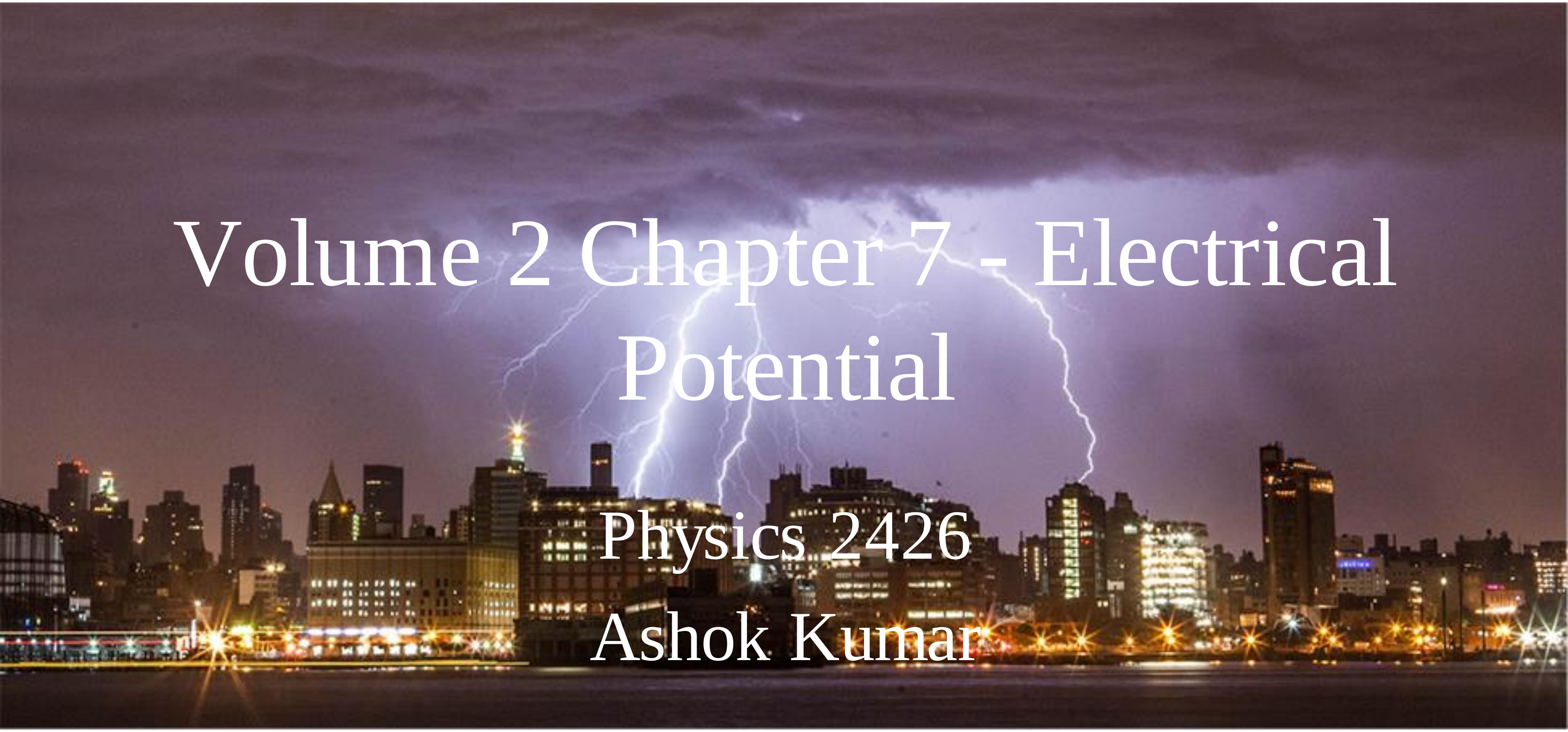


Volume 2 Chapter 7 - Electrical Potential

Physics 2426

Ashok Kumar

Electrical Potential and Energy

- In this chapter we will extend our study of electricity to include the definition of the Electrical Potential, which we call Voltage.
- This will allow us to use conservation of energy calculations for solving electrical problems.
- Voltage is a fundamental property for electrical systems.

Electrical Potential

- Electrical potential, or Voltage, is related to potential energy in the same way the the electric field is related to the electrical force.
- $\vec{E} = {}^{\vec{F}_{q+}}/_{q+}$, where $\vec{F}_{q+}$ is the force on the test charge and $q+$ is the magnitude of the test charge.
- So, the electrical potential, V, is defined in terms of potential energy, U, by the formula
- $V = {}^{U}/_{q+}$
- The unit of electrical potential is Volts.

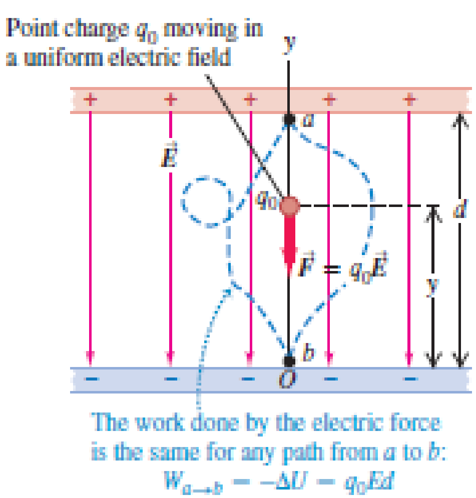
Work done by a conservative force

Potential energy at initial position

$$W_{a \to b} = U_a - U_b = -(U_b - U_a) = \underbrace{-\Delta U}_{\text{Negative of change in potential energy}} \qquad (23.2)$$

Potential energy at final position

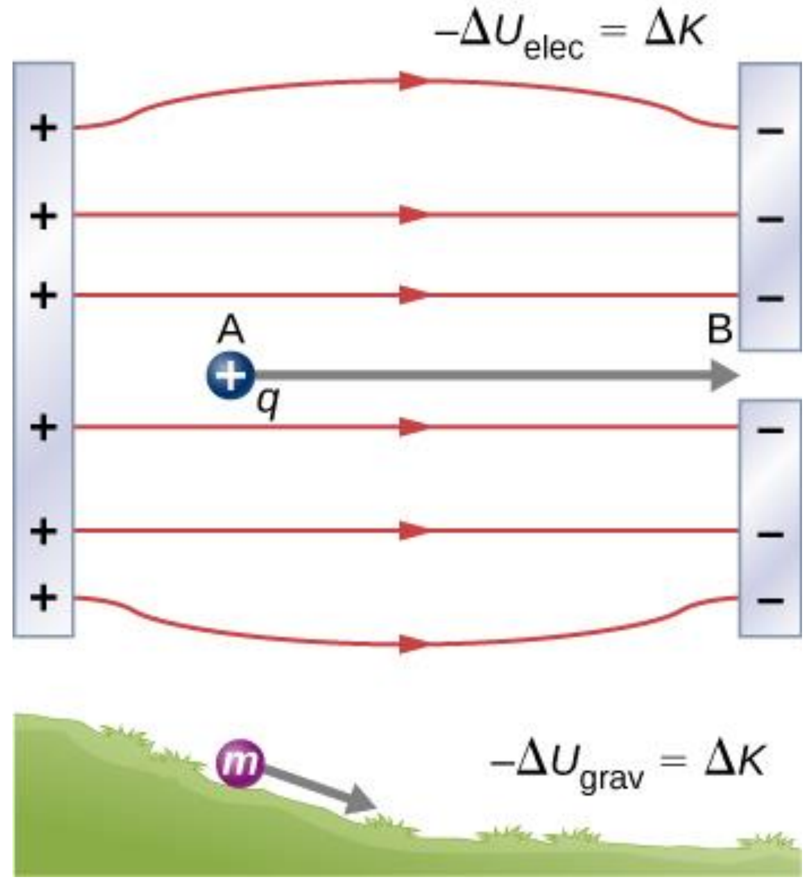
$$K_a + U_a = K_b + U_b$$



$$W_{a \to b} = -\Delta U = -(U_b - U_a) = -(q_0 E y_b - q_0 E y_a) = q_0 E (y_a - y_b)$$

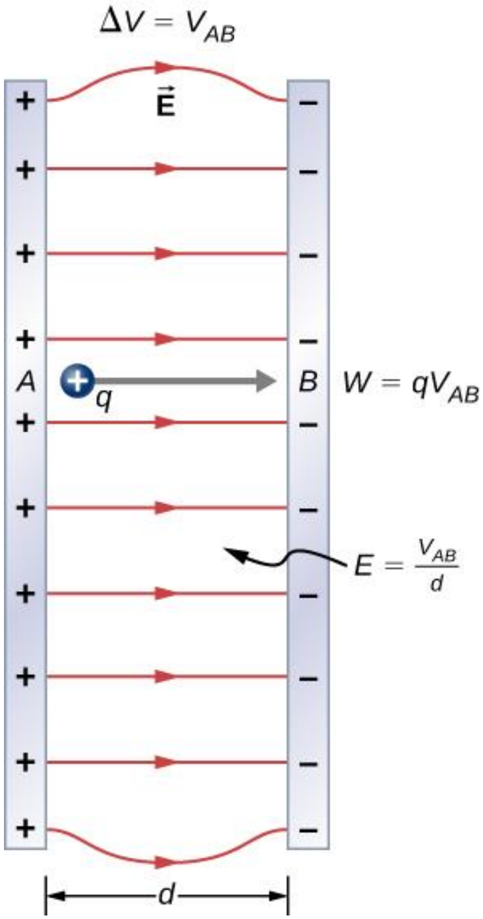
Electrical Potential

- From our work-energy equation,
- $\Delta \boldsymbol{U} = -\boldsymbol{W} = -\vec{\boldsymbol{F}} \bullet \vec{\boldsymbol{d}}$
- The work done by a constant electric field on a moving charge is just $-q\vec{E} \bullet \vec{d}$
- So our change in potential energy is just
- $\Delta \boldsymbol{U} = -\boldsymbol{q}\vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}}$



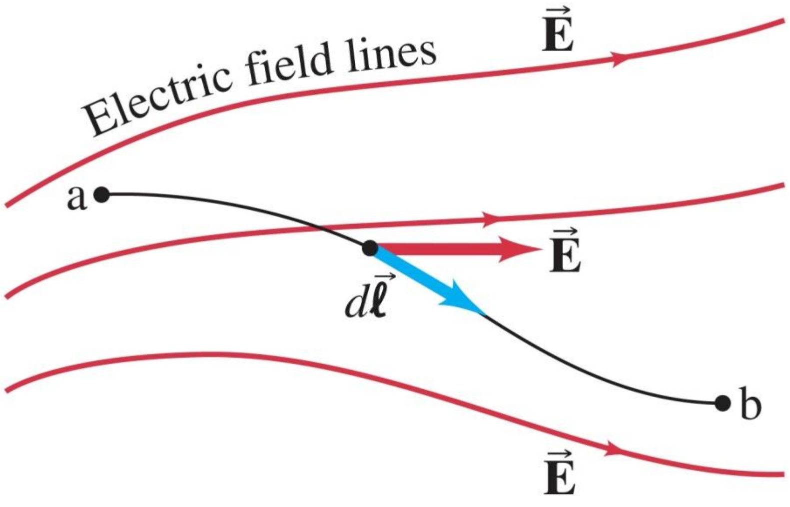
Electrical Potential

- We define the electric potential as
- $\Delta \boldsymbol{V} = \frac{\Delta \boldsymbol{U}}{\boldsymbol{q}} = -\vec{\boldsymbol{E}} \bullet \vec{\boldsymbol{d}}$
- For a constant field or more generally
- $V_b - V_a = \int_a^b -\vec{\boldsymbol{E}} \bullet \boldsymbol{d}\vec{\boldsymbol{l}}$



The general relationship between a conservative force and potential energy:

$$U_b - U_a = -\int_a^b \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{\ell}}.$$



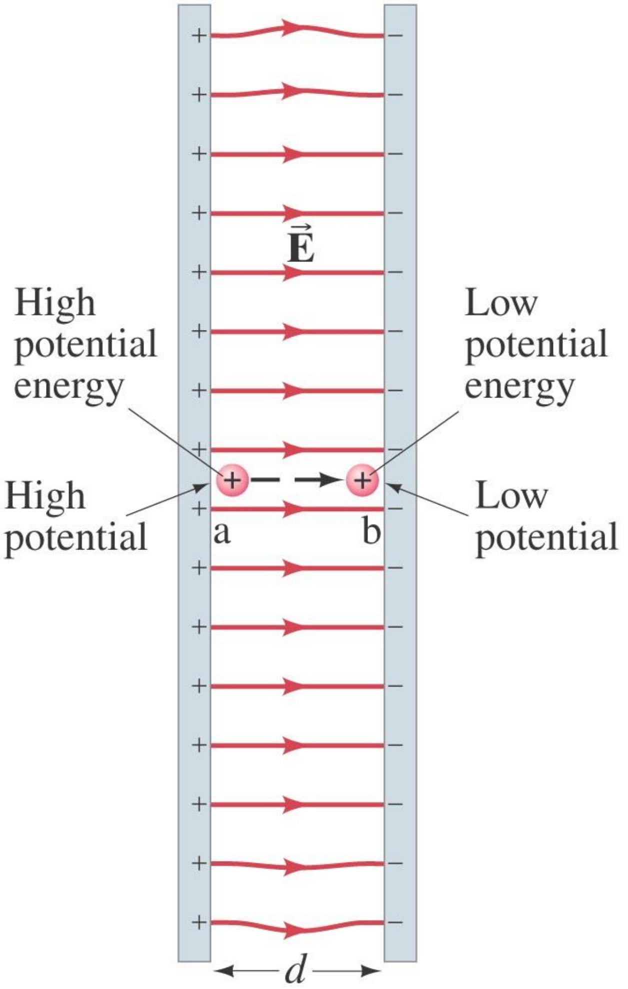
Substituting the potential difference and the electric field:

$$V_{ba} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}.$$

23-2 Relation between Electric Potential and Electric Field

The simplest case is a uniform field:

$$V_{\mathrm{ba}} = -Ed.$$



A proton (charge $+e = 1.602 \times 10^{-19}$ C) moves a distance $d = 0.50$ m in a straight line between points a and b in a linear accelerator. The electric field is uniform along this line, with magnitude $E = 1.5 \times 10^7$ V/m $= 1.5 \times 10^7$ N/C in the direction from a to b. Determine (a) the force on the proton; (b) the work done on it by the field; (c) the potential difference $V_a - V_b$.

SOLUTION

IDENTIFY and SET UP: This problem uses the relationship between electric field and electric force. It also uses the relationship among force, work, and potential-energy difference. We are given the electric field, so it is straightforward to find the electric force on the proton. Calculating the work is also straightforward because $\vec{E}$ is uniform, so the force on the proton is constant. Once the work is known, we find $V_a - V_b$ from Eq. (23.13).

EXECUTE: (a) The force on the proton is in the same direction as the electric field, and its magnitude is

$$F = qE = (1.602 \times 10^{-19}\ \text{C})(1.5 \times 10^7\ \text{N/C}) = 2.4 \times 10^{-12}\ \text{N}$$

(b) The force is constant and in the same direction as the displacement, so the work done on the proton is

$$\begin{aligned} W_{a \to b} &= Fd = (2.4 \times 10^{-12}\ \text{N})(0.50\ \text{m}) \\ &= 1.2 \times 10^{-12}\ \text{J} \\ &= (1.2 \times 10^{-12}\ \text{J})\frac{1\ \text{eV}}{1.602 \times 10^{-19}\ \text{J}} \\ &= 7.5 \times 10^6\ \text{eV} = 7.5\ \text{MeV} \end{aligned}$$

(c) From Eq. (23.13) the potential difference is the work per unit charge, which is

$$\begin{aligned} V_a - V_b &= \frac{W_{a \to b}}{q} = \frac{1.2 \times 10^{-12}\ \text{J}}{1.602 \times 10^{-19}\ \text{C}} \\ &= 7.5 \times 10^6\ \text{J/C} = 7.5 \times 10^6\ \text{V} = 7.5\ \text{MV} \end{aligned}$$

We can get this same result even more easily by remembering that 1 electron volt equals 1 volt multiplied by the charge e. The work done is 7.5×10^6 eV and the charge is e, so the potential difference is $(7.5 \times 10^6\ \text{eV})/e = 7.5 \times 10^6$ V.

EVALUATE: We can check our result in part (c) by using Eq. (23.17) or Eq. (23.18). The angle ϕ between the constant field $\vec{E}$ and the displacement is zero, so Eq. (23.17) becomes

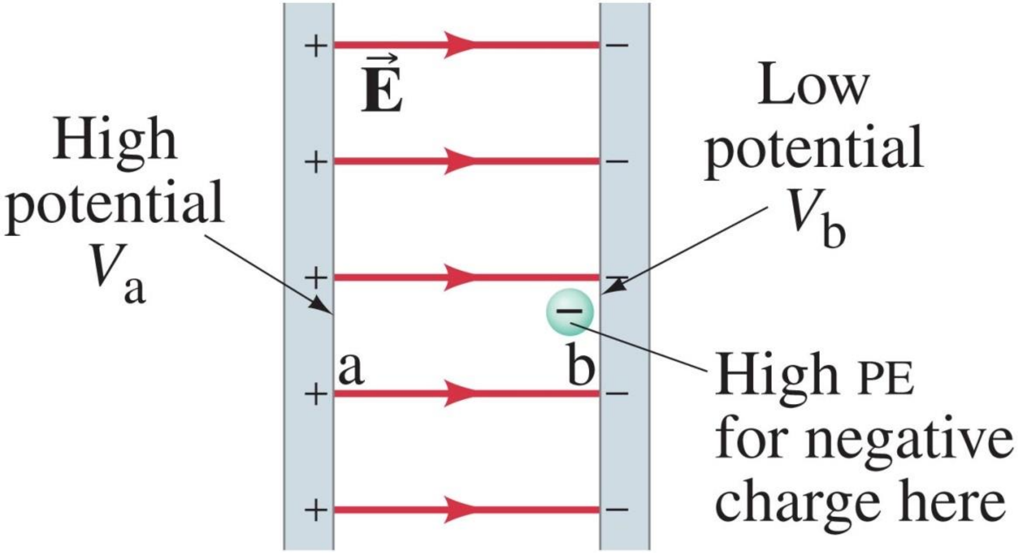
$$V_a - V_b = \int_a^b E\cos\phi\, dl = \int_a^b E\, dl = E\int_a^b dl$$

The integral of dl from a to b is just the distance d, so we again find

$$V_a - V_b = Ed = (1.5 \times 10^7\ \text{V/m})(0.50\ \text{m}) = 7.5 \times 10^6\ \text{V}$$

Conceptual Example 23-1: A negative charge.

Suppose a negative charge, such as an electron, is placed near the negative plate at point b, as shown here. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?



CONCEPTUAL EXAMPLE 23–1 **A negative charge.** Suppose a negative charge, such as an electron, is placed near the negative plate in Fig. 23–1, at point b, shown here in Fig. 23–2. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?

RESPONSE An electron released at point b will move toward the positive plate. As the electron moves toward the positive plate, its potential energy *decreases* as its kinetic energy gets larger, so $U_a < U_b$ and $\Delta U = U_a - U_b < 0$. But note that the electron moves from point b at low potential to point a at higher potential: $V_{ab} = V_a - V_b > 0$. (Potentials V_a and V_b are due to the charges on the plates, not due to the electron.) The sign of ΔU and ΔV are opposite because of the negative charge.

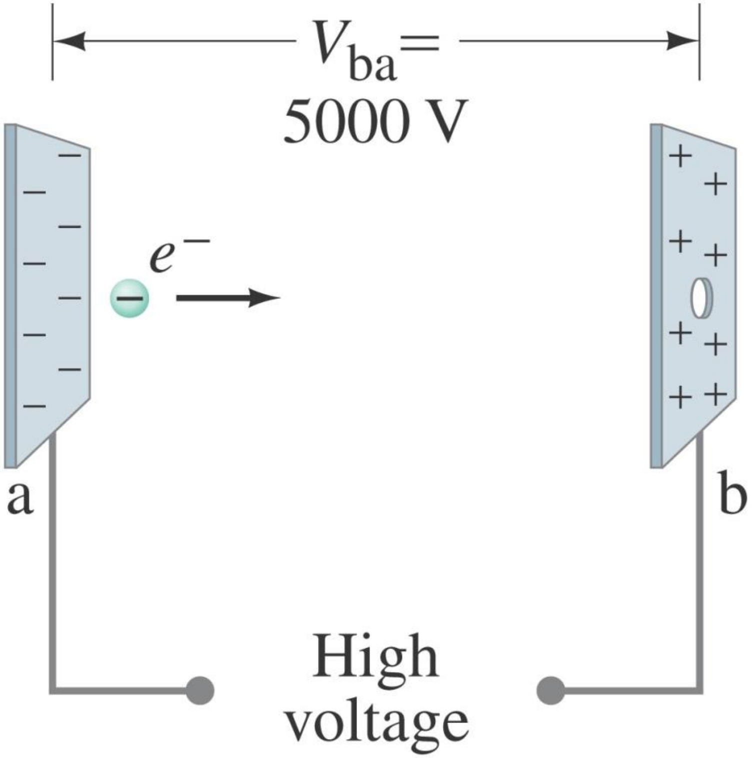
2. (I) How much work does the electric field do in moving a proton from a point with a potential of +185 V to a point where it is −55 V?

2. The work done by the electric field can be found from Eq. 23-2b.

$$V_{ba} = -\frac{W_{ba}}{q} \rightarrow W_{ba} = -qV_{ba} = -\left(1.60\times10^{-19}\,\mathrm{C}\right)\left[-55\,\mathrm{V} - 185\,\mathrm{V}\right] = \boxed{3.84\times10^{-17}\,\mathrm{J}}$$

Example 23-2: Electron in CRT.

Suppose an electron in a cathode ray tube is accelerated from rest through a potential difference $V_b - V_a = V_{ba} =$ +5000 V. (a) What is the change in electric potential energy of the electron? (b) What is the speed of the electron ($m = 9.1 \times 10^{-31}$ kg) as a result of this acceleration?



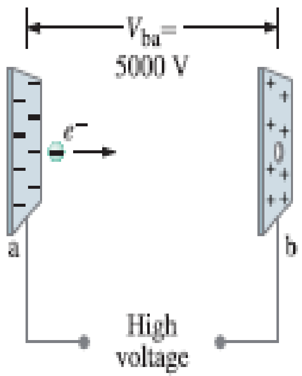


FIGURE 23–4 Electron accelerated in CRT. Example 23–2.

