

Business Dynamism, Sectoral Reallocation and Productivity in a Pandemic*

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Abstract

Asymmetric effects across sectors are the distinctive features of the Covid-19 shock. An Epidemiological-Industry Dynamic model with heterogeneous firms and endogenous firms dynamics mimics the deep recession suffered by sectors characterized by high exposure, the reallocation of entry and exit opportunities across sectors, and the dynamics of aggregate productivity during the first wave of the pandemic. The cleansing effect induced by the Covid-19 crisis is sector-specific. Monetary policy and sticky wages are central ingredients to capture reallocation effects. Social distancing, by smoothing out cleansing in the social sector, slows down the reallocation process and prolongs the recession, but saves lives.

Keywords: Covid-19; Productivity; Entry; Reallocation.

JEL classification: E3, L16, I3.

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1 Introduction

The crisis caused by the COVID-19 pandemic is like no other. It stands out for being a mixture of both demand and supply shocks and for its asymmetric impact across sectors and countries, as recently stressed by the [IMF World Economic Outlook \(April, 2021\)](#). Moreover, the IMF flagship publication highlights the effects of the pandemic on sectoral and aggregate productivity (see also, e.g., [Bloom et al., 2020a](#)).

We study a model of firm dynamics, in the spirit of [Melitz \(2003\)](#) and [Clementi and Palazzo \(2016\)](#), which we extend to allow for three key features that proved to be important to address the asymmetric effects across sectors and firms entailed by the pandemic: a) an epidemiological block, b) two sectors with different degree of exposure to the disease, and c) real effects of monetary policy through nominal wage contracts.

We use this framework to explain three facts that characterized the initial phase of the pandemic. The first one is that sectors where market activity involves exposure to the virus experienced a much sharper downturn, relative to trend, than the overall economy. The second one is that in sectors characterized by high exposure to the virus, there was a substantial drop, with respect to trend, in new business applications, followed by a rebound in the third quarter of 2020. The third fact concerns the dynamic pattern of aggregate productivity. The latter increased up to Summer/Fall of 2020, to drop sharply afterwards, and to start growing again at the beginning of 2021.

To capture these facts, we build an Epidemiological-Industry Dynamic model with the following features.

First, the economic block of the DSGE model builds on the heterogeneous-firms literature with endogenous firm dynamics à la [Melitz \(2003\)](#), augmented for nominal rigidities as in [Bilbiie et al. \(2007\)](#) and [Colciago and Silvestrini \(2022\)](#). Firms face initial uncertainty concerning their future productivity when making an investment decision to enter the market. Following [Bilbiie et al. \(2012\)](#), firm entry is subject to sunk product development costs, which investors pay in expectation of future profits. Firms join the market up to the point where the expected value of their newly created product equals its sunk cost. After entry, firms' production depends on their productivity levels. Firms face fixed production costs. As a result, given aggregate conditions, firms with idiosyncratic productivity levels below a specific threshold will be forced to discontinue production and stay inactive until production becomes profitable again. [Fairlie \(2020\)](#) finds that after widespread small business closures until April 2020, there were indeed reopening in May and an additional rebound in June.

Second, we add to our DSGE model an epidemiological block consisting of a SIR model à la [Kermack and McKendrick \(1927\)](#), augmented by a

feedback from the economic behavior to the dynamics of the pandemic, which makes the transmission rate of the virus affected endogenously by individual work and consumption choices.¹

Third, consistently with the sector classification that we propose in the descriptive analysis, we model a two-sector economy with a social and a non-social sector, both populated by an endogenous mass of heterogeneous firms that produce differentiated goods, which are aggregated into sectoral goods. The defining feature of social goods is that their consumption requires social interaction with other individuals. High social interaction translates into faster transmission of the disease. The heterogeneity in the perceived risk of infection in consumption between the social and the non-social sector allows us to micro-found the asymmetric propagation of the Covid-19 shock as an optimal behavioral response of the households to the pandemic, without modelling the pandemic shock as an artificial exogenous asymmetric demand or supply shock. Indeed, during the pandemic, individuals voluntarily substitute away from the consumption of social goods, inducing an endogenous reduction in the demand of those goods as documented by [Yan et al. \(2021\)](#). Analyzing Survey evidence in five European countries in the summer of 2020, [Hodbold et al. \(2021\)](#) report that households identified infection risk as the main reason for changing their consumption behavior. The relevance of the behavioural response of US households to the pandemic shock is evaluated by [Goolsbee and Syverson \(2021\)](#). They use data on foot traffic at 2.25 million individual businesses and find greater avoidance of and substitution away from establishments with higher potential transmission contacts. Their results indicate that, besides legal shutdowns, the behavioral response of households to the pandemic accounts for a large share of the overall change in consumption choices.

Fourth, to analyze the potential role of monetary policy during the pandemic, our framework assumes nominal rigidities in the form of sticky wages.

Fifth, to consider the role of production network and sectoral spillovers, the model incorporates a roundabout production structure wherein firms use the outputs of other firms as a factor of production.²

Next we briefly summarize our findings. In response to the COVID-19 shock, our model economy features reallocation effects within and across

¹Several frameworks highlight and quantify the importance of internalizing the behavioral response of the households in the description of the pandemics, see [Bisin and Moro \(2021\)](#)

²The literature (see, e.g., [Christiano et al., 2011](#); [Ascari et al., 2018](#)) showed that a roundabout production structure - a feature which [Christiano \(2016\)](#) refers to as “firms networking” after [Acemoglu et al. \(2015\)](#) - can act as an amplification source for real shocks, especially in presence of nominal rigidities (see, e.g., [Basu, 1995](#); [Huang et al., 2004](#); [Nakamura and Steinsson, 2010](#)).

sectors that are absent in a standard one-sector, homogeneous-firms framework. Specifically, since the disease is transmitted through demand interactions in the social sector, in response to the outburst of the crisis households' expenditure shifts towards the non-social sector, through a substitution channel. Thus, consistently with our first empirical fact, the output of the social sector declines more severely than aggregate output. Such a decline results in cleansing in the social sector, together with a reallocation of entry opportunities to the temporarily more profitable non-social sector. The reallocation process featured in our frameworks allows to successfully reproduce the pattern of business dynamism that we described above.

The reason is as follows. Due to the drop in revenues, break even in the social sector requires higher idiosyncratic productivity. This affects both the entry and exit margins. Indeed, only firms with higher productivity will find convenient to enter in the social sector, resulting in a drop in the number of potential entrants. Turning to the exit margin, there is cleansing of low-productivity firms, which become temporarily idle, causing an increase in the effective exit rate and in average sectoral productivity.³ Opposite dynamics with respect to those just described characterized the non-social sector. The reallocation of demand to the non-social sector implies that firms with lower idiosyncratic productivity will now break-even on their costs. As a result, we will observe a higher number of potential entrants, a lower effective exit, and a drop in average sectoral productivity.

Our analysis suggests that one distinctive feature of the Covid-19 crisis is that the cleansing effect on business dynamism that typically characterizes recessions is sector-specific. Cleansing in the social sector, together with reallocation across sectors, are the key dimensions to consider in order to explain the empirical dynamics of aggregate labor productivity during the pandemic. Our analysis indicates that the evolution of aggregate labor productivity observed in the data results from two driving forces. The first one derives from the change in average productivity within each of the two sectors in response to the demand reallocation. The second one derives from composition effects due to the change in the relative size of sectors. We show that both forces can be found in the data. The interaction between these two forces determines the dynamics of aggregate productivity. Bloom et al. (2020b) find that, during the pandemic, intersectoral reallocation had a stronger impact on productivity than in previous recessionary episodes in the UK.

Our model exhibits fluctuations in sectoral hours worked that reflect the dynamics of sectoral output. Hence, the model does not feature an extensive margin of labor, i.e. number of employed people, but just an intensive one. As such, it is not designed to capture the reallocation of

³The effective exit considers both idle firms that stop producing and firms that exogenously exit the market.

employment, i.e. workers flows across sectors or across firms within sectors. Nevertheless, the output and firm dynamics of our model imply that the US labor market experienced relevant changes during the first few months of the pandemic recession. Consistently, [Garcia-Cabo et al. \(2022\)](#) and [Anayi et al. \(2021\)](#) find that labor reallocation across sectors reached a historical high during the pandemic. Turning to reallocation within sectors, [Andrews et al. \(2021\)](#) use near-real-time data for Australia, New Zealand, and the United Kingdom to show that during the initial phase of the pandemic high-productivity firms were more likely to expand and low-productivity firms were more likely to contract. Additionally, they find that the reallocation-productivity link was initially much stronger in industries hard-hit by the pandemic. These findings, while not relative to the US, but to three major OECD countries, broadly support the transmission mechanism featured in our model.

Two notable results regard monetary policy. The first one is that monetary policy is a powerful tool in our setting. The second one is that monetary policy has a crucial role when it comes to explain the asymmetric reaction of business dynamism across sectors. [Lepetit and Fuentes-Albero \(2020\)](#) find that interest rate policies have little power to counteract the recessionary effect of the pandemic in one-sector business cycle models. The reason is that intertemporal substitution in consumption is impaired by the fear of contagion. The power of monetary policy is re-established in our asymmetric two-sector framework. A lower interest rate promotes intertemporal substitution, as in the standard one-sector New Keynesian model. However, fear of contagion, rather than impairing it, channels the additional demand to the non-social sector, where the likelihood of contagion is lower. As a result, an accommodating monetary policy promotes reallocation across sectors and leads to a faster recovery. The reallocation across sectors induced by monetary policy is a crucial ingredient to replicate the differing patterns of business creation across sectors observed during the pandemic. The persistent decline of the real interest rate in response to the crisis, that our model features in the presence of nominal wage rigidities, supports demand, and favors the reallocation of business opportunities to the non-social sector. Absent the endogenous response of the real rate, we would simply observe business destruction in both sectors.

Our model features a positive relationship between the size of the social sector and the severity of the recession. As a result, it is consistent with the observation that countries with a larger social sector suffered a stronger downturn (see, e.g., [International Monetary Fund, 2021](#)). Additionally, we find that social distancing, by smoothing out fluctuations in aggregate productivity, leads to a slower reallocation across sectors, and to a prolonged recession with respect to the case where no measures are taken, but it saves lives. Finally, our results are consistent with the imposition of containment

policies, e.g. a partial lockdown of the social sector, where the latter has the effect of exacerbating the reallocation process.

The remainder of the paper is as follows. Section 2 illustrates the three facts that motivate the analysis, while Section 3 introduces the theoretical model. Section 4 discusses the calibration strategy. Section 5 presents the results of the benchmark simulation, and Section 6 describes the response of aggregate productivity. Section 7 analyses the role of monetary policy. Section 8 presents the results from four further experiments: (i) introduction of social distancing measures, (ii) addition of exogenous lockdown policies, (iii) models with different relative sector sizes, and (iv) robustness checks on the elasticity of substitution between sectors. Section 9 concludes. Key derivations are reported in the Appendix to the main text, together with the empirical evidence on aggregate and sectoral inflation, while other technicalities are left to the Online Appendix.

2 Stylized Facts and Related Literature

To illustrate the facts addressed in this paper, we initially assign industries either to the socially-intensive sector, indexed by (s), or to the non-social sector, (ns), following the partition proposed by [Kaplan et al. \(2020\)](#). The first fact regards the contraction of the social sector. We aggregate the real GDP of social industries, and plot in Figure 1 its deviations from trend (dashed line) alongside the deviations from trend of aggregate real GDP (solid line), over the period 2019:Q1 to 2021:Q3. In the second quarter of 2020, at the outset of the pandemic, the contraction of aggregate GDP was about 9 percent relative to trend, while that of the social sector measured about 16 percent relative to trend. The heavier decline in the GDP of social sectors persisted until the first quarter of 2021.

The asymmetric effect of the Pandemic shock leads to the second empirical fact explained by our framework, namely the reallocation of business opportunities from social sectors to less exposed sectors. We consider statistics concerning business applications in the period 2019:1-2021:3. These data provide monthly measures of new business applications and formations in the United States.⁴ Specifically, we analyze business applications with planned wages. These are high-propensity business applications, that are applications with a much higher likelihood of becoming employer businesses with respect to the typical business application, given the intention to pay wages. Data are available for 2-Digit NAICS sectors. Figures 2 and

⁴The BFS is based on applications for Employer Identification Numbers (EINs). Businesses that hire employees need an EIN for payroll tax purposes. The monthly BFS data cover the period starting from July 2004 (2004:Q3) at a monthly frequency. Monthly BFS data are released approximately 11-12 days after the end of the observed month.

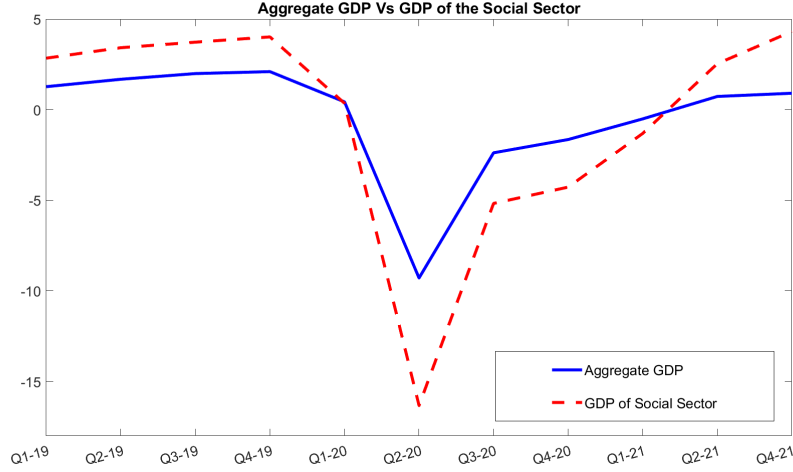


Figure 1: Cyclical components: Real GDP in the Social Sectors and Real aggregate US GDP

The figure displays percentage deviations from trend of the real GDP in the Social Sectors and the Real aggregate US GDP. The trend has been computed with an HP filter with parameter 1600, applied to quarterly data from 2005. From publicly available BEA Data.

