

Crypto and e-Voting: Homomorphisms, Zero-Knowledge Proofs

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Administrivia

Project 1: Going online tomorrow

Some coding, some written bits — get started early

Proper use of Piazza

Please don't post partial solutions (code, hash values, etc.) without marking those posts as "private" to the instructors.

Our last crypto lecture!

On your own: go read up on how RSA works

Modern cryptographers seem to have concluded that DH-style is the way to go

Instead, this lecture is a whirlwind through several modern crypto topics

Zero-knowledge proofs

Mixnets

Homomorphic cryptography

Application to electronic voting

All of these things are deep topics

Your mission: understand some of the clever things that crypto can do

Reminder: Diffie-Hellman & ElGamal Crypto

Modular arithmetic 101

We're working in \mathbb{Z}_p^* , the integers in [1, p)

$$2+3 = 5 \pmod{7}$$

 $2+4 = 6 \pmod{7}$
 $2+5 = 0 \pmod{7}$ Forbidden!
 $2*3 = 6 \pmod{7}$
 $2*4 = 1 \pmod{7}$
 $6*6 = 1 \pmod{7}$

Note: \mathbb{Z}_p^* is closed under multiplication but not addition.

Modular arithmetic 101

In \mathbb{Z}_p^* , we want to find *generators* such that

$$g^1, g^2, \dots, g^{p-1}$$

cover all the elements in the group.

Example, for p=7:

g=2 is not a generator, but g=3 is.

Discrete logarithms

Back to the regular integers, say I give you $q=5^{8437591243259543}$ and ask you to take $\log_5 q$

Logarithms, over integers, are tractable. But what about in \mathbb{Z}_p^* ?

No known efficient solution to DLog problem.

Diffie-Hellman (1976)

```
Alice: random a \in \mathbb{Z}_p^*
  Bob: random b \in \mathbb{Z}_p^*
Public : generator g \in \mathbb{Z}_p^*
A \rightarrow B: g^a
B \rightarrow A : g^b
 Alice: computes (g^b)^a = g^{ab}
   Bob: computes (g^a)^b = g^{ab}
   Eve: knows g^a, g^b, cannot compute g^{ab}
```

ElGamal encryption (1984)

Non-deterministic cryptosystem (different r every time)

$$E(g^{a}, r, M) = \langle g^{r}, (g^{a})^{r} M \rangle$$

$$D(g^{r}, g^{ar} M, a) = \frac{g^{ar} M}{(g^{r})^{a}}$$

$$= M$$

group generator

M plaintext (message)

random (chosen at encryption time)

(private) decryption key

ga (public) encryption key

Digital signature algorithm (1991)

Similar idea to ElGamal encryption, just a bit more involved

$$r = (g^k \bmod p) \bmod q$$

$$s = k^{-1} (H(M) + xr) \bmod q$$

$$Sign(x, k, M) = \langle r, s \rangle$$

$$Verify(y, r, s) :$$

$$w = s^{-1} \bmod q$$

$$u_1 = H(M) \cdot w \bmod q$$

$$u_2 = r \cdot w \bmod q$$

$$v = ((g^{u_1} y^{u_2}) \bmod p) \bmod q$$

$$valid if $v = r$$$

 $k = \text{random} \in [1, q)$

```
g group generator p large prime (1000+ bits) q large-ish prime (~160 bits) divisible by p-1 M plaintext (message) k random (chosen at encryption time) k (private) decryption key (chosen randomly k (public) encryption key (mod k)
```

Digital signature algorithm (1991)

Similar idea to ElGamal encryption, just a bit more involved

$$k = \operatorname{random} \in [1,q)$$

$$r = (g^k \operatorname{mod} p) \operatorname{mod} q$$

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$$w = s^{-1} \operatorname{mod} q$$

$$u_1 = H(M) \cdot w \operatorname{mod} q$$

$$r = (g^k \operatorname{mod} p) \operatorname{mod} q$$

$$r$$

Fun: if *k* isn't random and unique, attacker can recover *x*. (PS3 attack, Dec. 2010). Solution: derive *k* from *x* and *H*(*M*) deterministically. [RFC 6979]

Homomorphic property

Anybody can combine two ciphertexts to get a new one.

$$E(M_{1}) \oplus E(M_{2}) = \langle g^{r_{1}}, (g^{a})^{r_{1}}M_{1} \rangle \oplus \langle g^{r_{2}}, (g^{a})^{r_{2}}M_{2} \rangle$$

$$= \langle g^{r_{1}}g^{r_{2}}, (g^{a})^{r_{1}}M_{1}(g^{a})^{r_{2}}M_{2} \rangle$$

$$= g^{r_{1}+r_{2}}, g^{a(r_{1}+r_{2})}M_{1}M_{2}$$

$$= E(M_{1}M_{2})$$

```
egin{array}{ll} g & group generator \ M & plaintext (message) \ r & random (chosen at encryption time) \ a & (private) decryption key \ g^a & (public) encryption key \end{array}
```

Homomorphic vote tallying

Change messages to counters, additive in exponent of g. "Exponential ElGamal"

$$E(v_1) \oplus E(v_2) = \langle g^{r_1}, (g^a)^{r_1} g^{v_1} \rangle \oplus \langle g^{r_2}, (g^a)^{r_2} g^{v_2} \rangle$$

$$= \langle g^{r_1+r_2}, g^{a(r_1+r_2)} g^{v_1+v_2} \rangle$$

$$= E(v_1+v_2)$$

```
g group generator
```

V plaintext (counters)

r random (chosen at encryption time)

a (private) decryption key

ga (public) encryption key

Violation of encryption semantics?

If I know M_1 and M_2 and $E(M_1) \oplus E(M_2) = E(M_1M_2)$ then I can find other messages where I know their encryption!

Solution: Padding

Optimal Asymmetric Encryption Padding (OAEP) -

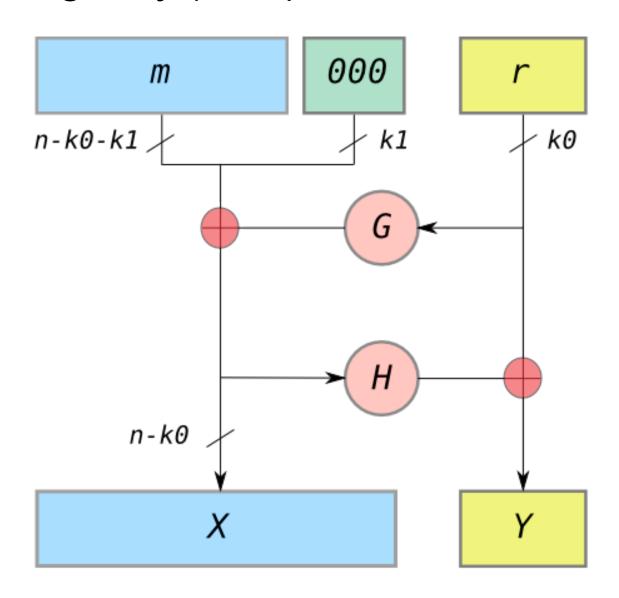
Belare and Rogaway (1995)

m - message (plaintext)

r - random number

G, H - cryptographic hash functions

X, Y - the message that gets encrypted



Cool trick: reencryption

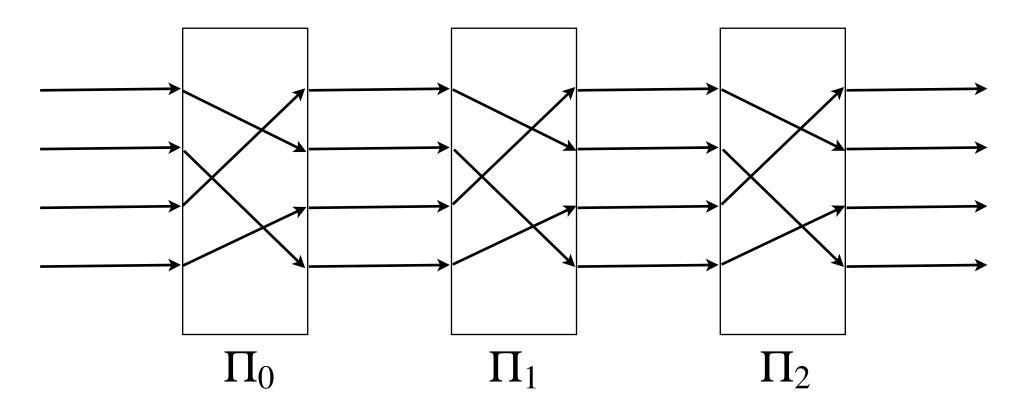
$$E(M) \oplus E(0) = E(M)^*$$

Anybody can "reencrypt" a message.

(New random number introduced from E(0).)

Reencryption mixnets

Permutations Π_i , where output is reencrypted.

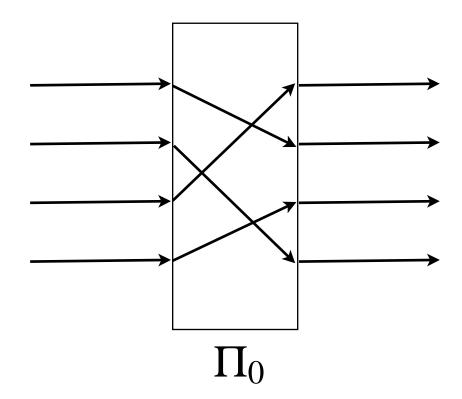


Each mix permutes/reencrypts.