EXAMPLE 23–2 **Electron in CRT.** Suppose an electron in a cathode ray tube (Section 23–9) is accelerated from rest through a potential difference $V_b - V_a = V_{ba} = +5000\ \text{V}$ (Fig. 23–4). (*a*) What is the change in electric potential energy of the electron? (*b*) What is the speed of the electron $(m = 9.1 \times 10^{-31}\ \text{kg})$ as a result of this acceleration?

APPROACH The electron, accelerated toward the positive plate, will decrease in potential energy by an amount $\Delta U = qV_{ba}$ (Eq. 23–3). The loss in potential energy will equal its gain in kinetic energy (energy conservation).

SOLUTION (*a*) The charge on an electron is $q = -e = -1.6 \times 10^{-19}$ C. Therefore its change in potential energy is

$$\Delta U = qV_{ba} = (-1.6 \times 10^{-19}\ \text{C})(+5000\ \text{V}) = -8.0 \times 10^{-16}\ \text{J}.$$

The minus sign indicates that the potential energy decreases. The potential difference V_{ba} has a positive sign since the final potential V_b is higher than the initial potential V_a. Negative electrons are attracted toward a positive electrode and repelled away from a negative electrode.

(*b*) The potential energy lost by the electron becomes kinetic energy K. From conservation of energy (Eq. 8–9a), $\Delta K + \Delta U = 0$, so

$$\begin{aligned} \Delta K &= -\Delta U \\ \tfrac{1}{2}mv^2 - 0 &= -q(V_b - V_a) = -qV_{ba}, \end{aligned}$$

where the initial kinetic energy is zero since we are given that the electron started from rest. We solve for v:

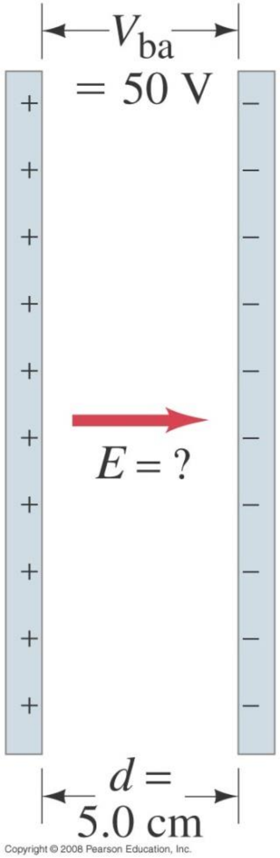
$$v = \sqrt{-\frac{2qV_{ba}}{m}} = \sqrt{-\frac{2(-1.6 \times 10^{-19}\ \text{C})(5000\ \text{V})}{9.1 \times 10^{-31}\ \text{kg}}} = 4.2 \times 10^{7}\ \text{m/s}.$$

NOTE The electric potential energy does not depend on the mass, only on the charge and voltage. The speed *does* depend on m.

Electrical Potential

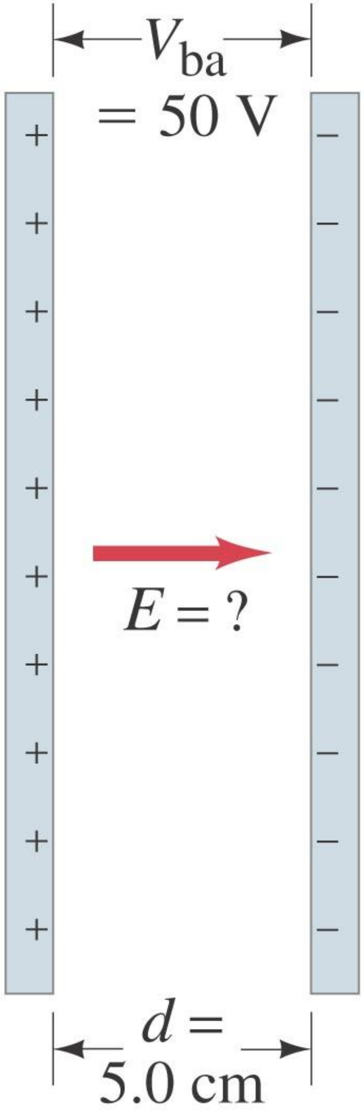
- $\Delta V = \Delta U/q$
- 1 Volt = 1 Joule/Coulomb

- Note that $E = \Delta V/d$
- So the units for electric field are Volts/meter or N/C.
- At right, the field would be 10 V/cm



Example : Electric field obtained from voltage.

Two parallel plates are charged to produce a potential difference of 50 V. If the separation between the plates is 0.050 m, calculate the magnitude of the electric field in the space between the plates.



EXAMPLE 23–3 **Electric field obtained from voltage.** Two parallel plates are charged to produce a potential difference of 50 V. If the separation between the plates is 0.050 m, calculate the magnitude of the electric field in the space between the plates (Fig. 23–6).

APPROACH We apply Eq. 23–4b to obtain the magnitude of E, assumed uniform.

SOLUTION The electric field magnitude is $E = V_{ba}/d = (50\,\text{V}/0.050\,\text{m}) = 1000\,\text{V/m}$.

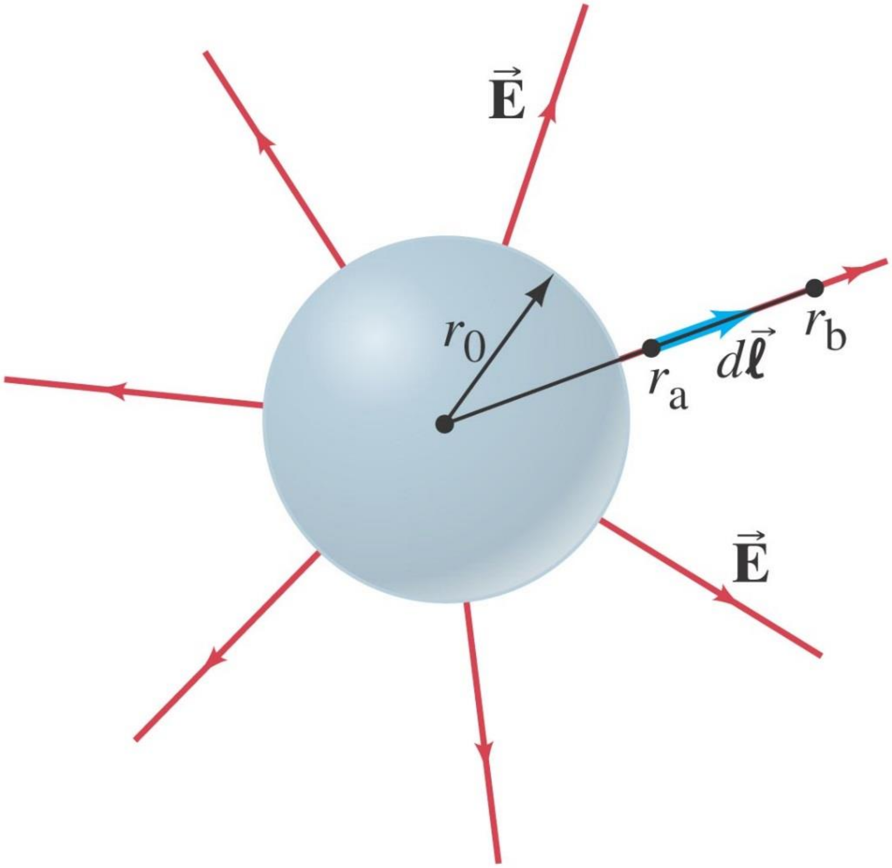
6. (I) The electric field between two parallel plates connected to a 45-V battery is 1300 V/m. How far apart are the plates?

6. The distance between the plates is found from Eq. 23-4b, using the magnitude of the electric field.

$$|E| = \frac{V_{ba}}{d} \rightarrow d = \frac{V_{ba}}{|E|} = \frac{45\,\mathrm{V}}{1300\,\mathrm{V/m}} = \boxed{3.5\times10^{-2}\,\mathrm{m}}$$

Example : Charged conducting sphere.

Determine the potential at a distance r from the center of a uniformly charged conducting sphere of radius r_0 for (a) $r > r_0$, (b) $r = r_0$, (c) $r < r_0$. The total charge on the sphere is Q.



EXAMPLE 23–4 **Charged conducting sphere.** Determine the potential at a distance r from the center of a charged conducting sphere of radius r_0 for (*a*) $r > r_0$, (*b*) $r = r_0$, (*c*) $r < r_0$. The total charge on the sphere is Q.

APPROACH The charge Q is distributed over the surface of the sphere since it is a conductor. We saw in Example 22–3 that the electric field outside a conducting sphere is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \qquad [r > r_0]$$

and points radially outward (inward if $Q < 0$). Since we know $\vec{\mathbf{E}}$, we can start by using Eq. 23–4a.

SOLUTION (*a*) We use Eq. 23–4a and integrate along a radial line with $d\vec{\boldsymbol{\ell}}$ parallel to $\vec{\mathbf{E}}$ (Fig. 23–7) between two points which are distances r_a and r_b from the sphere's center:

$$V_b - V_a = -\int_{r_a}^{r_b} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

and we set $d\ell = dr$. If we let $V = 0$ for $r = \infty$ (let's choose $V_b = 0$ at $r_b = \infty$), then at any other point r (for $r > r_0$) we have

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}. \qquad [r > r_0]$$

We will see in the next Section that this same equation applies for the potential a distance r from a single point charge. Thus the electric potential outside a spherical conductor with a uniformly distributed charge is the same as if all the charge were at its center.

(*b*) As r approaches r_0, we see that

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0} \qquad [r = r_0]$$

at the surface of the conductor.

(*c*) For points within the conductor, $E = 0$. Thus the integral, $\int \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$, between $r = r_0$ and any point within the conductor gives zero change in V. Hence V is constant within the conductor:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0}. \qquad [r \le r_0]$$

The whole conductor, not just its surface, is at this same potential. Plots of both E and V as a function of r are shown in Fig. 23–8 for a positively charged conducting sphere.

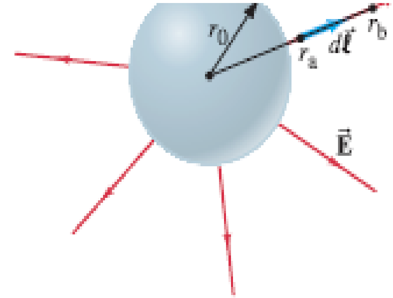
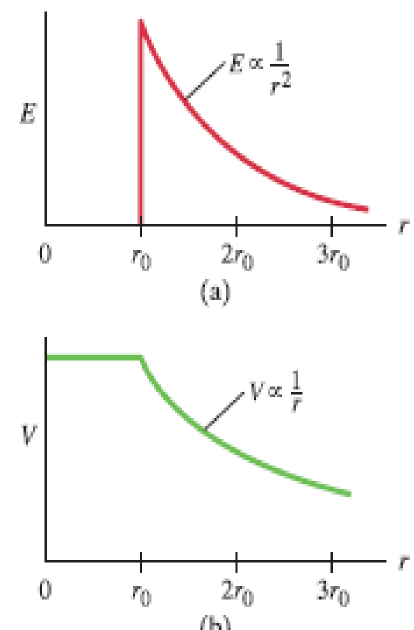


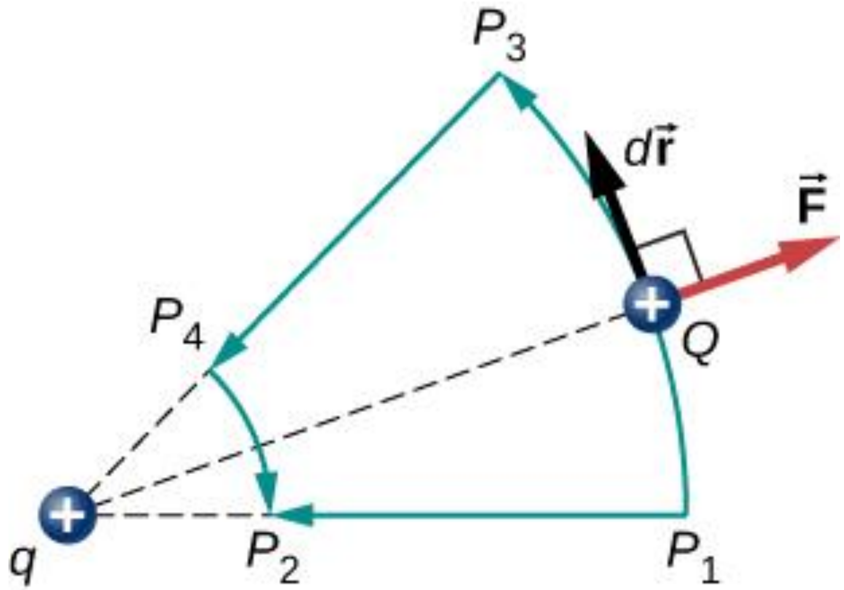
FIGURE 23–7 Example 23–4. Integrating $\vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}$ for the field outside a spherical conductor.

FIGURE 23–8 (a) E versus r, and (b) V versus r, for a positively charged solid conducting sphere of radius r_0 (the charge distributes itself on the surface); r is the distance from the center of the sphere.



Electrical Potential - Point Charge

- For a point charge,
- $E = k\frac{Q}{r^2}$
- Which we integrate to get
- $V = -kQ\int\frac{dr}{r^2} = \frac{kQ}{r} + c$
- Taking $V = 0$ @ $r = \infty$
- $V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 \cdot r}$



By integrating the electric field as in Eq. (23.17), find the potential at a distance r from a point charge q.

SOLUTION

IDENTIFY and SET UP: We let point a in Eq. (23.17) be at distance r and let point b be at infinity (**Fig. 23.14**). As usual, we choose the potential to be zero at an infinite distance from the charge q.

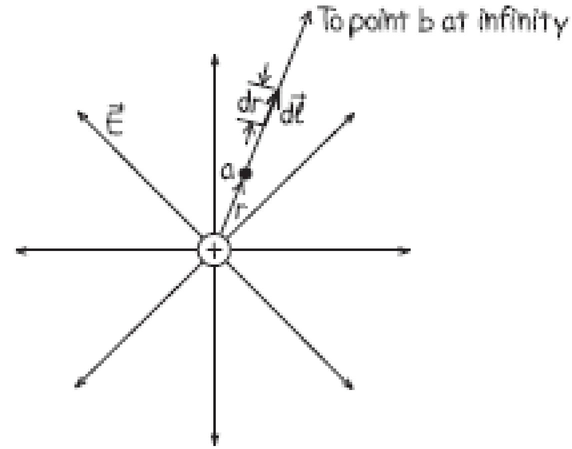
EXECUTE: To carry out the integral, we can choose any path we like between points a and b. The most convenient path is a radial line as shown in Fig. 23.14, so that $d\vec{l}$ is in the radial direction and has magnitude dr. Writing $d\vec{l} = \hat{r}dr$, we have from Eq. (23.17)

$$V - 0 = V = \int_r^{\infty} \vec{E} \cdot d\vec{l}$$

$$= \int_r^{\infty} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot \hat{r} dr = \int_r^{\infty} \frac{q}{4\pi\epsilon_0 r^2} dr$$

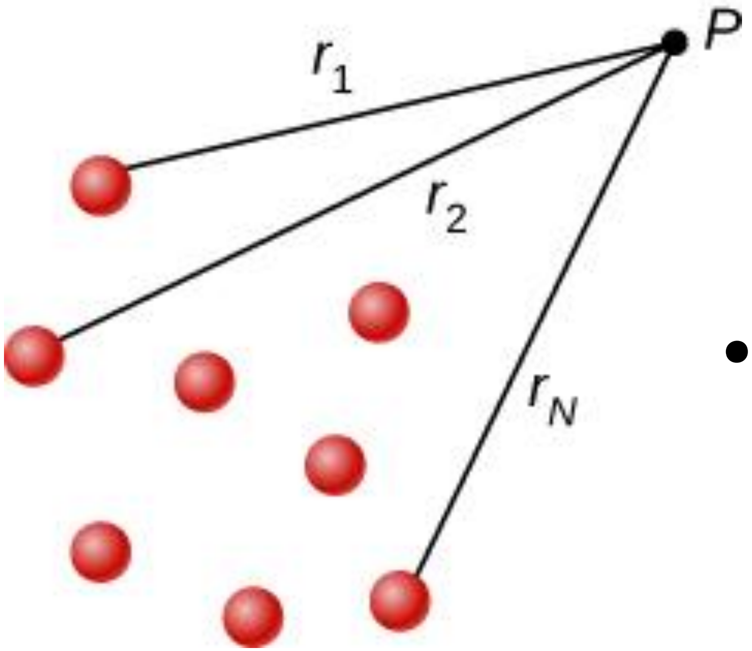
$$= -\frac{q}{4\pi\epsilon_0 r}\Big|_r^{\infty} = 0 - \left(-\frac{q}{4\pi\epsilon_0 r}\right) = \frac{q}{4\pi\epsilon_0 r}$$

23.14 Calculating the potential by integrating $\vec{E}$ for a single point charge.



EVALUATE: Our result agrees with Eq. (23.14) and is correct for positive or negative q.

Electrical Potential for Multiple Charges

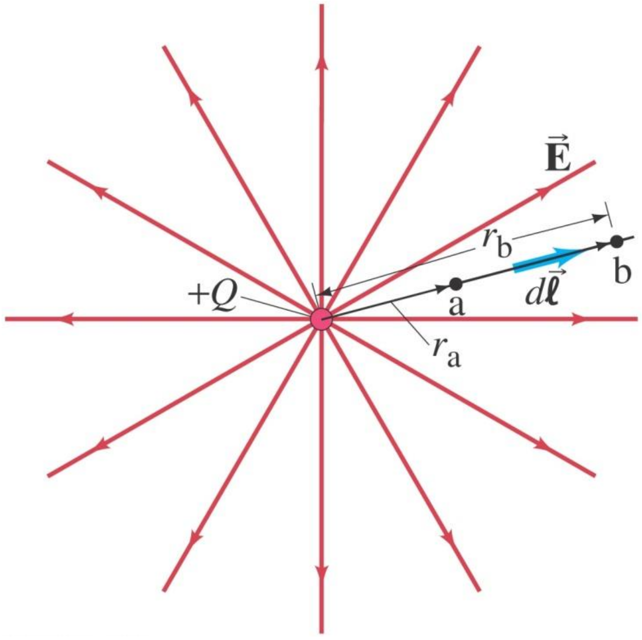


- For individual charges, we simply add the potentials due to the various charges.
- $V_p = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \cdots + \frac{kq_n}{r_n}$

Electric Potential Due to Point Charges

To find the electric potential due to a point charge, we integrate the field along a field line:

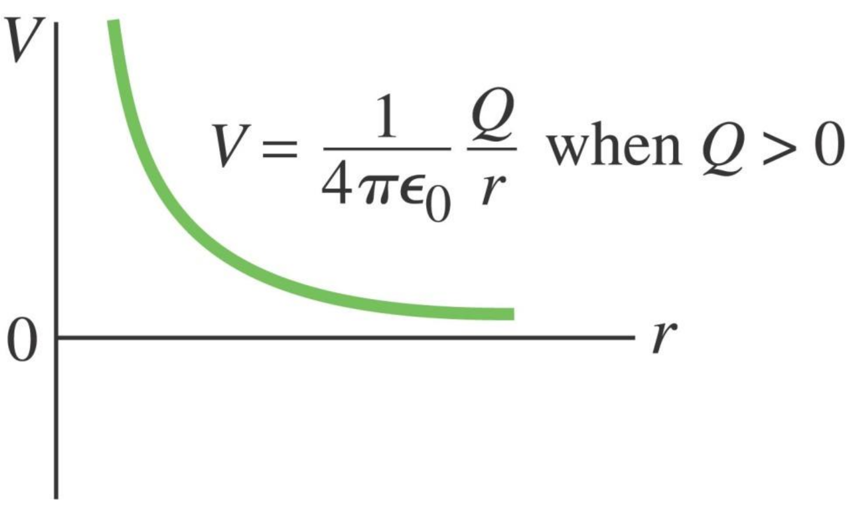
$$V_b - V_a = -\int_{r_a}^{r_b} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{Q}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{1}{r^2}\,dr = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right).$$

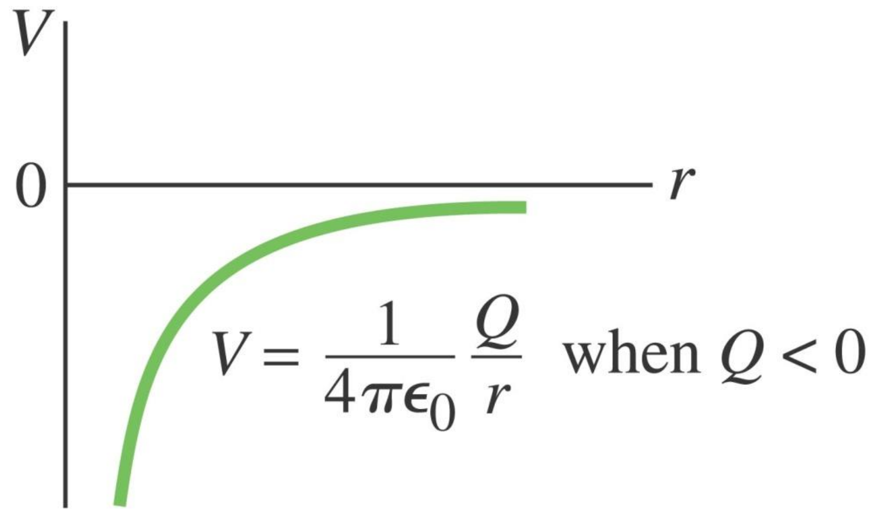


Electric Potential Due to Point Charges

Setting the potential to zero at *r* = ∞ gives the general form of the potential due to a point charge:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}. \qquad \left[\begin{array}{c}\text{single point charge;}\\ V = 0 \text{ at } r = \infty\end{array}\right]$$





23-3 Electric Potential Due to Point Charges

Example 23-6: Work required to bring two positive charges close together.

What minimum work must be done by an external force to bring a charge q = 3.00 μC from a great distance away (take $r = \infty$) to a point 0.500 m from a charge Q = 20.0 μC?

EXAMPLE 23–6 **Work required to bring two positive charges close together.** What minimum work must be done by an external force to bring a charge $q = 3.00\,\mu\text{C}$ from a great distance away (take $r = \infty$) to a point 0.500 m from a charge $Q = 20.0\,\mu\text{C}$?