3 display percentage deviations from trend of business applications with planned wages in the (*s*) sector and the (*ns*) sector, respectively. Indeed, Business Formation Statistics (BFS) data show a clear reallocation pattern since the outburst of the pandemic.

In the early phase of the crisis, that is March-April 2020, there was a drop, with respect to trend, in business applications in the (*s*) sector, followed by a slight rebound in the summer of the same year. Additionally, after a mild impact response, a surge in business applications in the (*ns*) sector can be observed in the same early phase of the crisis.

The data indicate that the pandemic represents a large shock to the (*s*) sector. The shift in entry opportunities originated from a strong reallocation of consumption expenditures toward the non-social sectors, with possible long lasting effects, as suggested by [Hodbold et al. \(2021\)](#).

This asymmetric response of (potential) entry in the two sectors hides different cases: displaced workers reinventing themselves as entrepreneurs in safer industries, actual entry of new firms, exiting firms opening/reopening establishments in the same sectors but in an online fashion - a common phenomenon often referred to as *intensive* entry, see [Bahaj et al. \(2022\)](#).⁵

⁵This last feature is potentially relevant quantitatively: industries as Truck and transportation and, in particular, Non-store retail, alone responsible of nearly one-third of the jump in total applications from 2019 to 2021, experienced a large restructuring toward online businesses, see [Decker and Haltiwanger \(2022\)](#).



Figure 2: Business Applications in the Social Sectors

The figure displays percentage deviations from trend of business applications with planned wages in the sectors characterised by a high exposure to the virus, i.e. the social sectors. Observations come from the BFS at a monthly frequency. The trend has been computed with an HP filter with parameter 14400, applied to monthly data from 2004.

Although our goal is not to capture these different nuances, the model is still able to distinguish some of these mechanism and capture them in a simplified way. While the between sector reallocation from the aforementioned shift in consumption expenditure is explored in depth, the within sector reallocation is also consistent with our results. The observed restructuring from in person to online businesses in the data is echoed in our framework by the fact that small and unproductive firms in the social sector shrink or leave the market at the benefit of larger and more productive incumbents. This aligns with the fact that larger firms were more digitalized to begin with and/or managed to adapt quicker and more effectively to an online environment, as shown by [Bloom et al. \(2020b\)](#) and the survey in [Criscuolo \(2021\)](#).

The third fact explained by our approach concerns the dynamic of aggregate productivity during the pandemic. Figure 4 displays percentage deviations from trend of output over hours in the non-farm business sector. Labor productivity displayed a peculiar pattern during the pandemic. It increased up to Summer/Fall of 2020, to drop sharply afterwards, and to start growing again at the beginning of 2021. Theory and evidence suggest that a substantial fraction of aggregate productivity growth is accounted for by the reallocation of resources from lower-productivity to higher-productivity firms. The effect of the pandemic on business dynamism could affect productivity through at least three channels: cleansing, reallocation, and pro-

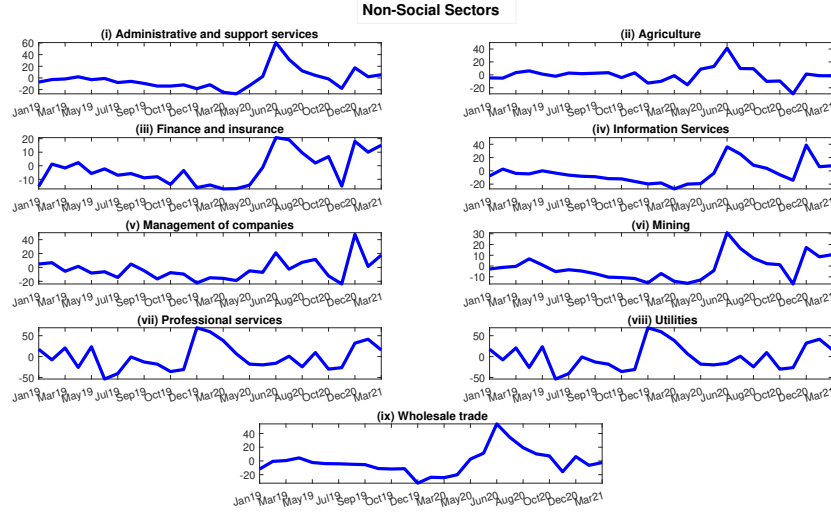


Figure 3: Business Applications in the Non-Social Sectors

The figure displays percentage deviations from trend of business applications with planned wages in the sectors characterised by a low degree of social interaction, i.e. the non-social sectors. Observations come from the BFS at a monthly frequency. The trend has been computed with an HP filter with parameter 14400, applied to monthly data from 2004.

duction networks.

First, the pandemic shock could actually improve sectoral productivity via a standard cleansing effect during recessions, namely by inducing exit of less productive businesses. Second, reallocation of activity from social towards non-social sectors affects the relative number of firms in those sectors through entry and exit with an, *a priori*, unclear effect on aggregate productivity. Thus, business dynamism determines a composition effect, within and across sectors, that is important in shaping the dynamics of sectoral and aggregate productivity. Finally, sectoral spillovers and production network could act as an important amplification mechanism of the previous effects. [Foster et al. \(2001\)](#) document that firm entry and exit are an especially critical component of productivity dynamics induced through reallocation. In our framework firms entry and exit, together with the reduction in the relative size of the social sector, play a critical role at explaining the dynamics of aggregate productivity during the pandemic.

The literature studying the effects of the COVID-19 pandemic through the lenses of macroeconomic models is already vast and rapidly expanding. [Eichenbaum et al. \(2020a\)](#) and [Eichenbaum et al. \(2020b\)](#) integrate the SIR model in a general equilibrium setting, in order to study the interaction between the economic system and the pandemic. [Baqaee and Farhi \(2020\)](#), and [Guerrieri et al. \(2020\)](#) study the relative importance of demand and



Figure 4: Labor productivity during the pandemic

The figure displays percentage deviations from trend of labor productivity, measure as the ratio between output and total hours worked in the non-farm business sector. The trend has been computed with an HP filter with parameter 1600, applied to quarterly data from 2004.

supply shocks. Various authors, such as [Alfaro et al. \(2020\)](#), [Toxvaerd \(2020\)](#), [Moser and Yared \(2020\)](#), [Alvarez et al. \(2020\)](#) and [Jones et al. \(2020\)](#) identify externalities in individual distancing decisions, and study optimal lockdown and social distancing policies. Considering a two-sector economy, [Guerrieri et al. \(2021\)](#) design optimal monetary policy in response to asymmetric shocks that shift demand from a sector to the other.

Much less vast, but closely related to our paper, is the literature studying the role of microeconomic heterogeneity at shaping the interaction between the epidemic and the economy. Considering heterogeneity across households, [Kaplan et al. \(2020\)](#) integrate the SIR model into a framework with income and wealth inequality, as well as occupational and sectoral heterogeneity, to study the distributional and welfare effects associated with the US policy response to the pandemic. The health and economic policies they consider entail large and heterogeneous welfare costs across households. [Hur \(2020\)](#) integrates a SIR model into an heterogeneous agent-life cycle economy. He designs Pareto-improving mitigation policies, and shows that the latter can reduce deaths by nearly 60 percent relative to a no mitigation scenario. [Eichenbaum et al. \(2021\)](#) develop a two agents model consistent with the observation that the health of low wage workers has been disproportionately affected by the pandemic.

Our paper is complementary to these analysis, since it considers heterogeneity in the supply side of the economy. We integrate the SIR epidemiological model into a New Keynesian Industry Dynamic framework with two

sectors, heterogeneous firms and endogenous entry and exit dynamics in order to study the effects of the pandemic and social distancing measures on productivity and business dynamism. Firm level heterogeneity is modeled similarly to [Clementi and Palazzo \(2016\)](#), [Hamano and Zanetti \(2017\)](#), and [Rossi \(2019\)](#). The focus on the first wave of the epidemic allows to isolate the economic effects resulting from the behavioral response of households to the pandemic, consistently with the propagation mechanism in our model. As argued by [Eichenbaum et al. \(2021\)](#), after the first wave, expectations about possible vaccinations and changes in fiscal policy played a large role in affecting people's behavior.

3 The Model

The economy features two sectors indexed by (q) , where $(q) = (s)$ identifies the social sector that produces a good whose consumption require social interactions, while and $(q) = (ns)$ identifies the non-social sector. Each sector is characterized by ex-ante heterogeneous firms, which produce a good in different varieties and compete monopolistically. The length of the mass of firms in each sector is determined endogenously by firms' entry and exit, which are modeled at the sectoral level.

The economy features a unitary continuum of homogeneous households or families, who use the final good for consumption and investment purposes. Each family is populated by a unitary continuum of ex-ante homogeneous individuals. Individuals' ex-post heterogeneity comes from their contagion status. The evolution of the disease is governed by a standard SIR model. The transmission rate of the virus depends endogenously on individual working decisions, and on consumption of the social good.

3.1 SIR and Contagion

The epidemiological block of our framework is based on the SIR model by [Kermack and McKendrick \(1927\)](#). In each time period t , individuals can be in one of four epidemiological states. The total number of people susceptible to the disease is \mathbb{S}_t . \mathbb{I}_t represents the aggregate number of infected individuals. Let \mathbb{D}_t denote the cumulative mass of dead people, then the total number of recovered individuals at time t is $1 - \mathbb{I}_t - \mathbb{D}_t - \mathbb{S}_t$. Since the population is initially normalized to one, these terms represent also the fractions over the initial population.⁶

⁶For clarification, in any period t the number of infected individuals \mathbb{I}_t also represents the fraction of infected individuals with respect to the initial unitary population, while the fraction of infected over the current population is $\mathbb{I}_t / (1 - \mathbb{D}_t)$, and the same holds for \mathbb{S}_t and \mathbb{R}_t .

We follow the approach by [Jones et al. \(2020\)](#) and [Kaplan et al. \(2020\)](#), and assume that households' members are not aware of their epidemiological state. As in [Eichenbaum et al. \(2020a\)](#) and [Eichenbaum et al. \(2020b\)](#), susceptible people become infected in either of three ways: by purchasing social goods, working, and through random interactions unrelated to economic activities. The evolution of the disease is internalized by the household. Thus, in the following, we use the calligraphic italics letters to describe the contagion types within the households, i.e. \mathcal{I}_t vs. \mathbb{I}_t and so on. The number of newly infected household's members at time t , \mathcal{T}_t , is given by:

$$\mathcal{T}_t = \mathcal{S}_t \mathbb{I}_t \pi_1 c_t(s) C_t(s) + \mathcal{S}_t \mathbb{I}_t \pi_2 l_t^s L_t^d + \pi_3 \mathcal{S}_t \mathbb{I}_t. \quad (1)$$

When considering exposure, household scales it to \mathbb{I}_t and not to \mathcal{I}_t , thus neglecting the effects of individual choices on the aggregate pool of infected. As emphasized by [Jones et al. \(2020\)](#), this implies an externality that results in a lower than optimal mitigation effort by households.

The term $\mathcal{S}_t \mathbb{I}_t \pi_1 c_t(s) C_t(s)$ indicates the number of newly infected household's due to shopping activities, where \mathcal{S}_t denotes the number of susceptible members within the household, π_1 is a multiplier that scales the probability of becoming infected as a result of consumption activities, $c_t(s)$ is the individual consumption, i.e. the consumption of each living household's member, of the social good, and $C_t(s)$ is aggregate consumption of the social good.⁷ The heterogeneity in the perceived risk of infection in consumption between the social and the non-social sector allows us to micro-found the asymmetric propagation of the Covid-19 shock as an optimal behavioral response of the households to the pandemic.

The term $\mathcal{S}_t \mathbb{I}_t \pi_2 l_t^s L_t^d$ yields the number of newly infected household's members due to work activities. Specifically, π_2 reflects the probability of becoming infected as a result of interactions on the workplace, l_t^s denotes individual labor supply and L_t^d represents aggregate labor demand. Note that, differently from consumption, labor in both the social and the non-social sector entail the same risk of contagion.

Finally, the exogenous component of the transition equation (1) comes from the number of random pairings between susceptible household's members and infected people. These meetings result in $\pi_3 \mathcal{S}_t \mathbb{I}_t$ newly infected household's members. Equation (1) is central in our framework. It affects the behavioral response of households to the pandemic, which, in turn, leads to a reallocation of demand toward the non-social sector.

Given \mathcal{T}_t , the number of household's members in the alternative epidemiological states evolve according to:

⁷A higher contagion risk in the social sector is the only ingredient required to generate reallocation across sectors. This assumption is consistent with the classifications of industries adopted in Section 2. Without loss of generality, we normalize to zero the contagion risk from consumption of the non-social good.

$$\mathcal{S}_{t+1} = \mathcal{S}_t - \mathcal{T}_t, \quad (2)$$

$$\mathcal{I}_{t+1} = \mathcal{I}_t + \mathcal{T}_t - (\pi_r + \pi_d) \mathcal{I}_t, \quad (3)$$

$$\mathcal{R}_{t+1} = \mathcal{R}_t + \pi_r \mathcal{I}_t, \quad (4)$$

$$\mathcal{D}_{t+1} = \mathcal{D}_t + \pi_d \mathcal{I}_t, \quad (5)$$

where π_r and π_d are the probability of infected to recover and to die, respectively.