Must prove output corresponds to input.

Non-solution: reveal the mix

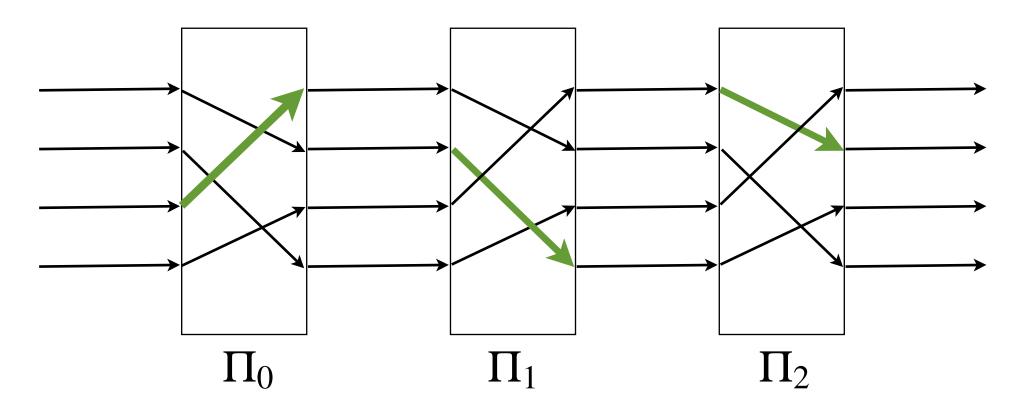
Publish the random numbers and the permutation.



Eliminates benefit of randomization.

Randomized partial checking

Effective across larger mixes. (Jakobsson, Jules, Rivest '02)



Say we're mixing 1 million ballots, each mix reveals 1%. After five mixes, 99.99% chance that all ballots reencrypted at least once.

Zero-knowledge proofs (ZKP)

want to prove you know something

while revealing nothing

generalized format

prover: commit to something (e.g., reencryption mix output)

verifier: challenge the prover

prover: respond to the challenge

Example: Hamiltonian paths

Prover: "I know a HP over graph G." Compute graph isomorphism H. Publish G, H.

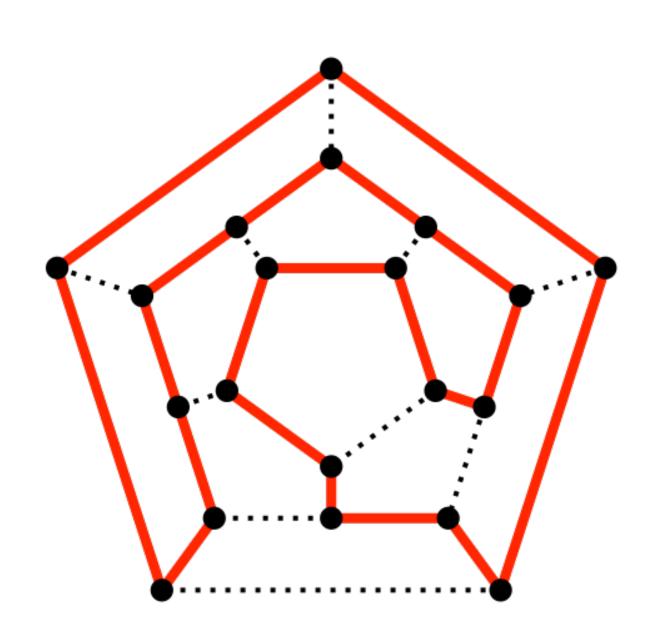
Verifier: Coin toss.

Heads: tell HP over H.

Tails: tell isomorphism G to H.

(Repeat N times.)

If prover doesn't know HP, verifier catches with high probability.

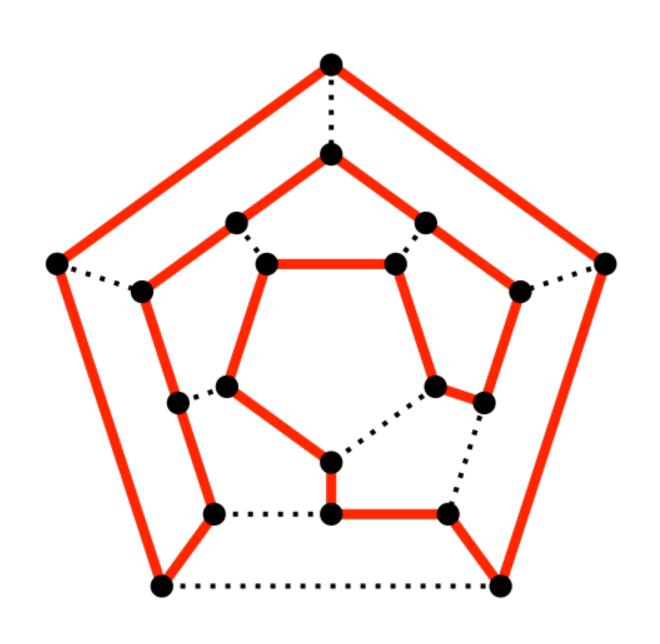


Non-interactive ZK proofs

Prover: Precompute N isomorphisms (H_1 to H_N) and hash them. Hash function yields coin tosses for virtual challenger. Then output the results.

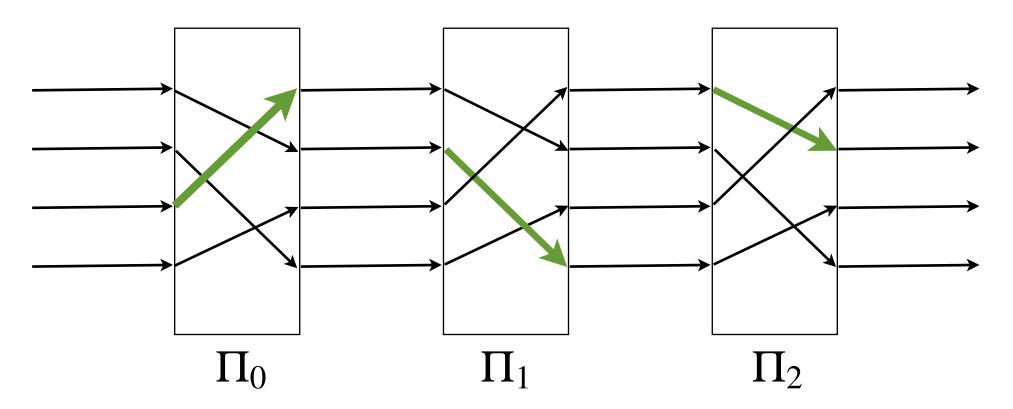
(Assumes good hash functions.)

This is an example of the *Fiat-Shamir heuristic* (1986).



NIZK variant for mixes

Hash the output of the permutation/reencryption. Use those bits to select which edges get revealed.



Say we're mixing 1 million ballots, each mix reveals 1%. After five mixes, 99.99% chance that all ballots reencrypted at least once.

Evil machine: E(bignum)?

Must prove ciphertext corresponds to well-formed plaintext. (Example, prove counters are zero or one.)

We need another ZK tool: Chaum-Pedersen proofs.

Prover knows: $(g, g^x), (h, h^x)$

Wants to prove that these two tuples share x

But x is a secret!

Chaum-Pedersen proofs (1992)

Goal: demonstrate $(g, g^x), (h, h^x)$

P: choose random $w \in \mathbb{Z}_p^*$, compute $(A = g^w, B = h^w)$ Send (A,B) to V

V: pick a random number c (challenge), send to P

P: compute R = w + xcsend R to V

V: Compute
$$A(g^{x})^{c} = g^{w}g^{xc}$$

$$= g^{w+xc}$$

$$= g^{R}$$

$$B(h^{x})^{c} = h^{w}h^{xc}$$

$$= h^{w+xc}$$

$$= h^{R}$$

Goal: demonstrate $(g, g^x), (h, h^x)$

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$$= h^{w+xc}$$

$$= h^{R}$$

Goal: demonstrate $(g, g^x), (h, h^x)$

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Goal: demonstrate $(g, g^x), (h, h^x)$

- **P**: choose random $w \in \mathbb{Z}_p^*$, compute $(A = g^w, B = h^w)$ Send(A,B) to P choses fake c, R s.t. $A = g^R(g^{xc})^{-1}$.
- V: pick a random number (c (challenge), send to P
- **P**: compute R = w + xcsend R to V
- V: Compute

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Goal: demonstrate (g, g^x), (h, h^x)
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 ${m V}$: pick a random number c (challenge), send to P

P: compute R = w + xcsend R to V Observer can compute $A(g^x)^c$...

