3x_3 \\ & 2x_1 + 3x_2 + x_3 \leq 8 \\ & 2x_1 - 2x_2 + x_3 \geq -12 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

ANSWER: Simplex iterations (tableau)

Written assessment, February 15, 2018

B

Last name ______________________ **First name** ______________________

WRITE THE REQUIRED INFORMATION ONLY IN THE SPACES AVAILABLE

ANSWER: Dual problem

ANSWER: Complementary slackness and optimal dual solution

Written assessment, February 15, 2018 **B**

Last name______________________________ **First name**______________________

WRITE THE REQUIRED INFORMATION ONLY IN THE SPACES AVAILABLE

Exercise 2 Step 1, value 10.
A fruit wholesaler is looking for the best method to use its refrigerated storage system. Let $J = \{1, \dots, n\}$ be the set of the fruit batchs to be stored, each associated with a size s_j $(j \in J)$. There is a set $I = \{1, \dots, m\}$ of identical refrigerators of capacity C, that can be activated in the storage system. Let $Q = \{1 \dots, q\}$ denote a set of customers. Each customer $k \in Q$ is associated with the fruit batches of the set $S_k \subset J$, with $S_k \cap S_h = \emptyset$ for $k, h \in Q$, $k \neq h$. The fruit batch of a customer must be stored into a single refrigerator $i \in I$, or in refrigerators which are not pairwise contiguous (i.e, if a batch is in refrigerator i no batch of the same customer can be in refrigerator $i + 1$). Write a linear program aimed to define a storing plan for all the batches in the storage system, while minimizing the number of refrigerators used.
Step 2, value 4. Modify the above model as follows. Assume that all the refrigerators can be used (i.e., we do not want any more to minimize the number of refrigerators). We want to find a 'balanced' loading of the refrigerators such that the absolute difference of load in the the odd and even refrigerators is minimized.

ANSWER: Variables

ANSWER: Objective function step 1

ANSWER: Objective function step 2

Written assessment, February 15, 2018

B

Last name____________________________ **First name**________________________

WRITE THE REQUIRED INFORMATION ONLY IN THE SPACES AVAILABLE

ANSWER: Constraints

Written assessment, February 15, 2018

B

Last name______________________________ **First name**________________________

WRITE THE REQUIRED INFORMATION ONLY IN THE SPACES AVAILABLE

Exercise 3 (value 6).
Consider a 0-1 knapsack problem with 8 items, with weights $w = (5, 6, 6, 10, 4, 12, 7)$, profits $p = (130, 120, 115, 190, 75, 220, 125)$ and bin capacity 25. Solve the problem with the branch and bound method.

ANSWER: Branch-and-bound tree.

Answers **B**

First and Last name ______________________________

Exercise 1

$$
\begin{aligned}
\min \quad & -2x_1 + 4x_2 - 3x_3 \\
& 2x_1 + 3x_2 + x_3 \le 8 \\
& 2x_1 - 2x_2 + x_3 \ge -12 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

x_1	x_2	x_3	x_4	x_5		
-2	4	-3	0	0	0	$-z$
2	3	1	1	0	8	x_4
-2	2	-1	0	1	12	x_5

x_1	x_2	x_3	x_4	x_5		
0	7	-2	1	0	8	$-z$
1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	4	x_1
0	5	0	1	1	20	x_5

x_1	x_2	x_3	x_4	x_5		
4	13	0	3	0	24	$-z$
2	3	1	1	0	8	x_3
0	5	0	1	1	20	x_5

$x = (0, 0, 8, 0, 20)$ $z_P = -24$

$$
\begin{aligned}
\min -2x_1 + 4x_2 - 3x_3 & & \\
-2x_1 - 3x_2 - x_3 &\ge -8 \\
2x_1 - 2x_2 + x_3 &\ge -12 \\
x_1, x_2, x_3 &\ge 0
\end{aligned}
$$

$$
\begin{aligned}
\max -8u_1 - 12u_2 & & \\
-2u_1 + 2u_2 &\le -2 \\
-3u_1 - 2u_2 &\le 4 \\
-u_1 + u_2 &\le -3 \\
u_1, u_2 &\ge 0
\end{aligned}
$$

$$
\begin{cases}
(-2x_1 - 3x_2 - x_3 + 8)u_1 = 0 \\
(2x_1 - 2x_2 + x_3 + 12)u_2 = 0 \\
(-2u_1 + 2u_2 + 2)x_1 = 0 \\
(2u_1 - 2u_2 - 4)x_2 = 0 \\
(-u_1 + u_2 + 3)x_3 = 0
\end{cases}
\qquad
\begin{cases}
(0)u_1 = 0 \\
(20)u_2 = 0 \\
(-2u_1 + 2u_2 + 2)\ 0 = 0 \\
(2u_1 - 2u_2 - 4)\ 0 = 0 \\
(-u_1 + u_2 + 3) = 0
\end{cases}
\qquad
\begin{cases}
... \\
u_2 = 0 \\
... \\
... \\
u_1 = 3
\end{cases}
$$

