Intermediate Macroeconomics
TA II

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Plan for Today

- Derive the general Solow growth model with and without human capital
- Derive a model with efficiency wages

Solow Growth Model - Framework

Last TA we assumed this Cobb-Douglas:

$$Y_t = B(K_t)^{\alpha}(L_t)^{1-\alpha} \quad \textit{with} \quad B > 0 \quad \textit{and} \quad 0 < \alpha < 1$$

This week one small twist: B is not a constant but it evolves over time.

$$Y_t = B_t(K_t)^{\alpha}(L_t)^{1-\alpha} \quad \textit{with} \quad 0 < \alpha < 1$$

The production function is rewritten as a labor augmenting technology:

$$Y_t = B_t(K_t)^{\alpha}(L_t)^{1-\alpha} = (K_t)^{\alpha}(B_t^{1/(1-\alpha)}L_t)^{1-\alpha} = (K_t)^{\alpha}(A_t L_t)^{1-\alpha}$$

with $0 < \alpha < 1$ and $A_t = B_t^{1/(1-\alpha)}$. The behaviour of the technology is exogenous:

$$A_{t+1} = (1+g)A_t \quad \textit{with} \quad g > -1$$

The remaining parts of the model are kept as in the previous week.

Solow Growth Model - Framework (II)

Define output per worker as $y_t = \frac{Y_t}{L_t}$ and capital per worker as $k_t = \frac{K_t}{L_t}$.

$$y_t = \frac{Y_t}{L_t} = \frac{1}{L_t}(K_t)^\alpha (A_t L_t)^{1-\alpha} = \left(\frac{K_t}{L_t}\right)^\alpha \left(A_t \frac{L_t}{L_t}\right)^{1-\alpha} = k_t^\alpha A_t^{1-\alpha}$$

Taking the log on both sides for period t and $t-1$:

$$\ln(y_t) = \ln(k_t^\alpha A_t^{1-\alpha}) = \ln(k_t^\alpha) + \ln(A_t^{1-\alpha}) = \alpha \ln(k_t) + (1-\alpha)\ln(A_t)$$

and

$$\ln(y_{t-1}) = \alpha \ln(k_{t-1}) + (1-\alpha)\ln(A_{t-1})$$

Taking the difference:

$$\ln(y_t) - \ln(y_{t-1}) = \alpha(\ln(k_t) - \ln(k_{t-1})) + (1-\alpha)(\ln(A_t) - \ln(A_{t-1}))$$

Solow Growth Model - Framework (III)

First difference in log terms is a proxy for the growth rate. Indeed:

$$\ln(y_t) - \ln(y_{t-1}) = \ln\left(\frac{y_t}{y_{t-1}}\right) = \ln\left(\frac{(1+g_t^y)y_{t-1}}{y_{t-1}}\right) = \ln(1+g_t^y) \approx g_t^y$$

Hence:

$$g_t^y \approx \alpha g_t^k + (1-\alpha)g$$

Solow Growth Model - Framework (IV)

The Complete Model

- $Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$
- $r_t = \alpha B_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} = \alpha A_t^{1-\alpha} \left(\frac{K_t}{L_t}\right)^{\alpha-1} = \alpha \left(\frac{K_t}{A_t L_t}\right)^{\alpha-1}$
- $w_t = (1-\alpha) B_t \left(\frac{K_t}{L_t}\right)^{\alpha} = (1-\alpha) A_t^{1-\alpha} \left(\frac{K_t}{L_t}\right)^{\alpha} = (1-\alpha) A_t \left(\frac{K_t}{A_t L_t}\right)^{\alpha}$
- $S_t = sY_t$
- $K_{t+1} = (1-\delta)K_t + S_t$
- $L_{t+1} = (1+n)L_t$
- $A_{t+1} = (1+g)A_t$

Given L_0, K_0, A_0, α, s, δ, n and g

Solow Growth Model - Dynamics and Steady State

In the last TA, we used per-worker terms to compute the steady state in which c^*, y^* and k^* are constant.

On the other hand, the aggregate C_t^*, Y_t^* and K_t^* are growing at a rate n in the steady state as we can see, for instance, from the definition $y^* = \frac{Y_t^*}{L_t^*}$ since L_t^* is growing at a rate n.

