Written assessment, September 14, 2017 **A**

First and Last name ______________________

Exercise 1 (value 13)
Mrs Smith must decide how to invest her saving amounting to B€, in the following T years. She has identified the following opportunities.
A) Single year investments. This is a set K^I of investments that can be activated and deactivated as many times as we want, but each investment has the minimum duration of one year. Each investment $k \in K^I$ can be activated with any amount between m_k^I and M_k^I euros ($m_k^I < M_k^I$). The revenue of investment $k \in K^I$ is r_k^I% each year.
B) Two years investments. This is a set K^{II} of investments that can be activated and deactivated as many times as we want, but each investment has the minimum duration of two years. This kind of investment cannot be activated in year $T-1$. Each investment $k \in K^{II}$ can be activated with any amount between m_k^{II} and M_k^{II} euros ($m_k^{II} < M_k^{II}$). The revenue of investment $k \in K^{II}$ is r_k^{II}% each year.
In addition, at most two investments from the set $\mathcal{O} \subseteq K^I \cup K^{II}$ can be active in the same year. Mrs Smith wants to plan the investments for the next T years, maximizing the total revenue at the end of the period.

Exercise 2 (value 8)
Consider the following PLC problem. Solve it with the simplex method, using the Bender's rule, than write the dual problem and compute the corresponding solution.

$$
\begin{aligned}
\min -2x_1 + x_3 & \\
-x_1 - 3x_2 + x_3 + x_4 &= 6 \\
-x_1 + x_2 + 2x_3 + x_4 &= 12 \\
x_1, \ldots, x_4, &\geq 0
\end{aligned}
$$

Exercise 3 (value 7)
Consider a knapsack problem with 5 objects and a bin of capacity C=28. The object's profits and weights are, respectively, are $p = (20, 25, 16, 11, 5)$ $w = (10, 15, 12, 10, 5)$. Solve the problem with the branch-and-bound method.

Exercise 1

Variables

$$
\begin{aligned}
x_{kt} &= \text{amount of euros invested in } k \in K^I \text{ in year } t;\\
y_{kt} &= \text{amount of euros invested in } k \in K^{II} \text{ in year } t;\\
\hat{x}_{kt} &= 1 \text{ if investment } k \in K^I \text{ is activated in year } t, 0 \text{ otherwise;}\\
\hat{y}_{kt} &= 1 \text{ if investment } k \in K^{II} \text{ is activated in year } t, 0 \text{ otherwise;}\\
B_t &= \text{budget available for investments at the beginning of year } t.
\end{aligned}
$$

$$\max\ z = \sum_{t=1}^{T} \sum_{k \in K^I} r_k^I x_{kt} + \sum_{t=1}^{T-1} \sum_{k \in K^{II}} r_k^{II} y_{kt}$$

$$B_1 = B \tag{1}$$

$$B_2 = B_1 + \sum_{k \in K^I} y_{k,1} r_k^I \tag{2}$$

$$B_t = B_{t-1} + \sum_{k \in K^{II}} y_{k,t-2} * (1 + r_k^{II}) - \sum_{k \in K^{II}} y_{k,t-1} + \sum_{k \in K^I} y_{k,t-1} r_k^I \quad t = 3, \dots, T \tag{3}$$

$$\sum_{k \in K^I} x_{kt} + \sum_{k \in K^{II}} y_{kt} \leq B_t \tag{4}$$

$$x_{kt} \leq M_k^I\ \widehat{x}_{kt} \quad k \in K^I, t = \ \dots, T \tag{5}$$

$$y_{kt} \leq M_k^{II}\ \widehat{y}_{kt} \quad k \in K^{II}, t = \ \dots, T-1 \tag{6}$$

$$x_{kt} \geq m_k^I\ \widehat{x}_{kt} \quad k \in K^I, t = \ \dots, T \tag{7}$$

$$y_{kt} \geq m_k^{II}\ \widehat{y}_{kt} \quad k \in K^{II}, t = \ \dots, T-1 \tag{8}$$

$$\sum_{k \in \mathcal{O}} (\widehat{x}_{k1} + \widehat{y}_{k1}) \leq 2 \tag{9}$$

$$\sum_{k \in \mathcal{O}} (\widehat{x}_{kt} + \widehat{y}_{kt} + \widehat{y}_{k,t-1}) \leq 2 \quad t = 2, \dots, T-1 \tag{10}$$

$$\sum_{k \in \mathcal{O}} (\widehat{x}_{kT} + \widehat{y}_{k,t-1}) \leq 2 \tag{11}$$

$$x_{kt} \geq 0 k \in K^I, \quad t = 1, \dots, T \tag{12}$$

$$y_{kt} \geq 0 k \in K^{II}, \quad t = 1, \dots, T-1 \tag{13}$$

$$B_t \geq 0 \quad t = 1, \dots, T \tag{14}$$

$$\widehat{x}_{kt} \in \{0,1\} \quad k \in K^I, \quad t = 1, \dots, T \tag{15}$$

$$\widehat{y}_{kt} \in \{0,1\} \quad k \in K^{II}, \quad t = 1, \dots, T-1 \tag{16}$$

Exercise 2

$$\begin{array}{rcl} \min -2x_1 + x_3 & & \\ -x_1 - 3x_2 + x_3 + x_4 & = & 6 \\ -x_1 + x_2 + 2x_3 + x_4 & = & 12 \\ x_1, \ldots, x_4, & \geq & 0 \end{array}$$