V: Compute

$$\begin{array}{rcl}
A(g^{x})^{c} &=& g^{w}g^{xc} \\
&=& g^{w+xc} \\
&=& g^{R}
\end{array}$$

$$\begin{array}{rcl}
B(h^{x})^{c} &=& h^{w}h^{xc} \\
&=& h^{w+xc} \\
&=& h^{R}$$

```
Goal: demonstrate (g, g^x), (h, h^x)
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- V: pick a random number c (challenge), send to P
- **P**: compute R = w + xcsend R to V Observer can compute $A(g^x)^c$...

V: Compute
$$A(g^x)^c = g^w g^{xc}$$
 $B(h^x)^c = h^w h^{xc}$ $= g^{w+xc}$ $= g^R$ $= h^R$

ZK protocols only work when "live" (or use Fiat-Shamir heuristic for non-interactive)

C-P for vote testing

Can I prove a vote is zero or one? First, how about "proving" it's zero using C-P.

Want to verify $\langle g^r, g^{ar}g^v \rangle$ for a specific value of v?

"Prove we know r" (or we could symmetrically prove we know a)

Do C-P protocol where (g, g^x) , (h, h^x) becomes (g, g^r) , $\left(g^a, \frac{g^{ar}g^{v}}{g^v}\right)$

We could do this for any value of v

Cramer-Damgård-Schoenmakers (1996)

Can run two Chaum-Pedersen (or any two ZK proofs like this) simultaneously, one "real" and one "simulated".

First, fake a proof (e.g., for v = 1) in advance.

Then, announce the first message for both protocols. Challenger sends c, prover announced a split c_0, c_1 where $c_0 + c_1 = c$, then executes both ZK protocols.

Verifier cannot tell which one was real vs. simulated, but knows that **one** of them was real.

Crypto summary

At the end of the day, **any** election observer can now:

- verify every single ballot for being "well-formed" (valid ElGamal tuple, encrypted zero-or-one, etc.)
- add together all the ballots (homomorphically)
- verify a proof of the tally (Chaum-Pedersen again) (only the election authority can generate this)

But we have no idea if the original ciphertext corresponded to the **intent of the voter** (versus evil machine flipping votes).

One newer, useful trick: ballot challenge

Catch a machine if it cheats!

Benaloh challenges [2006]

Benaloh challenges [2006]

voter makes selections

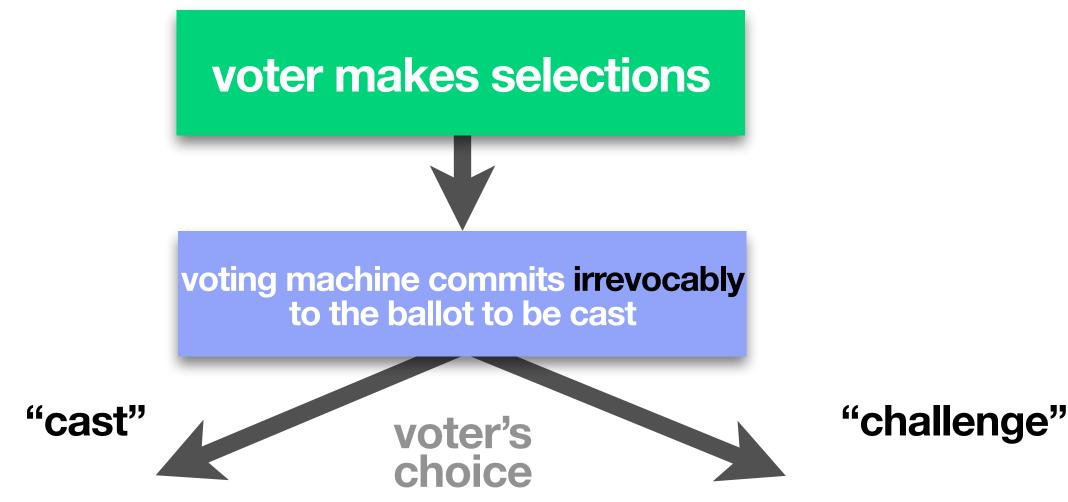
Benaloh challenges [2006]

voter makes selections

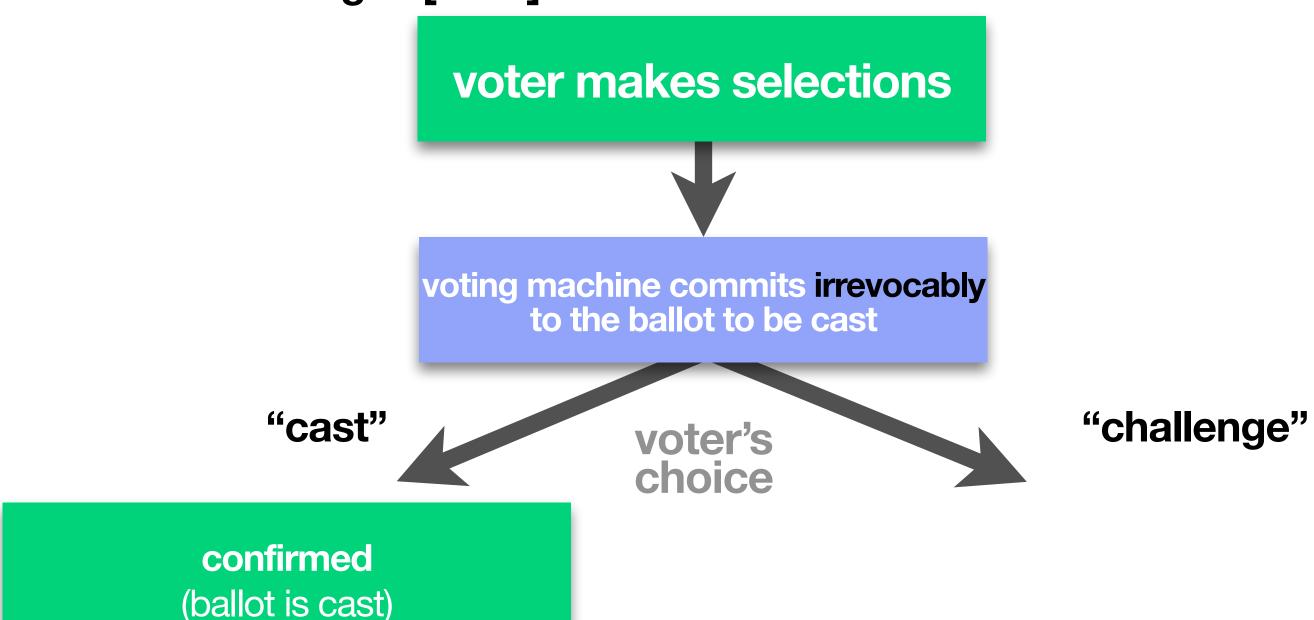


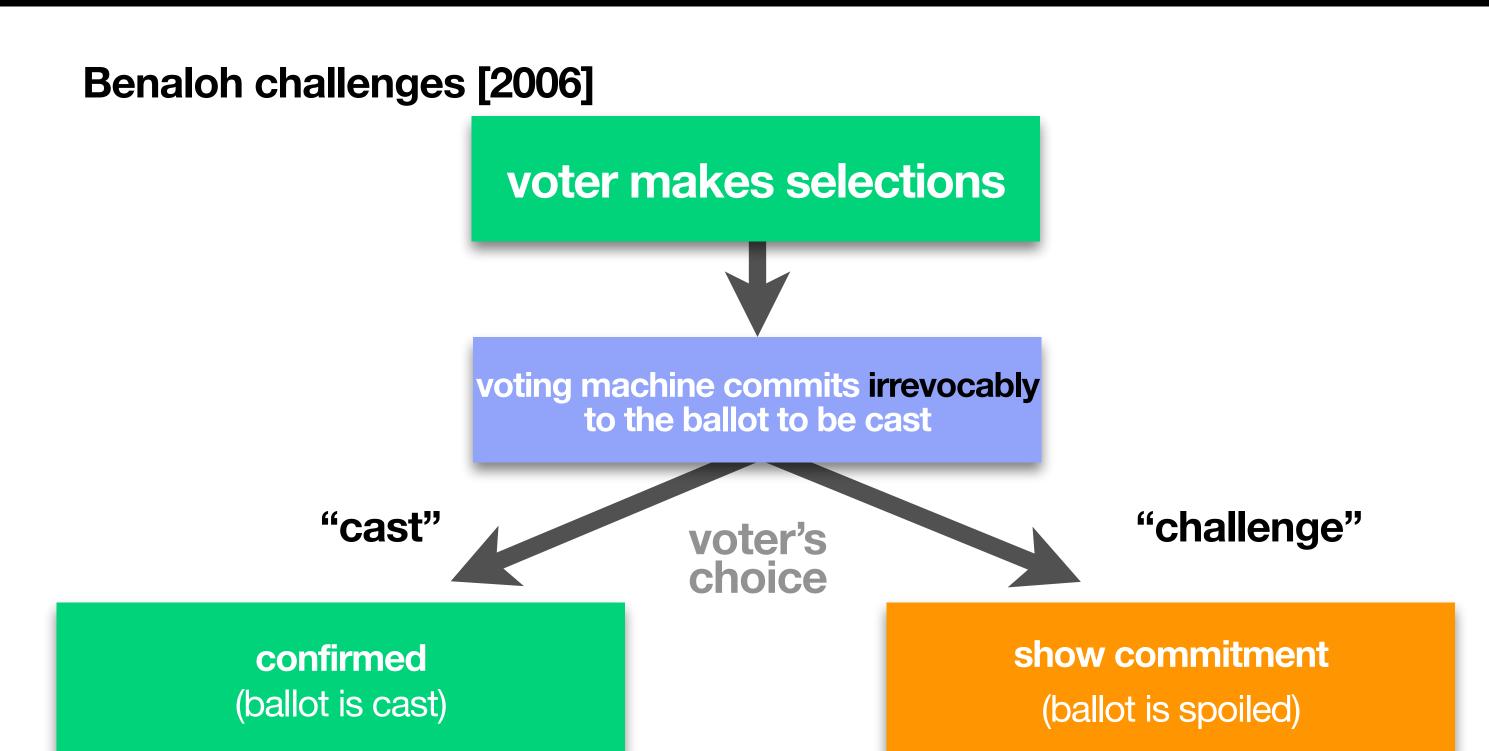
voting machine commits irrevocably to the ballot to be cast

Benaloh challenges [2006]



Benaloh challenges [2006]





ElGamal extra goodies

Two ways to decrypt:

$$E(g^{a}, r, M) = \langle g^{r}, (g^{a})^{r} M \rangle$$

$$D(g^{r}, g^{ar} M, a) = \frac{g^{ar} M}{(g^{r})^{a}}$$

$$D(g^{r}, g^{ar} M, r) = \frac{g^{ar} M}{(g^{a})^{r}}$$

