Mathematical Inequalities Statistics (academic discipline)

What is a real-world application of Chebyshev's inequality?

Suppose you are an insurance company. You receive claims of random sizes at random times from your customers. Based on the claims you have received so far, you want to get an idea about how large the claims are likely to be in the future, so you can set aside enough reserves to cover the claims of the upcoming calendar year, say.

To keep things simple, you assume that the claims are independent and identically distributed. So each claim is assumed to be a random draw from a single, unknown distribution.

Without knowing anything about this underlying distribution, Chebyshev's inequality immediately tells you that you can be at least 90% sure that future claims will be within three standard deviations away from the mean. You can get pretty good estimates of the mean and the standard deviation from the claims you have observed so far. Once you have those estimates, you immediately have a 90% prediction interval for future claims, courtesy of Chebyshev's inequality, *without assuming anything about the underlying distribution* (except finite mean and variance, of course).

You can get much tighter prediction intervals for future claims if you are willing to assume a parametric structure for the claims. If you assume, for example, that the claims are drawn from a Gamma distribution, then you can simply estimate the parameters of the Gamma distribution from your observations and compute a tighter 90% prediction interval than the relatively wide one provided by Chebyshev's inequality. The risk with this approach is, of course, that the underlying distribution may not be Gamma at all. The real distribution may be log-normal, say, or Pareto, both of which have heavier probability tails than the Gamma distribution, in which case your Gamma assumption would lead you to dangerously underestimate the sizes of future claims. Chebyshev's inequality gives you a much more reliable, albeit wider, prediction interval.

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Ido Levin, studied Mathematics
Answered Jan 5, 2016

Thanks for the A2A (Although there is already a good old answer by Sami Umut Can).

Chebyshev's inequality bounds the probability that a random variable will deviates from its expectation:

$\forall \alpha P(|X-\mu| \geq \alpha\sigma) \leq \frac{1}{\alpha^2}$ where μ and σ are the expected value and standard deviation of X, respectively.

Now, lets say you take an exam and you know that the grades are normalized to $\mu = 70$ and $\sigma = 10$. You want to know what's the probability you will fail (you assume that the grades are independent and that the distribution is symmetric, which is a rough assumption). Then $P_{\text{fail}} = P(X \leq 50) = 0.5P(|X-\mu| \geq 2\sigma) < 0.125$.

Luciano Robino, works at Centro Atomico Bariloche
Answered Oct 22, 2014

As Sami Umut Can stated. The idea of Chebyshev's inequality is to get as much information from you data when you have no clue how to describe a random variable that you have measured. It constitutes the minuimum amount of information you can get from any random variable distribution

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