Operations Research, June 15, 2017 A

Last name____________________ First name____________________

Exercise 1 (value 12)

A B2C (Business to Consumer) operator sells goods online, so they must be organized to transport thousand of goods to different customers located in different sites. Let $N = \{1, \dots, n\}$ denote the set of current customers, each requiring a single unit of good. The goods are stored in a set $M = \{1, \dots, m\}$ of depots located in different countries. The transport is performed in two/three phases. In the first phase the goods are transported from the depots to a set $S = \{1, \dots, s\}$ of satellite warehouses, closes to the main cities. In the second phase the goods are either transported from a satellite to the customers, or from a satellite to another satellite. The third phase occurs when a transport satellite-satellite has been performed and involve the transport from the second satellite to the customers. Let $d'_{ij} (i \in M, j \in S)$ denote the distance from depot i and satellite j. Let $d''_{jh} (j \in S, h \in S, j \neq h)$ denote the distance from satellite j and satellite h. Let $d'''_{jk} (j \in S, k \in N)$ denote the distance from satellite j and customer k. To simplify the problem suppose that all the goods are identical and that each depot $i \in M$ contains m_i products. Moreover, each satellite $j \in S$ can manage at most s_j incoming/outcoming goods. The cost of transport are given by the distance travelled times a flat cost $\overline{c}$ (independent of the number of goods transferred.)

Write a linear program to help the company to define an optimal plan to transport the goods from depots to customers, satisfying all constraints and minimizing the total cost.

Now suppose that the cost of a transport between a depot i and a satellite j is given by $c_1 d'_{ij}$ if the quantity transferred from i to j is smaller or equal to Δ, and otherwise, it is given by $c_2 d''_{ij}$, with $c_1 > c_2$ (note that the cost is independent of the specific number of products transferred. It only changes with the threshold Δ.)

Improve the above model with the new objective function.

Exercise 2 (value 5)

Consider the following PLI problem. Solve it with the branch-and-bound standard.

$$
\begin{aligned}
\min \quad & x_1 + x_2 \\
& 4x_1 + 5x_2 \leq 40 \\
& 2x_1 + x_2 \geq 4 \\
& -3x_1 + 8x_2 \geq 6 \\
& x_1, x_2 \geq 0 \text{ integer}
\end{aligned}
$$

Exercise 3 (value 5)

Consider the PLI problem of exercise 2, remove the integrality constraint, write the dual problem and solve it with the simplex method.

Exercise 4 (value 5)
Write a GLPK or XPRESS implementation for the following problem

$$\min\ z = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \notin R}}^{m} c_{ij} x_{ij} + \sum_{j=1}^{m} \text{maxC}\ y_j$$

$$\sum_{\substack{j=1 \\ j \notin R}}^{m} x_{ij} \geq Q_i \quad i = 1, \ldots, n; \tag{16}$$

$$\sum_{i=1}^{n} x_{i,j} \geq 1 \quad \forall j \in R; \tag{17}$$

$$\sum_{j \in R} y_j \geq 1; \tag{18}$$

$$\sum_{i=1}^{n} x_{ij} \leq \text{maxC}\ n\ y_j \quad j = 1, \ldots, m; \tag{19}$$

$$x_{ij} \geq 0 \quad i = 1, \ldots, n, j = 1, \ldots, m; \tag{20}$$

$$y_j \in \{0, 1\} \quad j = 1, \ldots, m; \tag{21}$$

where

$$\text{maxC} = \max_{i=1,\ldots,n, j=1,\ldots,m} c_{ij}$$

.

Exercise 1

$$
\begin{aligned}
x'_{ij} &= \text{number of goods transferred form } i \in M \text{ to } j \in S \\
x''_{jh} &= \text{number of goods transferred form } j \in M \text{ to } h \in S \\
x'''_{jk} &= \text{number of goods transferred form } j \in S \text{ to } k \in N \\
y'_{ij} &= 1 \text{ if goods are transferred form } i \in M \text{ to } j \in S\text{; 0 otherwise} \\
y''_{jh} &= 1 \text{ if goods are transferred form } j \in S \text{ to } h \in S\text{; 0 otherwise} \\
y'''_{jh} &= 1 \text{ if goods are transferred form } j \in S \text{ to } h \in S\text{; 0 otherwise}
\end{aligned}
$$

$$\min \ z = \sum_{i \in M} \sum_{j \in S} \overline{c} d'_{ij} y'_{ij} + \sum_{j \in S} \sum_{h \in S} \overline{c} d''_{jh} y''_{jh} + \sum_{j \in S} \sum_{k \in N} \overline{c} d'''_{jh} y'''_{jh}$$

$$\sum_{j \in S} x'_{ij} \leq m_i \quad \forall i \in M; \tag{22}$$

$$\sum_{i \in M} x'_{ij} + \sum_{h \in S, h \neq j} x''_{hj} \leq s_j \quad \forall j \in S; \tag{23}$$

