RICERCA OPERATIVA

Written assessment, July 19, 2018 A

First and Last name ____________________________

Exercise 1 (value 13)
An American e-commerce company has decided to start delivering parcels with drones in one of the most important cities for their business. To do so they need to locate a set of drone houses from where drones can start to deliver parcels. The set of possible drone houses is H. Marketing forecasted a set of customers U, each one with a unitary request.

Each drone house $h \in H$ has a cost c_h of activation. Considering that all deliveries are done in parallel, each drone house has also a capacity Q_h in number of deliveries (drones) that can be associated to it. Each drone starts from its drone house, serves only one customer (a drone can deliver a parcel at a time), and return to the drone house. The flying time between house $h \in H$ and customer $i \in U$ is t_{hi}. The same time is spent flying back from i to h. To compute the total delivery time of a parcel we have also to include the launching time la, the service time s, at the customer, and the landing time ld when returned at the house. The drones, that are all of the same type, have a battery that has a duration E. Hence a drone cannot perform a delivery whose time exceeds E. Each time the drone starts from the house it is supposed a new battery is loaded on it, so it has a full working time E. The cost of the fly is given by the flying time multiplied by the coefficient α.

Help the Logistics Manager to write a PLI model in order to locate the drone houses to serve all the customers respecting the constraints and minimizing the total costs.

Due to agreements with customers, the following constraints must be introduced in the model: (i) if drone house 1 serves customer a, then customers b and c must be served by other drone houses; (ii) the number of customers associated to any drone house must be at most double of the number of customers assigned to any other used house.

Exercise 2 (value 8)
Consider the following PLC problem. Solve it with the simplex method, using the Bland's rule. Verify the solution by solving the problem with the graphic method.

$$
\begin{aligned}
\max \quad & x_1 - 2x_2 \\
& x_1 - x_2 \leq 1 \\
& -2x_1 + x_2 \leq 1 \\
& x_1, x_2 \text{ free}
\end{aligned}
$$

NOTE: A tableau in its standard form requires non negative variables, therefore, the model must be appropriately transformed before starting the computation.

Exercise 3 (value 7)
Consider the following PLI model:

$$
\begin{aligned}
\max \quad & x_1 + 4x_2 \\
& 7x_1 - 5x_2 \leq 35 \\
& 3x_1 + 5x_2 \leq 30 \\
& -x_1 + 2x_2 \leq 3 \\
& x_1, x_2 \geq 0, \textit{integer}.
\end{aligned}
$$

Solve it with the standard Branch-and-Bound algorithm. Solve each relaxed subproblems graphically. Perform the first branching on variable x_1.

Assignment July 19, 2018 **A**

First and Last name ____________________

Exercise 1

Variables

$y_h = 1$ if drone house $h \in H$ is used, 0 otherwise.
$x_{hi} = 1$ if customer $i \in U$ is served from drone house $h \in H$, 0 otherwise.

Model

$$
\begin{aligned}
\min z \; &= \sum_{h \in H} c_h y_h + \alpha \sum_{h \in H} \sum_{i \in U} 2t_{hi} x_{hi} \\
&\sum_{i \in U} x_{hi} \le Q_h y_h \quad h \in H \\
&(2t_{hi} + s + la + ld) x_{hi} \le E \quad h \in H, i \in U \\
&\sum_{h \in H} x_{hi} = 1 \quad i \in U \\
&x_{hi} \in \{0,1\} \quad h \in H, i \in U \\
&y_h \in \{0,1\} \quad h \in H
\end{aligned}
$$

$$
(i) \; 2x_{1a} \le \sum_{h \in H, h \neq 1} (x_{hb} + x_{hc})
$$

$$
(ii) \; \sum_{i \in U} x_{hi} \le 2 \sum_{i \in U} x_{ki} \quad h, k \in H, h \neq k
$$

