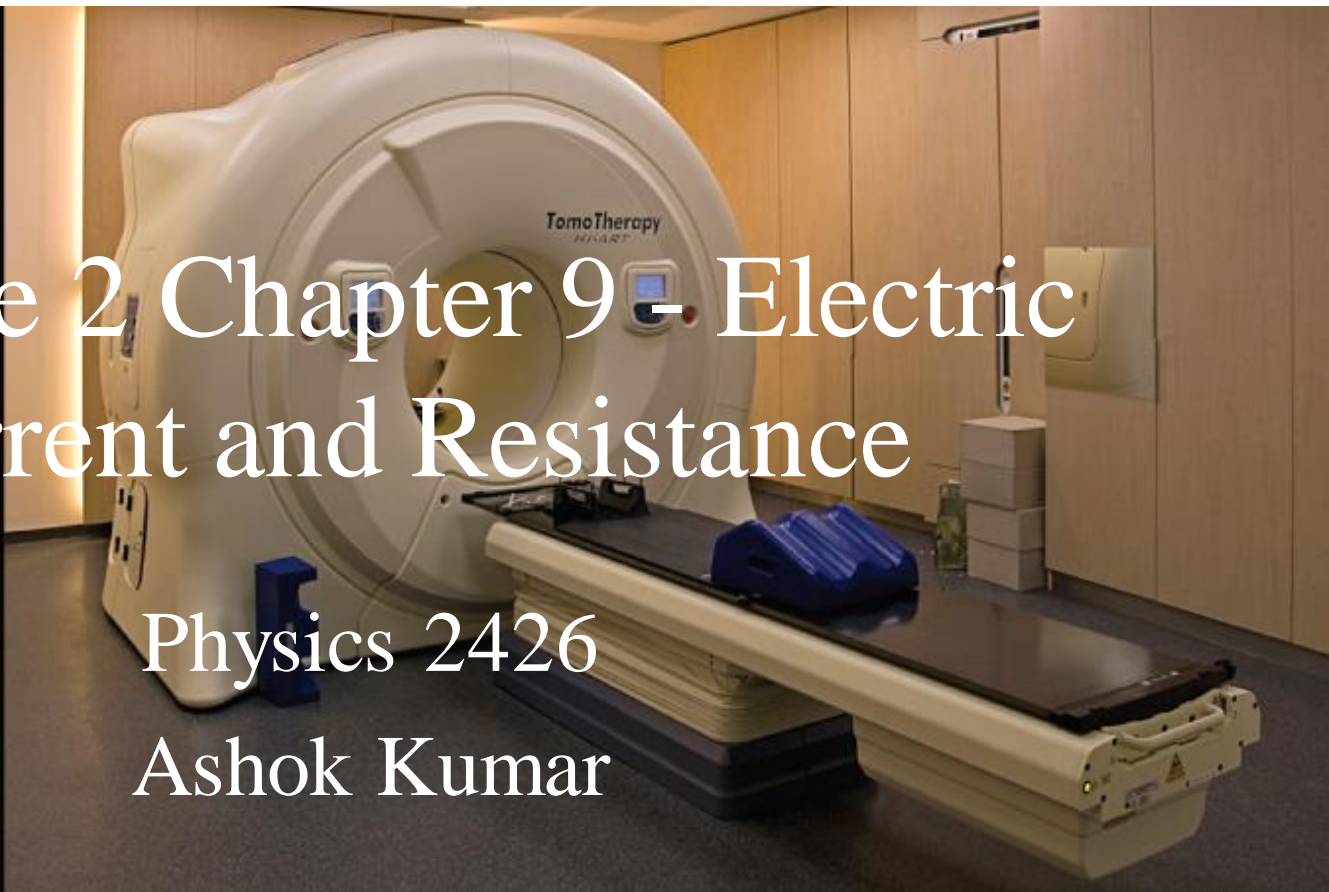




# Volume 2 Chapter 9 – Electric Current and Resistance

Physics 2426  
Ashok Kumar



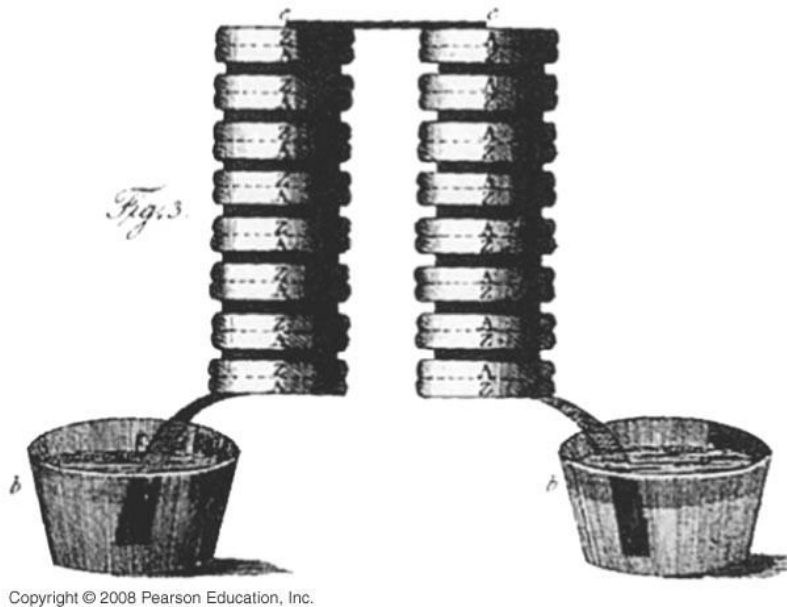
# Electrical Potential

- Electrical potential is related to potential energy in the same way the electric field is related to the electrical force.
- $E = F/q$ ; where  $F$  is the force on the test charge and  $q$  is the magnitude of the test charge.
- Similarly, the electrical potential  $V$  is given by the formula.
- $V = U/q$
- The units of electrical potential is Volts.

# How is Electrical Potential Produced?

- While there was clearly electrical potential associated with friction or Triboelectricity (and there were generators built), the first true voltage source was Volta's Battery.
- Batteries use the difference in the electron affinity of conductors to generate an electrical potential from a chemical mix.

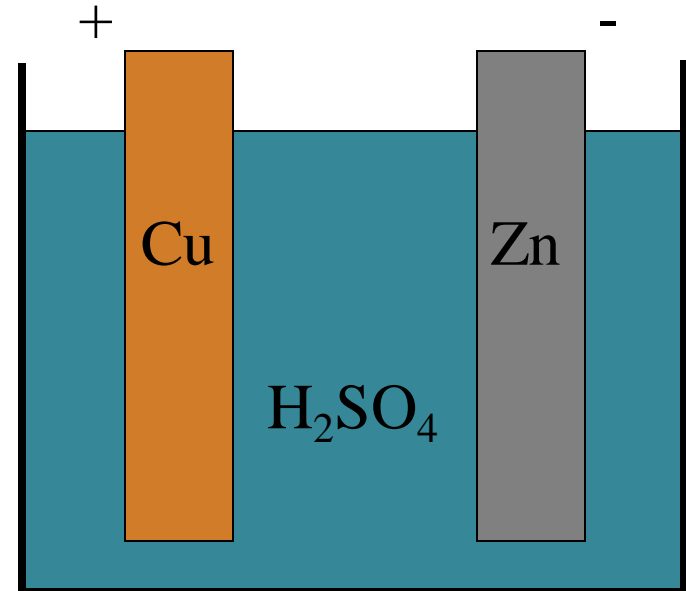
# Batteries and Direct Current



- Alessandro Volta discovered that plates of Copper and Zinc placed in a solution of Sulfuric Acid would produce an electrical potential difference between the two metals.
- He produced the first real battery by making a stack of Cu and Zn plates separated by paper soaked in saltwater or mild acid.

