Written assessment, January 27, 2022

Last and first name ______________________________

Exercise 1 (value 12, time 22)
A large private telecom company is going to improve its network by means of a set S of communication satellites. Each satellite $i \in S$ is able to transfer g_i Gigabytes per second, has a weight of w_i kilograms and a cost of cs_i euro. The company identified a set A of areas on the earth's surface, that must be served by the satellites. Each area $j \in A$ requires a total communication speed of G_j Gigabytes per second. Each satellite can be used for communication in a single area. The satellites are launched into space from of the bases of set B, located in different countries. A satellite launched from a base $k \in B$ can be used to communicate only with the areas in set $A_k \subset A$. Each base k has r_k rockets available. Each rocket $h = 1, \ldots, r_k$ can load at most W_{hk} kilograms. The objective of the company is to satisfy the communication requirements at minimum cost. (Tip: you might use a set of variables to define the satellites covering each area, and a second set to define from which area and on which rocket each satellites is launched).

Exercise 2 (value 9, time 18) Given the following LP problem:

$$
\begin{aligned}
\min \quad & 3x_1 + 3x_2 \\
& x_1 - 6x_2 \leq -1 \\
& 2x_1 + 3x_2 \geq 4 \\
& x_1 \geq 3 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Solve it using the dual simplex method. Always choose the row with the most negative r.h.s. for pivoting. Given the complementary slackness conditions of the LP above derive the optimal dual solution.

$$
\begin{aligned}
(x_1 - 6x_2 + 1)u_1 &= 0 \\
(2x_1 + 3x_2 - 4)u_2 &= 0 \\
(x_1 - 3)u_3 &= 0 \\
(-u_1 + 2u_2 + u_3 - 3)x_1 &= 0 \\
(6u_1 + 3u_2 - 3)x_2 &= 0
\end{aligned}
$$

Exercise 3 (value 7, time 12) Solve the following Knapsack Problem: weight $w_j = (5, 3, 2)$, profit $p_j = (4, 1, 2)$, and capacity $C = 6$.
Use a Dynamic Programming where the **states** correspond to the possible **profit levels**. (Tip: you can use M for infinity and E for empty set). Report all the iterations giving the states and their values. Report the optimal solution and its profit.

Written assessment, January 10, 2022

Last and first name ______________________________

Exercise 1

Variables

$x_{ij} = 1$ if satellite i serves area j, 0 otherwise.
$y_{ihk} = 1$ if satellite i is launched from base k using rocket h, 0 otherwise.

Model

$$\min z = \sum_{i\in S}\sum_{j\in A} cs_i x_{ij} + \sum_{i\in S}\sum_{k\in B}\sum_{h=1}^{r_k} cr_{hk} y_{ihk}$$

$$\sum_{i\in S} g_i x_{ij} \geq G_j \qquad j \in A$$

$$\sum_{j\in A} x_{ij} \leq 1 \qquad i \in S$$

$$\sum_{i\in S} w_i y_{ihk} \leq W_{hk} \qquad k \in B, h = 1,\ldots,r_k$$

$$\sum_{k\in B: j\in A_k}\sum_{h=1}^{r_k} y_{ihk} \geq x_{ij} \qquad i \in S, j \in A$$

$$x_{ij} \in \{0,1\} \qquad i \in S, j \in A$$

$$y_{ihk} \in \{0,1\} \qquad i \in S, k \in B, h = 1,\ldots,r_k$$

Exercise 2

x_1	x_2	x_3	x_4	x_5		
3	3	0	0	0	0	$-z$
1	-6	1	0	0	-1	x_3
-2	$\boxed{-3}$	0	1	0	-4	x_4
-1	0	0	0	1	-3	x_5

x_1	x_2	x_3	x_4	x_5		
1	0	0	1	0	-4	$-z$
5	0	1	-2	0	7	x_3
2/3	1	0	-1/3	0	4/3	x_2
$\boxed{-1}$	0	0	0	1	-3	x_5

x_1	x_2	x_3	x_4	x_5		
0	0	0	1	1	-7	$-z$
0	0	1	$\boxed{-2}$	5	-8	x_3
0	1	0	-1/3	2/3	-2/3	x_2
1	0	0	0	-1	3	x_1

x_1	x_2	x_3	x_4	x_5		
0	0	1/2	0	7/2	-11	$-z$
0	0	-1/2	1	-5/2	4	x_4
0	1	-1/6	0	-1/6	2/3	x_2
1	0	0	0	-1	3	x_1

$x = (3, 2/3, 0, 4, 0)$ $z_P = 11$

Dual solution:
$u = (1/2, 0, 7/2)$ $z_D = 11$

Exercise 3

$w_j = (5,\ 3,\ 2)$, $p_j = (4,\ 1,\ 2)$, $C = 6$. $P = 4 + 1 + 2 = 7$

	0	1	2	3	4	5	6	7
f_0	0	M	M	M	M	M	M	M
f_1	0	M	M	M	5	M	M	M
f_2	0	3	M	M	5	8	M	M
f_3	0	3	2	5	$\boxed{5}$	8	7	10

	0	1	2	3	4	5	6	7
J_0	E	E	E	E	E	E	E	E
J_1	E	E	E	E	$\{1\}$	E	E	E
J_2	E	$\{2\}$	E	E	$\{1\}$	$\{1,2\}$	E	E
J_3	E	$\{2\}$	$\{3\}$	$\{2,3\}$	$\boxed{\{1\}}$	$\{1,2\}$	$\{1,3\}$	$\{1,2,3\}$

The optimal solution $x = \{1\}$ has profit 4 and weight 5.