Exercise 2

$$\begin{aligned} \max \quad & x_1 - 2x_2 \\ & x_1 - x_2 \leq 1 \\ & -2x_1 + x_2 \leq 1 \end{aligned}$$

$$\begin{aligned} \max \quad & x_1^+ - x_1^- - 2x_2^+ + 2x_2^- \\ & x_1^+ - x_1^- - x_2^+ + x_2^- \leq 1 \\ & -2x_1^+ + 2x_1^- + x_2^+ - x_2^- \leq 1 \\ & x_1^+, x_1^-, x_2^+, x_2^- \geq 0 \end{aligned}$$

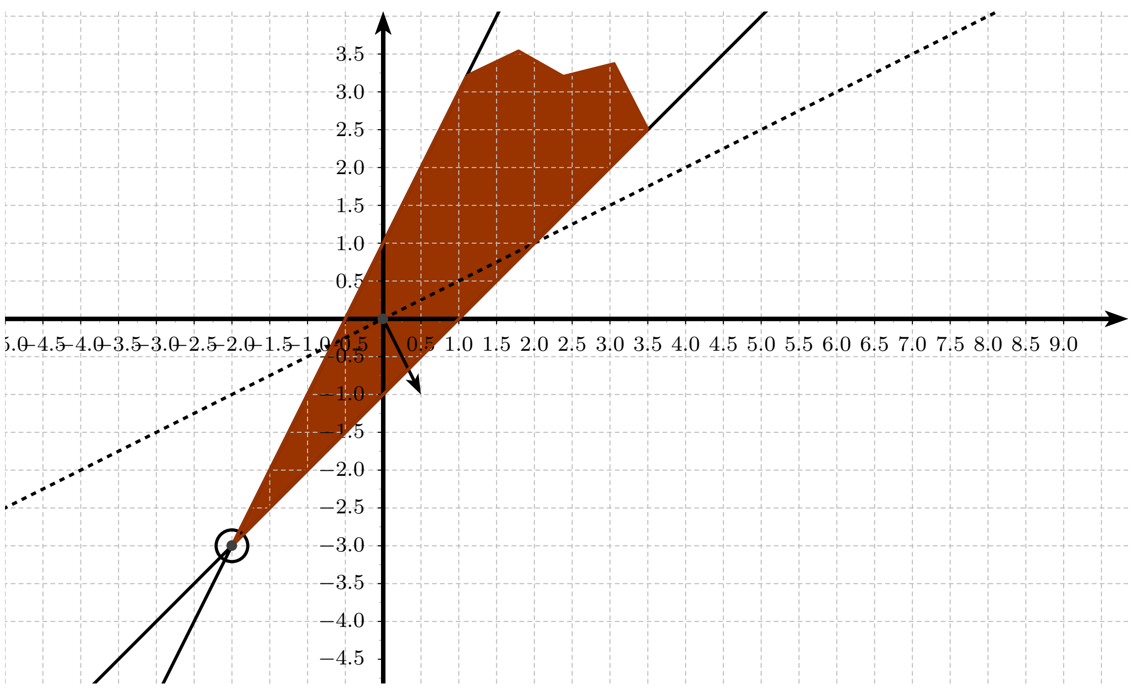
x_1	x_2	x_3	x_4	x_5	x_6		
1	-1	-2	2	0	0	0	$-z$
1	-1	-1	1	1	0	1	x_5
-2	2	1	-1	0	1	1	x_6

x_1	x_2	x_3	x_4	x_5	x_6		
0	0	-1	1	-1	0	-1	$-z$
1	-1	-1	1	1	0	1	x_1
0	0	-1	1	2	1	3	x_6

x_1	x_2	x_3	x_4	x_5	x_6		
-1	1	0	0	-2	0	-2	$-z$
1	-1	-1	1	1	0	1	x_4
-1	1	0	0	1	1	2	x_6