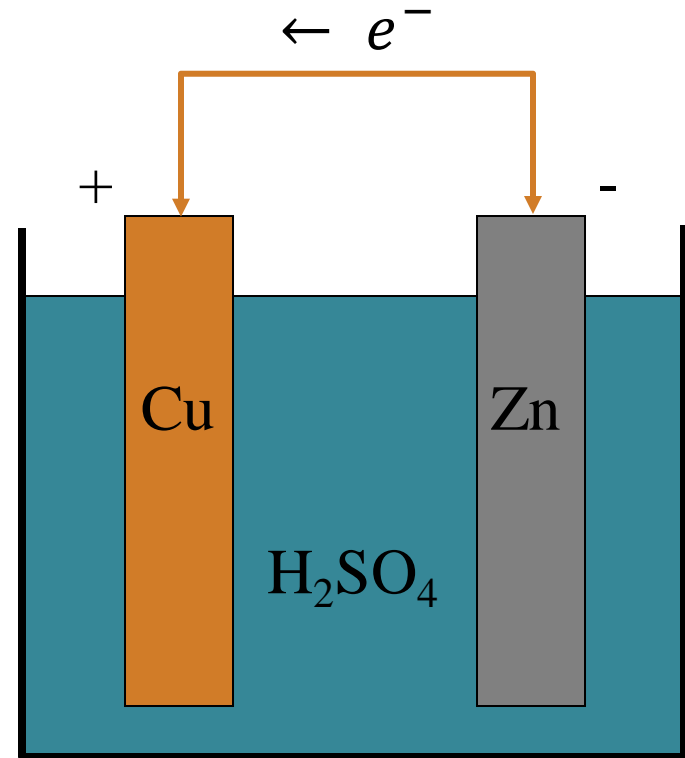
# Electrolytic Cells

- In this arrangement, the acid will attack the zinc, converting some it to zinc sulfate.
- Positive Zn ions are dissolved into the electrolyte
- This reaction proceeds until the (-) charge on the Zn becomes high enough to stop it.



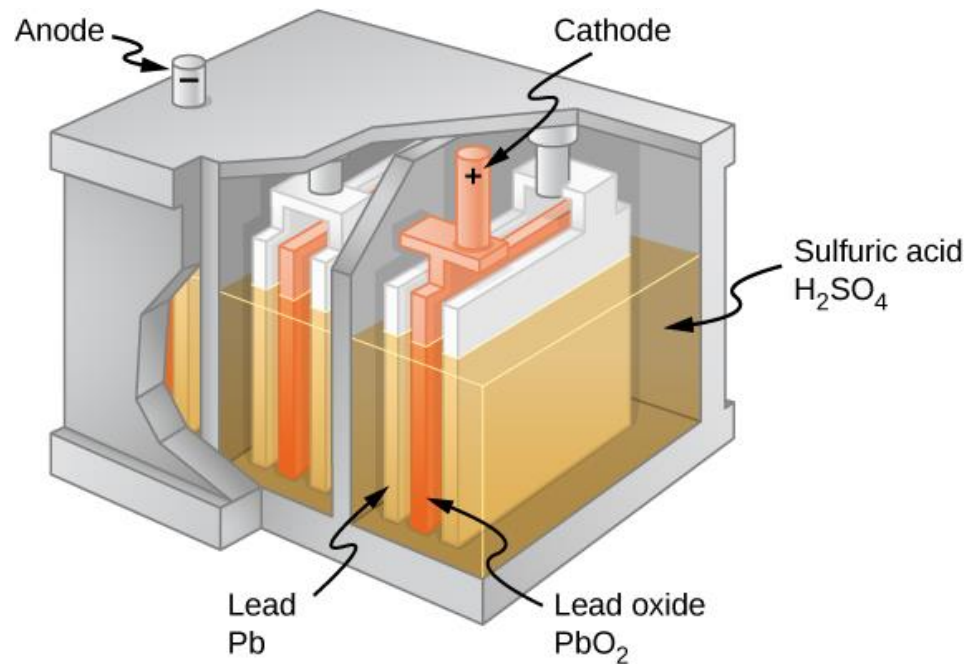
# Electrolytic Cells

- A wire connected between the Cu & Zn plates will allow the excess charge on the Zn to flow to the Cu.
- The chemical reaction resumes and more electrons are supplied as more Zn ions are dissolved.
- This process continues until the circuit is broken or the zinc consumed.

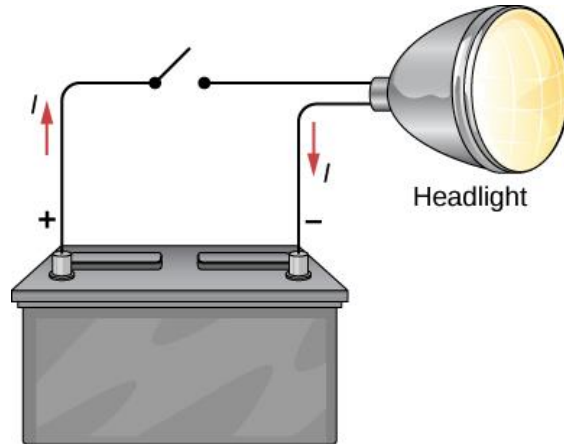


# Batteries vs. Cells

- An electrolytic cell uses two conducting plates in a electrolyte to produce a voltage. (D-cell)
- Typical voltages for a cell are  $\sim 1 - 2$  volts. The cell at right would produce 2.2 V
- Originally, a battery meant a voltage source formed by connecting 2 or more cells in series to make a higher voltage. A 12 V car battery has 6 cells.
- Today the term battery has become generic.

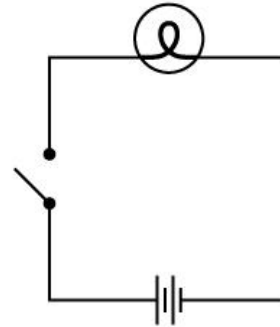


# Electrical Circuits & Current

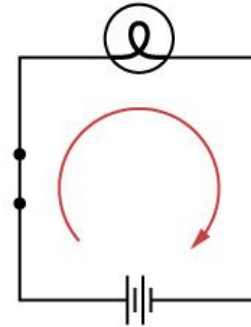


V battery

(a)



(b)

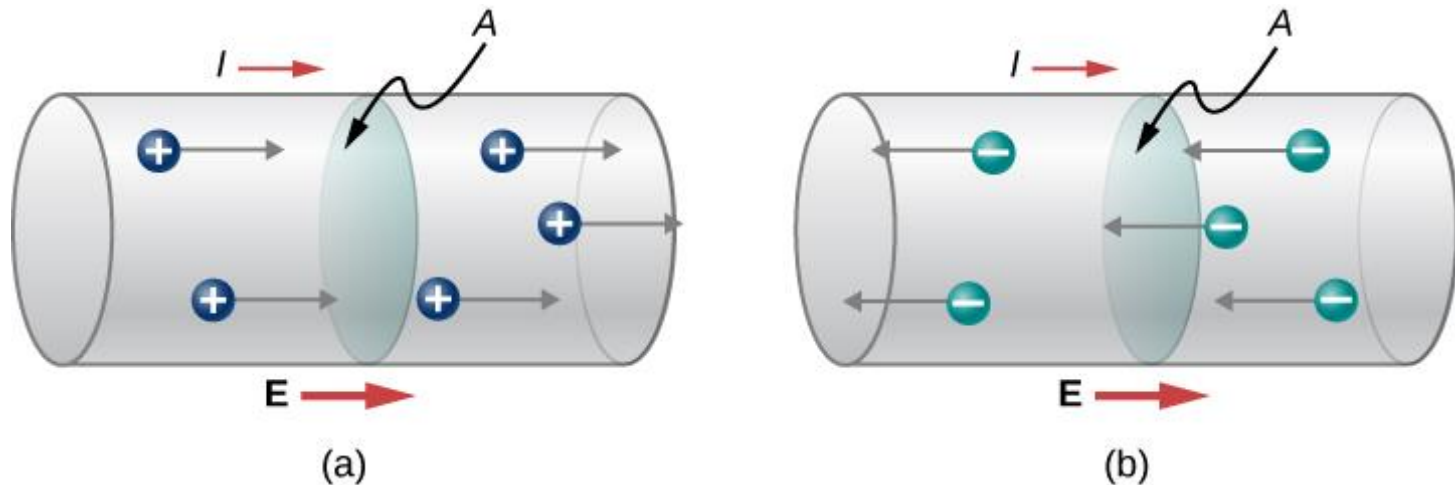


(c)

- We define an electrical circuit as any continuous conducting path.
- When a battery is connected to a circuit, the voltage (pressure) drives electrons (fluid) through the conducting path (pipes).