```
g group generator
```

random (chosen at encryption time)

*(*private) decryption key

g (public) encryption key

ElGamal extra goodies

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$$D(g^r, g^{ar}M, r) = \frac{g^{ar}M}{(g^a)^r}$$

Only election officials (or trustees) have *a* (or shares of *a*) to decrypt totals.

group generator

M plaintext (message)

random (chosen at encryption time)

a (private) decryption key

g^a (public) encryption key

ElGamal extra goodies

Two ways to decrypt:

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group generator

M plaintext (message)

random (chosen at encryption time)

a (private) decryption key

g^a (public) encryption key

When challenged, machine reveals random nonces (r), observer can decrypt.

Challenging the machine

When challenged, the machine must reveal r

We can then decrypt this ballot (only) and see if it's what we expected to see

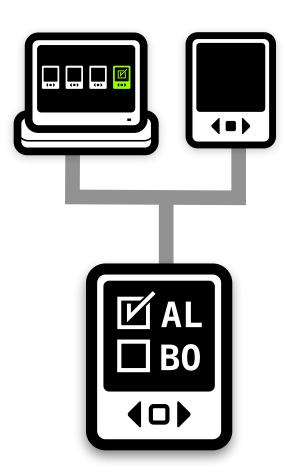
In Benaloh, the encrypted ballot is on paper

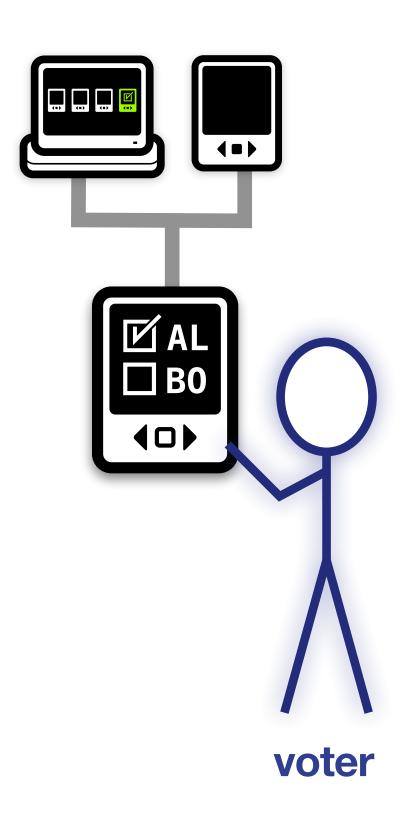
An **irrevocable** output medium decrypting requires additional equipment

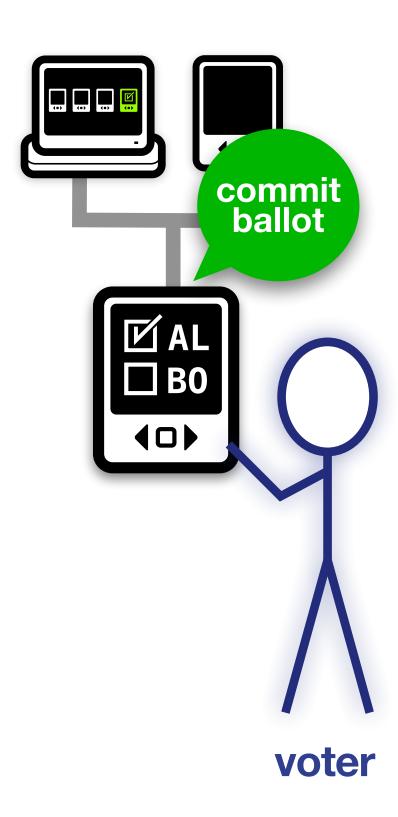
VoteBox (2008) has an irrevocable publishing system

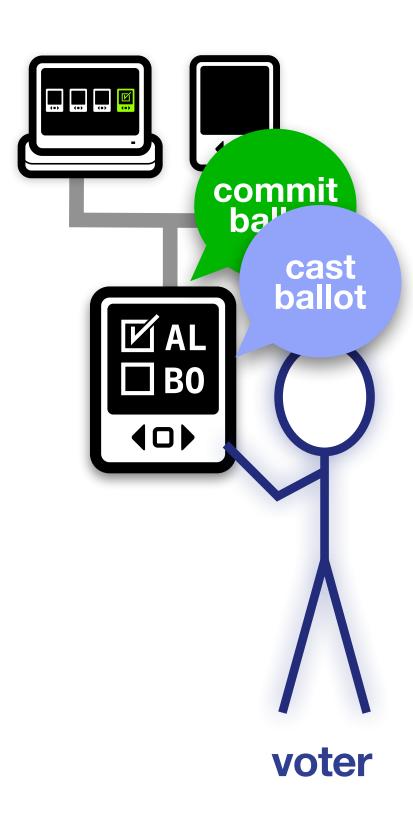
In-precinct LAN, where all machines replicate the history, plus hash entanglement.

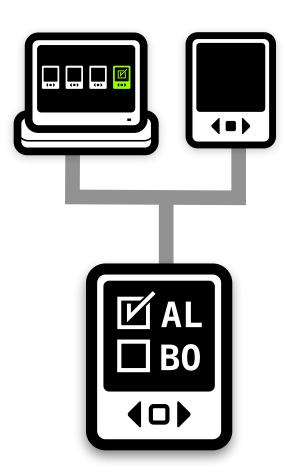
Helios (also 2008) just showed it to you on the web



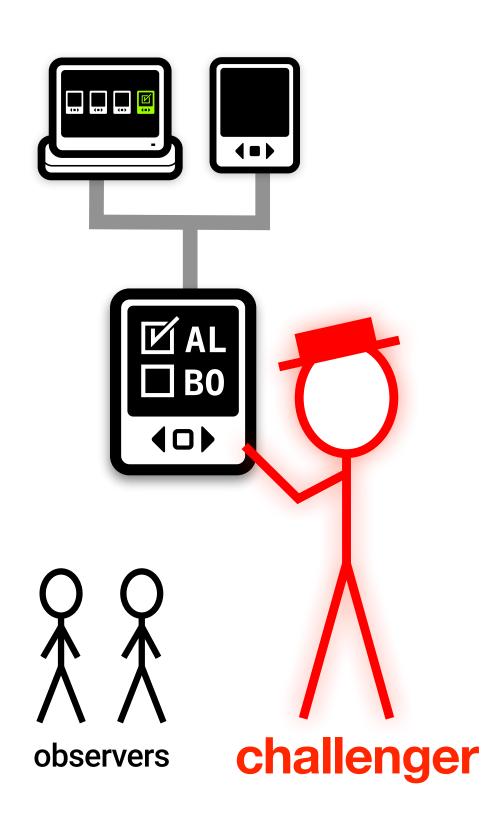






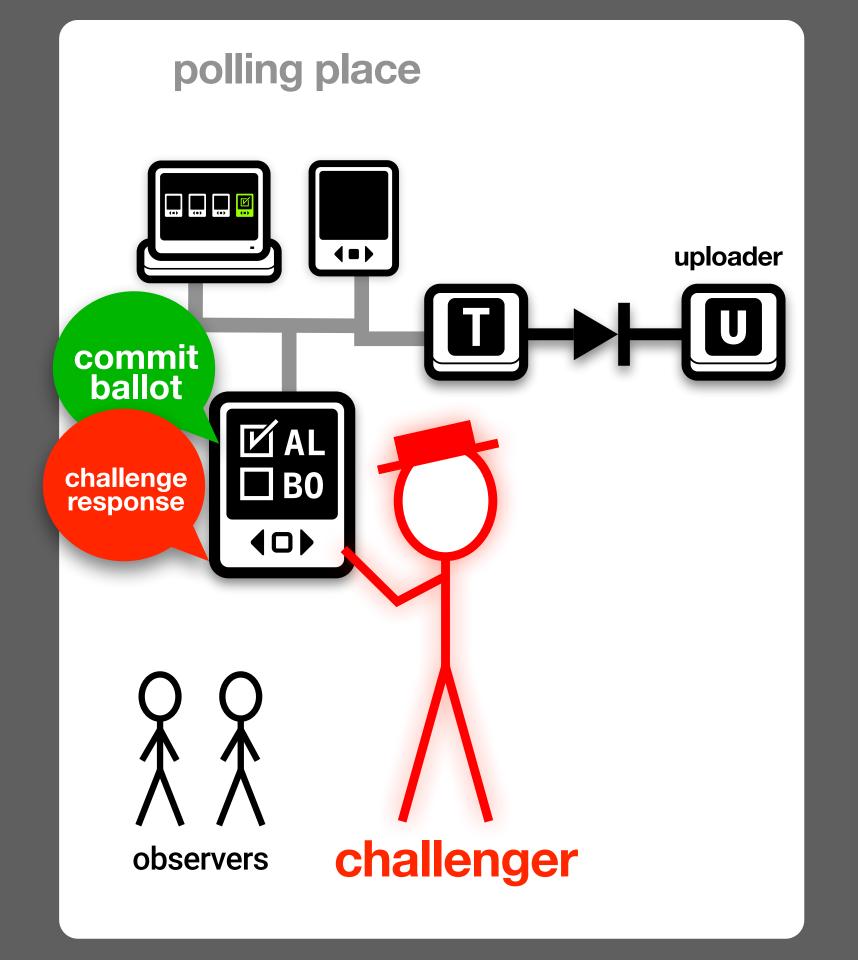


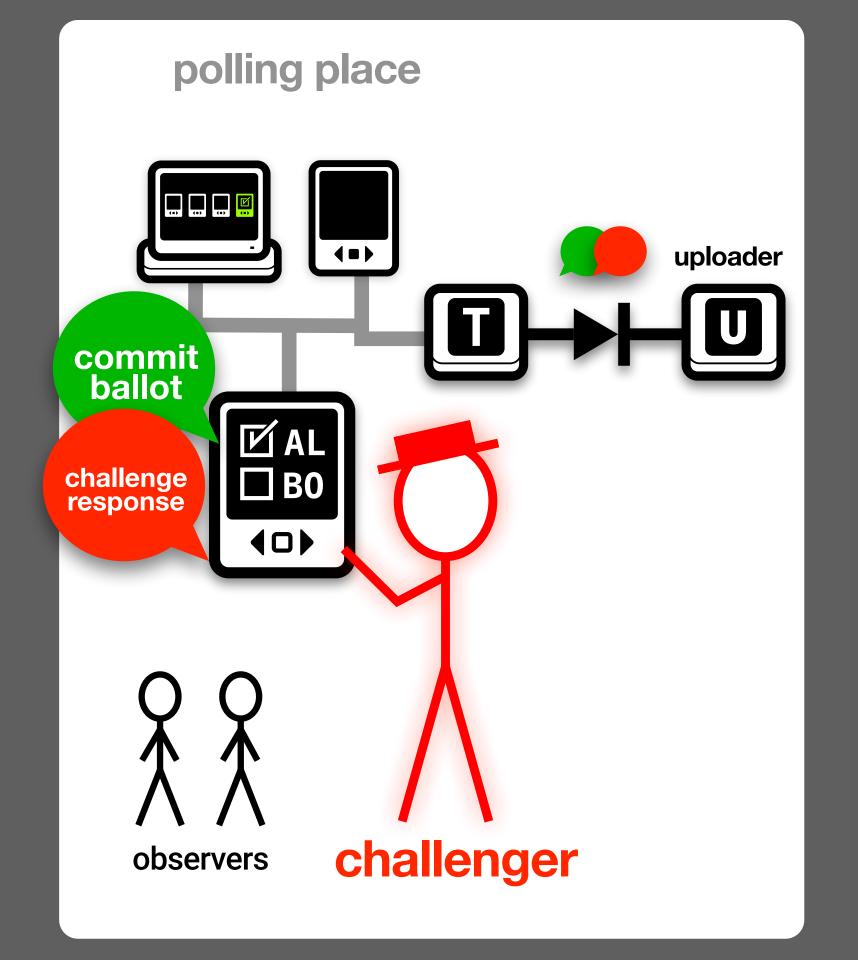
polling place **1**□**)** challenger

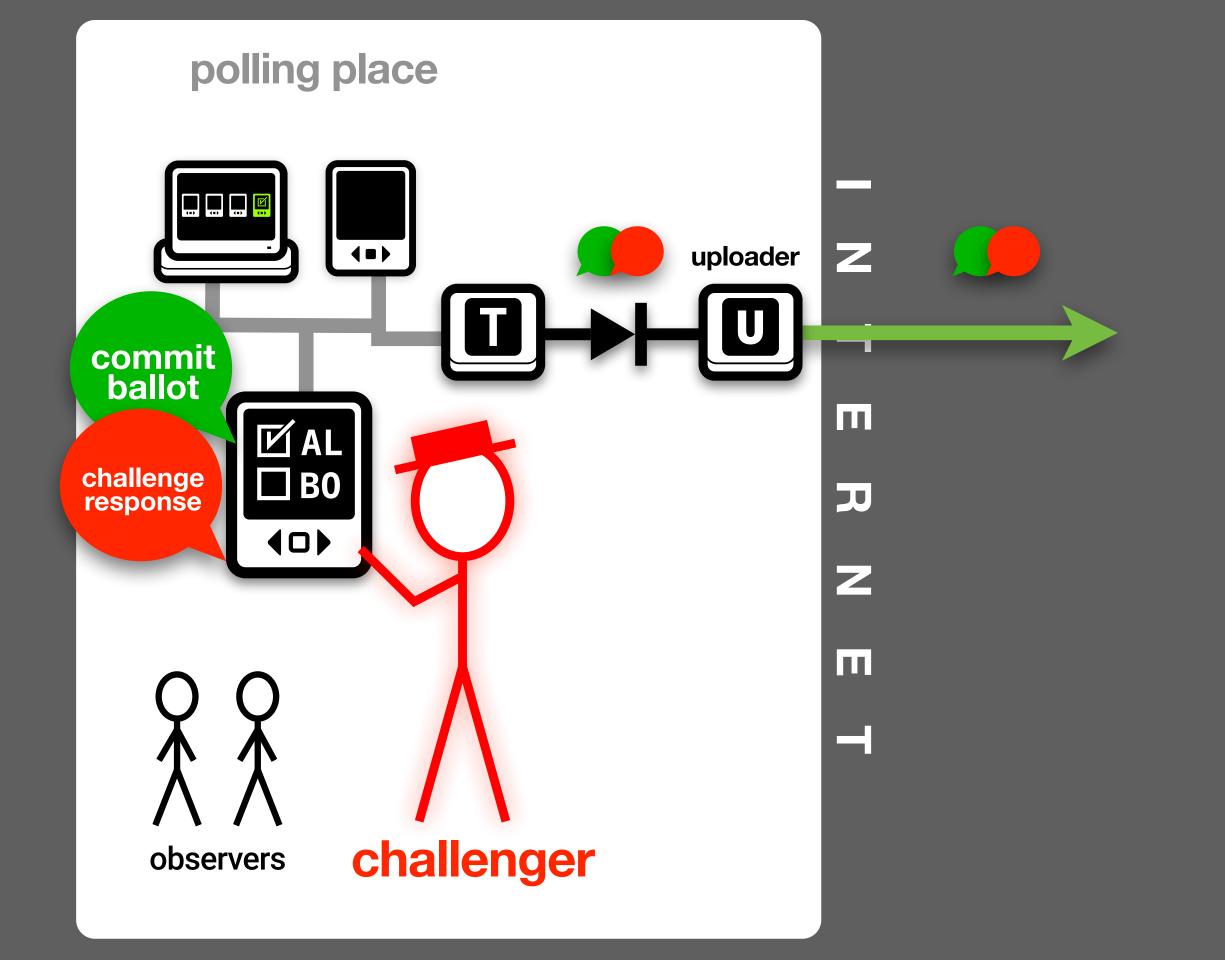


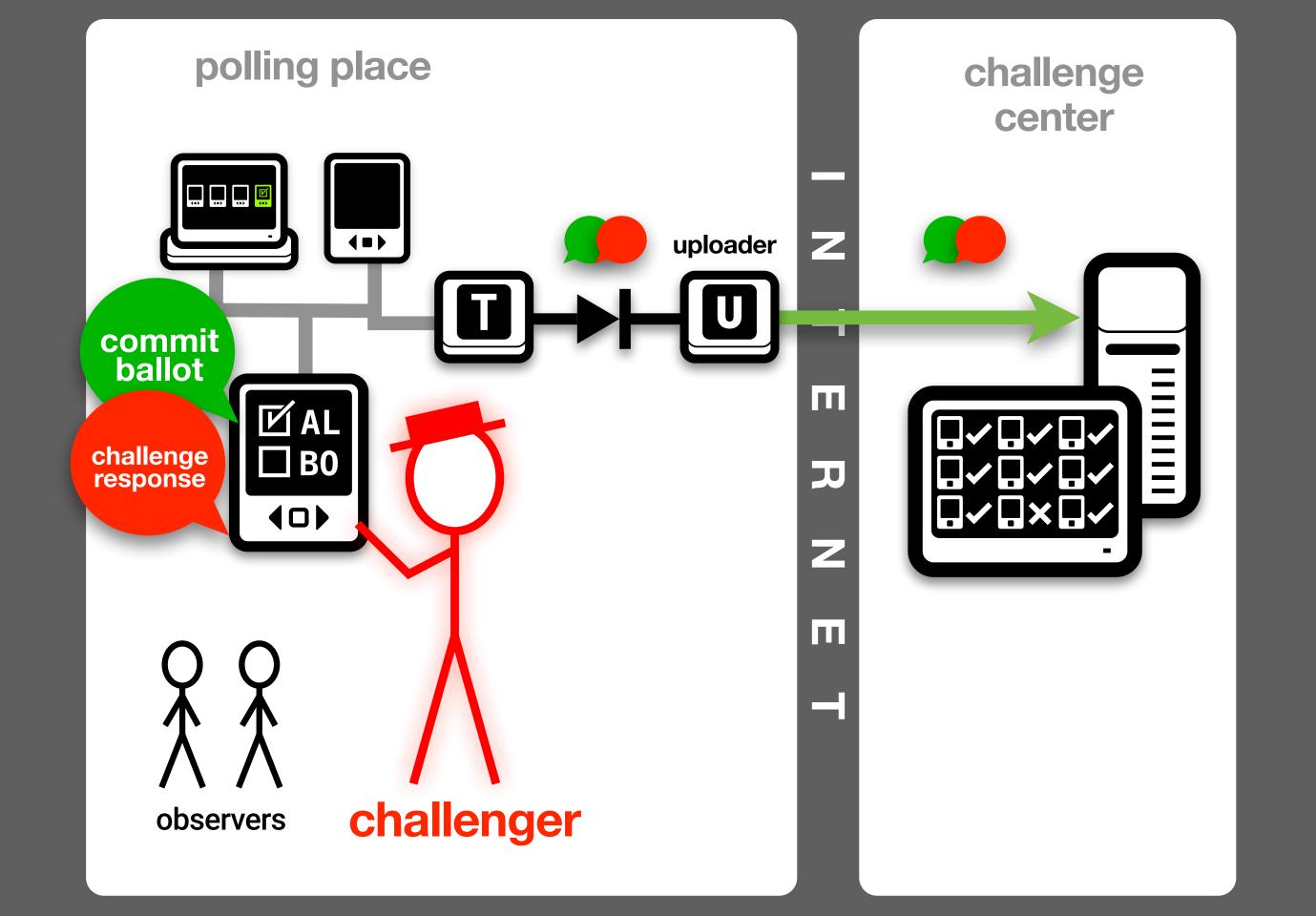
polling place commit ballot **(|** challenger observers

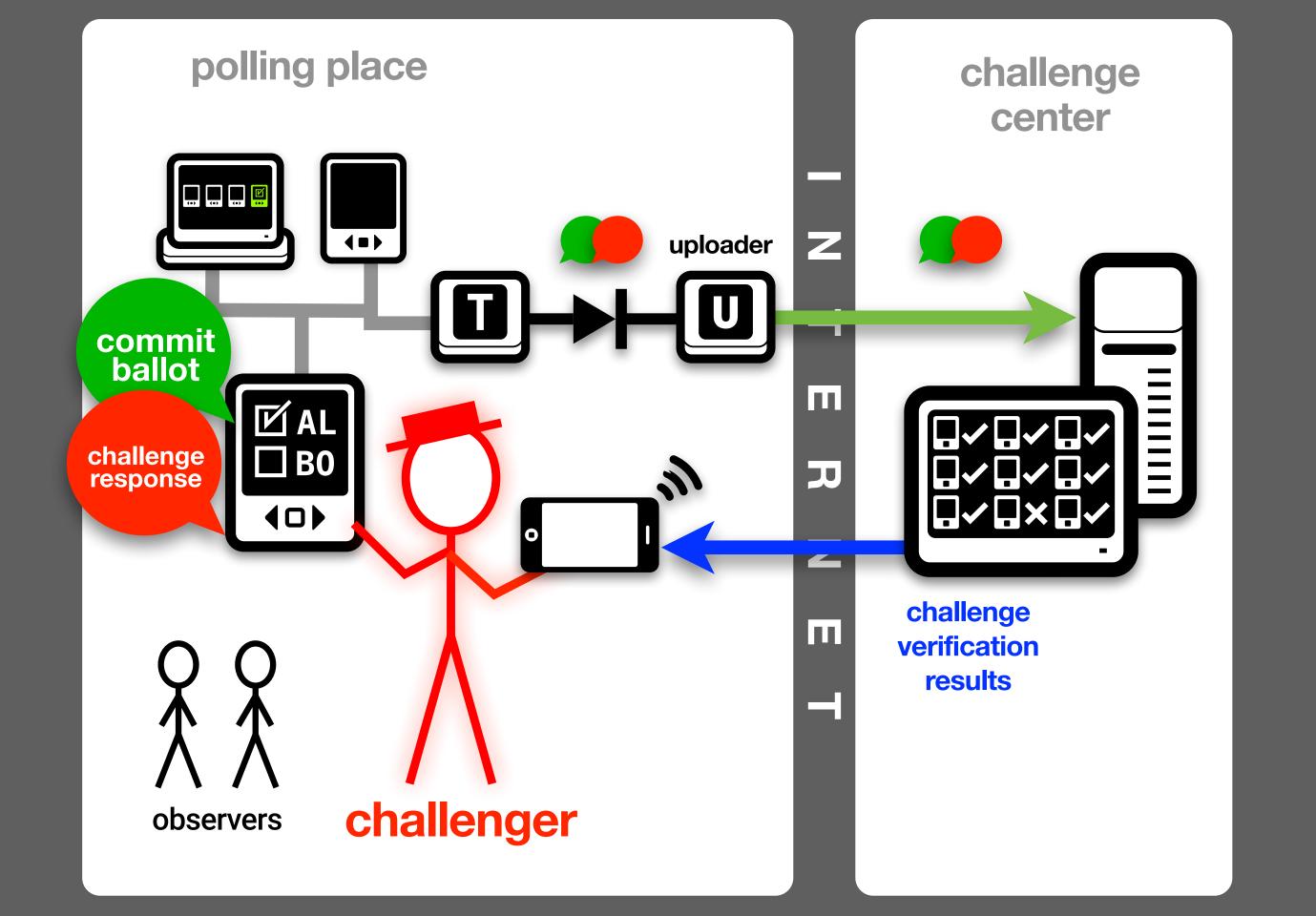
polling place commit ballot challenge response **□** B0 **(**| challenger observers

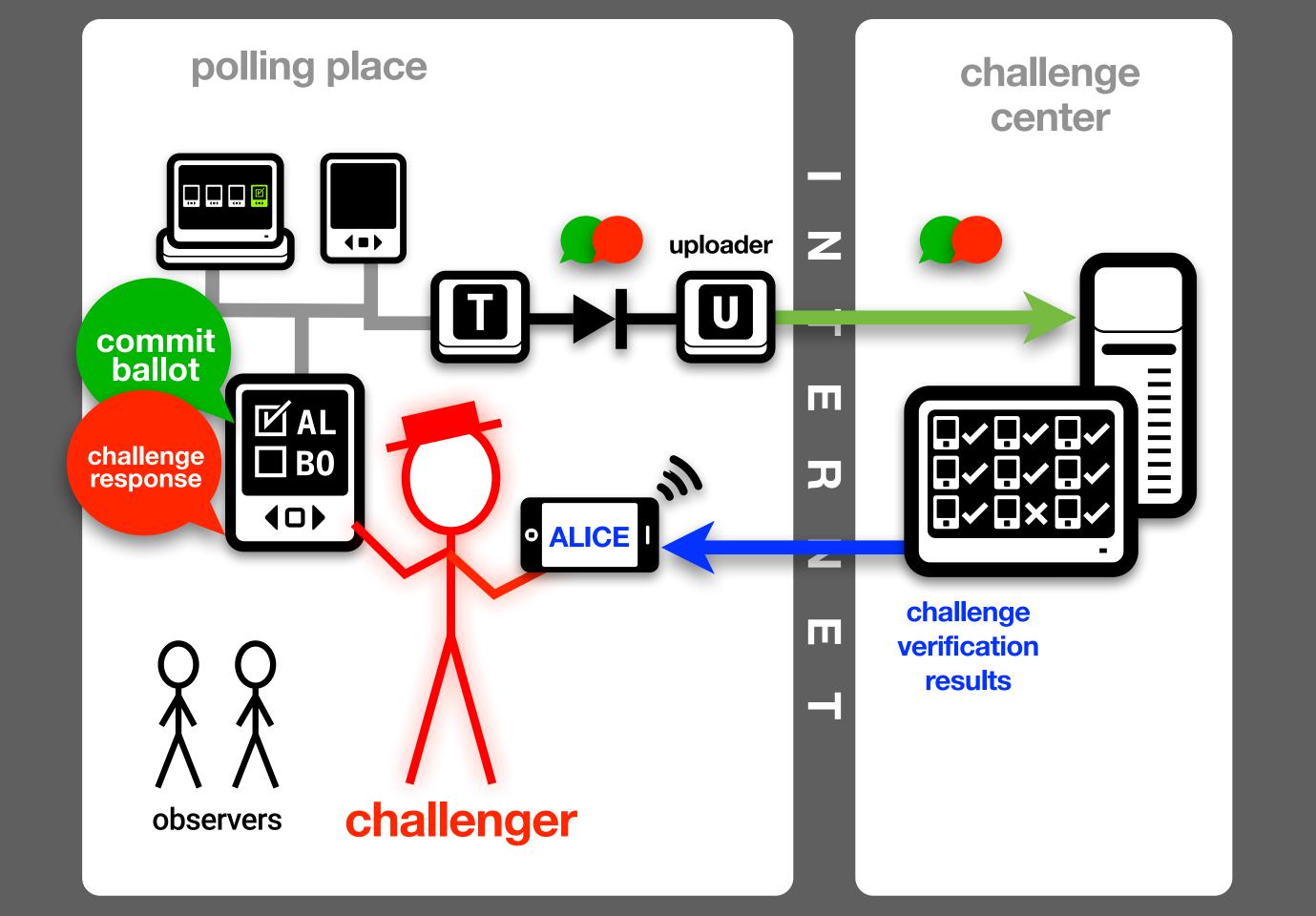












STAR challenges (2013)

Commitment: ciphertext broadcast to terminals

