Assignment, January 9, 2017 **A**

First and Last name ____________________

Exercise 1 (value 12)
The *Physical Internet* (PI) is a new concept assuming that the goods can be routed in the logistic network as communication packets in the Digital Internet. With this paradigm the resources (links, hubs etc) are shared among several actors. A possible implementation of PI in an urban area is as follows. There are n *sources* which send goods and $m \leq n$ *sinks* which receive the goods. Suppose, for sake of simplicity, that each source i sends exactly one good to the given sink $j(i)$. The goods are initially sent to a set $F = \{1, \dots, f\}$ of *consolidation centers.* From the consolidation centers the goods can be routed directly to the sink or to a set $T = \{1, \dots, t\}$ of *transit points.* The vehicles used may be different (e.g., we can use electric vehicle in the inner city). Sending a good from source i to consolidation center $k \in F$ cost c^I_{ik}. Sending goods from consolidation center $k \in F$ to sink j cost $c^{II}_{k,j}$ for each good and the vehicle used has a (small) capacity of q goods. Sending goods from consolidation center $k \in F$ to transit point $h \in T$ cost $c^{III}_{k,h}$ for each good and the vehicle used has a capacity of Q goods. Sending goods from transit point $h \in T$ to sink j cost $c^{IV}_{h,j}$ for each good.

Write a linear program aimed to send all goods with the overall minimum cost.

Now suppose that if a transit point is used by more than one consolidation center, there is an additional cost s to be paid for sharing the facility. Modify the model to include the new cost.

Exercise 2 (value 8)
Consider the following PLC problem. Solve it with the simplex method, than write the dual problem and compute the corresponding optimal solution using the complementary slackness conditions.

$$
\begin{aligned}
\min \quad & 2x_1 + 4x_2 - 3x_3 \\
& -2x_1 - 3x_2 \leq 15 \\
& x_1 + 2x_2 + x_3 \leq 12 \\
& 2x_1 + x_2 - x_3 \geq 4 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Exercise 3 (value 7)
Implement the following mathematical model using a modelling language (GLPK or XPRESS)

$$
\min \ z = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} p_{ij} x_{ij} + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} c_{ij} f_{ij}
$$

$$
f_{ij} \leq Q x_{ij} \quad i, j = 1, \dots, n : i \neq j; \tag{13}
$$

$$
\sum_{j=2}^{n} f_{1j} \geq s; \tag{14}
$$

$$
\sum_{\substack{i=1 \\ i \neq 2}}^{n} f_{ij} - \sum_{\substack{i=1 \\ i \neq 2}}^{n} f_{ji} = 0 \quad j = 1, \dots, n; \tag{15}
$$

$$
\sum_{\substack{i=1 \\ i \notin S}}^{n} \sum_{j \in S} f_{ij} \leq t; \tag{16}
$$

$$
x_{ij} \in \{0, 1\} \quad i, j = 1, \dots, n; \tag{17}
$$

$$
f_{ij} \geq 0 \quad i, j = 1, \dots, n; \tag{18}
$$

$$
\tag{19}
$$

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Exercise 1

$$
\begin{aligned}
x_{ik} &= \text{1 if a good is sent from source } i \text{ to consolidation center } k\\
y_{ikj} &= \text{1 if a good originated by source } i \text{ is sent from consolidation center } k \text{ to sink } j\\
w_{ikh} &= \text{1 if a good originated by source } i \text{ is sent from consolidation center } k \text{ to transit point } h\\
z_{ihj} &= \text{1 if a good originated by source } i \text{ is sent from transit point } h \text{ to sink } j
\end{aligned}
$$

$$\min \; z = \sum_{i=1}^{n}\sum_{k=1}^{f} c^{I}_{ik} x_{ik} + \sum_{k=1}^{f}\sum_{i=1}^{n} c^{II}_{kj(i)} y_{ikj(i)} + \sum_{k=1}^{f}\sum_{i=1}^{n}\sum_{h=1}^{t} c^{III}_{kh} w_{ikh} + \sum_{h=1}^{t}\sum_{i=1}^{n} c^{IV}_{hj(i)} z_{ihj(i)}$$

