Goodbye, Hazen; Hello, Kozeny-Carman

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Goodbye, Hazen; Hello, Kozeny-Carman

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Abstract: The century-old Hazen formula for predicting the permeability of sand is based only on the D_{10} particle size. Whereas, the half-century-old Kozeny-Carman formula is based on the entire particle size distribution, the particle shape, and the void ratio. As a consequence, the Hazen formula is less accurate than the Kozeny-Carman formula. It is recommended that the former be retired and the latter be adopted.

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Hazen Formula

About a century ago, Hazen (1892, 1911) developed the following empirical formula for predicting the permeability (or hydraulic conductivity) of saturated sands:

$$k = C_H D_{10}^2 \tag{1}$$

where k=permeability (cm/s); C_H=Hazen empirical coefficient; and D_{10}=particle size for which 10% of the soil is finer (cm).

Although Hazen developed his formula for the design of sand filters for water purification [i.e., loose, clean sands with a coefficient of uniformity D_{60}/D_{10}, less than about 2 (Terzaghi and Peck 1964)], it is frequently used to estimate the permeability of in situ soil.

The value of C_H is usually assumed to be equal to 100, but the following ranges have been reported in geotechnical textbooks:

- 41 to 146: Taylor (1948, p. 112),
- 100 to 150: Leonards (1962, p. 119),
- 100 to 1,000: Mansur and Kaufman (1962, p. 260–261),
- 100 to 150: Terzaghi and Peck (1964, p. 44),
- 90 to 120: Cedergren (1967, p. 42),
- 1 to 42: Lambe and Whitman (1969, p. 290),
- 40 to 120: Holtz and Kovacs (1981, pp. 209–212),
- 50 to 200: Terzaghi et al. (1996, pp. 73–74),
- 100 to 150: Das (1997, p. 153), and
- 80 to 120: Coduto (1999, pp. 226–227).

Thus, the published value of C_H ranges from 1 to 1,000. The formula's applicability is generally limited to $0.01\ \text{cm} < D_{10} < 0.3\ \text{cm}$ (Hazen 1892, 1911; Holtz and Kovacs 1981; Coduto 1999).

Nearly all of the geotechnical textbooks cited herein reference the Hazen formula, with two notable exceptions: Mansur and Kaufman (1962) and Mitchell (1976, 1993). Mansur and Kaufman do not actually report the values of C_H shown above but instead plot D_{10} versus permeability (based on field tests in sands along the middle and lower Mississippi River). Their log-log plot (Fig. 314) shows a rough correlation between D_{10} and k; for $0.01\ \text{cm} < D_{10} < \sim 0.07\ \text{cm}$, the implied value of C_H varies from 100 to 1,000. Furthermore, for $D_{10} > \sim 0.04\ \text{cm}$, k is not proportional to D_{10}^2: at $D_{10} = 0.1\ \text{cm}$, $k \propto D_{10}^{0.7}$. Mitchell does not reference Hazen, and only discusses Kozeny-Carman.

None of the geotechnical textbooks referenced herein mention that the Hazen empirical coefficient corresponds to a water temperature of 10°C. Perhaps all of the writers have tacitly concluded that the Hazen formula is so inaccurate that to correct for temperature would be superfluous. Hazen himself used a compound coefficient equal to $C_H(0.70 + 0.03\,T)$, where T is the temperature in degrees celsius. In his own words, ''I have found that the friction [i.e., resistance to flow] also varies with the temperature, being twice as great at the freezing point as at summer heat ...'' (Hazen 1892, p. 553; 1911, p. 200). Thus, at 20°, the compound coefficient is equal to $1.3\,C_H$, and the permeability would be 30% greater than at 10°.

Kozeny-Carman Formula

About a half-century ago, Kozeny (1927) and Carman (1938, 1956) developed the following semiempirical, semitheoretical formula for predicting the permeability of porous media:

$$k = (\gamma/\mu)(1/C_{\text{K-C}})(1/S_0^2)[e^3/(1+e)] \tag{2}$$

where γ=unit weight of permeant; μ=viscosity of permeant; $C_{\text{K-C}}$=Kozeny-Carman empirical coefficient; S_0=specific surface area per unit volume of particles (1/cm); and e=void ratio.

