Written assessment, January 11, 2018 C

First and Last name ____________________

Exercise 1 (value 8)
Consider the following PLC problem and solve it using the Simplex algorithm with the Bland rule. Than, write the dual of the problem, draw the feasible region and clearly indicate the points corresponding to the basis used in the Simplex iterations.

$$\begin{aligned} \min \quad & 2x_1 + x_2 + 3x_3 \\ & -2x_1 + 3x_2 + x_3 \leq 5 \\ & -x_1 - 2x_2 + x_3 \geq 8 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Exercise 2 (value 14).
The transport operator CITYFAST is specialized in good distribution in large metropolis. The distribution into the city requires two transports. A first transport from the suppliers (outside the city) to a city distribution center, and a second transport form the distribution center to the customers. Each supplier provides only one type of good. Let I be the set of suppliers, J the set of possible distribution centers and K be the set of customers. The first transport is managed by an external company. CITYFAST pays c'_{ij} euro for each kilogram transported from supplier $i \in I$ to distribution center $j \in J$. Instead, for the second transport, CITYFAST uses its own fleet of identical *ecological* trucks, each with a given capacity C. The cost for a *trip* from the distribution center $j \in J$ to customer $k \in K$ (and return) is c''_{jk}. Each trip can deliver goods from several suppliers in order to use the truck capacity efficiently. Each trip serve a single customer. Each customer asks for d_{ik} Kg of the good provided by supplier $i \in I$ (with d_{ik} possibly zero if the good is not required). If a distribution center j is used the company has to pay a fixed *una-tantum* amount of s_j euro.

Help the company to find an optimal transport policy by writing a Linear Program aimed to satisfy all the demands, while minimizing the total cost. (*Suggestion: use a three index variable (supplier-distribution center-customer) to track the flows of goods.)* Improve the previous model by adding the following constraint: given a subset of customers: $A \subseteq K$, it is required that all the customers of A are served by a single distribution center which is the same for all these customers.

Exercise 3 (value 6).
Write a GLPK or XPRESS code implementing the following model

$$\begin{aligned} \min z = & \sum_{j\in J} s_j\delta_j + \sum_{i=1}^{n}\sum_{j\in J} v_{ij}x_{ij} + \sum_{j\in J}\sum_{k=2}^{n-1} w_{jk}y_{jk} & \\ & \sum_{j\in J} x_{ij} = d_i & i = 1,\dots,n \\ & \sum_{i=1}^{n} x_{ij} \leq C y_{jk} & k = 1,\dots,n, j \in J \\ & \sum_{i=1}^{n} x_{ij} \leq M\delta_j & j \in J \\ & x_{ij} \geq 0 & i = 1,\dots,n, j \in J \\ & y_{jk} \geq 0 \text{ integer} & j \in J, k = 1,\dots,n \\ & \delta_j \in \{0,1\} & j \in J \end{aligned}$$

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Answers

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Exercise 1

$$
\begin{array}{ll}
\min & 2x_1 + x_2 + 3x_3 \\
 & -2x_1 + 3x_2 + x_3 \leq 5 \\
 & -x_1 - 2x_2 + x_3 \geq 8 \\
 & x_1, x_2, x_3 \geq 0
\end{array}
$$

x_1	x_2	x_3	x_4	x_5		
2	1	3	0	0	0	$-z$
-2	3	1	1	0	5	x_4
1	2	-1	0	1	-8	x_5

x_1	x_2	x_3	x_4	x_5		
5	7	0	0	3	-24	$-z$
-1	5	0	1	1	-3	x_4
-1	-2	1	0	-1	8	x_3

x_1	x_2	x_3	x_4	x_5		
0	32	0	5	8	-39	$-z$
1	-5	0	-1	-1	3	x_1
0	-7	1	-1	-2	11	x_3

$x = (3, 0, 11, 0, 0)$ $z_P = 39$

$$
\begin{array}{ll}
\min & 2x_1 + x_2 + 3x_3 \\
 & 2x_1 - 3x_2 - x_3 \geq -5 \\
 & -x_1 - 2x_2 + x_3 \geq 8 \\
 & x_1, x_2, x_3 \geq 0
\end{array}
\qquad
\begin{array}{ll}
\max & -5u_1 + 8u_2 \\
 & 2u_1 - u_2 \leq 2 \\
 & -3u_1 - 2u_2 \leq 1 \\
 & -u_1 + u_2 \leq 3 \\
 & u_1, u_2 \geq 0
\end{array}
$$

