RICERCA OPERATIVA

Written assessment , June 29, 2017 A

Last name______________________ First name______________________

Exercise 1 (value 14)
A manager has to decide how to store the products into a new warehouse. There are n different kinds of product, and each kind j consists of q_j units. Moreover a kind j has an associated priority p_j which is higher when the frequency of picking is greater. The warehouse consists of identical shelves, numbered from 1 to m, each with a capacity V. Each item of kind j occupies a volume v_j. The guiding rule to decide in which shelf we have to store a product is that the products with higher priority must be stored in the shelves closest to the picking area. The distance of the shelves from the picking area is proportional to the number of the shelf. More precisely, the measure of the quality of the solution is as follows: if one or more products of kind j are stored in shelf i, their contribution to the measure is $i \cdot p_j$ (independently of the number of products stored in i). It is required to write a Linear Programming model to help the manager to decide how to store all products, by minimizing the above measure, and respecting the capacity constraints. Moreover, there are additional requirements due to the relations among the products: (i) if a product of a set $R \subset \{1, \dots, n\}$ is stored in shelf i, there must be at least a product of a set $S \subset \{1, \dots, n\}$ stored in a shelf whose distance from i is smaller or equal to d; (ii) if more than τ products of kind $\bar{\jmath}$ are stored in shelf i, there must be no product of a set $Q \subset \{1, \dots, n\}$ stored in the same shelf.

Exercise 2 (value 7)
Consider the following PLC problem. Solve it with the simplex method, write the dual and solve the dual using the optimality conditions.

$$
\begin{aligned}
\min \quad & 4x_1 + 6x_2 + 3x_3 \\
& 4x_1 + 2x_2 - 2x_3 \leq 2 \\
& 6x_1 + x_2 \geq 6 \\
& x_1 + 2x_3 \leq 10 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Exercise 3 (value 5)
Write a GLPK or XPRESS model implementing the following PLI problem.

$$\min\ z = \sum_{j=1}^{n} c_{ij} x_{0j}$$

$$\sum_{j=1}^{n+1} x_{0j} - \sum_{j=0}^{n} x_{j,n+1} = 0; \tag{11}$$

$$\sum_{j=0}^{n+1} x_{ij} - \sum_{j=0}^{n+1} x_{ji} = 0 \quad i = 1, \dots, n; \tag{12}$$

$$\sum_{\substack{i=0 \\ i \notin S_k}}^{n+1} \sum_{j \in S_k} x_{ij} \leq 1 \quad k = 1, \dots, p; \tag{13}$$

$$\sum_{i \in S_k} \sum_{j \in S_k} x_{ij} \geq 1 \quad k = 1, \dots, p; \tag{14}$$

$$x_{ij} \in \{0, 1\} \quad i, j = 0, \dots, n+1, i \neq j; \tag{15}$$

$$\tag{16}$$

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First and Last name ____________________

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Exercise 1

$$
\begin{aligned}
x_{ij} &= \text{1 if products of kind } j \text{ are sotred in shelf } i\text{; 0 otherwise} \\
y_{ij} &= \text{number of products of kind } j \text{ stored in shelf } i \\
\delta_i &= \text{ if } \tau \text{ or more products of kind } \overline{\jmath} \text{ are in } i\text{; 0 otherwise}
\end{aligned}
$$

$$
\begin{aligned}
\min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n} i \cdot p_j x_{ij} & & \\
\sum_{i=1}^{m} y_{ij} = q_j & \quad j = 1,\dots,n; & (17)\\
\sum_{j=1}^{n} y_{ij} v_j \le V & \quad i = 1,\dots,m; & (18)\\
y_{ij} \le q_j x_{ij} & \quad i = 1,\dots,m, j = 1,\dots,n; & (19)\\
x_{ij} \le \sum_{h=max(1,i-d)}^{min(m,i+d)} \sum_{\ell\in S} x_{h\ell} & \quad i = 1,\dots,m, j \in R; & (20)\\
y_{i\overline{\jmath}} - \tau \le q_{\overline{\jmath}}\delta_i & \quad i = 1,\dots,m; & (21)\\
\sum_{j\in Q} x_{ij} \le |Q|(1-\delta_i) & \quad i = 1,\dots,m; & (22)\\
x_{ij} \in \{0,1\} & \quad i = 1,\dots,m, j = 1,\dots,n; & (23)\\
y_{ij} \ge 0 & \quad i = 1,\dots,m, j = 1,\dots,n; & (24)\\
\delta_i \in \{0,1\} & \quad i = 1,\dots,m. & (25)\\
& & (26)
\end{aligned}
$$

