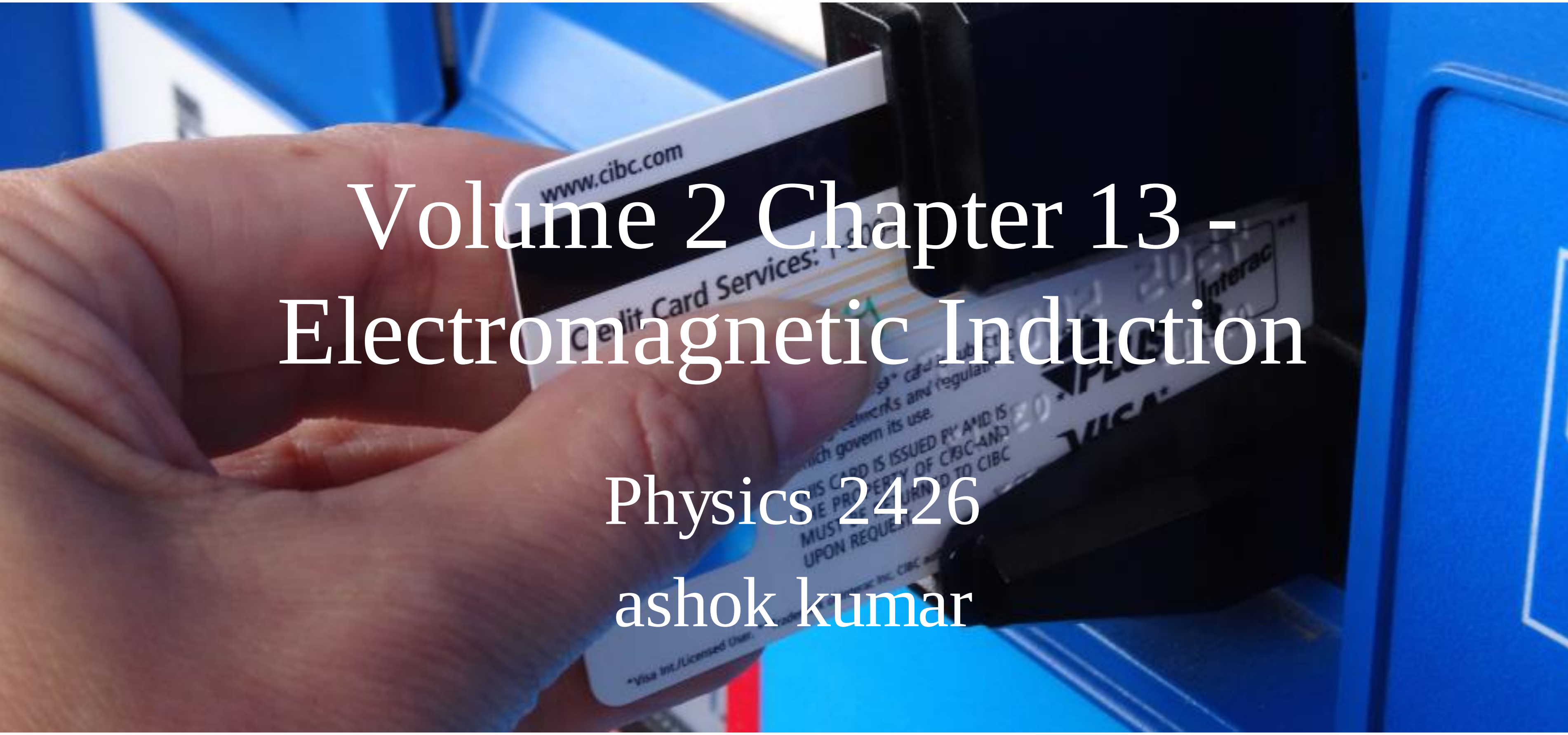
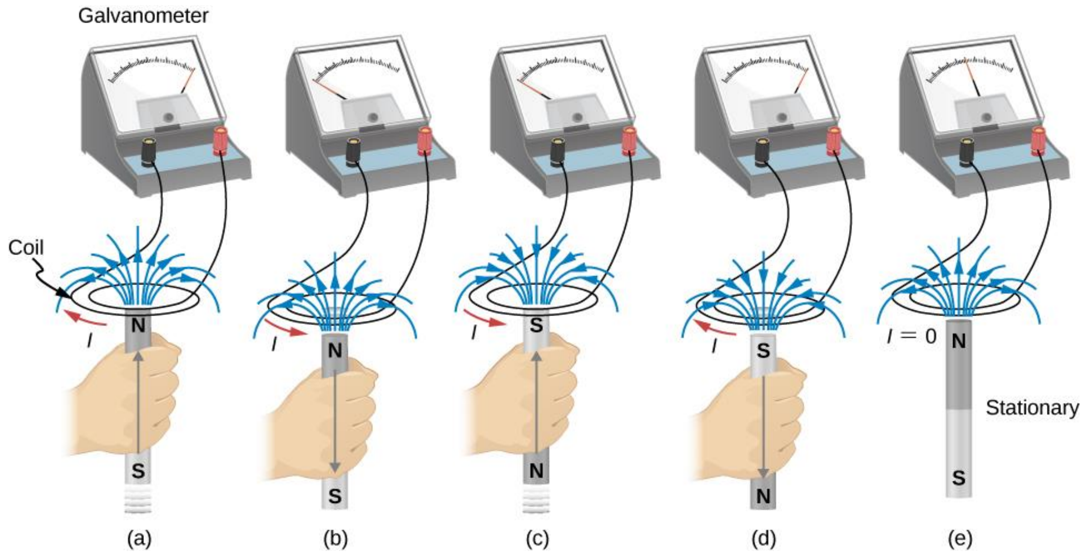


Volume 2 Chapter 13 - Electromagnetic Induction

Physics 2426

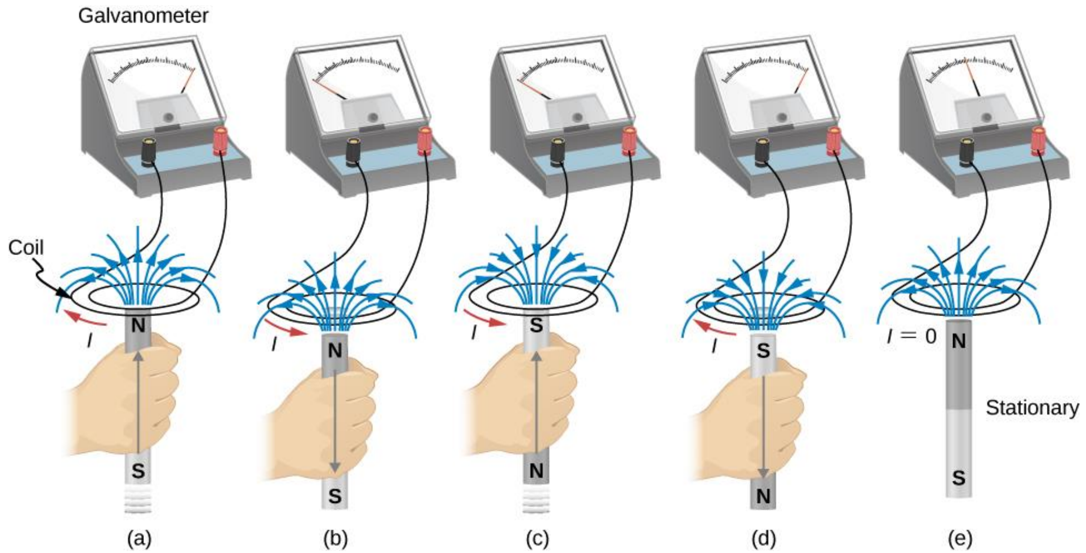
ashok kumar

Induced EMF



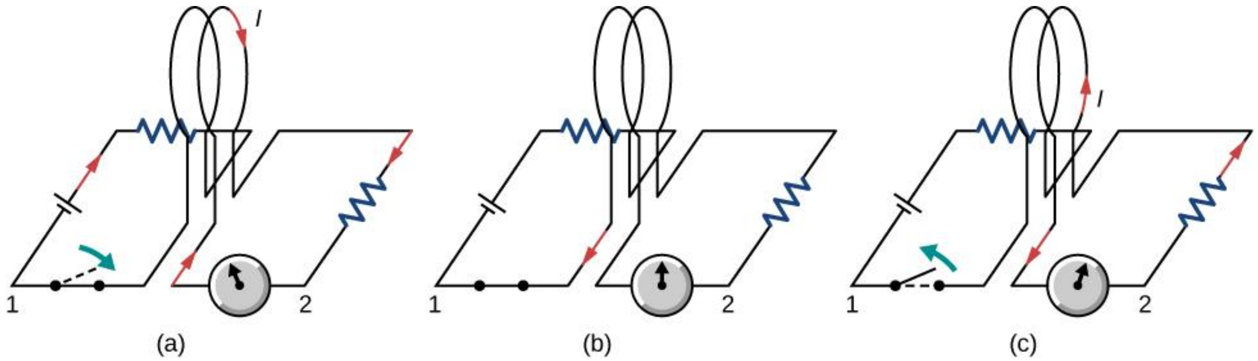
- Faraday used the equipment above to ask the question, ‘Can a magnetic field generate an electric current?’
- He found that the galvanometer only moved as he moved the magnet (or the coil).

Induced EMF



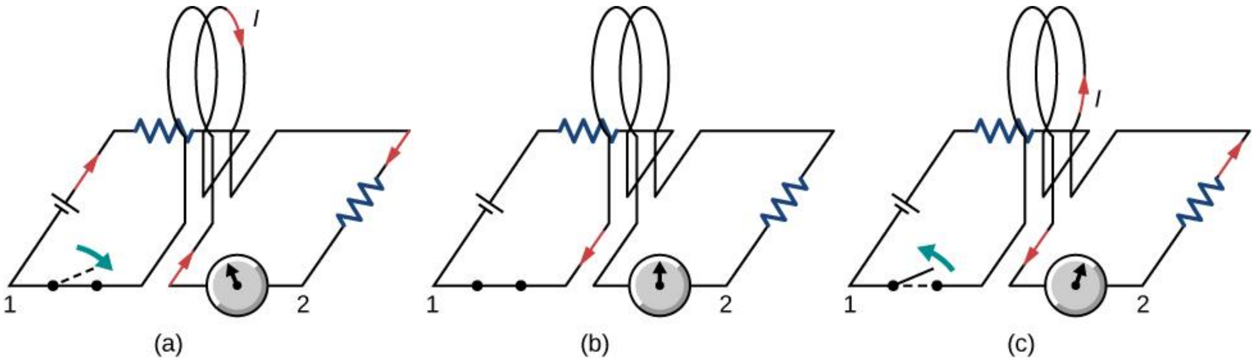
- He concluded that only a changing magnetic flux through the coil could produce an electric current.
- This implies that an electrical current must be due to a voltage or EMF which is induced in the loop by the changing magnetic flux.

Induced EMF



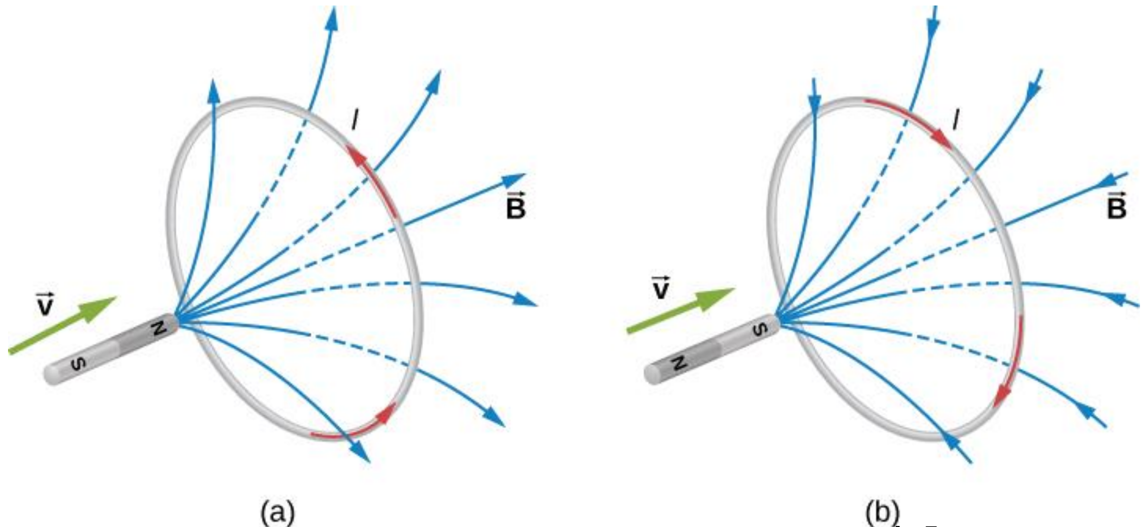
- His later experiments showed that by opening or closing a switch to change the current in one loop, an electrical current could be induced in a second loop.
- This process is called induction and is an important tool in modern electrical systems.

Induced EMF



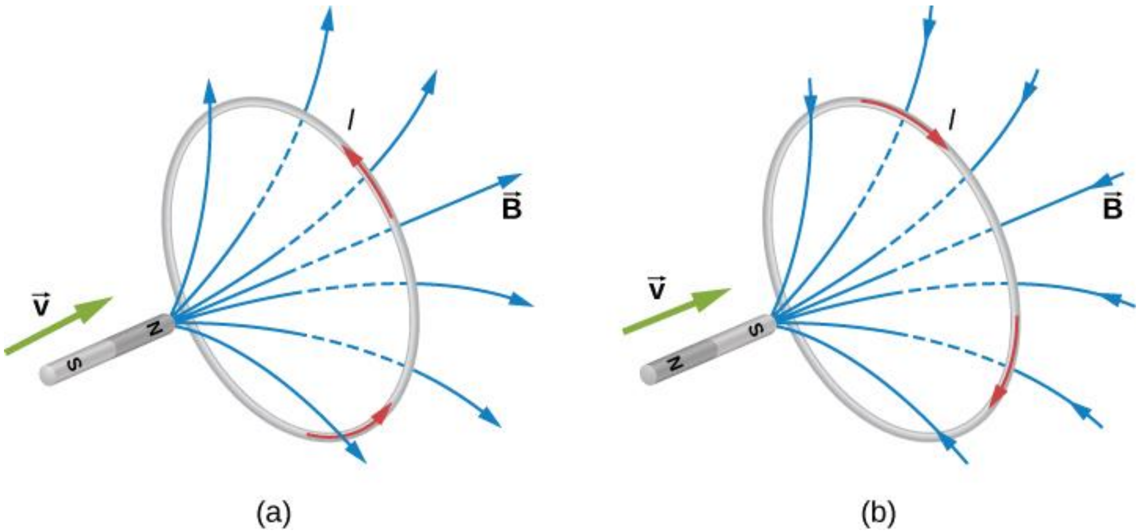
- Faraday's work can be summed up as follows:
- The EMF induced in a region is equal to the rate of change of the magnetic flux.
- $\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$
- The units of EMF are Volts (J/C).

Lenz's Law



- Why is there a negative sign in $\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$?
- Lenz found that **the direction of the induced current is such that it will produce its own magnetic field which will <u>oppose</u> original change in flux**.
- Note: A current that enhanced $\Delta\Phi$ would violate conservation of energy.

