The software house ICT_2.0 is going to plan its activities for the next months. The projects in charge to ICT_2.0 require to execute a set $\mathcal{T}$ of n tasks in this period. The company has m employees with different skills. Each task j can be done by a subset of the employees. The time required by employee i to perform task j is t_{ij}, with $t_{ij} = \infty$ if the employee has not the skills required for task j. The cost of one time unit of an employee i is c_i euro. Each employee i has a maximum working time T_i. If a task is not assigned to one of the employees it must be made by some external supplier. There are K possible suppliers. Each supplier k can execute at most M_k tasks. A task j executed by supplier k costs d_{kj} euro. For commercial reasons, if a task is assigned to a supplier k with $k > 1$, then some task must be assigned to the preferred supplier $k = 1$. Moreover, given the sets $P_1 \subset \mathcal{T}$ and $P_2 \subset \mathcal{T}$, with $P_1 \cap P_2 = \emptyset$, corresponding to two different and competing projects, no task of P_1 (res. P_2) can be assigned to an employee that executes some task of P_2 (rsp. P_1).

Write a Linear Programming model which satisfies all constraints minimizing the total cost.

Solution

x_{ij} = 1 if task j is executed by employee i; 0 otherwise

y_{kj} = 1 if task j is executed by supplier k; 0 otherwise

$p1_i$ = 1 if employee i executes one or more tasks from project $P1$; 0 otherwise

$p2_i$ = 1 if employee i executes one or more tasks from project $P2$; 0 otherwise

$$\min \sum_{i=1}^{m}\sum_{j=1}^{n} c_i t_{ij} x_{ij} + \sum_{k=1}^{K}\sum_{j=1}^{n} y_{kj} d_{kj} \tag{1}$$

$$\sum_{i=1}^{m} x_{ij} + \sum_{k=1}^{K} y_{kj} = 1 \quad j = 1, 2, \ldots, n \tag{2}$$

$$\sum_{j=1}^{n} t_{ij} x_{ij} \leq T_i \quad i = 1, 2 \ldots, m \tag{3}$$

$$\sum_{j=1}^{n} y_{kj} \leq M_k \quad k = 1, 2 \ldots, K \tag{4}$$

$$\sum_{k=2}^{K}\sum_{j=1}^{n} y_{kj} \leq m \sum_{j=1}^{n} y_{1j} \tag{5}$$

$$\sum_{j \in P1} x_{ij} \leq |P1| p1_i \quad i = 1, 2, \ldots, m \tag{6}$$

$$x \sum_{j \in P2} x_{ij} \leq |P2| p2_i \quad i = 1, 2, \ldots, m \tag{7}$$

$$p1_i + p2_i \leq 1 \quad i = 1, 2, \ldots, m \tag{8}$$

$$x_{ij} \in \{0, 1\} \quad i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \tag{9}$$

$$y_{kj} \in \{0, 1\} \quad k = 1, 2, \ldots, K, j = 1, 2, \ldots, n \tag{10}$$

$$p1_i \in \{0, 1\} \quad i = 1, 2, \ldots, m \tag{11}$$

$$p2_i \in \{0, 1\} \quad i = 1, 2, \ldots, m \tag{12}$$