$$\sum_{k=1}^{f} x_{ik} = 1 \quad i = 1,\dots,n; \tag{20}$$

$$\sum_{k=1}^{f} y_{ikj(i)} + \sum_{h=1}^{t} z_{ihj(i)} = 1 \quad i = 1,\dots,n; \tag{21}$$

$$x_{ik} - y_{ikj(i)} - \sum_{h=1}^{t} w_{ikh} = 0 \quad i = 1,\dots,n, k = 1\dots,f; \tag{22}$$

$$\sum_{k=1}^{f} w_{ikh} - z_{ihj(i)} = 0 \quad i = 1,\dots,n, h = 1\dots,t; \tag{23}$$

$$\sum_{i=1}^{n} y_{ikj} \le q \quad k = 1,\dots,f, j = 1\dots,m; \tag{24}$$

$$\sum_{i=1}^{n} w_{ikh} \le Q \quad k = 1,\dots,f, h = 1\dots,t; \tag{25}$$

$$x_{ik} \in \{0,1\} \quad i = 1,\dots,n; k = 1,\dots,f; \tag{26}$$

$$y_{ikj} \in \{0,1\} \quad i = 1,\dots,n; k = 1,\dots,f; j = 1\dots,m; \tag{27}$$

$$w_{ikh} \in \{0,1\} \quad i = 1,\dots,n; k = 1,\dots,f; h = 1\dots,t; \tag{28}$$

$$z_{ihj} \in \{0,1\} \quad i = 1,\dots,n; h = 1,\dots,t; j = 1\dots,m; \tag{29}$$

$\delta_h \;=\;$ 1 if the transit point h is shared among two or more consolidation centers, 0 otherwise

$$\min \; z = \sum_{i=1}^{n}\sum_{k=1}^{f} c^{I}_{ik} x_{ik} + \sum_{k=1}^{f}\sum_{i=1}^{n} c^{II}_{kj(i)} y_{ikj(i)} + \sum_{k=1}^{f}\sum_{i=1}^{n}\sum_{h=1}^{t} c^{III}_{kh} w_{ikh} + \sum_{h=1}^{t}\sum_{i=1}^{n} c^{IV}_{hj(i)} z_{ihj(i)} + s\sum_{h=1}^{t}\delta_h$$

$$\dots$$

$$\sum_{i=1}^{n}\sum_{k=1}^{f} w_{ikh} - 1 \le n \cdot f \delta_h \quad h = 1,\dots,t; \tag{30}$$

$$\delta_h \in \{0,1\} \quad h = 1,\dots,t; \tag{31}$$

Exercise 2

$$\begin{array}{ll} \min & 2x_1 + 4x_2 - 3x_3 \\ & -2x_1 - 3x_2 \leq 15 \\ & x_1 + 2x_2 + x_3 \leq 12 \\ & 2x_1 + x_2 - x_3 \geq 4 \end{array}$$

FASE I

x_1	x_2	x_3	x_4	x_5	x_6	x_7		
-2	-1	1	0	0	1	0	-4	$-z$
-2	-3	0	1	0	0	0	15	x_4
1	2	1	0	1	0	0	12	x_5
2	1	-1	0	0	-1	1	4	x_7

x_1	x_2	x_3	x_4	x_5	x_6	x_7		
0	0	0	0	0	0	1	0	$-z$
0	-2	-1	1	0	-1	1	19	x_4
0	$\frac{3}{2}$	$\frac{3}{2}$	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	10	x_5
1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	2	x_1

x_1	x_2	x_3	x_4	x_5	x_6		
0	3	-2	0	0	1	-4	$-z$
0	-2	-1	1	0	-1	19	x_4
0	$\frac{3}{2}$	$\frac{3}{2}$	0	1	$\frac{1}{2}$	10	x_5
1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	2	x_1

x_1	x_2	x_3	x_4	x_5	x_6		
0	5	0	0	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{28}{3}$	$-z$
0	-1	0	1	$\frac{2}{3}$	$-\frac{2}{3}$	$\frac{77}{3}$	x_4
0	1	1	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{20}{3}$	x_3
1	1	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{16}{3}$	x_1

