ECE374 Fall2020

Homework3

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Q1

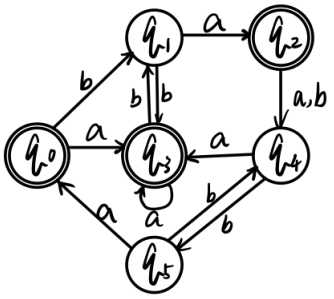
Ex1

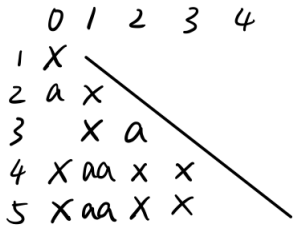
1) 1*(01*01*)*
2) 0*10*(10*10*)*
3) 1*(01*01*)* + 0*10*(10*10*)*
4) 0(00+11)*(01+10)((00+11)+(01+10)(00+11)*(01+10))* + 1((00+11)+(01+10)(00+11)*(01+10))*

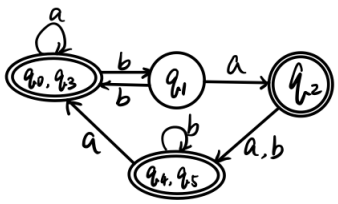
Ex2

5) ((a+b)(a+b)(a+b))*
6) (a+b)*+(a+c)*+(b+c)*
7) b*aa*bb*aa*b(bb*a*+aa*)

Q2







Q3

1. $L = \{a^p | p\ prime\ number\}$

proof:
fooling set F=a*
$\forall x = a^i, y = a^j \in F, i \neq j$, assume $i \leq j$
let the prime number $p \geq i$
$p + 0(j - i) = p$ is prime
$p + p(j - i) \geq p$ is not prime
$\therefore \exists integer\ k \in (0, p]\ s.t.$
$p + (k - 1)(j - i)$ is prime
$p + k(j - i)$ is not prime
let $z = a^{p+(k-1)j-ki}$
$xz = a^{p+(k-1)(j-i)} \in L$
$yz = a^{p+k(j-i)} \notin L$
$\therefore$ F is the fooling set of L
$\because$ F is infinite
$\therefore$ L is not regular

2. $L = \{a^{n^2} | n \geq 1\}$

proof:
fooling set $F = L = \{a^{n^2} | n \geq 1\}$
$\forall x = a^{i^2}, y = a^{j^2} \in F, i \neq j$, assume $i \leq j$
let $z = a^{2i+1}$
$xz = a^{i^2+2i+1} = a^{(i+1)^2} \in L$
$yz = a^{j^2+2i+1}$
the next square number after j^2 is $(j+1)^2 = j^2 + 2j + 1 > j^2 + 2i + 1 > j^2$
$\therefore j^2 + 2i + 1$ is not a square number
$\therefore yz \notin L$
$\therefore$ F is the fooling set of L
$\because$ F is infinite
$\therefore$ L is not regular