Ricerca Operativa

Assignment, February 2, 2017 — A

First and Last name ______________________________

Exercise 1 (value 12)
The town of Xu in an emerging country wants to plan the extension of the city to a neighbouring area. The new settlement has been divided into n small areas. Let V denote the set of all areas. For each pair of areas $i, j \in V$ which are considered in "proximity" by the city planners there is a link (i, j) in the link set E. The settlement must host private buildings and public facilities. There are m potential private buildings, each described by a volume requirement w_j and by a profit (for the municipality) of p_{ij} euro, if building j is built on area $i \in V$. The public facilities are of two kinds. The construction of a facility of kind one (resp. two) on area $i \in V$ cost c_i^I euros (resp. c_i^{II}) euros. Each private building must be in proximity with a facility of kind one, while only 50% of the private building must be in proximity with a facility of kind two. In each area we can build a single private building or a single public facility. The total volume of the private buildings cannot exceed the quantity Q. The cost for the public facilities cannot exceed the budget B.

Write a linear program to help the public administration to define an optimal plan while maximizing their profit.

Exercise 2 (value 9)
Consider the following PLC problem. Solve it with the simplex method. Write the dual problem and solve it using a graphical method. Using the primal and dual solution found, check their optimality using the complementary slackness conditions.

$$
\begin{aligned}
\min \quad & x_1 + 2x_2 + 6x_3 \\
& 3x_1 + 3x_2 + 2x_3 \leq 8 \\
& x_1 - 2x_2 + 4x_3 \geq 10 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Exercise 3 (value 7)
Consider the dual problem of exercise 2. Impose that the dual variable must be integral and solve it using the branch-and-bound standard by selecting for the first branch the variable u_2.

Assihnment, February 1, 2017 **A**

First and Last name ______________________________

Exercise 1

Data: let $\delta(i) = \{h \in V : (i,h) \in E\}$

x_{ij} = 1 if building i is built on area j, 0 otherwise

y_j^I = 1 if a facility of kind one is built on area j, 0 otherwise

y_j^{II} = 1 if a facility of kind two is built on area j, 0 otherwise

α_i = 1 if there is a private building in area j and there is at least a facilities of kind two in proximity 0 otherwise

$$\max\ z = \sum_{j \in V} \left(\sum_{i=1}^{m} p_{ij} x_{ij} - c_j^I y_j^I - c_j^{II} y_j^{II} \right)$$

$$\sum_{i=1}^{m} x_{ij} + y_j^I + y_j^{II} \leq 1 \quad \forall j \in V; \tag{23}$$

$$\sum_{h \in \delta(j)} y_h^I \geq \sum_{i=1}^{m} x_{ij} \quad \forall j \in V; \tag{24}$$

$$\alpha_j \leq \sum_{i=1}^{m} x_{ij} \quad \forall j \in V; \tag{25}$$

$$\alpha_j \leq \sum_{h \in \delta(j)} y_h^{II} \quad \forall j \in V; \tag{26}$$

$$\sum_{j \in V} \alpha_j \leq 0.5 \sum_{j \in V} \sum_{i=1}^{m} x_{ij}; \tag{27}$$

$$\sum_{j \in V} \sum_{i=1}^{m} w_i x_{ij} \leq Q; \tag{28}$$

$$\sum_{j \in V} (c_j^I y_j^I + c_j^{II} y_j^{II}) \leq B; \tag{29}$$

$$x_{ij} \in \{0,1\} \quad \forall j \in V, i = 1, \ldots, m; \tag{30}$$

$$y_j^I \in \{0,1\} \quad \forall j \in V; \tag{31}$$

$$y_j^{II} \in \{0,1\} \quad \forall j \in V; \tag{32}$$

$$\alpha_j \in \{0,1\} \quad \forall j \in V; \tag{33}$$

Exercise 2

$$\begin{aligned} \min \quad & x_1 + 2x_2 + 6x_3 \\ & 3x_1 + 3x_2 + 2x_3 \leq 8 \\ & x_1 - 2x_2 + 4x_3 \geq 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