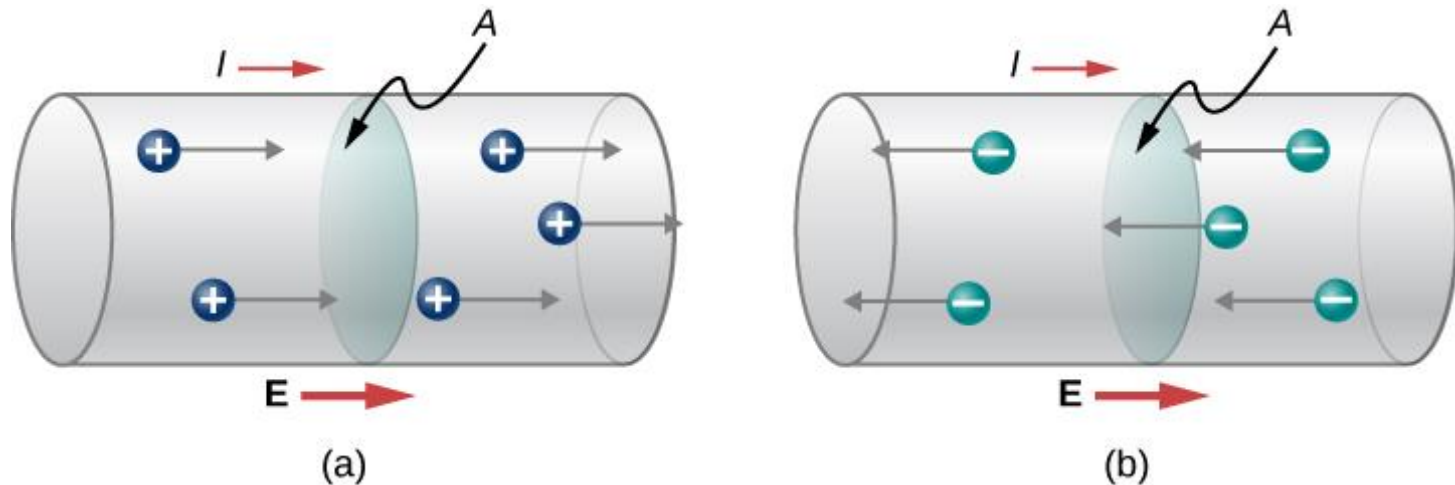


# Electrical Circuits & Current



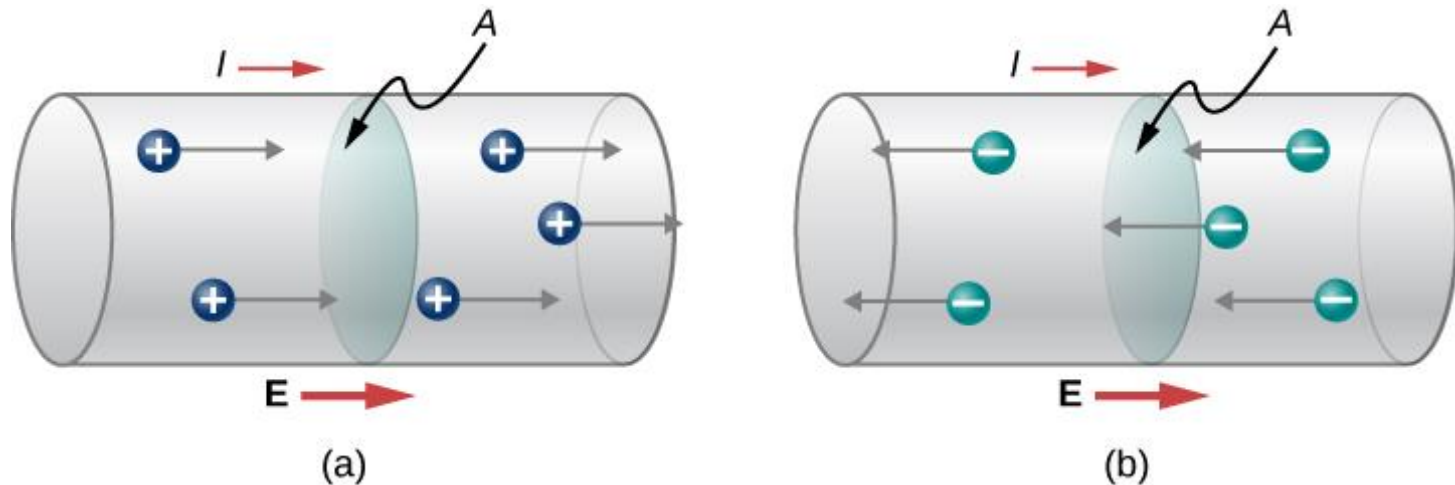
- Because current flow and batteries were discovered before the electron was discovered, it was convention to say that current flowed from the positive terminal of the battery to the negative.
- We now know that this is generally not true.
- Electrons flow (-) to (+).

# Electrical Circuits & Current



- For this class, we will use conventional current, assuming that the motion of positive charges (a) determines the direction of flow (+ to -).
- The magnitude of the current is the total charge which passes through a cross-section of the conductor in 1 second.

# Electrical Circuits & Current



- The SI unit of current is the Ampere or Amp
- 1 Ampere = 1 Coulomb of charge flow per second.
- (Actually the Ampere is the base unit and the coulomb is defined from it.)

**Electric current is the rate of flow of charge through a conductor:**

$$\bar{I} = \frac{\Delta Q}{\Delta t}.$$

**The instantaneous current is given by:**

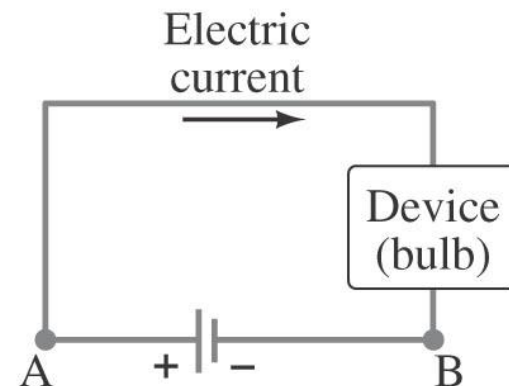
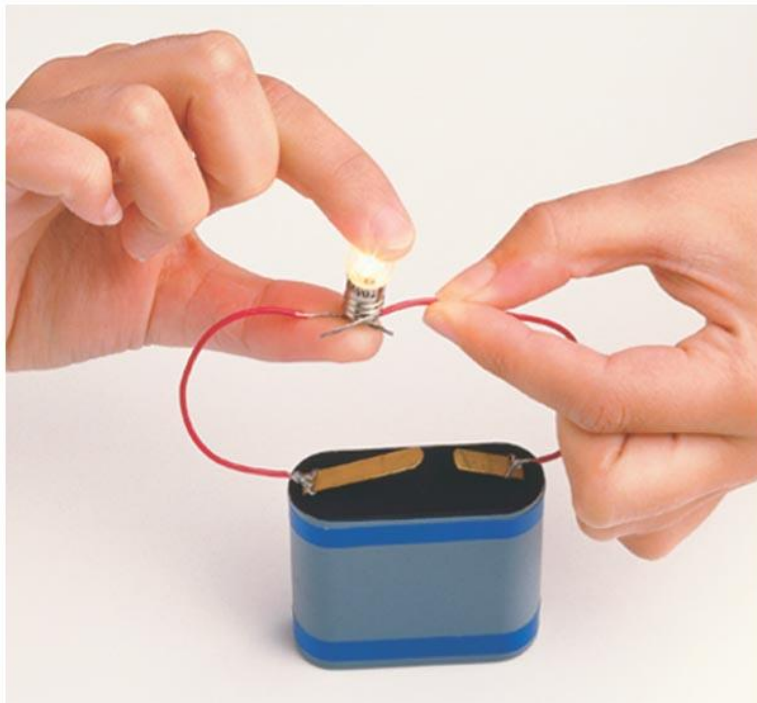
$$I = \frac{dQ}{dt}.$$