Lenz's Law

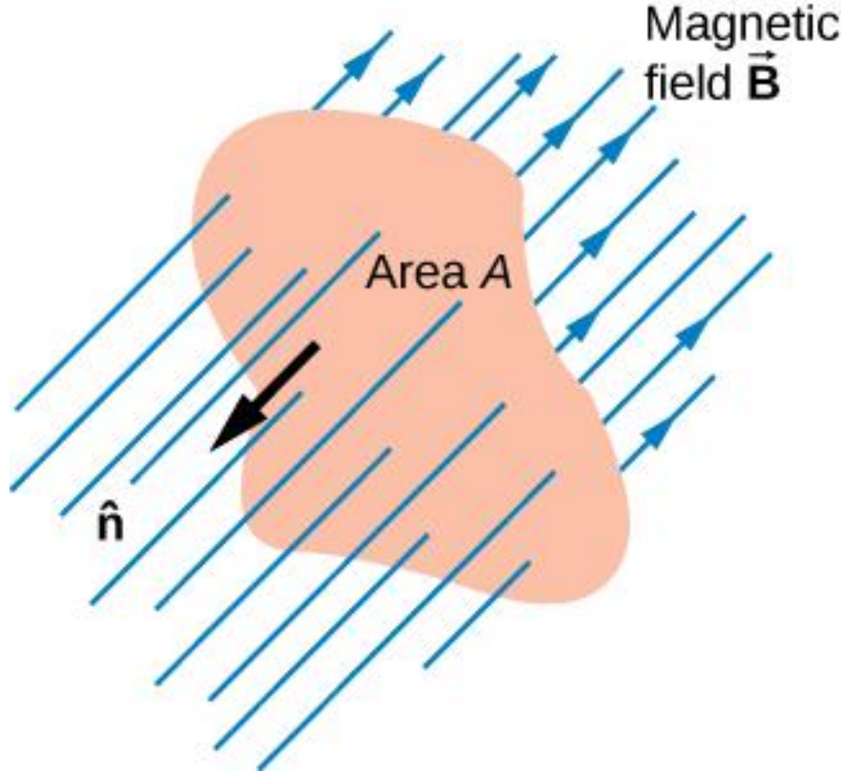


- This means that the induced EMF & current will act to reduce the changes in the inducing field.
- In (a) an N-field is increasing and the induced current will produce an opposing N-field.

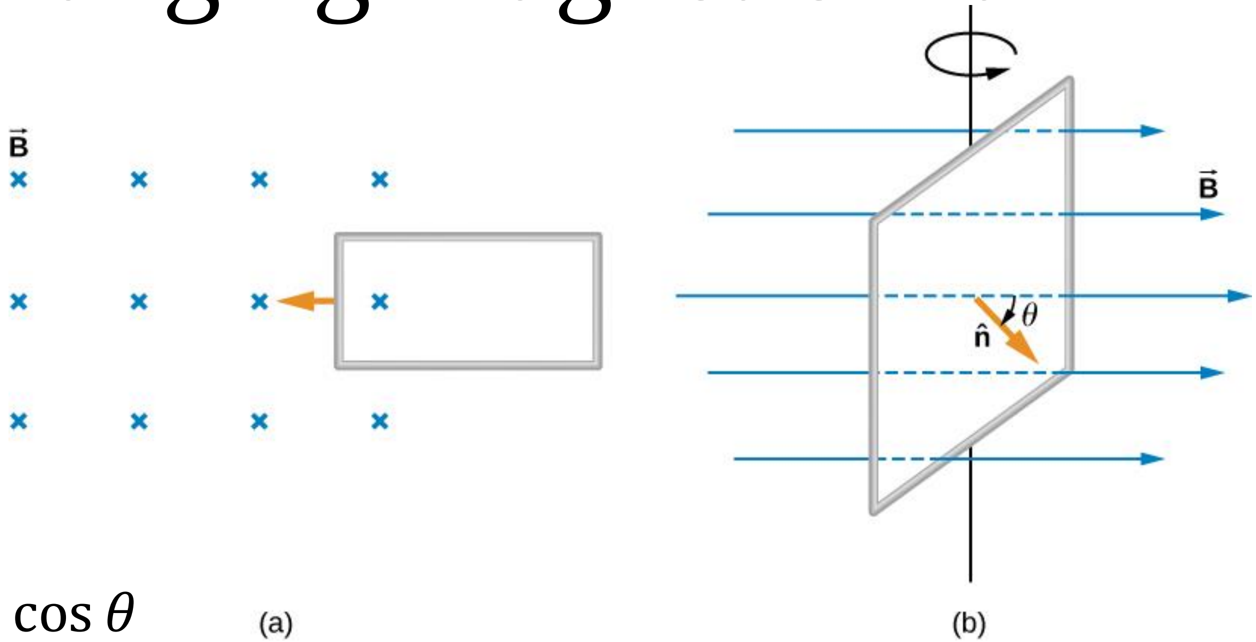
Magnetic Flux

The units for Φ_B: 1 Weber = $1 T \cdot m^2$

- To develop a formula for this effect, we need to calculate the number of magnetic field lines passing through an area, A.
- The result is similar to the result for the electric flux developed earlier.
- $\Phi_B = \vec{B} \bullet \vec{A} = BA\cos\theta$



Changing Magnetic Flux



- $\Phi_B = BA\cos\theta$
- There are three ways to change Φ and induce an EMF:
- B: We can change the field. (AC Current & Transformers)
- A: We can change the coil area. (Sliding Coil)
- $\cos\theta$: Or we can rotate the coil to change the angle. (Motors & generators)

Example 29-2: A loop of wire in a magnetic field.

A square loop of wire of side l = 5.0 cm is in a uniform magnetic field B = 0.16 T. What is the magnetic flux in the loop (a) when $\vec{B}$ is perpendicular to the face of the loop and (b) when $\vec{B}$ is at an angle of 30° to the area $\vec{A}$ of the loop? (c) What is the magnitude of the average current in the loop if it has a resistance of 0.012 Ω and it is rotated from position (b) to position (a) in 0.14 s?

EXAMPLE 29–2 **A loop of wire in a magnetic field.** A square loop of wire of side $\ell = 5.0\,\mathrm{cm}$ is in a uniform magnetic field $B = 0.16\,\mathrm{T}$. What is the magnetic flux in the loop (*a*) when $\vec{\mathbf{B}}$ is perpendicular to the face of the loop and (*b*) when $\vec{\mathbf{B}}$ is at an angle of 30° to the area $\vec{\mathbf{A}}$ of the loop? (*c*) What is the magnitude of the average current in the loop if it has a resistance of $0.012\,\Omega$ and it is rotated from position (*b*) to position (*a*) in 0.14 s?

APPROACH We use the definition $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$ to calculate the magnetic flux. Then we use Faraday's law of induction to find the induced emf in the coil, and from that the induced current $(I = \mathscr{E}/R)$.

SOLUTION The area of the coil is $A = \ell^2 = (5.0 \times 10^{-2}\,\mathrm{m})^2 = 2.5 \times 10^{-3}\,\mathrm{m}^2$, and the direction of $\vec{\mathbf{A}}$ is perpendicular to the face of the loop (Fig. 29–3).
(*a*) $\vec{\mathbf{B}}$ is perpendicular to the coil's face, and thus parallel to $\vec{\mathbf{A}}$ (Fig. 29–3), so

$$\begin{aligned}\Phi_B &= \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} \\ &= BA\cos 0° = (0.16\,\mathrm{T})(2.5 \times 10^{-3}\,\mathrm{m}^2)(1) = 4.0 \times 10^{-4}\,\mathrm{Wb}.\end{aligned}$$

(*b*) The angle between $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$ is 30°, so

$$\begin{aligned}\Phi_B &= \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} \\ &= BA\cos\theta = (0.16\,\mathrm{T})(2.5 \times 10^{-3}\,\mathrm{m}^2)\cos 30° = 3.5 \times 10^{-4}\,\mathrm{Wb}.\end{aligned}$$

(*c*) The magnitude of the induced emf is

$$\mathscr{E} = \frac{\Delta\Phi_B}{\Delta t} = \frac{(4.0 \times 10^{-4}\,\mathrm{Wb}) - (3.5 \times 10^{-4}\,\mathrm{Wb})}{0.14\,\mathrm{s}} = 3.6 \times 10^{-4}\,\mathrm{V}.$$

The current is then

$$I = \frac{\mathscr{E}}{R} = \frac{3.6 \times 10^{-4}\,\mathrm{V}}{0.012\,\Omega} = 0.030\,\mathrm{A} = 30\,\mathrm{mA}.$$

The minus signs in Eqs. 29–2a and b are there to remind us in which direction the induced emf acts. Experiments show that a current produced by an induced emf moves in a direction so that the magnetic field created by that current opposes the original change in flux.

1. (I) The magnetic flux through a coil of wire containing two loops changes at a constant rate from -58 Wb to $+38$ Wb in 0.42 s. What is the emf induced in the coil?

$\boxed{1.}$ The average induced emf is given by Eq. 29-2b.

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} = -N\frac{\Delta\Phi_B}{\Delta t} = -2\frac{38\,\text{Wb} - \left(-58\,\text{Wb}\right)}{0.42\,\text{s}} = \boxed{-460\,\text{V}}$$

2. (I) The north pole of the magnet in Fig. 29–36 is being inserted into the coil. In which direction is the induced current flowing through the resistor R?

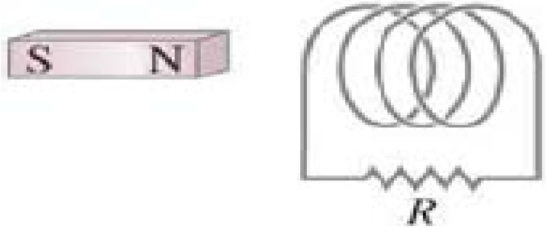


FIGURE 29–36 Problem 2.

As the magnet is pushed into the coil, the magnetic flux increases to the right. To oppose this increase, flux produced by the induced current must be to the left, so the induced current in the resistor will be $\boxed{\text{from right to left}}$.

6. (II) A 10.8-cm-diameter wire coil is initially oriented so that its plane is perpendicular to a magnetic field of 0.68 T pointing up. During the course of 0.16 s, the field is changed to one of 0.25 T pointing down. What is the average induced emf in the coil?

6. We choose up as the positive direction. The average induced emf is given by the "difference" version of Eq. 29-2a.

$$\mathscr{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{A\Delta B}{\Delta t} = -\frac{\pi(0.054\,\mathrm{m})^2(-0.25\,\mathrm{T} - 0.68\,\mathrm{T})}{0.16\,\mathrm{s}} = \boxed{5.3\times10^{-2}\,\mathrm{V}}$$

8. (II) (*a*) If the resistance of the resistor in Fig. 29–38 is slowly increased, what is the direction of the current induced in the small circular loop inside the larger loop? (*b*) What would it be if the small loop were placed outside the larger one, to the left?

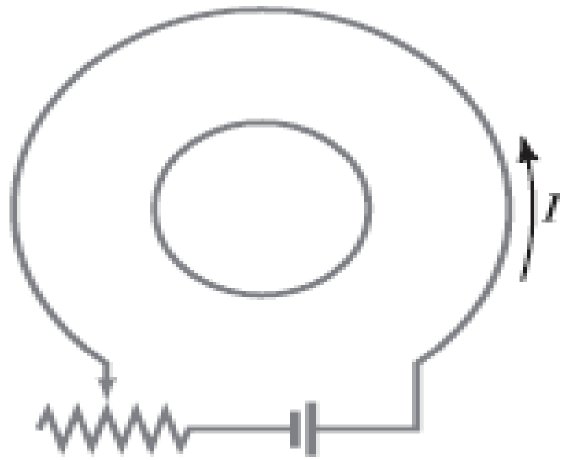


FIGURE 29–38 Problem 8.

8. (*a*) As the resistance is increased, the current in the outer loop will decrease. Thus the flux through the inner loop, which is out of the page, will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux out of the page, so the direction of the induced current will be $\boxed{\text{counterclockwise}}$.

(*b*) If the small loop is placed to the left, the flux through the small loop will be into the page and will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux into the page, so the direction of the induced current will be $\boxed{\text{clockwise}}$.

20. (II) The area of an elastic circular loop decreases at a constant rate, $dA/dt = -3.50 \times 10^{-2}\ \mathrm{m^2/s}$. The loop is in a magnetic field $B = 0.28\ \mathrm{T}$ whose direction is perpendicular to the plane of the loop. At $t = 0$, the loop has area $A = 0.285\ \mathrm{m^2}$. Determine the induced emf at $t = 0$, and at $t = 2.00\ \mathrm{s}$.

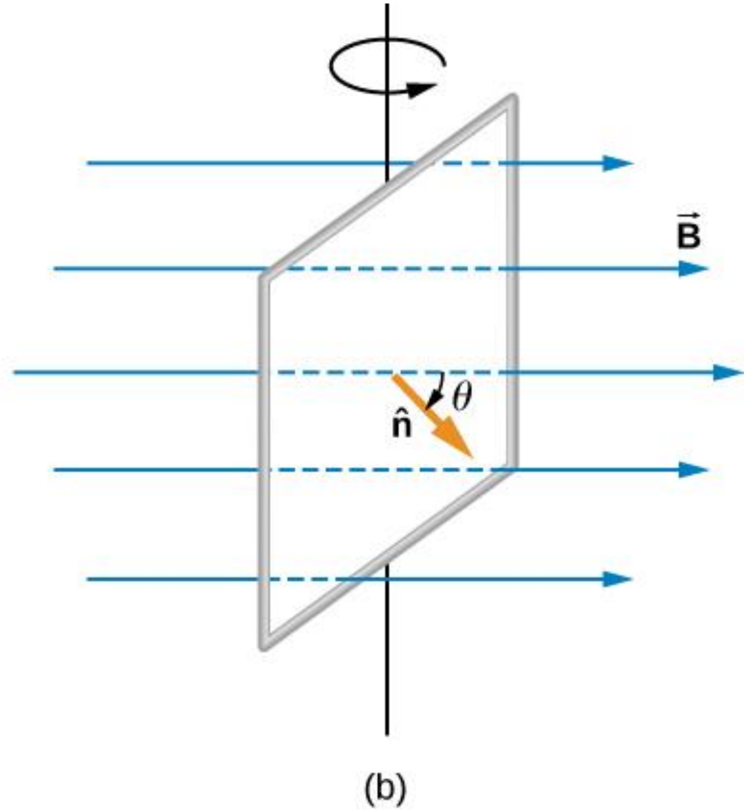
20. The induced emf is given by Eq. 29-2a. Since the field is uniform and is perpendicular to the area, the flux is simply the field times the area.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -(0.28\,\mathrm{T})\left(-3.50\times10^{-2}\,\mathrm{m^2/s}\right) = \boxed{9.8\,\mathrm{mV}}$$

Since the area changes at a constant rate, and the area has not shrunk to 0 at $t = 2.00$ s, the emf is the same for both times.