Happens when the ballot is printed, just like VoteBox

Challenge: voter deposits or keeps ballot

Challenger takes home printed ballot

Ballot randomness posted, anyone can decrypt (ballots not counted!)

Procedurally: same as a spoiled ballot

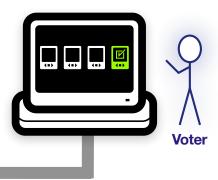
Big usability win

No need to ask the voter a challenge question Simple "live parallel testing"

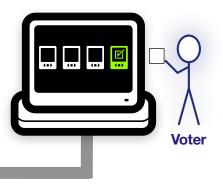
STAR Workflow: Registration



STAR Workflow: Registration



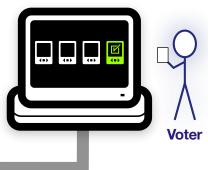
STAR Workflow: Registration





Workflow: Authorization

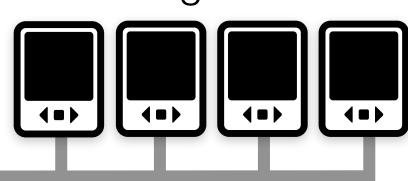


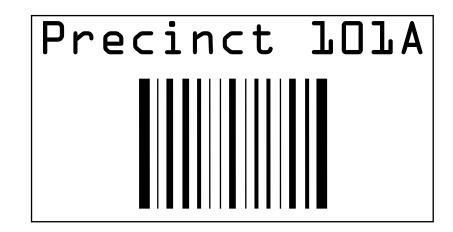






Voting terminals

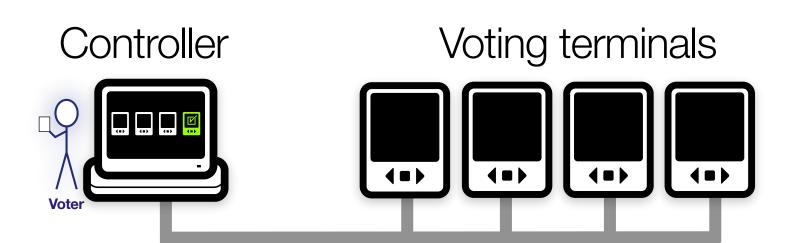


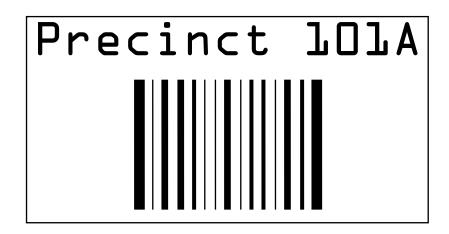


Workflow: Authorization



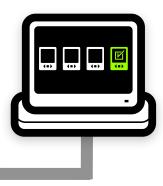


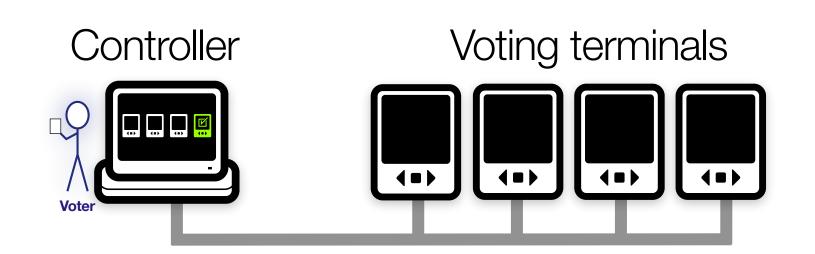




Workflow: Authorization

Registration



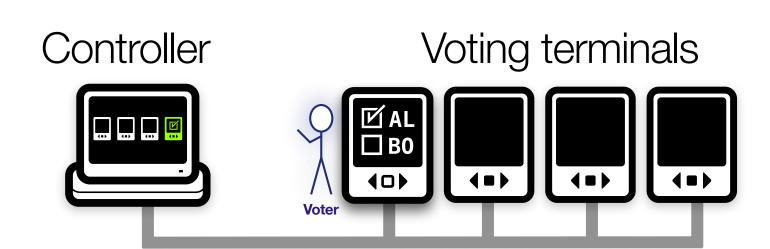


Auth: 52794

Similar to Hart InterCivic eSlate

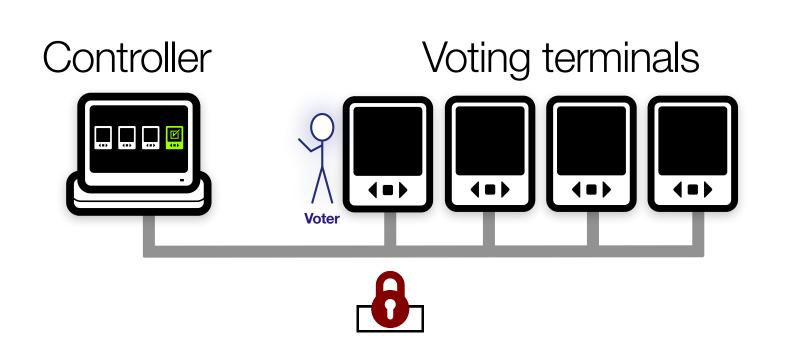
Workflow: Voting





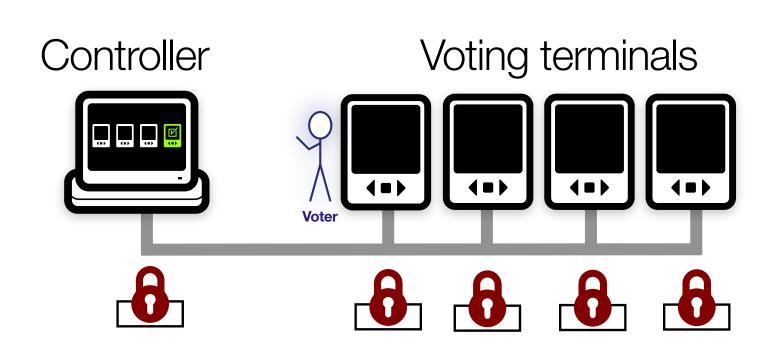
Workflow: Ciphertext copies distributed





Workflow: Ciphertext copies distributed





Workflow: Casting

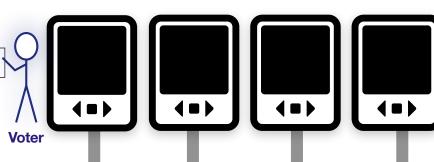
Registration

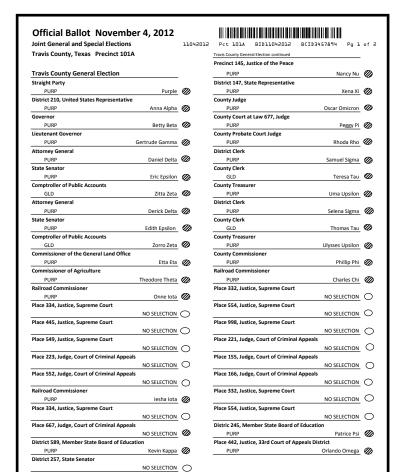


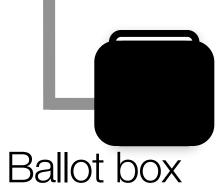




Voting terminals







Workflow: Casting

Registration

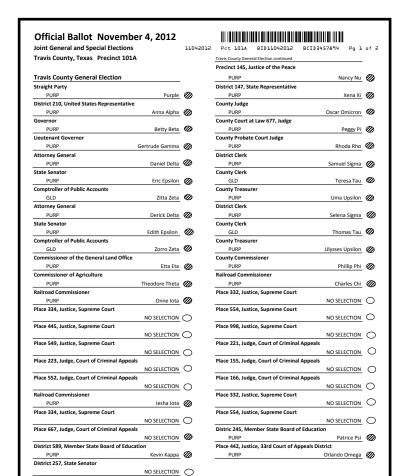


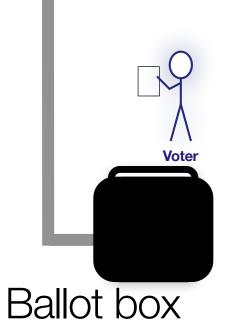
Controller



Voting terminals







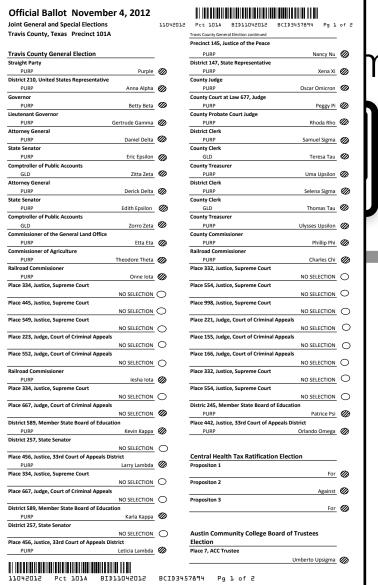
Networked ballot box

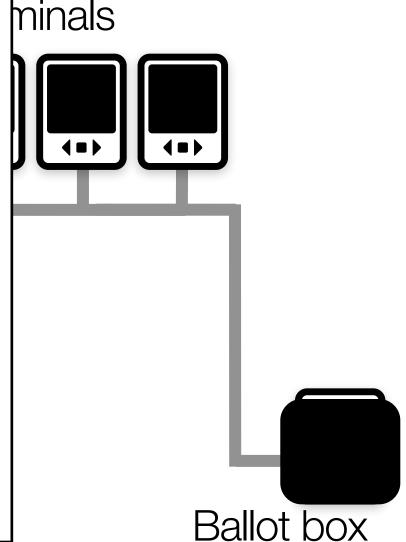
Notifies other machines that ballot wa

Ballot has random ID

Voter can spoil ballot and start over Usability win!

Ballot box has no UI





Deposit and done (just need a scanner for the ballot ID)

Networked ballot box

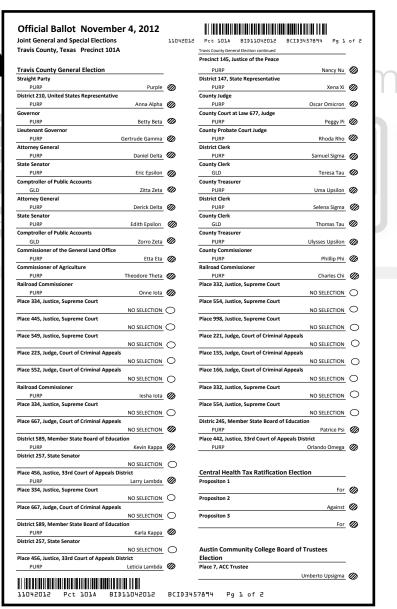
Notifies other machines that ballot wa