**Unit of electric current: the ampere, A:**

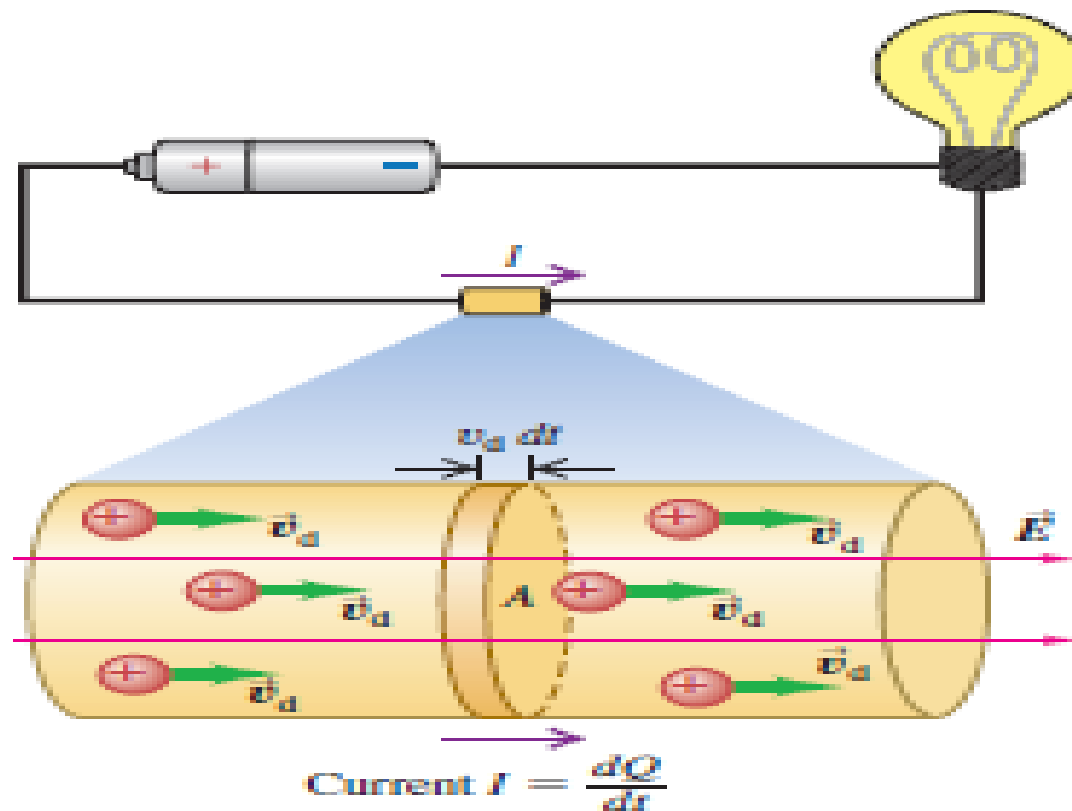
$$1 \text{ A} = 1 \text{ C/s.}$$

The SI unit of current is the **ampere**; one ampere is defined to be *one coulomb per second* ( $1 \text{ A} = 1 \text{ C/s}$ ). This unit is named in honor of the French scientist André Marie Ampère (1775–1836). When an ordinary flashlight (D-cell size) is turned on, the current in the flashlight is about 0.5 to 1 A; the current in the wires of a car engine's starter motor is around 200 A. Currents in radio and television circuits are usually expressed in *milliamperes* ( $1 \text{ mA} = 10^{-3} \text{ A}$ ) or *microamperes* ( $1 \mu\text{A} = 10^{-6} \text{ A}$ ), and currents in computer circuits are expressed in *nanoamperes* ( $1 \text{ nA} = 10^{-9} \text{ A}$ ) or *picoamperes* ( $1 \text{ pA} = 10^{-12} \text{ A}$ ).

**A complete circuit is one where current can flow all the way around. Note that the schematic drawing doesn't look much like the physical circuit!**



**25.3** The current  $I$  is the time rate of charge transfer through the cross-sectional area  $A$ . The random component of each moving charged particle's motion averages to zero, and the current is in the same direction as  $\vec{E}$  whether the moving charges are positive (as shown here) or negative (see Fig. 25.2b).



## **Example 25-1: Current is flow of charge.**

**A steady current of 2.5 A exists in a wire for 4.0 min. (a) How much total charge passed by a given point in the circuit during those 4.0 min? (b) How many electrons would this be?**



**EXAMPLE 25-1** **Current is flow of charge.** A steady current of 2.5 A exists in a wire for 4.0 min. (a) How much total charge passed by a given point in the circuit during those 4.0 min? (b) How many electrons would this be?

**APPROACH** Current is flow of charge per unit time, Eqs. 25-1, so the amount of charge passing a point is the product of the current and the time interval. To get the number of electrons (b), we divide the total charge by the charge on one electron.

**SOLUTION** (a) Since the current was 2.5 A, or 2.5 C/s, then in 4.0 min (= 240 s) the total charge that flowed past a given point in the wire was, from Eq. 25-1a,

$$\Delta Q = I \Delta t = (2.5 \text{ C/s})(240 \text{ s}) = 600 \text{ C}.$$

(b) The charge on one electron is  $1.60 \times 10^{-19} \text{ C}$ , so 600 C would consist of

$$\frac{600 \text{ C}}{1.60 \times 10^{-19} \text{ C/electron}} = 3.8 \times 10^{21} \text{ electrons}.$$

## Conceptual Example 25-2: How to connect a battery.

What is wrong with each of the schemes shown for lighting a flashlight bulb with a flashlight battery and a single wire?