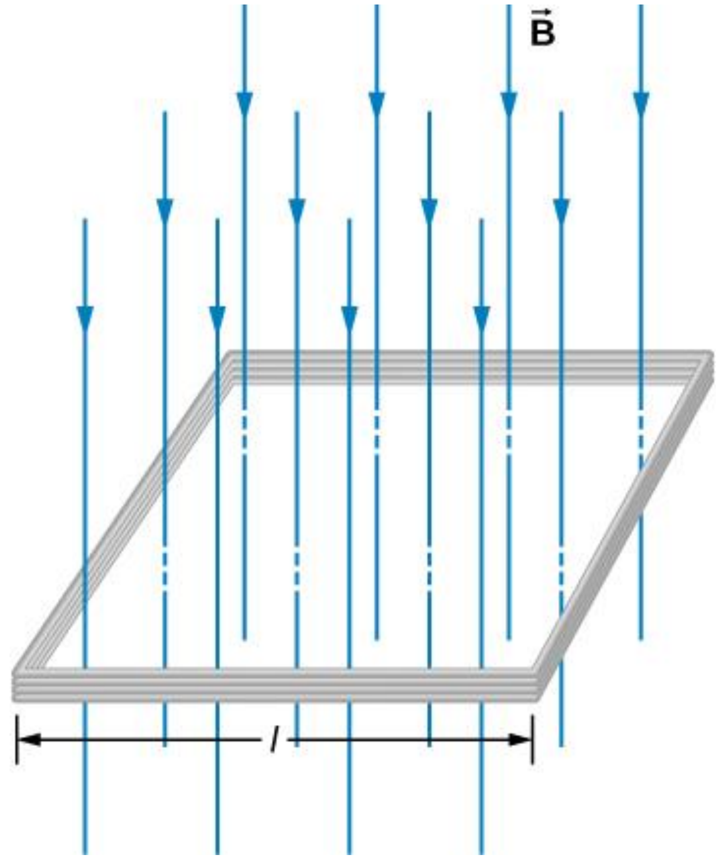
Faraday's & Law

- A square loop of wire 5.0 cm on a side is in a uniform field B = 0.16T.
- Calculate Φ_B when the loop and the field are at 0°.
- Calculate Φ_B when the loop and the field are at 30°.
- Calculate $\mathcal{E}$ when the loop rotates from 0° to 30° in 0.14s.
- Calculate I, if the loop resistance is 0.012 Ω



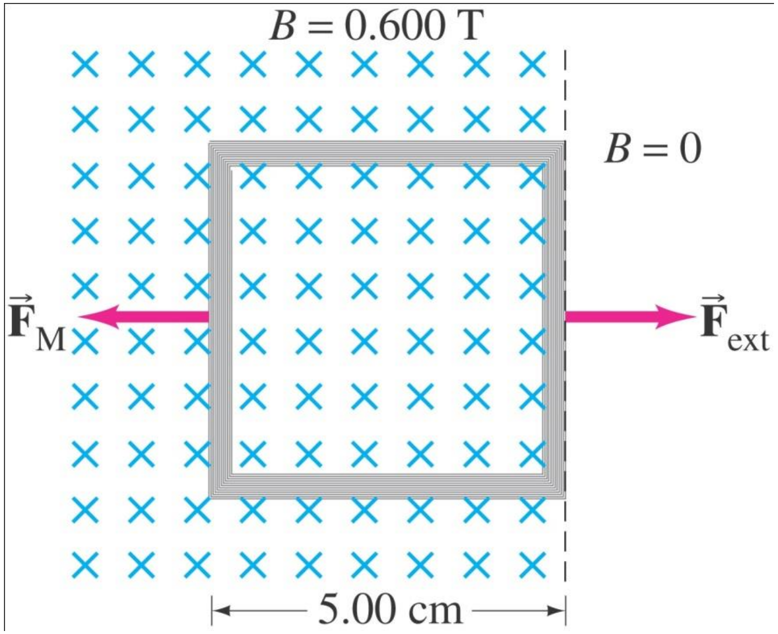
Faraday's Law of induction

- $\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$
- If we wind a coil with N loops, then the total EMF induced will be
- $\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$

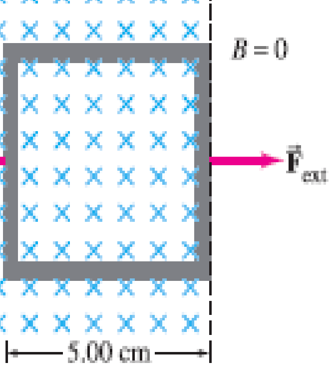


Example 29-5: Pulling a coil from a magnetic field.

A 100-loop square coil of wire, with side l = 5.00 cm and total resistance 100 Ω, is positioned perpendicular to a uniform 0.600-T magnetic field. It is quickly pulled from the field at constant speed (moving perpendicular to $\vec{B}$) to a region where B drops abruptly to zero. At t = 0, the right edge of the coil is at the edge of the field. It takes 0.100 s for the whole coil to reach the field-free region. Find (a) the rate of change in flux through the coil, and (b) the emf and current induced. (c) How much energy is dissipated in the coil? (d) What was the average force required (F_{ext})?



coil of wire, with side $\ell = 5.00\,\text{cm}$ and total resistance $100\,\Omega$, is positioned perpendicular to a uniform 0.600-T magnetic field, as shown in Fig. 29–10. It is quickly pulled from the field at constant speed (moving perpendicular to $\vec{\mathbf{B}}$) to a region where B drops abruptly to zero. At $t = 0$, the right edge of the coil is at the edge of the field. It takes 0.100 s for the whole coil to reach the field-free region. Find (*a*) the rate of change in flux through the coil, and (*b*) the emf and current induced. (*c*) How much energy is dissipated in the coil? (*d*) What was the average force required (F_{ext})?



9–10 Example 29–5. e coil in a magnetic field T is pulled abruptly to o a region where $B = 0$.

APPROACH We start by finding how the magnetic flux, $\Phi_B = BA$, changes during the time interval $\Delta t = 0.100\,\text{s}$. Faraday's law then gives the induced emf and Ohm's law gives the current.

SOLUTION (*a*) The area of the coil is $A = \ell^2 = (5.00 \times 10^{-2}\,\text{m})^2 = 2.50 \times 10^{-3}\,\text{m}^2$. The flux through one loop is initially $\Phi_B = BA = (0.600\,\text{T})(2.50 \times 10^{-3}\,\text{m}^2) = 1.50 \times 10^{-3}\,\text{Wb}$. After 0.100 s, the flux is zero. The rate of change in flux is constant (because the coil is square), equal to

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{0 - (1.50 \times 10^{-3}\,\text{Wb})}{0.100\,\text{s}} = -1.50 \times 10^{-2}\,\text{Wb/s}.$$

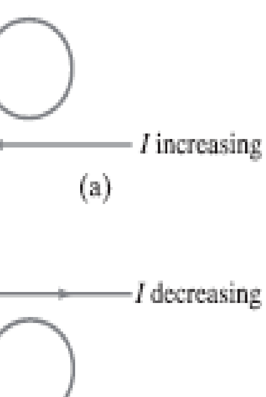
(*b*) The emf induced (Eq. 29–2) in the 100-loop coil during this 0.100-s interval is

$$\mathscr{E} = -N\frac{\Delta\Phi_B}{\Delta t} = -(100)(-1.50 \times 10^{-2}\,\text{Wb/s}) = 1.50\,\text{V}.$$

The current is found by applying Ohm's law to the 100-Ω coil:

$$I = \frac{\mathscr{E}}{R} = \frac{1.50\,\text{V}}{100\,\Omega} = 1.50 \times 10^{-2}\,\text{A} = 15.0\,\text{mA}.$$

9–11 Exercise B.



By Lenz's law, the current must be clockwise to produce more $\vec{\mathbf{B}}$ into the page and thus oppose the decreasing flux into the page.

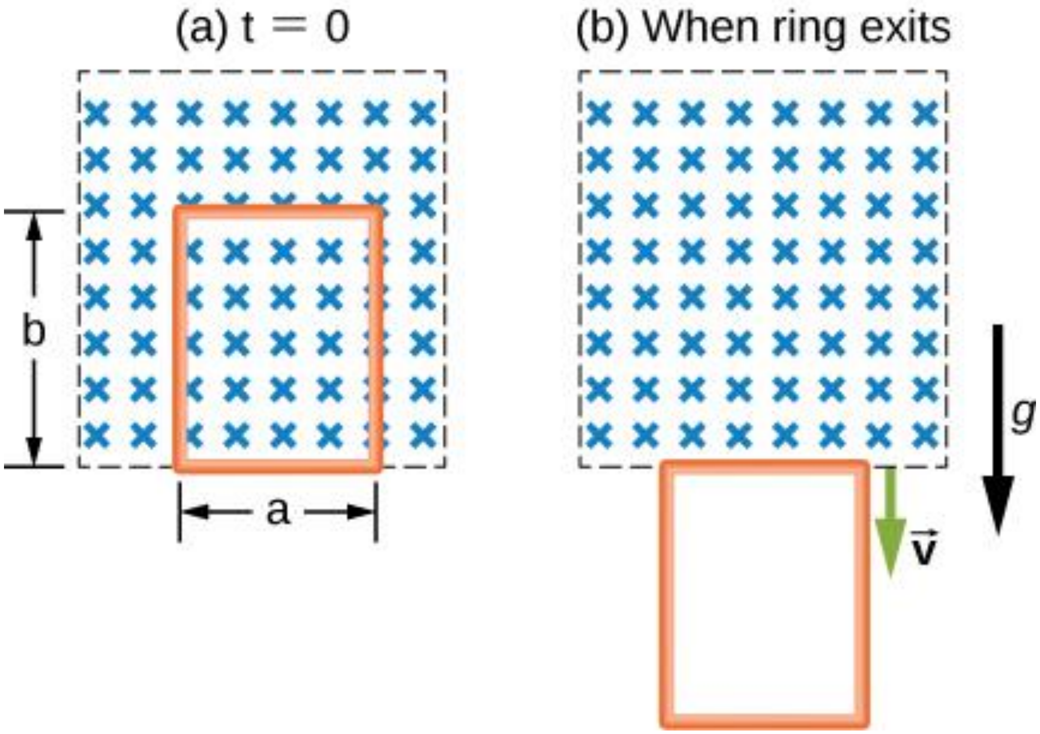
(*c*) The total energy dissipated in the coil is the product of the power $(= I^2R)$ and the time:

$$E = Pt = I^2Rt = (1.50 \times 10^{-2}\,\text{A})^2(100\,\Omega)(0.100\,\text{s}) = 2.25 \times 10^{-3}\,\text{J}.$$

(*d*) We can use the result of part (*c*) and apply the work-energy principle: the energy dissipated E is equal to the work W needed to pull the coil out of the field

Faraday's & Lenz's Law

- If the coil at right has 100 loops and 100 Ω resistance, how much force is required to pull it out of the field in 0.100 sec?
- Ans: 0.000450 N

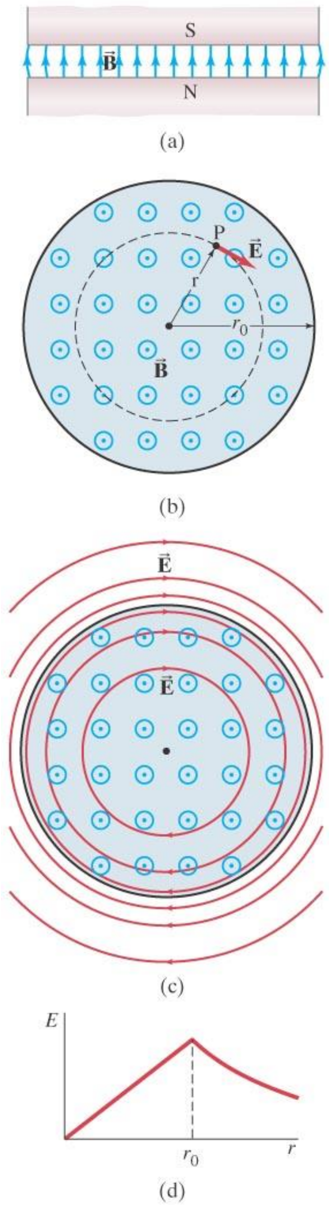


General form of Faraday's Law

- In earlier chapters, we found that when a current flows in a wire that there is an electric field and that this field gives rise to an electrical potential.
- $dV = \int \vec{E} \bullet d\vec{l}$
- We can use this to generalize Faraday's Law.

General form of Faraday's Law

- $\Delta V = \int \vec{E} \bullet d\vec{l}$
- For an EMF induced in a loop by a changing magnetic flux, we can integrate around the loop to get
- $\mathcal{E} = \oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt}$
- This form of the law implies that a changing Φ_B generates a local E-field whether or not any wires or charges are present.



E-field from a changing Φ_B

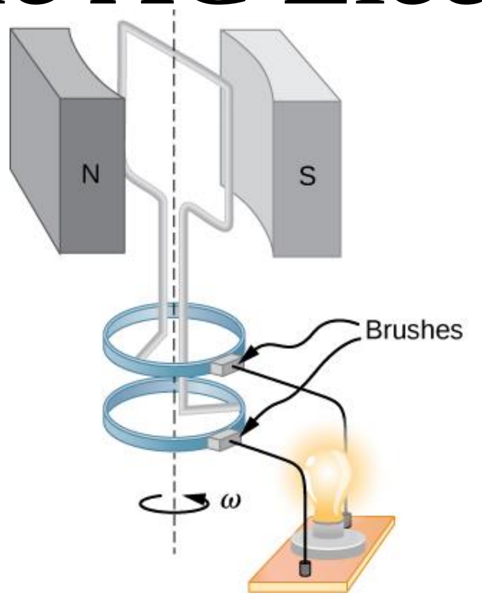
- Calculate the E-field generated by a changing flux.
- B in figure (a) is spatially uniform over a radius r_0 and is varying in time such that dB/dt is constant.
- $-\frac{d\Phi_B}{dt} = \oint \vec{E} \bullet d\vec{l} = 2\pi r E$, for any r
- Area of a circle of radius r is πr^2
- $\Phi_B = B \cdot A = \pi r^2 \cdot B$, for $r \le r_{max}$
- So $E = \frac{r}{2}\frac{dB}{dt}$, for $r \le r_{max}$

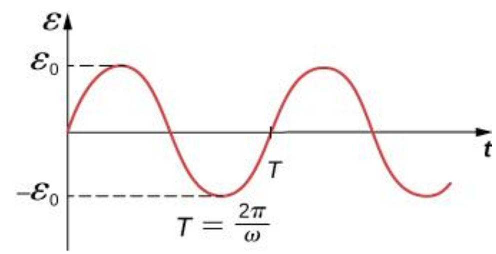
- And $\Phi_B = \pi r_{max}^2 \cdot B$, for $r \le r_{max}$
- So $E = \frac{r_{max}^2}{2r}\frac{dB}{dt}$, for $r > r_{max}$

General form of Faraday's Law

- $\mathcal{E} = \oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt}$
- We now have a set of equations which can completely define all possible properties of electric and magnetic fields.
- In chapter 16, we will combine these laws to produce a complete "Electromagnetic Theory"