APPROACH To find the work we cannot simply multiply the force times distance because the force is not constant. Instead we can set the change in potential energy equal to the (positive of the) work required of an *external* force (Chapter 8), and Eq. 23–3: $W = \Delta U = q(V_b - V_a)$. We get the potentials V_b and V_a using Eq. 23–5.

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SOLUTION The work required is equal to the change in potential energy:

$$\begin{aligned} W &= q(V_b - V_a) \\ &= q\left(\frac{kQ}{r_b} - \frac{kQ}{r_a}\right), \end{aligned}$$

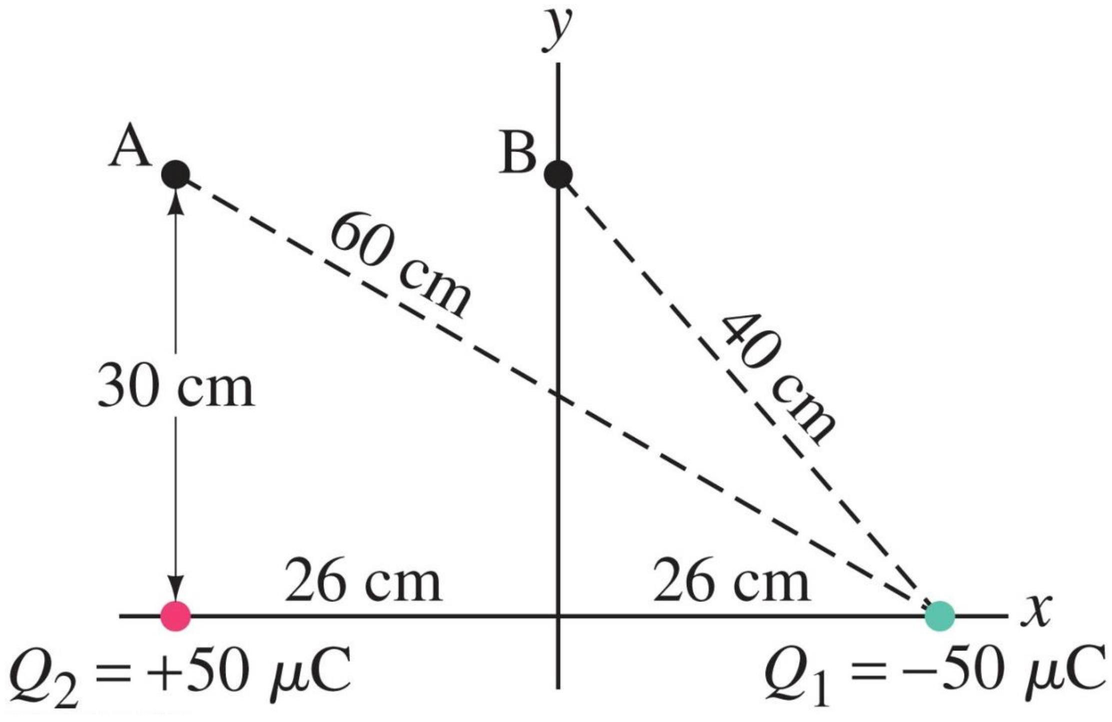
where $r_b = 0.500\text{ m}$ and $r_a = \infty$. The right-hand term within the parentheses is zero $(1/\infty = 0)$ so

$$W = (3.00 \times 10^{-6}\,\text{C})\frac{(8.99 \times 10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.00 \times 10^{-5}\,\text{C})}{(0.500\,\text{m})} = 1.08\text{ J}.$$

NOTE We could not use Eq. 23–4b here because it applies *only* to uniform fields. But we did use Eq. 23–3 because it is always valid.

Example : Potential above two charges.

Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point B.



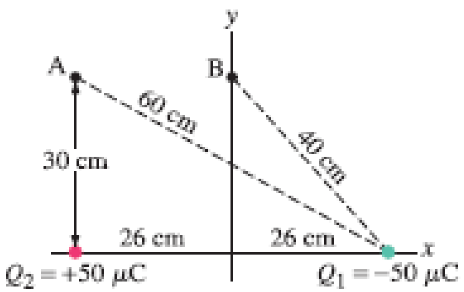


FIGURE 23–12 Example 23–7. (See also Example 21–8, Fig. 21–27.)

CAUTION

Potential is a scalar and has no components

EXAMPLE 23–7 **Potential above two charges.** Calculate the electric potential (*a*) at point A in Fig. 23–12 due to the two charges shown, and (*b*) at point B. [This is the same situation as Example 21–8, Fig. 21–27, where we calculated the electric field at these points.]

APPROACH The total potential at point A (or at point B) is the sum of the potentials at that point due to each of the two charges Q_1 and Q_2. The potential due to each single charge is given by Eq. 23–5. We do not have to worry about directions because electric potential is a scalar quantity. But we do have to keep track of the signs of charges.

SOLUTION (*a*) We add the potentials at point A due to each charge Q_1 and Q_2, and we use Eq. 23–5 for each:

$$\begin{aligned} V_A &= V_{A2} + V_{A1} \\ &= k\frac{Q_2}{r_{2A}} + k\frac{Q_1}{r_{1A}} \end{aligned}$$

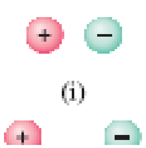
where $r_{1A} = 60\text{ cm}$ and $r_{2A} = 30\text{ cm}$. Then

$$\begin{aligned} V_A &= \frac{(9.0\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.0\times10^{-5}\,\text{C})}{0.30\,\text{m}} \\ &\quad + \frac{(9.0\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(-5.0\times10^{-5}\,\text{C})}{0.60\,\text{m}} \\ &= 1.50\times10^6\,\text{V} - 0.75\times10^6\,\text{V} \\ &= 7.5\times10^5\,\text{V}. \end{aligned}$$

(*b*) At point B, $r_{1B} = r_{2B} = 0.40\text{ m}$, so

$$\begin{aligned} V_B &= V_{B2} + V_{B1} \\ &= \frac{(9.0\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.0\times10^{-5}\,\text{C})}{0.40\,\text{m}} \\ &\quad + \frac{(9.0\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(-5.0\times10^{-5}\,\text{C})}{0.40\,\text{m}} \\ &= 0\,\text{V}. \end{aligned}$$

NOTE The two terms in the sum in (*b*) cancel for any point equidistant from Q_1 and Q_2 $(r_{1B} = r_{2B})$. Thus the potential will be zero everywhere on the plane equidistant between the two opposite charges. This plane where V is constant is called an equipotential surface.



(i)

32. (II) Two equal but opposite charges are separated by a distance d, as shown in Fig. 23–28. Determine a formula for $V_{BA} = V_B - V_A$ for points B and A on the line between the charges situated as shown.

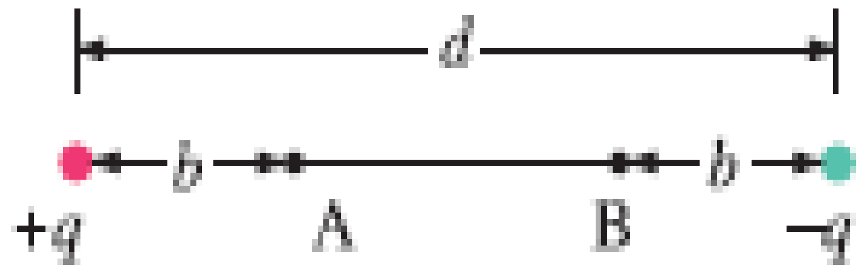


FIGURE 23–28
Problem 32.

32. Use Eq. 23-2b and Eq. 23-5.

$$V_{BA} = V_B - V_A = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d-b} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{b}\right) - \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{b} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{d-b}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{d-b} - \frac{1}{b} - \frac{1}{b} + \frac{1}{d-b}\right) = 2\frac{1}{4\pi\varepsilon_0}q\left(\frac{1}{d-b} - \frac{1}{b}\right) = \boxed{\frac{2q(2b-d)}{4\pi\varepsilon_0 b(d-b)}}$$

28. (II) Point a is 26 cm north of a −3.8 μC point charge, and point b is 36 cm west of the charge (Fig. 23–27). Determine (*a*) $V_b - V_a$, and (*b*) $\vec{E}_b - \vec{E}_a$ (magnitude and direction).

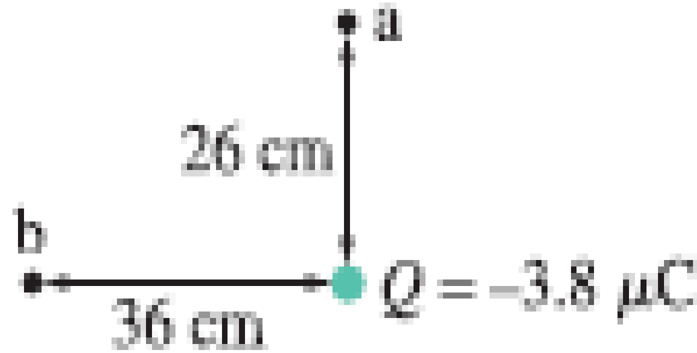


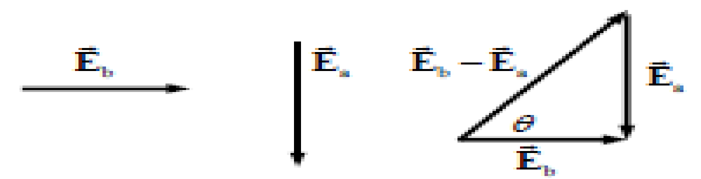
FIGURE 23–27 Problem 28.

28. (*a*) The potential due to a point charge is given by Eq. 23-5.

$$V_{ba} = V_b - V_a = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_b} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_a} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

$$= \left(8.99\times10^9\ \mathrm{N{\cdot}m^2/C^2}\right)\left(-3.8\times10^{-6}\,\mathrm{C}\right)\left(\frac{1}{0.36\,\mathrm{m}} - \frac{1}{0.26\,\mathrm{m}}\right) = \boxed{3.6\times10^4\ \mathrm{V}}$$

(*b*) The magnitude of the electric field due to a point charge is given by Eq. 21-4a. The direction of the electric field due to a negative charge is towards the charge, so the field at point a will point downward, and the field at point b will point to the right. See the vector diagram.



$$\vec{E}_b = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r_b^2}\hat{\mathbf{i}} = \frac{\left(8.99\times10^9\ \mathrm{N{\cdot}m^2/C^2}\right)\left(3.8\times10^{-6}\,\mathrm{C}\right)}{(0.36\,\mathrm{m})^2}\hat{\mathbf{i}} = 2.636\times10^5\ \mathrm{V/m}\,\hat{\mathbf{i}}$$

$$\vec{E}_a = -\frac{1}{4\pi\varepsilon_0}\frac{|q|}{r_a^2}\hat{\mathbf{j}} = -\frac{\left(8.99\times10^9\ \mathrm{N{\cdot}m^2/C^2}\right)\left(3.8\times10^{-6}\,\mathrm{C}\right)}{(0.26\,\mathrm{m})^2}\hat{\mathbf{j}} = -5.054\times10^5\ \mathrm{V/m}\,\hat{\mathbf{j}}$$

$$\vec{E}_b - \vec{E}_a = 2.636\times10^5\ \mathrm{V/m}\,\hat{\mathbf{i}} + 5.054\times10^5\ \mathrm{V/m}\,\hat{\mathbf{j}}$$

$$\left|\vec{E}_b - \vec{E}_a\right| = \sqrt{\left(2.636\times10^5\ \mathrm{V/m}\right)^2 + \left(5.054\times10^5\ \mathrm{V/m}\right)^2} = \boxed{5.7\times10^5\ \mathrm{V/m}}$$

$$\theta = \tan^{-1}\frac{-E_a}{E_b} = \tan^{-1}\frac{5.054\times10^5}{2.636\times10^5} = \boxed{62^\circ}$$

34. (II) Three point charges are arranged at the corners of a square of side ℓ as shown in Fig. 23–29. What is the potential at the fourth corner (point A), taking $V = 0$ at a great distance?

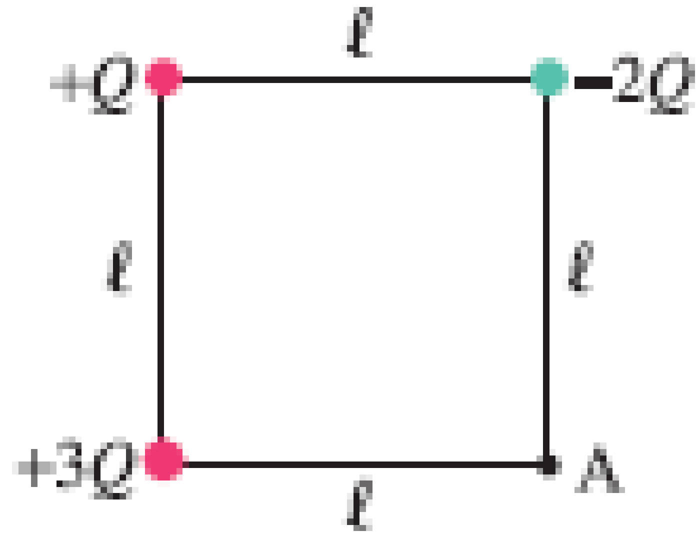
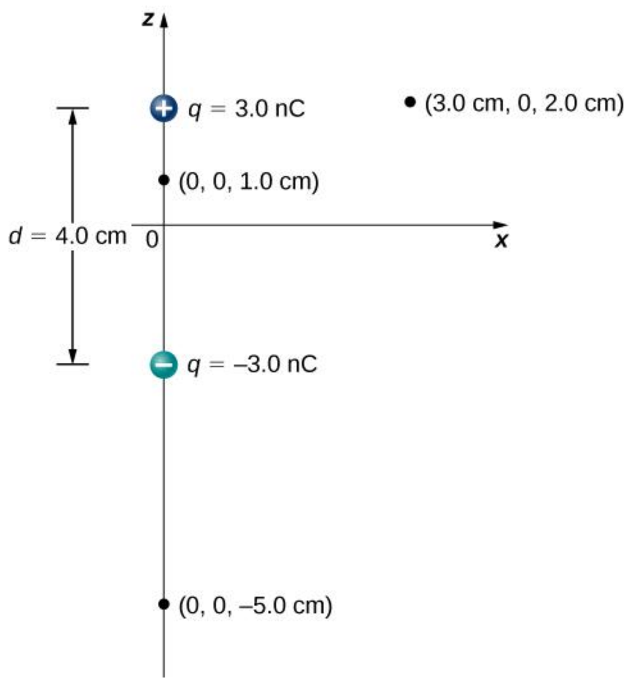


FIGURE 23–29 Problem 34.

The potential at the corner is the sum of the potentials due to each of the charges, using Eq. 23-5.

$$V=\frac{1}{4\pi\varepsilon_0}\frac{(3Q)}{\ell}+\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}\ell}+\frac{1}{4\pi\varepsilon_0}\frac{(-2Q)}{\ell}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\ell}\left(1+\frac{1}{\sqrt{2}}\right)=\boxed{\frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}Q}{2\ell}\left(\sqrt{2}+1\right)}$$

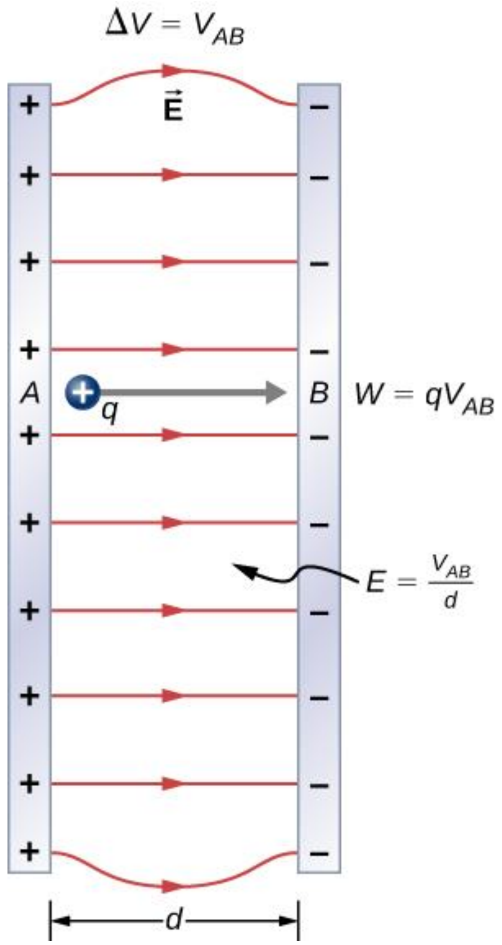
Electrical Potential for a dipole



- For each point, we take the difference in the potentials for each of the two charges.
- $V_p = \frac{k \cdot 3nC}{r_+} - \frac{k \cdot 3nC}{r_-}$
- What would be the potential along the x axis.

Electrical Potential - Summary

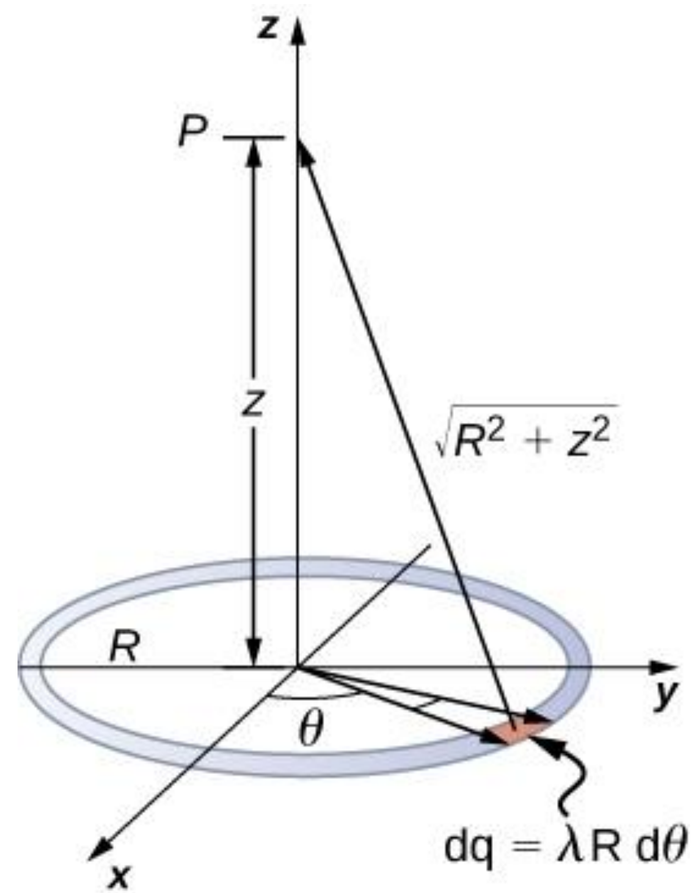
- In the presence of an electrical potential, a <u>positive</u> charge will move along a field line toward a point of lower potential (less positive or more negative).
- In the presence of an electrical potential, a <u>negative</u> charge will move along a field line toward a point of higher potential.



Electrical Potential – Distribution of Charge

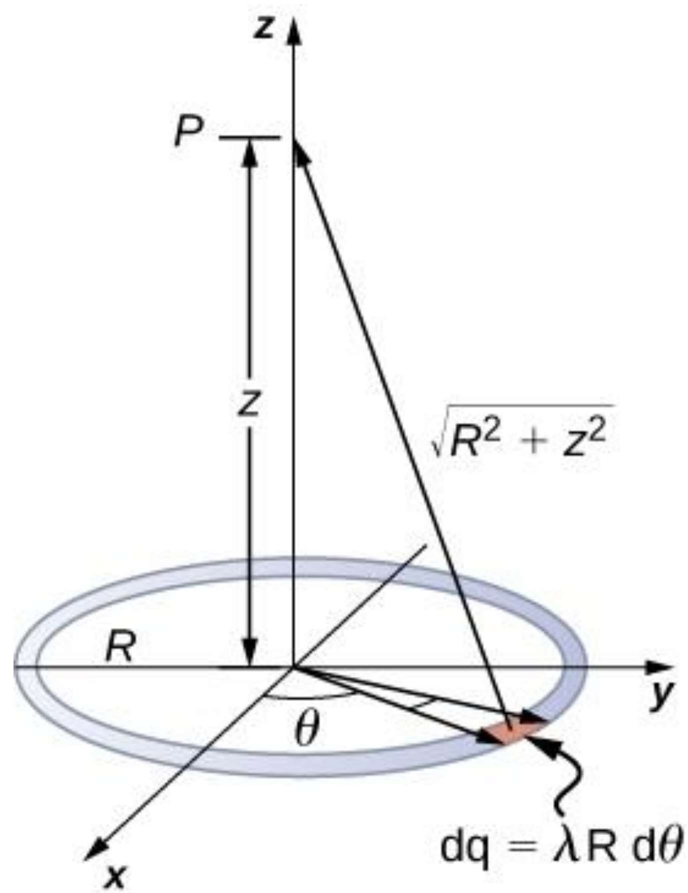
- For a set of point charges, the potential is simply the sum of the potentials for each charge.
- For a distribution of charge, we need to integrate over the distribution.