3.2 Households

The representative family is initially of size 1, while the mass of the living population within the household at time t is $1 - \mathcal{D}_t$. The time- t utility of the representative household is:

$$(1 - \mathcal{D}_t) \log(c_t) - (1 - \mathcal{D}_t) \nu \left(\frac{(I_t^s)^{1+\phi}}{1 + \phi} \right) - u_d \mathcal{D}_t, \quad (6)$$

where c_t is the individual consumption of the final good and u_d is the disutility from death, which includes the flow value of the psychological costs of death on surviving members. The final good is a composite good defined as a CES (Constant Elasticity of Substitution) function over the household's consumption levels of the social, $c_t(s)$, and the non-social good, $c_t(ns)$, as: $c_t = \left[\chi^{\frac{1}{\eta}} c_t(s)^{\frac{\eta-1}{\eta}} + (1 - \chi)^{\frac{1}{\eta}} c_t(ns)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$. Both $c_t(s)$ and $c_t(ns)$ are aggregators of goods produced in the social and non-social sector, respectively. The parameter χ captures the relative importance of the social good in the consumption basket and determines the steady state size of the sector, while the parameter $\eta > 1$ measures the elasticity of substitution between the social and the non-social goods.

In each time period t , agents can purchase any desired state-contingent nominal payment A_{t+1} in period $t + 1$ at the dollar cost $E_t \Lambda_{t,t+1} A_{t+1} / \pi_{t+1}$, where $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period $t + 1$ and t , and π_{t+1} denotes the inflation rate over the same period. Households choose consumption, hours of work, and how much to invest in state-contingent assets and in risky stocks $b_{t+1}(q)$. Stock ownership ensures to households a flow of dividend distributed by operative firms. The timing of investment in the stock market is as in [Bilbiie et al. \(2012\)](#) and [Chugh and Ghironi \(2011\)](#). At the beginning of period t , the household owns $b_t(q)$ shares of a sector mutual fund that represents the ownership of the $N_t(q)$ incumbents in sector (q) in period t , with $(q) = \{(s), (ns)\}$.

The period- t asset value of the portfolio of firms held in sector (q) is the total firms' value in sector (q) , given by the product between the average

value of a firm $\tilde{v}_t(q)$ and the existing mass of firms $N_t(q)$ in the same sector. To obtain the total value of the portfolio held by households, one needs to sum over the two sectoral funds.

During period t , the household purchases $b_{t+1}(q)$ shares in new sectoral funds to be carried to period $t+1$. Since the household does not know which firms will disappear from the market, it finances the continued operations of all incumbent firms as well as those of the new entrants, $N_t^e(q)$, although at the very end of period t a fraction of these firms disappears. The value of total stock market purchases is thus $\sum_{q=s,ns} \tilde{v}_t(q) (N_t(q) + N_t^e(q)) b_{t+1}(q)$.

Households derive income from three sources: labor, dividend, and from interests on loans to firms. We assume a continuum of differentiated labor inputs indexed by $j \in [0, 1]$. Wages are set by labor type specific unions, indexed by $j \in [0, 1]$. Given the nominal wage, W_t^j , set by union j , agents stand ready to supply as many hours to labor market j , L_t^j , as required by firms, that is:

$$L_t^j = (W_t^j / W_t)^{-\theta_w} L_t^d, \quad (7)$$

where θ_w is the elasticity of substitution between labor types, W_t is an aggregate nominal wage index, and L_t^d is aggregate labor demand. The latter can be obtained by integrating firms' individual labor demand over the distribution of idiosyncratic productivities. Agents are distributed uniformly across unions, hence aggregate demand for labor type j is spread uniformly across households. The labor market structure rules out differences in labor income between households without the need to resort to contingent markets for hours. The common labor income is given by:

$$\int_0^1 (w_t^j L_t^j) dj = L_t^d \int_0^1 w_t^j (w_t^j / w_t)^{-\theta_w} dj. \quad (8)$$

Stock ownership entitles households to dividend income. Operative firms distribute dividends, following the production and sales of varieties in the imperfectly competitive goods markets. Operative firms in sector (q) , that we denote as $N_{o,t}(q)$ and formally define below, are the firms that are actively producing in each sector at time t . As shown in the Online Appendix, total dividends received by a household in a sector can be written as $N_{o,t}(q) \tilde{e}_t(q)$, where $\tilde{e}_t(q)$ denotes average sectoral dividends, that is the amount of dividends distributed by the firm with average sectoral productivity.

Finally, a fraction of the resources of households is deposited to financial intermediaries that provide loans to firms. Firms use one-period loans to finance a fraction $\alpha_w \in [0, 1]$ of the wage bill in advance of production. In equilibrium, a real amount equal to $\alpha_w w_t L_t^d$ must be gathered for this purpose. The deposit yields a gross interest rate R_t . Interests on deposits are distributed to households at the end of each period t in a lump sum

fashion.

We can then write the flow budget constraint of the representative household:

$$\begin{aligned}
(1 - \mathcal{D}_t) \sum_{q=s,ns} \rho_t(q) c_t(q) + E_t r_{t,t+1} a_{t+1} + \\
\sum_{q=s,ns} \tilde{v}_t(q) (N_t(q) + N_t^e(q)) b_{t+1}(q) = L_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\theta_w} dj + \frac{a_t}{\pi_t} \\
+ \sum_{q=s,ns} (N_t(q) \tilde{v}_t(q) + N_{o,t}(q) \tilde{e}_t(q)) b_t(q) + (R_t - 1) \alpha_W w_t L_t^d, \quad (9)
\end{aligned}$$

where w_t denotes real wages and $\rho_t(q)$ is the price of the good produced in sector (q) expressed in real terms, that we define in Appendix C.

In the Online Appendix we show the solution of the infinitely-lived representative household's problem of maximising the present discounted value of (6) - where β is the subjective discount factor - subject to the flow of budget constraints (9) and the constraints coming from the SIR block.

The demand functions of the sectoral goods are key objects in our analysis, since they generate sectoral reallocation, by internalizing the asymmetric contagion risk that leads to the behavioral response of the household. These demands can be found from either the first order conditions of the households (Online Appendix) or by postulating the existence of a fictitious final good bundler (Appendix B), and they are taken as given in the firms' problem we present below. The demand for the production of the social good $Y_t(s)$ reads as:

$$\left(\frac{Y_t(s)}{Y_t} \right) = \chi \left[\lambda_t \rho_t(s) + \lambda_{\mathcal{T},t} \frac{\mathcal{S}_t \mathbb{I}_t}{1 - \mathcal{D}_t} \pi_1 C_t(s) \right]^{-\eta} \left(\frac{C_t}{(1 - \mathcal{D}_t)} \right)^{-\eta}, \quad (10)$$

where Y_t is aggregate production, λ_t is the Lagrange multiplier of the budget constraint, $\lambda_{\mathcal{T},t}$ represents the shadow cost of a new infected, $C_t(s)$ is aggregate consumption of the social good and C_t is aggregate consumption. As mentioned above, this demand departs from the standard CES demand since it internalizes the exposure to contagion.⁸ *Ceteris paribus*, a higher contagion risk, e.g. coming from a higher number of infected or from a larger consumption of the social good, depresses the demand for $Y_t(s)$.

⁸The demand function of the non-social good, that is good (ns) , is:

$$\left(\frac{Y_t(ns)}{Y_t} \right) = (1 - \chi) [\lambda_t \rho_t(ns)]^{-\eta} \left(\frac{C_t}{(1 - \mathcal{D}_t)} \right)^{-\eta}.$$

3.3 Firms

Each sector (q) is populated by a mass $N_t(q)$ of atomistic firms. Upon entry, firms draw a time invariant idiosyncratic productivity level, denoted by z , from a known distribution function, $g(z)$, which is identical across sectors and has a positive support. Within their sector of operation, the only source of heterogeneity across firms is the idiosyncratic productivity level, so that we can index firms within a sector with z .

Firms compete monopolistically within the sector and are subject to entry and exit. Each firm produces an imperfectly substitutable good $y_{z,t}(q)$, using the following constant return to scale production function with roundabout:

$$y_{z,t}(q) = Z_t z l_{z,t}(q)^{1-\alpha} X_{z,t}(q)^\alpha, \quad (11)$$

where the variable Z_t is an exogenous level of productivity, common to all firms. The two inputs are labor, $l_{z,t}(q)$, and an intermediate input, $X_{z,t}(q)$. The former is defined as a CES aggregator of differentiated labor inputs indexed by $j \in [0, 1]$, defined as:

$$l_{z,t} = \left(\int_0^1 (l_{z,t}^j)^{\frac{\theta_w-1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w-1}}, \quad (12)$$

where $\theta_w > 1$ is the degree of substitution between labor inputs. The latter is a composite of all the goods in the economy, combined through the same CES function as consumption. The goods $y_{z,t}(q)$ are input to the production of a sectoral bundle, $Y_t(q)$, by a sectoral good producer that operates in perfect competition. The latter adopts a CES production function defined as:

$$Y_t(q) = \left(\int_0^\infty N_t(q) y_{z,t}(q)^{\frac{\theta-1}{\theta}} g(z) dz \right)^{\frac{\theta}{\theta-1}}, \quad (13)$$

where $\theta > 1$ is the degree of substitution between goods within a specific sector.

We assume that firms finance a fraction $0 \leq \alpha_W \leq 1$ of their wage bill resorting to loans from financial intermediaries. Loans are reimbursed at the end of the period at the gross risk-free interest rate R_t . Additionally, firms face fixed costs of production $f_{x,t}$, defined in terms of the final good. The Online Appendix provides the technical derivations concerning the cost minimization and the profit maximization problem of firm z . The equilibrium optimal real price $\rho_{z,t}(q)$ is:

$$\rho_{z,t}(q) = \frac{\theta}{\theta-1} \frac{1}{Z_t z} \left(\frac{(\alpha_W R_t + 1 - \alpha_W) w_t}{1 - \alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha, \quad (14)$$

where the ratio $\frac{\theta}{\theta-1}$ represents the markup over real marginal costs under monopolistic competition with atomistic firms. Optimal pricing delivers real profits:

$$e_{z,t}(q) = \frac{1}{\theta} \rho_{z,t}(q)^{1-\theta} \rho_t(q)^\theta Y_t(q) - f_{x,t}. \quad (15)$$

3.4 Entry and Exit

Potential entrants must pay a sunk entry cost, $f_{e,t}(q)$, measured in units of the final good, to draw their individual productivity level, z , from a p.d.f. $g(z)$ common to both sectors. We assume that the entry costs take the form: $f_{e,t}(q) = \psi_0 + \psi_1 (N_t^e(q))^\gamma$. Entry costs are composed of a constant term, ψ_0 , and of a term which increases with the mass of potential entrants, $\psi_1 (N_t^e(q))^\gamma$. The variable term could be motivated by various factors, among which we can list congestion externalities, as in [Jaef and Lopez \(2014\)](#) and [Casares et al. \(2018\)](#), and diminishing quality in managerial ability, as in [Bergin et al. \(2018\)](#). Notice that when $\gamma = 0$ the entry cost is a constant.

Firms enter the market up to the point where the sunk cost of entry is equal, in expectation, to the value of discounted future profits. Since the idiosyncratic productivity z is unknown ex-ante, the expected value of discounted profits is evaluated using the value of the average firm in period t , $\tilde{v}_t(q)$.⁹ Thus, the free entry condition is $f_{e,t}(q) = \tilde{v}_t(q)$.

Due to the fixed costs of production, not all $N_t(q)$ firms have non negative profits, but just those with idiosyncratic productivity, z , above a certain minimum cut-off productivity level $z_t^c(q)$. Appendix [A](#) shows that $z_t^c(s)$ is given by:

$$z_t^c(q) = \Omega_t^c \left(\frac{1}{\rho_t(q)^\theta Y_t(q)} \right)^{\frac{1}{\theta-1}}, \quad (16)$$

where:

$$\Omega_t^c = \frac{\theta^{\frac{\theta}{\theta-1}}}{\theta-1} \frac{1}{Z_t} \left(\frac{(\alpha_W R_t + 1 - \alpha_W) w_t}{1 - \alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha (f_{x,t})^{\frac{1}{\theta-1}}$$

is common across sectors.¹⁰ Firms with idiosyncratic productivity lower

⁹Although in the following we define the notion of an active firm, i.e. an incumbent engaged in production with non-negative profits, note that the expected firm's value is computed considering every incumbent in the market and not just active firms. This is so since an inactive firm with zero current profits could become active in the future, conditional on surviving.

¹⁰The latter is affected by households' preferences, through θ . Additionally, it increases in the magnitude of both fixed cost, $f_{x,t}$, and in the common, across sectors, components of marginal costs of production, i.e. the real wage and the gross interest

than $z_t^c(q)$ become idle, as in [Ghironi and Kim \(2019\)](#) and [Colciago and Silvestrini \(2022\)](#). Idle firms discontinue production, but stand ready to join again the mass of operative firms when their idiosyncratic productivity becomes again larger than $z_t^c(q)$. This could be interpreted as an endogenous voluntary lockdown of not profitable firms.

On top of the endogenous inactivity margin, firms permanently exit the market when hit by an exogenous exit shock. The latter wipes out a fraction δ of existing firms in each period t , no matter if new entrants or incumbent, or if active or inactive. The exit rate, δ , is common across the two sectors.