In the general Solow model, it is handy to define the model in terms of effective worker: $\tilde{y}_t = Y_t/\tilde{L}_t = Y_t/(A_t L_t)$ and $\tilde{k}_t = K_t/\tilde{L}_t = K_t/(A_t L_t)$.

$$\tilde{y}_t = \frac{Y_t}{A_t L_t} = \frac{1}{A_t L_t} K_t^{\alpha} (A_t L_t)^{1-\alpha} = \left(\frac{K_t}{A_t L_t}\right)^{\alpha} \left(\frac{A_t L_t}{A_t L_t}\right)^{1-\alpha} = \tilde{k}_t^{\alpha}$$

Solow Growth Model - Dynamics and Steady State (II)

Divide the intertemporal budget constraint by $A_{t+1}L_{t+1} = A_{t+1}(1+n)L_t = (1+g)(1+n)A_t L_t$:

$$\frac{K_{t+1}}{A_{t+1}L_{t+1}} = s\frac{Y_t}{(1+g)(1+n)A_t L_t} + (1-\delta)\frac{K_t}{(1+g)(1+n)A_t L_t}$$

which gives the transition equation:

$$\tilde{k}_{t+1} = \frac{1}{(1+g)(1+n)}(s\tilde{k}_t^{\alpha} + (1-\delta)\tilde{k}_t)$$

Subtracting $\tilde{k}_t$ from both sides we can obtain the Solow equation:

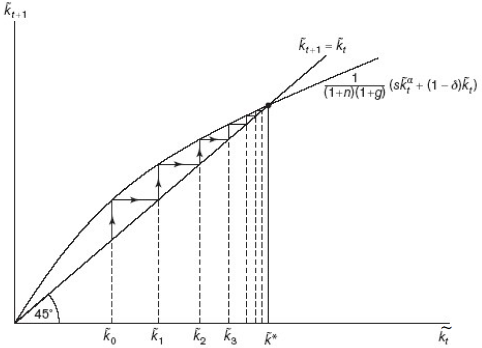
$$\tilde{k}_{t+1} - \tilde{k}_t = \frac{1}{(1+g)(1+n)}(s\tilde{k}_t^{\alpha} - (n+g+ng+\delta)\tilde{k}_t)$$

Solow Growth Model - Dynamics and Steady State (III)

Dynamics to the steady state:

$$\tilde{k}_{t+1} = \frac{1}{(1+n)(1+g)}\left(s\tilde{k}_t^{\alpha} + (1-\delta)\tilde{k}_t\right).$$

Transition diagram

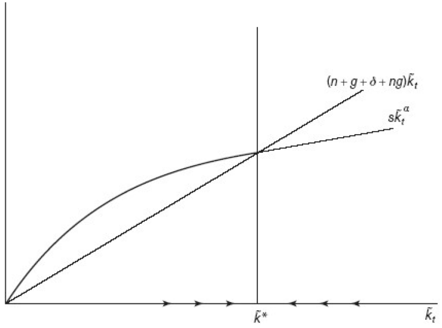


Solow Growth Model - Dynamics and Steady State (IV)

Dynamics to the steady state:

$$\tilde{k}_{t+1} - \tilde{k}_t = \frac{1}{(1+n)(1+g)}\left(s\tilde{k}_t^{\alpha} - (n+g+\delta+ng)\tilde{k}_t\right)$$

Solow diagram



Solow Growth Model - Dynamics and Steady State (V)

Steady State exists. We can solve for it by starting from the Solow equation imposing $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$:

- $\tilde{k}_{t+1} - \tilde{k}_t = \tilde{k}^* - \tilde{k}^* = 0 = \frac{1}{(1+g)(1+n)}(s(\tilde{k}^*)^\alpha - (n+g+gn+\delta)\tilde{k}^*)$

Hence:

- $s(\tilde{k}^*)^\alpha = (n+g+gn+\delta)\tilde{k}^* \rightarrow s(\tilde{k}^*)^{\alpha-1} = (n+g+gn+\delta) \rightarrow$
  $\tilde{k}^* = \left(\frac{n+g+gn+\delta}{s}\right)^{\frac{1}{\alpha-1}}$

Finally:

- $\tilde{k}^* = \left(\frac{s}{n+g+gn+\delta}\right)^{\frac{1}{1-\alpha}}$
- $\tilde{y}^* = (\tilde{k}^*)^\alpha = \left(\frac{s}{n+g+gn+\delta}\right)^{\frac{\alpha}{1-\alpha}}$
- $\tilde{c}^* = (1-s)\tilde{y}^* = (1-s)\left(\frac{s}{n+g+gn+\delta}\right)^{\frac{\alpha}{1-\alpha}}$

Solow Growth Model - Dynamics and Steady State (VI)

What about per worker steady state variables? Since $\tilde{k}_t = K_t / A_t L_t$

$$A_t \tilde{k}_t = A_t \frac{K_t}{A_t L_t} = \frac{K_t}{L_t} = k_t$$

- $k_t^* = A_t \tilde{k}^* = A_t \left(\frac{s}{n+g+gn+\delta} \right)^{\frac{1}{1-\alpha}}$
- $y_t^* = A_t \tilde{y}^* = A_t \left(\frac{s}{n+g+gn+\delta} \right)^{\frac{\alpha}{1-\alpha}}$
- $c_t^* = A_t \tilde{c}^* = A_t (1-s) \left(\frac{s}{n+g+gn+\delta} \right)^{\frac{\alpha}{1-\alpha}}$

Note that:

$$A_t = (1+g)A_{t-1} = (1+g)(1+g)A_{t-2} = (1+g)...(1+g)A_0 = (1+g)^t A_0$$

Hence, consumption, output and capital per worker grows at a rate g in the steady state.

Solow Growth Model - Dynamics and Steady State (VII)

Furthermore:

- $r_t^* = \alpha \left(\frac{K_t}{A_t L_t}\right)^{\alpha-1} = \alpha(\tilde{k}^*)^{\alpha-1} = \alpha \left(\frac{s}{n+g+gn+\delta}\right)^{\frac{\alpha-1}{1-\alpha}} = \alpha \left(\frac{s}{n+g+gn+\delta}\right)^{-1} = r^*$
- $w_t^* = (1-\alpha)A_t \left(\frac{K_t}{A_t L_t}\right)^{\alpha} = (1-\alpha)A_t(\tilde{k}^*)^{\alpha} = (1-\alpha)A_t \left(\frac{s}{n+g+gn+\delta}\right)^{\frac{\alpha}{1-\alpha}}$

Steady state wages are growing as well, while return on capital is constant.

This happens because workers are getting more productive over time, hence the higher wages (workers are paid at the marginal productivity of labor) and higher output (and capital and consumption).

Solow Growth Model Human Capital - Framework

Only investment in physical capital, what about human capital?

$$Y_t = K_t^{\alpha} H_t^{\phi} (A_t L_t)^{1-\alpha-\phi} \quad \textit{with} \quad 0 < \alpha < 1 \quad \textit{and} \quad 0 < \phi < 1$$

Every worker is endowed with human capital h_t and $H_t = h_t L_t$. Hence:

$$Y_t = K_t^{\alpha} H_t^{\phi} (A_t L_t)^{1-\alpha-\phi} = K_t^{\alpha} (h_t L_t)^{\phi} (A_t L_t)^{1-\alpha-\phi} = K_t^{\alpha} h_t^{\phi} A_t^{1-\alpha-\phi} L_t^{1-\alpha}$$

Production in per-worker terms is:

$$y_t = \frac{Y_t}{L_t} = \frac{1}{L_t} K_t^{\alpha} h_t^{\phi} A_t^{1-\alpha-\phi} L_t^{1-\alpha} = (\frac{K_t}{L_t})^{\alpha} h_t^{\phi} A_t^{1-\alpha-\phi} (\frac{L_t}{L_t})^{1-\alpha} = k_t^{\alpha} h_t^{\phi} A_t^{1-\alpha-\phi}$$

Solow Growth Model Human Capital - Framework (II)

Maximization of the firm is:

$$\max_{L_t, K_t} Y_t - w_t L_t - r_t K_t \quad s.t. \quad Y_t = K_t^{\alpha} H_t^{\phi} (A_t L_t)^{1-\alpha-\phi}$$

F.O.C.s of the firm are still:

- $w_t = F_L(K_t, L_t, H_t)$
- $r_t = F_K(K_t, L_t, H_t)$

Hence:

- $w_t = (1-\alpha) K_t^{\alpha} h_t^{\phi} A_t^{1-\alpha-\phi} L_t^{-\alpha} = (1-\alpha)\left(\frac{K_t}{A_t L_t}\right)^{\alpha}\left(\frac{h_t}{A_t}\right)^{\phi} A_t = (1-\alpha)\left(\frac{K_t}{A_t L_t}\right)^{\alpha}\left(\frac{H_t}{L_t A_t}\right)^{\phi} A_t$
- $r_t = \alpha K_t^{\alpha-1} h_t^{\phi} A_t^{1-\alpha-\phi} L_t^{1-\alpha} = \alpha\left(\frac{K_t}{A_t L_t}\right)^{\alpha-1}\left(\frac{h_t}{A_t}\right)^{\phi} = \alpha\left(\frac{K_t}{A_t L_t}\right)^{\alpha-1}\left(\frac{H_t}{L_t A_t}\right)^{\phi}$

Solow Growth Model Human Capital - Framework (III)

Savings are still $S_t = Y_t - C_t$ but investment is split between investment in physical and in human capital, with same depreciation:

$$I_t^K + I_t^H = S_t$$

and

- $K_{t+1} = (1-\delta)K_t + I_t^K$
- $H_{t+1} = (1-\delta)H_t + I_t^H$

Constant fractions of income saved for human and physical capital:

- $I_t^K = s^k Y_t$
- $I_t^H = s^H Y_t$

Hence

- $S_t = (s^k + s^H)Y_t$
- $C_t = (1 - s^k - s^H)Y_t$

The Complete Model

- $Y_t = K_t^{\alpha} H_t^{\phi} (A_t L_t)^{1-\alpha-\phi}$
- $r_t = \alpha \left(\frac{K_t}{A_t L_t}\right)^{\alpha-1} \left(\frac{H_t}{L_t A_t}\right)^{\phi}$
- $w_t = (1-\alpha) \left(\frac{K_t}{A_t L_t}\right)^{\alpha} \left(\frac{H_t}{L_t A_t}\right)^{\phi} A_t$
- $K_{t+1} = (1-\delta) K_t + s^K Y_t$
- $H_{t+1} = (1-\delta) H_t + s^H Y_t$
- $L_{t+1} = (1+n) L_t$
- $A_{t+1} = (1+g) A_t$

Given L_0, K_0, A_0, H_0, α, ϕ, s^K, s^H, δ, n and g

Solow Growth Model Human Capital - Steady State

As for the general Solow model, let's define the model in terms of effective workers: $\tilde{y}_t = Y_t/\tilde{L}_t = Y_t/(A_tL_t)$, $\tilde{k}_t = K_t/\tilde{L}_t = K_t/(A_tL_t)$ and $\tilde{h}_t = H_t/\tilde{L}_t = H_t/(A_tL_t)$

$$\tilde{y}_t = \frac{Y_t}{A_tL_t} = \frac{1}{A_tL_t}K_t^{\alpha}H_t^{\phi}(A_tL_t)^{1-\alpha-\phi} =$$

$$= \left(\frac{K_t}{A_tL_t}\right)^{\alpha}\left(\frac{H_t}{A_tL_t}\right)^{\phi}\left(\frac{A_tL_t}{A_tL_t}\right)^{1-\alpha-\phi} = \tilde{k}_t^{\alpha}\tilde{h}_t^{\phi}$$

Solow Growth Model Human Capital - Steady State (II)

Now we have two laws of motion. Dividing the law of motion for physical capital by $A_{t+1}L_{t+1} = A_{t+1}(1+n)L_t = (1+g)(1+n)A_tL_t$:

$$\frac{K_{t+1}}{A_{t+1}L_{t+1}} = s^K \frac{Y_t}{(1+g)(1+n)A_tL_t} + (1-\delta)\frac{K_t}{(1+g)(1+n)A_tL_t}$$

which gives the transition equation:

$$\tilde{k}_{t+1} = \frac{1}{(1+g)(1+n)}(s^K \tilde{k}_t^{\alpha} \tilde{h}_t^{\phi} + (1-\delta)\tilde{k}_t)$$