First iteration $x = (0, 0, 0, 5, -8)$

$$
\begin{cases}
(2x_1 - 3x_2 - x_3 + 5)u_1 = 0 \\
(-x_1 - 2x_2 + x_3 - 8)u_2 = 0 \\
(2u_1 - u_2 - 2)x_1 = 0 \\
(-3u_1 - 2u_2 - 1)x_2 = 0 \\
(-u_1 + u_2 - 3)x_3 = 0
\end{cases}
\qquad
\begin{cases}
(5)u_1 = 0 \\
(-8)u_2 = 0 \\
(2u_1 - u_2 - 2)x_1 = 0 \\
(-3u_1 - 2u_2 - 1)x_2 = 0 \\
(-u_1 + u_2 - 3)x_3 = 0
\end{cases}
$$

$u = (0, 0)$

Second iteration $x = (0, 0, 8, -3, 0)$

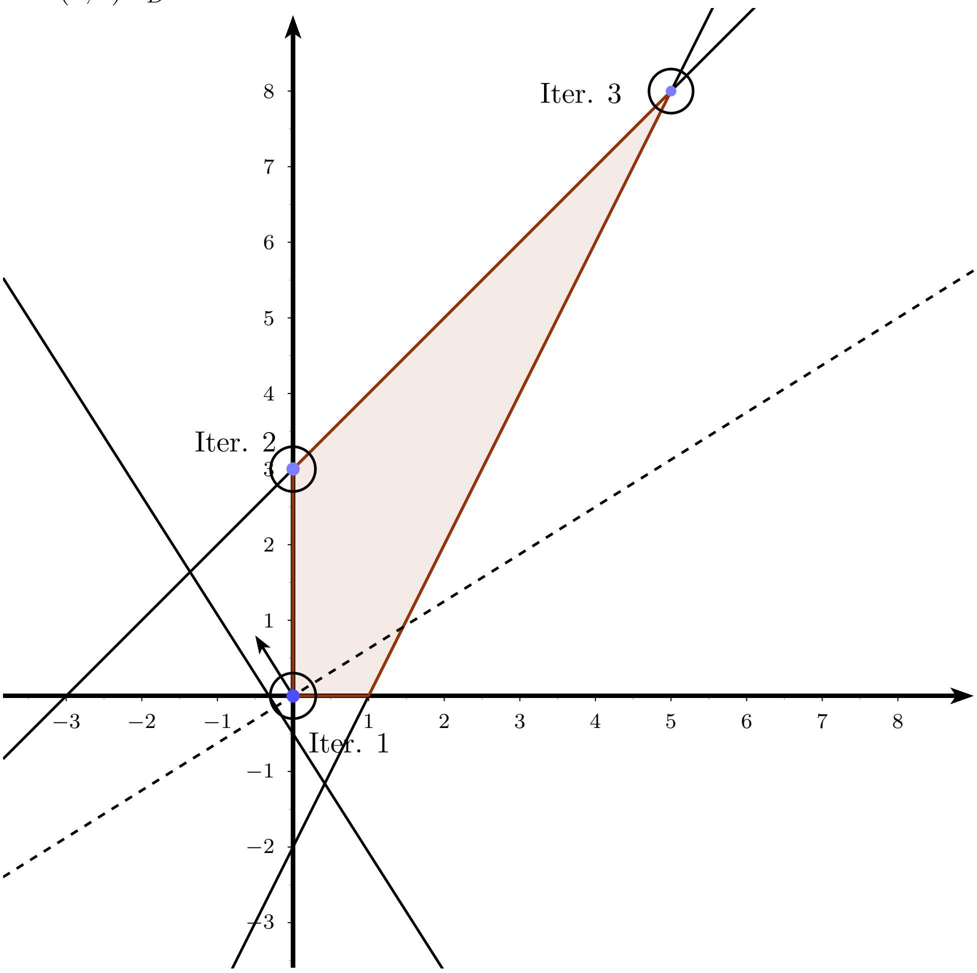
$$
\begin{cases}
(-3)u_1 = 0 \\
(0)u_2 = 0 \\
(2u_1 - u_2 - 2)x_1 = 0 \\
(-3u_1 - 2u_2 - 1)x_2 = 0 \\
-u_1 + u_2 = 3
\end{cases}
$$