In this framework, the fraction of firms that becomes idle in period t represents the endogenous component of the exit rate. Indeed, in order to be actively operating, firms must be endowed with an idiosyncratic productivity level above the cut-off. We denote the mass of operative firms at time t as $N_{o,t}(q)$. The latter is formally defined as:

$$N_{o,t}(q) = N_t(q) \Pr[z > z_t^c(q)] = [1 - G(z_t^c(q))] N_t(q) \quad \text{for } (q) = \{(s), (ns)\},$$

where $G(z)$ is the cumulative distribution function associated to $g(z)$: $G(z) = \int_0^z g(x)dx$. Importantly for our purposes, cut-off productivities levels, and thus the size of the mass of operative firms, is directly affected by the pandemic. The reason is the following. In response to the outburst of the pandemic, agents curtail consumption of the social good, partially substituting it with that of the non social good. As a result, output increases in the non-social sector and decreases in the social one. The demand reallocation translates in a higher cut-off productivity threshold, $z_t^c(s)$, in the social sector and in a lower cut-off productivity in the non-social one. As we show in the remained of the analysis, changes in cut-off productivities due to the reallocation of demand ultimately affect both aggregate productivity and business dynamism.

To conclude the description of the entry and exit processes we assume, as in [Bilbiie et al. \(2012\)](#) and many other studies in the literature, a one time period to build, i.e. a one period lag between the decision to enter the market and the beginning of production. This period represents the amount of time required to set up production facilities. As a result, the number of firms in each sector evolves according to:

$$N_t(q) = (1 - \delta) (N_{t-1}(q) + N_{t-1}^e(q)) \quad \text{for } (q) = \{(s), (ns)\}, \quad (17)$$

where $N_{t-1}(q)$ is the mass of firms in sector (q) in period $t - 1$ and $N_{t-1}^e(q)$ denotes the mass of potential entrants between periods $t - 1$ and t in the rate.

same sector.

3.5 Labor Unions, Monetary Policy and Aggregation

Nominal wage rigidities are modeled according to the Calvo (1983) mechanism. In each period a union faces a constant probability $(1 - \alpha^*)$ of reoptimizing the wage. The optimal nominal wage in sector j set at time t , that we denote with W_t^* , is chosen to maximize agents' lifetime utilities.¹¹ Due to symmetry, the newly reset wage is identical across labor markets.

The Central Bank sets the nominal interest rate, R_t , according to the following Taylor rule with smoothing:

$$\left(\frac{R_t}{R}\right) = \left[\left(\frac{\pi_t}{\pi}\right)^{\varphi_\pi} \left(\frac{Y_t}{Y}\right)^{\varphi_Y}\right]^{1-\varphi_R} \left(\frac{R_{t-1}}{R}\right)^{\varphi_R}, \quad (18)$$

where variables without time subscript denote steady state values. For simplicity, we assume that the steady state gross inflation rate equals one.

In equilibrium, the representative household holds the entire portfolio of firms and the trade of state-contingent asset trade is nil. As a result, $b_{t+1}(q) = b_t(q) = 1$, and $a_{t+1} = a_t = 0$, so that:

$$C_t + N_t^e(s)\tilde{v}_t(s) + N_t^e(ns)\tilde{v}_t(ns) = (\alpha_W R_t + 1 - \alpha_W) w_t L_t^d + N_{o,t}(s)\tilde{c}_t(s) + N_{o,t}(ns)\tilde{e}_t(ns). \quad (19)$$

Y_t is either consumed, used as intermediate input in the production process or used to cover fixed costs of production and entry costs, thus:

$$Y_t = C_t + X_t + (N_{o,t}(s) + N_{o,t}(ns)) f_{x,t} + N_t^e(s) f_{e,t}(s) + N_t^e(ns) f_{e,t}(ns). \quad (20)$$

Finally, the representative household assumption implies: $\mathbb{I}_t = \mathcal{I}_t$, $\mathbb{S}_t = \mathcal{S}_t$, $\mathbb{D}_t = \mathcal{D}_t$ and $\mathbb{R}_t = \mathcal{R}_t$.

To obtain tractable results, a Pareto distribution is assumed for the p.d.f. $g(z)$ with minimum $zmin$ and tail parameter κ . This assumption simplifies considerably several equilibrium conditions and allows us to compute analytical solutions. Following Melitz (2003), a special average productivity is defined over operating firms. In our case, however, the special average productivity is sector-specific and it is defined as $\tilde{z}_t(q)$. The special average productivity allows to represent each sector as one populated by a mass of homogeneous firms $N_{o,t}(q)$, each of which endowed with idiosyncratic productivity $\tilde{z}_t(q)$, as we show in the Online Appendix. Thanks to the properties of the Pareto distribution, we can write $\tilde{z}_t(q)$ as a function

¹¹See the Online Appendix for details.

of the cut-off productivity, $z_t^c(q)$, as follows:

$$\tilde{z}_t(q) = \left[\frac{1}{1 - G(z_t^c(q))} \int_{z_t^c(q)}^{\infty} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta-1}} = \Gamma z_t^c(q), \quad (21)$$

where $\Gamma = \left[\frac{\kappa}{\kappa - (\theta - 1)} \right]^{\frac{1}{\theta-1}}$ and $1 - G(z_t^c(q)) = \left(\frac{z_{min}}{z_t^c(q)} \right)^\kappa$. The latter illustrates that changes in the cut-off productivity levels, due either to the pandemic or to other exogenous disturbances, lead to changes in average sectoral productivities.

4 Calibration

The time period is a week. The discount factor β equals $0.98^{1/52}$, as in [Eichenbaum et al. \(2020a\)](#). The coefficient measuring the disutility of labor, ν , is set to 1, while the inverse Frisch elasticity of labor supply equals 4, as in many other studies of the business cycle. We set $u_d = 10$ such that the value of a statistical life (VSL) is included between 1 and 50 millions dollars, depending on the definition of income in our model, in all the experiments we consider.¹²

In order to match the US empirical level of 10% job destruction rate, the weekly exit probability δ is calibrated to a value of 0.00211. The aggregate productivity level Z_t is normalized to 1. In the benchmark calibration $\chi = 1/2$, such that the two sectors have the same size. The elasticity of substitution between the social and the non-social good, η , is 1.5, as estimated by [Edmond et al. \(2015\)](#). On the other hand, the elasticity of substitution between goods belonging to the same sector is $\theta = 3.8$, following [Bernard et al. \(2003\)](#), who calibrated the value of θ to fit US plant and macro trade data. The selected value of θ entails a price markup equal to 35%, within the range estimated by [De Loecker and Eeckhout \(2018\)](#). The elasticity of substitution across labor types, θ_w , equals 4, which implies a steady state wage markup equal to 33%.

Turning to the parameters that determine entry frictions, ψ_0 is normalized to 1, as in [Bilbiie et al. \(2012\)](#). We set the elasticity of entry cost equal to $\gamma = 1.5$, in line with the estimate of [Gutierrez Gallardo et al. \(2019\)](#), who exploit the co-movement between industry-level entry rates and stock prices to pin down this parameter. The parameter ψ_1 is such that the steady state ratio between investment and GDP, $\frac{f_e(s)N_e(s) + f_e(ns)N_e(ns)}{Y - X}$, is approximately

¹²To calibrate the psychological cost of death, u_d , we consider the VSL which measures how much the average US citizen is willing to pay for a reduction in mortality rates equivalent to saving one life on average. [Greenstone and Nigam \(2020\)](#) estimate the VSL to be 11.5 million dollars.

15%. As in [Ghironi and Kim \(2019\)](#), given our equilibrium entry costs, we calibrate the entry costs to fixed costs of production $\frac{f_e(s)+f_e(ns)}{2f_x}$ to 4.5, as in [Collard-Wexler \(2013\)](#).

The parameterization of the productivity distribution is as follows. We normalize $zmin$ to 1 with no loss of generality. In the spirit of [Gabaix \(2011\)](#) and [Di Giovanni and Levchenko \(2012\)](#), our sectors can be defined as granular when $1 < \frac{\kappa}{\theta-1} < 2$. Given the value of θ , we set $\kappa = 6$. In this case, as discussed by [Colciago and Silvestrini \(2022\)](#), our sectors are just short of being granular, but the Herfindal-Hirschman Index (HHI) of concentration is well defined.

The calibration of the SIR model follows [Eichenbaum et al. \(2020a\)](#), which is based on data on the infection from South Korea. Assuming that the average duration of the disease is 14 days, the recovery probability π_d and the death probability π_r are calibrated such that $\pi_r + \pi_d = 7/14$. Based on the evidence, the death probability is fixed at 0.2%, i.e. $\pi_d = 7(0.002/14)$. The parameters of the transition equation which govern, respectively, the risk of infection from consumption π_1 , from work π_2 and from the mere interaction between susceptible and infected within the household π_3 , are calibrated such that the initial contagion is due for 1/6 to consumption activities, for 1/6 to working activities and for 2/3 to random interactions. Finally, the parameters are calibrated such that the exogenous SIR framework converges to the so-called Merkel scenario, where 60% of the population is either recovered or dead. The 60% threshold is regarded to deliver herd immunity.

The Calvo parameter α^* is set to 0.98 in order to observe, on average, one wage change per year. The parameters of the Taylor Rule are $\vartheta_\pi = 1.5$, $\vartheta_Y = 0.5/52$, and $\vartheta_R = 0.8$, as in [Eichenbaum et al. \(2020a\)](#). In the benchmark calibration firms don't borrow to pay wages, i.e. α_w is set to 0, and the Cobb-Douglas parameter α is 1/3. We begin our simulation from an initial infection seed of 0.1%, i.e. $\mathbb{I}_0 = 1e^{-3}$. For reader's convenience, [Table 1](#) reports the values of the calibrated parameters.

5 The Asymmetric Transmission of an Epidemic

In this section, we discuss the impact of an epidemic in our two-sector model. [Figure 5](#) displays percentage deviations from the steady state of key macro variables in response to the pandemic shock.¹³

Consumption and output drop substantially. As described by [Eichenbaum et al. \(2020a\)](#), the pandemic entails both negative demand and sup-

¹³Unless otherwise stated, the Figures that follows report percentage deviations of variables from their respective steady state.

Table 1: Calibration of exogenous parameters

Param.	Value	Target
SIR		
π_r	0.499	Duration infection 14 days, Eichenbaum et al. (2020a)
π_d	0.001	Mortality rate of 0.2%, Eichenbaum et al. (2020a)
π_2	0.143	1/6 contagion from labor, Eichenbaum et al. (2020a)
π_1	9.254	1/6 from consumption, Eichenbaum et al. (2020a)
π_3	0.5	2/3 from interaction, Eichenbaum et al. (2020a)
\mathbb{I}_0	0.001	Seed infected 0.1%, Eichenbaum et al. (2020a)
$(\mathbb{R} + \mathbb{D})_{end}$	0.6	60% Merkel scenario, Eichenbaum et al. (2020a)
u_d	10	VSL \approx 10 millions, Greenstone and Nigam (2020)
Nominal Stickiness		
α^*	0.98	Wages change once a year, Eichenbaum et al. (2020a)
ϑ_π	1.5	Standard for weekly, Eichenbaum et al. (2020a)
ϑ_Y	0.5/52	Standard for weekly, Eichenbaum et al. (2020a)
ϑ_R	0.8	Standard smoothing, Christiano et al. (2005)
Firms		
α	1/3	Standard for Cobb-Douglas production
ψ_0	1	Normalization, Bilbiie et al. (2012)
ψ_1	1000	Investment to GDP Ratio \approx 15%
γ	1.5	Entry and stocks, Gutierrez Gallardo et al. (2019)
f_x	0.47	Ratio $f_e/f_x \approx$ 4.5, Collard-Wexler (2013)
α_w	0	No working capital constraint
δ	0.00211	Yearly job destruction rate \approx 10%, Colciago (2016)
Z	1	Normalization
θ_w	4	Wage markup of 33%
$zmin$	1	Normalization
κ	6	Almost granular, Colciago and Silvestrini (2022)
Households		
χ	0.5	Ex-ante homogeneous sectors
η	1.5	Inter-sectoral substitutability, Edmond et al. (2015)
θ	3.8	US plant data in Bernard et al. (2003)
ν	1	Normalization
ϕ	4	Frisch elasticity as in King and Rebelo (1999)
β	$0.98^{1/52}$	\approx 4% yearly interest rate, Eichenbaum et al. (2020a)

Notes: The table summarizes the calibration of the exogenous parameters. The second column describes the value assigned to the parameters. The third column describes the targets of the calibration and their sources.

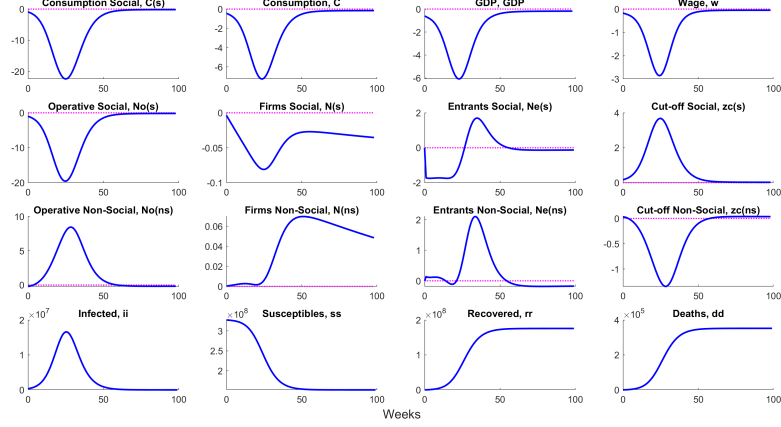


Figure 5: IRFs of the main macroeconomic variables to the pandemic shock.

ply shocks. The demand shock arises because agents optimally curtail their consumption to limit their exposure to the virus. For the same reason, a negative supply shock results from the reduction in households' desire to work.

In the presence of endogenous business dynamism, heterogeneous firms, and sectors characterized by different degree of social contact, the pandemic shock spurs additional effects. Specifically, agents curtail their consumption of the s -good because of fear of contagion. As a result, output in the s -sector drops to a much larger extent than aggregate output, consistently with the US evidence displayed in Figure 1.