x_1	x_2	x_3	x_4	x_5		
1	2	6	0	0	0	$-z$
3	3	2	1	0	8	x_4
-1	2	-4	0	1	-10	x_5

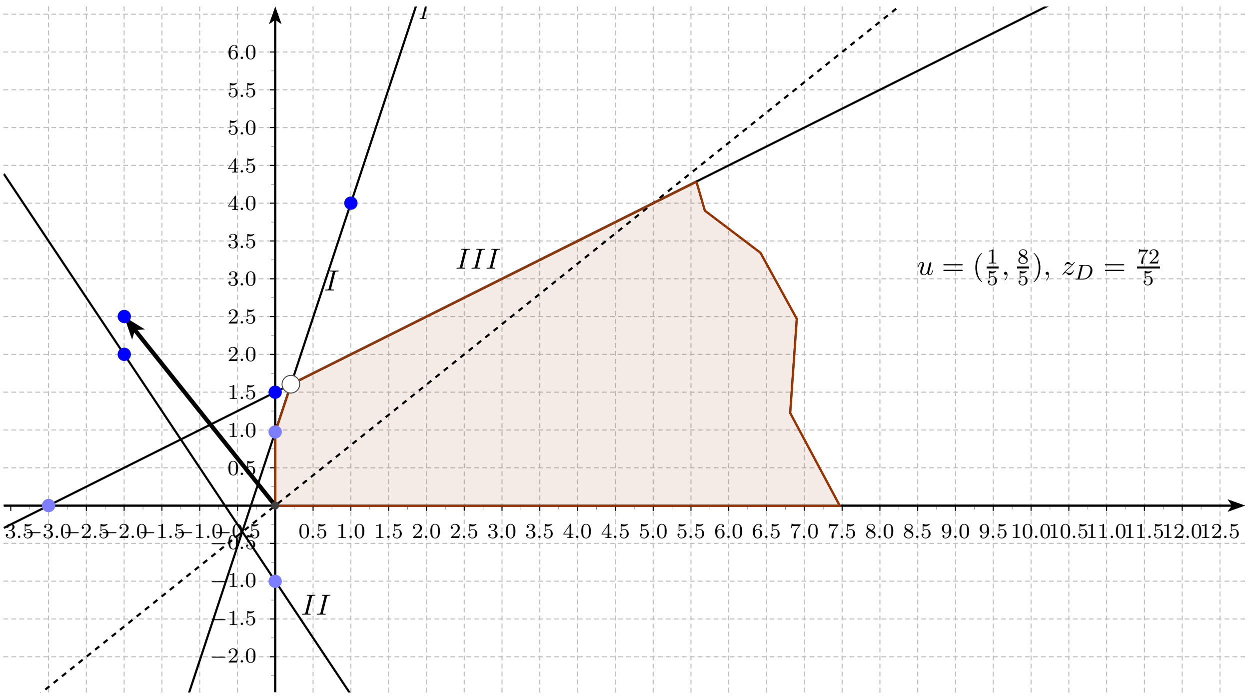
x_1	x_2	x_3	x_4	x_5		
0	4	2	0	1	-10	$-z$
0	9	-10	1	3	-22	x_4
1	-2	4	0	-1	10	x_1

x_1	x_2	x_3	x_4	x_5		
0	$\frac{29}{5}$	0	$\frac{1}{5}$	$\frac{8}{5}$	$-\frac{72}{5}$	$-z$
0	$-\frac{9}{10}$	1	$-\frac{1}{10}$	$-\frac{3}{10}$	$\frac{11}{5}$	x_3
1	$\frac{8}{5}$	0	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{6}{5}$	x_1

$x = (\frac{6}{5}, 0, \frac{11}{5}, 0, 0)$, $z_P = \frac{72}{5}$

$$\begin{aligned} \min \quad & x_1 + 2x_2 + 6x_3 \\ & 3x_1 + 3x_2 + 2x_3 \leq 8 \\ & x_1 - 2x_2 + 4x_3 \geq 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