- $V = \frac{1}{4\pi\varepsilon_0}\int \frac{dQ}{r}$

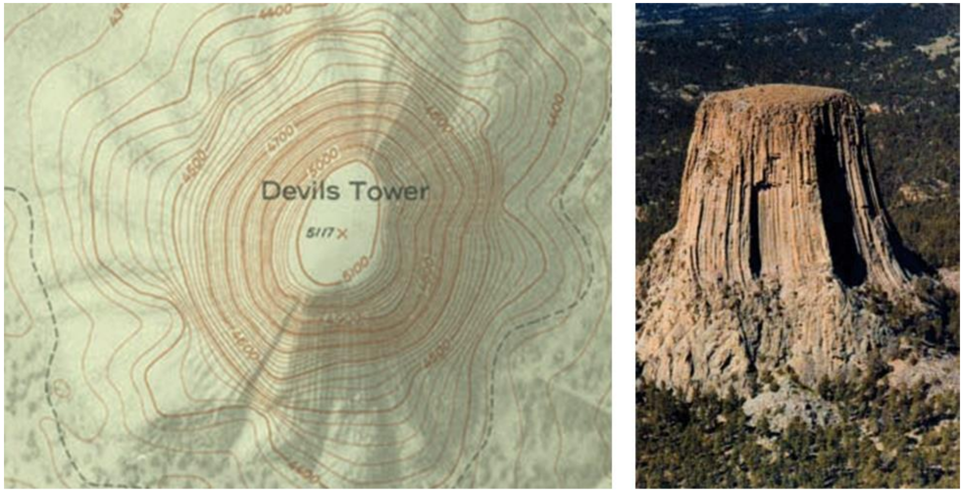


Electrical Potential – Distribution of Charge

- Along the center line of the ring at right, r is constant.
- $r = (x^2 + R^2)^{1/2}$
- So $\int dQ = Q$, and
- $V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{(x^2+R^2)^{1/2}}$



Equipotential Surfaces

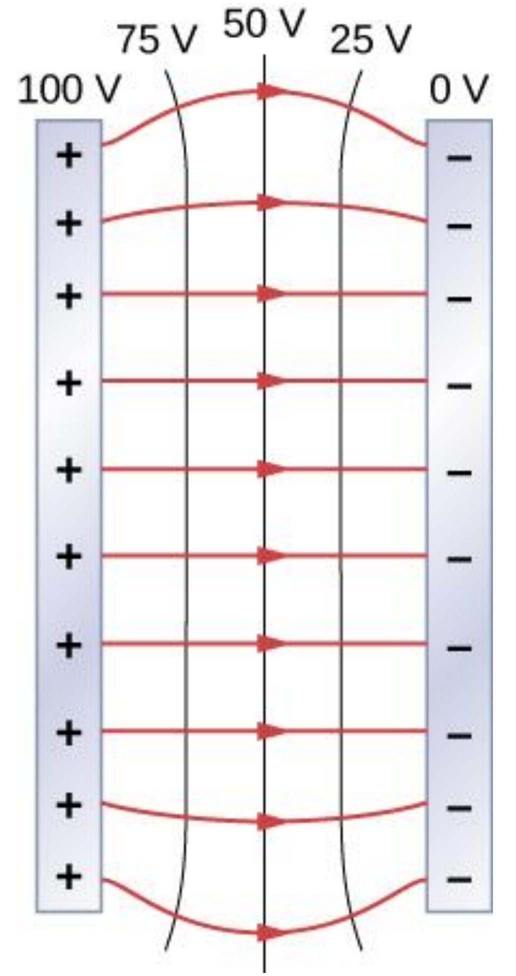


(a) (b)

- Equipotential means equal or same potential energy.
- The altitude map above shows lines of equal height which have equal gravitational potential energy.

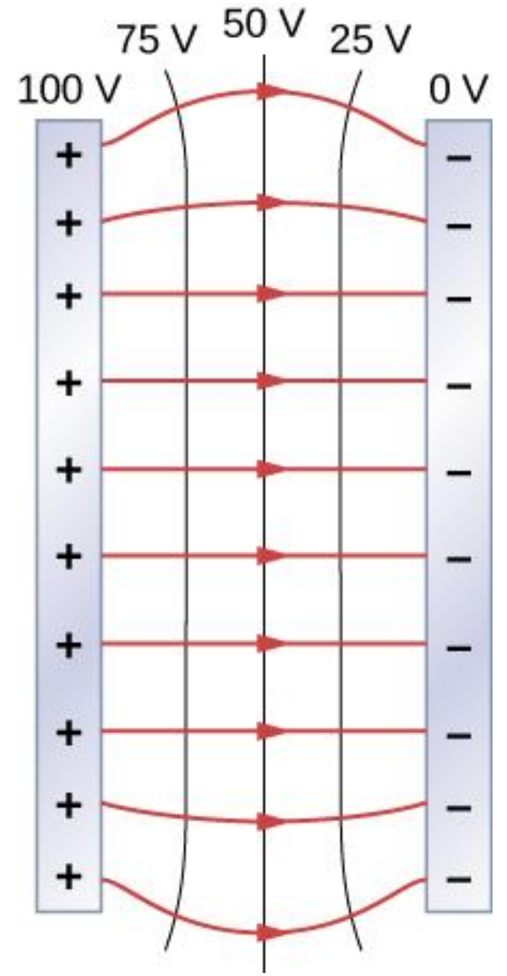
Equipotential Surfaces

- In an electric field, an Equipotential surface is locus of points having the same electrical potential.
- Therefore, no work is done when moving a charge from point to point on the equipotential.

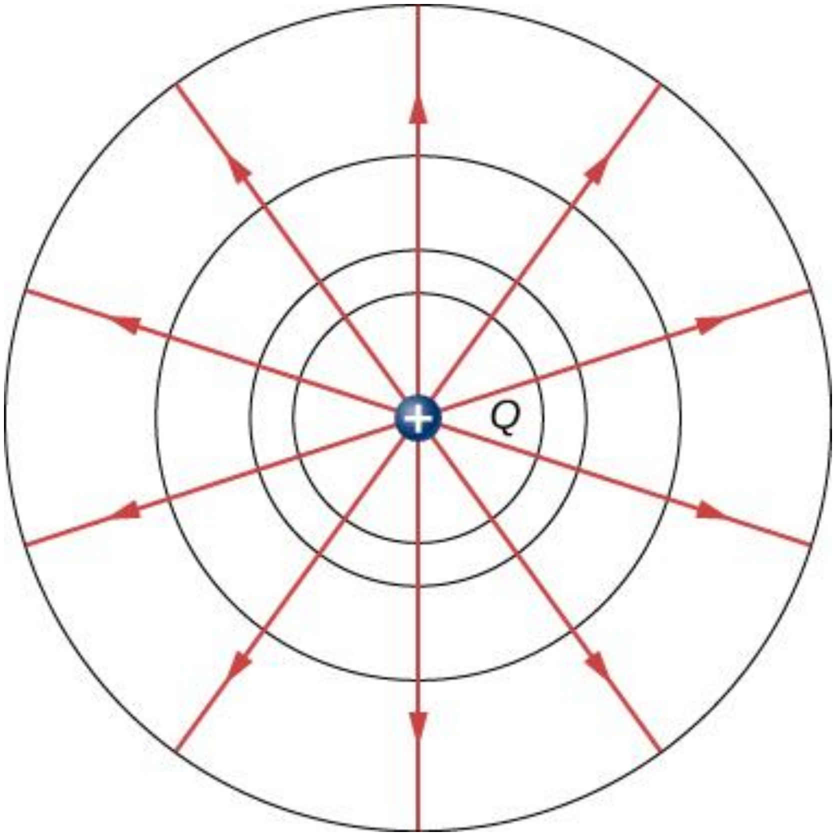


Equipotential Surfaces

- No work implies that $\vec{E} \bullet d\vec{l} = 0$, so the equipotential surface must be perpendicular to the local field.
- For parallel plates, the equipotential surfaces will also be parallel planes.

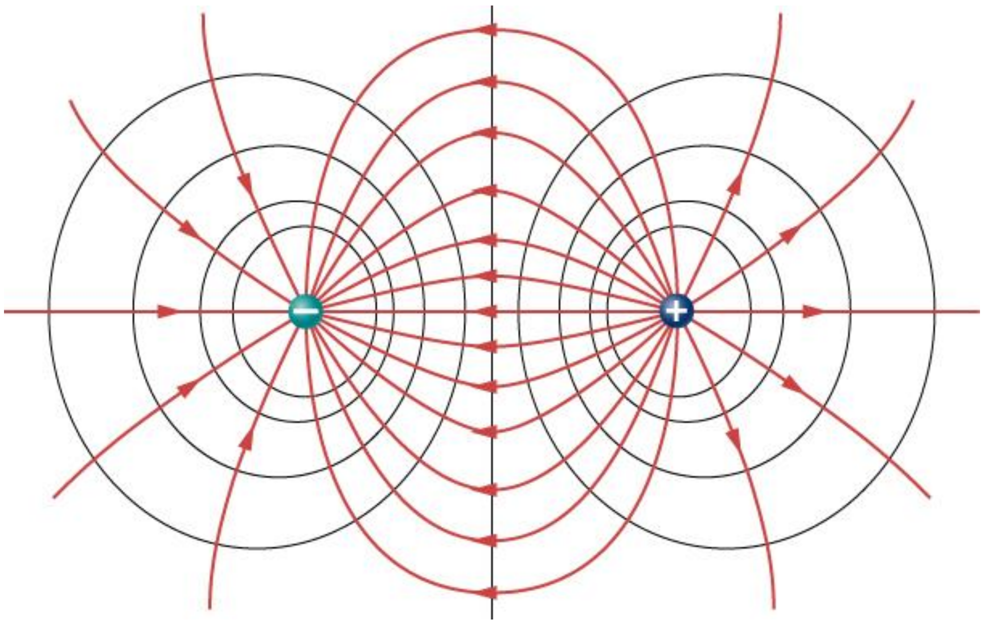


Equipotential Surfaces



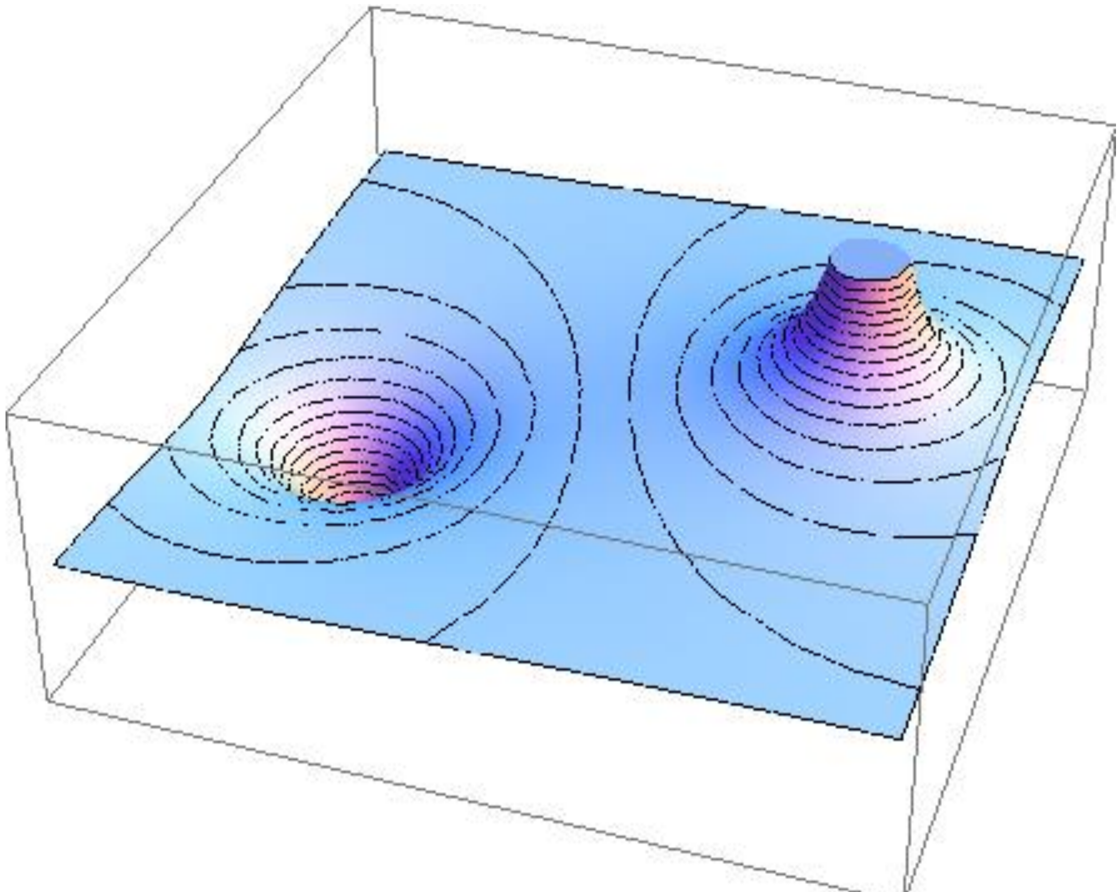
- What about a point source?
- $V = kq/r$
- This formula implies that every point which is the same distance d from the point charge, the potential V is the same, $V = kq/d$.
- The equipotentials are concentric spheres with the charge at the center.

Equipotential Surfaces for a dipole

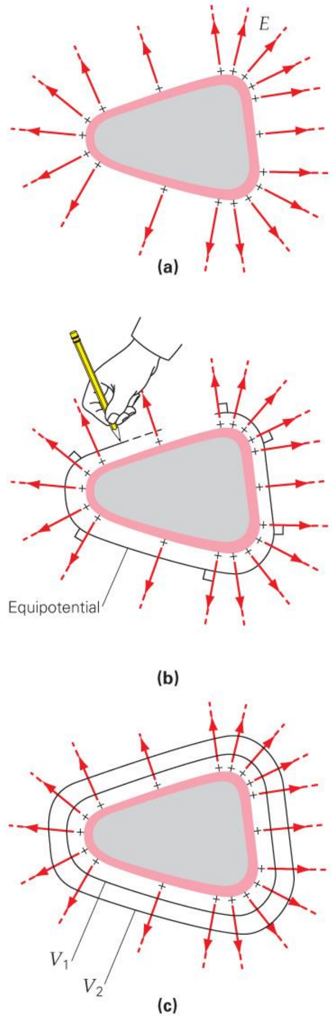


- Equipotential lines are blue & field lines are red.
- Very near the charges, the surfaces are nearly spheres.
- Half-way in-between the charges, the surface is a plane (V_2).

Equipotential for a Dipole

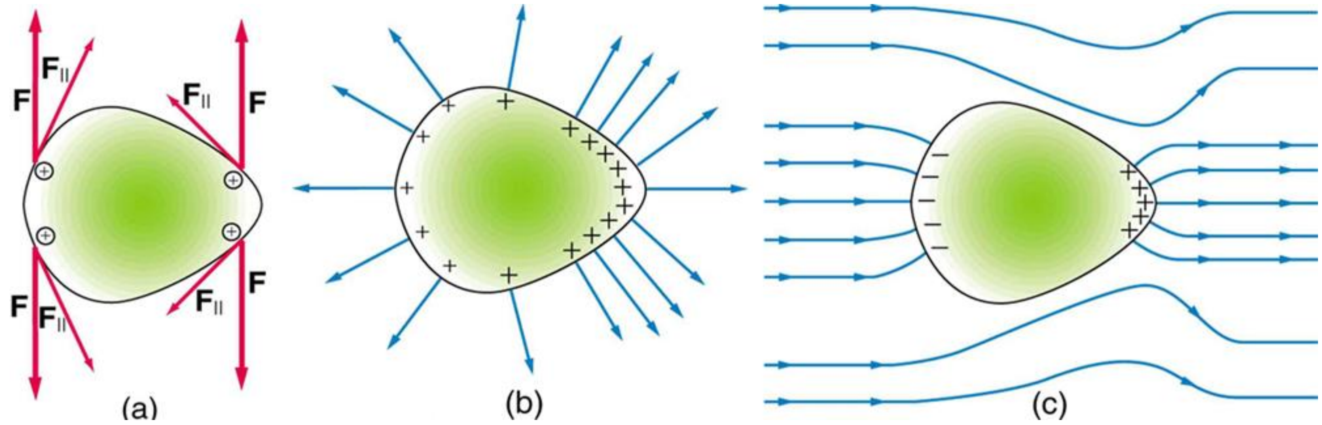


Equipotential Surfaces for a conductor



- In the last chapter, we learned that the field near the surface of a conductor must be perpendicular to the surface.
- So, the surface of a conductor is an equipotential and the nearby equipotential surfaces lie parallel to the surface.

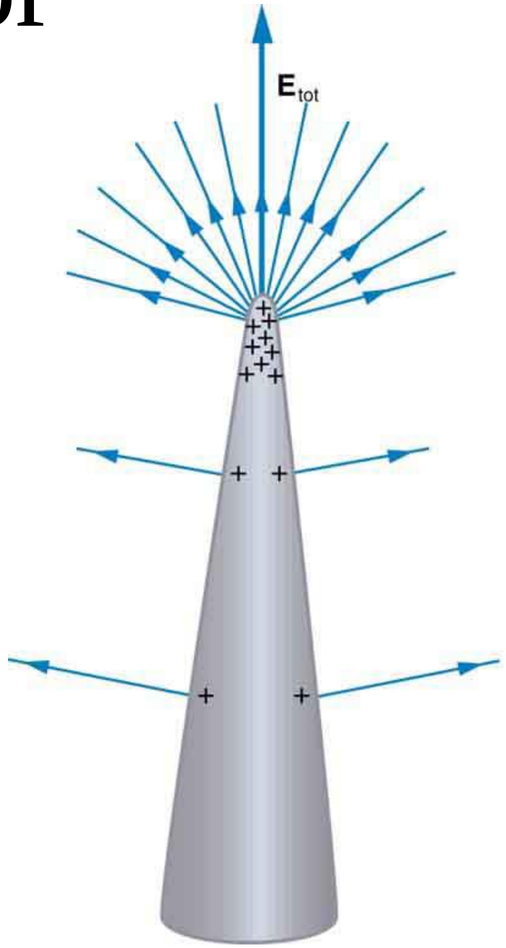
Electric Field for a Pointed Conductor



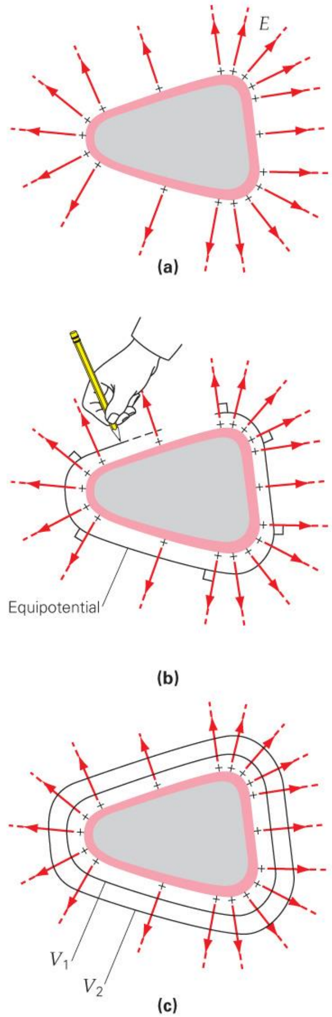
- **Figure 6: Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature. (a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is $F_{||}$ that moves the charges apart once they have reached the surface. (b) $F_{||}$ is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c) An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end.**
- From: OpenStax College. (2012, June 6). Conductors and Electric Fields in Static Equilibrium. Retrieved from the Connexions Web site: http://cnx.org/content/m42317/1.3/

Electric Field for a Pointed Conductor

- **Figure 7: A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor.**
- **Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.**
- From: OpenStax College. (2012, June 6). Conductors and Electric Fields in Static Equilibrium. Retrieved from the Connexions Web site: http://cnx.org/content/m42317/1.3/



Equipotential Surfaces - Summary



- Equipotentials are surfaces around a set of charges which are all at the same potential.
- Equipotential surfaces are always perpendicular to the electric field lines which pass through them.
- Around a point source they are spheres.
- Between parallel plates they are planes.
- The surface of a conductor is, by definition, an equipotential.

Determining E (vector) from V

- From our relationship between Voltage and field, we can develop a formula for finding E from dV.
- $dV = -\vec{E} \bullet d\vec{l}$, or $E_l = -\frac{dV}{dl}$
- So the change in V as we move a distance dl is just the component of E (Ecosθ) along that infinitesimal path.
- This means that we can find the x, y, & z components of the E vector by finding the change in V in each of the 3 directions.

Determining E (vector) from V

- Notice that since we use $dV = -\vec{E} \bullet d\vec{l}$, that dV will be largest if $d\vec{l}$ is chosen to be in the direction of $\vec{E}$, which is perpendicular to the equipotential surfaces of V.
- We need a mathematical operation which picks out the direction of maximum increase or decrease of a function, V in this case.
- This mathematical process is the gradient operator, grad or ∇.

Determining E (vector) from V

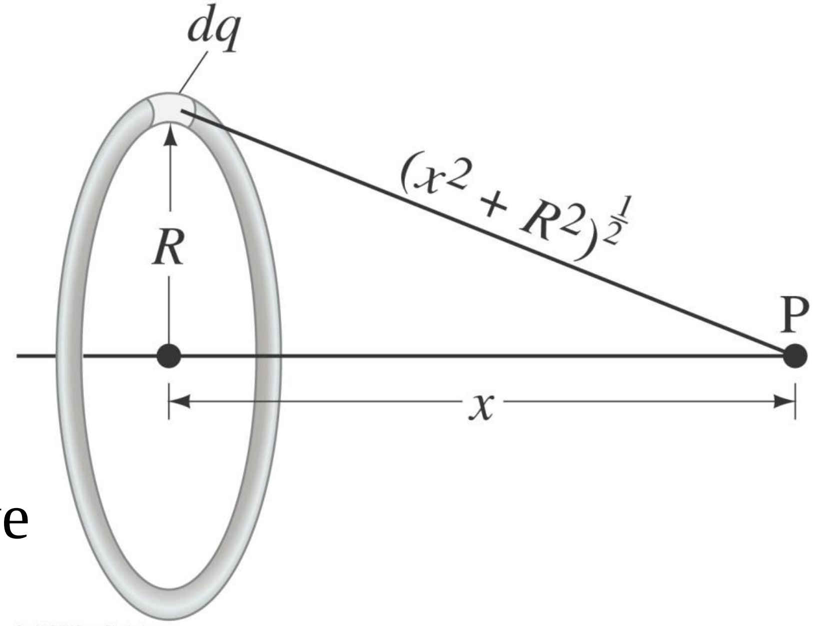
- We construct the grad operation by successively replacing dl with dx, dy, & dz.
- Using these three differentials, we can find the three components of $\vec{E}$.
- $E_x = -\frac{\delta V}{\delta x}, E_y = -\frac{\delta V}{\delta y}, E_z = -\frac{\delta V}{\delta z}$
- Where the symbol δ indicates a ‘partial derivative’.
- δV/δ*x* means that we differentiate V with respect to x while holding y & z constant.