$$\sum_{i \in M} x'_{ij} + \sum_{h \in S, h \neq j} x''_{hj} = \sum_{h \in S, h \neq j} x''_{jh} + \sum_{k \in N} x'''_{jk} \quad \forall j \in S; \tag{24}$$

$$\sum_{j \in S} x'''_{jk} = 1 \quad \forall j \in S; \tag{25}$$

$$x'_{ij} \leq m_i \ y'_{ij} \quad \forall i \in M, j \in S; \tag{26}$$

$$x''_{jh} \leq s_j \ y''_{jh} \quad \forall j \in S, h \in S; \tag{27}$$

$$x'''_{jk} \leq y'_{jk} \quad \forall j \in S, k \in N; \tag{28}$$

$$x'_{ij} \geq 0 \text{ integer} \quad \forall i \in M, \forall j \in S; \tag{29}$$

$$x''_{jh} \geq 0 \text{ integer} \quad \forall j \in S, \forall h \in S; \tag{30}$$

$$x'''_{jk} \geq 0 \text{ integer} \quad \forall j \in S, \forall k \in N; \tag{31}$$

$$y'_{ij} \in \{0, 1\} \quad \forall i \in M, \forall j \in S; \tag{32}$$

$$y''_{jh} \in \{0, 1\} \quad \forall j \in S, \forall h \in S; \tag{33}$$

$$y'''_{jk} \in \{0, 1\} \quad \forall j \in S, \forall k \in N; \tag{34}$$

$$
\begin{aligned}
\alpha_{ij} &= 1 \text{ if goods are transferred form } i \in M \text{ to } j \in S \text{ and } x'_{ij} \leq \Delta\text{; 0 otherwise} \\
\beta_{ij} &= 1 \text{ if goods are transferred form } i \in M \text{ to } j \in S \text{ and } x'_{ij} > \Delta\text{; 0 otherwise}
\end{aligned}
$$

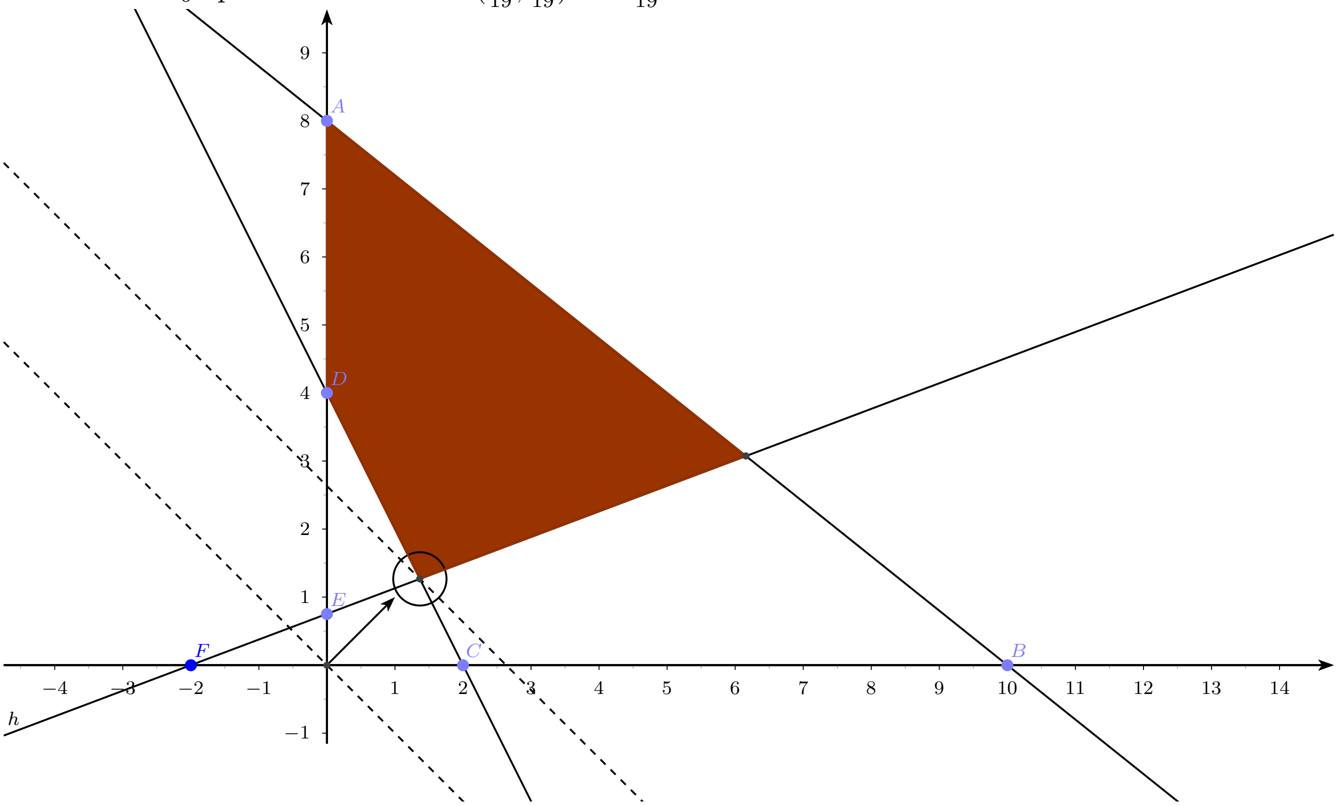
$$\min \ z = \sum_{i \in M} \sum_{j \in S} (c_1 \alpha_{ij} + c_2 \beta_{ij}) + \sum_{j \in S} \sum_{h \in S} \overline{c} d''_{jh} y''_{jh} + \sum_{j \in S} \sum_{k \in N} \overline{c} d'''_{jh} y'''_{jh}$$