$u = (3, 0)$ $z_D = -24$

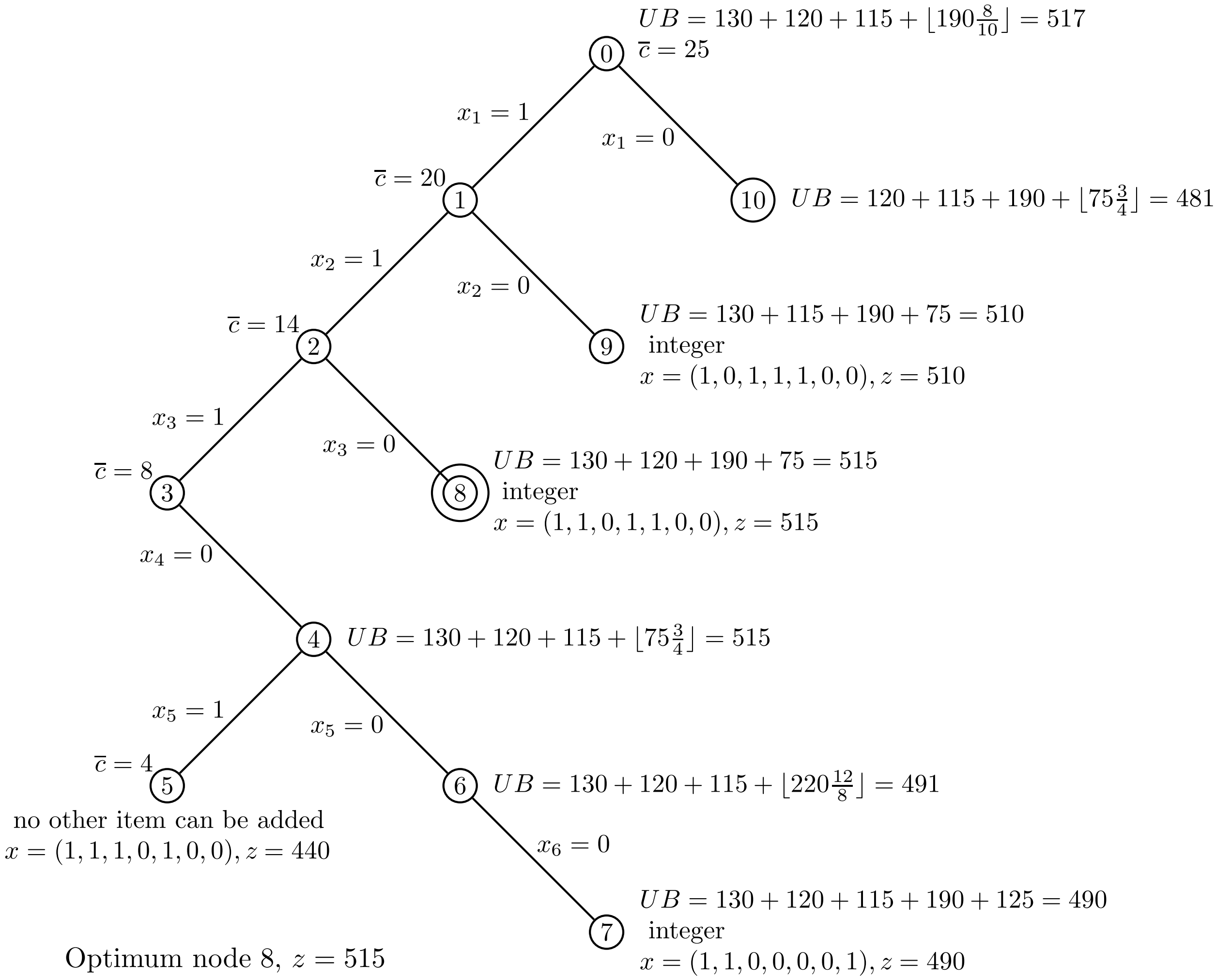
Exercise 2
Variables

$$x_{ijk} = 1 \text{ if fruit batch } j \text{ is stored in refrigerator } i\text{; 0 otherwise}$$
$$z_i = 1 \text{ if refrigerator } i \text{ is user; 0 otherwise}$$

$$\min z = \sum_{i \in I} z_i \tag{15}$$
$$\sum_{i \in I} x_{ij} = 1 \qquad j \in J \tag{16}$$
$$\sum_{j \in J} s_j x_{ij} \leq C z_i \qquad i \in I \tag{17}$$
$$n(1 - x_{ij}) \geq \sum_{\ell \in S_p, \ell \neq j} x_{i+1\ell} \qquad q \in Q, j \in S_q, i \in I \tag{18}$$
$$x_{ij} \in \{0, 1\} \qquad i \in I, j \in J \tag{19}$$
$$z_i \in \{0, 1\} \qquad i \in I \tag{20}$$

$$\min \Delta \tag{21}$$
$$\sum_{i \in I} x_{ij} = 1 \qquad j \in J \tag{22}$$
$$\sum_{j \in J} s_j x_{ij} \leq C \qquad i \in I \tag{23}$$
$$x_{ij} + x_{i+1,j} \leq 1 \qquad q \in Q, j \in S_q, i \in I \tag{24}$$
$$\sum_{i \in I: i \text{ odd}} s_j x_{ij} - \sum_{i \in I, i \text{ even}} s_j x_{ij} \leq \Delta \tag{25}$$
$$-\sum_{i \in I: i \text{ odd}} s_j x_{ij} + \sum_{i \in I, i \text{ even}} s_j x_{ij} \leq \Delta \tag{26}$$
$$x_{ij} \in \{0, 1\} \qquad i \in I, j \in J \tag{27}$$
$$\tag{28}$$

Exercise 3



Optimum node 8, $z = 515$