Determining E (vector) from V

- Once we have the components, we can write $\vec{E}$ in unit vector form to get:
- $\vec{E} = -\frac{\delta V}{\delta x}\hat{\imath} - \frac{\delta V}{\delta y}\hat{\jmath} - \frac{\delta V}{\delta z}\hat{k}$
- This is the mathematical operation is called a gradient function, symbolized by ∇.
- $\nabla = +\frac{\delta}{\delta x}\hat{\imath} + \frac{\delta}{\delta y}\hat{\jmath} + \frac{\delta}{\delta z}\hat{k}$
- $\vec{E} = -\nabla V = -Gradient\ of\ V = -Grad\ V$

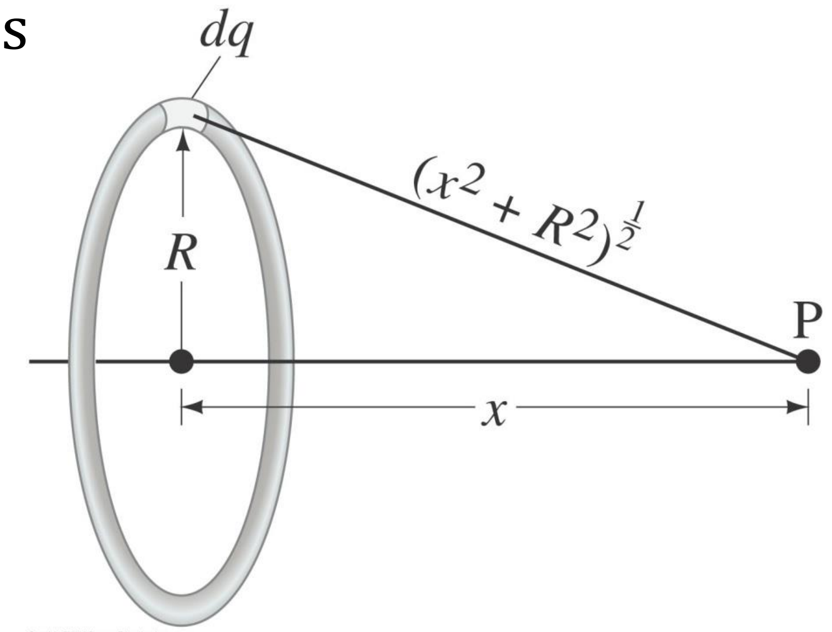
E from V for a ring of charge.

- $V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{(x^2+R^2)^{1/2}}$
- $E_x = \frac{\delta V}{\delta x}$
- $E_x = -\frac{Q}{4\pi\varepsilon_0} \frac{\delta}{\delta x} (x^2 + R^2)^{-1/2}$
- $E_x = -\frac{Q}{4\pi\varepsilon_0} \frac{x}{(x^2+R^2)^{3/2}}$
- Which is the same result that we got by integrating E_x directly.



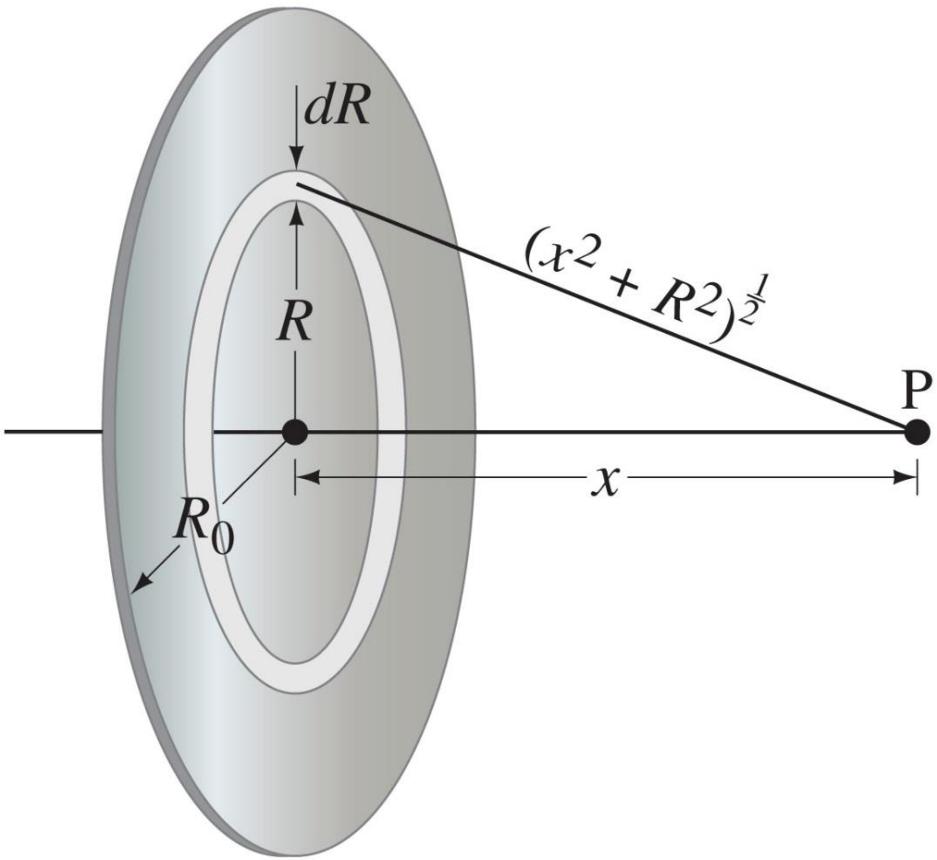
E from V for a ring of charge.

- We said that E_y was zero on the basis of symmetry, so let's see
- $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x^2+R^2)^{1/2}}$
- $E_y = \frac{\delta V}{\delta y}$
- $E_y = -\frac{Q}{4\pi\varepsilon_0}\frac{\delta}{\delta y}(x^2+R^2)^{-1/2}$
- $E_y = 0$



Example Potential due to a charged disk.

A thin flat disk, of radius R_0, has a uniformly distributed charge Q. Determine the potential at a point P on the axis of the disk, a distance x from its center.



EXAMPLE 23–9 **Potential due to a charged disk.** A thin flat disk, of radius R_0, has a uniformly distributed charge Q, Fig. 23–15. Determine the potential at a point P on the axis of the disk, a distance x from its center.

APPROACH We divide the disk into thin rings of radius R and thickness dR and use the result of Example 23–8 to sum over the disk.

SOLUTION The charge Q is distributed uniformly, so the charge contained in each ring is proportional to its area. The disk has area πR_0^2 and each thin ring has area $dA = (2\pi R)(dR)$. Hence

$$\frac{dq}{Q} = \frac{2\pi R\, dR}{\pi R_0^2}$$

so

$$dq = Q\frac{(2\pi R)(dR)}{\pi R_0^2} = \frac{2QR\, dR}{R_0^2}.$$

Then the potential at P, using Eq. 23–6b in which r is replaced by $(x^2 + R^2)^{\frac{1}{2}}$, is

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{(x^2+R^2)^{\frac{1}{2}}} = \frac{2Q}{4\pi\epsilon_0 R_0^2}\int_0^{R_0}\frac{R\, dR}{(x^2+R^2)^{\frac{1}{2}}} = \frac{Q}{2\pi\epsilon_0 R_0^2}(x^2+R^2)^{\frac{1}{2}}\Big|_{R=0}^{R=R_0}$$

$$= \frac{Q}{2\pi\epsilon_0 R_0^2}\left[(x^2+R_0^2)^{\frac{1}{2}} - x\right].$$

NOTE For $x \gg R_0$, this formula reduces to

$$V \approx \frac{Q}{2\pi\epsilon_0 R_0^2}\left[x\left(1+\frac{1}{2}\frac{R_0^2}{x^2}\right) - x\right] = \frac{Q}{4\pi\epsilon_0 x}.$$

This is the formula for a point charge, as we expect.

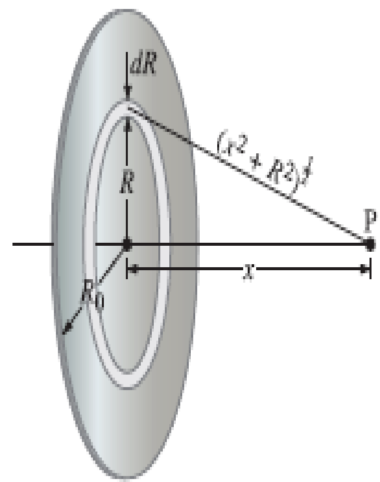


FIGURE 23–15 Example 23–9. Calculating the electric potential at point P on the axis of a uniformly charged thin disk.

Find the potential at a distance r from a very long line of charge with linear charge density (charge per unit length) λ.

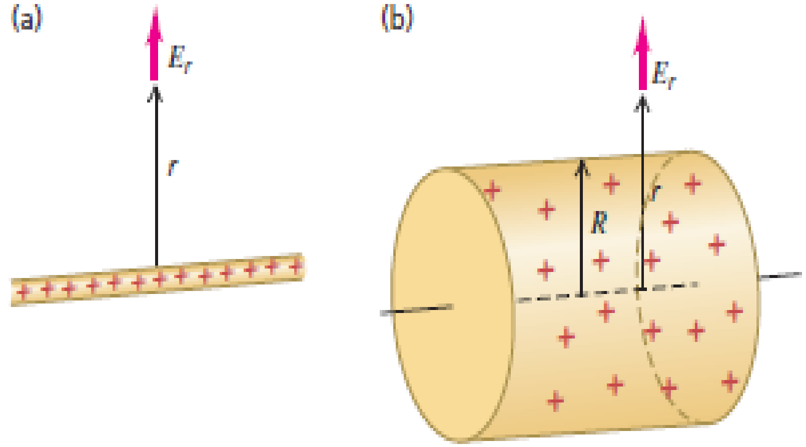
SOLUTION

IDENTIFY and SET UP: In both Example 21.10 (Section 21.5) and Example 22.6 (Section 22.4) we found that the electric field at a radial distance r from a long straight-line charge (**Fig. 23.19a**) has only a radial component $E_r = \lambda/2\pi\epsilon_0 r$. We use this expression to find the potential by integrating $\vec{E}$ as in Eq. (23.17).

EXECUTE: Since the field has only a radial component, we have $\vec{E} \cdot d\vec{l} = E_r dr$. Hence from Eq. (23.17) the potential of any point a with respect to any other point b, at radial distances r_a and r_b from the line of charge, is

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E_r dr = \frac{\lambda}{2\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_b}{r_a}$$

23.19 Electric field outside (a) a long, positively charged wire and (b) a long, positively charged cylinder.



If we take point b at infinity and set $V_b = 0$, we find that V_a is *infinite* for any finite distance r_a from the line charge: $V_a = (\lambda/2\pi\epsilon_0)\ln(\infty/r_a) = \infty$. This is *not* a useful way to define

V for this problem! The difficulty is that the charge distribution itself extends to infinity.

Instead, as recommended in Problem-Solving Strategy 23.1, we set $V_b = 0$ at point b at an arbitrary but *finite* radial distance r_0. Then the potential $V = V_a$ at point a at a radial distance r is given by $V - 0 = (\lambda/2\pi\epsilon_0)\ln(r_0/r)$, or

$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_0}{r}$$

EVALUATE: According to our result, if λ is positive, then V decreases as r increases. This is as it should be: V decreases as we move in the direction of $\vec{E}$.

From Example 22.6, the expression for E_r with which we started also applies outside a long, charged conducting cylinder with charge per unit length λ (Fig. 23.19b). Hence our result also gives the potential for such a cylinder, but only for values of r (the distance from the cylinder axis) equal to or greater than the radius R of the cylinder. If we choose r_0 to be the radius R, so that $V = 0$ when $r = R$, then at any point for which $r > R$,

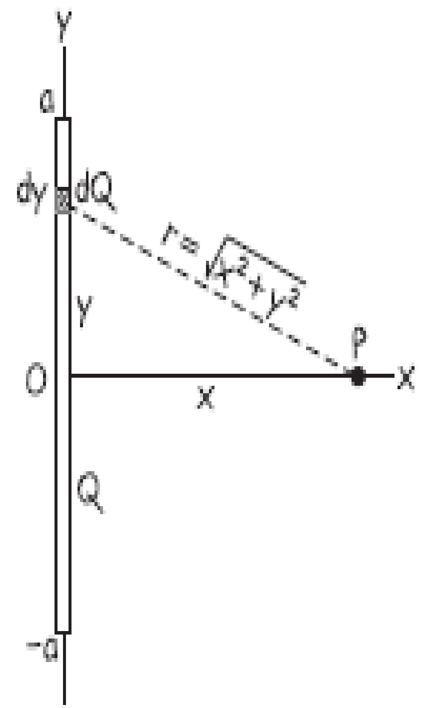
$$V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{R}{r}$$

Inside the cylinder, $\vec{E} = \mathbf{0}$, and V has the same value (zero) as on the cylinder's surface.

EXAMPLE 23.12 POTENTIAL OF A LINE OF CHARGE

Positive electric charge Q is distributed uniformly along a line of length $2a$ lying along the y-axis between $y = -a$ and $y = +a$ (**Fig. 23.21**). Find the electric potential at a point P on the x-axis at a distance x from the origin.

23.21 Our sketch for this problem.



SOLUTION

IDENTIFY and SET UP: This is the situation of Example 21.10 (Section 21.5), where we found an expression for the electric field $\vec{E}$ at an arbitrary point on the x-axis. We can find V at point P by using Eq. (23.16) to integrate over the charge distribution. Unlike the situation in Example 23.11, each charge element dQ is a *different* distance from point P, so the integration will take a little more effort.

EXECUTE: As in Example 21.10, the element of charge dQ corresponding to an element of length dy on the rod is $dQ = (Q/2a)dy$. The distance from dQ to P is $\sqrt{x^2 + y^2}$, so the contribution dV that the charge element makes to the potential at P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{\sqrt{x^2 + y^2}}$$

To find the potential at P due to the entire rod, we integrate dV over the length of the rod from $y = -a$ to $y = a$:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{dy}{\sqrt{x^2 + y^2}}$$

You can look up the integral in a table. The final result is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\ln\left(\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right)$$

EVALUATE: We can check our result by letting x approach infinity. In this limit the point P is infinitely far from all of the charge, so we expect V to approach zero; you can verify that it does.

We know the electric field at all points along the x-axis from Example 21.10. We invite you to use this information to find V along this axis by integrating $\vec{E}$ as in Eq. (23.17).

The Electron Volt

- The <u>electron volt</u> is an <u>energy unit</u> defined as the <u>kinetic energy</u> gained by an electron (or any singly charged particle) when it is accelerated through a potential of volt.
- We can calculate the increase in kinetic energy from the decrease in potential energy
- $1eV = -\Delta U = -q \cdot \Delta V = -(-1.6x10^{-19}C)(1J/C)$
- $1eV = 1.6x10^{-19}J$

Electron Volts

The magnitude e of the electron charge can be used to define a unit of energy that is useful in many calculations with atomic and nuclear systems. When a particle with charge q moves from a point where the potential is V_b to a point where it is V_a, the change in the potential energy U is

$$U_a - U_b = q(V_a - V_b) = qV_{ab}$$

If charge q equals the magnitude e of the electron charge, 1.602×10^{-19} C, and the potential difference is $V_{ab} = 1$ V, the change in energy is

$$U_a - U_b = (1.602 \times 10^{-19}\ \text{C})(1\ \text{V}) = 1.602 \times 10^{-19}\ \text{J}$$

This quantity of energy is defined to be 1 **electron volt** (1 eV):

$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

The multiples meV, keV, MeV, GeV, and TeV are often used.

End of Volume 2 Chapter 7

- Read the Chapter Summary -
- Complete the Homework for Ch 23 (Vol 2 Ch 7)

Backup slides

4. (II) The work done by an external force to move a $-9.10\ \mu C$ charge from point a to point b is 7.00×10^{-4} J. If the charge was started from rest and had 2.10×10^{-4} J of kinetic energy when it reached point b, what must be the potential difference between a and b?

4. By the work energy theorem, the total work done, by the external force and the electric field together, is the change in kinetic energy. The work done by the electric field is given by Eq. 23-2b.

$$W_{\text{external}} + W_{\text{electric}} = \text{KE}_{\text{final}} - \text{KE}_{\text{initial}} \rightarrow W_{\text{external}} - q\left(V_b - V_a\right) = \text{KE}_{\text{final}} \rightarrow$$

$$\left(V_b - V_a\right) = \frac{W_{\text{external}} - \text{KE}_{\text{final}}}{q} = \frac{7.00 \times 10^{-4}\,\text{J} - 2.10 \times 10^{-4}\,\text{J}}{-9.10 \times 10^{-6}\,\text{C}} = \boxed{-53.8\,\text{V}}$$

Since the potential difference is negative, we see that $V_a > V_b$.

In **Fig. 23.15** a dust particle with mass $m = 5.0 \times 10^{-9}$ kg = 5.0 μg and charge $q_0 = 2.0$ nC starts from rest and moves in a straight line from point a to point b. What is its speed v at point b?

SOLUTION

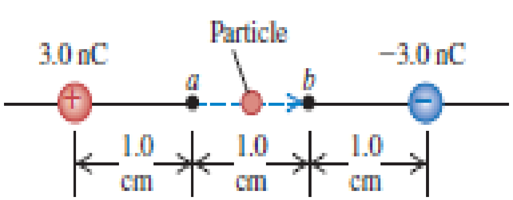
IDENTIFY and SET UP: Only the conservative electric force acts on the particle, so mechanical energy is conserved: $K_a + U_a = K_b + U_b$. We get the potential energies U from the corresponding potentials V from Eq. (23.12): $U_a = q_0 V_a$ and $U_b = q_0 V_b$.

EXECUTE: We have $K_a = 0$ and $K_b = \frac{1}{2}mv^2$. We substitute these and our expressions for U_a and U_b into the energy-conservation equation, then solve for v. We find

$$0 + q_0 V_a = \tfrac{1}{2}mv^2 + q_0 V_b$$

$$v = \sqrt{\frac{2q_0(V_a - V_b)}{m}}$$

23.15 The particle moves from point a to point b; its acceleration is not constant.



We calculate the potentials from Eq. (23.15), $V = q/4\pi\epsilon_0 r$:

$$V_a = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \times \left(\frac{3.0 \times 10^{-9}\ \mathrm{C}}{0.010\ \mathrm{m}} + \frac{(-3.0 \times 10^{-9}\ \mathrm{C})}{0.020\ \mathrm{m}}\right)$$

$$= 1350\ \mathrm{V}$$

$$V_b = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \times \left(\frac{3.0 \times 10^{-9}\ \mathrm{C}}{0.020\ \mathrm{m}} + \frac{(-3.0 \times 10^{-9}\ \mathrm{C})}{0.010\ \mathrm{m}}\right)$$

$$= -1350\ \mathrm{V}$$

$$V_a - V_b = (1350\ \mathrm{V}) - (-1350\ \mathrm{V}) = 2700\ \mathrm{V}$$

Finally,

$$v = \sqrt{\frac{2(2.0 \times 10^{-9}\ \mathrm{C})(2700\ \mathrm{V})}{5.0 \times 10^{-9}\ \mathrm{kg}}} = 46\ \mathrm{m/s}$$

EVALUATE: Our result makes sense: The positive dust particle speeds up as it moves away from the +3.0-nC charge and toward the −3.0-nC charge. To check unit consistency in the final line of the calculation, note that 1 V = 1 J/C, so the numerator under the radical has units of J or $\mathrm{kg \cdot m^2/s^2}$.

An electric dipole consists of point charges $q_1 = +12\text{ nC}$ and $q_2 = -12\text{ nC}$ placed 10.0 cm apart (**Fig. 23.13**). Compute the electric potentials at points a, b, and c.

SOLUTION

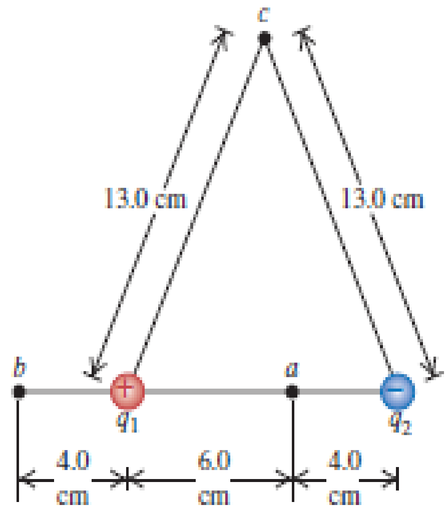
IDENTIFY and SET UP: This is the same arrangement as in Example 21.8, in which we calculated the electric *field* at each point by doing a *vector* sum. Here our target variable is the electric *potential* V at three points, which we find by doing the *algebraic* sum in Eq. (23.15).

EXECUTE: At point a we have $r_1 = 0.060\text{ m}$ and $r_2 = 0.040\text{ m}$, so Eq. (23.15) becomes

$$V_a = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$$
$$= (9.0 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\text{ C}}{0.060\text{ m}}$$
$$+ (9.0 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-12 \times 10^{-9}\text{ C})}{0.040\text{ m}}$$
$$= 1800\text{ N}\cdot\text{m/C} + (-2700\text{ N}\cdot\text{m/C})$$
$$= 1800\text{ V} + (-2700\text{ V}) = -900\text{ V}$$

In a similar way you can show that the potential at point b (where $r_1 = 0.040\text{ m}$ and $r_2 = 0.140\text{ m}$) is $V_b = 1930\text{ V}$ and that the potential at point c (where $r_1 = r_2 = 0.130\text{ m}$) is $V_c = 0$.

23.13 What are the potentials at points a, b, and c due to this electric dipole?



EVALUATE: Let's confirm that these results make sense. Point a is closer to the -12-nC charge than to the $+12$-nC charge, so the potential at a is negative. The potential is positive at point b, which is closer to the $+12$-nC charge than the -12-nC charge. Finally, point c is equidistant from the $+12$-nC charge and the -12-nC charge, so the potential there is zero. (The potential is also equal to zero at a point infinitely far from both charges.)

Comparing this example with Example 21.8 shows that it's much easier to calculate electric potential (a scalar) than electric field (a vector). We'll take advantage of this simplification whenever possible.

Example 23-5: Breakdown voltage.

In many kinds of equipment, very high voltages are used. A problem with high voltage is that the air can become ionized due to the high electric fields: free electrons in the air (produced by cosmic rays, for example) can be accelerated by such high fields to speeds sufficient to ionize O_2 and N_2 molecules by collision, knocking out one or more of their electrons. The air then becomes conducting and the high voltage cannot be maintained as charge flows. The breakdown of air occurs for electric fields of about 3.0×10^6 V/m. (a) Show that the breakdown voltage for a spherical conductor in air is proportional to the radius of the sphere, and (b) estimate the breakdown voltage in air for a sphere of diameter 1.0 cm.

EXAMPLE 23–5 **Breakdown voltage.** In many kinds of equipment, very high voltages are used. A problem with high voltage is that the air can become ionized due to the high electric fields: free electrons in the air (produced by cosmic rays, for example) can be accelerated by such high fields to speeds sufficient to ionize O_2 and N_2 molecules by collision, knocking out one or more of their electrons. The air then becomes conducting and the high voltage cannot be maintained as charge flows. The breakdown of air occurs for electric fields of about $3 \times 10^6\ \mathrm{V/m}$. (*a*) Show that the breakdown voltage for a spherical conductor in air is proportional to the radius of the sphere, and (*b*) estimate the breakdown voltage in air for a sphere of diameter 1.0 cm.