$u = (0, 3)$

Third iteration $x = (3, 0, 11, 0, 0)$ $z_P = 39$

$$\begin{cases} (0)u_1 = 0 \\ (0)u_2 = 0 \\ 2u_1 - u_2 = 2 \\ (-3u_1 - 2u_2 - 1)0 = 0 \\ -u_1 + u_2 = 3 \end{cases}$$

$u = (5, 8)$ $z_D = 39$



Exercise 2
Variables

x_{ijk} = kilograms of the good of supplier i transported from distribution center j to customer k
y_{jk} = number of trips from distribution center j to customer k
δ_j = 1 if distribution center j is used; 0 otherwise
M = a big number (constant)
z_{jk} = 1 if distribution center j serve a customer $k \in A$, 0 otherwise

$$\min z = \sum_{j \in J} s_j \delta_j + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} c'_{ij} x_{ijk} + \sum_{j \in J} \sum_{k \in K} c''_{jk} y_{jk} \tag{21}$$

$$\sum_{j \in J} x_{ijk} = d_{ik} \qquad k \in K, i \in I \tag{22}$$

$$\sum_{i \in I} x_{ijk} \leq C y_{jk} \qquad k \in K, j \in J \tag{23}$$

$$\sum_{i \in I} \sum_{k \in K} x_{ijk} \leq M \delta_j \qquad j \in J \tag{24}$$

$$x_{ijk} \geq 0 \qquad i \in I, j \in J, k \in K \tag{25}$$

$$y_{jk} \geq 0 \text{ integer} \qquad j \in J, k \in K \tag{26}$$

$$\delta_j \in \{0, 1\} \qquad j \in J \tag{27}$$

$$y_{jk} \leq M z_{jk} \qquad k \in A, j \in J \tag{28}$$

$$\sum_{j \in J} z_{jk} = 1 \qquad k \in A \tag{29}$$

$$z_{jk} = z_{jh} \qquad k, h \in A, j \in J \tag{30}$$

Exercise 3

```
param n, integer, > 0;
param C, integer, > 0;

set I  := 1..n;
set J ;
set K  := 2..n-1;

param d{i in I}, >= 0;
param s{j in J}, >= 0;
param v{i in I, j in J}, >= 0;
param w{ j in J, k in K}, >= 0;

param M := sum{i in I} d[i];

var  x{i in I, j in I},   >= 0;
var  y{j in J, k in I},   >= 0, integer;
var  delta{j in J},   >= 0, binary;

minimize z: sum{j in J} s[j]* delta[j] + sum{i in I, j in J} v[i,j] * x[i,j] +
sum{j in J, k in K} w[j,k] * y[j,k];
s.t.
      V1{i in I}: sum{j in J}    x[i,j] = d[i];
      V2{k in I, j in J}: sum{i in I} x[i,j] - C * y[j,k] <= 0;
      V3{j in J}: sum{i in I} x[i,j] <= M * delta[j];

solve;

data;
param n := 5;
param C := 100;
set J := 1 2 3;
param d := 1 100 2 150 3 200 4 130 5 170;
param s := 1 10 2 12 3 8;
param v :  1  2  3 :=
      1    5  4  8
      2    1 99  5
      3   99  8  2
      4    1  5  1
      5    1  1  99;
param w :  2  3  4  :=
      1    1  7  6
      2    9  4  5
      3    8  1  2 ;
end;
```