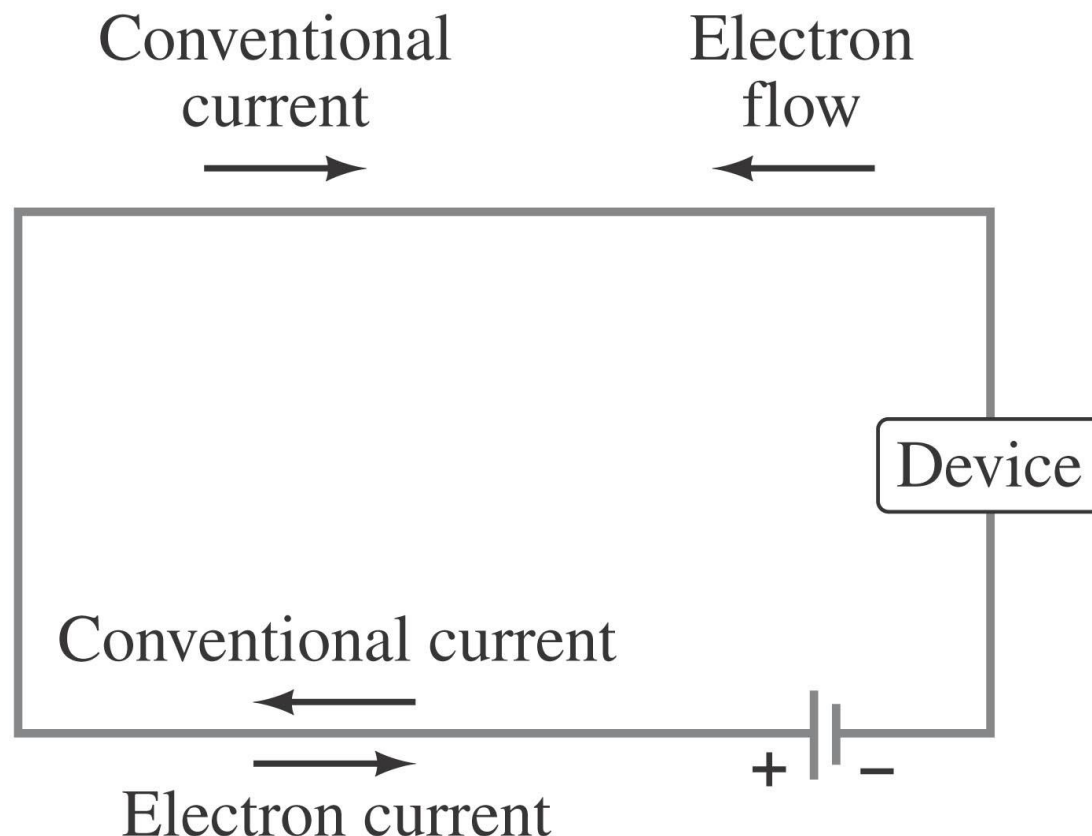
**CONCEPTUAL EXAMPLE 25–2** **How to connect a battery.** What is wrong with each of the schemes shown in Fig. 25–7 for lighting a flashlight bulb with a flashlight battery and a single wire?

**RESPONSE** (a) There is no closed path for charge to flow around. Charges might briefly start to flow from the battery toward the lightbulb, but there they run into a “dead end,” and the flow would immediately come to a stop.

(b) Now there is a closed path passing to and from the lightbulb; but the wire touches only one battery terminal, so there is no potential difference in the circuit to make the charge move.

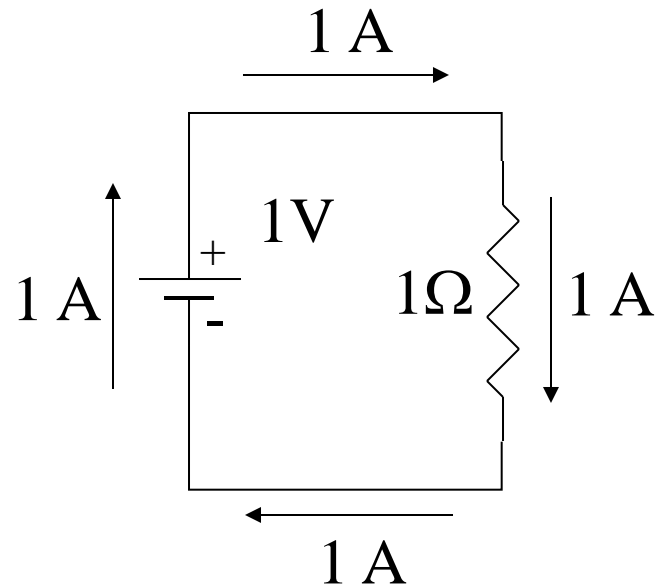
(c) Nothing is wrong here. This is a complete circuit: charge can flow out from one terminal of the battery, through the wire and the bulb, and into the other terminal. This scheme will light the bulb.

**By convention, current is defined as flowing from + to -. Electrons actually flow in the opposite direction, but not all currents consist of electrons.**



# Electrical Current & Resistance

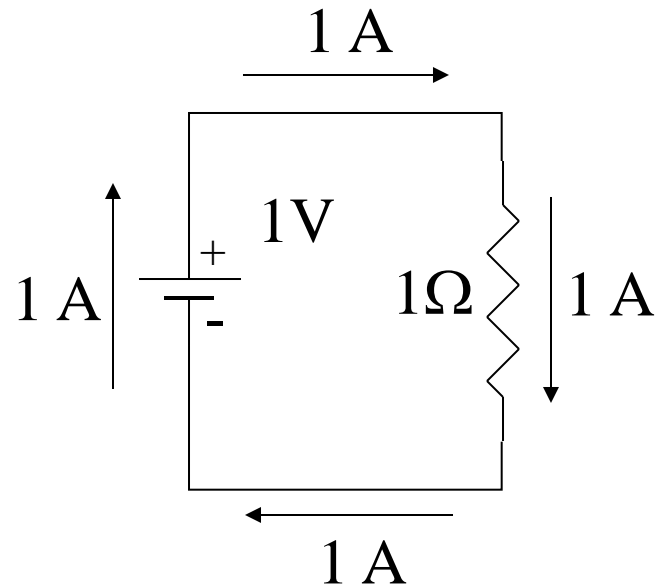
- Electricity does not flow through a wire without some friction.
- As the electrons drift along, they are in rapid thermal motion, colliding with the atoms in the conductor.
- This process dissipates energy, so there is a voltage (electrical potential) loss around the circuit.



# Electrical Current & Resistance

## Simple Circuits

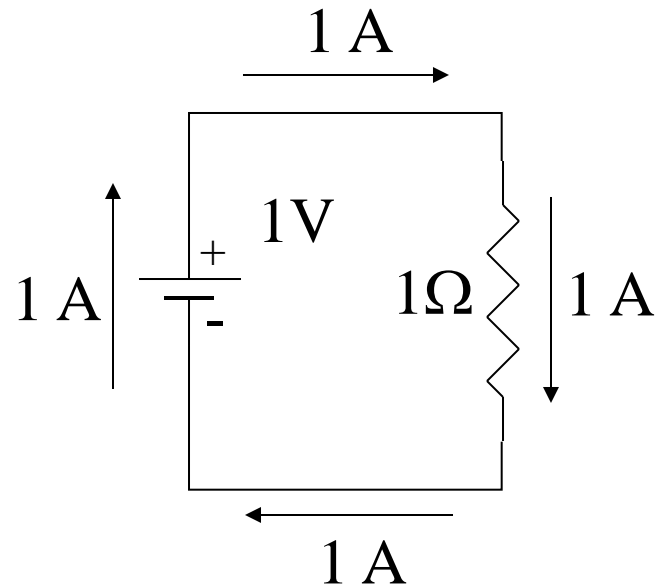
- To characterize this friction & loss we define the concept of electrical resistance.
- 1 volt will drive 1 Amp of current through a circuit having 1 Ohm ( $\Omega$ ) of resistance.
- $1 \Omega = 1 \text{ Volt}/1 \text{ Amp}$



# Electrical Current & Resistance

## Simple Circuits

- This linear relationship between voltage, resistance and current is called Ohm's Law.
- $V = I \cdot R$
- Where  $R$  is a constant of proportionality, independent of voltage and current.



**Experimentally, it is found that the current in a wire is proportional to the potential difference between its ends:**

$$I \propto V.$$

The following equation is often called Ohm's law:

Relationship among  
voltage, current,  
and resistance:

$$V = IR$$

Diagram illustrating the relationship between voltage (V), current (I), and resistance (R) in Ohm's law equation (25.11):

- Voltage between ends of conductor (V)
- Resistance of conductor (R)
- Current in conductor (I)

(25.11)



# 25-3 Ohm's Law: Resistance and Resistors

**The ratio of voltage to current is called the resistance:**

$$I = \frac{V}{R}.$$

$$V = IR.$$

- .....
4. (I) What is the resistance of a toaster if 120 V produces a current of 4.2 A?

4. Solve Eq. 25-2a for resistance.

$$R = \frac{V}{I} = \frac{120 \text{ V}}{4.2 \text{ A}} = \boxed{29 \Omega}$$

- (II) A 4.5-V battery is connected to a bulb whose resistance is  $1.6\ \Omega$ . How many electrons leave the battery per minute?

7.

