RICERCA OPERATIVA

Operations Research February 22, 2016 A

**First and Last name** _____

**Exercise 1** ( value 12)
The manager of a public gross market for fruit and vegetables wants to reorganize the layout of the warehouse that hosts the local market. There are $n$ potential companies interested to have a space in the market. Each company $i = 1 \dots n$ requires a space of $m_i$ square meters and has an associated priority $p_i$. The warehouse has $M$ square meters available for the companies. The companies are grouped in categories. There are $m$ categories. Let $C_j, j = 1, \dots, m$ be the set of all companies of category $j$ (each company belongs to a unique category). Since $\sum_{i=1}^{n} m_i > M$ not all the companies can be accepted in the market, hence the manager will select a group of companies that give a total priority not smaller than a threshold $P$. The last category $m$ differentiate from the previous one, in the sense that if it is not possible to give it at least $\widetilde{m}$ square meters, than no company at all is selected, for this category. the manager wants to give each category almost the same space. This is done by minimizing the sum of the absolute differences between the space given to a category and the "ideal" space $M/m$. Write a Linear Programming model to help the manager to select the companies to be allocated in the market.

**Exercise 2** (value 7)
Consider the following primal PLC problem, write the dual problem and solve it drawing the region of the feasible solutions. Find the optimal solution of the primal problem using the complementary slackness conditions.

$$\begin{aligned} \min \quad & 10x_1 + 15x_2 - 10x_3 \\ & x_1 - 5x_3 \geq 3 \\ & x_1 - 3x_2 + 2x_3 \leq 2 \\ & x_1, \dots, x_3 \geq 0 \end{aligned}$$

**Exercise 3** (value 7)
Consider the directed graph with arc costs given by the following matrix and find the shortest path from vertex 8 to all other vertices reporting the steps of the algorithm in the table on the rear of the sheet.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | - | 3 | - | 2 | - | - | 6 | 4 |
| 2 | - | - | 2 | - | - | - | 2 | - |
| 3 | - | 1 | - | 4 | - | 4 | 5 | 5 |
| 4 | 3 | - | - | - | 1 | - | - | - |
| 5 | 1 | - | - | 2 | - | - | - | - |
| 6 | - | 2 | - | - | - | - | - | 1 |
| 7 | - | - | 1 | 1 | - | - | - | 1 |
| 8 | 4 | - | - | 2 | - | - | 3 | - |

RICERCA OPERATIVA

Operations Research February 22, 2016 B

**First and Last name** _____

**Exercise 1** ( value 12).
The manager of a public gross market for fruit and vegetables wants to reorganize the layout of the warehouse that hosts the local market. There are $n$ potential companies interested to have a space in the market. Each company $i = 1 \dots n$ requires a space of $m_i$ square meters and has an associated priority $p_i$. The companies are grouped in categories. The warehouse has $S$ square meters available for the companies. There are $m$ categories. Let $C_j, j = 1, \dots, m$ be the set of all companies of category $j$ (each company belongs to a unique category). Since $\sum_{i=1}^{n} m_i > S$ not all the companies can be accepted in the market, hence the manager will select some companies so that the total priority of the included companies of a each category are similar. This is done by minimizing the sum of the absolute differences between the total priority of the selected companies in a category and the "ideal" priority $\sum_{i=1}^{n} p_i/m$. Moreover we want that the used square meters are at least $\widetilde{M}$. The last category $m$ differentiate from the previous one, in the sense that if we give it at least $\overline{m}$ square meters, than all companies of this category must be selected. Write a Linear Programming model to help the manager to select the companies to be allocated in the market.

**Exercise 2** (value 7)
Consider the following PLC problem and solve it using the Primal Simplex algorithm. Write the dual problem, drawn the feasible region and find its optimal solution.

$$\begin{aligned} \min \quad & 4x_1 + 8x_2 - 4x_3 \\ & -x_1 + 2x_3 \leq 2 \\ & -2x_1 + 2x_2 + x_3 \geq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

**Exercise 3** (value 7)
Consider the directed graph with arc costs given by the following matrix and find the shortest path from vertex 8 to all other vertices reporting the steps of the algorithm in the table on the rear of the sheet.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | - | 2 | - | 2 | - | - | 4 | 4 |
| 2 | - | - | 1 | 2 | - | - | - | - |
| 3 | - | 1 | - | 2 | - | 4 | 5 | 4 |
| 4 | 3 | - | - | - | 1 | - | - | - |
| 5 | 1 | - | - | - | - | - | - | - |
| 6 | - | 2 | - | - | - | - | - | 1 |
| 7 | - | - | 1 | 3 | - | - | - | 1 |
| 8 | 6 | - | - | 2 | - | - | 1 | - |