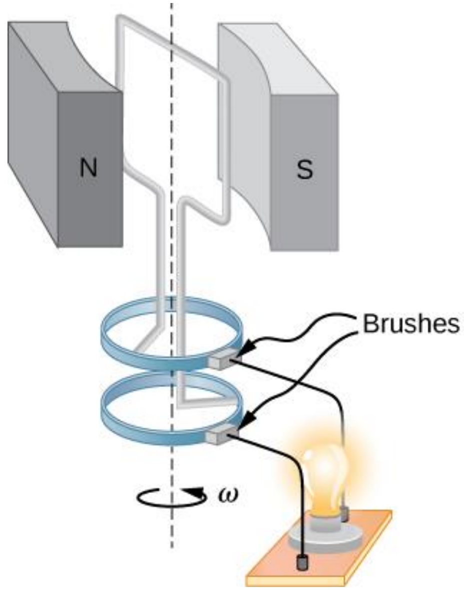
The AC Electric Generator

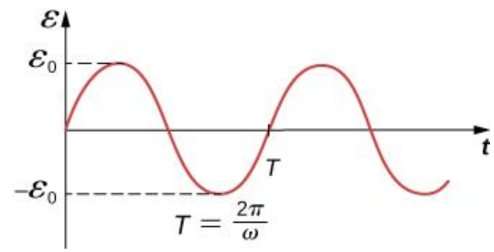




- Suppose we rotate a coil at a constant angular velocity, ω, in a uniform magnetic field.
- We can write the angle between the loop and **B** as
- $\theta = \omega \cdot t = 2\pi f \cdot t$

The AC Electric Generator

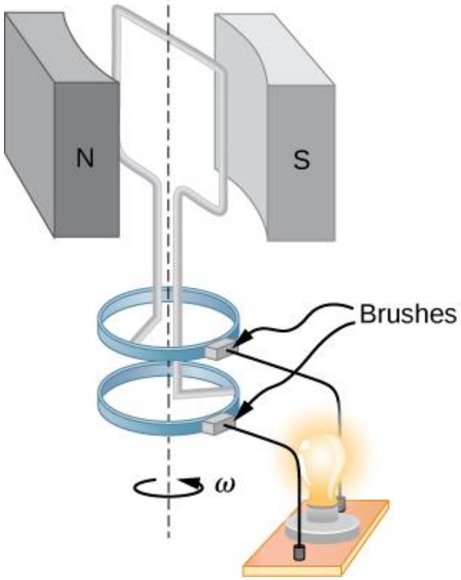




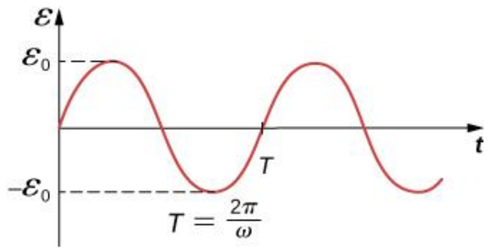
- Plugging this into the flux formula, we get
- $\Phi(t) = B \cdot A\cos(\omega t) = B \cdot A\cos(2\pi f t)$

- [Looks like a simple harmonic oscillator, doesn't it!]

The AC Electric Generator



$$\mathcal{E}_0 = N \cdot 2\pi f \cdot BA = N\omega BA$$



- The induced EMF will be
- $\mathcal{E} = -N\dfrac{d\Phi}{dt} = -NBA\dfrac{d\cos(2\pi f t)}{dt}$
- $\mathcal{E} = -N \cdot 2\pi f \cdot BA \cdot \sin(2\pi f t) = -\mathcal{E}_0 \sin \omega t$

38. (II) A simple generator has a 480-loop square coil 22.0 cm on a side. How fast must it turn in a 0.550-T field to produce a 120-V peak output?

38. From Eq. 29-4, the peak voltage is $\mathcal{E}_{\text{peak}} = NB\omega A$. Solve this for the rotation speed.

$$\mathcal{E}_{\text{peak}} = NB\omega A \quad \rightarrow \quad \omega = \frac{\mathcal{E}_{\text{peak}}}{NBA} = \frac{120\,\text{V}}{480(0.550\,\text{T})(0.220\,\text{m})^2} = 9.39\,\text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{9.39\,\text{rad/s}}{2\pi\,\text{rad/rev}} = \boxed{1.49\,\text{rev/s}}$$

Example 29-9: An ac generator.

The armature of a 60-Hz ac generator rotates in a 0.15-T magnetic field. If the area of the coil is $2.0 \times 10^{-2}\ m^2$, how many loops must the coil contain if the peak output is to be $\%_0 = 170$ V?

EXAMPLE 29–9 **An ac generator.** The armature of a 60-Hz ac generator rotates in a 0.15-T magnetic field. If the area of the coil is $2.0 \times 10^{-2}\,\mathrm{m}^2$, how many loops must the coil contain if the peak output is to be $\mathscr{E}_0 = 170\,\mathrm{V}$?

APPROACH From Eq. 29–4 we see that the maximum emf is $\mathscr{E}_0 = NBA\omega$.

SOLUTION We solve Eq. 29–4 for N with $\omega = 2\pi f = (6.28)(60\,\mathrm{s}^{-1}) = 377\,\mathrm{s}^{-1}$:

$$N = \frac{\mathscr{E}_0}{BA\omega} = \frac{170\,\mathrm{V}}{(0.15\,\mathrm{T})(2.0 \times 10^{-2}\,\mathrm{m}^2)(377\,\mathrm{s}^{-1})} = 150\ \text{turns}.$$

Figure 29.8a shows a simple *alternator,* a device that generates an emf. A rectangular loop is rotated with constant angular speed ω about the axis shown. The magnetic field $\vec{B}$ is uniform and constant. At time $t = 0$, $\phi = 0$. Determine the induced emf.

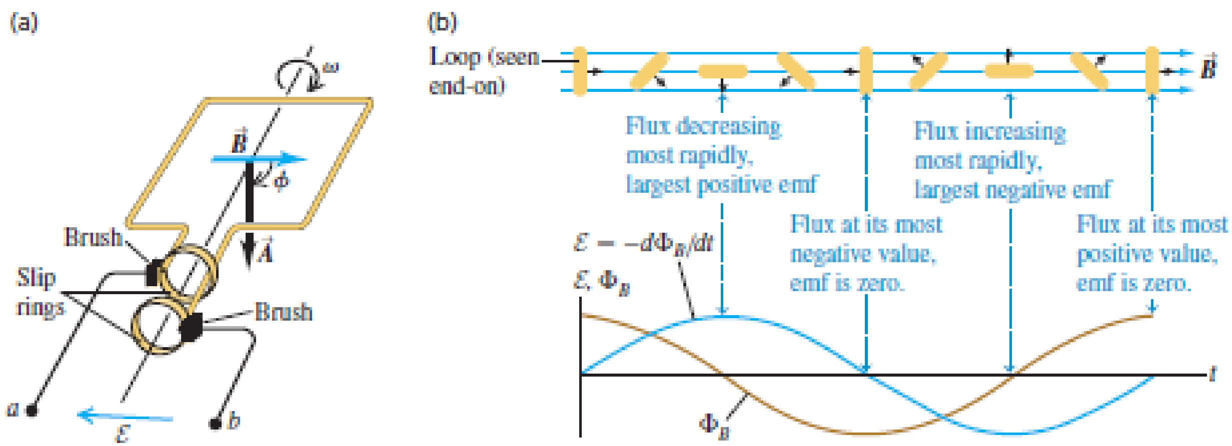
SOLUTION

IDENTIFY and SET UP: The magnetic field $\vec{B}$ and the loop area A are constant, but the flux through the loop varies because the loop rotates and so the angle ϕ between $\vec{B}$ and the area vector $\vec{A}$ changes (Fig. 29.8a). Because the angular speed is constant and $\phi = 0$ at $t = 0$, the angle as a function of time is $\phi = \omega t$.

EXECUTE: The magnetic field is uniform over the loop, so the magnetic flux is $\Phi_B = BA\cos\phi = BA\cos\omega t$. Hence, by Faraday's law [Eq. (29.3)] the induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA\cos\omega t) = \omega BA\sin\omega t$$

29.8 (a) Schematic diagram of an alternator. A conducting loop rotates in a magnetic field, producing an emf. Connections from each end of the loop to the external circuit are made by means of that end's slip ring. The system is shown at the time when the angle $\phi = \omega t = 90°$. (b) Graph of the flux through the loop and the resulting emf between terminals a and b, along with the corresponding positions of the loop during one complete rotation.



Continued

29.9 A commercial alternator uses many loops of wire wound around a barrel-like structure called an armature. The armature and wire remain stationary while electromagnets rotate on a shaft (not shown) through the center of the armature. The resulting induced emf is far larger than would be possible with a single loop of wire.

The alternator in Example 29.3 produces a sinusoidally varying emf and hence an alternating current. **Figure 29.10a** shows a *direct-current* (dc) *generator* that produces an emf that always has the same sign. The arrangement of split rings, called a *commutator*, reverses the connections to the external circuit at angular positions at which the emf reverses. Figure 29.10b shows the resulting emf. Commercial dc generators have a large number of coils and commutator segments, smoothing out the bumps in the emf so that the terminal voltage is not only one-directional but also practically constant. This brush-and-commutator arrangement is the same as that in the direct-current motor discussed in Section 27.8. The motor's *back emf* is just the emf induced by the changing magnetic flux through its rotating coil. Consider a motor with a square, 500-turn coil 10.0 cm on a side. If the magnetic field has magnitude 0.200 T, at what rotation speed is the *average* back emf of the motor equal to 112 V?

SOLUTION

IDENTIFY and SET UP: As far as the rotating loop is concerned, the situation is the same as in Example 29.3 except that we now have N turns of wire. Without the commutator, the emf would alternate between positive and negative values and have an average value of zero (see Fig. 29.8b). With the commutator, the emf is never negative and its average value is positive (Fig. 29.10b). We'll use our result from Example 29.3 to obtain an expression for this average value and solve it for the rotational speed ω.

29.10 (a) Schematic diagram of a dc generator, using a split-ring commutator. The ring halves are attached to the loop and rotate with it. (b) Graph of the resulting induced emf between terminals a and b. Compare to Fig. 29.8b.

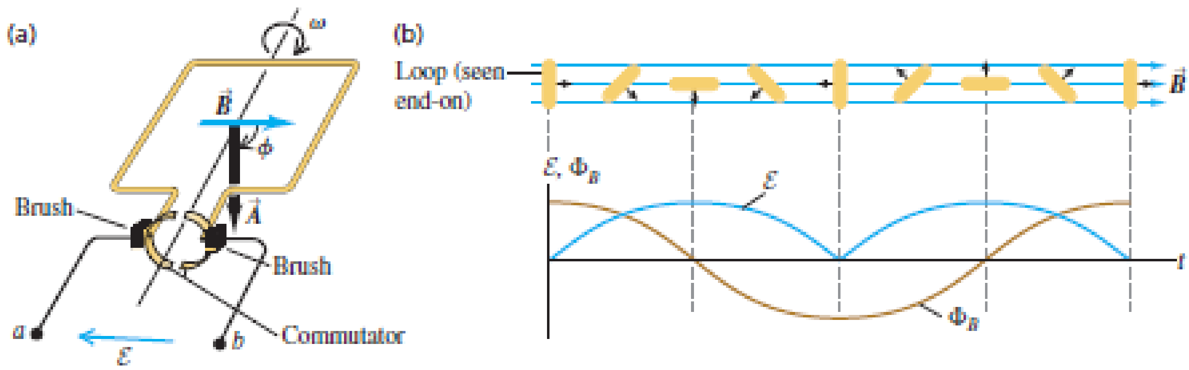
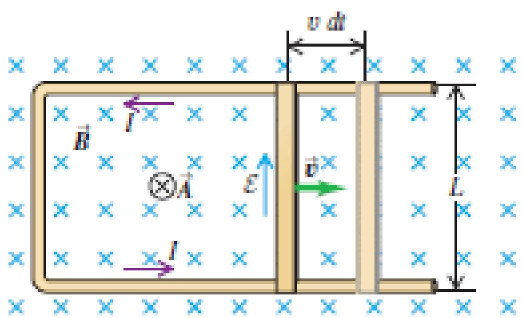


Figure 29.11 shows a U-shaped conductor in a uniform magnetic field $\vec{B}$ perpendicular to the plane of the figure and directed *into* the page. We lay a metal rod (the "slidewire") with length L across the two arms of the conductor, forming a circuit, and move it to the right with constant velocity $\vec{v}$. This induces an emf and a current, which is why this device is called a *slidewire generator*. Find the magnitude and direction of the resulting induced emf.

SOLUTION

IDENTIFY and SET UP: The magnetic flux changes because the area of the loop—bounded on the right by the moving rod—is increasing. Our target variable is the emf $\mathcal{E}$ induced in this expanding loop. The magnetic field is uniform over the area of the loop, so we can find the flux from $\Phi_B = BA\cos\phi$. We choose the area vector $\vec{A}$ to point straight into the page, in the same direction as $\vec{B}$. With this choice a positive emf will be one that is directed clockwise around the loop. (You can check this with the right-hand rule: Using your right hand, point your thumb into the page and curl your fingers as in Fig. 29.6.)

29.11 A slidewire generator. The magnetic field $\vec{B}$ and the vector area $\vec{A}$ are both directed into the figure. The increase in magnetic flux (caused by an increase in area) induces the emf and current.



EXECUTE: Since $\vec{B}$ and $\vec{A}$ point in the same direction, the angle $\phi = 0$ and $\Phi_B = BA$. The magnetic field magnitude B is constant, so the induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt}$$

To calculate dA/dt, note that in a time dt the sliding rod moves a distance $v\,dt$ (Fig. 29.11) and the loop area increases by an amount $dA = Lv\,dt$. Hence the induced emf is

$$\mathcal{E} = -B\frac{Lv\,dt}{dt} = -BLv$$

The minus sign tells us that the emf is directed *counterclockwise* around the loop. The induced current is also counterclockwise, as shown in the figure.

EVALUATE: The emf of a slidewire generator is constant if $\vec{v}$ is constant. Hence the slidewire generator is a *direct-current* generator. It's not a very practical device because the rod eventually moves beyond the U-shaped conductor and loses contact, after which the current stops.

In the slidewire generator of Example 29.5, energy is dissipated in the circuit owing to its resistance. Let the resistance of the circuit (made up of the moving slidewire and the U-shaped conductor that connects the ends of the slidewire) at a given point in the slidewire's motion be R. Find the rate at which energy is dissipated in the circuit and the rate at which work must be done to move the rod through the magnetic field.

SOLUTION

IDENTIFY and SET UP: Our target variables are the *rates* at which energy is dissipated and at which work is done. Energy is dissipated in the circuit at the rate $P_{\text{dissipated}} = I^2R$. The current I in

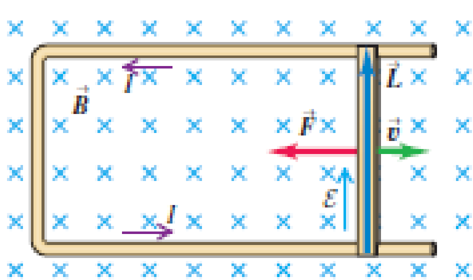
Continued

the circuit equals $|\mathcal{E}|/R$; we found an expression for the induced emf $\mathcal{E}$ in this circuit in Example 29.5. There is a magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ on the rod, where $\vec{L}$ points along the rod in the direction of the current. **Figure 29.12** shows that this force is opposite to the rod velocity $\vec{v}$; to maintain the motion, whoever is pushing the rod must apply a force of equal magnitude in the direction of $\vec{v}$. This force does work at the rate $P_{\text{applied}} = Fv$.

