RICERCA OPERATIVA

Written assessment, July 17, 2017 **A**

First and Last name ______________________

Exercise 1 (value 6)
Consider the following PLC problem and solve it using the Simplex algorithm with the Bland rule.

$$
\begin{aligned}
\min \quad & -4x_1 + 6x_2 + 3x_3 - 4x_4 \\
& 2x_1 - x_2 = 6 \\
& x_1 + 2x_3 + 3x_4 = 10 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

Exercise 2 (value 6)
Write the dual of the problem described at exercise 1, draw the feasible region and find the optimal dual solution using the complementary slackness conditions.

Exercise 3 (value 10).
A tile factory wants to organize the storage area of the pallets containing final products. There is a set I of pallets that have to be stored in a set L of possible locations. In each location one can store up to m pallets one over the other. Over each pallet of a special "fragile" set F one can store at most two pallets.

Write a linear programming model to help the factory to find a feasible storage for all pallets (without objective function).

Let σ denote a sorting of the the pallets, i.e., $\sigma(i) < \sigma(j)$ indicates that pallet i precedes pallet j. In a second time the pallets will be picked from the storage area one after the other accordingly to the above ordering. If pallet i with $\sigma(i) < \sigma(j)$ is stored under pallet j (in the same location) before picking i one must move pallet j. Improve the linear programming model by adding an objective function which minimizes the movement of pallets.

Exercise 4 (value 6)
Write a GLPK or XPRESS model for the following PLI model.

$$\min \sum_{i \in D} O_i^d y_i^d + \sum_{i \in S} O_i^s y_i^s + \sum_{(i,j) \in A} c_{ij} f_{ij} \tag{10}$$

$$\sum_{i \in D} f_{ij} - \sum_{i \in C} f_{ji} = 0 \qquad j \in S \tag{11}$$

$$\sum_{i \in S} f_{ij} \geq d_j \qquad j \in C \tag{12}$$

$$\sum_{j \in S} f_{ij} \leq Q_i^d y_i^d \qquad i \in D \tag{13}$$

$$\sum_{i \in D} f_{ij} \leq Q_j^s y_j^s \qquad j \in S \tag{14}$$

$$\sum_{i \in S} h_{ij}^c = 1 \qquad j \in C \tag{15}$$

$$f_{ij} \leq Q_i^s h_{ij}^c \qquad i \in S, j \in C \tag{16}$$

$$h_{ij}^c \in \{0,1\} \qquad i \in S, j \in C \tag{17}$$

$$y_i^d \in \{0,1\} \qquad i \in D \tag{18}$$

$$y_i^s \in \{0,1\} \qquad i \in S \tag{19}$$

$$f_{ij} \geq 0 \qquad (i,j) \in A \tag{20}$$

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Answers

First and Last name ______________________

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Exercise 1

$$\begin{aligned} \min \quad & -4x_1 + 6x_2 + 3x_3 - 4x_4 \\ & 2x_1 - x_2 = 6 \\ & x_1 + 2x_3 + 3x_4 = 10 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

FASE I

x_1	x_2	x_3	x_4	x_1^a	x_2^a		
-3	1	-2	-3	0	0	-16	$-\xi$
2	-1	0	0	1	0	6	x_1^a
1	0	2	3	0	1	10	x_2^a

x_1	x_2	x_3	x_4	x_1^a	x_2^a		
0	$-\frac{1}{2}$	-2	-3	$\frac{3}{2}$	0	-7	$-\xi$
1	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	3	x_1
0	$\frac{1}{2}$	2	3	$-\frac{1}{2}$	1	7	x_2^a

x_1	x_2	x_3	x_4	x_1^a	x_2^a		
0	0	0	0	1	1	0	$-\xi$
1	0	2	3	0	1	10	x_1
0	1	4	6	-1	2	14	x_2