When the permeant is water at 20°, $\gamma/\mu = 9.93 \times 10^4$ 1/cm s. [At 10°, $\gamma/\mu = 7.64 \times 10^4$ 1/cm s. Thus, the ratio of 20° to 10° is 1.3, the same as Hazen used in his compound coefficient. For the effect of temperature on the viscosity of water, see Lambe (1965, p. 148) or Terzaghi et al. (1996 p. 73).] Carman (1956, p. 14) reported the value of $C_{\text{K-C}}$ as being equal to 4.8 ± 0.3 for uniform spheres; $C_{\text{K-C}}$ is usually taken to be equal to 5. Thus, Eq. (2) becomes

$$k = 1.99 \times 10^4 (1/S_0^2)[e^3/(1+e)] \quad [\text{for } 20°\text{C}] \tag{3}$$

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The Kozeny-Carman formula appears in some geotechnical textbooks, such as Leonards, Lambe and Whitman, Mitchell, and Das; but not in many others: Cedergren, Peck et al. (1974), Holtz and Kovacs, Terzaghi et al., and Coduto. And yet, the Kozeny-Carman formula is known to be more accurate than the Hazen formula (e.g., Loudon 1952). The explanation for this paradox is at least twofold: First, most geotechnical engineers do not regularly measure specific surface area (SSA), and, thus, they are not used to it. [SSA is usually expressed in m^2/g; by also measuring the specific gravity of the soil particles, S_0 in Eqs. (2) and (3) can be simply calculated from the SSA.] Although soil scientists routinely measure SSA using nitrogen gas adsorption (e.g., Hillel 1980), there is presently no ASTM standard for soil (there are several standards for measuring the SSA of other materials).

Second, none of the geotechnical textbooks referenced herein explain that S_0 can be simply estimated from the particle size distribution. For example, if a soil consists of uniform spheres of diameter D (cm), $S_0 = \text{area/volume} = (\pi D^2)/[(\pi D^3/6)] = 6/D$. Thus, Eq. (3) becomes

$$k = 552 D^2 [e^3/(1+e)] \tag{4}$$

If the soil consists of nonuniform spheres, then the effective diameter D_{eff} can be calculated from the particle size distribution

$$D_{\text{eff}} = 100\% \bigg/ \left[\sum (f_{\text{i}}/D_{\text{ave i}})\right] \tag{5}$$

where f_{i}=fraction of particles between two sieve sizes; larger [l] and smaller [s] (%); and

$$D_{\text{ave i}} = \text{average particle size between two sieve sizes (cm)}$$
$$= D_{\text{li}}^{0.5} \times D_{\text{si}}^{0.5} \tag{6}$$

Then,

$$S_0 = 6/D_{\text{eff}} \tag{7}$$

Obviously, the smaller particles have the most influence on the calculated D_{eff} and S_0.

Finally, to account for the angularity of the individual soil particles, a shape factor SF can be introduced

$$S_0 = \text{SF}/D_{\text{eff}} \tag{8}$$

Collecting terms, Eq. (3) can be expressed as

$$k = 1.99 \times 10^4 \left(100\% \bigg/ \left\{ \sum [f_{\text{i}}/(D_{\text{li}}^{0.5} \times D_{\text{si}}^{0.5})] \right\} \right)^2 (1/\text{SF}^2) \times [e^3/(1+e)] \tag{9}$$

Eq. (9) is similar to versions of the Kozeny-Carman formula presented in Fair and Hatch (1933—note the date), Loudon, Carman (1956), and Todd (1959) [interestingly, Eq. (9) does not appear in Todd (1980)]. Fair and Hatch suggested the following values for the shape factor, SF: spherical—6.0; rounded—6.1; worn—6.4; sharp—7.4; and angular—7.7. Loudon suggested the following values: rounded—6.6; medium angularity—7.5; and angular—8.4.

Eq. (9) can be improved slightly by noting that although $D_{\text{ave i}} = D_{\text{li}}^{0.5} \times D_{\text{si}}^{0.5}$ [Eq. (6)], $S_{0\text{ i}}/\text{SF} = (1/D_{\text{i}})_{\text{ave}} \neq (1/D_{\text{ave i}})$. Assuming the particle size distribution is log-linear between each pair of sieve sizes, it can be shown that $(1/D_{\text{i}})_{\text{ave}} = 1/(D_{\text{li}}^{0.404} \times D_{\text{si}}^{0.595})$. Thus, Eq. (9) becomes

$$k = 1.99 \times 10^4 \left(100\% \bigg/ \left\{ \sum [f_{\text{i}}/(D_{\text{li}}^{0.404} \times D_{\text{si}}^{0.595})] \right\} \right)^2 (1/\text{SF}^2) \times [e^3/(1+e)] \tag{10}$$

Hazen versus Kozeny-Carman

As noted above, the published value of the Hazen coefficient C_H varies from 1 to 1,000; that is, three orders of magnitude. This alone should have disqualified the Hazen formula. Nonetheless, it has continued to be used because of its simplicity and ease of memorization.