EXECUTE: First we'll calculate $P_{\text{dissipated}}$. From Example 29.5, $\mathcal{E} = -BLv$, so the current in the rod is $I = |\mathcal{E}|/R = Blv/R$. Hence

$$P_{\text{dissipated}} = I^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$$

29.12 The magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ that acts on the rod due to the induced current is to the left, opposite to $\vec{v}$.



To calculate P_{applied}, we first calculate the magnitude of $\vec{F} = I\vec{L} \times \vec{B}$. Since $\vec{L}$ and $\vec{B}$ are perpendicular, this magnitude is

$$F = ILB = \frac{BLv}{R}LB = \frac{B^2L^2v}{R}$$

The applied force has the same magnitude and does work at the rate

$$P_{\text{applied}} = Fv = \frac{B^2L^2v^2}{R}$$

EVALUATE: The rate at which work is done is exactly *equal* to the rate at which energy is dissipated in the resistance.

CAUTION **You can't violate energy conservation** You might think that reversing the direction of $\vec{B}$ or of $\vec{v}$ would allow the magnetic force $\vec{F} = I\vec{L} \times \vec{B}$ to be in the *same* direction as $\vec{v}$. This would be a neat trick. Once the rod was moving, the changing magnetic flux would induce an emf and a current, and the magnetic force on the rod would make it move even faster, increasing the emf and current until the rod was moving at tremendous speed and producing electrical power at a prodigious rate. If this seems too good to be true and a violation of energy conservation, that's because it is. Reversing $\vec{B}$ also reverses the sign of the induced emf and current and hence the direction of $\vec{L}$, so the magnetic force still opposes the motion of the rod; a similar result holds true if we reverse $\vec{v}$.

12. (II) Part of a single rectangular loop of wire with dimensions shown in Fig. 29–40 is situated inside a region of uniform magnetic field of 0.650 T. The total resistance of the loop is 0.280 Ω. Calculate the force required to pull the loop from the field (to the right) at a constant velocity of 3.40 m/s. Neglect gravity.

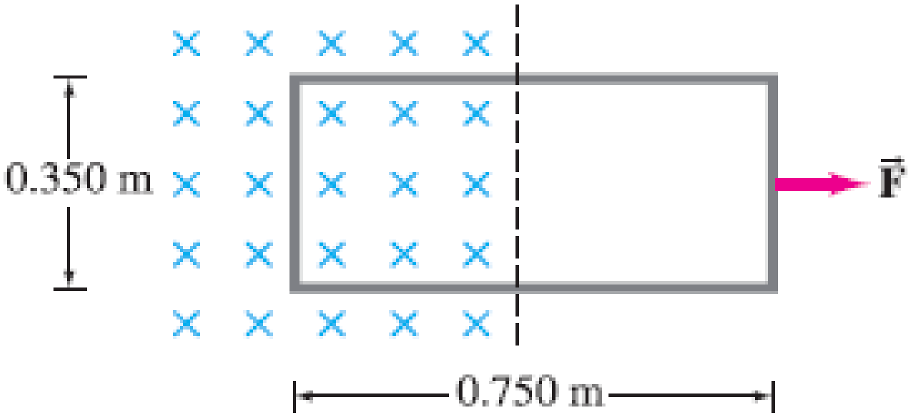


FIGURE 29–40 Problem 12.

12. As the loop is pulled from the field, the flux through the loop decreases, causing an induced EMF whose magnitude is given by Eq. 29-3, $\mathcal{E} = B\ell v$. Because the inward flux is decreasing, the induced flux will be into the page, so the induced current is clockwise, given by $I = \mathcal{E}/R$. Because this current in the left-hand side of the loop is in a downward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, given by $F = I\ell B$.

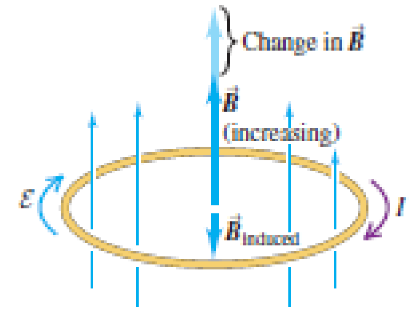
$$F = I\ell B = \frac{\mathcal{E}}{R}\ell B = \frac{B\ell v}{R}\ell B = \frac{B^2\ell^2 v}{R} = \frac{(0.650\,\mathrm{T})^2(0.350\,\mathrm{m})^2(3.40\,\mathrm{m/s})}{0.280\,\Omega} = \boxed{0.628\,\mathrm{N}}$$

In **Fig. 29.13** there is a uniform magnetic field $\vec{B}$ through the coil. The magnitude of the field is increasing, so there is an induced emf. Use Lenz's law to determine the direction of the resulting induced current.

SOLUTION

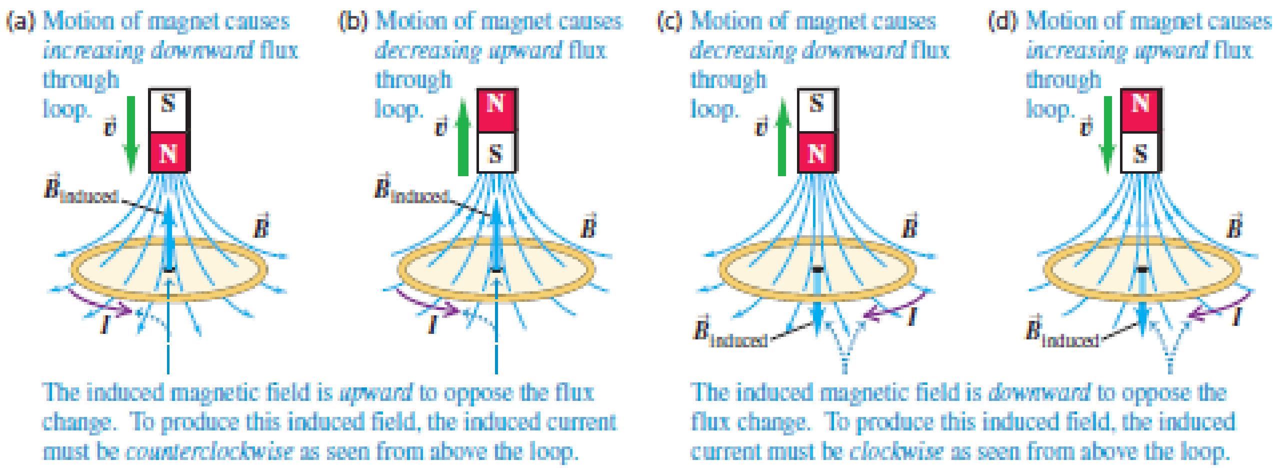
This situation is the same as in Example 29.1 (Section 29.2). By Lenz's law the induced current must produce a magnetic field $\vec{B}_{\text{induced}}$ inside the coil that is downward, opposing the change in flux. From the right-hand rule we described in Section 28.5 for the direction of the magnetic field produced by a circular loop,

29.13 The induced current due to the change in $\vec{B}$ is clockwise, as seen from above the loop. The added field $\vec{B}_{\text{induced}}$ that it causes is downward, opposing the change in the upward field $\vec{B}$.



Continued

29.14 Directions of induced currents as a bar magnet moves along the axis of a conducting loop. If the bar magnet is stationary, there is no induced current.



Suppose the moving rod in Fig. 29.15b is 0.10 m long, the velocity v is 2.5 m/s, the total resistance of the loop is 0.030 Ω, and B is 0.60 T. Find the motional emf, the induced current, and the force acting on the rod.

SOLUTION

IDENTIFY and SET UP: We'll find the motional emf $\mathcal{E}$ from Eq. (29.6) and the current from the values of $\mathcal{E}$ and the resistance R. The force on the rod is a *magnetic* force, exerted by $\vec{B}$ on the current in the rod; we'll find this force by using $\vec{F} = I\vec{L} \times \vec{B}$.

EXECUTE: From Eq. (29.6) the motional emf is

$$\mathcal{E} = vBL = (2.5\ \text{m/s})(0.60\ \text{T})(0.10\ \text{m}) = 0.15\ \text{V}$$

The induced current in the loop is

$$I = \frac{\mathcal{E}}{R} = \frac{0.15\ \text{V}}{0.030\ \Omega} = 5.0\ \text{A}$$

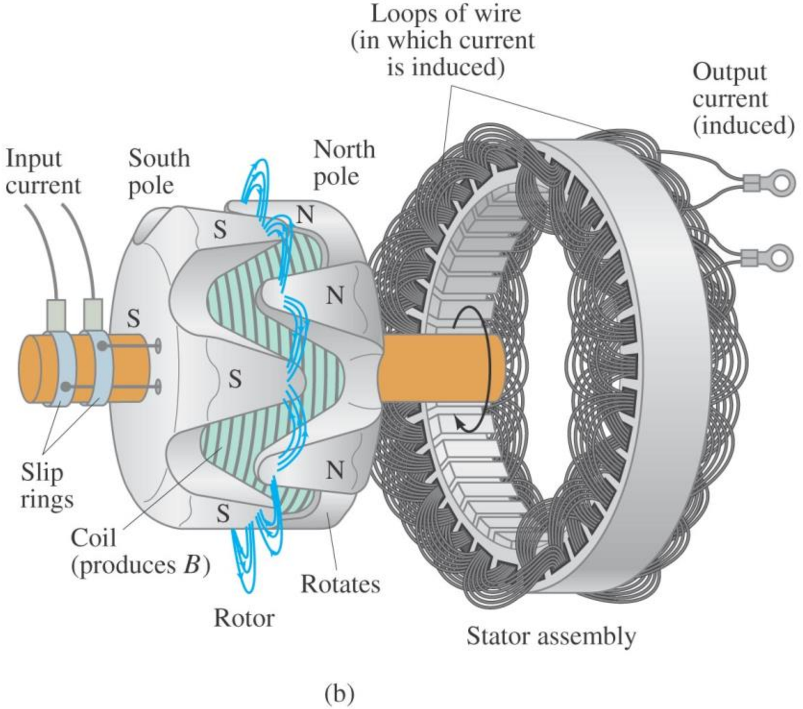
In the expression for the magnetic force, $\vec{F} = I\vec{L} \times \vec{B}$, the vector $\vec{L}$ points in the same direction as the induced current in the rod (from b to a in Fig. 29.15). The right-hand rule for vector products shows that this force is directed *opposite* to the rod's motion. Since $\vec{L}$ and $\vec{B}$ are perpendicular, the force has magnitude

$$F = ILB = (5.0\ \text{A})(0.10\ \text{m})(0.60\ \text{T}) = 0.30\ \text{N}$$

EVALUATE: We can check our answer for the direction of $\vec{F}$ by using Lenz's law. If we take the area vector $\vec{A}$ to point into the plane of the loop, the magnetic flux is positive and increasing as the rod moves to the right and increases the area of the loop. Lenz's law tells us that a force appears to oppose this increase in flux. Hence the force on the rod is to the left, opposite its motion.

Your Car’s Alternator

- Your car’s electrical system is run by a battery.
- To keep the battery charged, an alternator, turned by the car’s engine is used.
- The battery supplies current to the rotating electromagnet, the ‘rotor’ which then induces an EMF and current in the surrounding coils, the ‘stator’.
- The resulting AC current is converted to DC by a rectifier circuit.

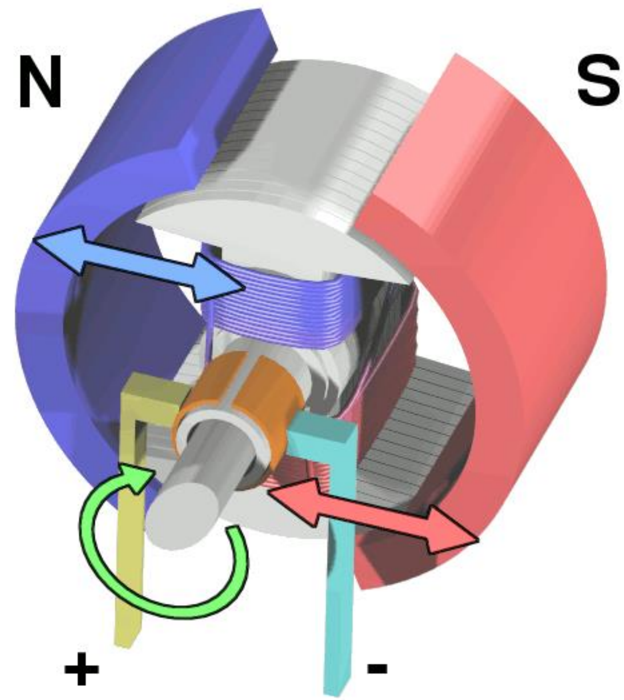


Electric Power Generation

- The picture at right shows the typical modern generator.
- Now days, the coils sit still and the magnets rotate around the coils.
- This eliminates the need for slip rings which are a major source of wear and problems.

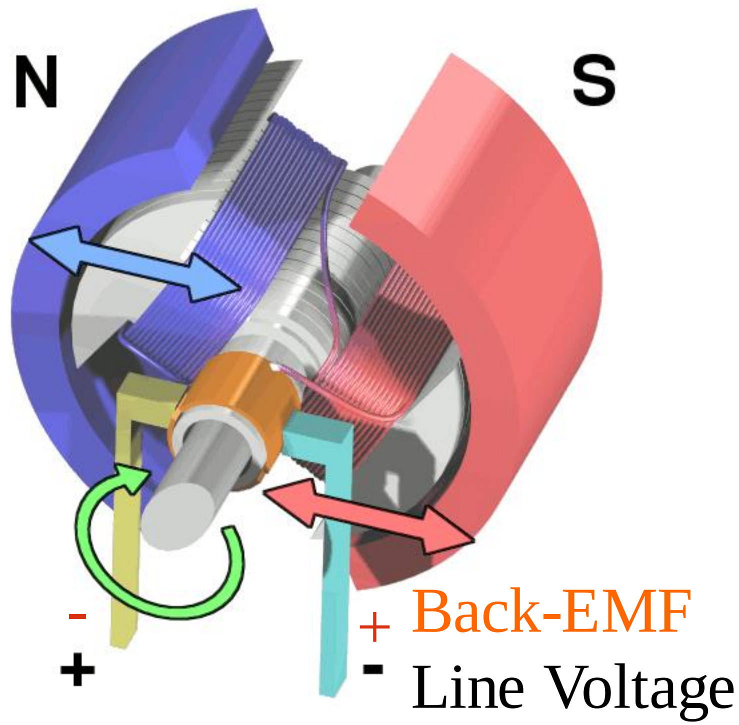
Another Look at Motors

- There is really no difference in the basic construction of an electric motor and an electric generator.
- Both consist of coils of wire that turn in a magnetic field.



Another Look at Motors

- That means that when a motor is turning it is generating an EMF.
- Since an induced EMF acts to prevent the changing B-field flux, the induced EMF will be opposite the applied voltage that is running the motor.
- This is referred to as the 'back-EMF' of the motor.