For this reason, entry becomes less profitable in the s -sector. Due to the drop in revenues, break-even in the s -sector requires higher idiosyncratic productivity. This, in turn, has two effects. The first one is an increase in the effective exit rate coming from the inactivity margin, the second one is a drop in the number of potential entrants. Indeed, only firms with higher productivity will find convenient to continue production or to newly enter in the s -sector.¹⁴ Both effects increase average productivity in the social sector. Opposite forces, with respect to those just described, characterized the ns -sector: firms with lower idiosyncratic productivity will now break-even on their costs.¹⁵ Thus, the reallocation of demand to the non-social sector results in a lower effective exit, a higher number of potential entrants, and a drop in average sectoral productivity. Thus, entry opportunities are shifted away from temporary less profitable sectors and concentrated in

¹⁴Note that small and unproductive firms that fall below the cut-off productivity exit only temporarily, until market conditions improve. This is consistent with the re-opening of small businesses we observe in the data, see Fairlie (2020)

¹⁵The heterogeneous effects of the pandemic on firms survival have been extensively documented, see for instance Fernández Cerezo et al. (2021).

more profitable ones.

This pattern qualitatively reflect the dynamics of US data on business applications reported in Figures 2 and 3 and the drop in production of Figure 1. Indeed, the data show a drop in business applications in social sectors. On the contrary, business applications in non-social sectors are characterize by a rise, after an initially flat response.¹⁶

Exit is aligned as well, both regarding the sectoral heterogeneity and the fact that small and unproductive firms - our framework implies a one-to-one correspondence between firm size and firm productivity - were hit the hardest. The economic literature on the Covid-19 pandemic has revealed a strong negative correlation between firm size and exit rates.¹⁷ Here, we extend this evidence directly inspecting the 2020 Business Dynamics Statistics (BDS) Datasets from the US Census Bureau. The BDS shows how exit rates for small firms (1 to 4 employees) remained high through 2020, and particularly so in social sectors. Their exit rate is often in a 10% – 20% range, significantly higher than the one for larger firms.¹⁸ Similarly, studies by Crane et al. (2022) in the US and Gourinchas et al. (2020) in Europe have found that small firms exhibit exit rates higher than average, particularly so those operating in industries highly impacted by the pandemic. Similar evidence for the US can be found in Bartik et al. (2020) and Fairlie (2020).

¹⁶Note that while the fluctuations in entry in the model qualitatively reflect the dynamics on business applications, the former should not be quantitatively compared to the latter for two main reasons. First, the magnitude of the fluctuations in business applications were not reflected in the Q2 and Q3 entry data from the Business Employment Dynamics, as noticed by Bilbiie and Melitz (2020). While these latter data confirm the reallocation of entry opportunities across social and non-social sectors, they feature fluctuations in establishment openings of an amplitude ranging from one quarter (construction) to half (accommodation services) of those characterizing business application data. Second, our model studies the implications of the endogenous behavioral response of households to the shock, and thus abstract from government measures to contain the spread of the virus - such as lockdown measures - that obviously impacted on actual fluctuations in entry and other macroeconomic variables. Sections 8.1 and 8.2 discuss the impact of the introduction of social distancing measures and lockdown in our model, respectively.

¹⁷This correlation is evident across multiple sectors and countries, regardless of government policies implemented, e.g. in spite of lockdown severity or the magnitude of firms' subsidies.

¹⁸The average exit rate for small firms in social sectors is 18%, higher than the historical average as well as the time trend for social sectors, and higher than the average in the non-social sectors for 2020. More precisely, the lowest exit rate among small firms in 2020 is 10.6% from NAICS-2 sector 22 Utilities, a non-social sector, closely followed by another non-social sector (Finance and Insurance). On the other hand, the highest exit rate is 26.1% from a social sector, NAICS-2 sector 72 Accommodation and Food, followed by Warehousing and Transportation, again a social sector. Moreover, this exit rate is significantly higher than the one for other size bins, rarely above 5%, both during and before Covid, for the latter see Tian (2018).

Moreover, [Muzi et al. \(2022\)](#) provides complementary evidence of the direct link between firm exit and productivity, particularly during the Covid-19 crisis. [Muzi et al. \(2022\)](#) use a panel from 34 economies and 4 continents to demonstrate a negative relationship between labor productivity and firm exit. They suggest that the COVID-induced recession led to the elimination of unproductive enterprises, i.e. we observe productivity cleansing.¹⁹ Overall, the evidence suggests that firm exit can be productivity-enhancing, and it surely was during the recent economic downturns.

The dynamics just described occurred in the early phases of the pandemic, prior to the introduction of legal lockdown measures. The timing of events suggests that the behavioral response of agents to the spread of the disease had a key role in the reallocation process, in line with the findings by [Goolsbee and Syverson \(2021\)](#).

Finally, the dynamics characterizing the epidemics are those typically observed in the data, where the response of the number of infected people to the diffusion of the disease displays an inverted U shape. After the peak in the number of infected, the economy converges to a steady state where aggregate output, consumption, and entry shrink permanently due to the decline in population.

6 Aggregate Productivity

In this section, we discuss the empirical pattern displayed by aggregate labor productivity during the COVID-19 pandemic. Then, we show that our model qualitatively replicates the observed dynamics. Specifically, we consider a simplified version of our framework to show analytically that aggregate productivity is a size-weighted harmonic mean of sectoral average productivities, corrected for the sectoral shares of operating firms.

Using this measure as a proxy for aggregate labor productivity in our benchmark model, we argue that the sector-specific cleansing effect and the reallocation of demand and across sectors are key to explain the empirical dynamics of aggregate labor productivity. Note that, to simplify the intuition, sectors are ex-ante homogeneous in our baseline model. This allows us to eliminate any initial size effects that would complicate the interpretation of the relative movements. However, given that the US has different sector sizes, we re-calibrate the model to account for this fact. At this

¹⁹[Grover and Karplus \(2021\)](#) have shown that effective management practices, such as monitoring, increase the likelihood of survival for manufacturing firms during Covid. Using a dataset of Belgian enterprises, [Konings et al. \(2022\)](#) show a large positive contribution of firm exit to aggregate productivity growth in 2020, the largest in magnitude over the entire horizon they observe. Additionally, [Cros et al. \(2021\)](#) claim that, even though the exit of firms during the crisis was depressed, it had a positive impact on productivity. See [Criscuolo \(2021\)](#) for an extensive survey on the topic

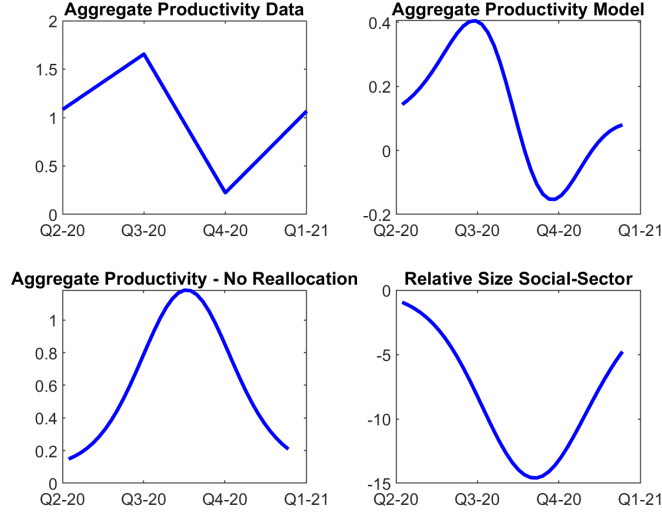


Figure 6: Top Panels: aggregate productivity in the model and in the data during the pandemic. Bottom Panels: productivity decomposition.

end of this section, we show that our qualitative results are robust to the alternative specification, while the quantitative match drastically improves.

The top left panel of Figure 6 reproduces the empirical series of aggregate labor productivity displayed in Figure 4.²⁰ Labor productivity had a peculiar pattern during the pandemic. It increased up to summer/fall of 2020, to drop sharply afterwards, and to start growing again at the beginning of 2021.

To provide economic intuition about the response of labor productivity to the pandemic shock, we simplify our model to consider the case where there is no contagion from consumption, that is where $\pi_1 = 0$, and where labor is the only factor of production. As a result of the latter assumption, the model features no network effects. In this simplified case, as detailed in Appendix D, the quantity of final output produced in the economy can be written as:

$$Y_t = N_{o,t}^{\frac{1}{\theta-1}} Z_t \tilde{Z}_t L_t^d, \quad (22)$$

where L_t^d are aggregate hours of work, $N_{o,t}$ denotes the total number of operating firms in the economy, and \tilde{Z}_t defines the endogenous component

²⁰Note that, in order to align all panels to the first of April 2020, we discarded the first month of observations from the simulated model, as we assume that the behavioral response started in March.

of aggregate productivity. The latter is given by:

$$\tilde{Z}_t = \left(\chi \rho_t(s)^{-\eta} \omega_t(s)^{\frac{1}{1-\theta}} \frac{1}{\tilde{z}_t(s)} + (1-\chi)(1-\omega_t(s))^{\frac{1}{1-\theta}} \rho_t(ns)^{-\eta} \frac{1}{\tilde{z}_t(ns)} \right)^{-1}, \quad (23)$$

where $\omega_t(q) = N_{o,t}(q)/N_{o,t}$. In the restricted model, the endogenous component of aggregate productivity, \tilde{Z}_t , is a weighted harmonic average of the average sectoral productivities, $\tilde{z}_t(s)$ and $\tilde{z}_t(ns)$, corrected for a measure of the sectoral fractions of operative firms, that is $\omega_t(s)^{\frac{1}{1-\theta}}$. The weights are represented by the relative size of sectors, $\chi \rho_t(s)^{-\eta}$ and $(1-\chi) \rho_t(ns)^{-\eta}$. We next use equation (23) to construct a measure of aggregate productivity in the fully-fledged model.²¹ The latter is displayed in the top right panel of Figure 6.

When using equation (23) to build labor productivity, our model delivers a sinusoidal-shaped series that mirrors closely that in the data, at least from a qualitative standpoint. The reason for this relative success, is that our model accounts for both sector-specific cleansing in the social sector, and for the reallocation of demand across sectors that characterized the pandemic. To see this, consider the two bottom panels of the figure. The bottom left panel plots again labor productivity as in (23), but shutting down the reallocation channel. Specifically, we assume that $\omega_t(q)$ and the sectors' relative size in equation (23) remain constant at their pre-pandemic steady state values. The bottom right panel, instead, plots the change of the relative weight of the social sector in response to the pandemic shock.

A joint reading of two lower panels suggests that the response of aggregate productivity to the pandemic shock is the result of two driving forces. The first one results from the change in sectoral average productivity in the two sectors. The second force is a composition effect due to the reallocation of demand across sectors, that alters their relative size. The interaction between these two forces determines the overall productivity dynamics. In the initial part of the sample, the cleansing in the social sector dominates, because the relative size of the social sector displayed a limited decline. As a result overall productivity rises (even if much less than without reallocation, lower left panel). In the central part of the sample, instead, the decline in the relative size of the social sector becomes quantitatively sizeable, thus the composition effect dominates, leading to a sharp decline in aggregate

²¹ As argued in Ghironi and Melitz (2005) and Bilbiie et al. (2012) when using the model for empirical statements, one has to recognize that empirically relevant variables, as opposed to welfare-consistent concepts, net out the effect of changes in the range of available varieties. Unfortunately, in our rich benchmark framework, we cannot analytically pin-down the relationship between welfare consistent and empirically relevant variables. On the contrary, we can identify that relationship in the simplified framework, and for that reason we use the endogenous measure of productivity obtained in the latter case as a proxy for aggregate productivity in the benchmark model.

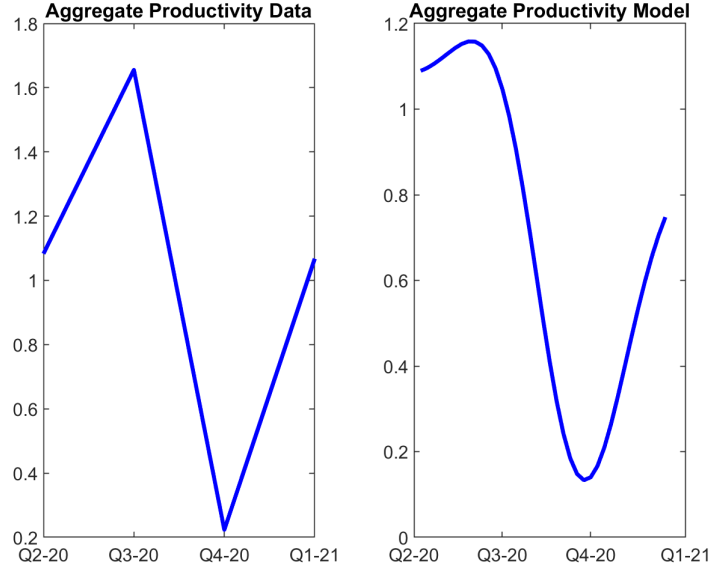


Figure 7: Aggregate productivity in the data and in the model (US)

productivity (much stronger - and earlier - than without reallocation). As a result, aggregate productivity overshoots and then approaches the long-run level of productivity from below at the end of sample, as the relative size of the social sector converges to steady state, pushing up again productivity.

To take into account that the US has different sector sizes, we now first present the same series for labor productivity computed in a model where the relative size of the social sector in the model matches the one of the US in the data, implying $\chi = 0.34$. Second, we discuss how both the aggregate and the sectoral dynamics generated by the model resemble the empirical counterparts.