APPROACH The electric potential at the surface of a spherical conductor of radius r_0 (Example 23–4), and the electric field just outside its surface, are

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0} \quad \text{and} \quad E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_0^2}.$$

SOLUTION (*a*) We combine these two equations and obtain

$$V = r_0 E. \qquad \text{[at surface of spherical conductor]}$$

(*b*) For $r_0 = 5 \times 10^{-3}\ \mathrm{m}$, the breakdown voltage in air is

$$V = (5 \times 10^{-3}\ \mathrm{m})(3 \times 10^6\ \mathrm{V/m}) \approx 15{,}000\ \mathrm{V}.$$

When high voltages are present, a glow may be seen around sharp points, known as a **corona discharge**, due to the high electric fields at these points which ionize air molecules. The light we see is due to electrons jumping down to empty lower states. **Lightning rods**, with their sharp tips, are intended to ionize the surrounding air when a storm cloud is near, and to provide a conduction path to discharge a dangerous high-voltage cloud slowly, over a period of time. Thus lightning rods, connected to the ground, are intended to draw electric charge off threatening clouds before a large buildup of charge results in a swift destructive lightning bolt.

11. (II) A uniform electric field $\vec{\mathbf{E}} = -4.20\,\text{N/C}\,\hat{\mathbf{i}}$ points in the negative x direction as shown in Fig. 23–25. The x and y coordinates of points A, B, and C are given on the diagram (in meters). Determine the differences in potential (*a*) V_{BA}, (*b*) V_{CB}, and (*c*) V_{CA}.

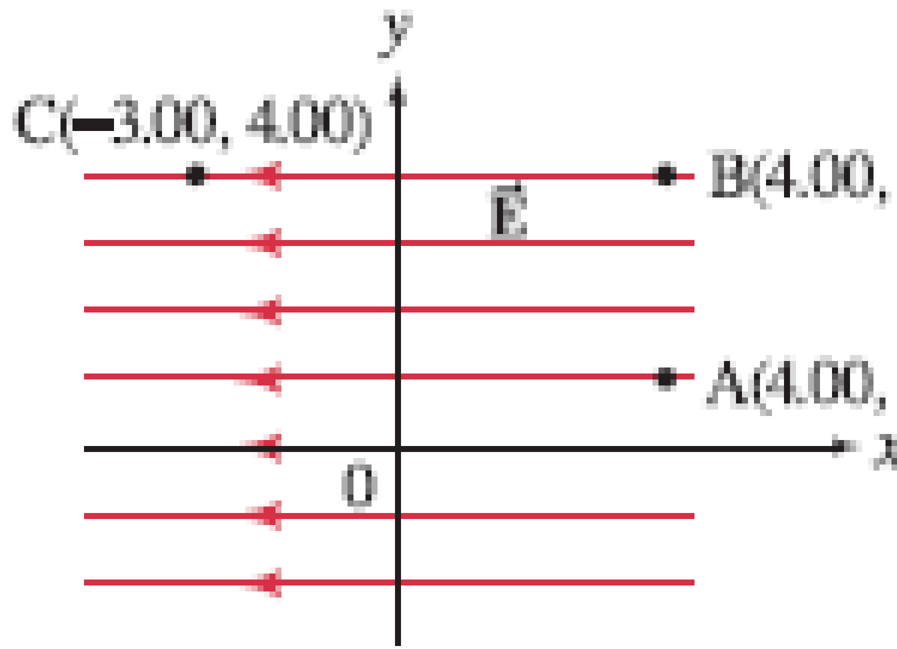


FIGURE 23–25
Problem 11.

11. Since the field is uniform, we may apply Eq. 23-4b. Note that the electric field always points from high potential to low potential.

(*a*) $\boxed{V_{BA} = 0}$. The distance between the two points is exactly perpendicular to the field lines.

(*b*) $V_{CB} = V_C - V_B = (-4.20\,\text{N/C})(7.00\,\text{m}) = \boxed{-29.4\,\text{V}}$

(*c*) $V_{CA} = V_C - V_A = V_C - V_B + V_B - V_A = V_{CB} + V_{BA} = -29.4\,\text{V} + 0 = \boxed{-29.4\,\text{V}}$

18. (II) Estimate the electric field in the membrane wall of a living cell. Assume the wall is 10 nm thick and has a potential of 0.10 V across it.

18. We assume the field is uniform, and so Eq. 23-4b applies.

$$E = \frac{V}{d} = \frac{0.10\,\text{V}}{10 \times 10^{-9}\,\text{m}} = \boxed{1 \times 10^{7}\ \text{V/m}}$$

19. (II) A nonconducting sphere of radius r_0 carries a total charge Q distributed uniformly throughout its volume. Determine the electric potential as a function of the distance r from the center of the sphere for (*a*) $r > r_0$ and (*b*) $r < r_0$. Take $V = 0$ at $r = \infty$. (*c*) Plot V versus r and E versus r.

(a) The electric field outside a charged, spherically symmetric volume is the same as that for a point charge of the same magnitude of charge. Integrating the electric field from infinity to the radius of interest will give the potential at that radius.

$$E(r \geq r_0) = \frac{Q}{4\pi\varepsilon_0 r^2} \quad ; \quad V(r \geq r_0) = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \left.\frac{Q}{4\pi\varepsilon_0 r}\right|_{\infty}^{r} = \boxed{\frac{Q}{4\pi\varepsilon_0 r}}$$

(b) Inside the sphere the electric field is obtained from Gauss's Law using the charge enclosed by a sphere of radius r.

$$4\pi r^2 E = \frac{Q}{\varepsilon_0}\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi r_0^3} \quad \rightarrow \quad E(r < r_0) = \frac{Qr}{4\pi\varepsilon_0 r_0^3}$$

Integrating the electric field from the surface to $r < r_0$ gives the electric potential inside the sphere.

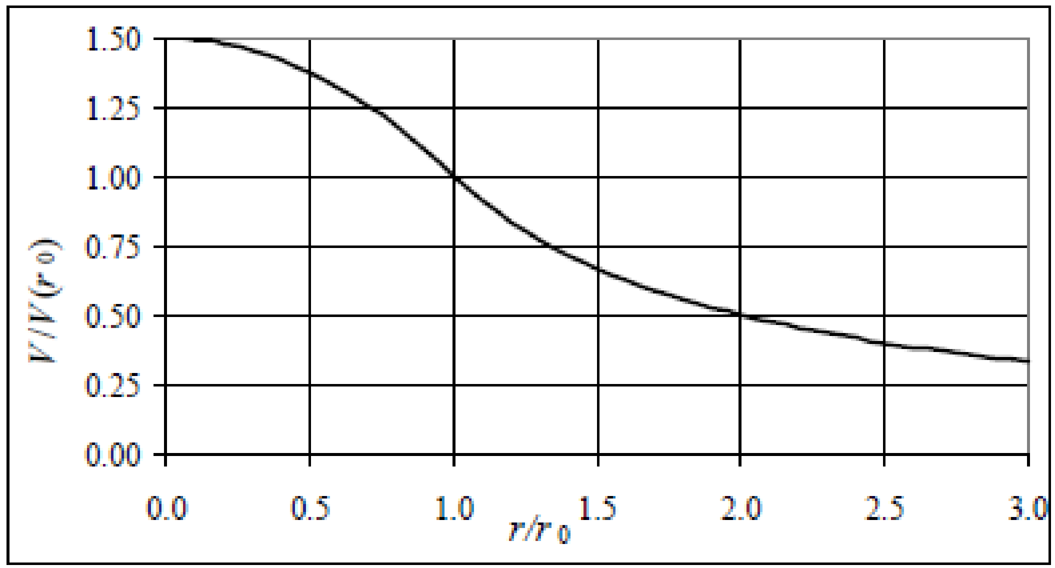
$$V(r < r_0) = V(r_0) - \int_{r_0}^{r} \frac{Qr}{4\pi\varepsilon_0 r_0^3}\,dr = \frac{Q}{4\pi\varepsilon_0 r_0} - \left.\frac{Qr^2}{8\pi\varepsilon_0 r_0^3}\right|_{r_0}^{r} = \boxed{\frac{Q}{8\pi\varepsilon_0 r_0}\left(3 - \frac{r^2}{r_0^2}\right)}$$

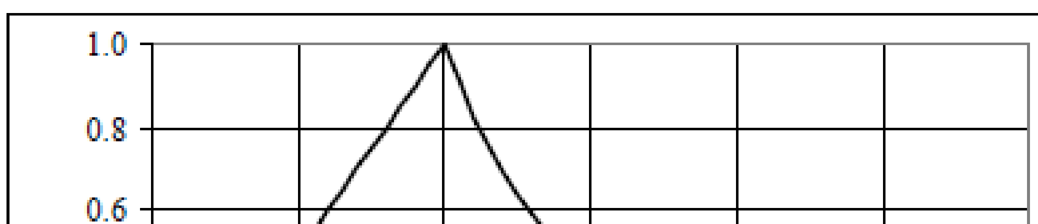
(c) To plot, we first calculate $V_0 = V(r = r_0) = \frac{Q}{4\pi\varepsilon_0 r_0}$ and $E_0 = E(r = r_0) = \frac{Q}{4\pi\varepsilon_0 r_0^2}$. Then we plot V/V_0 and E/E_0 as functions of r/r_0.

For $r < r_0$: $\quad V/V_0 = \dfrac{\dfrac{Q}{8\pi\varepsilon_0 r_0}\left(3 - \dfrac{r^2}{r_0^2}\right)}{\dfrac{Q}{4\pi\varepsilon_0 r_0}} = \frac{1}{2}\left(3 - \dfrac{r^2}{r_0^2}\right)$; $E/E_0 = \dfrac{\dfrac{Qr}{4\pi\varepsilon_0 r_0^3}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}} = \dfrac{r}{r_0}$

For $r > r_0$: $\quad V/V_0 = \dfrac{\dfrac{Q}{4\pi\varepsilon_0 r}}{\dfrac{Q}{4\pi\varepsilon_0 r_0}} = \dfrac{r_0}{r} = (r/r_0)^{-1}$; $E/E_0 = \dfrac{\dfrac{Q}{4\pi\varepsilon_0 r^2}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}} = \dfrac{r_0^2}{r^2} = (r/r_0)^{-2}$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.19c."





20. (III) Repeat Problem 19 assuming the charge density ρ_E increases as the square of the distance from the center of the sphere, and $\rho_E = 0$ at the center.

We assume the total charge is still Q, and let $\rho_E = kr^2$. We evaluate the constant k by calculating the total charge, in the manner of Example 22-5.

$$Q = \int \rho_E dV = \int_0^{r_0} kr^2 \left(4\pi r^2 dr\right) = \tfrac{4}{5} k\pi r_0^5 \quad \rightarrow \quad k = \frac{5Q}{4\pi r_0^5}$$

(*a*) The electric field outside a charged, spherically symmetric volume is the same as that for a point charge of the same magnitude of charge. Integrating the electric field from infinity to the radius of interest gives the potential at that radius.

$$E\left(r \geq r_0\right) = \frac{Q}{4\pi\varepsilon_0 r^2} \quad ; \quad V\left(r \geq r_0\right) = -\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r^2} dr = \left.\frac{Q}{4\pi\varepsilon_0 r}\right|_\infty^r = \boxed{\frac{Q}{4\pi\varepsilon_0 r}}$$

(*b*) Inside the sphere the electric field is obtained from Gauss's Law using the charge enclosed by a sphere of radius r.

$$4\pi r^2 E = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad ; \quad Q_{\text{encl}} = \int \rho_E dV = \frac{5Q}{4\pi r_0^5} \int_0^r r^2 \left(4\pi r^2 dr\right) = \frac{5Q}{4\pi r_0^5} \tfrac{4}{5} \pi r^5 = \frac{Qr^5}{r_0^5} \quad \rightarrow$$

$$E(r<r_0)=\frac{Q_{\text{encl}}}{4\pi\varepsilon_0 r^2}=\frac{Qr^3}{4\pi\varepsilon_0 r_0^5}$$

Integrating the electric field from the surface to $r<r_0$ gives the electric potential inside the sphere.

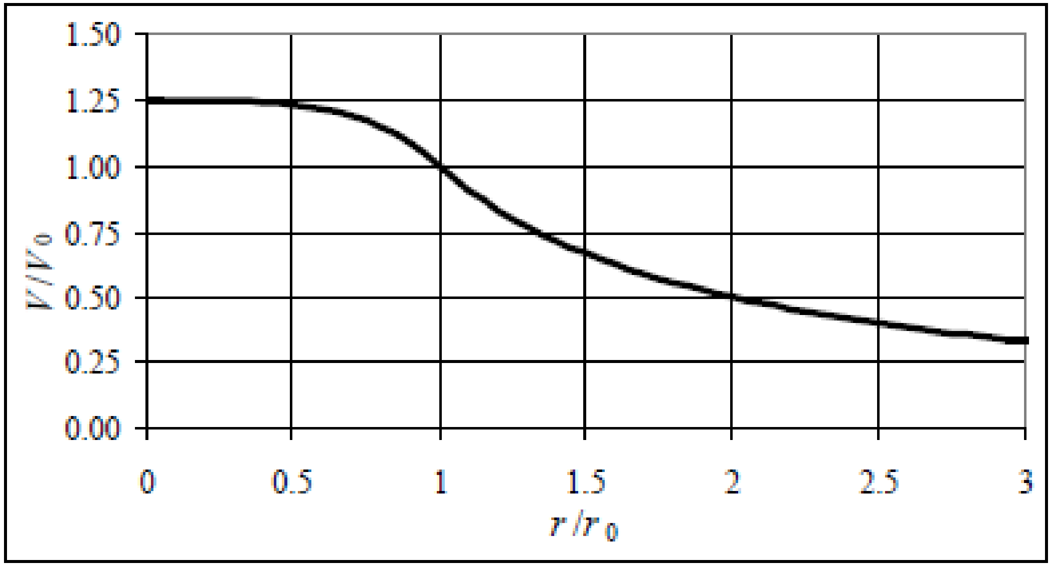
$$V(r<r_0)=V(r_0)-\int_{r_0}^{r}\frac{Qr^3}{4\pi\varepsilon_0 r_0^5}dr=\frac{Q}{4\pi\varepsilon_0 r_0}-\frac{Qr^4}{16\pi\varepsilon_0 r_0^5}\Bigg|_{r_0}^{r}=\frac{Q}{16\pi\varepsilon_0 r_0}\left(5-\frac{r^4}{r_0^4}\right)$$

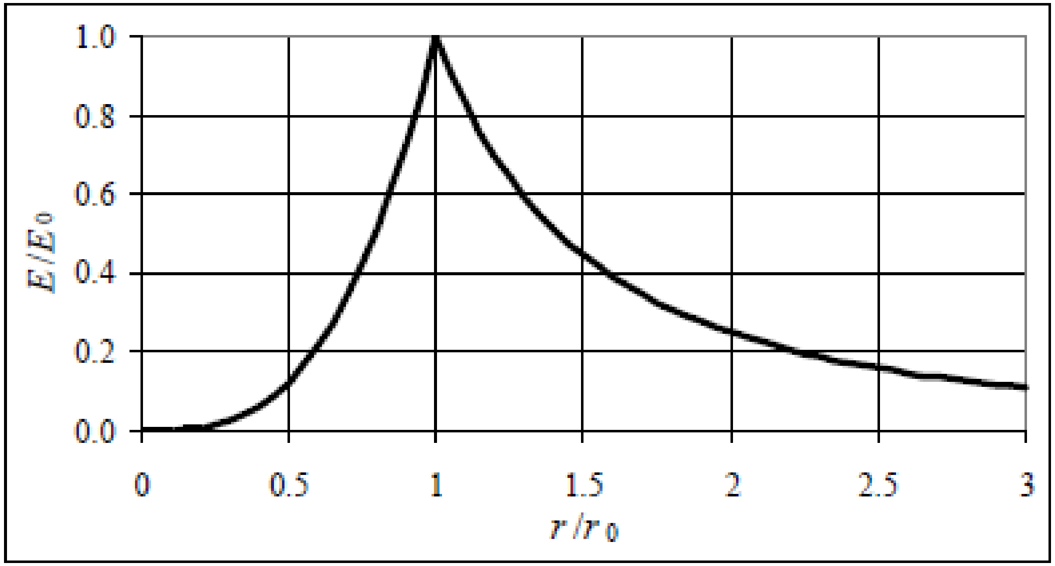
(c) To plot, we first calculate $V_0=V(r=r_0)=\dfrac{Q}{4\pi\varepsilon_0 r_0}$ and $E_0=E(r=r_0)=\dfrac{Q}{4\pi\varepsilon_0 r_0^2}$. Then we plot V/V_0 and E/E_0 as functions of r/r_0.

For $r<r_0$: $\quad V/V_0=\dfrac{\dfrac{Q}{16\pi\varepsilon_0 r_0}\left(5-\dfrac{r^4}{r_0^4}\right)}{\dfrac{Q}{4\pi\varepsilon_0 r_0}}=\frac{1}{4}\left(5-\dfrac{r^4}{r_0^4}\right)$; $E/E_0=\dfrac{\dfrac{Qr^3}{4\pi\varepsilon_0 r_0^5}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}}=\dfrac{r^3}{r_0^3}$

For $r>r_0$: $\quad V/V_0=\dfrac{\dfrac{Q}{4\pi\varepsilon_0 r}}{\dfrac{Q}{4\pi\varepsilon_0 r_0}}=\dfrac{r_0}{r}=(r/r_0)^{-1}$; $E/E_0=\dfrac{\dfrac{Q}{4\pi\varepsilon_0 r^2}}{\dfrac{Q}{4\pi\varepsilon_0 r_0^2}}=\dfrac{r_0^2}{r^2}=(r/r_0)^{-2}$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.20c."





21. (III) The volume charge density ρ_E within a sphere of radius r_0 is distributed in accordance with the following spherically symmetric relation

$$\rho_E(r) = \rho_0\left[1 - \frac{r^2}{r_0^2}\right]$$

where r is measured from the center of the sphere and ρ_0 is a constant. For a point P inside the sphere ($r < r_0$), determine the electric potential V. Let $V = 0$ at infinity.

21. We first need to find the electric field. Since the charge distribution is spherically symmetric, Gauss's law tells us the electric field everywhere.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2}$$

If $r < r_0$, calculate the charge enclosed in the manner of Example 22-5.

$$Q_{\text{encl}} = \int \rho_E dV = \int_0^r \rho_0\left[1 - \frac{r^2}{r_0^2}\right]4\pi r^2 dr = 4\pi\rho_0\int_0^r\left[r^2 - \frac{r^4}{r_0^2}\right]dr = 4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^5}{5r_0^2}\right]$$

The total charge in the sphere is the above expression evaluated at $r = r_0$.

$$Q_{\text{total}} = 4\pi\rho_0\left[\frac{r_0^3}{3} - \frac{r_0^5}{5r_0^2}\right] = \frac{8\pi\rho_0 r_0^3}{15}$$

Outside the sphere, we may treat it as a point charge, and so the potential at the surface of the sphere is given by Eq. 23-5, evaluated at the surface of the sphere.

$$V\left(r = r_0\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{total}}}{r_0} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{8\pi\rho_0 r_0^3}{15}}{r_0} = \frac{2\rho_0 r_0^2}{15\varepsilon_0}$$

The potential inside is found from Eq. 23-4a. We need the field inside the sphere to use Eq. 23-4a. The field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = E dr$.

$$E\left(r < r_0\right) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^5}{5r_0^2}\right]}{r^2} = \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^3}{5r_0^2}\right]$$

$$V_r - V_{r_0} = -\int_{r_0}^{r} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\int_{r_0}^{r} E\, dr = -\int_{r_0}^{r} \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^3}{5r_0^2}\right]dr = -\frac{\rho_0}{\varepsilon_0}\left[\frac{r^2}{6} - \frac{r^4}{20r_0^2}\right]_{r_0}^{r}$$

$$V_r = V_{r_0} + \left(-\frac{\rho_0}{\varepsilon_0}\left[\frac{r^2}{6} - \frac{r^4}{20r_0^2}\right]_{r_0}^{r}\right) = \frac{2\rho_0 r_0^2}{15\varepsilon_0} - \frac{\rho_0}{\varepsilon_0}\left[\left(\frac{r^2}{6} - \frac{r^4}{20r_0^2}\right) - \left(\frac{r_0^2}{6} - \frac{r_0^4}{20r_0^2}\right)\right]$$

$$= \boxed{\frac{\rho_0}{\varepsilon_0}\left(\frac{r_0^2}{4} - \frac{r^2}{6} + \frac{r^4}{20r_0^2}\right)}$$

22. (III) A hollow spherical conductor, carrying a net charge $+Q$, has inner radius r_1 and outer radius $r_2 = 2r_1$ (Fig. 23–26). At the center of the sphere is a point charge $+Q/2$. (*a*) Write the electric field strength E in all three regions as a function of r. Then determine the potential as a function of r, the distance from the center, for (*b*) $r > r_2$, (*c*) $r_1 < r < r_2$, and (*d*) $0 < r < r_1$. (*e*) Plot both V and E as a function of r from $r = 0$ to $r = 2r_2$.

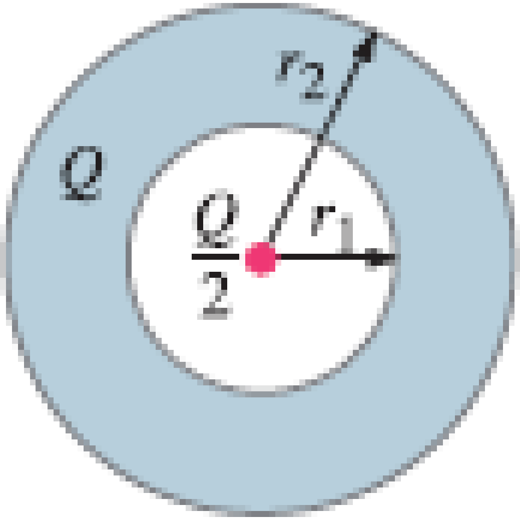


FIGURE 23–26 Problem 22.

22. Because of the spherical symmetry of the problem, the electric field in each region is the same as that of a point charge equal to the net enclosed charge.

(a) For $r > r_2$: $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_{\text{encl}}}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\frac{3}{2}Q}{r^2} = \boxed{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r^2}}$

For $r_1 < r < r_2$: $E = \boxed{0}$, because the electric field is 0 inside of conducting material.