Another Look at Motors

- The slower an electric motor turns, the less back-EMF it generates.
- If a motor is allowed to run without a load, it will spin fast enough that the back-EMF nearly cancels the applied voltage.
- As the motor is required to turn a greater load, it slows down, back-EMF decreases and the motor uses more power. [$I \cdot R = V_{applied} - E_{back}$]

29-5 Back EMF and Counter Torque; Eddy Currents

An electric motor turns because there is a torque on it due to the current. We would expect the motor to accelerate unless there is some sort of drag torque.

That drag torque exists, and is due to the induced emf, called a back emf.

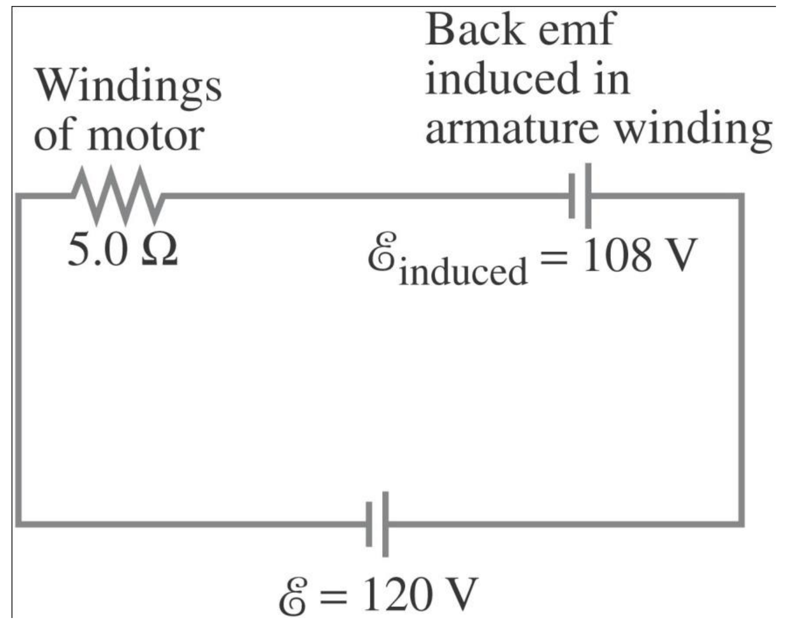
*42. (I) A motor has an armature resistance of $3.05\ \Omega$. If it draws 7.20 A when running at full speed and connected to a 120-V line, how large is the back emf?

42. When the motor is running at full speed, the back emf opposes the applied emf, to give the net across the motor.

$$\mathscr{E}_{\text{applied}} - \mathscr{E}_{\text{back}} = IR \;\rightarrow\; \mathscr{E}_{\text{back}} = \mathscr{E}_{\text{applied}} - IR = 120\,\text{V} - (7.20\,\text{A})(3.05\,\Omega) = \boxed{98\,\text{V}}$$

Example 29-10: Back emf in a motor.

The armature windings of a dc motor have a resistance of 5.0 Ω. The motor is connected to a 120-V line, and when the motor reaches full speed against its normal load, the back emf is 108 V. Calculate (a) the current into the motor when it is just starting up, and (b) the current when the motor reaches full speed.



EXAMPLE 29–10 **Back emf in a motor.** The armature windings of a dc motor have a resistance of 5.0 Ω. The motor is connected to a 120-V line, and when the motor reaches full speed against its normal load, the back emf is 108 V. Calculate (*a*) the current into the motor when it is just starting up, and (*b*) the current when the motor reaches full speed.

APPROACH As the motor is just starting up, it is turning very slowly, so there is no induced back emf. The only voltage is the 120-V line. The current is given by Ohm's law with $R = 5.0\,\Omega$. At full speed, we must include as emfs both the 120-V applied emf and the opposing back emf.

SOLUTION (*a*) At start up, the current is controlled by the 120 V applied to the coil's 5.0-Ω resistance. By Ohm's law,

$$I = \frac{V}{R} = \frac{120\,\mathrm{V}}{5.0\,\Omega} = 24\,\mathrm{A}.$$

(*b*) When the motor is at full speed, the back emf must be included in the equivalent circuit shown in Fig. 29–20. In this case, Ohm's law (or Kirchhoff's rule) gives

$$120\,\mathrm{V} - 108\,\mathrm{V} = I(5.0\,\Omega).$$

Therefore

$$I = \frac{12\,\mathrm{V}}{5.0\,\Omega} = 2.4\,\mathrm{A}.$$

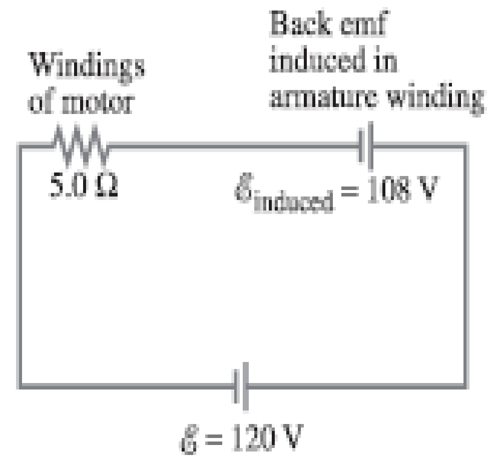


FIGURE 29–20 Circuit of a motor showing induced back emf. Example 29–10.

Conceptual Example 29-11: Motor overload.

When using an appliance such as a blender, electric drill, or sewing machine, if the appliance is overloaded or jammed so that the motor slows appreciably or stops while the power is still connected, the device can burn out and be ruined. Explain why this happens.

CONCEPTUAL EXAMPLE 29–11 **Motor overload.** When using an appliance such as a blender, electric drill, or sewing machine, if the appliance is overloaded or jammed so that the motor slows appreciably or stops while the power is still connected, the device can burn out and be ruined. Explain why this happens.

RESPONSE The motors are designed to run at a certain speed for a given applied voltage, and the designer must take the expected back emf into account. If the rotation speed is reduced, the back emf will not be as high as expected ($\mathscr{E} \propto \omega$, Eq. 29–4), and the current will increase, and may become large enough that the windings of the motor heat up to the point of ruining the motor.

29-6 Transformers and Transmission of Power

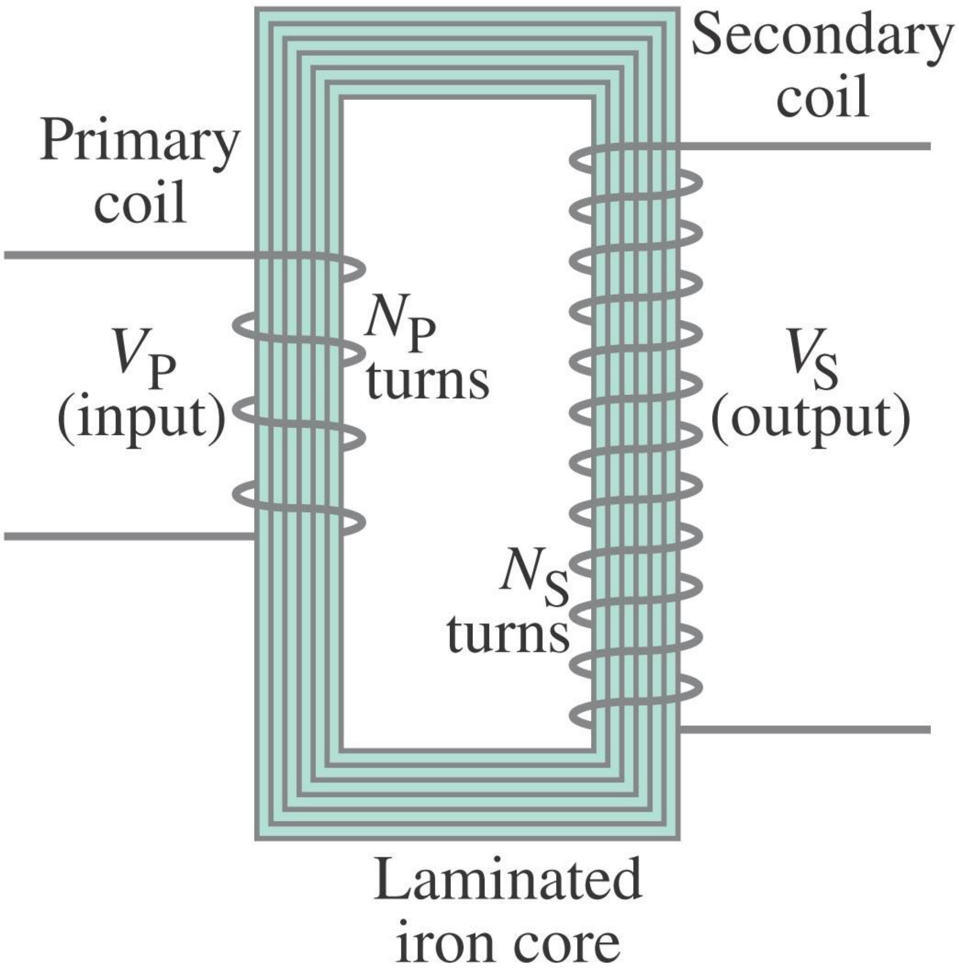
A transformer consists of two coils, either interwoven or linked by an iron core. A changing emf in one induces an emf in the other.

The ratio of the emfs is equal to the ratio of the number of turns in each coil:

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}.$$

29-6 Transformers and Transmission of Power

This is a step-up transformer – the emf in the secondary coil is larger than the emf in the primary:



29-6 Transformers and Transmission of Power

Energy must be conserved; therefore, in the absence of losses, the ratio of the currents must be the inverse of the ratio of turns:

$$\frac{I_S}{I_P} = \frac{N_P}{N_S}.$$

46. (I) A transformer has 620 turns in the primary coil and 85 in the secondary coil. What kind of transformer is this, and by what factor does it change the voltage? By what factor does it change the current?

46. Because $N_S < N_P$, this is a $\boxed{\text{step-down}}$ transformer. Use Eq. 29-5 to find the voltage ratio, and Eq. 29-6 to find the current ratio.

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{85\text{ turns}}{620\text{ turns}} = \boxed{0.14} \qquad \frac{I_S}{I_P} = \frac{N_P}{N_S} = \frac{620\text{ turns}}{85\text{ turns}} = \boxed{7.3}$$

48. (II) A model-train transformer plugs into 120-V ac and draws 0.35 A while supplying 7.5 A to the train. (*a*) What voltage is present across the tracks? (*b*) Is the transformer step-up or step-down?

48. (*a*) Use Eqs. 29-5 and 29-6 to relate the voltage and current ratios.

$$\frac{V_S}{V_P}=\frac{N_S}{N_P}\ ;\ \frac{I_S}{I_P}=\frac{N_P}{N_S}\quad\rightarrow\quad\frac{V_S}{V_P}=\frac{I_P}{I_S}\quad\rightarrow\quad V_S=V_P\frac{I_P}{I_S}=(120\,\text{V})\frac{0.35\,\text{A}}{7.5\,\text{A}}=\boxed{5.6\,\text{V}}$$

(*b*) Because $V_S<V_P$, this is a $\boxed{\text{step-down}}$ transformer.

29-6 Transformers and Transmission of Power

Example 29-12: Cell phone charger.

The charger for a cell phone contains a transformer that reduces 120-V ac to 5.0-V ac to charge the 3.7-V battery. (It also contains diodes to change the 5.0-V ac to 5.0-V dc.) Suppose the secondary coil contains 30 turns and the charger supplies 700 mA. Calculate (a) the number of turns in the primary coil, (b) the current in the primary, and (c) the power transformed.

EXAMPLE 29–12 **Cell phone charger.** The charger for a cell phone contains a transformer that reduces 120-V (or 240-V) ac to 5.0-V ac to charge the 3.7-V battery (Section 26–4). (It also contains diodes to change the 5.0-V ac to 5.0-V dc.) Suppose the secondary coil contains 30 turns and the charger supplies 700 mA. Calculate (*a*) the number of turns in the primary coil, (*b*) the current in the primary, and (*c*) the power transformed.

APPROACH We assume the transformer is ideal, with no flux loss, so we can use Eq. 29–5 and then Eq. 29–6.

SOLUTION (*a*) This is a step-down transformer, and from Eq. 29–5 we have

$$N_{\mathrm{P}} = N_{\mathrm{S}}\frac{V_{\mathrm{P}}}{V_{\mathrm{S}}} = \frac{(30)(120\,\mathrm{V})}{(5.0\,\mathrm{V})} = 720\ \text{turns.}$$

(*b*) From Eq. 29–6

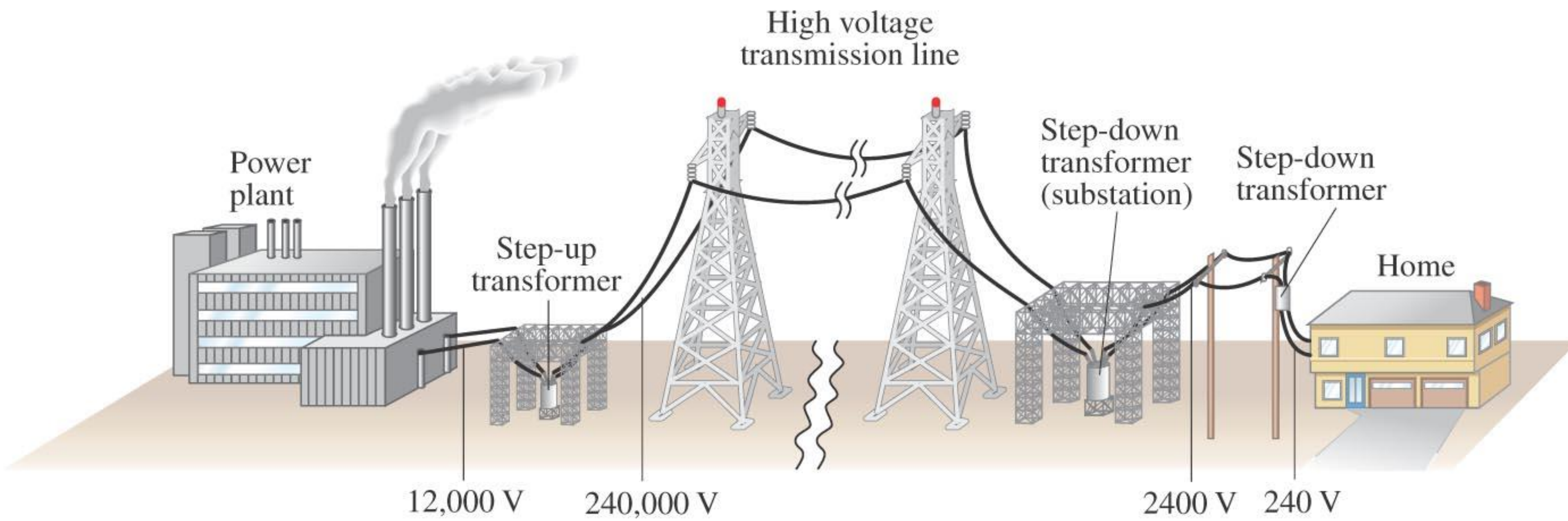
$$I_{\mathrm{P}} = I_{\mathrm{S}}\frac{N_{\mathrm{S}}}{N_{\mathrm{P}}} = (0.70\,\mathrm{A})\left(\frac{30}{720}\right) = 29\,\mathrm{mA}.$$

(*c*) The power transformed is

$$P = I_{\mathrm{S}}V_{\mathrm{S}} = (0.70\,\mathrm{A})(5.0\,\mathrm{V}) = 3.5\,\mathrm{W}.$$

29-6 Transformers and Transmission of Power

Transformers work only if the current is changing; this is one reason why electricity is transmitted as ac.