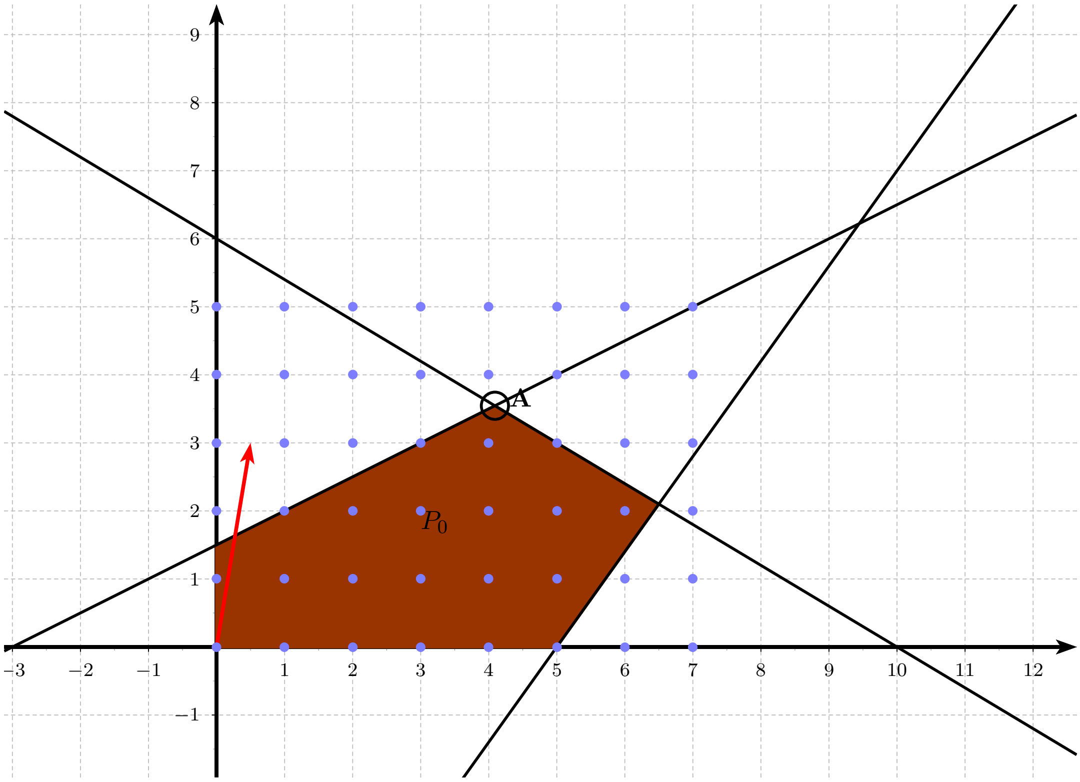
x_1	x_2	x_3	x_4	x_5	x_6		
0	0	0	0	-3	-1	-4	$-z$
0	0	-1	1	2	1	3	x_4
-1	1	0	0	1	1	2	x_2

