RICERCA OPERATIVA

Written assessment, September 4, 2018 A

First and Last name ________________________________

Exercise 1 (value 13)
After canceling the Baseball World cup in 2011, the International Baseball Federation (IBAF) is planning to organize the Baseball World cup again in September 2019. After the qualification phase, we know that m teams are qualified to take part to the world cup. Let $I = \{1, \ldots, m\}$ denote the set of all teams. IBAF is now organizing the group stage (=*fase a gironi*). The groups of teams may include **either** 4 teams **or** 6 teams. Let G denote the set of all groups, $G4 \subset G$ denote the set of all 4-teams groups and $G6 \subset G$ the set of all 6-teams groups.

These teams are categorized by continent of origin. Due to a market analysis, IBAF knows that: (i) every group must have at least one Asian team; (ii) the teams of the same continent in each group cannot be more than the half of the size of the group. Let $I_{Asia} \subset I$ be the set of Asian teams. Let C denote the set of continents and let $I(h) \subset I$ be the set of teams of continent $h \in C$.

Help the IBAF in planning the world-cup group stage by writing an Integer Linear Programming model that assigns every team to a group, respecting the constraints, and minimizing the number of groups used, as follows. Minimize the number of 6 teams groups; if two solutions have the minimum number of 6 teams groups select the solution with the smallest number of 4 team groups.

Exercise 2 (value 8)
Consider the following PLC problem. Solve it with the two-phase method and follow the Bland's rule.

$$
\begin{aligned}
\max \quad & z = 10x_1 + 4x_2 \\
& x_1 + x_2 \geq 6 \\
& 5x_1 - 6x_2 \leq 30 \\
& x_1 + 2x_2 \leq 20 \\
& -2x_1 + x_2 \leq 6 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Draw the region of the feasible solutions and draw the solutions obtained during the simplex algorithm computation.

Exercise 3 (value 7)
Write a GLPK or XPRESS model corresponding to the following mathematical model

$$
\max \ z = \sum_{i \in N} \sum_{j=1}^{m} \sum_{k \in S} i x_{ij}^k - \sum_{k \in S} v^k \rho^k
$$

$$
\begin{aligned}
\sum_{k \in S} x_{ij}^k &\geq a && i \in N, j = 1, \ldots, m && (1) \\
\sum_{j=1}^{m} \sum_{i \in N | i \neq j} x_{ij}^k &\geq k\rho^k && k \in S && (2) \\
\sum_{i \in N} \sum_{k \in S} x_{ij}^k &\leq Q^j && j = 1, \ldots, m && (3) \\
\rho^k + \rho^h &\leq 1 && k \in S^1, h \in S^2, k \neq h && (4) \\
\rho^k &\in \{0, 1\} && k \in S && (5) \\
x_{ij}^k &\geq 0, integer && i \in N, j = 1, \ldots, m, k \in S. && (6)
\end{aligned}
$$

Exercise 1

Variables

$x_{ig} = 1$ if team $i \in I$ is assigned to the group $g \in G$, 0 otherwise.
$y_g = 1$ if group $g \in G$ is used, 0 otherwise.

Model

$$\min z = |G| \sum_{g \in G6} y_g + \sum_{g \in G4} y_g \tag{7}$$

$$\sum_{g \in G} x_{ig} = 1 \qquad i \in I \tag{8}$$

$$\sum_{i \in I} x_{ig} = 4y_g \qquad g \in G4 \tag{9}$$

$$\sum_{i \in I} x_{ig} = 6y_g \qquad g \in G6 \tag{10}$$

$$\sum_{i \in I(Asia)} x_{ig} \geq y_g \qquad g \in G \tag{11}$$

$$\sum_{i \in I(h)} x_{ig} \leq 2y_g \qquad g \in G4, h \in C \tag{12}$$

$$\sum_{i \in I(h)} x_{ig} \leq 3y_g \qquad g \in G6, h \in C \tag{13}$$

$$x_{ig} \in \{0,1\} \qquad i \in I, g \in G \tag{14}$$

$$y_g^\delta \in \{0,1\} \qquad g \in G, \delta \in \{4,6\}. \tag{15}$$

Exercise 2

$$\begin{aligned} \min \quad & z = -10x_1 - 4x_2 \\ & x_1 + x_2 \geq 6 \\ & 5x_1 - 6x_2 \leq 30 \\ & x_1 + 2x_2 \leq 20 \\ & -2x_1 + x_2 \leq 6 \\ & x_1, x_2 \geq 0 \end{aligned}$$