Use Ohm's Law, Eq. 25-2a, to find the current. Then use the definition of current, Eq. 25-1a, to calculate the number of electrons per minute.

$$I = \frac{V}{R} = \frac{\Delta Q}{\Delta t} = \frac{4.5 \text{ V}}{1.6 \Omega} = \frac{2.8 \text{ C}}{\text{s}} \times \frac{1 \text{ electron}}{1.60 \times 10^{-19} \text{ C}} \times \frac{60 \text{ s}}{1 \text{ min}} = \boxed{1.1 \times 10^{21} \frac{\text{electrons}}{\text{minute}}}$$

8. (II) A bird stands on a dc electric transmission line carrying 3100 A (Fig. 25-34). The line has  $2.5 \times 10^{-5} \Omega$  resistance per meter, and the bird's feet are 4.0 cm apart. What is the potential difference between the bird's feet?



**FIGURE 25-34**  
Problem 8.

8. Find the potential difference from the resistance and the current.

$$R = (2.5 \times 10^{-5} \Omega/\text{m})(4.0 \times 10^{-2} \text{ m}) = 1.0 \times 10^{-6} \Omega$$

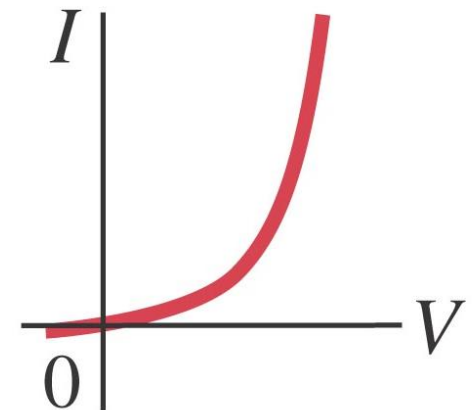
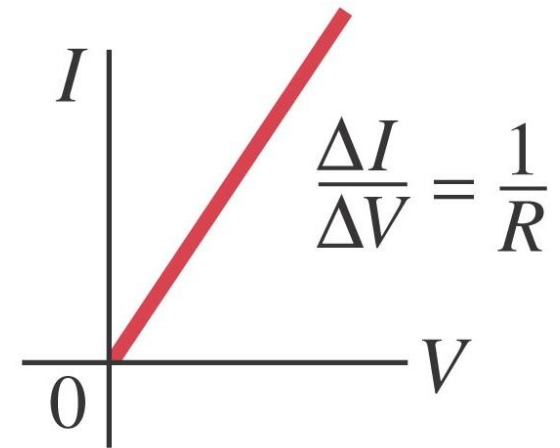
$$V = IR = (3100 \text{ A})(1.0 \times 10^{-6} \Omega) = \boxed{3.1 \times 10^{-3} \text{ V}}$$

# 25-3 Ohm's Law: Resistance and Resistors

In many conductors, the resistance is independent of the voltage; this relationship is called Ohm's law. Materials that do not follow Ohm's law are called nonohmic.

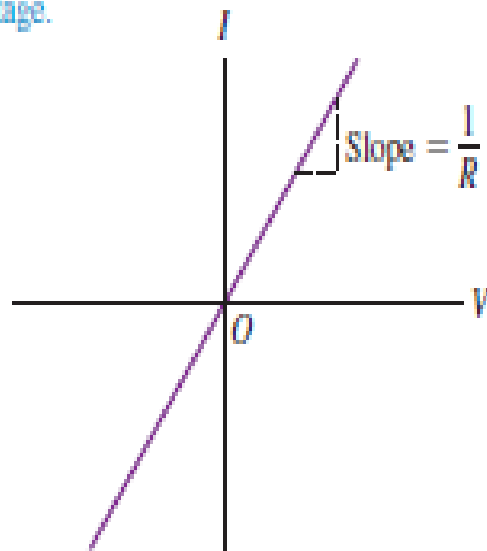
Unit of resistance:  
the ohm,  $\Omega$ :

$$1 \Omega = 1 \text{ V/A.}$$



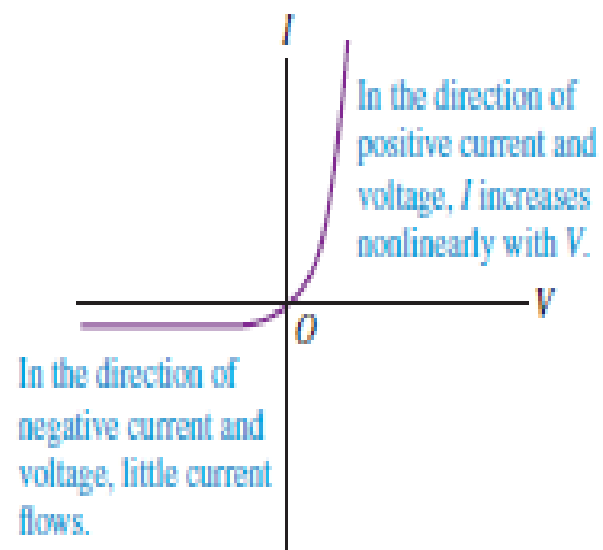
(a)

**Ohmic resistor** (e.g., typical metal wire): At a given temperature, current is proportional to voltage.



(b)

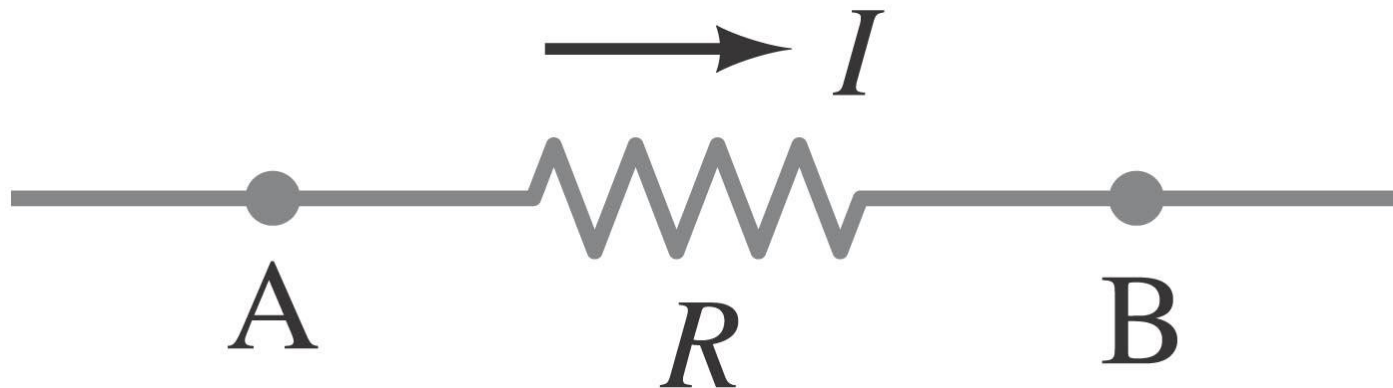
**Semiconductor diode: a nonohmic resistor**



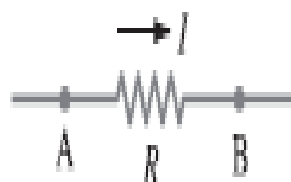


## Conceptual Example 25-3: Current and potential.

Current  $I$  enters a resistor  $R$  as shown. (a) Is the potential higher at point A or at point B? (b) Is the current greater at point A or at point B?



