RICERCA OPERATIVA

Written assessment, July 19, 2023

Last name, First name ______________________________

Exercise 1 (value 9)

Containers have to be loaded on a ship. There is a set C of containers that have to be stored in a set L of possible locations. In each location $l \in L$, up to m containers can be stacked on top of each other (being 1 the lower level). Let σ denote an order of the containers, based on the disembark of the containers at destination ports: $\sigma(i) \prec \sigma(j)$ means that the container i will be disembarked before container j. In case container j is stacked on top of container i in a same location $l \in L$ and $\sigma(i) \prec \sigma(j)$, then container j will have to be moved (and placed back in the same location l) in order to unload container i.

Write a linear programming model to help the Captain to load his vessel in such a way that the total container moves for unloading operations is minimized.
(*Suggestion: use a variable to define the position of each container, and a second variable to know if i is below j, in the same location, when $\sigma(i) \prec \sigma(j)$*).

Exercise 2 (value 10)

Consider the following LP problem.

$$
\begin{aligned}
\min z = \quad & 2x_1 + 5x_2 - 3x_3 + 10x_4 \\
& -x_1 - 2x_2 + x_3 - 4x_4 = 110 \\
& x_1 + 10x_2 - 2/3x_3 + 5x_4 = 80 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

The optimal solution is $x^* = (460, 0, 570, 0)$.
Perform sensitivity analysis in order to understand how much the objective function coefficient of the variable x_1 can change without altering the current optimal basis. Explain the procedure.

Exercise 3 (value 8)

Find the optimal solution of the following knapsack problem using a method of your choice (write which method you are using): $n = 4, c = 32$, $(p_j, w_j) = [(20, 3), (30, 14), (20, 25), (9, 18)]$.

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Solution sketch

Exercise 1

$$x_{ilk} = 1 \text{ if container } i \text{ is positioned in location } l \text{ at level } k; 0 \text{ otherwise}$$

$$y_{ij} = 1 \text{ if container } i \text{ is below container } j \text{ in a same location }; 0 \text{ otherwise}$$

$$
\begin{array}{lll}
\min & \displaystyle\sum_{i\in C}\ \sum_{j\in C:\sigma(i)\prec\sigma(j)} y_{ij} & \\
& \displaystyle\sum_{l\in L}\sum_{k=1}^{m} x_{ilk} = 1 & i\in C \\
& \displaystyle\sum_{i\in C} x_{ilk} \le 1 & l\in L, k\in\{1,\dots,m\} \\
& \displaystyle\sum_{i\in C} x_{ilk} \le \sum_{j\in C} x_{jl(k-1)} & l\in L, k\in\{2,\dots,m\} \\
& \displaystyle y_{ij} \ge x_{ilk} + \sum_{h=k+1}^{m} x_{jlh} - 1 & i,j\in C, i\ne j, l\in L, k\in\{1,\dots,m-1\} \\
& x_{ilk}\in\{0,1\} & i\in C, l\in L, k\in\{1,2,\dots,m\} \\
& y_{ij}\in\{0,1\} & i,j\in C, l\ne j
\end{array}
$$

Exercise 2

We recall that the basis does not change if $c_F^T - (c_B^T + \Delta c_B^T)B^{-1}F \ge 0$

$$B = \begin{bmatrix} -1 & 1 \\ 1 & -2/3 \end{bmatrix} \Rightarrow B^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix}$$

$$B^{-1}F = \begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} -2 & -4 \\ 10 & 5 \end{bmatrix} = \begin{bmatrix} 26 & 7 \\ 24 & 3 \end{bmatrix}$$

$$c_F^T - (c_B^T + \Delta c_B^T)B^{-1}F = \begin{bmatrix} 5, & 10 \end{bmatrix} - \begin{bmatrix} 2+\Delta c_1, & -3 \end{bmatrix}\begin{bmatrix} 26 & 7 \\ 24 & 3 \end{bmatrix} = \begin{bmatrix} 25-26\Delta c_1, & 5-7\Delta c_1 \end{bmatrix}$$

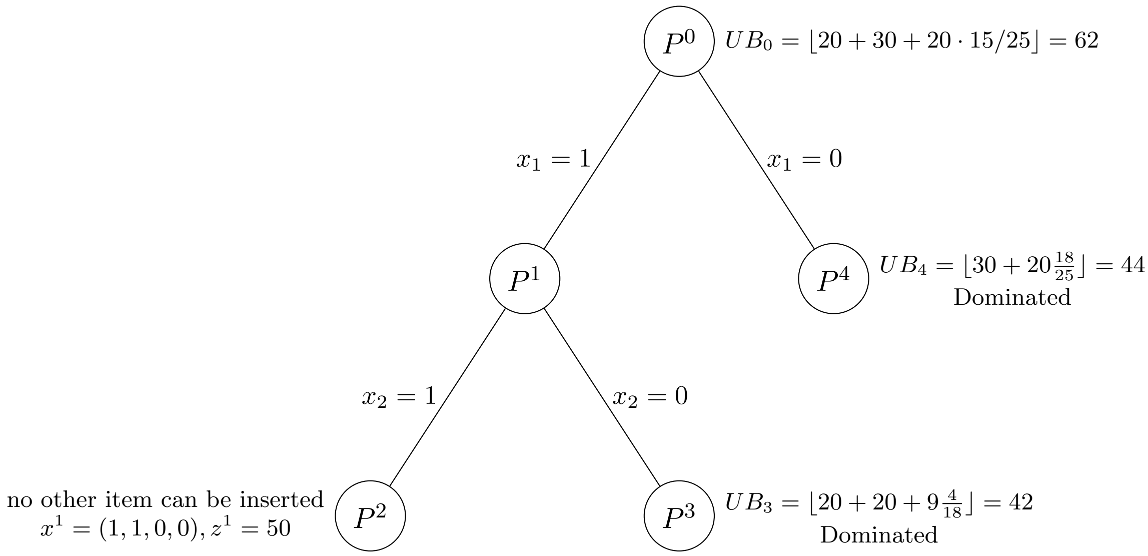
$$\begin{cases} 25 - 26\Delta c_1 \ge 0 \\ 5 - 7\Delta c1 \ge 0 \end{cases} \Rightarrow \begin{cases} \Delta c_1 \le 25/26 \\ \Delta c_1 \le 5/7 \end{cases} \Rightarrow\ \Delta c_1 \le 5/7$$

Exercise 3

$n = 4, c = 32,\ (p_j, w_j) = [(20,3),(30,14),(20,25),(9,18)]$.

$p_j = (20, 30, 20, 9)\ w_j = (3, 14, 25, 18),\ c = 32$

We use the Branch and Bound method. The objects are in the correct order for applying this method.



The optimal solution is $x^1 = (1, 1, 0, 0)$ with cost $z^1 = 50$.