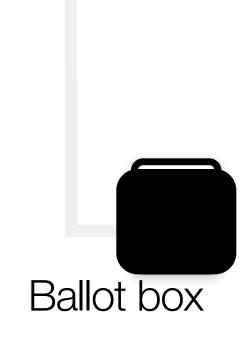
Ballot has random ID

Voter can spoil ballot and start over

Usability win!

Ballot box has no UI





Deposit and done (just need a scanner for the ballot ID)

Post-election verification

Separate page to take home

Ballot hash for lookup on public bulletin board

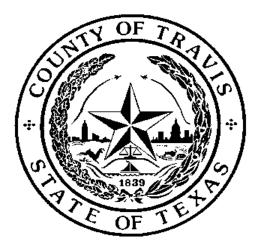
Cast ballot: ciphertext will match

Challenge ballot: plaintext also present, verifiable

Homomorphic tallying: provably includes all ballots on the bulletin board

Hash chains: also publicly verifiable

YOUR VOTE COUNTS



Thank you for voting!

Take this confirmation of voting with you

Verify your ballot at:

www.star-vote.org/ballot/HV1235Z7568RK84

Or, scan this code with your phone



Find your code on the STAR-Vote website to ensure that your vote was recorded correctly.

Look for Election results and other tools for confirming the election at:

www.traviscountvelections.org

Voting Date: October 30, 2012 Voting Terminal: UI12345

Location: Randall's South Mopac

Risk limiting audits

Random sampling of individual paper ballots

Each should exactly match up with electronic records Required in many states, growing in popularity

 Requires touching tens of ballots, rarely hundreds, unlikely more

Mitigates malicious software in the ballot scanners

Discrepancies require more samples, degenerates to a full manual recount of the ballots



Voting + Cryptography

Microsoft's open 'ElectionGuard' SDK aims to secure democratic elections

ElectionGuard is open source and built to make sure votes are verifiable, secure, and auditable.

DAN THORP-LANCASTER 6 May 2019



How not to do e2e cryptography

Ceci n'est pas une preuve

The use of trapdoor commitments in Bayer-Groth proofs and the implications for the verifiabilty of the Scytl-SwissPost Internet voting system*

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March 12, 2019