**FIGURE 25-10** Example 25-3.



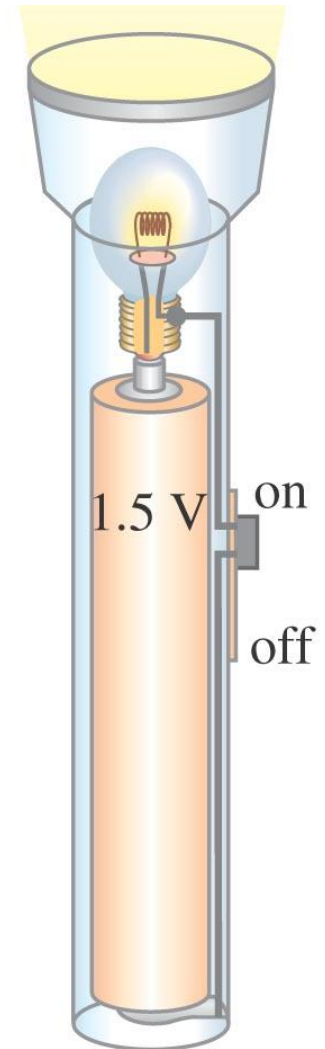
**CONCEPTUAL EXAMPLE 25-3** **Current and potential.** Current  $I$  enters a resistor  $R$  as shown in Fig. 25-10. (a) Is the potential higher at point A or at point B? (b) Is the current greater at point A or at point B?

**RESPONSE** (a) Positive charge always flows from + to −, from high potential to low potential. Think again of the gravitational analogy: a mass will fall down from high gravitational potential to low. So for positive current  $I$ , point A is at a higher potential than point B.

(b) Conservation of charge requires that whatever charge flows into the resistor at point A, an equal amount of charge emerges at point B. Charge or current does not get “used up” by a resistor, just as an object that falls through a gravitational potential difference does not gain or lose mass. So the current is the same at A and B.

## Example 25-4: Flashlight bulb resistance.

A small flashlight bulb draws 300 mA from its 1.5-V battery. (a) What is the resistance of the bulb? (b) If the battery becomes weak and the voltage drops to 1.2 V, how would the current change?



**EXAMPLE 25-4 Flashlight bulb resistance.** A small flashlight bulb (Fig. 25-11) draws 300 mA from its 1.5-V battery. (a) What is the resistance of the bulb? (b) If the battery becomes weak and the voltage drops to 1.2 V, how would the current change?

**APPROACH** We can apply Ohm's law to the bulb, where the voltage applied across it is the battery voltage.

**SOLUTION** (a) We change 300 mA to 0.30 A and use Eq. 25-2a or b:

$$R = \frac{V}{I} = \frac{1.5 \text{ V}}{0.30 \text{ A}} = 5.0 \, \Omega.$$

(b) If the resistance stays the same, the current would be

$$I = \frac{V}{R} = \frac{1.2 \text{ V}}{5.0 \, \Omega} = 0.24 \text{ A} = 240 \text{ mA},$$

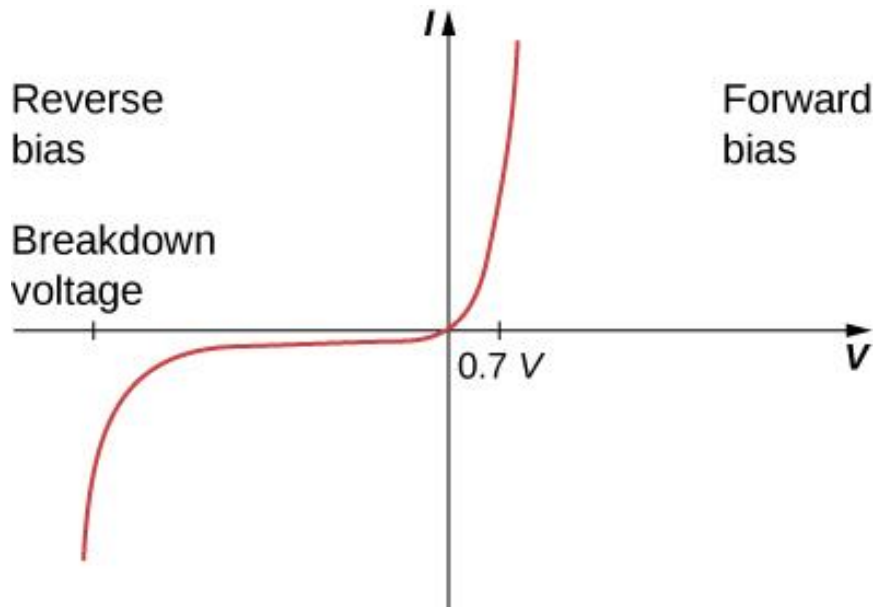
or a decrease of 60 mA.

**NOTE** With the smaller current in part (b), the bulb filament's temperature would be lower and the bulb less bright. Also, resistance does depend on temperature (Section 25-4), so our calculation is only a rough approximation.



**FIGURE 25-11** Flashlight (Example 25-4). Note how the circuit is completed along the side strip.

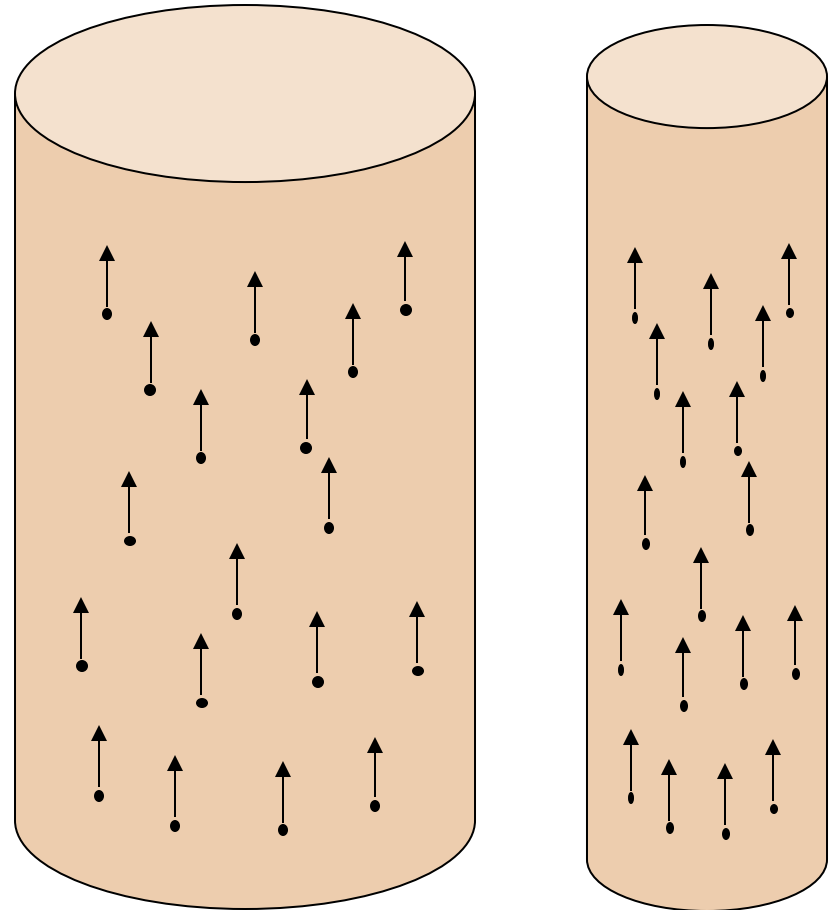
# Ohm's Law



- $V = I \cdot R$
- Most materials obey Ohm's Law and have a fixed resistance (a). This includes the resistors that we will use in the lab.
- However, some materials and devices do not and are said to be 'non-Ohmic'
- Large temperature changes can strongly affect  $R$