FASE II

x_1	x_2	x_3	x_4		
0	0	-13	-28	-44	$-z$
1	0	2	3	10	x_1
0	1	4	6	14	x_2

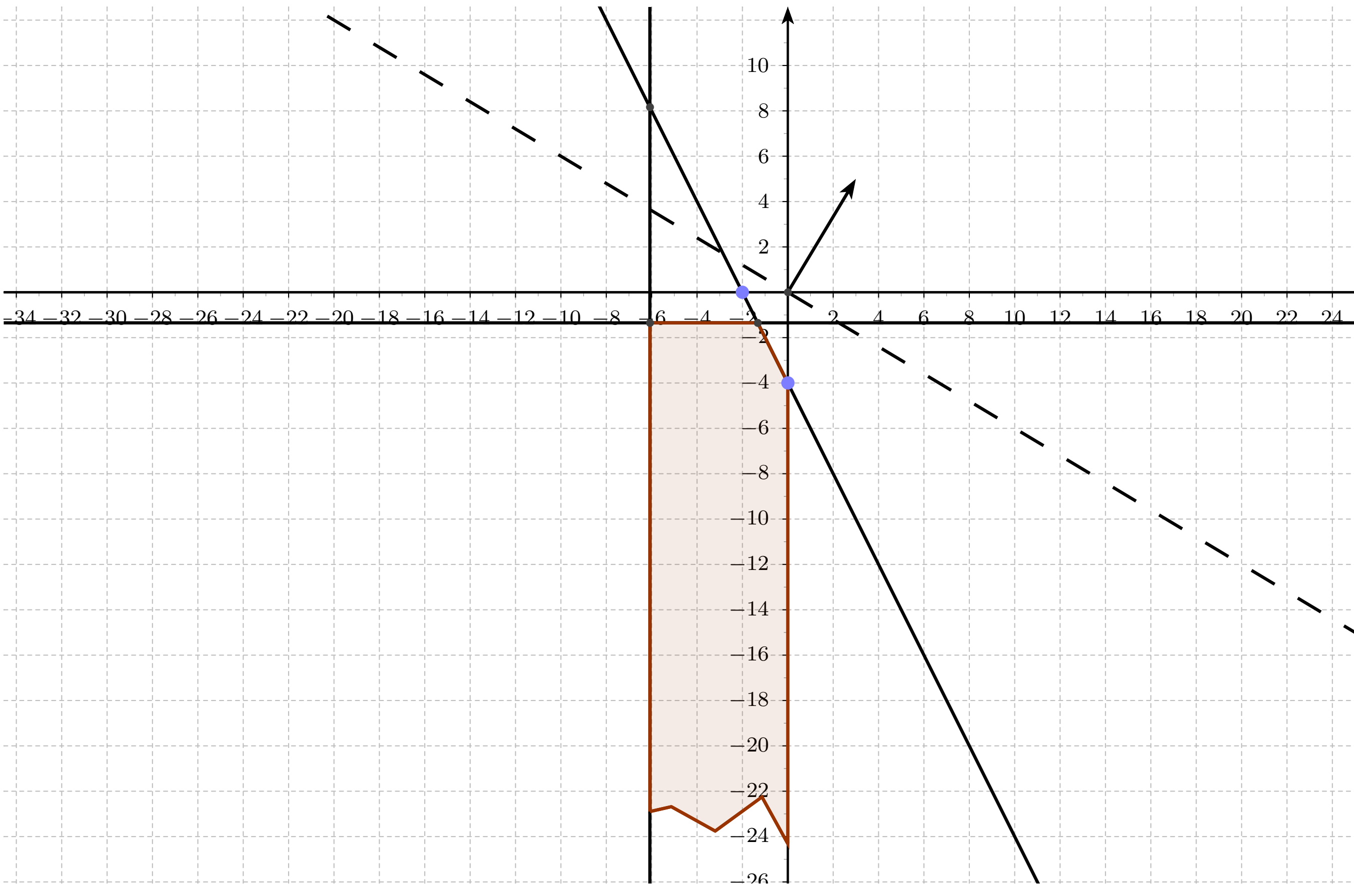
x_1	x_2	x_3	x_4		
0	$\frac{13}{4}$	0	$-\frac{17}{2}$	$\frac{3}{2}$	$-z$
1	$-\frac{1}{2}$	0	0	3	x_1
0	$\frac{1}{4}$	1	$\frac{3}{2}$	$\frac{7}{2}$	x_3

