RICERCA OPERATIVA

Written assessment, February 21, 2024

Last name, First name ________________________________

Exercise 1

A set of courses needs to be scheduled in a certain number of available rooms. To avoid clashes, a room cannot be allocated to more than one course at the same time, while a course is given in exactly one room. Besides, the room allocated to the course shall be large enough to host all the students of that course. The set of the rooms is denoted by R, the set of the courses is denoted by C, and the set of teachers is denoted by T. The set of courses given by teacher $t \in T$ is denoted by $C_t \subseteq C$. Parameter S_r denotes the capacity (number of seats) of room $r \in R$. Parameter N_c denotes the number of students of course $c \in C$. Parameter P_{cr} denotes the teachers' preferences for having course c in room r. Higher values represent higher preferences. Write an integer linear program to help the planner to find the best scheduling, with the objective of *maximising* the sum of the preferences given to the teacher which receives the *minimum* preferences.

Exercise 2

Consider the following problem

$$
\begin{aligned}
\max \quad & 7x_1 + 9x_2 + 5x_3 \\
& -1/2x_1 + 3/2x_2 + x_3 = 12 \\
& 7x_1 + x_2 \leq 4 \\
& x_2 \leq 4 \\
& x_1, x_2, x_3 \geq 0, \text{integer}
\end{aligned}
$$

Solve it using the standard branch-and-bound method for PLI. Always apply the Bland's rule in the tableaus and, if needed, branch on the fractional variable with smaller index. Eventually explore a maximum of 4 nodes of the decision tree.

Exercise 3

Consider a Knapsack problem with four objects characterized by the following profits $p = (1, 2, 3, 2)$ and weights $w = (10, 5, 8, 7)$, and with a container of capacity $c = 19$. Calculate the optimal solution using dynamic programming. Report all the steps of the algorithm together with the final solution and its cost.

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Solution sketch

Exercise 1

$x_{cr} =$ 1 if course c is allocated in room r, 0 otherwise
$z =$ minimum value of total preference among the teachers

$$
\begin{array}{ll}
\max z & \\
z \le \sum_{c \in C_t} \sum_{r \in R} p_{cr} x_{cr} & t \in T \\
\sum_{r \in R} x_{cr} = 1 & c \in C \\
\sum_{c \in C} x_{cr} \le 1 & r \in R \\
x_{cr} \le \dfrac{S_r}{N_c} & c \in C, r \in R \\
x_{cr} \in \{0,1\} & c \in C, r \in R \\
z \ge 0 &
\end{array}
$$

Exercise 2

We solve the linear relaxation at the root node of the search tree, after having added the required slack variables.

x_1	x_2	x_3	s_1	s_2		
7	9	5	0	0	0	$-z$
-1/2	3/2	1	0	0	12	x_3
7	1	0	1	0	4	s_1
0	1	0	0	1	4	s_2

We take the tableau into the basis form.

x_1	x_2	x_3	s_1	s_2		
19/2	3/2	0	0	0	-60	$-z$
-1/2	3/2	1	0	0	12	x_3
$\boxed{7}$	1	0	1	0	4	s_1
0	1	0	0	1	4	s_2

x_1	x_2	x_3	s_1	s_2		
0	1/7	0	-19/14	0	-458/7	$-z$
0	11/7	1	1/14	0	86/7	x_3
1	$\boxed{1/7}$	0	1/7	0	4/7	x_1
0	1	0	0	1	4	s_2

x_1	x_2	x_3	s_1	s_2		
-1	0	0	-3/2	0	-66	$-z$
-11	0	1	-3/2	0	6	x_3
7	1	0	1	0	4	x_2
-7	0	0	-1	1	0	s_2

The optimal solution of the root of the search tree is integer, so we can stop.
The optimal integer solution is $x = (0, 4, 6)$ with $z = 66$.

Exercise 3

$P = 1 + 2 + 3 + 2 = 8$

	0	1	2	3	4	5	6	7	8
f_0	0	∞	∞	∞	∞	∞	∞	∞	∞
f_1	0	10	∞	∞	∞	∞	∞	∞	∞
f_2	0	10	5	15	∞	∞	∞	∞	∞
f_3	0	10	5	8	18	13	23	∞	∞
f_4	0	10	5	8	12	$\boxed{13}$	23	20	30

	0	1	2	3	4	5	6	7	8
J_0	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
J_1	$\emptyset$	$\{1\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
J_2	$\emptyset$	$\{1\}$	$\{2\}$	$\{1, 2\}$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
J_3	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1, 3\}$	$\{2, 3\}$	$\{1, 2, 3\}$	$\emptyset$	$\emptyset$
J_4	$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{2, 4\}$	$\boxed{\{2,3\}}$	$\{1, 2, 3\}$	$\{2, 3, 4\}$	$\{1, 2, 3, 4\}$

The optimal solution $x = \{2, 3\}$ has profit 5 and weight 13.