For $0 < r < r_1$: $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_{\text{encl}}}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\frac{1}{2}Q}{r^2} = \boxed{\dfrac{1}{8\pi\varepsilon_0}\dfrac{Q}{r^2}}$

(b) For $r > r_2$, the potential is that of a point charge at the center of the sphere.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{3}{2}Q}{r} = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r}},\ r > r_2$$

c) For $r_1 < r < r_2$, the potential is constant and equal to its value on the outer shell, because there is no electric field inside the conducting material.

$$V = V(r = r_2) = \boxed{\frac{3}{8\pi\varepsilon_0}\frac{Q}{r_2}}, \ r_1 < r < r_2$$

d) For $0 < r < r_1$, we use Eq. 23-4a. The field is radial, so we integrate along a radial line so that $\vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = E dr$.

$$V_r - V_{r_1} = -\int_{r_1}^{r} \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}} = -\int_{r_1}^{r} E\, dr = -\int_{r_1}^{r} \frac{1}{8\pi\varepsilon_0}\frac{Q}{r^2} dr = \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

$$V_r = V_{r_1} + \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_1}\right) = \frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{2r_1} + \frac{1}{r}\right) = \boxed{\frac{Q}{8\pi\varepsilon_0}\left(\frac{1}{r_2} + \frac{1}{r}\right)}, \ 0 < r < r_1$$

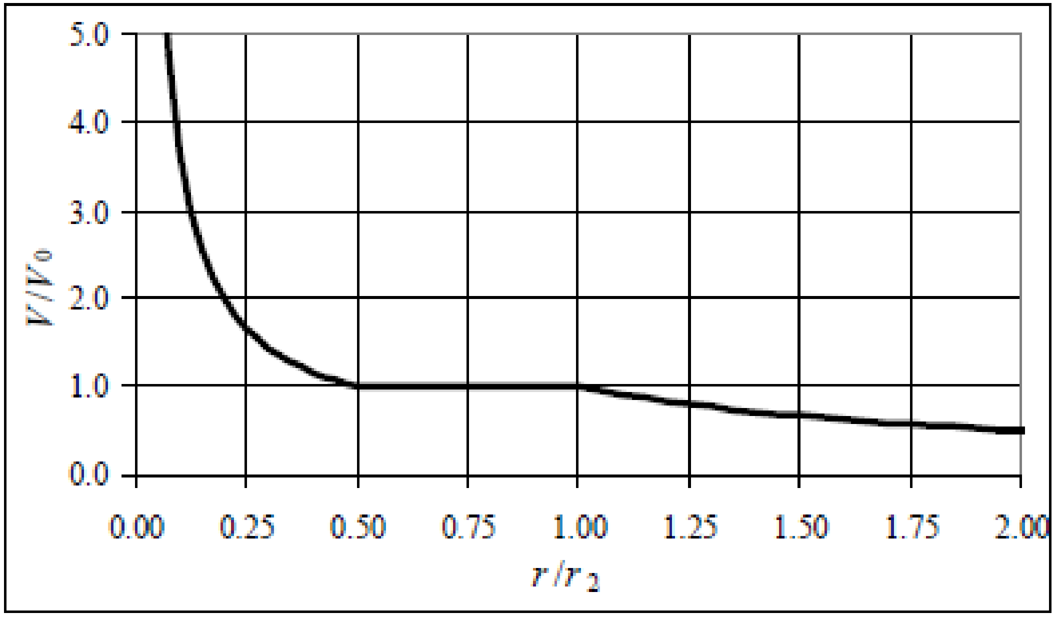
e) To plot, we first calculate $V_0 = V(r = r_2) = \dfrac{3Q}{8\pi\varepsilon_0 r_2}$ and $E_0 = E(r = r_2) = \dfrac{3Q}{8\pi\varepsilon_0 r_2^2}$. Then we plot V/V_0 and E/E_0 as functions of r/r_2.

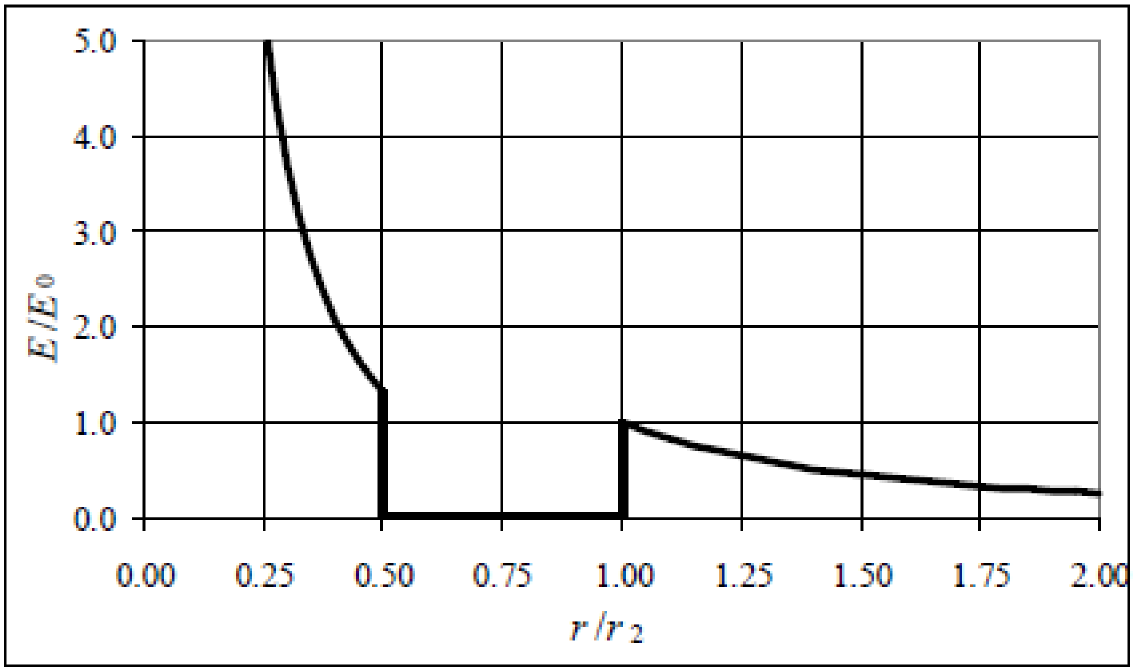
For $0 < r < r_1$: $$\frac{V}{V_0} = \frac{\dfrac{Q}{8\pi\varepsilon_0}\left(\dfrac{1}{r_2}+\dfrac{1}{r}\right)}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = \tfrac{1}{3}\left[1+(r/r_2)^{-1}\right]\ ;\ \frac{E}{E_0} = \frac{\dfrac{1}{8\pi\varepsilon_0}\dfrac{Q}{r^2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = \tfrac{1}{3}\frac{r_2^2}{r^2} = \tfrac{1}{3}(r/r_2)^{-2}$$

For $r_1 < r < r_2$: $$\frac{V}{V_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r_2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = 1\ ;\ \frac{E}{E_0} = \frac{0}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = 0$$

For $r > r_2$: $$\frac{V}{V_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2}} = \frac{r_2}{r} = (r/r_2)^{-1}\ ;\ \frac{E}{E_0} = \frac{\dfrac{3}{8\pi\varepsilon_0}\dfrac{Q}{r^2}}{\dfrac{3Q}{8\pi\varepsilon_0 r_2^2}} = \frac{r_2^2}{r^2} = (r/r_2)^{-2}$$

The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH23.XLS," on tab "Problem 23.22e."

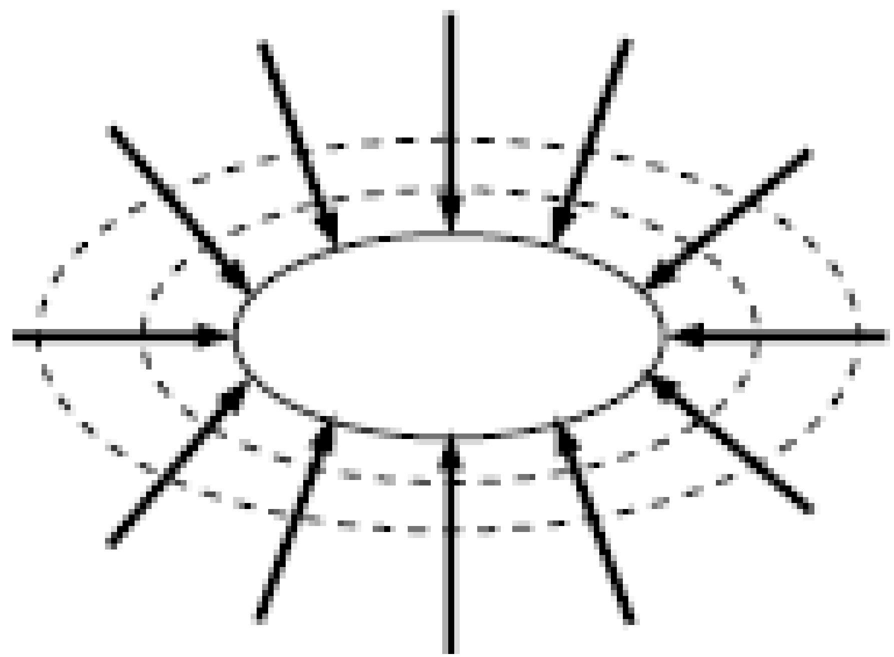




5.0
4.0
3.0
2.0
1.0
0.0
E/E_0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
r/r_2

42. (I) Draw a conductor in the shape of a football. This conductor carries a net negative charge, $-Q$. Draw in a dozen or so electric field lines and equipotential lines.

42.



44. (II) A metal sphere of radius $r_0 = 0.44\ \mathrm{m}$ carries a charge $Q = 0.50\ \mu\mathrm{C}$. Equipotential surfaces are to be drawn for 100-V intervals outside the sphere. Determine the radius r of (*a*) the first, (*b*) the tenth, and (*c*) the 100th equipotential from the surface.

44. The potential at the surface of the sphere is $V_0 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r_0}$. The potential outside the sphere is $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r} = V_0\dfrac{r_0}{r}$, and decreases as you move away from the surface. The difference in potential between a given location and the surface is to be a multiple of 100 V.

$$V_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0} = \left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{0.50\times10^{-6}\,\mathrm{C}}{0.44\,\mathrm{m}}\right) = 10{,}216\,\mathrm{V}$$

$$V_0 - V = V_0 - V_0\frac{r_0}{r} = (100\,\mathrm{V})n \quad\to\quad r = \frac{V_0}{[V_0 - (100\,\mathrm{V})n]}r_0$$

(*a*) $r_1 = \dfrac{V_0}{[V_0 - (100\,\mathrm{V})1]}r_0 = \dfrac{10{,}216\,\mathrm{V}}{10{,}116\,\mathrm{V}}(0.44\,\mathrm{m}) = \boxed{0.444\,\mathrm{m}}$

Note that to within the appropriate number of significant figures, this location is at the surface of the sphere. That can be interpreted that we don't know the voltage well enough to be working with a 100-V difference.

(*b*) $r_{10} = \dfrac{V_0}{[V_0 - (100\,\mathrm{V})10]}r_0 = \dfrac{10{,}216\,\mathrm{V}}{9{,}216\,\mathrm{V}}(0.44\,\mathrm{m}) = \boxed{0.49\,\mathrm{m}}$

(*c*) $r_{100} = \dfrac{V_0}{[V_0 - (100\,\mathrm{V})100]}r_0 = \dfrac{10{,}216\,\mathrm{V}}{216\,\mathrm{V}}(0.44\,\mathrm{m}) = \boxed{21\,\mathrm{m}}$

Example 23-10: Point charge equipotential surfaces.

For a single point charge with Q = 4.0 × 10^{-9} C, sketch the equipotential surfaces (or lines in a plane containing the charge) corresponding to V_1 = 10 V, V_2 = 20 V, and V_3 = 30 V.

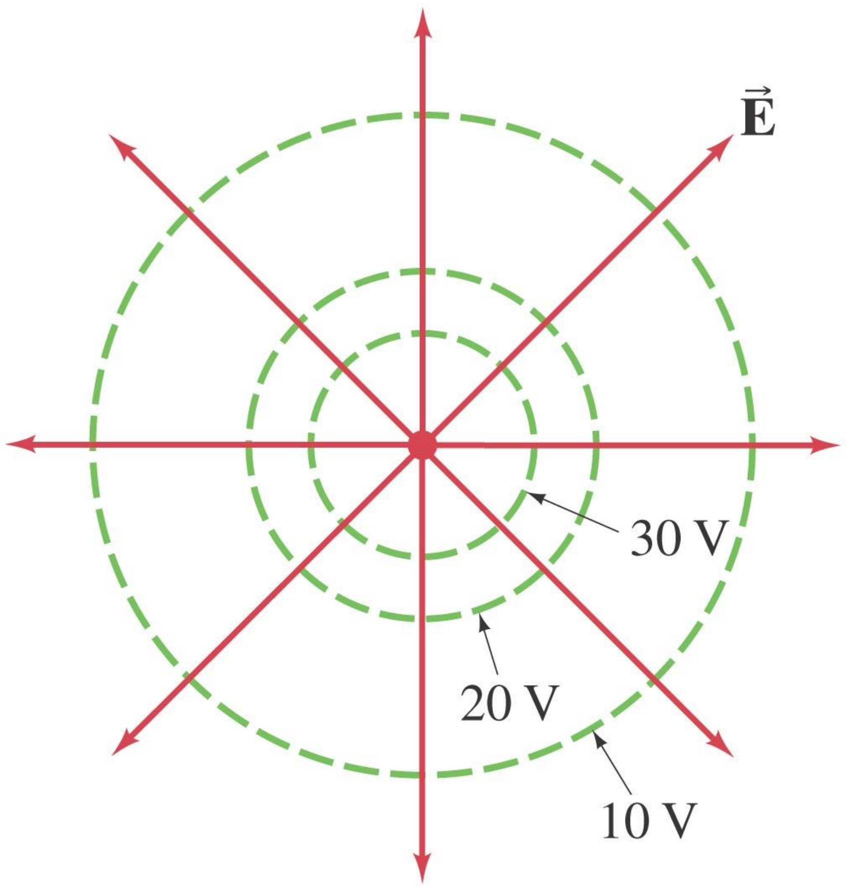
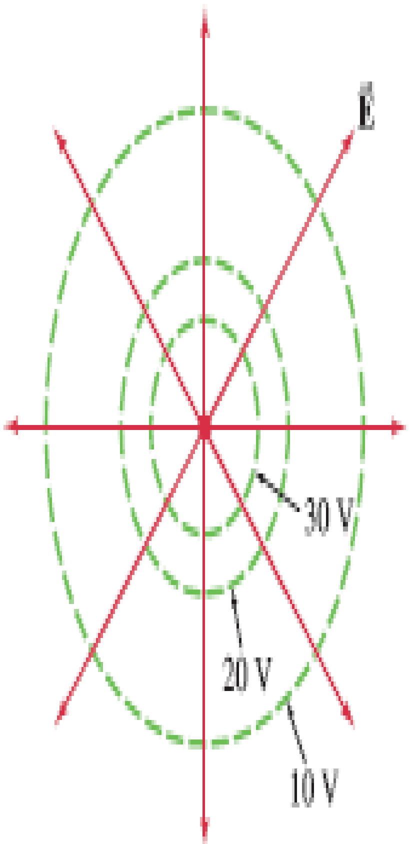


FIGURE 23–17 Example 23–10. Electric field lines and equipotential surfaces for a point charge.



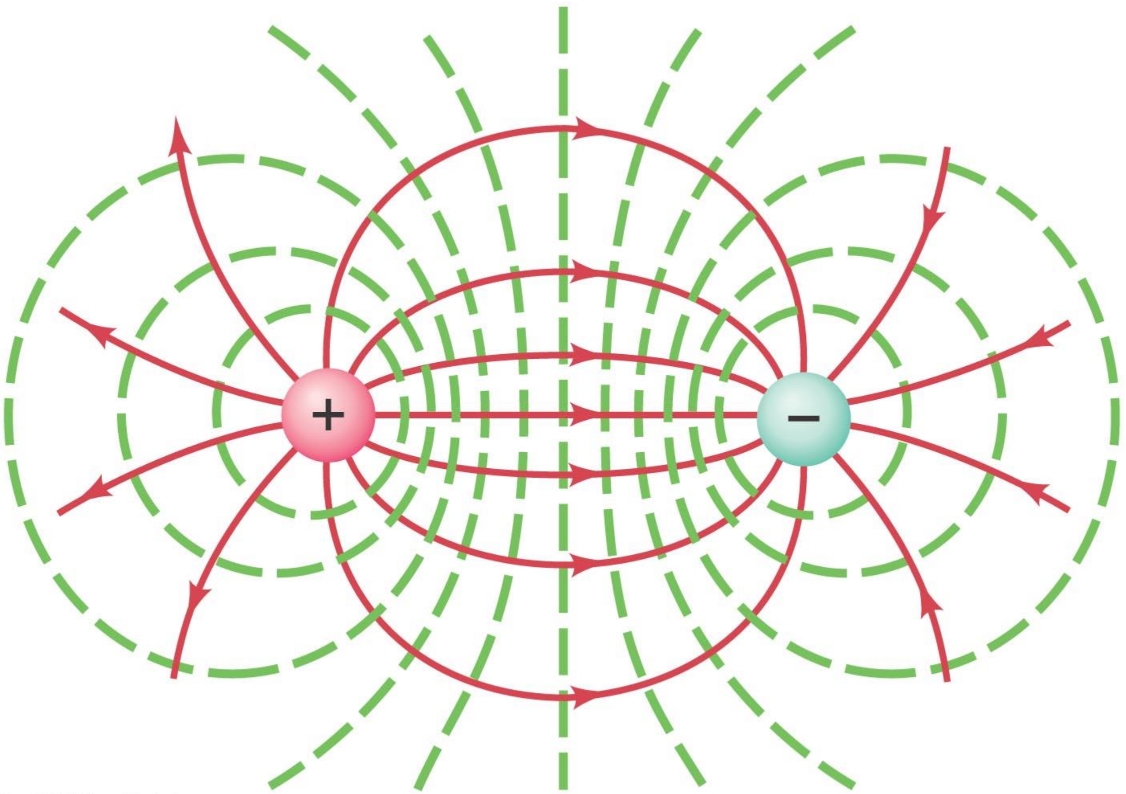
EXAMPLE 23–10 **Point charge equipotential surfaces.** For a single point charge with $Q = 4.0 \times 10^{-9}\,\mathrm{C}$, sketch the equipotential surfaces (or lines in a plane containing the charge) corresponding to $V_1 = 10\,\mathrm{V}$, $V_2 = 20\,\mathrm{V}$, and $V_3 = 30\,\mathrm{V}$.

APPROACH The electric potential V depends on the distance r from the charge (Eq. 23–5).

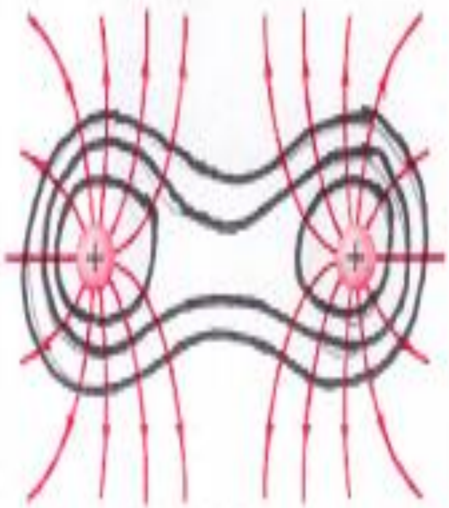
SOLUTION The electric field for a positive point charge is directed radially outward. Since the equipotential surfaces must be perpendicular to the lines of electric field, they will be spherical in shape, centered on the point charge, Fig. 23–17. From Eq. 23–5 we have $r = (1/4\pi\epsilon_0)(Q/V)$, so that for $V_1 = 10\,\mathrm{V}$, $r_1 = (9.0 \times 10^9\,\mathrm{N\cdot m^2/C^2})(4.0 \times 10^{-9}\,\mathrm{C})/(10\,\mathrm{V}) = 3.6\,\mathrm{m}$, for $V_2 = 20\,\mathrm{V}$, $r_2 = 1.8\,\mathrm{m}$, and for $V_3 = 30\,\mathrm{V}$, $r_3 = 1.2\,\mathrm{m}$, as shown.

NOTE The equipotential surface with the largest potential is closest to the positive charge. How would this change if Q were negative?

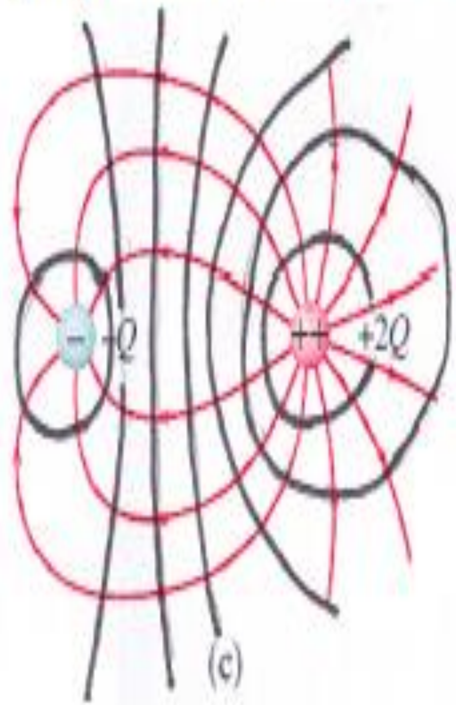
Equipotential surfaces are always perpendicular to field lines; they are always closed surfaces (unlike field lines, which begin and end on charges).



The equipotential lines (in black) are perpendicular to the electric field lines (in red).



