20210908

1. **MM-value=13/time=25**

   ESSAY | 13 points | 0.10 penalty | editor

   A clever university student wants to write a mathematical model to optimize her participation to the examinations to the n courses she attended. The examinations have been fixed by the teachers, along a period of d days. Note that at least one examination has been fixed for each course by the correspondent teacher in the given period. To each element e_{ij} of the $n \times d$ matrix E is given value 1 if course $i = 1, \ldots, n$ fixed an exam on day $j = 1, \ldots, d$, 0 otherwise. The student is not allowed to try more than two times the examination of the same course; moreover, the student is not allowed to attend more than one exam per day.

   Write a mathematical model to allow the student to select a feasible exam plan, by maximizing the number of courses whose exam has been tried at least once. Clearly define all the used variables.

   Notes for grader:

   - - x_{ij}: 1 if the student undergo exam of course i on day j, 0 otherwise;
     - y_i: 1 if the student tries at least one time the exam of course i, 0 otherwise;

   $$\begin{aligned}
   \max \ z = \ & \sum_{i=1}^{n} y_i \\
   & \sum_{j=1}^{d} x_{ij} \leq 2y_i, \quad i = 1, \ldots, n \\
   & y_i \leq \sum_{j=1}^{d} x_{ij}, \quad i = 1, \ldots, n \\
   & \sum_{i=1}^{n} x_{ij} \leq 1, \quad j = 1, \ldots, d \\
   & x_{ij} \leq e_{ij}, \quad i = 1, \ldots, n, j = 1, \ldots, d \\
   & x_{ij} \in \{0, 1\}, \quad i = 1, \ldots, n, j = 1, \ldots, d \\
   & y_i \in \{0, 1\}, \quad i = 1, \ldots, n
   \end{aligned}$$

2. **PLC-value=7/time=15**

ESSAY | 7 points | 0.10 penalty | editor

Consider the following LP problem:

$$
\begin{array}{rlllllll}
\min z = & 3x_1 & +3x_2 & +x_3 & & & & \\
 & 3x_1 & & +4x_3 & -x_4 & & = 1 \\
 & 2x_1 & +x_2 & +x_3 & & -x_5 & = 3 \\
 & x_1, & x_2, & x_3, & x_4, & x_5 & \geq 0.
\end{array}
$$

In the optimal solution the variables in the base are x_3 and x_4

Add to the first equation variable $x_6 \geq 0$ (thus obtaining $3x_1 + 4x_3 - x_4 + x_6 = 1$).

Is the solution still optimal ? Justify your answer. (*Suggestion: the solution remains optimal if the reduced cost of x_6 is ...*

Notes for grader:

- The reduced cost for x_6 is:
  $\overline{c}_6 = c_6 - c_B^T B^{-1} A_6 = 0 - [1, 0] \begin{bmatrix} 0 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$
  The current solution is still optimal.

3. **BB-value=8/time=15**

ESSAY | 8 points | 0.10 penalty | editor

Consider the integer problem

$$
\begin{array}{rllllll}
\min z = & 3x_1 & +3x_2 & +x_3 & & \\
 & 2x_1 & -6x_2 & +4x_3 & & = 1 \\
 & 2x_1 & +3x_2 & +x_3 & +x_4 & = 3 \\
 & x_1, & x_2, & x_3, & x_4 & \geq 0, \mathit{integer}.
\end{array}
$$

And the optimal solution of its linear relaxation:

x_1	x_2	x_3	x_4		
5/2	9/2	0	0	−1/4	$-z$
1/2	−3/2	1	0	1/4	x_3
3/2	9/2	0	1	11/4	x_4

Compute the Gomory's cut associated to the first equation and add it to the tableau: compute the new relaxed solution. Is it optimal for the integer problem ?

Notes for grader:

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	x_1	x_2	x_3	x_4	x_5		
	5/ 2	9/ 2	0	0	0	-1/ 4	$-z$
	1/ 2	- 3/ 2	1	0	0	11/ 4	x_3
	3/ 2	9/ 2	0	1	0	11/ 4	x_4
	- 1/ 2	- 1/ 2	0	0	1	- 1/ 4	x_5

x_1	x_2	x_3	x_4	x_5		
0	2	0	0	5	- 3/ 2	$-z$
0	-2	1	0	1	0	x_3
0	3	0	1	3	2	x_4
1	1	0	0	-2	1/ 2	x_1

The solution is not optimal for the integer problem, since variable x_1 has a fractional value.

Total of marks: 28