29-6 Transformers and Transmission of Power

Example 29-13: Transmission lines.

An average of 120 kW of electric power is sent to a small town from a power plant 10 km away. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is transmitted at (a) 240 V and (b) 24,000 V.

EXAMPLE 29–13 **Transmission lines.** An average of 120 kW of electric power is sent to a small town from a power plant 10 km away. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is transmitted at (*a*) 240 V and (*b*) 24,000 V.

APPROACH We cannot use $P = V^2/R$ because if R is the resistance of the transmission lines, we don't know the voltage drop along them; the given voltages are applied across the lines plus the load (the town). But we can determine the current I in the lines $(= P/V)$, and then find the power loss from $P_L = I^2R$, for both cases (*a*) and (*b*).

SOLUTION (*a*) If 120 kW is sent at 240 V, the total current will be

$$I = \frac{P}{V} = \frac{1.2 \times 10^5\,\mathrm{W}}{2.4 \times 10^2\,\mathrm{V}} = 500\,\mathrm{A}.$$

The power loss in the lines, P_L, is then

$$P_L = I^2R = (500\,\mathrm{A})^2(0.40\,\Omega) = 100\,\mathrm{kW}.$$

Thus, over 80% of all the power would be wasted as heat in the power lines!

(*b*) If 120 kW is sent at 24,000 V, the total current will be

$$I = \frac{P}{V} = \frac{1.2 \times 10^5\ \text{W}}{2.4 \times 10^4\ \text{V}} = 5.0\ \text{A}.$$

The power loss in the lines is then

$$P_L = I^2 R = (5.0\ \text{A})^2 (0.40\ \Omega) = 10\ \text{W},$$

which is less than $\frac{1}{100}$ of 1%: a far better efficiency!

An average of 120 kW of electric power is sent from a power plant. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is sent at (a) 240 V, and (b) 24,000 V.

(a) $$I = \frac{P}{V} = \frac{1.2\times10^5 W}{2.4\times10^2 V} = 500A$$

$$P_L = I^2R = (500A)^2(0.40\Omega) = 100kW$$

83% loss!!

(b) $$I = \frac{P}{V} = \frac{1.2\times10^5 W}{2.4\times10^4 V} = 5.0A$$

$$P_L = I^2R = (5.0A)^2(0.40\Omega) = 10W$$

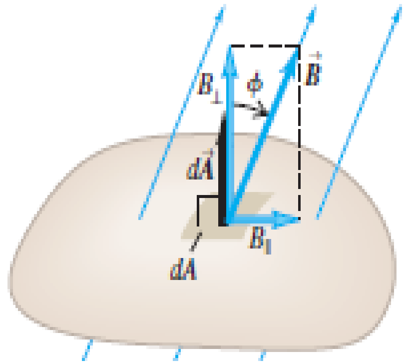
0.0083% loss

The common element in all induction effects is changing magnetic flux through a circuit. Before stating the simple physical law that summarizes all of the kinds of experiments described in Section 29.1, let's first review the concept of magnetic flux Φ_B (which we introduced in Section 27.3). For an infinitesimal-area element $d\vec{A}$ in a magnetic field $\vec{B}$ (**Fig. 29.3**), the magnetic flux $d\Phi_B$ through the area is

$$d\Phi_B = \vec{B} \cdot d\vec{A} = B_\perp \, dA = B \, dA \cos\phi$$

where $B_\perp$ is the component of $\vec{B}$ perpendicular to the surface of the area element and ϕ is the angle between $\vec{B}$ and $d\vec{A}$. (As in Chapter 27, be careful to distinguish between two quantities named "phi," ϕ and Φ_B.) The total magnetic flux Φ_B through a finite area is the integral of this expression over the area:

29.3 Calculating the magnetic flux through an area element.

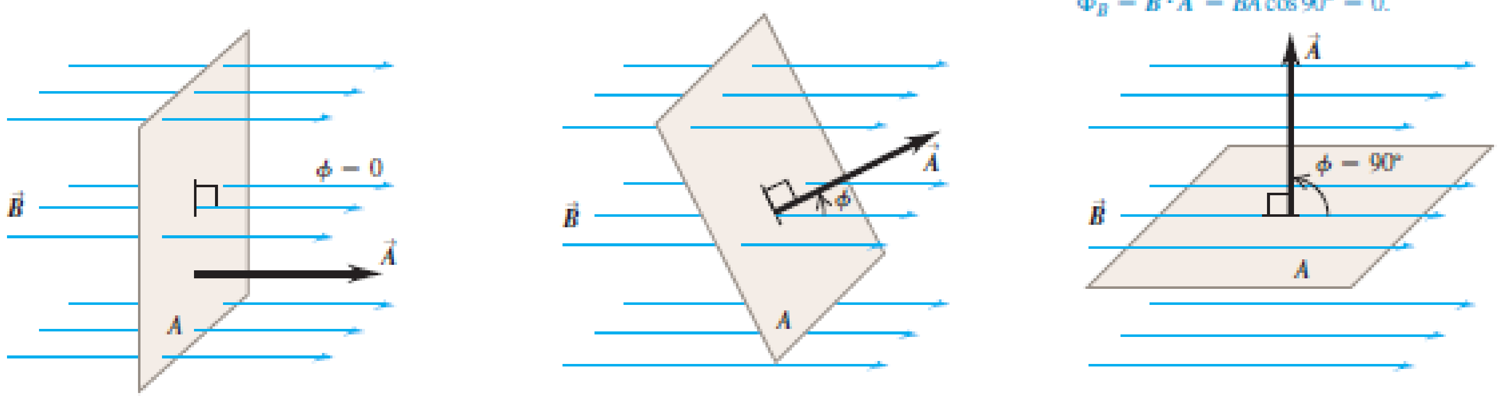


29.4 Calculating the flux of a uniform magnetic field through a flat area. (Compare to Fig. 22.6, which shows the rules for calculating the flux of a uniform *electric* field.)

Surface is face-on to magnetic field:
- $\vec{B}$ and $\vec{A}$ are parallel (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 0$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA$.

Surface is tilted from a face-on orientation by an angle ϕ:
- The angle between $\vec{B}$ and $\vec{A}$ is ϕ.
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\phi$.

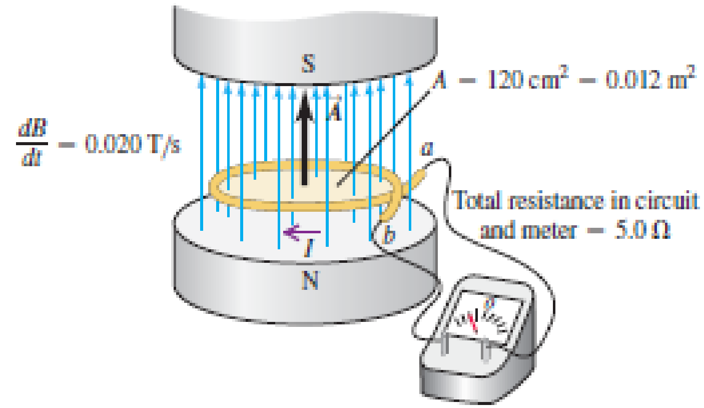
Surface is edge-on to magnetic field:
- $\vec{B}$ and $\vec{A}$ are perpendicular (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 90°$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos 90° = 0$.



Faraday's law: The induced emf in a closed loop ... $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ ... equals the negative of the time rate of change of magnetic flux through the loop.

The magnetic field between the poles of the electromagnet in **Fig. 29.5** is uniform at any time, but its magnitude is increasing at the rate of 0.020 T/s. The area of the conducting loop in the field is 120 cm^2, and the total circuit resistance, including the meter, is 5.0 Ω. (a) Find the induced emf and the induced current in the circuit. (b) If the loop is replaced by one made of an insulator, what effect does this have on the induced emf and induced current?

29.5 A stationary conducting loop in an increasing magnetic field.



SOLUTION

IDENTIFY and SET UP: The magnetic flux Φ_B through the loop changes as the magnetic field changes. Hence there will be an induced emf $\mathcal{E}$ and an induced current I in the loop. We calculate Φ_B from Eq. (29.2), then find $\mathcal{E}$ by using Faraday's law. Finally, we calculate I from $\mathcal{E} = IR$, where R is the total resistance of the circuit that includes the loop.

EXECUTE: (a) The area vector $\vec{A}$ for the loop is perpendicular to the plane of the loop; we take $\vec{A}$ to be vertically upward. Then $\vec{A}$ and $\vec{B}$ are parallel, and because $\vec{B}$ is uniform the magnetic flux through the loop is $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos 0 = BA$. The area $A = 0.012\ \text{m}^2$ is constant, so the rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = \frac{dB}{dt}A = (0.020\ \text{T/s})(0.012\ \text{m}^2)$$
$$= 2.4 \times 10^{-4}\ \text{V} = 0.24\ \text{mV}$$

This, apart from a sign that we haven't discussed yet, is the induced emf $\mathcal{E}$. The corresponding induced current is

$$I = \frac{\mathcal{E}}{R} = \frac{2.4 \times 10^{-4}\ \text{V}}{5.0\ \Omega} = 4.8 \times 10^{-5}\ \text{A} = 0.048\ \text{mA}$$

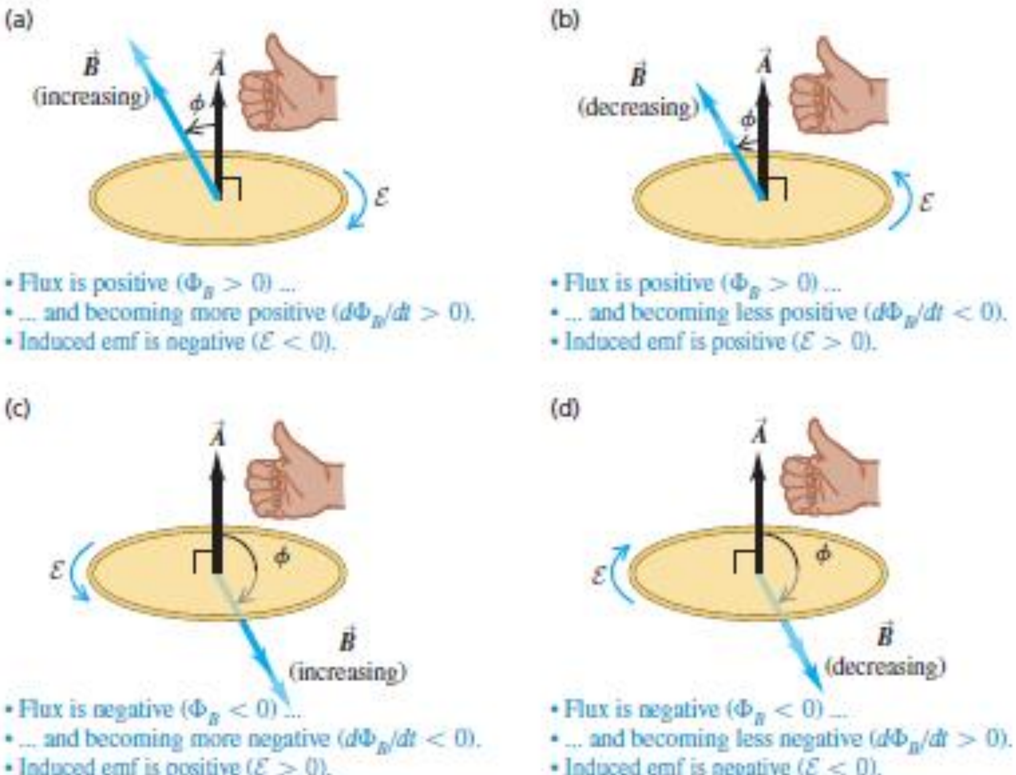
(b) By changing to an insulating loop, we've made the resistance of the loop very high. Faraday's law, Eq. (29.3), does not involve the resistance of the circuit in any way, so the induced *emf* does not change. But the *current* will be smaller, as given by the equation $I = \mathcal{E}/R$. If the loop is made of a perfect insulator with infinite resistance, the induced current is zero. This situation is analogous to an isolated battery whose terminals aren't connected to anything: An emf is present, but no current flows.

EVALUATE: We can verify unit consistency in this calculation by noting that the magnetic-force relationship $\vec{F} = q\vec{v} \times \vec{B}$ implies that the units of $\vec{B}$ are the units of force divided by the units of (charge times velocity): $1\ \text{T} = (1\ \text{N})/(1\ \text{C} \cdot \text{m/s})$. The units of magnetic flux are then $(1\ \text{T})(1\ \text{m}^2) = 1\ \text{N} \cdot \text{s} \cdot \text{m/C}$, and the rate of change of magnetic flux is $1\ \text{N} \cdot \text{m/C} = 1\ \text{J/C} = 1\ \text{V}$. Thus the unit of $d\Phi_B/dt$ is the volt, as required by Eq. (29.3). Also recall that the unit of magnetic flux is the weber (Wb): $1\ \text{T} \cdot \text{m}^2 = 1\ \text{Wb}$, so $1\ \text{V} = 1\ \text{Wb/s}$.

Direction of Induced emf

We can find the direction of an induced emf or current by using Eq. (29.3) together with some simple sign rules. Here's the procedure:

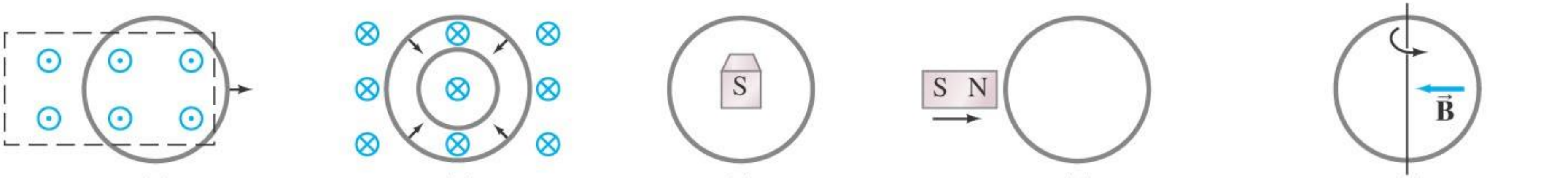
1. Define a positive direction for the vector area $\vec{A}$.
2. From the directions of $\vec{A}$ and the magnetic field $\vec{B}$, determine the sign of the magnetic flux Φ_B and its rate of change $d\Phi_B/dt$. **Figure 29.6** shows several examples.
3. Determine the sign of the induced emf or current. If the flux is increasing, so $d\Phi_B/dt$ is positive, then the induced emf or current is negative; if the flux is decreasing, $d\Phi_B/dt$ is negative and the induced emf or current is positive.
4. Finally, use your right hand to determine the direction of the induced emf or current. Curl the fingers of your right hand around the $\vec{A}$ vector, with your right thumb in the direction of $\vec{A}$. If the induced emf or current in the circuit is *positive,* it is in the same direction as your curled fingers; if the induced emf or current is *negative,* it is in the opposite direction.