The implementation of the commitment scheme in the SwissPost-Scytl mixnet uses a trapdoor commitment scheme, which allows an authority who knows the trapdoor values to generate a shuffle proof transcript that passes verification but actually alters votes. We give two examples of details of how this could be used. The first example allows the first mix to use the trapdoors to substitute votes for which it knows the randomness used to generate the encrypted vote. The second example does not even require knowledge of the random factors used to generate the votes, and could be used by the last mix in the sequence.

Swiss Post Public Intrusion Test

Undetectable Attack Against Vote Integrity and Secrecy

ROLF HAENNI

Bern University of Applied Sciences

1 Theoretical Background

Commitment schemes are usually perfectly hiding and computationally binding. This means that no information about the message can be derived from the commitment and that the commitment can not be opened to a message other than the original one. The Pedersen commitment scheme achieves these properties by computing $c = G^m H^r$ in a multiplicative cyclic group, for which the discrete logarithm assumption (DL) is believed to hold. This scheme is perfectly binding, because a randomization $r' \neq r$ exists for any other message $m' \neq m$ such that $c = G^{m'}H^{r'}$. This means that c could potentially be opened to all q messages from \mathbb{Z}_q , but this requires the computation of the discrete logarithm. The Swiss Post voting protocol works with a subgroup $G_q \subseteq \mathbb{Z}_p^*$ of integers modulo p, where $q = |G_q|$ denotes the prime order of the subgroup, $m \in \mathbb{Z}_q$ the message, and $r \in \mathbb{Z}_q$ the randomization. Both G and H are elements of G_q .

A pre-condition for the scheme to be computationally binding is the *independence* of the two values $G, H \in G_q \setminus \{1\}$ (in a group of prime order q, both values are generators of G_q). Independence means that respective discrete logarithms $h = \log_G H$ and $g = \log_H G$ are unknown to everyone. Otherwise, for example if $h = \log_G H$ is known to the person who created c, then c can be rewritten as