x_1	x_2	x_3	x_4		
0	$\frac{14}{3}$	$\frac{17}{3}$	0	$\frac{64}{3}$	$-z$
1	$-\frac{1}{2}$	0	0	3	x_1
0	$\frac{1}{6}$	$\frac{2}{3}$	1	$\frac{7}{3}$	x_4

Exercise 2

$$\begin{array}{ll} \min & -4x_1 + 6x_2 + 3x_3 - 4x_4 \\ & 2x_1 - x_2 = 6 \\ & x_1 + 2x_3 + 3x_4 = 10 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{array}$$

$$\begin{array}{ll} \max & 6u_1 + 10u_2 \\ & 2u_1 + u_2 \leq -4 \\ & -u_1 \leq 6 \\ & 2u_2 \leq 3 \\ & 3u_2 \leq -4 \end{array}$$



$$\begin{cases} (2x_1 - x_2 - 6)u_1 = 0 \\ (x_1 + 2x_3 + 3x_4 - 10)u_2 = 0 \\ (2u_1 + u_2 + 4)x_1 = 0 \\ (-u_1 - 6)x_2 = 0 \\ (2u_2 - 3)x_3 = 0 \\ (3u_2 + 4)x_4 = 0 \end{cases} \qquad \begin{cases} (0)u_1 = 0 \\ (0)u_2 = 0 \\ (2u_1 + u_2 + 4)x_1 = 0 \\ (-u_1 - 6)0 = 0 \\ (2u_2 - 3)0 = 0 \\ (3u_2 + 4)x_4 = 0 \end{cases}$$

$$\begin{cases} 2u_1 + u_2 = -4 \\ 3u_2 = -4 \end{cases} \qquad \begin{cases} u_1 = -4/3 \\ u_2 = -4/3 \end{cases}$$

Exercise 3

$$\begin{aligned} x_{i\ell k} &= \text{1 if pallet } i \text{ is stored in location } \ell \text{ at level } k\text{; 0 otherwise} \\ y_{ij} &= \text{1 if pallet below pallet } j \text{ (in the same location); 0 otherwise} \end{aligned}$$

$$\min \sum_{i \in I} \sum_{j \in I : \sigma(i) < \sigma(j)} y_{ij} \tag{21}$$

$$\sum_{\ell \in L} \sum_{k=1}^{m} x_{i\ell k} = 1 \quad i \in I \tag{22}$$

$$\sum_{i \in I} x_{i\ell k} \leq 1 \quad \ell \in L, k = 1, \ldots, m \tag{23}$$

$$\sum_{i \in I} x_{i\ell k} \geq \sum_{i \in I} x_{i\ell(k+1)} \quad \ell \in L, k = 1, \ldots, m-1 \tag{24}$$

$$m(1 - x_{i\ell k}) \geq \sum_{h=k+1}^{m} \sum_{j \in I : j \neq i} x_{j\ell h} - 2 \quad i \in F, \ell \in L, k = 1, \ldots, m-3 \tag{25}$$

$$x_{i\ell k} + \sum_{h=k+1}^{m} x_{j\ell k} - 1 \leq y_{ij} \quad i, j \in I, i \neq j, \ell \in L, k = 1, \ldots, m-1 \tag{26}$$

$$x_{i\ell k} \in \{0, 1\} \quad i \in I, \ell \in L, k = 1, \ldots, m \tag{27}$$

$$y_{ij} \in \{0, 1\} \quad i, j \in I : i \neq j \tag{28}$$

$$\tag{29}$$