RICERCA OPERATIVA

Operations Research February 22, 2016 A

**First and Last name** _____

**Exercise 1**

$x_i$ = 1 if company $i$ is selected; 0 otherwise

$z_j$ = absolute difference between $M/m$ and the space allocated to category $j$

$\delta$ = 1 if we select companies from category $m$, requiring at least $\widetilde{m}$ square meters; 0 otherwise

$$\min \sum_{j=1}^{m} z_j$$

$$\sum_{i=1}^{n} p_i x_i \geq P, \tag{1}$$

$$\sum_{i=1}^{n} m_i x_i \leq M, \tag{2}$$

$$\sum_{i \in C_m} m_i x_i \geq \widetilde{m}\delta, \tag{3}$$

$$\sum_{i \in C_m} x_i \leq n\delta \tag{4}$$

$$\sum_{i \in C_j} m_i x_i - \frac{M}{m} \leq z_j, \quad j = 1, \ldots, m \tag{5}$$

$$\frac{M}{m} - \sum_{i \in C_j} m_i x_i \leq z_j, \quad j = 1, \ldots, m \tag{6}$$

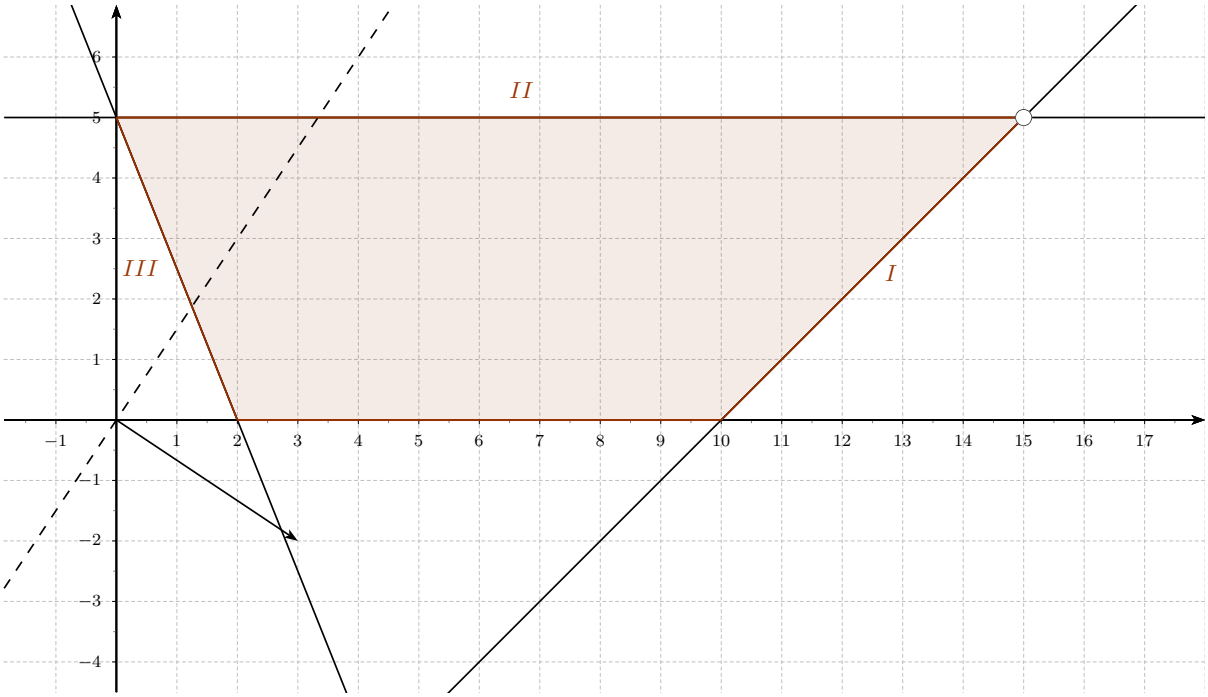
$$x_i \in \{0, 1\} \quad i = 1, \ldots, n \tag{7}$$

$$z_j \geq 0, \quad j = 1, \ldots, m \tag{8}$$

$$\delta_i \in \{0, 1\}, \quad i \in P \tag{9}$$

**Exercise 2**

$$\begin{aligned} \min \quad & 10x_1 + 15x_2 - 10x_3 \\ & x_1 - 5x_3 \geq 3 \\ & x_1 - 3x_2 + 2x_3 \leq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned} \qquad \begin{aligned} \max \quad & 3u_1 - 2u_2 \\ & u_1 - u_2 \leq 10 \\ & 3u_2 \leq 15 \\ & 5u_1 + 2u_2 \geq 10 \\ & u_1, u_2 \geq 0 \end{aligned}$$




$$\begin{cases} (x_1 - 5x_3 - 3)u_1 = 0 \\ (x_1 - 3x_2 + 2x_3 - 2)u_2 = 0 \\ (u_1 - u_2 - 10)x_1 = 0 \\ (3u_2 - 15)x_2 = 0 \\ (5u_1 + 2u_2 - 10)x_3 = 0 \end{cases} \qquad \begin{cases} (x_1 - 5x_3 - 3)15 = 0 \\ (x_1 - 3x_2 + 2x_3 - 2)5 = 0 \\ (0)x_1 = 0 \\ (0)x_2 = 0 \\ (45)x_3 = 0 \end{cases}$$