$$x'_{ij} \geq \Delta(1 - \alpha_{ij}) \quad \forall i \in M, j \in S; \tag{35}$$

$$x'_{ij} - \Delta \leq m_i \beta_{ij} \quad \forall i \in M, j \in S; \tag{36}$$

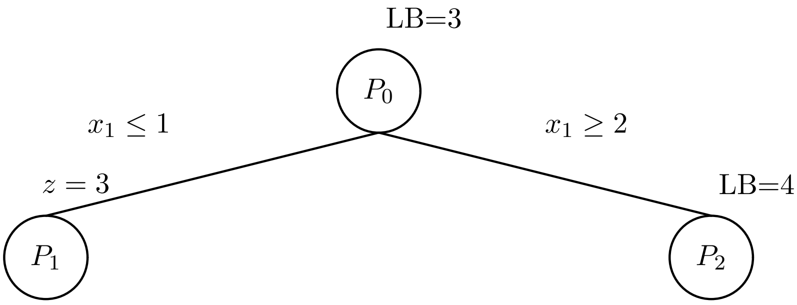
Exercise 2

Root node P_0 optimal solution $x = (\frac{26}{19}, \frac{24}{19})$ $z = \frac{50}{19}$ $LB = 3$



Node P_1 optimal solution $x = (1, 2)$ $z = 3$

Node P_2 optimal solution $x = (2, \frac{3}{2})$ $z = \frac{7}{2}$ $LB = 4$



Exercise 3

$$\begin{array}{ll} \min & x_1 + x_2 \\ & -4x_1 - 5x_2 \geq -40 \\ & 2x_1 + x_2 \geq 4 \\ & -3x_1 + 8x_2 \geq 6 \\ & x_1, x_2 \geq 0 \end{array}$$

$$\begin{array}{ll} \max & -40u_1 + 4u_2 + 6u_3 \\ & -4u_1 + 2u_2 - 3u_3 \leq 1 \\ & -5u_1 + u_2 + 8u_3 \leq 1 \\ & u_1, u_2, u_3 \geq 0 \end{array}$$

x_1	x_2	x_3	x_4	x_5		
-40	4	6	0	0	0	$-z$
-4	2	-3	1	0	1	x_4
-5	1	8	0	1	1	x_5

x_1	x_2	x_3	x_4	x_5		
-32	0	12	-2	0	-2	$-z$
-2	1	$-\frac{3}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	x_2
-3	0	$\frac{19}{2}$	$-\frac{1}{2}$	1	$\frac{1}{2}$	x_5

x_1	x_2	x_3	x_4	x_5		
$-\frac{536}{19}$	0	0	$-\frac{26}{19}$	$-\frac{24}{19}$	$-\frac{50}{19}$	$-z$
$-\frac{47}{19}$	1	0	$\frac{8}{19}$	$\frac{3}{19}$	$\frac{11}{19}$	x_2
$-\frac{6}{19}$	0	1	$-\frac{1}{19}$	$\frac{2}{19}$	$\frac{1}{19}$	x_3

Exercise 4

```
/* Written in GNU MathProg */

param n, integer, > 0;
param m, integer, > 0;

set I := 1..n;
set J := 1..m;
set R;
param Q{i in I}, >= 0;
param c{i in I, j in J}, >= 0;
param maxC := max{i in I, j in J} c[i,j];

var  x{i in I, j in J},   >= 0;
var  y{ j in J},   >= 0, binary;

minimize z: sum{i in I, j in J: j not in R} c[i,j]* x[i,j] + sum{j in J} maxC*y[j];

s.t.
      V1{i in I}: sum{j in J: j not in R} x[i,j] >= Q[i];
      V2{j in R}: sum{i in I} x[i,j] >= 1;
      V3: sum{j in R} y[j] >= 1 ;
      V4{j in J}: sum{i in I} x[i,j] <=  maxC*n*y[j];
solve;

printf "\n";
printf "\nx[i,j]\n";
for{i in I} {
 printf {j in J}  "%4d ", x[i,j];
printf "\n";
}
printf "\ny[j]\n";
 printf {j in J}  "%4d ", y[j];
printf "\n\n-----z = %g\n\n",z;

data;

param n := 3;
param m := 5;
```

```
param Q := [1]  10   [2]   20  [3]  15 ;
param c : 1  2  3  4  5:=
      1  10 12  8  4 11
      2   3  5 11  6  9
      3   5  4 10  7  4;
set R := 1 4;
end;
```