CSCE 222 Discrete Structures for Computing – Fall 2023
Hyunyoung Lee

Problem Set 2

Due dates: Electronic submission of *yourLastName-yourFirstName-hw2.tex* and *yourLastName-yourFirstName-hw2.pdf* files of this homework is due on **Monday, 9/18/2023 11:59 p.m.** on `https://canvas.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two files are missing, you will receive zero points for this homework.**

Name: (Manas Navale) **UIN: (333006797)**

Resources. (All people, books, articles, web pages, etc. that have been consulted when producing your answers to this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic signature: (Manas Navale)

Total 100 points.

The intended formatting is that this first page is a cover page and each problem solved on a new page. You only need to fill in your solution between the `\begin{solution}` and `\end{solution}` environment. Please do not change this overall formatting.

Checklist:
- ☐ Did you type in your name and UIN?
- ☐ Did you disclose all resources that you have used? (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit both the .tex and .pdf files of your homework to each correct link on Canvas?

Problem 1. (5+5 = 10 points) Section 2.6, Exercise 2.53 (a) and (c). Explain.

Solution.

(a) 0 values.
(the equation $a^n + b^n = c^n$ has no values when n is ≥ 3)
(b) 1 values.
(3, 4, 5)

Problem 2. ($5 + 5 = 10$ points) Section 2.6, Exercise 2.54 (b) and (c)

Solution.

(b)For every value of x, there exists a value of y such that x is less than y.
(c) For all values of x and for all values of z, there exists a value of y such that if x is less than z, then both x is less than y and y is less than z.

Problem 3. ($5 + 5 = 10$ points) Section 2.7, Exercise 2.58 (a) and (e)

Solution.
(a)$(\forall x \exists y (P(x) \wedge \neg Q(y)))$
(e)$(\forall x \forall y (P(x) \vee Q(y)))$

Problem 4. ($5 + 5 = 10$ points) Section 2.7, Exercise 2.59 (d) and (e)

Solution.
(d)For all integers a, there exists an integer b such that ($a + b \neq 1001$).
(e)There exists a positive integer a such that for all positive integers b, ($b \geq a$).

Problem 5. (15 points) Section 2.9, Exercise 2.73 [Hint: Use the property of "consecutive integers" and the definition of an "odd integer".]

Solution.

$(n = m + 1)$
$(m + n = m + (m + 1))$
$(m + n = 2m + 1)$
Since m is an integer, 2m + 1 is also an integer, and an odd integer is an integer that can be expressed in the form $(2m + 1)$

Problem 6. (15 points) Section 2.9, Exercise 2.80

Solution.

if $(m + n > 100)$, then $(m > 40)$ or $(n > 60)$
The negation of this is $(m \leq 40)$ and $(n \leq 60)$
to prove it by contraposition we need to show the negation implies the negation of the original statement.
So $(m + n > 100)$ is false which means $(m + n \leq 100)$.
Since $(m \leq 40)$ and $(n \leq 60)$ then we can combine the two inequalities $(m+n \leq 100)$
So if $(m \leq 40)$ and $(n \leq 60)$ then $(m + n \leq 100)$ which is the negation of the original statement.
So by contraposition this means that $(m + n > 100)$, and $(m > 40)$ or $(n > 60)$

Problem 7. (15 points) Section 2.9, Exercise 2.84

Solution.

simplifying the equation ($42m + 70n = 1000$) to ($3m + 5n = 71$)

If both 3m and 5n leave a remainder of 2 when divided by their respective divisors, then 3m+5n would leave a remainder of 4 when divided by 3. But 71 divided by 3 leaves a remainder of 2, which is a contradiction.

Problem 8. (15 points) Section 3.3, Exercise 3.20 [Hint: Use the definitions of $\subseteq$, $\cup$, and the power set.]

Solution.

Using X be an arbitrary element in $P(A) \cup P(B)$. This implies that X is an element of either $P(A)$ or $P(B)$.
Using $X \in P(A)$, which means X is a subset of A
Using $X \in P(B)$, which means X is a subset of B.
Now, consider the union $A \cup B$. Any subset of $A \cup B$ contains elements from both A and B.
Since X is a subset of either A or B it is also a subset of $A \cup B$ because A and B are both subsets of $A \cup B$. Therefore, X is an element of $P(A \cup B)$.
So we have shown that any arbitrary element X in $P(A) \cup P(B)$ is also an element of $P(A \cup B)$. Thus, $P(A) \cup P(B) \subseteq P(A \cup B)$.