Figure 7 depicts a comparison between the labor productivity observed in the empirical data and the predictions of our model. The predictions of our model match the behavior of the aggregate labor productivity in the data, both in terms of the timing and, to a large extent, the magnitude of the path. Moreover, the model can explain the underlying sectoral dynamics that, according to our model, shape the dynamics of aggregate productivity, i.e. (i) the sector-specific cleansing and (ii) the asymmetric change in sectors' size.

Regarding the first mechanism, the cleansing in the data was, indeed, sector-specific in the Bureau of Labor Statistics (BLS) data, as implied by our model. In 2020, social sectors exhibit, on average, a growth rate of labor productivity which is at least 1.5 percentage points above the trend.

On the contrary, non-social sectors are 0.5% below the historical average.²² Furthermore, note that, consistently with our simulation, in the data the cleansing of the social sectors is stronger in absolute value than the sullyng of the non-social sectors.

Regarding the second mechanism, the change in the relative size of the sectors implied by the model is consistent with the data, too. Social sectors shrank (more) with respect to the non-social ones according to several observable measures. This reallocation of business opportunities is apparent, for instance, from the entry and production patterns shown in Figure 1 and in Figures 2 and 3.

While the matching surely improved by calibrating the initial sectors share to the US data, quantitatively the positive peaks leading to $Q3 - 20$ or $Q1 - 21$ are weaker in our simulation than in the data. Our model goes a long way in explaining the empirical behavior, but it abstracts from some important factors, as, for instance, lockdown policies. While we endogenize the lockdown policy as an optimal response of the households, alternatively one could also superimpose an exogenous lockdown. We do that in Section 8.2, showing that an additional (exogenous) lockdown could improve the quantitative match of the model, especially regarding the aforementioned missing peak leading to $Q3$, as it would exacerbate the initial reallocation and cleansing effects.

Furthermore, it is worth noting that our baseline model generates a simple one-waved pandemic, although Section 8.1 extend our analysis introducing social distancing. Allowing for the possibility of a resurgence of the pandemic in the winter of 2020 – 2021, due to the relaxation of social distancing measures, could also enhance the match of the model for $Q1 - 21$, as again this could foster reallocation and, thus, cleansing.

In short, our analysis shows that the composition effect due to reallocation is key to generate a sinusoidal shape of the response of aggregate productivity. Thus, neglecting the reallocation of demand across sectors that characterized the pandemic, as we do in the bottom left panel of Figure 6, entails counterfactual dynamics in labor productivity.

7 Monetary Policy in an Epidemic

This section studies the role of monetary policy during the pandemic. Monetary policy is very powerful in influencing the dynamics of output and the

²²From BLS data we obtain yearly information on labor productivity growth for many sectors and industries of the US economy (from 2 to 6 NAICS digits level of granularity) over the period 2005 – 2020. Then, we take unweighted yearly averages across all social sectors and contrast them to the non-social ones, following the classification of Kaplan et al. (2020). Finally, we compare the growth in the first year of Covid, i.e. 2020, to the average growth rate in the previous years.

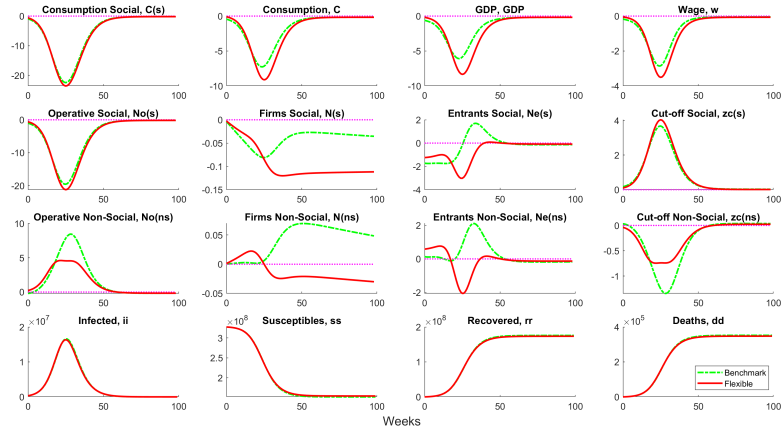


Figure 8: IRFs of the main macroeconomic variables to the pandemic shock. Sticky Wages Vs. Flexible Wages.

reallocation of activity across sectors.

Figure 8 compares the dynamics of our benchmark specification with the ones under flexible wages. Dynamics of output and consumption are similar across specifications, although the recession is stronger in the flexible prices case. Importantly, there is a substantial difference in business dynamism between the two cases. Under flexible wages, we do not observe the reallocation of entry opportunities that characterize US data. Indeed, few periods after the shock, entry diminishes sharply in both sectors. Despite the sizeable reduction in the productivity cut-off, the recession is so severe to induce a contraction in the number of entrants also in the non-social sector. While powerless in the flexible wage case, in presence of nominal rigidities monetary policy affects the real interest rate.²³ Specifically, in our benchmark case, the monetary response is such that the real rate decreases persistently. This supports consumption during the crisis, leading to a milder recession with respect to the flexible wages scenario. This stands in contrast with the transmission mechanism featured in one sector New Keynesian models. In that context, [Lepetit and Fuentes-Albero \(2020\)](#) find that intertemporal substitution is muted due to fear of contagion. As a result monetary policy can do little to boost economy activity. On the contrary, in our two sector economy, the monetary stimulus leads to additional demand that flows to the non-social sector. Hence, by sustaining aggregate demand through a blunt - and not tailored - instrument as the real interest rate, monetary policy induces a larger reallocation.

²³While the real interest rate surges slightly on impact to stay flat for one year before starting decreasing under flexible wages, with nominal rigidities the real interest rate drops on impact and keep reducing for around one year and half before reverting to steady state.

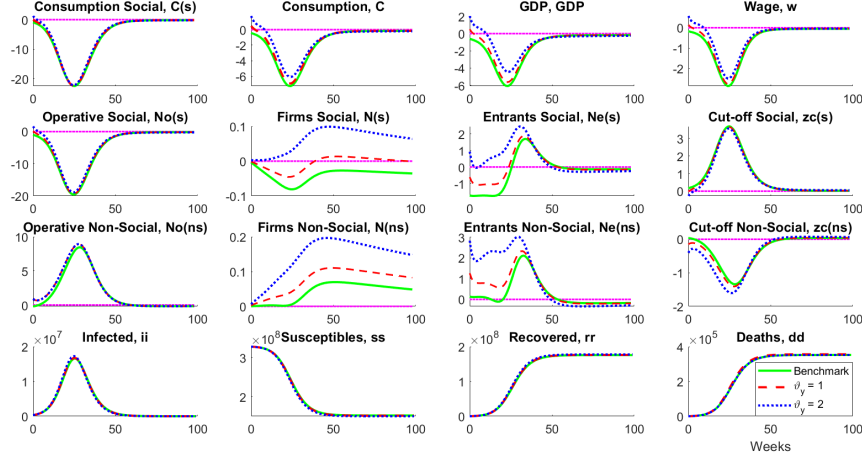


Figure 9: IRFs of the main macroeconomic variables to the pandemic shock under different Values of the output response coefficient in the interest rate rule.

Guerrieri et al. (2021) study the optimal degree of monetary accommodation in response to an asymmetric shock in a two sector economy. They find that, when labor is mobile between sectors, the optimal monetary easing induces a faster reallocation across sectors. While studying the optimality of interest rate policies goes beyond the scope of this paper, we point out that the dynamics of the real interest rate is a key ingredient to replicate the sectoral business dynamism observed in US data.

To further investigate this point, Figure 9 shows model dynamics under alternative calibrations of the output gap coefficient response in the interest rate rule. A larger weight on output in the Taylor rule leads to a stronger reduction in the real rate in response to the crisis and, through the demand channel, to a milder recession and hence a stronger reallocation. The larger is ϑ_y , the lower is the cut-off in the non-social sector at the outset of the pandemic. Hence, entry and the number of firms respond strongly and are always above steady state. The social sector also benefits from a lower recession, because the dynamics of both entry and the number of firms raises with ϑ_y almost uniformly - in the sense that they exhibit a similar shape, but just shifted upwards.

8 Further Results

This section discusses the role of social distancing measures, of an exogenous lockdown, of the economic structure of a country, in terms of the relative size of the social sector, and of the elasticity of substitution be-

tween sectors. We do not discuss the effect of roundabout production in the main text since our analysis suggests that the network linkages - as far as modelled as standard and symmetric roundabout - did not have strong economic effects during the pandemic.

8.1 Social Distancing

The Social distancing (SD, henceforth) measures implemented by various government around the globe, were aimed at reducing the diffusion of the disease along the three possible contagion channels that we consider: consumption, work and random encounters between susceptible and infected agents. In other words, they aimed at reducing the probabilities π_1 , π_2 and π_3 .

We compare the benchmark simulation without SD to three alternative scenarios, that differ only in the duration of containment measures. Under all scenarios characterized by SD, we assume a generalized 20% cut in social contacts with respect to the benchmark case. Figure 10 displays transition dynamics under the cases we consider. Green dashed lines refer to the benchmark transition with no containment measures, blue solid lines to the case in which containment measures are imposed for 6 months, yellow dotted lines to the case in which social distancing applies for 1 year, and finally red dotted-dashed lines refer to the case in which SD is imposed permanently, i.e. for the whole transition.

Independently of their length, SD measures dampen the peak effect of the recession on consumption, output and other macroeconomic variables with respect to the benchmark no-SD scenario. Nevertheless, they extend the duration of the recession, and the more so, the longer their duration. The reason is that SD smooths out the reallocation process across sectors with respect to the benchmark case. Indeed, fluctuations in sectoral productivities are dampened and last longer with respect to what observed in the baseline scenario.

Turning to epidemiological effects, containment measures are effective at diminishing the number of infected people, but their effects on the number of dead individuals depend on their duration. Specifically, when measures are imposed for half a year the effects are simply delayed. Indeed, with respect to our no-SD baseline scenario the difference in the number of dead individuals at the end of the transition is minor. However, we do not consider the possibility that the temporary strategy would prove effective in buying time to authorities for a reorganization of health care facilities, and for the development of a vaccine or a cure of the disease. In the case of a more extended SD policy, where measures are imposed for a year, we observe a substantial reduction in the loss of lives. Lifting containment measures after a year, however, leads to a second wave of infection as

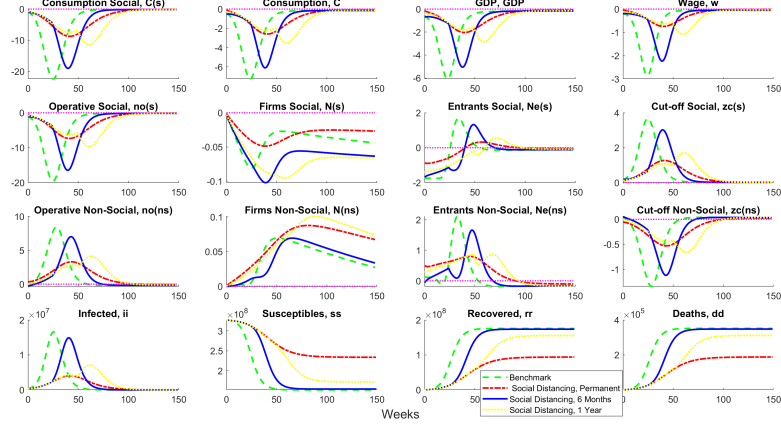


Figure 10: Simulation Benchmark vs. Social Distancing

suggested by the dynamics of the number of infected.

8.2 Exogenous Lockdown

In this section, we present the results regarding aggregate labor productivity when we superimpose an exogenous lockdown.

Figure 11 displays aggregate labor productivity in the data and in the baseline model, as presented in Figure 7, and labor productivity in an alternative model with an exogenous lockdown. The lockdown is modelled by scaling down the productivity level in the social sector: while the common productivity is still Z_t in the non-social sector, in the social sector it is now ιZ_t , where $\iota = 1$ outside of the lockdown. We impose a lockdown for approximately two months, starting from the end of March 2020/beginning of April 2020, setting $\iota = 0.98$. The drop in the social sector common productivity level increases even further the cut-off productivity in the social sector, forcing firms to go inactive.

Hence, as mentioned already in Section 6, superimposing an exogenous lockdown exacerbate the reallocation process in our framework. Although the effects on several aggregate variables (not shown) is mild, this can still be helpful to improve the quantitative matching of the model. For, instance, in Figure 11 the peak leading to Q3 is stronger.

8.3 Economic Structure

To highlight the key mechanisms of our model, in our benchmark specification we calibrated the two sectors to have equal size, in order to avoid any initial size effect. However, in the US the social sectors represent approximately one quarter of the total value added: in Section 6, we show that

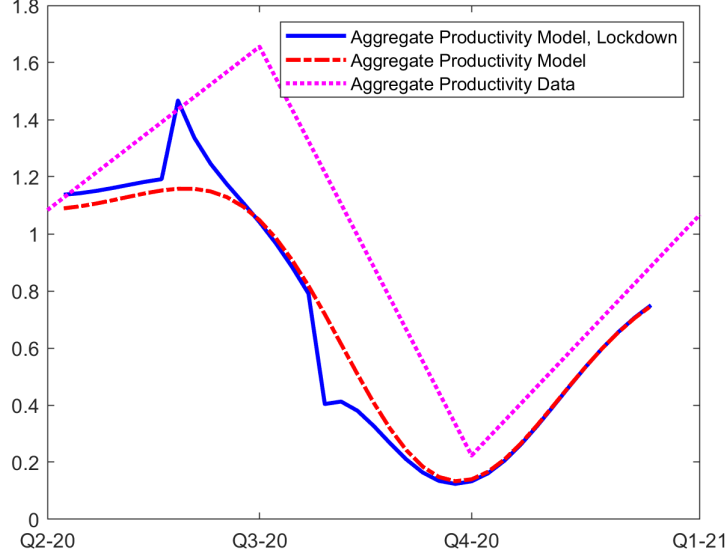


Figure 11: Aggregate Labor Productivity, Benchmark vs. Lockdown vs. Data

our results are robust to this characterization. Here, we highlight the role of sectors' sizes, echoing the [IMF World Economic Outlook \(April, 2021\)](#), which argues that output losses have been particularly severe for countries with a large relative size of high-contact sectors.