$$\begin{aligned} \min \quad & -z = -10x_1 - 4x_2 \\ & x_1 + x_2 - x_3 = 6 \\ & 5x_1 - 6x_2 + x_4 = 30 \\ & x_1 + 2x_2 + x_5 = 20 \\ & -2x_1 + x_2 + x_6 = 6 \\ & x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{aligned}$$

$$\begin{aligned} \min \quad & -z = -10x_1 - 4x_2 \\ & x_1 + x_2 - x_3 + x_7 = 6 \\ & 5x_1 - 6x_2 + x_4 = 30 \\ & x_1 + 2x_2 + x_5 = 20 \\ & -2x_1 + x_2 + x_6 = 6 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \end{aligned}$$

Phase I

x_1	x_2	x_3	x_4	x_5	x_6	x_7		
0	0	0	0	0	0	1	0	$-\xi$
1	1	-1	0	0	0	1	6	
5	-6	0	1	0	0	0	30	
1	2	0	0	1	0	0	20	
-2	1	0	0	0	1	0	6	

x_1	x_2	x_3	x_4	x_5	x_6	x_7		
-1	-1	1	0	0	0	0	-6	$-\xi$
1	1	-1	0	0	0	1	6	x_7
5	-6	0	1	0	0	0	30	x_4
1	2	0	0	1	0	0	20	x_5
-2	1	0	0	0	1	0	6	x_6

x_1	x_2	x_3	x_4	x_5	x_6	x_7		
0	0	0	0	0	0	1	0	$-\xi$
1	1	-1	0	0	0	1	6	x_1
0	-11	5	1	0	0	-5	0	x_4
0	1	1	0	1	0	-1	14	x_5
0	3	-2	0	0	1	2	18	x_6