$x_1^- = 2, x_2^- = 3 \quad \Rightarrow \quad x_1 = -2, x_2 = -3,\ z = 4$

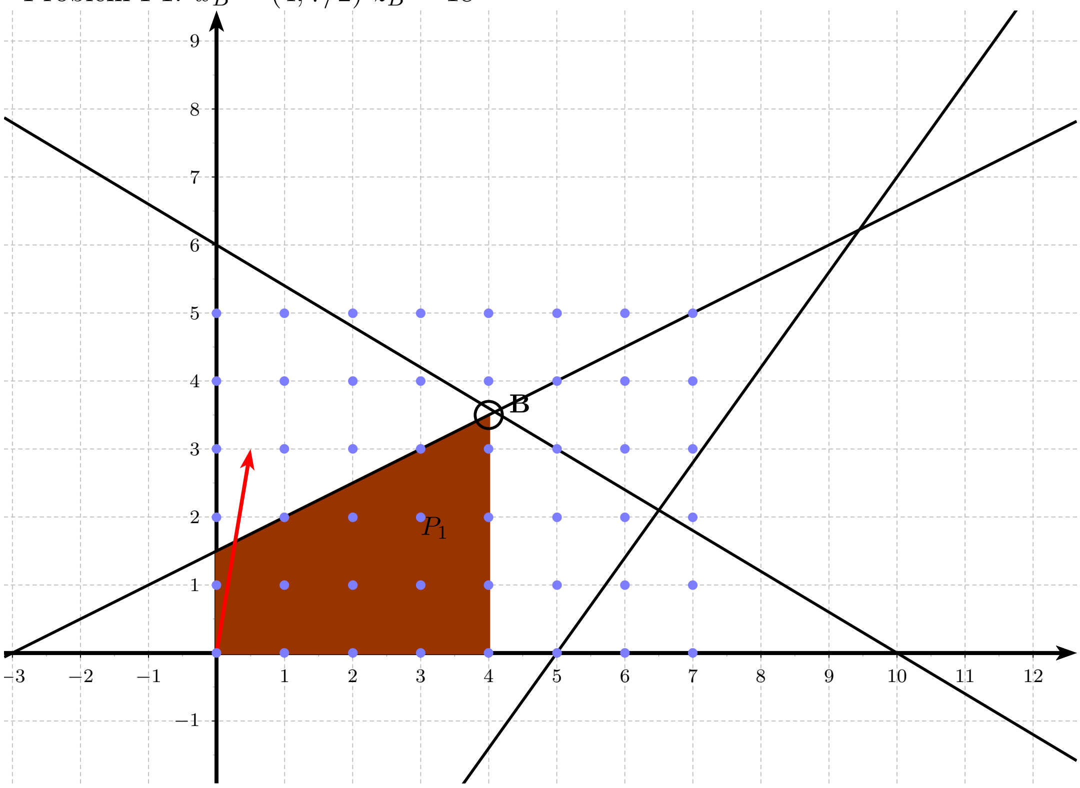


Exercise 3

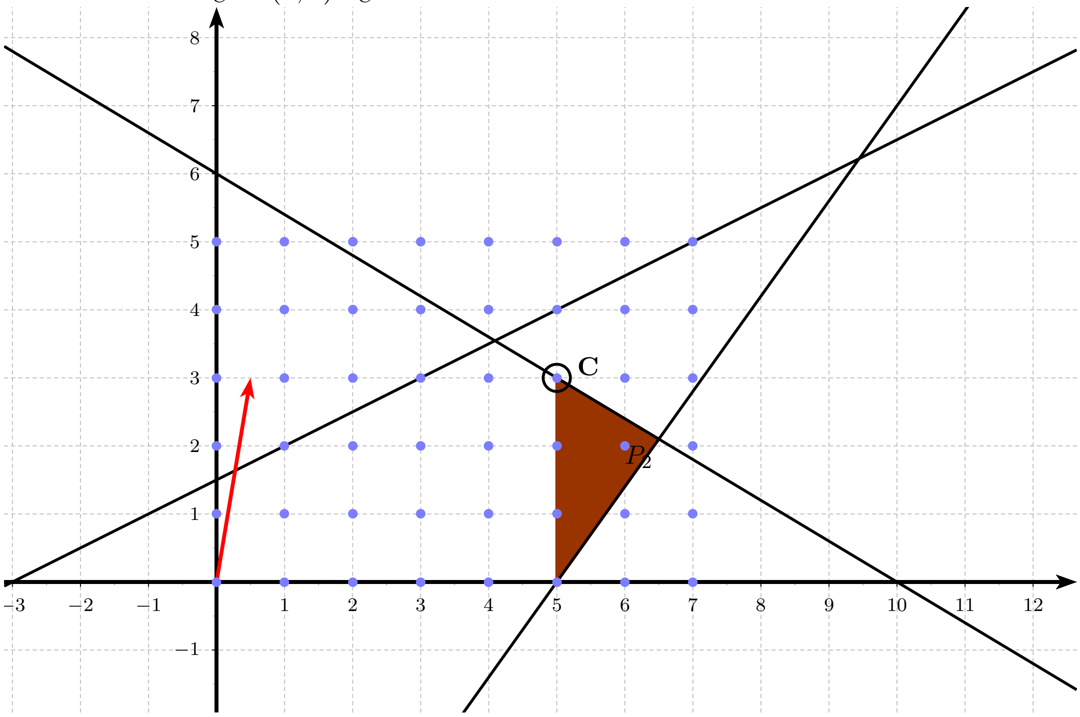
Problem P0: $x_A = (45/11, 39/11)$ $z_A = \lfloor 201/11 \rfloor = 18$