29-2 Faraday's Law of Induction; Lenz's Law

Conceptual Example 29-4: Practice with Lenz's law.

In which direction is the current induced in the circular loop for each situation?



(a) Pulling a round loop to the right out of a magnetic field which points out of the page

(b) Shrinking a loop in a magnetic field pointing into the page

(c) S magnetic pole moving from below, up toward the loop

(d) N magnetic pole moving toward loop in the plane of the page

(e) Rotating the loop by pulling the left side toward us and pushing the right side in; the magnetic field points from right to left

CONCEPTUAL EXAMPLE 29–4 **Practice with Lenz's law.** In which direction is the current induced in the circular loop for each situation in Fig. 29–9?

RESPONSE (*a*) Initially, the magnetic field pointing out of the page passes through the loop. If you pull the loop out of the field, magnetic flux through the loop decreases; so the induced current will be in a direction to maintain the decreasing flux through the loop: the current will be counterclockwise to produce a magnetic field outward (toward the reader).

(*b*) The external field is into the page. The coil area gets smaller, so the flux will decrease; hence the induced current will be clockwise, producing its own field into the page to make up for the flux decrease.

(*c*) Magnetic field lines point into the S pole of a magnet, so as the magnet moves toward us and the loop, the magnet's field points into the page and is getting stronger. The current in the loop will be induced in the counterclockwise direction in order to produce a field $\vec{\mathbf{B}}$ *out* of the page.

(*d*) The field is in the plane of the loop, so no magnetic field lines pass through the loop and the flux through the loop is zero throughout the process; hence there is no change in external magnetic flux with time, and there will be no induced emf or current in the loop.

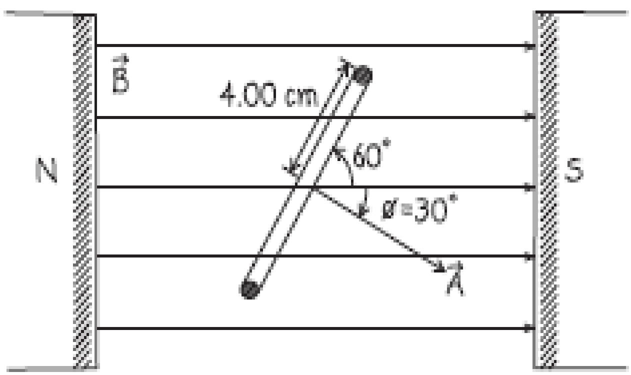
(*e*) Initially there is no flux through the loop. When you start to rotate the loop, the external field through the loop begins increasing to the left. To counteract this change in flux, the loop will have current induced in a counterclockwise direction so as to produce its own field to the right.

A 500-loop circular wire coil with radius 4.00 cm is placed between the poles of a large electromagnet. The magnetic field is uniform and makes an angle of 60° with the plane of the coil; it decreases at 0.200 T/s. What are the magnitude and direction of the induced emf?

SOLUTION

IDENTIFY and SET UP: Our target variable is the emf induced by a varying magnetic flux through the coil. The flux varies because the magnetic field decreases in amplitude. We choose the area vector $\vec{A}$ to be in the direction shown in **Fig. 29.7**. With this choice, the geometry is similar to that of Fig. 29.6b. That figure will help us determine the direction of the induced emf.

29.7 Our sketch for this problem.



EXECUTE: The magnetic field is uniform over the loop, so we can calculate the flux from Eq. (29.2): $\Phi_B = BA\cos\phi$, where $\phi = 30°$. In this expression, the only quantity that changes with time is the magnitude B of the field, so $d\Phi_B/dt = (dB/dt)A\cos\phi$.

CAUTION Remember how ϕ is defined You may have been tempted to say that $\phi = 60°$ in this problem. If so, remember that ϕ is the angle between $\vec{A}$ and $\vec{B}$, *not* the angle between $\vec{B}$ and the plane of the loop.

From Eq. (29.4), the induced emf in the coil of $N = 500$ turns is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{dB}{dt}A\cos\phi$$
$$= -500(-0.200\ \mathrm{T/s})\pi(0.0400\ \mathrm{m})^2(\cos 30°) = 0.435\ \mathrm{V}$$

The positive answer means that when you point your right thumb in the direction of area vector $\vec{A}$ (30° below field $\vec{B}$ in Fig. 29.7), the positive direction for $\mathcal{E}$ is in the direction of the curled fingers of your right hand. If you viewed the coil from the left in Fig. 29.7 and looked in the direction of $\vec{A}$, the emf would be clockwise.

EVALUATE: If the ends of the wire are connected, the direction of current in the coil is in the same direction as the emf—that is, clockwise as seen from the left side of the coil. A clockwise current increases the magnetic flux through the coil, and therefore tends to oppose the decrease in total flux. This is an example of Lenz's law, which we'll discuss in Section 29.3.

27. (I) The moving rod in Fig. 29–12b is 13.2 cm long and generates an emf of 120 mV while moving in a 0.90-T magnetic field. What is its speed?

27. The velocity is found from Eq. 29-3.

$$\mathcal{E} = B\ell v \rightarrow v = \frac{\mathcal{E}}{B\ell} = \frac{0.12\,\mathrm{V}}{(0.90\,\mathrm{T})(0.132\,\mathrm{m})} = \boxed{1.0\,\mathrm{m/s}}$$

33. (II) A conducting rod rests on two long frictionless parallel rails in a magnetic field $\vec{\mathbf{B}}$ ($\perp$ to the rails and rod) as in Fig. 29–44. (*a*) If the rails are horizontal and the rod is given an initial push, will the rod travel at constant speed even though a magnetic field is present? (*b*) Suppose at $t = 0$, when the rod has speed $v = v_0$, the two rails are connected electrically by a wire from point a to point b. Assuming the rod has resistance R and the rails have negligible resistance, determine the speed of the rod as a function of time. Discuss your answer.

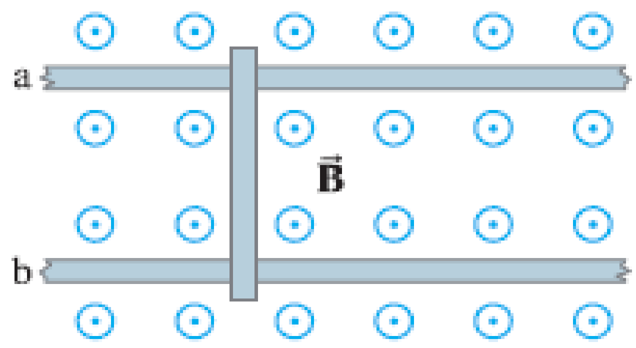
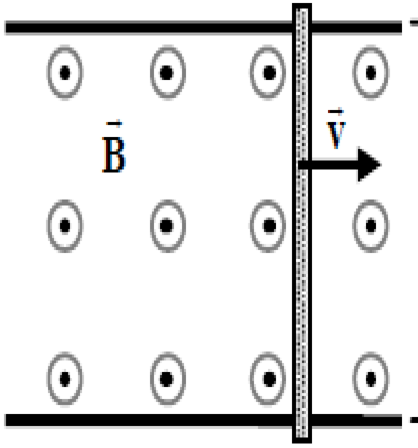


FIGURE 29–44 Problems 33 and 34.

33. (*a*) As the rod moves through the magnetic field an emf will be built up across the rod, but no current can flow. Without the current, there is no force to oppose the motion of the rod, so $\boxed{\text{yes,}}$ the rod travels at constant speed.



(*b*) We set the force on the moving rod, obtained in Example 29-8, equal to the mass times the acceleration of the rod. We then write the acceleration as the derivative of the velocity, and by separation of variables we integrate the velocity to obtain an equation for the velocity as a function of tim

$$F = ma = m\frac{dv}{dt} = -\frac{B^2\ell^2}{R}v \rightarrow \frac{dv}{v} = -\frac{B^2\ell^2}{mR}dt$$

$$\int_{v_0}^{v}\frac{dv'}{v'} = -\frac{B^2\ell^2}{mR}\int_0^t dt' \rightarrow \ln\frac{v}{v_0} = -\frac{B^2\ell^2}{mR}t \rightarrow \boxed{v(t) = v_0 e^{-\frac{B^2\ell^2}{mR}t}}$$

The magnetic force is proportional to the velocity of the rod and opposes the motion. This results in an exponentially decreasing velocity.

Assignment for Vol 2 Ch 13

- Read optional sections and Chapter 13 Summary -
- Complete Homework for chapter 13

50. (II) If 65 MW of power at 45 kV (rms) arrives at a town from a generator via 3.0-Ω transmission lines, calculate (*a*) the emf at the generator end of the lines, and (*b*) the fraction of the power generated that is wasted in the lines.

50. (*a*) The current in the transmission lines can be found from Eq. 25-10a, and then the emf at the end of the lines can be calculated from Kirchhoff's loop rule.

$$P_{\text{town}} = V_{\text{rms}} I_{\text{rms}} \quad \rightarrow \quad I_{\text{rms}} = \frac{P_{\text{town}}}{V_{\text{rms}}} = \frac{65\times10^6\,\text{W}}{45\times10^3\,\text{V}} = 1444\,\text{A}$$

$$\mathcal{E} - IR - V_{\text{output}} = 0 \quad \rightarrow$$

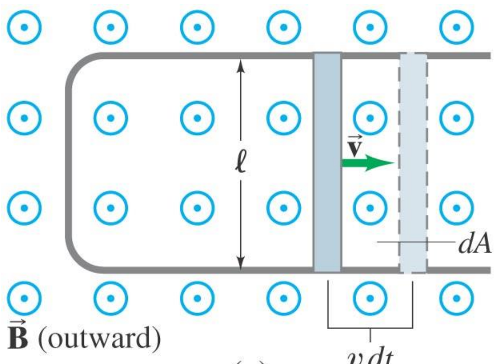
$$\mathcal{E} = IR + V_{\text{output}} = \frac{P_{\text{town}}}{V_{\text{rms}}} R + V_{\text{rms}} = \frac{65\times10^6\,\text{W}}{45\times10^3\,\text{V}}(3.0\,\Omega) + 45\times10^3\,\text{V} = 49333\,\text{V} = \boxed{49\,\text{kV}\,(\text{rms})}$$

(*b*) The power loss in the lines is given by $P_{\text{loss}} = I_{\text{rms}}^2 R$.

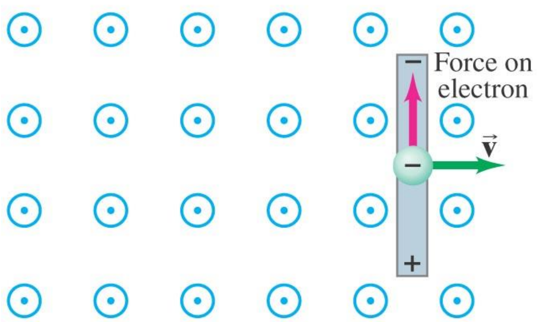
$$\text{Fraction wasted} = \frac{P_{\text{loss}}}{P_{\text{total}}} = \frac{P_{\text{loss}}}{P_{\text{town}} + P_{\text{loss}}} = \frac{I_{\text{rms}}^2 R}{P_{\text{town}} + I_{\text{rms}}^2 R} = \frac{(1444\,\text{A})^2(3.0\,\Omega)}{65\times10^6\,\text{W} + (1444\,\text{A})^2(3.0\,\Omega)}$$

$$= \boxed{0.088} = 8.8\%$$

Faraday's Law vs. F = q**v** × **B**



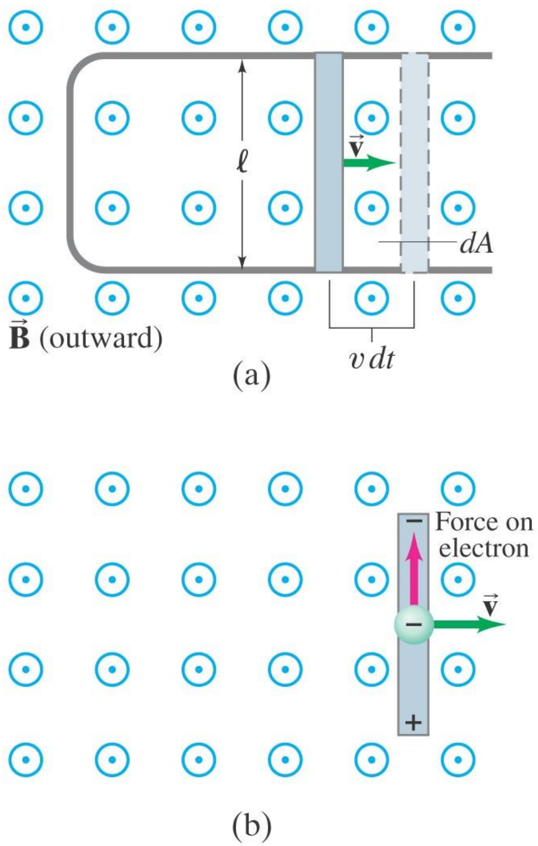
(a)



(b)

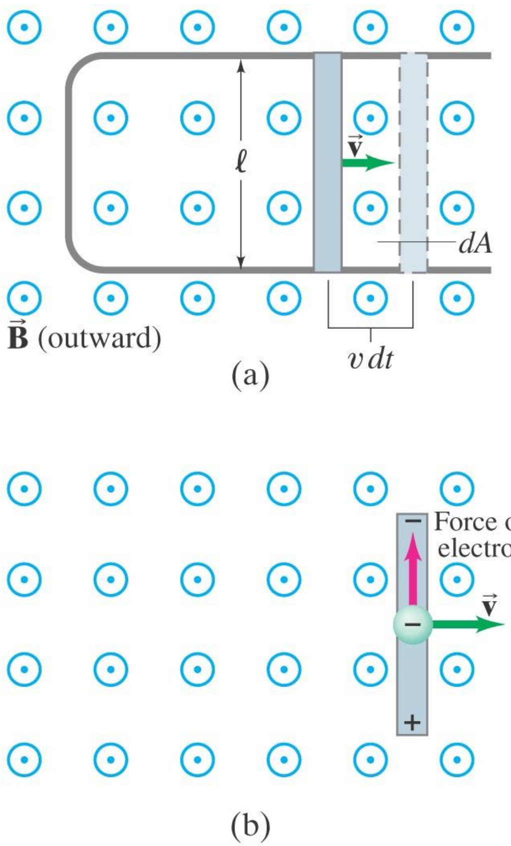
- If the metal rod at left is moving through a field B, we can show that Faraday's Law produces the same result as our earlier force rule.
- Whether or not the rod is in contact with a stationary conducting loop, it will produce a force
- F = qvB.

Faraday's Law vs. $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$



- The work done pushing the electrons a distance ℓ is just
- $W = F\ell = qvB\ell$.
- The voltage developed is
  - $\mathscr{E} = W/q = B\ell v$

Faraday's Law vs. $F = q\mathbf{v} \times \mathbf{B}$



- $\mathcal{E} = d\Phi_B/dt$
- $= B \cdot dA/dt$
- $dA = \ell \cdot vdt$
- So,
- $\mathcal{E} = B\ell v$