$u = (15, 5), z_D = 35, \quad x = (3, 1/3, 0), z_P = 35$

**Exercise 3**

| | predecessors | | | | | | | | labels | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 8 | 8 | | | 8 | | | 8 | | 4 | | | **2** | | | 3 | |
| 8,4 | 8 | | | 8 | 4 | | 8 | | 4 | | | - | **3** | | 3 | |
| 8,4,5 | 8 | | | 8 | 4 | | 8 | | 4 | | | - | - | | **3** | |
| 8,4,5,7 | 8 | | 7 | 8 | 4 | | 8 | | **4** | | 4 | - | - | | - | |
| 8,4,5,7,1 | 8 | 1 | 7 | 8 | 4 | | 8 | | - | 7 | **4** | - | - | | - | |
| 8,4,5,7,1,3 | 8 | 3 | 7 | 8 | 4 | 3 | 8 | | - | **5** | - | - | - | 8 | - | |
| 8,4,5,7,1,3,2 | 8 | 3 | 7 | 8 | 4 | 3 | 8 | | - | - | - | - | - | 8 | - | |
| 8,4,5,7,1,3,2,6 | 8 | 3 | 7 | 8 | 4 | 3 | 8 | | 4 | 5 | 4 | 2 | 3 | 8 | 3 | |

RICERCA OPERATIVA

Operations Research February 1, 2016 B

**First and Last name** _____

**Exercise 1**

$x_i$ = 1 if company $i$ is selected; 0 otherwise

$z_j$ = absolute difference between $\sum_{i=1}^{n} p_i/m$ and the total priority of a category $j$

$\delta$ = 1 if we give at least $\overline{m}$ square meters to the last category ; 0 otherwise

$$\min \sum_{j=1}^{m} z_j$$

$$\sum_{i=1}^{n} m_i x_i \geq \widetilde{M}, \tag{1}$$

$$\sum_{i=1}^{n} m_i x_i \leq S, \tag{2}$$

$$\sum_{i \in C_m} m_i x_i \leq \overline{m} - 1 + S\delta, \tag{3}$$

$$\sum_{i \in C_m} x_i \geq |C_m|\delta \tag{4}$$

$$\sum_{i \in C_j} p_i x_i - \frac{\sum_{i=1}^{n} p_i}{m} \leq z_j, \quad j = 1, \ldots, m \tag{5}$$

$$\frac{\sum_{i=i}^{n} p_1}{m} - \sum_{i \in C_j} p_i x_i \leq z_j, \quad j = 1, \ldots, m \tag{6}$$

$$x_i \in \{0, 1\} \quad i = 1, \ldots, n \tag{7}$$

$$z_j \geq 0, \quad j = 1, \ldots, m \tag{8}$$

$$\delta \in \{0, 1\}, \tag{9}$$

**Exercise 2**

**FASE II**

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | | |
|---|---|---|---|---|---|---|---|
| 2 | -2 | -1 | 0 | 1 | 0 | 0 | $-z$ |
| -1 | 0 | 2 | 1 | 0 | 0 | 2 | $x_4$ |
| -2 | (2) | 1 | 0 | -1 | 1 | 2 | $x_6$ |

