RICERCA OPERATIVA

Operations Research February 1, 2016 A

First and Last name _____

Exercise 1 (value 12).
Electron is a company which distributes and sells electrical power. The company has to design the distribution network in a new region. In the region there is a set P of power *generation plants* where each plant $i \in P$ can give at most p_i kWh each day. The distribution network starts from the generation plants and has two additional layers denoted by sets D and C. In the *primary* layer D each distribution node $j \in D$ receives the electricity from a fixed set $P_j \subseteq P$ of generation plants and sends it to some of the *customer* nodes in C, after a reduction of the voltage. The physical connection between each distribution node $j \in D$ and each generation plant $i \in P_j$ already exists and the cost to be payed is only for the energy transferred. A kWh transferred from i to j costs c^I_{ij} Euro The connections between each distribution node $j \in D$ and each customer $k \in C$ have to be designed and build. Therefore the total cost to be payed is a fixed cost of s_{jk} Euro to set up the connection and a variable cost of c^{II}_{jk} Euro for each kWh transferred from j to k. To ensure the continuity of the service each customer must be linked to at least two distribution nodes. The maximum number of kWh that can be processed in a day by a node $j \in D$ is cap_j. A customer $k \in C$ requires q_k kWh each day. Write a Linear Programming model to help the company to design the distribution network that minimizes the costs, respecting all the constraints.
Now consider the possibility to chose if a generation plant has to be activated or not. The activation of a plant $i \in P$ costs a_i Euro and each kWh generated by this plant costs g_i Euro. Modify the above model to find the choice for the production plants that minimizes the total cost.

Exercise 2 (value 6)
Consider the following PLC problem and solve it using the Primal Simplex algorithm. Write the dual problem, drawn the feasible region and find its optimal solution.

$$\begin{aligned} \max \quad & 3x_2 - 2x_3 \\ & 2x_2 + 2x_3 + 3x_4 \leq 12 \\ & x_1 + 5x_2 - 5x_3 = 4 \\ & x_1, \ldots, x_4 \geq 0 \end{aligned}$$

Exercise 3 (value 7)
A project manager has identified the activities, precedences and duration of a project (see tables below). Draw the graph which represents the project and determine the critical path.

activity	A	B	C	D	E	F	G	H	I	L	M
duration	2	3	5	4	3	1	5 q	4	3	6	1

Precedences $A \prec B$; $A \prec H$; $B \prec C$; $C \prec M$; $D \prec E$; $D \prec H$;
$F \prec G$; $F \prec H$; $F \prec L$; $G \prec I$; $H \prec I$; $I \prec M$;