We can do the same for the human capital to obtain the transition equation for $\tilde{h}_t$:

$$\tilde{h}_{t+1} = \frac{1}{(1+g)(1+n)}(s^H \tilde{k}_t^{\alpha} \tilde{h}_t^{\phi} + (1-\delta)\tilde{h}_t)$$

Solow Growth Model Human Capital - Steady State (III)

With the two transitions equations above we can obtain two Solow equations:

$$\tilde{k}_{t+1} - \tilde{k}_t = \frac{1}{(1+g)(1+n)}(s^K \tilde{k}_t^{\alpha} \tilde{h}_t^{\phi} - (n+g+ng+\delta)\tilde{k}_t)$$

and

$$\tilde{h}_{t+1} - \tilde{h}_t = \frac{1}{(1+g)(1+n)}(s^H \tilde{k}_t^{\alpha} \tilde{h}_t^{\phi} - (n+g+ng+\delta)\tilde{h}_t)$$

Solow Growth Model Human Capital - Steady State (IV)

We impose $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$ and $\tilde{h}_{t+1} = \tilde{h}_t = \tilde{h}^*$ to find the steady state values:

$$0 = \frac{1}{(1+g)(1+n)}(s^K(\tilde{k}^*)^\alpha(\tilde{h}^*)^\phi - (n+g+ng+\delta)\tilde{k}^*)$$

and

$$0 = \frac{1}{(1+g)(1+n)}(s^H(\tilde{k}^*)^\alpha(\tilde{h}^*)^\phi - (n+g+ng+\delta)\tilde{h}^*)$$

Hence:
$s^K(\tilde{k}^*)^{\alpha-1}(\tilde{h}^*)^\phi = (n+g+ng+\delta) \rightarrow (\tilde{h}^*)^\phi = \frac{(n+g+ng+\delta)}{s^K(\tilde{k}^*)^{\alpha-1}} \rightarrow$
$\tilde{h}^* = \left(\frac{(n+g+ng+\delta)}{s^K(\tilde{k}^*)^{\alpha-1}}\right)^{\frac{1}{\phi}}$

Substitute this quantity in the second equation:

We had:

$$\tilde{h}^* = \left(\frac{(n+g+ng+\delta)}{s^K(\tilde{k}^*)^{\alpha-1}} \right)^{\frac{1}{\phi}}$$

Substitute this quantity in the second equation:

$$0 = \frac{1}{(1+g)(1+n)} (s^H (\tilde{k}^*)^{\alpha} (\tilde{h}^*)^{\phi} - (n+g+ng+\delta)\tilde{h}^*)$$

Thus:

$$s^H (\tilde{k}^*)^{\alpha} \left(\frac{(n+g+ng+\delta)}{s^K(\tilde{k}^*)^{\alpha-1}} \right) = (n+g+ng+\delta) \left(\frac{(n+g+ng+\delta)}{s^K(\tilde{k}^*)^{\alpha-1}} \right)^{\frac{1}{\phi}}$$

Simplifying:

$$s^H \tilde{k}^* \left(\frac{1}{s^K} \right) = \left(\frac{(n+g+ng+\delta)}{s^K(\tilde{k}^*)^{\alpha-1}} \right)^{\frac{1}{\phi}} \rightarrow (\tilde{k}^*)^{\phi} (\tilde{k}^*)^{\alpha-1} = \left(\frac{(n+g+ng+\delta)}{s^K (s^H)^{\phi} (s^K)^{-\phi}} \right)$$

Finally:

$$\tilde{k}^* = \left(\frac{(n+g+ng+\delta)}{(s^H)^{\phi}(s^K)^{1-\phi}}\right)^{1/(\phi+\alpha-1)} \rightarrow \tilde{k}^* = \left(\frac{(s^H)^{\phi}(s^K)^{1-\phi}}{(n+g+ng+\delta)}\right)^{\frac{1}{1-\phi-\alpha}}$$

Plug $\tilde{k}^*$ back to obtain steady state human capital per effective worker:

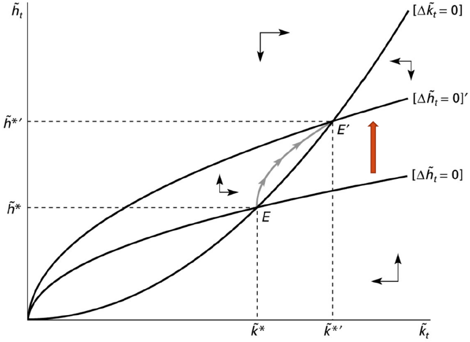
$$\tilde{h}^* = \left(\frac{(n+g+ng+\delta)}{s^K(\tilde{k}^*)^{\alpha-1}}\right)^{\frac{1}{\phi}} = \left(\frac{(n+g+ng+\delta)}{s^K\left(\frac{(s^H)^{\phi}(s^K)^{1-\phi}}{(n+g+ng+\delta)}\right)^{\frac{\alpha-1}{1-\phi-\alpha}}}\right)^{\frac{1}{\phi}} =$$

$$\left(\frac{(s^K)^{-1}\left((s^H)^{\phi}(s^K)^{1-\phi}\right)^{\frac{1-\alpha}{1-\phi-\alpha}}}{(n+g+ng+\delta)^{-1}(n+g+ng+\delta)^{\frac{1-\alpha}{1-\phi-\alpha}}}\right)^{\frac{1}{\phi}} =$$

$$\left(\frac{(s^H)^{\frac{\phi(1-\alpha)}{1-\phi-\alpha}}(s^K)^{\frac{\phi+\alpha-1}{1-\phi-\alpha}+\frac{(1-\phi)(1-\alpha)}{1-\phi-\alpha}}}{(n+g+ng+\delta)^{\frac{\phi+\alpha-1}{1-\phi-\alpha}+\frac{1-\alpha}{1-\phi-\alpha}}}\right)^{\frac{1}{\phi}} = \left(\frac{(s^H)^{1-\alpha}(s^K)^{\alpha}}{(n+g+ng+\delta)}\right)^{\frac{1}{1-\phi-\alpha}}$$

Comparative statics:

An increase in s_H



Transition dynamics

$s_H \uparrow \longrightarrow \widetilde{h^*} \uparrow$

$$\tilde{y}_t = \tilde{k}_t^{\alpha} \tilde{h}_t^{\varphi}.$$

$\tilde{y} \uparrow$

$$K_{t+1} - K_t = s_k Y_t - \delta K_t$$

$\tilde{k} \uparrow \longrightarrow \tilde{y}_t = \tilde{k}_t^{\alpha} \tilde{h}_t^{\varphi}. \uparrow$

Model with efficiency wages - Framework

We have n firms producing differentiated products under monopolistic competition. Demand function for product i is:

$$D(P_i) = Y_i = \left(\frac{P_i}{P}\right)^{-\sigma} \frac{Y}{n} \quad with \quad \sigma > 1$$

where Y is aggregate demand, P_i is the price of good i, P is the price index and σ is the elasticity of substitution between goods.
Same production function:

$$Y_i = a_i L_i$$

where

$$a_i = a\left(\frac{W_i}{P}\right) = (w_i - v)^{\eta} \quad with \quad 0 < \eta < 1$$

and W_i is the wage paid by firm i and v the outside option of the worker.

Model with efficiency wages - Framework (II)

Maximization of firm i is:

$$\max_{p_i, w_i} \frac{P_i Y_i}{P} - \frac{W_i L_i}{P} = Y_i p_i - w_i \frac{Y_i}{a_i} = Y_i \left(p_i - \frac{w_i}{(w_i - v)^\eta} \right) =$$

$$= p_i^{-\sigma} \frac{Y}{n} \left(p_i - \frac{w_i}{(w_i - v)^\eta} \right)$$

F.O.C. with respect to real price p_i

$$-\sigma p_i^{-\sigma-1} \frac{Y}{n} \left(p_i - \frac{w_i}{(w_i - v)^\eta} \right) + p_i^{-\sigma} \frac{Y}{n} = 0$$

Simplifying:

$\sigma \left(1 - \frac{w_i}{p_i (w_i - v)^\eta} \right) = 1 \rightarrow \sigma - 1 = \frac{\sigma w_i}{p_i (w_i - v)^\eta} \rightarrow p_i = \frac{\sigma}{\sigma - 1} \frac{w_i}{(w_i - v)^\eta}$

Model with efficiency wages - Framework (III)