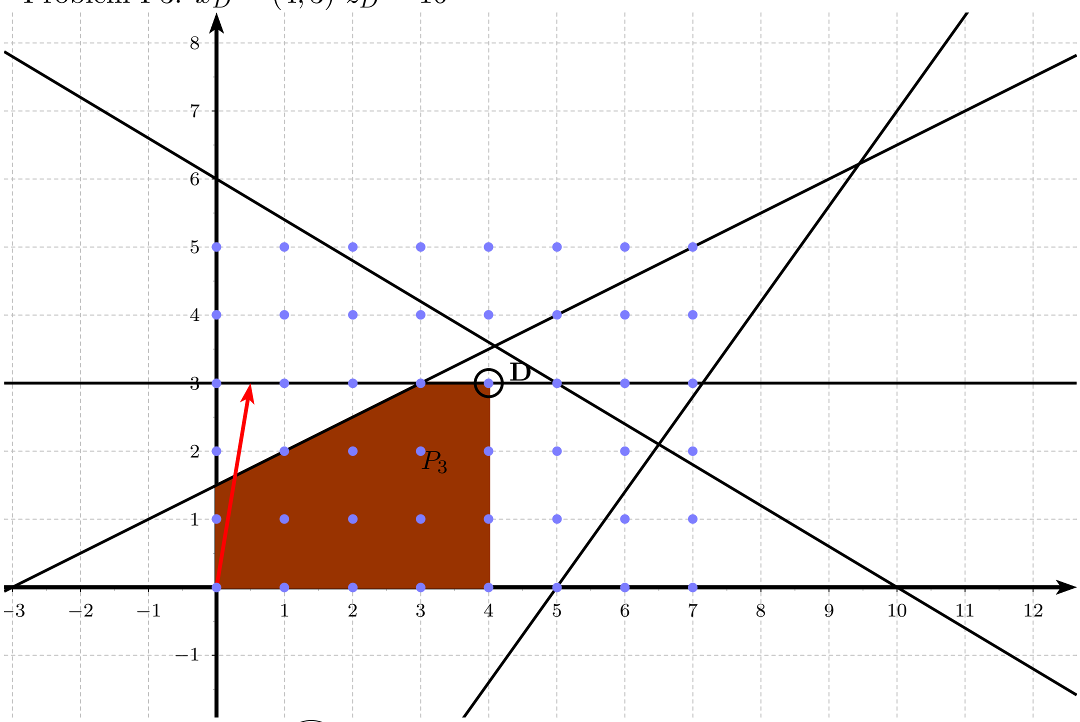
Problem P1: $x_B = (4, 7/2)$ $z_B = 18$

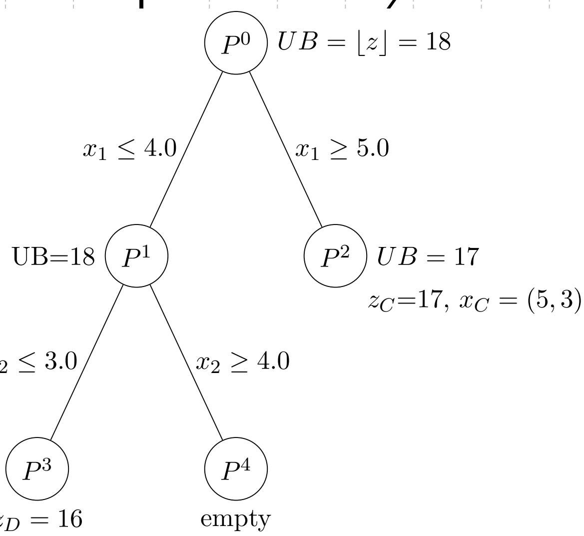


Problem P2: $x_C = (5,3)$ $z_C = 17$



Problem P3: $x_D = (4,3)$ $z_D = 16$





$x^* = (5,3), z^* = 17.$