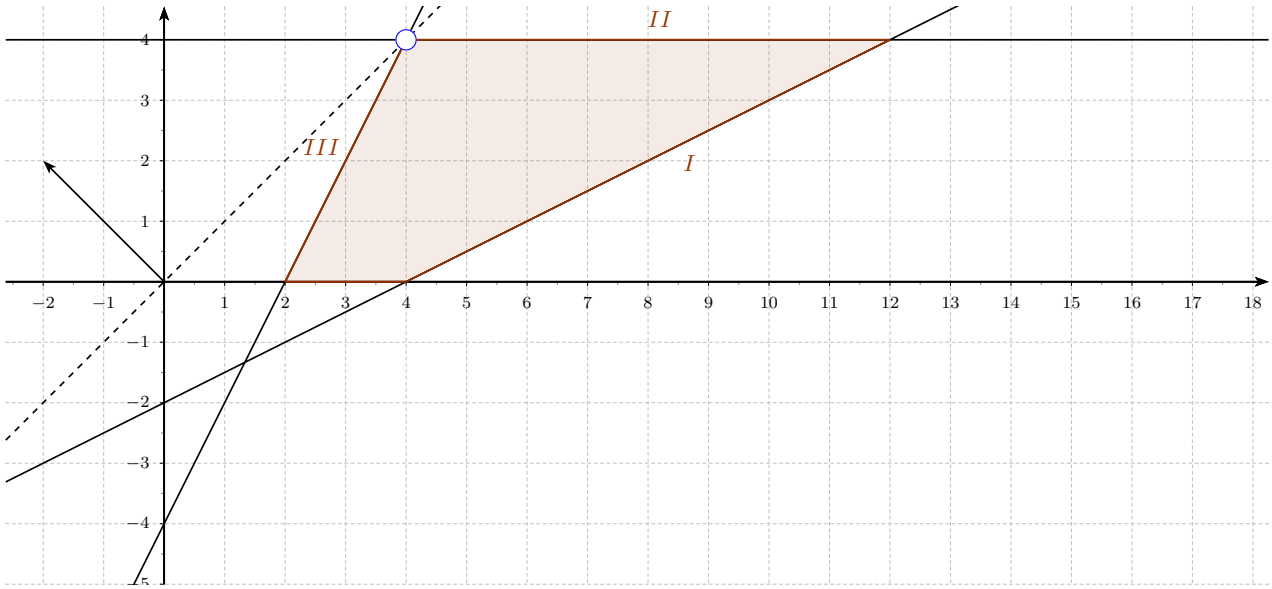
| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | $-z$ |
| -1 | 0 | 2 | 1 | 0 | 0 | 2 | $x_4$ |
| -1 | 1 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $\frac{1}{2}$ | 1 | $x_2$ |

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 12 | 0 | -8 | 0 | 4 | -8 | $-z$ |
| -1 | 0 | (2) | 1 | 0 | 2 | $x_4$ |
| -1 | 1 | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 1 | $x_2$ |

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | | |
|---|---|---|---|---|---|---|
| 8 | 0 | 0 | 4 | 4 | 0 | $-z$ |
| $-\frac{1}{2}$ | 0 | 1 | $\frac{1}{2}$ | 0 | 1 | $x_3$ |
| $-\frac{3}{4}$ | 1 | 0 | $-\frac{1}{4}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $x_2$ |

$$\begin{aligned} \min \quad & 4x_1 + 8x_2 - 4x_3 \\ & -x_1 + 2x_3 \leq 2 \\ & -2x_1 + 2x_2 + x_3 \geq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

$$\begin{aligned} \max \quad & -2u_1 + 2u_2 \\ & u_1 - 2u_2 \leq 4 \\ & 2u_2 \leq 8 \\ & 2u_1 - u_2 \geq 4 \\ & u_1, u_2 \geq 0 \end{aligned}$$




$$\begin{cases} (-x_1 + 2x_3 - 2)u_1 = 0 \\ (-2x_1 + 2x_2 + x_3 - 2)u_2 = 0 \\ (u_1 - 2u_2 - 4)x_1 = 0 \\ (2u_2 - 8)x_2 = 0 \\ (2u_1 - u_2 - 4)x_3 = 0 \end{cases} \qquad \begin{cases} (-x_1 + 2x_3 - 2)4 = 0 \\ (-2x_1 + 2x_2 + x_3 - 2)4 = 0 \\ (-8)x_1 = 0 \\ (0)x_2 = 0 \\ (45)x_3 = 0 \end{cases}$$

$u = (4, 4), z_D = 0, \quad x = (0, 1/2, 3/5), z_P = 35$

**Exercise 3**

| | predecessors | | | | | | | | labels | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 8 | 8 | | | 8 | | | 8 | | 6 | | | 2 | | | **1** | |
| 8,7 | 8 | | 7 | 8 | | | 8 | | 6 | | **2** | 2 | | | - | |
| 8,7,3 | 8 | 3 | 7 | 8 | | 3 | 8 | | 6 | 3 | - | **2** | | 6 | - | |
| 8,7,3,4 | 4 | 3 | 7 | 8 | 4 | 3 | 8 | | 5 | **3** | - | - | 3 | 6 | - | |
| 8,7,3,4,2 | 4 | 3 | 7 | 8 | 4 | 3 | 8 | | 5 | - | - | - | **3** | 6 | - | |
| 8,7,3,4,2,5 | 5 | 3 | 7 | 8 | 4 | 3 | 8 | | **4** | - | - | - | - | 6 | - | |
| 8,7,3,4,2,5,1 | 5 | 3 | 7 | 8 | 4 | 3 | 8 | | - | - | - | - | - | **6** | - | |
| 8,7,3,4,2,5,1,6 | 5 | 3 | 7 | 8 | 4 | 3 | 8 | | 4 | 3 | 2 | 2 | 3 | 6 | 1 | |