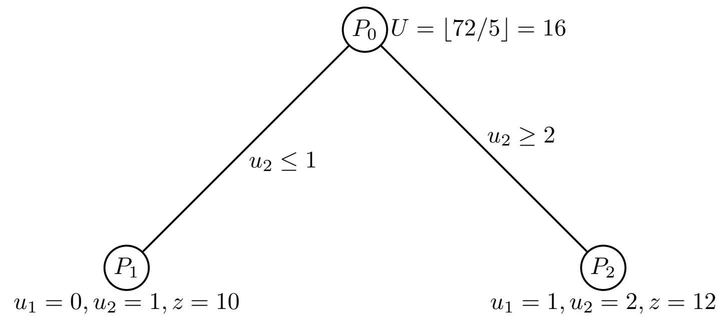
$$\begin{aligned} \max -8u_1 + 10u_2 & & \\ -3u_1 + u_2 &\leq 1 \\ -3u_1 - 2u_2 &\leq 2 \\ -2u_1 + 4u_2 &\leq 6 \\ u_1, u_2 &\geq 0 \end{aligned}$$

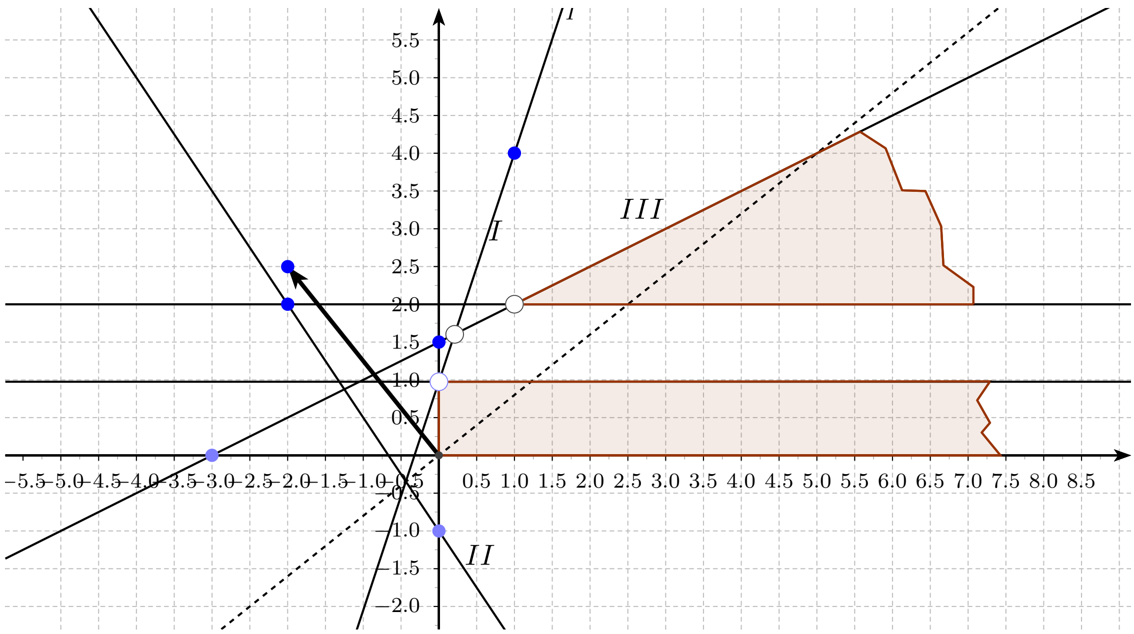


$$\begin{cases} (3x_1 + 3x_2 + 2x_3 - 8)u_1 = 0 \\ (x_1 - 2x_2 + 4x_3 - 10)u_2 = 0 \end{cases} \qquad \begin{matrix} x_2 = 0 \\ u_1, u_2 > 0 \end{matrix} \quad \Rightarrow \quad \begin{cases} 3x_1 + 2x_3 = 8 \\ x_1 + 4x_3 = 10 \end{cases} \begin{cases} x_1 = \frac{6}{5} \\ x_3 = \frac{11}{5} \end{cases}$$

$$\begin{cases} (-3u_1 + u_2 - 1)x_1 = 0 \\ (-3u_1 - 2u_2 - 2)x_2 = 0 \\ (-2u_1 + 4u_2 - 6)x_3 = 0 \end{cases} \qquad x_1, x_3 > 0 \quad \Rightarrow \quad \begin{cases} -3u_1 + u_2 = 1 \\ -2u_1 + 4u_2 = 6 \end{cases} \begin{cases} u_1 = \frac{1}{5} \\ u_2 = \frac{8}{5} \end{cases}$$

Exercise 3





Optimal integer solution $u = (1, 2), z_D = 12$