We compare the response of our model economy to the pandemic under two alternative scenarios concerning the relative size of the social sector. The first one is meant to mimic the US. [Kaplan et al. \(2020\)](#) report that the US value added share of social sectors is approximately 0.26. We set $\chi_{US} = 0.34$ to match this target. This is the calibration used for the analysis of labor productivity at the end of Section 6.

The second scenario is meant to represent the case of an economy with a larger social sector. To do so, we set $\chi = 1 - \chi_{US} = 0.66$. The relative sizes of social sectors across scenarios sum to 1, so that the aggregate size of the economy is the same in the two cases. Because of this reason, the percentage variation in aggregate consumption and GDP in Figure 12 are directly comparable across the two scenarios. Red solid lines in Figure 12 refer to the case of an economy characterized by a large social sector, while green dotted-dashed refer to the case of a small social sector. Consistently with the evidence uncovered by the IMF, output and consumption losses are larger in the former case.

The initial reaction is more inertial in the economy where the social sector is large, because the effects of the pandemic shock is partially compensated by a strong investment - i.e. new firms - in the small non-social

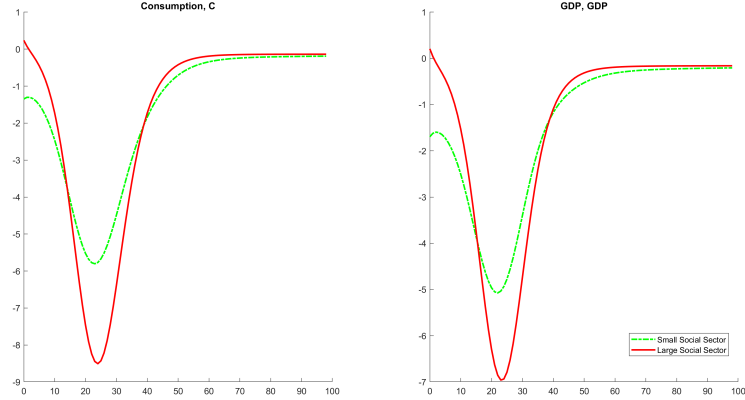


Figure 12: Alternative Sizes of the Social Sector

sector whose size is foreseen to expand substantially (in relative terms). In contrast, when the social sector is small the reallocation is limited because the pandemic hits an already small sector where only the very productive firms stay in business. In this case, the cleansing effect in the social sector is relatively larger, i.e., the percentage variation in the productivity cutoff is higher. The effect is similar to that of a standard contractionary shock, and consumption drops on impact.

The relative size of sectors is reflected also in the epidemiological block of the model (not shown) - again comparable across models because of the same initial economy size across the two scenarios. Despite the larger aggregate contraction, the social sector remains larger in a more-social economy. A larger social sector entails a higher number of infected both at the peak and during the transition, which ultimately results in a higher death toll.

8.4 Elasticity

In this section, we test how a change in the elasticity of substitution between sectors, i.e. η , can potentially affect our results.²⁴ Below, we compare the baseline simulation to ones in a high- η economy, $\eta = 2$, and in a low- η environment, $\eta = 1.2$.

Whenever η is low, the substitutability between sectors is low. Note, indeed, that when $\eta \rightarrow 1$, the sectoral aggregator converges to a Cobb-Douglas, where the relative sectors' size is fixed by a function of χ . Due to the lower elasticity, the response of the variables is muted, as shown by the blue curves in Figure 13. In particular, the reduction in consumption of the

²⁴We also test how results change in response to changes in the elasticity of substitution within sectors/between firms, i.e. θ . However, in this case results are qualitatively almost identical, except for small deviations on impact.

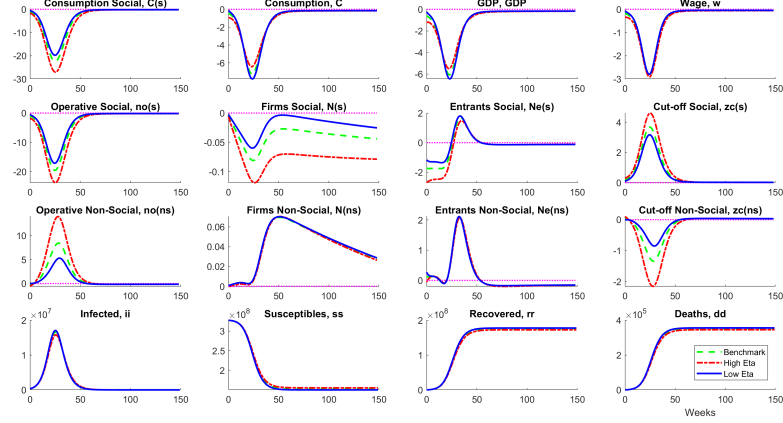


Figure 13: Alternative Elasticity of Substitution between Sectors

social sector is lower, as well as the reallocation as a whole. This results in a lower drop in entry in the social sector and a weaker partial lockdown. Moreover, sectoral productivity reacts less, as both the cleansing in the social and the sullyng in the non-social sector are milder. Due to this lower elasticity, which increases the costs of reallocation as sectoral goods are less substitutable, aggregate GDP and consumption drop by more than in the baseline.

Conversely, the red dotted lines in Figure 13 represent the case of a high substitutability economy. Here, reallocation is more prominent, as households can easily substitute the riskier social good with the safer non-social one. This creates a stronger drop in entry, consumption and production in the social sector, with respect to the baseline, which leads to a more pronounced change in sectoral productivity. Thanks to this easier reallocation, which renders the economy more flexible to sectoral shocks, the drop in GDP is milder.

9 Conclusions

A key dimension to consider in order to understand firms economic exposure to the COVID-19 pandemic is their sector of operation. Considering US statistics on Business formation, we argue that during the pandemic business startups reallocated from sectors characterized by social interaction to those characterized by low social contact. To rationalize this fact, we build an Epidemiological-Industry Dynamic model characterized by firms with heterogeneous productivity and endogenous business dynamism. The epidemiological block of the model consists of a SIR model.

We showed that in response to the outburst of the pandemic, our frame-

work reproduces the reallocation of entry opportunities across sectors observed in US data. The latter entails a response in sectoral and aggregate productivity. At the sector level, there is cleansing of low-productivity firms in the social sector, and the opposite in the non-social one. Accounting for productivity movement at the sectoral level does not, however, suffice to explain the empirical pattern of aggregate labor productivity during the pandemic. We showed that a paramount ingredient to replicate the latter is to capture the reallocation of demand and of operative firms across sectors that we observed during the pandemic.

Monetary policy, through its effect on the real interest rate, affected business dynamism during the Covid-19 crisis. Indeed, the latter cannot be replicated in a the flexible-wage counterpart of our model. Finally, our framework could naturally rationalize the different quantitative effects of the Covid-19 shock observed in countries characterized by different relative sizes of the social vs. non-social sectors.

The analysis does not feature the reorganization of production caused by the crisis, e.g. through the acceleration of the digital uptake or teleworking. For this reason, we focused on the first wave of the pandemic, when the response of households played the lion's role at shaping macroeconomic dynamics. In our view, those factors would quantitatively amplify, but not qualitatively alter, the reallocation process across sectors that we described in the paper, and its productivity effects, as shown for social distancing and lockdown measures. Identifying the interplay between firm dynamics, the digital uptake, and teleworking for the dynamics of aggregate productivity during the pandemic is a promising avenue for future research.

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Appendix

A Productivity Cut-off

Firms turn inactive when, by producing, they would make negative profits. Using this, we can define a cut-off productivity level, one for each sector, below which firms become idle. Setting equilibrium real profits equal to zero we get:

$$f_{x,t} = \frac{1}{\theta} \rho_{z,t}(q)^{1-\theta} \rho_t(q)^\theta Y_t(q)$$

or:

$$\left(\frac{f_{x,t}}{\rho_t(q)^\theta Y_t(q)} \right)^{\frac{1}{1-\theta}} \theta^{\frac{1}{1-\theta}} = \rho_{z,t}(q)$$

substituting the real price $\rho_{z,t}$, evaluated at the cut-off z_c :

$$\frac{\theta}{\theta-1} \frac{1}{Z_t z_t^c(q)} \left(\frac{(\alpha_W R_t + 1 - \alpha_W) w_t}{1 - \alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha = \left(\frac{f_{x,t}}{\rho_t(q)^\theta Y_t(q)} \right)^{\frac{1}{1-\theta}} \theta^{\frac{1}{1-\theta}}$$

Solving for the sectoral cut-off productivity $z_t^c(q)$:

$$z_t^c(q) = \frac{\theta^{\frac{\theta}{\theta-1}}}{\theta-1} \frac{1}{Z_t} \left(\frac{(\alpha_W R_t + 1 - \alpha_W) w_t}{1 - \alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^\alpha \left(\frac{f_{x,t}}{\rho_t(q)^\theta Y_t(q)} \right)^{\frac{1}{\theta-1}}$$

which is the formula we use in the main text.

B Fictitious Bundler of Y_t

The demand functions for the sectoral outputs $Y_t(q)$ can be obtained from a fictitious bundler that maximizes, in real terms:

$$Y_t - \rho_t(s) Y_t(s) - \rho_t(ns) Y_t(ns)$$

subject to:

$$Y_t = \left[\chi^{\frac{1}{\eta}} Y_t(s)^{\frac{\eta-1}{\eta}} + (1 - \chi)^{\frac{1}{\eta}} Y_t(ns)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

and to:

$$\mathcal{T}_t = \mathcal{S}_t \mathbb{I}_t \pi_1 c_t(s) C_t(s) + \mathcal{S}_t \mathbb{I}_t \pi_2 l_t^s L_t^d + \pi_3 \mathcal{S}_t \mathbb{I}_t$$

The Lagrangian is:

$$\begin{aligned} \mathbb{L} = & Y_t - \rho_t(s) Y_t(s) - \rho_t(ns) Y_t(ns) + \\ & + \bar{\lambda}_t \left(\left[\chi^{\frac{1}{\eta}} Y_t(s)^{\frac{\eta-1}{\eta}} + (1 - \chi)^{\frac{1}{\eta}} Y_t(ns)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} - Y_t \right) + \end{aligned}$$

$$+\bar{\lambda}_{t,t} \left(\mathcal{T}_t - \mathcal{S}_t \mathbb{I}_t \pi_1 c_t(s) C_t(s) - \mathcal{S}_t \mathbb{I}_t \pi_2 l_t^s L_t^d - \pi_3 \mathcal{S}_t \mathbb{I}_t \right)$$

The first order condition with respect to $Y_t(s)$ is:

$$-\rho_t(s) + \bar{\lambda}_t \frac{\eta}{1-\eta} Y_t^{\frac{1}{\eta}} \chi^{\frac{1}{\eta}} \frac{\eta-1}{\eta} Y_t(s)^{\frac{-1}{\eta}} - \bar{\lambda}_{t,t} \mathcal{S}_t \mathbb{I}_t \pi_1 c_t(s) \frac{\delta C_t(s)}{\delta Y_t(s)} = 0$$

Since $Y_t(s) = C_t(s) + X_t(s) + f_{x,t} N_{o,t}(s) + f_{e,t} N_t^e(s)$ and $c_t(s)(1 - \mathcal{D}_t) = C_t(s)$, this can be written as:

$$\bar{\lambda}_t Y_t^{\frac{1}{\eta}} \chi^{\frac{1}{\eta}} Y_t(s)^{\frac{-1}{\eta}} = \rho_t(s) + \bar{\lambda}_{t,t} \frac{\mathcal{S}_t \mathbb{I}_t}{1 - \mathcal{D}_t} \pi_1 C_t(s)$$

The Lagrange multiplier $\bar{\lambda}_t$ represents the real value in terms of increased revenues of an extra unit of Y_t (indeed, without contagion, the condition would be real marginal cost of $Y_t(s)$, i.e. $\rho_t(s)$, equal to the marginal benefit of $Y_t(s)$, which is the marginal benefit of Y_t times the marginal product of $Y_t(s)$, i.e. $\frac{\delta Y_t}{\delta Y_t(s)}$). The first is equal to 1. For the household, the value of one extra unit of Y_t is hence $1 \cdot \frac{\delta C_t}{\delta Y_t} \frac{\delta c_t}{\delta C_t} \frac{\delta U}{\delta c_t}$, which is $\frac{1 - \mathcal{D}_t}{C_t}$.

