Assignment, February 17, 2017

A

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Last name________________________ First name________________________

Exercise 1 (value 12)
The national company for electric energy is going to plan the use of its power generators for a time horizon T of time periods. Let N be the set of power generators and let M be the set of customers. For each customer $j \in M$ and each time period $t \in T$ it is known the forecasted demand d_{jt}. For each power generator $i \in N$ it is know the maximum power P_i that can be produced in each time period, and the set $S(i) \subseteq M$ of the customers that may be served by generator i. A unit of energy produced by generator $i \in N$ in period $t \in T$ has a cost g_{it}. The energy produced by generator $i \in N$ can be transferred to customer $j \in M$ only if a *link* has been established between i and j. The installation of a link (i, j) has cost c_{ij}. To have a robust service, in case of failure of a power generator, it is required that each customer is linked to at least 2 generators.

Write a linear program to help the company to define an optimal plan to install the links among generators and customers, and to provide the required energy to each customer, while minimizing the total cost.

Improve the above model by adding the following constraints: in each time period each customer can receive energy from no more than three generators;

Exercise 2 (value 8)
Consider the following PLC problem. Solve it with the simplex method.

$$
\begin{aligned}
\min \quad & x_1 + 2x_2 + 1x_3 \\
& 2x_1 + 3x_2 - 2x_3 = 20 \\
& x_1 - 2x_2 + 3x_3 = 12 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Add the integrality constraints for the variables x and apply the cutting plane method with Gomory's cuts to look for an integer solution. Always choose the first row available to generate a cut. Stop the algorithm after adding at most two cuts.

Exercise 3 (value 8)
Consider a 0-1 knapsack problem with 5 objects and a bin of capacity 30. The object's profits are: $p_j = (7, 12, 11, 8, 9)$, while the object's weights are $w_j = (4, 7, 7, 5, 9)$. Find the optimal solution using the branch-and-bound method.

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First and Last name ______________________________

Exercise 1

$$
\begin{aligned}
x_{ij} &= \text{1 if generator } i \in N \text{ is linked to customer } j \in S_(i)\text{, 0 otherwise}\\
f_{ijt} &= \text{quantity of energy that generator } i \text{ gives to customer } j\text{, 0 otherwise}\\
\delta_{ijt} &= \text{1 if generator } i \text{ provides energy to customer } j \text{ at time } t\text{, 0 otherwise}
\end{aligned}
$$

$$
\min \; z = \sum_{i \in N} \sum_{j \in S(i)} c_{ij} x_{ij} + \sum_{i \in N} \sum_{j \in S(i)} \sum_{t \in T} g_{it} f_{ijt}
$$

$$
\sum_{i \in N : j \in S(i)} f_{ijt} \geq d_{jt} \quad \forall j \in M, \forall t \in T; \tag{18}
$$

$$
\sum_{t \in T} f_{ijt} \leq (|T| \max_{t \in T} d_{jt}) x_{ij} \quad \forall i \in N, \forall j \in S(i); \tag{19}
$$

$$
\sum_{j \in S(i)} f_{ijt} \leq P_i \quad \forall i \in N, \forall t \in T; \tag{20}
$$

$$
\sum_{i \in N : i \in S(i)} x_{ij} \geq 2 \quad \forall j \in M; \tag{21}
$$

$$
f_{ijt} \leq d_{jt} \delta_{ijt} \quad \forall i \in N, \forall j \in S(i), \forall t \in T; \tag{22}
$$

$$
\sum_{i \in N} \delta_{ijt} \leq 3 \quad \forall j \in S(i), t \in T; \tag{23}
$$

$$
x_{ij} \in \{0, 1\} \quad \forall i \in N, \forall j \in S(i); \tag{24}
$$

$$
f_{ijt} \geq 0 \quad \forall i \in N, \forall j \in M, \forall t \in T; \tag{25}
$$

$$
\delta_{ijt} \in \{0, 1\} \quad \forall i \in N, \forall j \in S(i), \forall t \in T; \tag{26}
$$

Exercise 2

$$\begin{aligned} \min \quad & x_1 + 2x_2 + 1x_3 \\ & 2x_1 + 3x_2 - 2x_3 = 20 \\ & x_1 - 2x_2 + 3x_3 = 12 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

FASE I

x_1	x_2	x_3	x_1^a	x_2^a		
-3	-1	-1	0	0	-32	$-\xi$
2	3	-2	1	0	20	x_1^a
1	-2	3	0	1	12	x_2^a

x_1	x_2	x_3	x_1^a	x_2^a		
0	$\frac{7}{2}$	-4	$\frac{3}{2}$	0	-2	$-\xi$
1	$\frac{3}{2}$	-1	$\frac{1}{2}$	0	10	x_1
0	$-\frac{7}{2}$	4	$-\frac{1}{2}$	1	2	x_2^a

x_1	x_2	x_3	x_1^a	x_2^a		
0	0	0	1	1	0	$-\xi$
1	$\frac{5}{8}$	0	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{21}{2}$	x_1
0	$-\frac{7}{8}$	1	$-\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	x_3

FASE II

x_1	x_2	x_3		
0	$\frac{9}{4}$	0	-11	$-z$
1	$\frac{5}{8}$	0	$\frac{21}{2}$	x_1
0	$-\frac{7}{8}$	1	$\frac{1}{2}$	x_3