F.O.C. with respect to w_i

$$-p_i^{-\sigma}\frac{Y}{n}\left(\frac{(w_i-v)^{\eta}-w_i\eta(w_i-v)^{\eta-1}}{(w_i-v)^{2\eta}}\right)=0$$

Simplifying:
$\frac{1-w_i\eta(w_i-v)^{-1}}{(w_i-v)^{\eta}}=0\rightarrow 1-w_i\eta(w_i-v)^{-1}=0\rightarrow(w_i-v)=w_i\eta$

which gives:

$$w_i=\frac{v}{1-\eta}$$

Model with efficiency wages - Framework (IV)

Given the market structure, and that, by symmetry, $w = w_i$, we can define the outside option as:

$$v = ub + (1-u)w$$

Hence:

$$w_i = w = \frac{v}{1-\eta} = \frac{ub + (1-u)w}{1-\eta}$$

Solving for w:

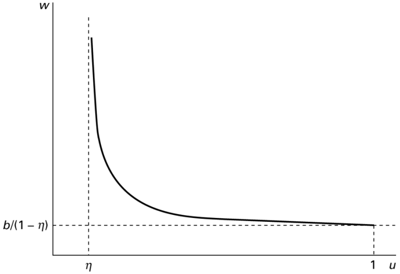
$$w\left(1 - \frac{1-u}{1-\eta}\right) = w\frac{u-\eta}{1-\eta} = \frac{ub}{1-\eta} \rightarrow w = \frac{u}{u-\eta}b \rightarrow w = \frac{1}{1-\frac{\eta}{u}}b \text{ with } u > \eta$$

Model with efficiency wages - Framework (V)

Equilibrium wage:

$$w = \frac{1}{1 - \frac{\eta}{u}} b$$

Wage is increasing in b and in η, decreasing in u



Model with efficiency wages - Framework (VI)

Due to symmetry, all the firms set the same price P_i. Moreover, $P = P_i$ since here the price index is the average of the prices charged by the n firms. The real price is $p_i = P_i/P = 1$. Hence:

$$p_i = 1 = \frac{\sigma}{\sigma - 1}\frac{w}{(w-v)^\eta} = m\frac{w}{(w-v)^\eta}$$

Thus:

$$mw = (w-v)^\eta$$

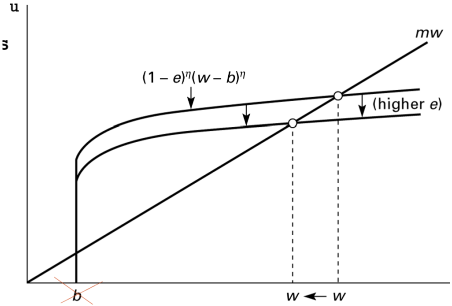
Plugging in the definition of v:

$$mw = (w - ub - (1-u)w)^\eta = (uw - ub)^\eta = [u(w-b)]^\eta$$

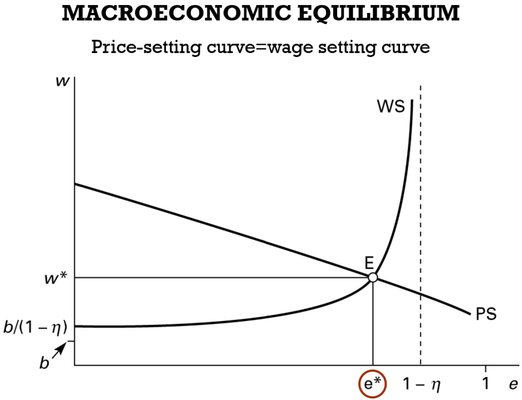
Model with efficiency wages - Framework (VII)

This gives the price equation:

$$mw = \left[u\left(w-b\right)\right]^{\eta} = \left(1-e\right)^{\eta}\left(w-b\right)^{\eta}$$



Putting the two curves together we find the equilibrium wage and employment rate:



Model with efficiency wages - Framework (IX)

In the model, the supply of labor is inelastic at L, employment is thus e^*L.
In every firm, the labor employed is e^*L/n

Individual productivity is
$a^* = (w^* - v)^\eta = (w^* - (1 - e^*)b - e^*w^*)^\eta = [(w^* - b)(1 - e^*)]^\eta$.
Hence, the individual production is a^*e^*L/n

Finally, total production is a^*e^*L