On the other hand, the Lagrange multiplier $\bar{\lambda}_{t,t}$ represents the costs in terms of newly infected of having one extra unit of $Y_t(s)$. This is equal to $\lambda_{t,t} \frac{\delta C_t(s)}{\delta Y_t(s)} = \lambda_{\mathcal{T},t}$. Finally, since the household owns the bundler, we rescale both multipliers by $1/\lambda_t$ to express everything in terms of utility. Thus:

$$\frac{1 - \mathcal{D}_t}{C_t} \frac{1}{\lambda_t} Y_t^{\frac{1}{\eta}} \chi^{\frac{1}{\eta}} Y_t(s)^{\frac{-1}{\eta}} = \rho_t(s) + \frac{\lambda_{\mathcal{T},t}}{\lambda_t} \frac{\mathcal{S}_t \mathbb{I}_t}{1 - \mathcal{D}_t} \pi_1 C_t(s)$$

This gives:

$$Y_t^{\frac{1}{\eta}} \chi^{\frac{1}{\eta}} Y_t(s)^{\frac{-1}{\eta}} = \left[\lambda_t \rho_t(s) + \lambda_{\mathcal{T},t} \frac{\mathcal{S}_t \mathbb{I}_t}{1 - \mathcal{D}_t} \pi_1 C_t(s) \right] \frac{C_t}{1 - \mathcal{D}_t}$$

Raising to the power of $-\eta$:

$$\frac{Y_t(s)}{Y_t} = \chi \left[\lambda_t \rho_t(s) + \lambda_{\mathcal{T},t} \frac{\mathcal{S}_t \mathbb{I}_t}{1 - \mathcal{D}_t} \pi_1 C_t(s) \right]^{-\eta} \left(\frac{C_t}{1 - \mathcal{D}_t} \right)^{-\eta}$$

With the same reasoning, and knowing that $C_t(ns)$ does not impact the endogenous contagion rate, the first order condition for $Y_t(ns)$ is:

$$\frac{Y_t(ns)}{Y_t} = (1 - \chi) [\lambda_t \rho_t(ns)]^{-\eta} \left(\frac{C_t}{1 - \mathcal{D}_t} \right)^{-\eta}$$

Those are the two demand constraints used in the main text.

C Aggregate price index

The aggregate price index must be such that

$$P_t C_t = P_t(s) C_t(s) + P_t(ns) C_t(ns)$$

substituting for the demand functions:

$$\begin{aligned} P_t C_t &= P_t(s) \chi (1 - \mathcal{D}_t)^\eta \left[\lambda_t \rho_t(s) + \lambda_{\tau,t} \frac{\mathcal{S}_t \mathbb{I}_t}{1 - \mathcal{D}_t} \pi_1 C_t(s) \right]^{-\eta} C_t^{1-\eta} + \\ &+ P_t(ns) (1 - \chi) (1 - \mathcal{D}_t)^\eta (\lambda_t \rho_t(ns))^{-\eta} C_t^{1-\eta} \end{aligned}$$

or

$$\begin{aligned} P_t &= \left(\frac{C_t}{1 - \mathcal{D}_t} \right)^{-\eta} \left\{ P_t(s) \chi \left[\lambda_t \rho_t(s) + \lambda_{\tau,t} \frac{\mathcal{S}_t \mathbb{I}_t}{1 - \mathcal{D}_t} \pi_1 C_t(s) \right]^{-\eta} + \right. \\ &\quad \left. + P_t(ns) (1 - \chi) (\lambda_t \rho_t(ns))^{-\eta} \right\} \end{aligned}$$

Notice that when $\mathbb{I}_t = 0$, $\mathcal{D}_t = 0$ and, thus, $\lambda_t = \frac{1}{C_t}$:

$$\begin{aligned} P_t &= \chi P_t(s)^{1-\eta} P_t^\eta + (1 - \chi) P_t(ns)^{1-\eta} P_t^\eta \\ P_t &= [\chi P_t(s)^{1-\eta} + (1 - \chi) P_t(ns)^{1-\eta}]^{\frac{1}{1-\eta}} \end{aligned}$$

which is the traditional price index under CES production function.

D Analytical Derivation of Aggregate Productivity in the Simplified Model

The first step in our derivation is to show that the aggregate price index can be written as $P_t = N_{o,t}^{1/(1-\theta)} \tilde{P}_t$, where $N_{o,t} \equiv N_{o,t}(s) + N_{o,t}(ns)$ and \tilde{P}_t is an average of producers' prices. When $\pi_1 = 0$ the price index equation reduces to:

$$\left(\frac{1 - \mathcal{D}_t}{\lambda_t C_t} \right)^{-\eta} = \left\{ \chi \left[[N_{o,t}(s)]^{\frac{1}{1-\theta}} \tilde{\rho}_t(s) \right]^{1-\eta} + [N_{o,t}(ns)]^{\frac{1-\eta}{1-\theta}} \tilde{\rho}_t(ns)^{1-\eta} (1 - \chi) \right\}$$

or, in nominal terms using that $\tilde{\rho}_t(q) = \tilde{p}_t(q)/P_t$:²⁵

$$P_t = \left\{ \chi [N_{o,t}(s)]^{\frac{1-\eta}{1-\theta}} \tilde{p}_t(s)^{1-\eta} + (1-\chi) [N_{o,t}(ns)]^{\frac{1-\eta}{1-\theta}} \tilde{p}_t(ns)^{1-\eta} \right\}^{\frac{1}{1-\eta}}$$

Using the definition of $N_{o,t}$ provided above we get:

$$P_t = N_{o,t}^{\frac{1}{1-\theta}} \left\{ \chi \omega_s^{\frac{1-\eta}{1-\theta}} \tilde{p}_t(s)^{1-\eta} + (1-\chi)(1-\omega_s)^{\frac{1-\eta}{1-\theta}} \tilde{p}_t(ns)^{1-\eta} \right\}^{\frac{1}{1-\eta}} = N_{o,t}^{\frac{1}{1-\theta}} \tilde{P}_t$$

where $\omega_s = N_{o,t}(s)/N_{o,t}$. Note that \tilde{P}_t is a form of weighted average of the average producers' prices in the two sectors $\tilde{p}_t(s)$ and $\tilde{p}_t(ns)$. Given the price index, we aggregate the production function to obtain a notion of aggregate labor productivity. From the sectoral demand constraint and the definition of sectoral production we get:

$$Y_t(s) = \chi \rho_t(s)^{-\eta} Y_t = N_{o,t}(s)^{\frac{1}{\theta-1}} Z_t \tilde{z}_t(s) L_t(s)$$

and similar results for the non-social sector. Solving for $L_t(q)$ we get:

$$\chi \rho_t(s)^{-\eta} Y_t N_{o,t}(s)^{\frac{1}{1-\theta}} \frac{1}{Z_t \tilde{z}_t(s)} = L_t(s)$$

and

$$(1-\chi) \rho_t(ns)^{-\eta} Y_t N_{o,t}(ns)^{\frac{1}{1-\theta}} \frac{1}{Z_t \tilde{z}_t(ns)} = L_t(ns)$$

Summing side by side:

$$\frac{Y_t}{Z_t} \left(\chi \rho_t(s)^{-\eta} N_{o,t}(s)^{\frac{1}{1-\theta}} \frac{1}{\tilde{z}_t(s)} + (1-\chi) \rho_t(ns)^{-\eta} N_{o,t}(ns)^{\frac{1}{1-\theta}} \frac{1}{\tilde{z}_t(ns)} \right) = L_t^d$$

Using again the definition of $N_{o,t}$ and of ω_s this becomes:

$$\frac{Y_t}{Z_t} N_{o,t}^{\frac{1}{1-\theta}} \left(\chi \omega_s^{\frac{1}{1-\theta}} \rho_t(s)^{-\eta} \frac{1}{\tilde{z}_t(s)} + (1-\chi)(1-\omega_s)^{\frac{1}{1-\theta}} \rho_t(ns)^{-\eta} \frac{1}{\tilde{z}_t(ns)} \right) = L_t^d$$

Solving for Y_t :

$$Y_t = N_{o,t}^{\frac{1}{\theta-1}} Z_t \left(\chi \omega_s^{\frac{1}{1-\theta}} \rho_t(s)^{-\eta} \frac{1}{\tilde{z}_t(s)} + (1-\chi)(1-\omega_s)^{\frac{1}{1-\theta}} \rho_t(ns)^{-\eta} \frac{1}{\tilde{z}_t(ns)} \right)^{-1} L_t^d =$$

²⁵Provided that $(1 - \mathcal{D}_t)/(\lambda_t C_t)$ is equal to 1. This can be proven under mild assumptions or it can be obtained by re-scaling the Lagrange multiplier of the household. Thus, in the following we omit this term from the derivations.

$$= N_{o,t}^{\frac{1}{\theta-1}} Z_t \tilde{Z}_t L_t^d$$

The aggregate labor productivity \tilde{Z}_t is a weighted harmonic average of the average sectoral productivities $\tilde{z}_t(s)$ and $\tilde{z}_t(ns)$. The statistics we compute and we use for the comparison with the aggregate labor productivity in the data is:

$$\frac{\left(\frac{Y_t P_t}{\tilde{P}_t}\right)}{L_t^d} = Z_t \tilde{Z}_t$$

The reason is the following: in the data, real variables in units of consumption are obtained by deflating the nominal quantities with price deflators as the CPI. However, these deflators, by being based on averages of producers' prices over a semi-fixed bundle of goods, are conceptually more similar to the average producer price \tilde{P}_t than to the consumer welfare-based price index P_t .²⁶ For this reason, *real* variables in the data do not correspond to $P_t X_t / P_t = X_t$ in the model but to $P_t X_t / \tilde{P}_t$, and these are the statistics we use. Note that this allows us to *correct* for the presence of love for variety in the model and directly use \tilde{Z}_t and $\tilde{z}_t(q)$ as measures of aggregate and sectoral productivity, respectively.

E Aggregate and Sectoral Inflation

This section presents the results of our simulation regarding inflation and its sectoral counterparts. Note that our goal is not to present a quantitative match for the observed empirical series for at least two reasons.

First, our model lacks the required depth to discuss inflation in a meaningful way: we model a quite standard Taylor rule, without sticky prices. Our main goal is to capture real adjustments of the economy, and our modelling assumptions follow this rationale.

Second, even with more a careful modelling of inflation and nominal frictions, the empirical measures of inflation are difficult to interpret: when there is no market for several goods due to the lockdown(s), and the weights used for the computation of the CPI are constantly changing with a lag, the reliability of the measure drops, in particular at the sectoral level.

Having said, in terms of timing and qualitative matching, we are still able to capture the up-down-up dynamics that characterized the inflation in the data. Figure 14 plot the monthly annualized inflation from the model and the data. The data counterpart is the CPI from the BLS, and variations are presented in percentage points from the baseline of January 2020.

²⁶For a deeper discussion on the topic, see [Ghironi and Melitz \(2005\)](#) and [Bilbiie et al. \(2012\)](#).

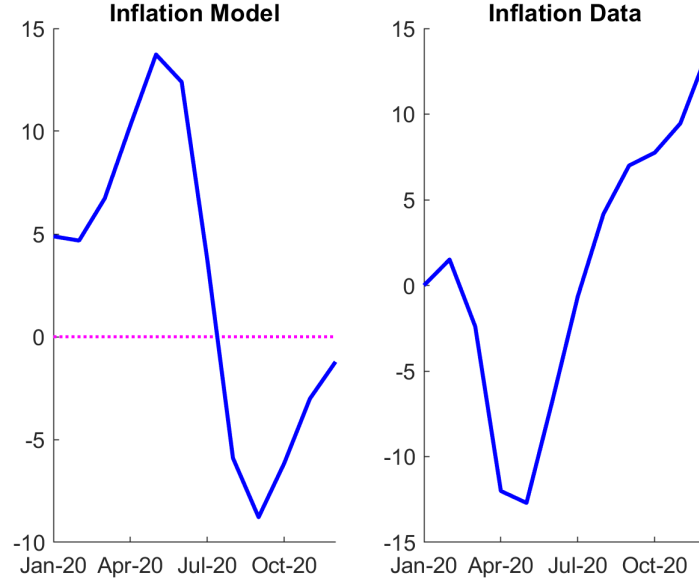


Figure 14: Inflation, Benchmark Model vs. Data

Overall, both the model and the data present an increase at the beginning of the pandemic, followed by a drop around the summer of 2020, to conclude with a further increase by the end of the year. The initial increase is much larger in the model, while the negative peak and the following rebound are much smaller. Moreover, the negative peak occurs earlier in the data than in the model.

Figure 15 plots the sectoral inflation dynamics from the social and non-social sectors in the model and compare them to the data. Since it is challenging to find a suitable empirical counterpart, we decided to simply look at several sectoral CPI, using different levels of granularity. Clearly, the magnitude varies significantly, and it is difficult to detect a common behavior. However, there are regularities across sectors that are broadly coherent with our model.

In the data, almost all social sectors present an increase in the CPI on impact/for the first months, consistently with the model, followed by a decrease around mid-2020. In some sectors, as transport and recreation, the drop is enough to change the sign of the response, as in our model, while for other sectors the drop is significantly milder. The opposite is true for the non-social sectors: the inflation of the non-social sector drops in the first half of the year both in the data and the model, to then recover, and turn positive, in the second half. Although the timing and magnitude strongly oscillates across sectors' type, the asymmetric pattern that can be recovered in the data is mirrored by the model.

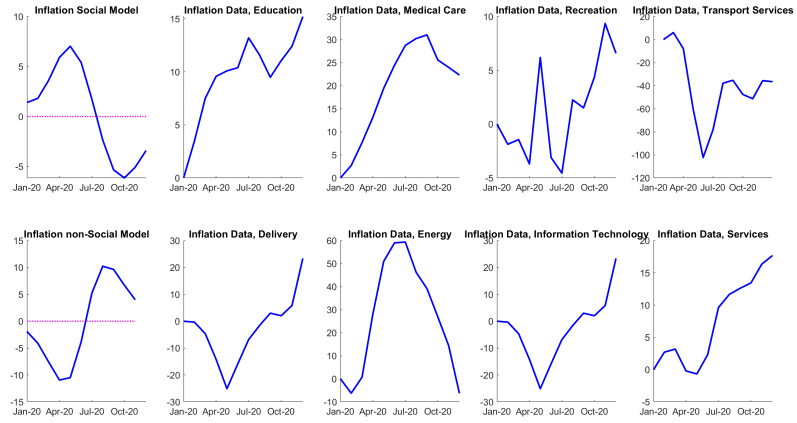


Figure 15: Sectoral Inflation, Benchmark Model vs. Data