Exercise 2

$$\begin{array}{ll} \min & 4x_1 + 6x_2 + 3x_3 \\ & 4x_1 + 2x_2 - 2x_3 \le 2 \\ & -6x_1 - x_2 \le -6 \\ & x_1 + 2x_3 \le 10 \\ & x_1, x_2, x_3 \ge 0 \end{array}$$

x_1	x_2	x_3	x_4	x_5	x_6		
4	6	3	0	0	0	0	$-z$
4	2	-2	1	0	0	2	x_4
-6	-1	0	0	1	0	-6	x_5
1	0	2	0	0	1	10	x_6

x_1	x_2	x_3	x_4	x_5	x_6		
0	$\frac{16}{3}$	3	0	$\frac{2}{3}$	0	-4	$-z$
0	$\frac{4}{3}$	-2	1	$\frac{2}{3}$	0	-2	x_4
1	$\frac{1}{6}$	0	0	$-\frac{1}{6}$	0	1	x_1
0	$-\frac{1}{6}$	2	0	$\frac{1}{6}$	1	9	x_6

x_1	x_2	x_3	x_4	x_5	x_6		
0	$\frac{22}{3}$	0	$\frac{3}{2}$	$\frac{5}{3}$	0	-7	$-z$
0	$-\frac{2}{3}$	1	$-\frac{1}{2}$	$-\frac{1}{3}$	0	1	x_3
1	$\frac{1}{6}$	0	0	$-\frac{1}{6}$	0	1	x_1
0	$\frac{7}{6}$	0	1	$\frac{5}{6}$	1	7	x_6

$x = (1, 0, 1, 0, 0, 7)$ $z_P = 7$

$$\begin{array}{ll} \min & 4x_1 + 6x_2 + 3x_3 \\ & 4x_1 + 2x_2 - 2x_3 \le 2 \\ & -6x_1 - x_2 \le -6 \\ & x_1 + 2x_3 \le 10 \\ & x_1, x_2, x_3 \ge 0 \end{array} \qquad \begin{array}{ll} \max & 2u_1 - 6u_2 + 10u_3 \\ & 4u_1 - 6u_2 + u_3 \ge 4 \\ & 2u_1 - u_2 \ge 6 \\ & -2u_1 + 2u_3 \ge 3 \\ & u_1, u_2, u_3 \le 0 \end{array}$$

$$\left\{ \begin{array}{ll} (4x_1 + 2x_2 - 2x_3 - 2)u_1 & = 0 \\ (-6x_1 - x_2 + 6)u_2 & = 0 \\ (x_1 + 2x_3 - 10)u_3 & = 0 \\ (4u_1 - 6u_2 + u_3 - 4)x_1 & = 0 \\ (2u_1 - u_2 - 6)x_2 & = 0 \\ (-2u_1 + 2u_3 - 3)x_3 & = 0 \end{array} \right. \qquad \left\{ \begin{array}{ll} (0)u_1 & = 0 \\ (0)u_2 & = 0 \\ (-7)u_3 & = 0 \\ (4u_1 - 6u_2 + u_3 - 4) & = 0 \\ (2u_1 - u_2 - 6)0 & = 0 \\ (-2u_1 + 2u_3 - 3) & = 0 \end{array} \right.$$

$$\left\{ \begin{array}{ll} -- & \\ -- & \\ u_3 = 0 & \\ (4u_1 - 6u_2 - 4) & = 0 \\ -- & \\ (-2u_1 - 3) & = 0 \end{array} \right. \qquad \left\{ \begin{array}{ll} -- & \\ -- & \\ u_3 & = 0 \\ u_2 & = -5/3 \\ -- & \\ u_1 & = -3/2 \end{array} \right.$$

$u = (-3/2, -5/3, 0)$ $z_D = 7$

Exercise 3

```
/* Written in GNU MathProg */

param n, integer, > 0;
param p, integer, > 0;

set I  := 0..n+1;
set Ip := 1..n;
set P  := 1..p;
set S{k in P};

param q{i in I}, >= 0;
param c{i in I, j in I}, >= 0;

var  x{i in I, j in I},   >= 0, binary;

minimize z: sum{j in I} c[0,j]* x[0,j];
s.t.
      V1: sum{j in I: j != 0}  x[0,j] - sum{j in I: j != n+1} x[j,n+1] = 0;
      V2{i in Ip}: sum{j in I} x[i,j] - sum{j in I} x[j,i] = q[i];
      V3{k in P}: sum{i in I diff S[k], j in S[k]} x[i,j] <= 1;
      V4{k in P}: sum{i in S[k], j in S[k]} x[i,j] >= 1;

solve;


end;
```