$x = (\frac{16}{3}, 0, \frac{20}{3}, \frac{77}{3}, 0, 0),\ z_P = -\frac{28}{3}$

$$\begin{array}{ll} \min & 2x_1 + 4x_2 - 3x_3 \\ & 2x_1 + 3x_2 \geq -15 \\ & -x_1 - 2x_2 - x_3 \geq -12 \\ & 2x_1 + x_2 - x_3 \geq 4 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

$$\begin{array}{rcl} \max -15u_1 - 12u_2 + 4u_3 & & \\ 2u_1 - u_2 + 2u_3 & \leq & 2 \\ 3u_1 - 2u_2 + u_3 & \leq & 4 \\ -u_2 - u_3 & \leq & -3 \\ u_1, u_2, u_3 & \geq & 0 \end{array}$$

$$\begin{cases} (2x_1 + 3x_2 + 15)u_1 = 0 \\ (-x_1 - 2x_2 - x_3 + 12)u_2 = 0 \\ (2x_1 + x_2 - x_3 - 4)u_3 = 0 \end{cases} \quad \begin{cases} (2u_1 - u_2 + 2u_3 - 2)x_1 = 0 \\ (3u_1 - 2u_2 + u_3 - 4)x_2 = 0 \\ (-u_2 - u_3 + 3)x_3 = 0 \end{cases} \quad \begin{cases} u_1 = 0 \\ -- \\ -- \\ -u_2 + 2u_3 = 2 \\ -- \\ u_2 + u_3 = 3 \end{cases}$$

$u = (0, \frac{4}{3}, \frac{5}{3}),\ z_D = -\frac{28}{3}$

Exercise 3

```
param n, integer, > 0;
param m, integer, > 0;
set I := 1..n;
set I2 := 2..n;
set S;
param Q;
param s;
param t;

param p{i in I, j in I}, >= 0;
param c{i in I, j in I}, >= 0;

var  x{i in I, j in I},  binary;
var  f{i in I, j in I},  >= 0;

minimize z: sum{i in I, j in I : i<>j} p[i,j]*x[i,j] + sum{i in I, j in I : i<>j} c[i,j]*f[i,j];

s.t.  xf_and_cap{i in I, j in I : i<>j}:  f[i,j] - Q*x[i,j] <=   0;
  out1: sum{j in I2} f[1,j] >= s;
  balance{j in I}: sum{i in I: i <> j} f[i,j] - sum{i in I: i <> j} f[j,i] = 0;
  flowS: sum{i in I diff S, j in S} f[i,j] <= t;
solve;

printf "\n--x";
for{i in I} {
   printf "\n%1d)",i;
   printf{j in I}  "%5d ",  x[i,j];
}
printf "\n--f";
for{i in I} {
   printf "\n%1d)",i;
   printf{j in I} "%5.2f ", f[i,j];
}
printf "\n\n-----z = %g\n\n",z;
printf "\n\n  ";

data;

param n := 7;
param p :  1   2  3   4  5  6   7 :=
1  1   0  3   4 20  5   0
2  0  10  4   4 10  5   0
3  0   0  1   4  0  5 100
4  2   3  1   4  0  5   0
5 10   0  9   4  0  5   0
6  3  10  4   4  0  5 100
7  5   0  1 100  0  5 100;

param c :  1   2  3   4  5  6   7 :=
1  1   0  1   2  0  5   1
2  0   1  1   2  1  5   0
3  0   0  1   0  0  0  10
4  2   1  0   0  0  0   0
5  1   0  2   1 50  1   0
6  1  1   2   1  0  5   0
7  1   0  2 100  0  5   1;
param Q := 10;
param s := 50;
param t := 20;
set S := 4 5 6;
end;
```