PHASE I

x_1	x_2	x_3	x_4	x_1^a	x_2^a		
2	2	-3	-2	0	0	-18	$-\xi$
-1	-3	1	1	1	0	6	x_1^a
-1	1	2	1	0	1	12	x_2^a

x_1	x_2	x_3	x_4	x_1^a	x_2^a		
-1	-7	0	1	3	0	0	$-\xi$
-1	-3	1	1	1	0	6	x_3
1	7	0	-1	-2	1	0	x_2^a

x_1	x_2	x_3	x_4	x_1^a	x_2^a		
0	0	0	0	1	1	0	$-\xi$
0	4	1	0	-1	1	6	x_3
1	7	0	-1	-2	1	0	x_1

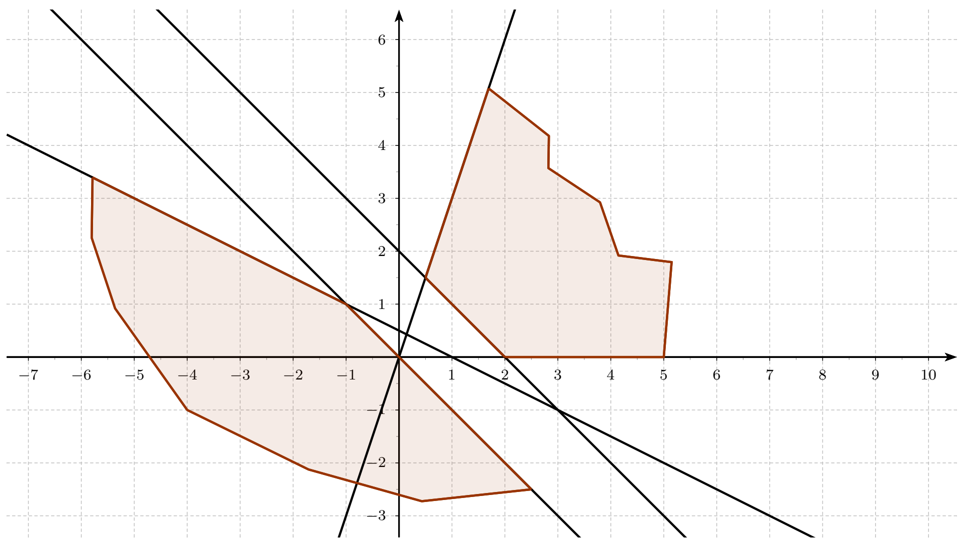
PHASE II

x_1	x_2	x_3	x_4		
0	10	0	-2	-6	$-z$
0	4	1	0	6	x_3
1	7	0	-1	0	x_1

The problem is unlimited

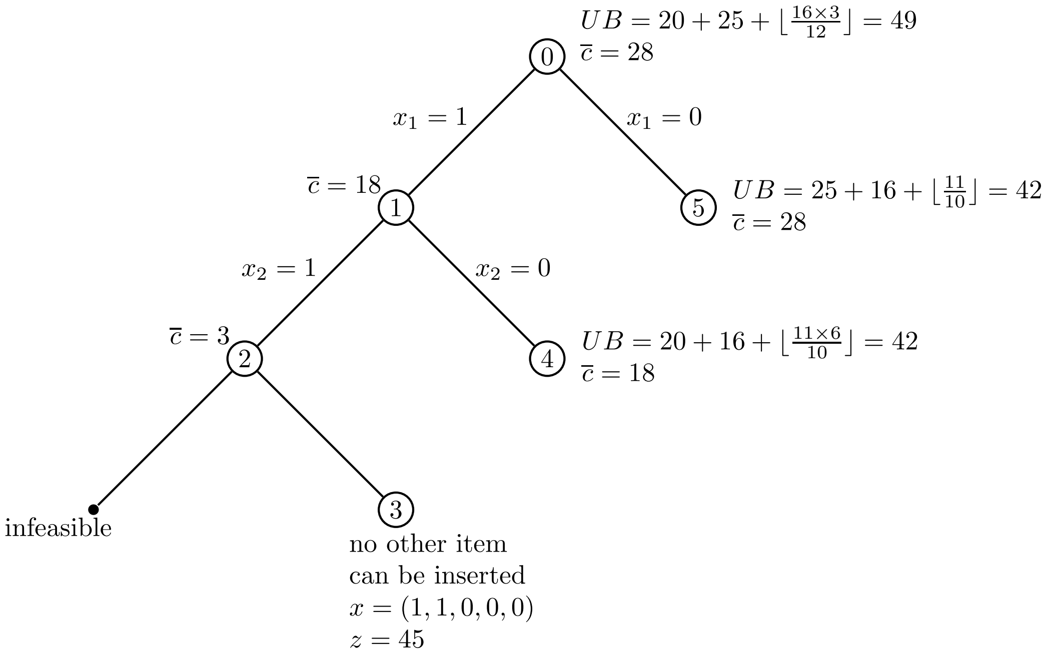
$$\begin{array}{rcl} \min -2x_1 + x_3 & & \\ -x_1 - 3x_2 + x_3 + x_4 & = & 6 \\ -x_1 + x_2 + 2x_3 + x_4 & = & 12 \\ x_1, \ldots, x_4, & \geq & 0 \end{array}$$

$$\begin{array}{rcl} \max\ 6u_1 + 12u_2 & & \\ u_1 + u_2 & \geq & 2 \\ -3u_1 + u_2 & \leq & 0 \\ u_1 + 2u_2 & \leq & 1 \\ u_1 + u_2 & \leq & 0 \end{array}$$



Problem is **empty**.

Exercise 3



Optimal solution: node 3, $z = 45$