$$c = G^m H^r = G^m (G^h)^r = G^{m+hr \bmod q}.$$

Therefore, to open c to a different message $m' \neq m$, the adversary can easily solve

$$m + hr \equiv m' + hr' \pmod{q}$$

to find the matching randomization $r' = (m - m')h^{-1} + r \mod q$. As a consequence, the binding property of the commitment scheme is completely broken in that case.

SWI swissinfo.ch

ONLINE DEMOCRACY

Swiss Post's e-voting system pulled for May votes

MAR 29, 2019 - 13:46



Swiss Post's e-voting system had been in use in four cantons: Basel City, Fribourg, Neuchâtel and Thurgau

(Keystone)

The e-voting system operated by Swiss Post will not be available for nationwide votes on May 19. This is the consequence of "critical errors" found during a public intrusion test, the Federal Chancellery and Swiss Post announced on Friday.

The Federal Chancellery <u>r</u> <u>said in a statement</u> it would review the licensing and certification procedures for e-voting systems. It added that it had no indication that these flaws had resulted in votes being manipulated in previous ballots.

Swiss Post's e-voting system had been in use in four cantons: Basel City, Fribourg, Neuchâtel and Thurgau.

^{*}Since making this work public, we have learned that the same issue was identified independently by Thomas Haines of NTNU, and also by Rolf Haenni of the Bern University of Applied Sciences, https://e-voting.bfh.ch/publications/2019/

Will ElectionGuard be better?

Talented researchers from Microsoft Research, others (including Wallach)

Multiple current implementations (open source, on GitHub today)

Your project 1 (going online tomorrow!) is based on ElectionGuard

The ElectionGuard model: plugins for existing voting machines

Targeted outreach to election equipment vendors

Still requires in-precinct voting

Sophisticated cryptography doesn't solve problems with internet voting