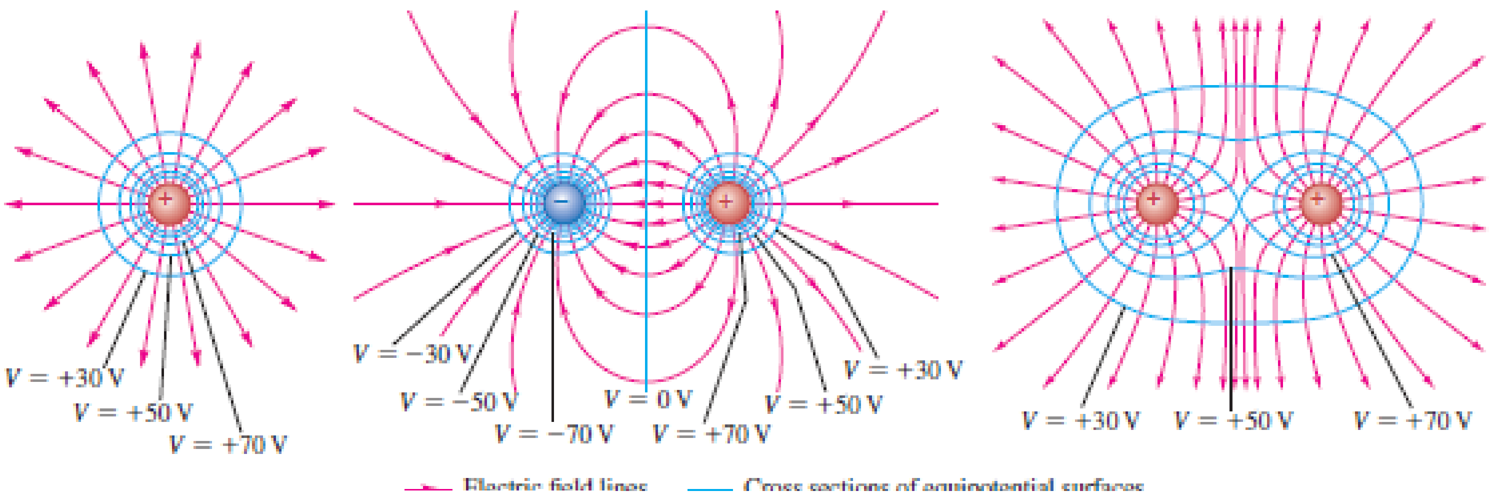
(b)



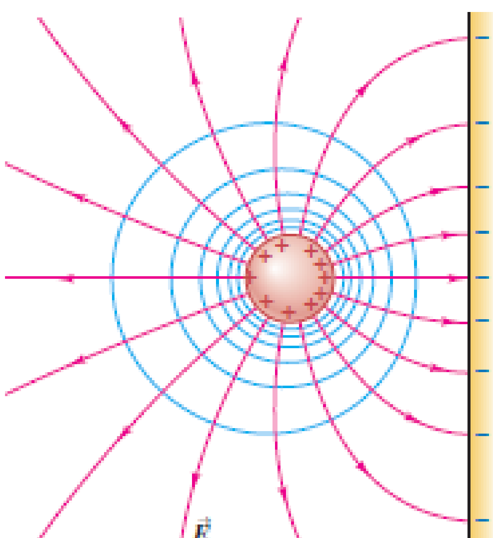
(c)

23.23 Cross sections of equipotential surfaces (blue lines) and electric field lines (red lines) for assemblies of point charges. There are equal potential differences between adjacent surfaces. Compare these diagrams to those in Fig. 21.28, which showed only the electric field lines.

(a) A single positive charge (b) An electric dipole (c) Two equal positive charges



23.24 When charges are at rest, a conducting surface is always an equipotential surface. Field lines are perpendicular to a conducting surface.

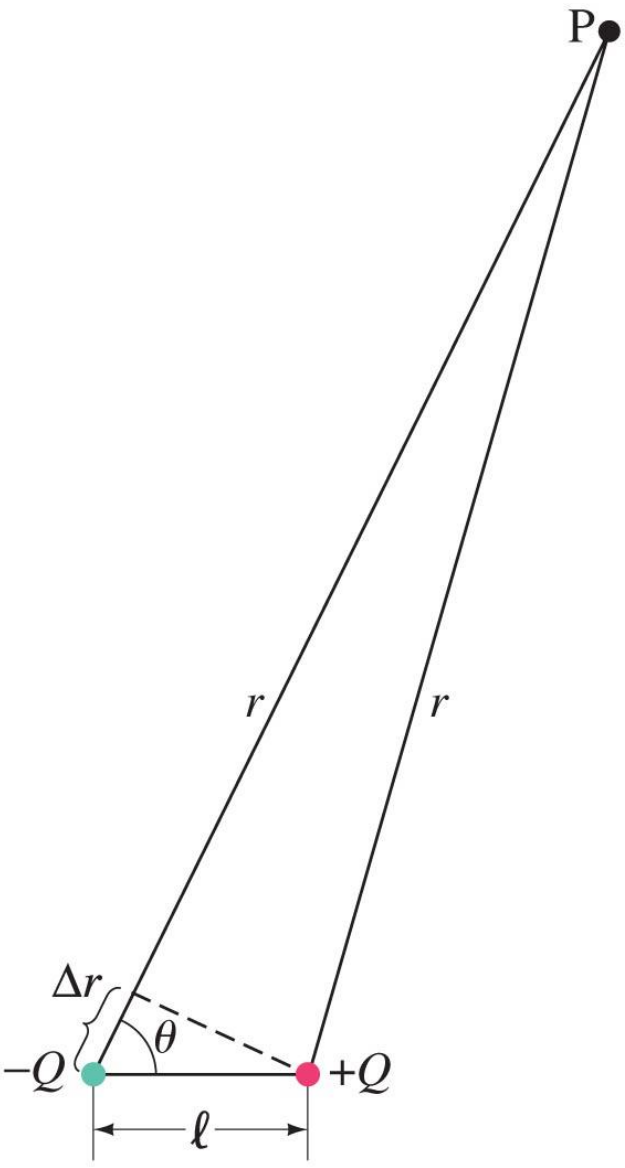


Equipotentials and Conductors

Here's an important statement about equipotential surfaces: **When all charges are at rest, the surface of a conductor is always an equipotential surface.** Since the electric field $\vec{E}$ is always perpendicular to an equipotential surface, we can prove this statement by proving that **when all charges are at rest, the electric field just outside a conductor must be perpendicular to the surface at every point** (**Fig. 23.24**). We know that $\vec{E} = \mathbf{0}$ everywhere inside the conductor; otherwise, charges would move. In particular, at any point just inside the surface the component of $\vec{E}$ tangent to the surface is zero. It follows that the tangential component of $\vec{E}$ is also zero just *outside* the surface. If it were not, a charge could

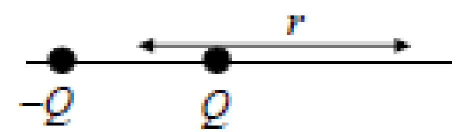
The potential due to an electric dipole is just the sum of the potentials due to each charge, and can be calculated exactly. For distances large compared to the charge separation:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q\ell\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}.$$



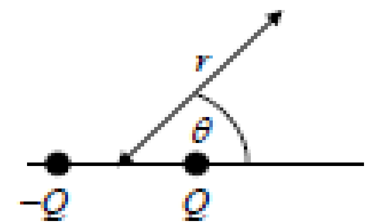
45. (II) Calculate the electric potential due to a tiny dipole whose dipole moment is 4.8×10^{-30} C·m at a point 4.1×10^{-9} m away if this point is (*a*) along the axis of the dipole nearer the positive charge; (*b*) 45° above the axis but nearer the positive charge; (*c*) 45° above the axis but nearer the negative charge. Let $V = 0$ at $r = \infty$.

45. The potential due to the dipole is given by Eq. 23-7.



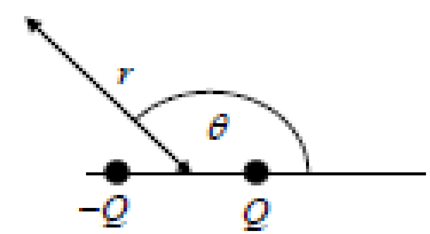
(*a*) $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^{9}\ \mathrm{N{\bullet}m^2/C^2}\right)\left(4.8\times10^{-30}\ \mathrm{C{\bullet}m}\right)\cos 0}{\left(4.1\times10^{-9}\ \mathrm{m}\right)^2}$$

$$= \boxed{2.6\times10^{-3}\ \mathrm{V}}$$



(*b*) $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^{9}\ \mathrm{N{\bullet}m^2/C^2}\right)\left(4.8\times10^{-30}\ \mathrm{C{\bullet}m}\right)\cos 45^\circ}{\left(4.1\times10^{-9}\ \mathrm{m}\right)^2}$$

$$= \boxed{1.8\times10^{-3}\ \mathrm{V}}$$



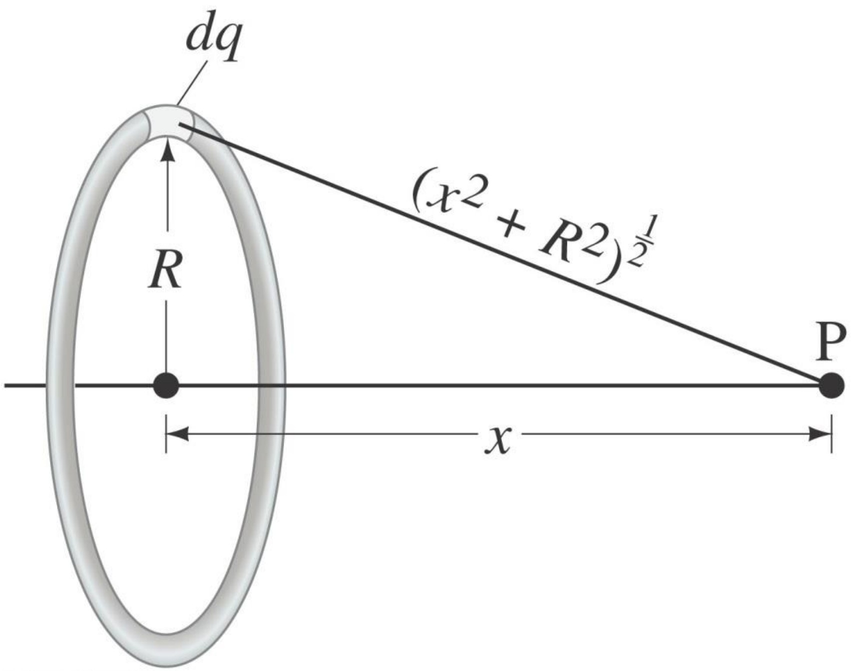
(*c*) $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{\left(8.99\times10^{9}\ \mathrm{N{\bullet}m^2/C^2}\right)\left(4.8\times10^{-30}\ \mathrm{C{\bullet}m}\right)\cos 135^\circ}{\left(1.1\times10^{-9}\ \mathrm{m}\right)^2}$$

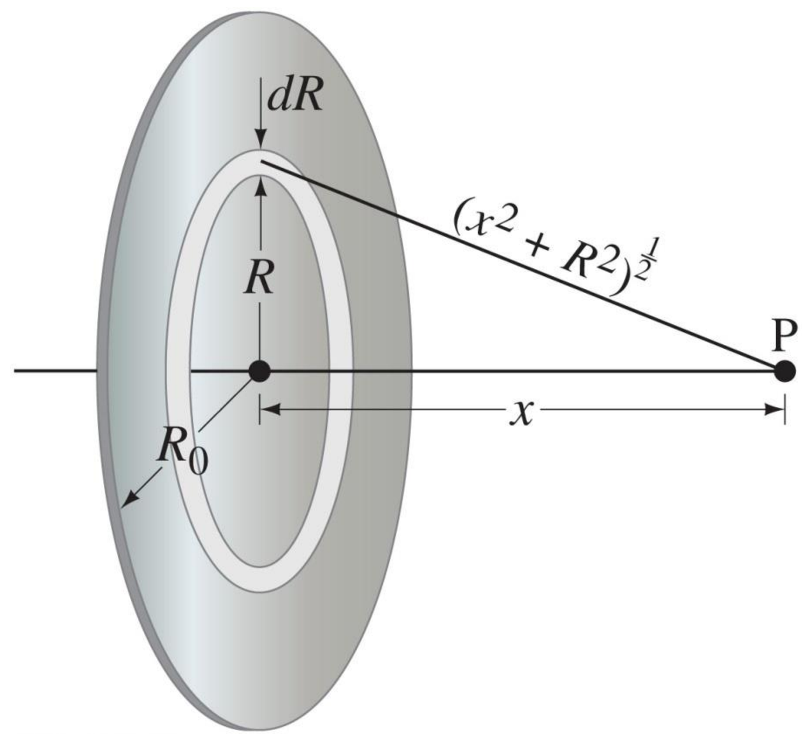
$$= \boxed{-1.8\times10^{-3}\ \mathrm{V}}$$

$\vec{E}$

Example 23-11: $\vec{E}$ for ring and disk.

Use electric potential to determine the electric field at point P on the axis of (a) a circular ring of charge and (b) a uniformly charged disk.





EXAMPLE 23–11 **$\vec{E}$ for ring and disk.** Use electric potential to determine the electric field at point P on the axis of (*a*) a circular ring of charge (Fig. 23–14) and (*b*) a uniformly charged disk (Fig. 23–15).

APPROACH We obtained V as a function of x in Examples 23–8 and 23–9, so we find E by taking derivatives (Eqs. 23–9).

SOLUTION (*a*) From Example 23–8,

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\left(x^2 + R^2\right)^{\frac{1}{2}}}.$$

Then

$$E_x = -\frac{\partial V}{\partial x} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2 + R^2\right)^{\frac{3}{2}}}.$$

This is the same result we obtained in Example 21–9.

(*b*) From Example 23–9,

$$V = \frac{Q}{2\pi\epsilon_0 R_0^2}\left[\left(x^2 + R_0^2\right)^{\frac{1}{2}} - x\right],$$

so

$$E_x = -\frac{\partial V}{\partial x} = \frac{Q}{2\pi\epsilon_0 R_0^2}\left[1 - \frac{x}{\left(x^2 + R_0^2\right)^{\frac{1}{2}}}\right].$$

For points very close to the disk, $x \ll R_0$, this can be approximated by

$$E_x \approx \frac{Q}{2\pi\epsilon_0 R_0^2} = \frac{\sigma}{2\epsilon_0}$$

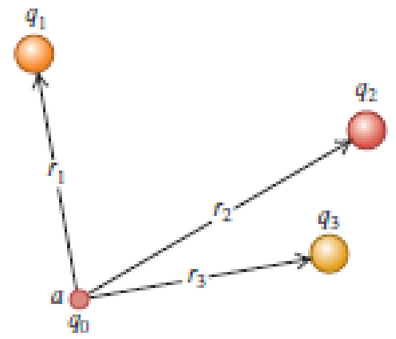
where $\sigma = Q/\pi R_0^2$ is the surface charge density. We also obtained these results in Chapter 21, Example 21–12 and Eq. 21–7.

51. (II) In a certain region of space, the electric potential is given by $V = y^2 + 2.5xy - 3.5xyz$. Determine the electric field vector, $\vec{\mathbf{E}}$, in this region.

51. We use Eq. 23-9 to find the components of the electric field.

$$E_x = -\frac{\partial V}{\partial x} = -2.5y + 3.5yz \ ; \ E_y = -\frac{\partial V}{\partial y} = -2y - 2.5x + 3.5xz \ ; \ E_z = -\frac{\partial V}{\partial z} = 3.5xy$$

$$\vec{\mathbf{E}} = \boxed{\left(-2.5y + 3.5yz\right)\hat{\mathbf{i}} + \left(-2y - 2.5x + 3.5xz\right)\hat{\mathbf{j}} + \left(3.5xy\right)\hat{\mathbf{k}}}$$



Electric potential energy of point charge q_0 and collection of charges $q_1, q_2, q_3, \ldots$

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Electric constant

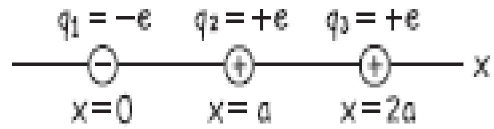
Distances from q_0 to $q_1, q_2, q_3, \ldots$

The potential energy of a charge in an electric potential is $U = qV$. To find the electric potential energy of two charges, imagine bringing each in from infinitely far away. The first one takes no work, as there is no field. To bring in the second one, we must do work due to the field of the first one; this means the potential energy of the pair is:

$$U = Q_2 V = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{12}}.$$

Two point charges are located on the x-axis, $q_1 = -e$ at $x = 0$ and $q_2 = +e$ at $x = a$. (a) Find the work that must be done by an external force to bring a third point charge $q_3 = +e$ from infinity to $x = 2a$. (b) Find the total potential energy of the system of three charges.

23.10 Our sketch of the situation after the third charge has been brought in from infinity.



SOLUTION

IDENTIFY and SET UP: Figure 23.10 shows the final arrangement of the three charges. In part (a) we need to find the work W that must be done on q_3 by an external force $\vec{F}_{ext}$ to bring q_3 in from infinity to $x = 2a$. We do this by using Eq. (23.10) to find the potential energy associated with q_3 in the presence of q_1 and q_2. In part (b) we use Eq. (23.11), the expression for the potential energy of a collection of point charges, to find the total potential energy of the system.

EXECUTE: (a) The work W equals the difference between (i) the potential energy U associated with q_3 when it is at $x = 2a$ and (ii) the potential energy when it is infinitely far away. The second of these is zero, so the work required is equal to U. The distances between the charges are $r_{13} = 2a$ and $r_{23} = a$, so from Eq. (23.10),

$$W = U = \frac{q_3}{4\pi\epsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) = \frac{+e}{4\pi\epsilon_0}\left(\frac{-e}{2a} + \frac{+e}{a}\right) = \frac{+e^2}{8\pi\epsilon_0 a}$$

This is positive, just as we should expect. If we bring q_3 in from infinity along the $+x$-axis, it is attracted by q_1 but is repelled more strongly by q_2. Hence we must do positive work to push q_3 to the position at $x = 2a$.

(b) From Eq. (23.11), the total potential energy of the three-charge system is

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{(-e)(e)}{a} + \frac{(-e)(e)}{2a} + \frac{(e)(e)}{a}\right] = \frac{-e^2}{8\pi\epsilon_0 a}$$

EVALUATE: Our negative result in part (b) means that the system has lower potential energy than it would if the three charges were infinitely far apart. An external force would have to do *negative* work to bring the three charges from infinity to assemble this entire arrangement and would have to do *positive* work to move the three charges back to infinity.

Example 23-12: Disassembling a hydrogen atom.

Calculate the work needed to "disassemble" a hydrogen atom. Assume that the proton and electron are initially separated by a distance equal to the "average" radius of the hydrogen atom in its ground state, 0.529×10^{-10} m, and that they end up an infinite distance apart from each other.

EXAMPLE 23–12 **Disassembling a hydrogen atom.** Calculate the work needed to "disassemble" a hydrogen atom. Assume that the proton and electron are initially separated by a distance equal to the "average" radius of the hydrogen atom in its ground state, 0.529×10^{-10} m, and that they end up an infinite distance apart from each other.

APPROACH The work necessary will be equal to the total energy, kinetic plus potential, of the electron and proton as an atom, compared to their total energy when infinitely far apart.

SOLUTION From Eq. 23–10 we have initially

$$U = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{(e)(-e)}{r} = \frac{-(8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})(1.60 \times 10^{-19}\,\mathrm{C})^2}{(0.529 \times 10^{-10}\,\mathrm{m})}$$

$$= -27.2(1.60 \times 10^{-19})\,\mathrm{J} = -27.2\,\mathrm{eV}.$$

This represents the potential energy. The total energy must include also the kinetic energy of the electron moving in an orbit of radius $r = 0.529 \times 10^{-10}$ m. From $F = ma$ for centripetal acceleration, we have

$$\frac{1}{4\pi\epsilon_0}\left(\frac{e^2}{r^2}\right) = \frac{mv^2}{r}.$$

Then

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{r}$$

which equals $-\frac{1}{2}U$ (as calculated above), so $K = +13.6\,\mathrm{eV}$. The total energy initially is $E = K + U = 13.6\,\mathrm{eV} - 27.2\,\mathrm{eV} = -13.6\,\mathrm{eV}$. To separate a stable hydrogen atom into a proton and an electron at rest very far apart ($U = 0$ at $r = \infty$, $K = 0$ because $v = 0$) requires $+13.6\,\mathrm{eV}$. This is, in fact, the measured ionization energy for hydrogen.

NOTE To treat atoms properly, we need to use quantum theory (Chapters 37 to 39). But our "classical" calculation does give the correct answer here.

EXERCISE F The kinetic energy of a 1000-kg automobile traveling 20 m/s (70 km/h) would be about (*a*) 100 GeV, (*b*) 1000 TeV, (*c*) 10^6 TeV, (*d*) 10^{12} TeV, (*e*) 10^{18} TeV.

56. (I) What is the speed of (*a*) a 1.5-keV (kinetic energy) electron and (*b*) a 1.5-keV proton?

56. The kinetic energy of the particle is given in each case. Use the kinetic energy to find the speed.

(*a*) $\frac{1}{2}mv^2 = K \rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1500\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = \boxed{2.3\times10^7\,\text{m/s}}$

(*b*) $\frac{1}{2}mv^2 = K \rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1500\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{1.67\times10^{-27}\,\text{kg}}} = \boxed{5.4\times10^5\,\text{m/s}}$

59. (II) Write the total electrostatic potential energy, U, for (*a*) four point charges and (*b*) five point charges. Draw a diagram defining all quantities.

59. Following the same method as presented in Section 23-8, we get the following results.

(*a*) 1 charge: No work is required to move a single charge into a position, so $U_1 = 0$.

2 charges: This represents the interaction between Q_1 and Q_2.

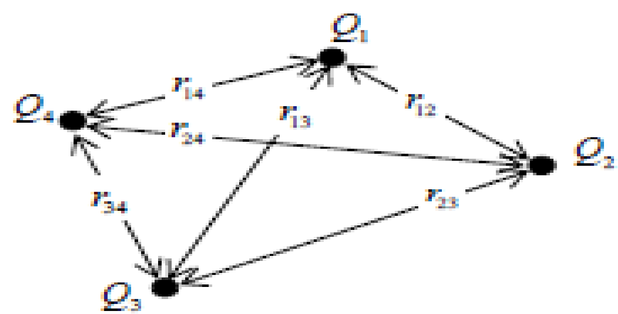
$$U_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}}$$

3 charges: This now adds the interactions between Q_1 & Q_3 and Q_2 & Q_3.

$$U_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right)$$

4 charges: This now adds the interaction between Q_1 & Q_4, Q_2 & Q_4, and Q_3 & Q_4.

$$\boxed{U_4 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_3Q_4}{r_{34}}\right)}$$



(*b*) 5 charges: This now adds the interaction between Q_1 & Q_5, Q_2 & Q_5, Q_3 & Q_5, and Q_4 & Q_5.

$$\boxed{U_5 = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_1Q_5}{r_{15}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_2Q_5}{r_{25}} + \frac{Q_3Q_4}{r_{34}} + \frac{Q_3Q_5}{r_{35}} + \frac{Q_4Q_5}{r_{45}}\right)}$$