

CSCE 222 Discrete Structures for Computing – Fall 2023

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Problem Set 1

Due dates: Electronic submission of *yourLastName-yourFirstName-hw1.tex* and *yourLastName-yourFirstName-hw1.pdf* files of this homework is due on **Tuesday, 9/5/2023 11:59 p.m.** on <https://canvas.tamu.edu>. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will receive zero points for this homework.**

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Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic signature: Manas Navale

Total 100 points.

The intended formatting is that this first page is a cover page and each problem solved on a new page. You only need to fill in your solution between the `\begin{solution}` and `\end{solution}` environment. Please do not change this overall formatting.

Checklist:

- ☐ Did you type in your name and UIN?
- ☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit both the .tex and .pdf files of your homework to each correct link on Canvas?

Problem 1. ($10 + 10 = 20$ points) Section 1.1, Exercise 1.3. For (b), give the knight's graph in a text format by giving all edges in the graph such that the knight's move from vertex v_i to vertex v_{i+1} is given as (v_i, v_{i+1}) . Once you have all of the edges written, you can also give the path in the form of $v_i - v_{i+1} - v_{i+2} - \dots$.

Use the common convention of expressing the columns and rows of a chess-board as a, b, and c, and 1, 2, and 3, respectively.

Solution. For part a, it is impossible for a knight to visit all 9 squares because of the way the knight moves. Since the board is 3×3 , even if the knight starts at a corner it can only go to an edge. If it starts at an edge it can only go to a corner. And if it starts at a center square it can only go to a center. It is unable to make the moves necessary to visit all 9 squares.

Part b:

(a1, c2)
 (a1, b3)
 (b1, a3)
 (b1, c3)
 (c1, a2)
 (c1, b3)
 (a2, c1)
 (a2, c3)
 (b2, a1)
 (b2, a3)
 (b2, c1)
 (b2, c3)
 (c2, a1)
 (c2, a3)
 (c3, a2)
 (c3, b1)

Path:

a1 - c2 - a3 - c1 - a2 - c3 - b1 - a3 - b2 - c3 - b2 - a1 - c2 - a1 - b3 - c1 - b3

Problem 2. (2 points \times 5 subproblems = 10 points) Section 2.1, Exercise 2.1

Solution. (a) is false because π is an irrational number but not the smallest
Eg. $\sqrt{2}$.

(b) is true because π is an irrational number as it can't be expressed as a ratio of two integers.

(c) is false, as the quadrature equation does have solutions $x=2-\sqrt{2}$, $x=2+\sqrt{2}$

(d) is false as 23 is not less than 23 and is actually equal to it.

(e) is not a statement, as it depends on the context of what x is.

All of the previous options were statements because they could be proved true or false.

Problem 3. (3 points \times 5 subproblems = 15 points) Section 2.1, Exercise 2.3

Solution. (a) is true because if $x = 0.111\dots$ and $110x = 1.111\dots$ then if you subtract $10x - x$ you get 1 which means that $9x = 1$ which means $x = 1/9$

(b) is false as 0.121212 can be expressed as the fraction $121/990$.

(c) is true because there are no other common factors that divide into 1111 and 11111111

(d) is true because the product of two negatives is positive so $(-1)(-1) = 1$

(e) is true since all positive whole numbers would continue indefinitely leading to infinity.

Problem 4. (2 points \times 2 subproblems = 4 points) Section 2.2, Exercise 2.7
(a) and (b)

Solution. (a) Albert cooks pasta, and Emmy is not happy
(b) If Albert cooks pasta, then Albert is happy and Emmy is also happy.

Problem 5. (3 points \times 2 subproblems = 6 points) Section 2.2, Exercise 2.8
(a) and (d)

Solution. (a) $C \rightarrow \neg S$
(d) $S \leftrightarrow \neg C$

Problem 6. (15 points) Section 2.2, Exercise 2.18. Use a truth table to show your reasoning.

Example \LaTeX source for how to draw a truth table is shown in the truth-table.tex and truth-table.pdf files.

Solution. if A is a knight, then the statement is true, and if A is a knave the statement is false. But there is not enough information to determine the status of B since there was no statement, and as a result A's status is uncertain as well

p	q	$p \wedge q$	$p \oplus q$
Knight	Knight	True	False
Knight	Knave	True	False
Knave	Knight	False	True
Knave	Knave	False	True

Problem 7. (10 points) Section 2.3, Exercise 2.25. Use a truth table.

Solution. As shown in the truth table $(A \rightarrow B) \wedge (B \rightarrow A)$ and $A \leftrightarrow B$ are the same for all combinations of A and B, showing that they are equivalent.

A	B	$A \rightarrow B$	$B \rightarrow A$	$(A \rightarrow B)$	$(B \rightarrow A)$	$(A \rightarrow B) \wedge (B \rightarrow A)$
T	T	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Problem 8. (20 points) Section 2.3, Exercise 2.26. Your answer should consist of a series of logical equivalences you learned in the text, and the final step must resolve to T . Do not use a truth table. Study the proofs of Proposition 2.8 (b) and (c) for the expected style of your answer. Watching the video “Problem Solving Exercise 1” in Module 2.1 will also be helpful.

Example L^AT_EX source for how to align the steps nicely is shown in the truth-table.tex and truth-table.pdf files.

Solution. STEPS

1. $(A \wedge (A \rightarrow B)) \rightarrow B$
2. $(A \wedge (\neg A \vee B)) \rightarrow B$
3. $((A \wedge \neg A) \vee (A \wedge B)) \rightarrow B$
4. $(\text{False} \vee (A \wedge B)) \rightarrow B$
5. $(A \wedge B) \rightarrow B$
6. B