20200625/Exercises1

1. **MM01-value=8**

   In a summer camp there are 70 children to be allocated to 10 groups, each one with an educator that must supervise exactly 7 people. The age of each child is $e_i, i = 1, \ldots 70$.

   We want to minimize the absolute value of the maximum difference between the average age of a group and the age of each child allocated to the group (example: if the 7 children of a group have ages 8,9,10,11,11,12,12 the maximum difference for this group is $|73/7-8| = 10.428 - 8 = 2.428$)

   Write an integer linear programming model in order to decide how to allocate the children to the groups. Clearly define the variables used.

   Notes: (not included in XML)

   - $x_{ig} = 1$ if child $i = 1, \ldots, 70$ is in group $g = 1 \in 10, 0$ otherwise.
     Δ maximum difference of age.

   - $$\min \Delta$$
     $$\Delta \geq \sum_{i=1}^{70} e_i x_{ig}/7 - e_j x_{jg}, \quad j = 1, \ldots, 70, g = 1, \ldots, 10$$
     $$\Delta \geq -\sum_{i=1}^{70} e_i x_{ig}/7 + e_j x_{jg}, \quad j = 1, \ldots, 70, g = 1, \ldots, 10$$
     $$\sum_{g=1}^{10} x_{ig} = 1 \quad i = 1, \ldots, 70$$
     $$\sum_{i=1}^{70} x_{ig} = 7 \quad g = 1, \ldots, 10$$
     $$x_{ig} \in \{0, 1\} \quad i = 1, \ldots, 70, g = 1, \ldots, 10$$
     $$\Delta \geq 0$$

2. **MM03-value=6**

   Consider this minimization problem:

$$\begin{aligned} \min\quad & 8x_1 + 3x_2 + x_3 \\ & 2x_1 + 7x_2 - 5x_3 = 4 \\ & 4x_1 - 3x_2 + x_3 \geq 8 \\ & x_1 \geq 0 \\ & x_2 \leq 0 \\ & x_3 \quad free \end{aligned}$$

   Write the corresponding canonical form.

   Notes: (not included in XML)

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$$\begin{aligned} \min\ & 8x_1 - 3x_2 - x_3^- + x_3^+ \\ & 2x_1 - 7x_2 + 5x_3^- - 5x_3^+ \geq 4 \\ & -2x_1 + 7x_2 - 5x_3^- + 5x_3^+ \geq -4 \\ & 4x_1 + 3x_2 - x_3^- + x_3^+ \geq 8 \\ & x_1, x_2, x_3^-, x_3^+ \geq 0 \end{aligned}$$

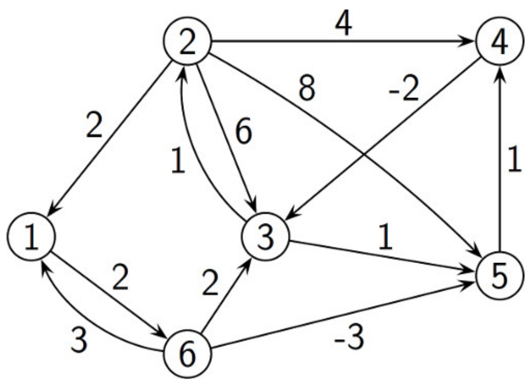
20200625/Exercises2

1. **MM02-value=6**

   Consider the graph below and the Shortest Path Problem from vertex 2. In the next table are reported the first iterations of the Bellman-Ford dynamic programming method to compute the shortest path. Continue to apply the method executing one iteration.

   Report the labels of iteration 3.

   Report the shortest path from 2 to 5 as results at the end of the third iteration



	$f(j)$						$pred$					
Iter	1	2	3	4	5	6	1	2	3	4	5	6
0	–	0	–	–	–	–	–	2	–	–	–	–
1	2	0	6	4	8	–	2	2	2	2	2	–
2	2	0	2	4	7	4	2	2	4	2	3	1
3												

Notes: (not included in XML)

Iter	$f(j)$						pred					
	1	2	3	4	5	6	1	2	3	4	5	6
0	-	0	-	-	-	-	-	2	-	-	-	-
1	2	0	6	4	8	-	2	2	2	2	2	-
2	2	0	2	4	7	4	2	2	4	2	3	1
3	2	0	2	4	1	4	2	2	4	2	6	1
4	2	0	2	2	1	4	2	2	4	5	6	1
5	2	0	0	2	1	4	2	2	4	5	6	1

  The shortest path at the end of iteration 3 has length 1 and is (2, 1, 6, 5)

2. **MM04-value=4**

   Consider the following LP model and write an implementation in GLPK or XPRESS

$$\begin{aligned} \max \quad & \sum_{i=2}^{n} \sum_{j \in R} c_{ij} x_{ij} \\ & \sum_{i=1}^{n-1} x_{ij} \leq b_j, j \in R \setminus \{7\} \\ & x_{ij} \geq 0, i = 1, \ldots, n, j \in R \end{aligned}$$

Notes: (not included in XML)

- ```
  param n integer > 0;
  set I := 1..n;
  set I2 := 2..n;
  set In1 := 1..n-1;
  set R;
  param c{i in I, j in R} integer > 0;
  param b{i in I} integer > 0;
  var x { i in I, j in R } >= 0 ;
  maximize z :  sum {i in I2, j in R} c[i,j]*x[i,j];
  C1{j in R: j <> 7} :  sum {i in In1} x[i,j] <= b[j];
  solve ;
  end;
  ```

3. **MM04-value=6**

   Consider the following LP problem.

$$\begin{aligned} \min \quad & 4x_1 + 5x_2 + 3x_3 \\ & 3x_1 + 4x_3 \geq 1 \\ & 2x_1 + x_2 + x_3 \geq 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

   We want to solve it with the two-phase method.

   - Write the problem to be solved in the first phase (with auxiliary variables).
   - Perform one iteration of the symplex algorithm on the tableau of the first phase problem, using the Bland's rule during the iteration.
  ```

- Is the obtained solution the optimal solution of the first phase problem? Justify your answer.

Notes: (not included in XML)

- First Phase problem:

$$\begin{aligned} \min\, & a_1 + a_2 \\ & 3x_1 + 4x_3 - x_4 + a_1 = 1 \\ & 2x_1 + x_2 + x_3 - x_5 + a_2 = 3 \\ & x_1, x_2, x_3, x_4, a_1, a_2 \geq 0 \end{aligned}$$

x_1	x_2	x_3	x_4	x_5	a_1	a_2		
-5	-1	-5	1	1	0	0	-4	$-\xi$
3	0	4	-1	0	1	0	1	a_1
2	1	1	0	-1	0	1	3	a_2

x_1	x_2	x_3	x_4	x_5	a_1	a_2		
0	-1	$\frac{5}{3}$	$-\frac{2}{3}$	1	$\frac{5}{3}$	0	$-\frac{7}{3}$	$-\xi$
1	0	$\frac{4}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$	x_1
0	1	$-\frac{5}{3}$	$\frac{2}{3}$	-1	$-\frac{2}{3}$	1	$\frac{7}{3}$	a_2

The solution after one iteration is: (1/3, 0, 0, 0, 0, 0, 7/3) with value 7/3.
This is not the optimal solution because, there is a negative reduced cost.