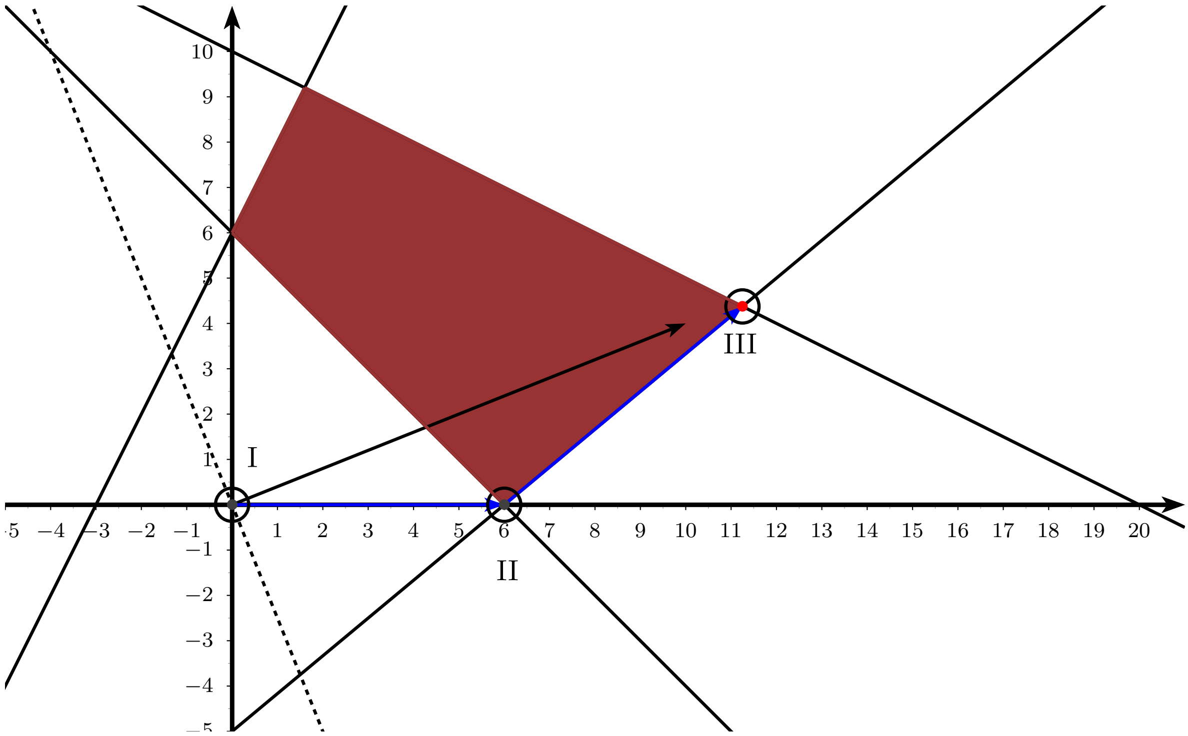
Phase II

x_1	x_2	x_3	x_4	x_5	x_6		
0	6	-10	0	0	0	60	
1	1	-1	0	0	0	6	x_1
0	-11	5	1	0	0	0	x_4
0	1	1	0	1	0	14	x_5
0	3	-2	0	0	1	18	x_6

x_1	x_2	x_3	x_4	x_5	x_6		
0	-16	0	2	0	0	60	
1	-6/5	0	1/5	0	0	6	x_1
0	-11/5	1	1/5	0	0	0	x_3
0	16/5	0	-1/5	1	0	14	x_5
0	-7/5	0	2/5	0	1	18	x_6

x_1	x_2	x_3	x_4	x_5	x_6		
0	0	0	1	5	0	130	
1	0	0	1/8	3/8	0	45/4	x_1
0	0	1	1/16	11/16	0	77/8	x_3
0	1	0	-1/16	5/16	0	35/8	x_2
0	0	0	5/16	7/16	1	193/8	x_6

$z^* = 130$, x^*=(45/4,35/8,77/8,0,0,193/8).



Exercise 3

```
/* Exercise 3, 2017 09 04 */

param m, integer, > 0;
set N
set J := 1..m;
set S;
set Sone;
set Stwo;

param a, >= 0;
param v{k in S}, >= 0;
param Q{j in J}, >= 0;


var  x{i in N, j in J, k in S}, integer ;
var  rho{k in S}, binary , >=0;

maximize z: sum{i in N, j in J, k in S} i*x[i,j,k] - sum{k in S} v[k]*rho[k];

s.t.  one{i in N, j in J}:  sum{k in S}x[i,j,k] >= a;
  two{k in S}: sum{i in N, j in J : i != j} x[i,j,k] >= k*rho[k];
  three{j in J}: sum{i in N, k in S} x[i,j,k] <= Q[j];
      four{k in Sone, h in Stwo: k != h}: rho[k] + rho[h] <= 1;
solve;

printf "\n";
for{i in I} {
   printf "\n%1d)",i;
   for{j in J}{
       printf "\n%1d)",j;
       printf{k in S} "%5d ", x[i,j,k];
    }
}
printf "\n";
printf{k in S} "%5d ", rho[k];

printf "%5d ", theta;

printf "\n\n-----z = %g\n\n",z;
printf "\n\n ";

data;
param m := 5;
set N := 1 2 3 4 5 6;
set S := 1 2 3 4 5 6;
set Sone:= 1 2 3;
set Stwo:= 3 4 5 6;
param a:= 2;
param v:= [1] 2 [2] 3 [3] 1 [4] 3 [5] 2 [6] 1;
param Q:= [1] 10 [2] 12 [3] 15 [4] 11 [5] 3 [6] 8;

end;
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