$x = (\frac{21}{2}, 0, \frac{1}{2})$, $z_P = 11$. Gomory's cut: $\frac{5}{8}x_2 \geq \frac{1}{2}$

x_1	x_2	x_3	x_4		
0	$\frac{9}{4}$	0	0	-11	$-z$
1	$\frac{5}{8}$	0	0	$\frac{21}{2}$	x_1
0	$-\frac{7}{8}$	1	0	$\frac{1}{2}$	x_3
0	$-\frac{5}{8}$	0	1	$-\frac{1}{2}$	x_4

x_1	x_2	x_3	x_4		
0	0	0	$\frac{18}{5}$	$-\frac{64}{5}$	$-z$
1	0	0	1	10	x_1
0	0	1	$-\frac{7}{5}$	$\frac{6}{5}$	x_3
0	1	0	$-\frac{8}{5}$	$\frac{4}{5}$	x_2

$x = (10, \frac{4}{5}, \frac{6}{5}, 0)$, $z_P = \frac{64}{5}$ Gomory's cut: $\frac{3}{5}x_2 \geq \frac{1}{5}$

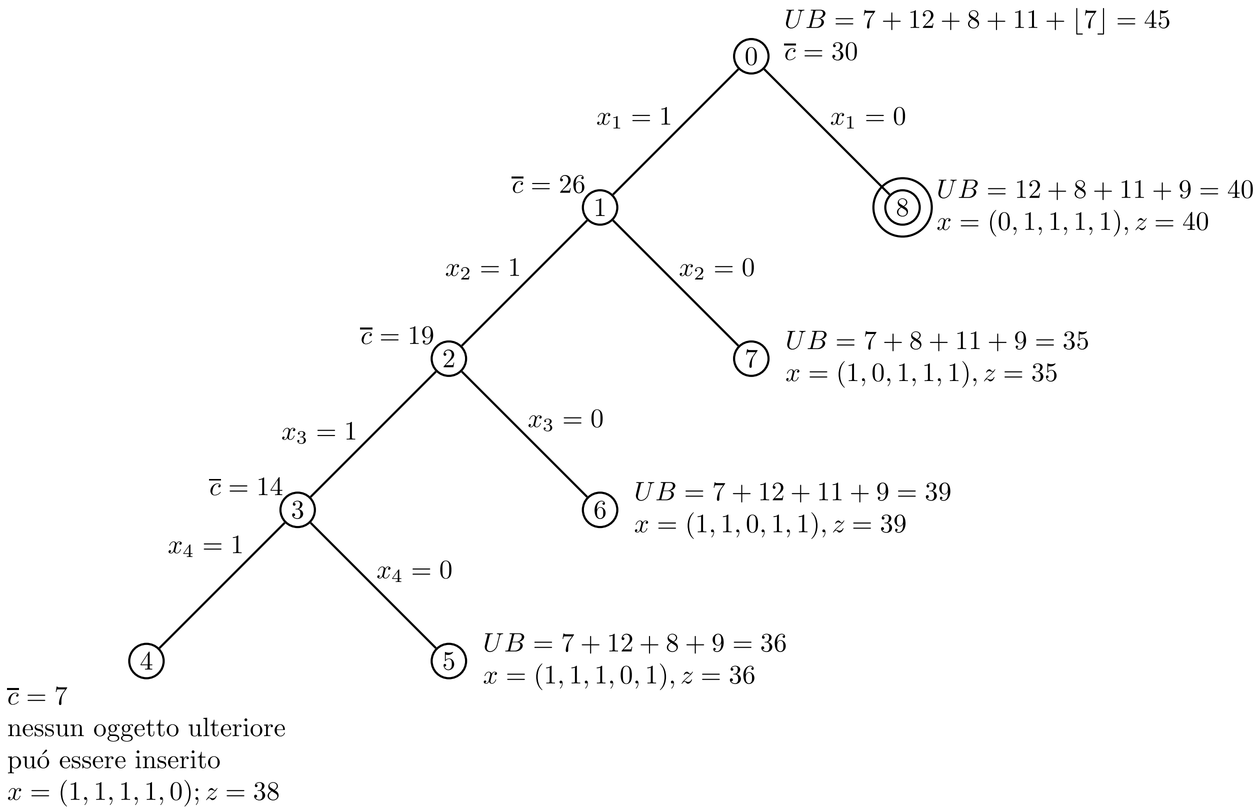
x_1	x_2	x_3	x_4	x_5		
0	0	0	$\frac{18}{5}$	0	$-\frac{64}{5}$	$-z$
1	0	0	1	0	10	x_1
0	0	1	$-\frac{7}{5}$	0	$\frac{6}{5}$	x_3
0	1	0	$-\frac{8}{5}$	0	$\frac{4}{5}$	x_2
0	0	0	$-\frac{3}{5}$	1	$-\frac{1}{5}$	x_5

x_1	x_2	x_3	x_4	x_5		
0	0	0	0	6	14	$-z$
1	0	0	0	$\frac{5}{3}$	$\frac{29}{3}$	x_1
0	0	1	0	$-\frac{7}{3}$	$\frac{5}{3}$	x_3
0	1	0	0	$-\frac{8}{3}$	$\frac{4}{3}$	x_2
0	0	0	1	$-\frac{5}{3}$	$\frac{1}{3}$	x_4

$x = (\frac{29}{3}, \frac{5}{3}, \frac{8}{3}, \frac{1}{3}, 0)$, $z_P = \frac{64}{5}$

Exercise 3

$p_j = (7, 12, 8, 11, 9)$ (sorted)
$w_j = (4, 7, 5, 7, 9)$
$c = 30$



Optimum node 8, $z = 40$