In the past, calculating the effective particle size diameter D_{eff} required for the Kozeny-Carman formula was considered by some geotechnical engineers to be time consuming, and this led to attempts to improve the Hazen formula. For example, Amer and Awad (1974) developed a permeability formula in which $k \propto (D_{60}/D_{10})^{0.6} D_{10}^{2.32} [e^3/(1+e)] = D_{60}^{0.6} D_{10}^{1.72} [e^3/(1+e)]$. But the price of improved accuracy was less simplicity and less ease of memorization.

Today, personal computers and spreadsheet software are nearly ubiquitous, making the calculation of D_{eff} extremely simple and fast. For that matter, the Kozeny-Carman formula could be easily incorporated into software that calculates and plots sieve analyses, such that the estimated permeability of a sandy layer could be automatically output. There is no longer any reason not to use the Kozeny-Carman formula.

Limitations of Kozeny-Carman Formula

The Kozeny-Carman formula is constrained by certain limitations (which also apply to the Hazen formula):

1. Fine side: The formula assumes there are no electrochemical reactions between the soil particles and the water. That means the formula is not appropriate for clayey soils, although it will work for nonplastic silts. [For an example of an empirical formula to predict the permeability of a clay, see Carrier and Beckman (1984).]
2. Coarse side: The formula assumes Darcian conditions; that is, laminar flow and a low pore water velocity, such that the inertia term in the Bernoulli energy equation can be ignored. These conditions apply in silts, sands, and even gravelly sands. But as the pore size increases and the velocity increases, turbulent flow and the inertia term must be taken into account.

   Hazen (1892, p. 554) also recognized this: ‘‘For gravels with effective sizes above 3 mm the friction varies in such a way as to make the application of a general formula very difficult. As the size increases beyond this point, the [quantity of flow] with a given head does not increase as rapidly as the square of the effective size; and with coarse gravels the [quantity of flow] varies as the square root of the head instead of directly with the head as in sands. The influence of temperature also becomes less marked with the coarse gravels.’’

   Thus, in gravels and coarser soils, a more advanced formula must be used. As an example, Scheidegger (1974, pp. 165–166) and Åberg (1992) use an expression in which the apparent permeability is a function of the hydraulic gradient: The maximum value of k_{apparent} is equal to $k_{\text{Kozeny-Carman}}$, which occurs when the hydraulic gradient is zero; and as the hydraulic gradient increases, k_{apparent} decreases.

3. Extreme particle shape: The formula assumes the soil particles are relatively compact. It is not appropriate for soils containing platy particles such as mica (and this is another reason it does not work for clayey soils). Furthermore, if the *measured* specific surface area is significantly higher than the *calculated* specific surface area, then the latter should be used to predict permeability. This condition occurs when the particles are extremely irregular with re-entrant surfaces and intragranular porosity. This results in ''dead end'' and ''bypassed'' flow channels that do not contribute to the effective specific surface area. [For example, lunar soil particles look like microscopic pieces of popcorn. The measured specific surface area is nearly eight times the calculated area (Carrier et al. 1991, pp. 480–481).]

   In fact, for extreme particle shapes, in which it is desired to determine the specific surface area for an industrial process (such as a catalyst), Carman recommended that the Kozeny-Carman formula be used in reverse: Measure the permeability and back-calculate the effective specific surface area.
4. Extreme particle size distribution: The formula is not appropriate if the particle size distribution has a long, flat tail in the fine fraction. As a practical matter, D_0 must be known or estimated in order to calculate D_{eff} [see Eqs. (5) and (6)].
5. Anisotropy: The formula does not explicitly account for anisotropy. Of course, in most deposits (both natural and manmade), the horizontal permeability k_h is greater than the vertical permeability k_v. Nearly all of the laboratory measurements which have been made to validate the formula and to establish the value of $C_{\text{K-C}}$ were based on the vertical permeability. Thus, $k_{\text{Kozeny-Carman}} \approx k_v$.

Conclusions

The Hazen formula [Eq. (1)] has been around for about a century. It is time to bid it goodbye, with thanks, and welcome the Kozeny-Carman formula [Eq. (10)], which, after all, has been around for half-a-century.

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