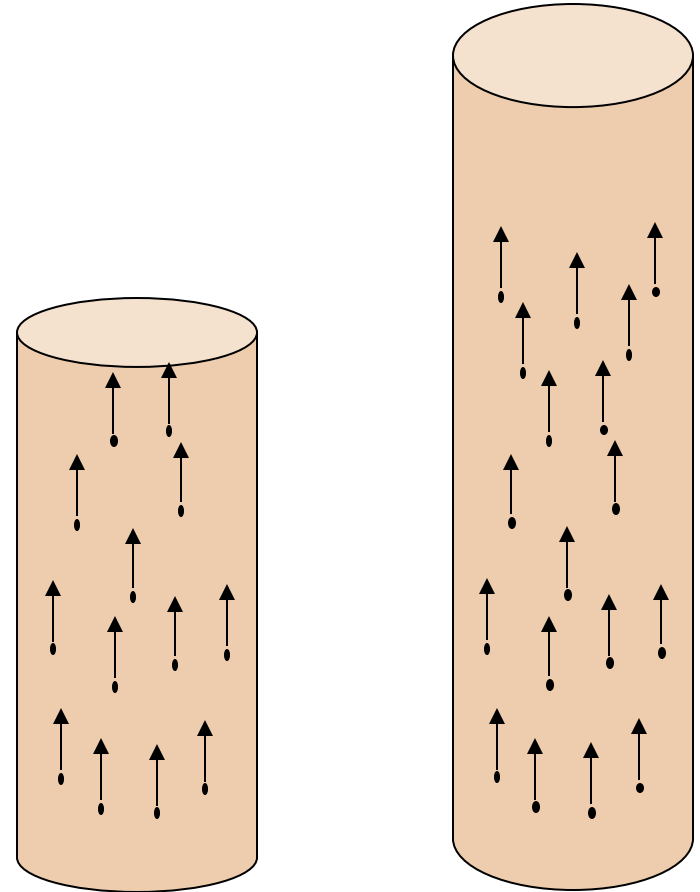
# Electrical Resistance of Materials

- Suppose we have two wires with different cross-sectional areas.
- Which one has the greater resistance?
- Since the smaller wire has less area, charge must flow faster to obtain the same current.
- As expected, this requires more force.
- The thinner wire has more resistance.



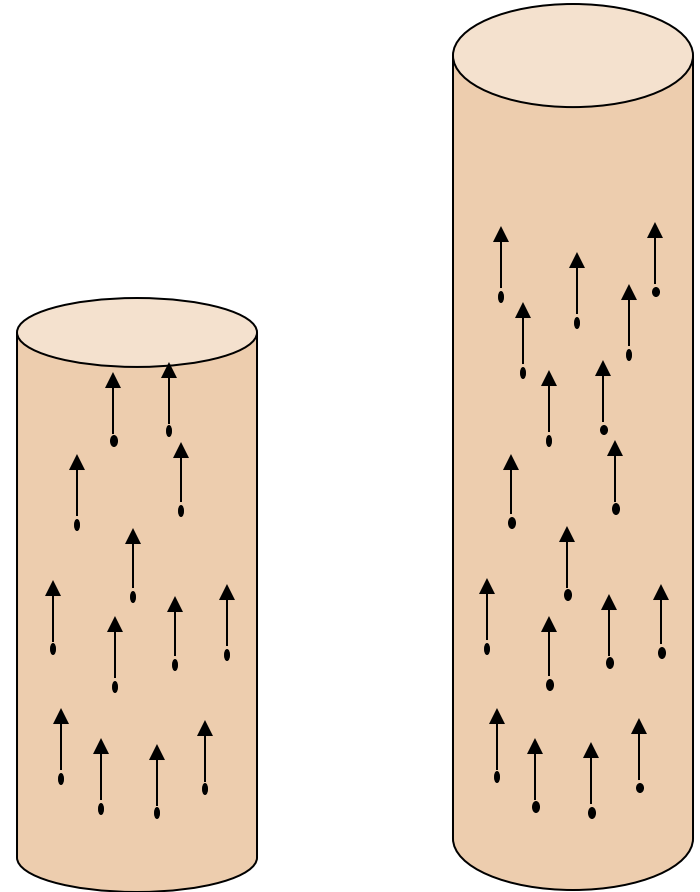
# Electrical Resistance of Materials

- What about the length of the wire?
- When the wire is longer, the electric field at any point is smaller, so there is less current flow.
- A longer wire has higher resistance.



# Electrical Resistance of Materials

- What about the material of the wire?
- Electrons have greater mobility in silver and copper and therefore have less resistance to current flow. Resistance is greater in iron for instance.
- We define a material property called resistivity,  $\rho$ , with units of Ohm•meter ( $\Omega\cdot\text{m}$ ).





**TABLE 25–1 Resistivity and Temperature Coefficients (at 20°C)**

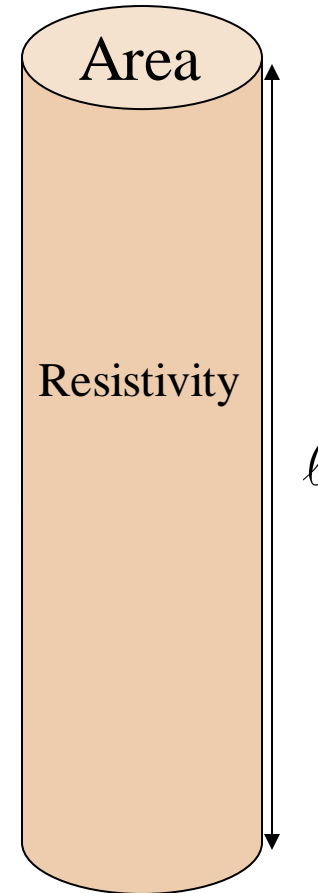
| Material                           | Resistivity,<br>$\rho$ ( $\Omega \cdot \text{m}$ ) | Temperature<br>Coefficient, $\alpha$ ( $^{\circ}\text{C}^{-1}$ ) |
|------------------------------------|--|--|
| <i>Conductors</i>                  |  |  |
| Silver                             | $1.59 \times 10^{-8}$                              | 0.0061   |
| Copper                             | $1.68 \times 10^{-8}$                              | 0.0068   |
| Gold                               | $2.44 \times 10^{-8}$                              | 0.0034   |
| Aluminum                           | $2.65 \times 10^{-8}$                              | 0.00429  |
| Tungsten                           | $5.60 \times 10^{-8}$                              | 0.0045   |
| Iron                               | $9.71 \times 10^{-8}$                              | 0.00651  |
| Platinum                           | $10.60 \times 10^{-8}$                             | 0.003927   |
| Mercury                            | $98.00 \times 10^{-8}$                             | 0.0009   |
| Nichrome (Ni, Fe, Cr alloy)        | $100.00 \times 10^{-8}$                            | 0.0004   |
| <i>Semiconductors</i> <sup>†</sup> |  |  |
| Carbon (graphite)                  | $(3 - 60) \times 10^{-5}$                          | −0.0005  |
| Germanium                          | $(1 - 500) \times 10^{-3}$                         | −0.05  |
| Silicon                            | 0.1 – 60   | −0.07  |
| <i>Insulators</i>                  |  |  |
| Glass                              | $10^9 - 10^{12}$                                   |  |
| Hard rubber                        | $10^{13} - 10^{15}$                                |  |

<sup>†</sup> Values depend strongly on the presence of even slight amounts of impurities.

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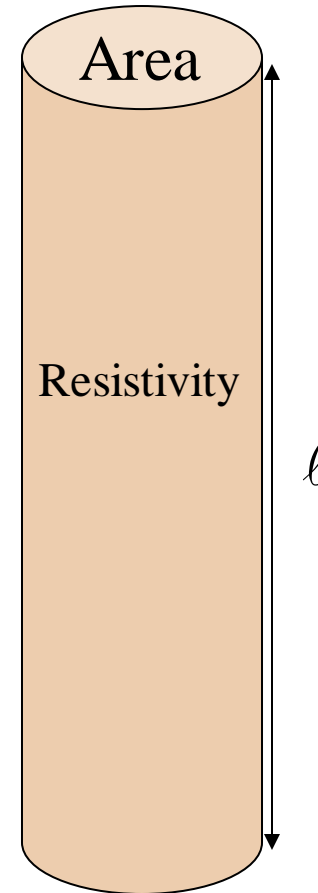
# Electrical Resistance of Materials

- From these discussions, we can write the formula for the resistance of a wire as
- $R = \rho \cdot \frac{l}{A}$
- The inverse of resistivity, called conductivity is useful in some electrical problems so we define it as
- $\sigma = 1/\rho$



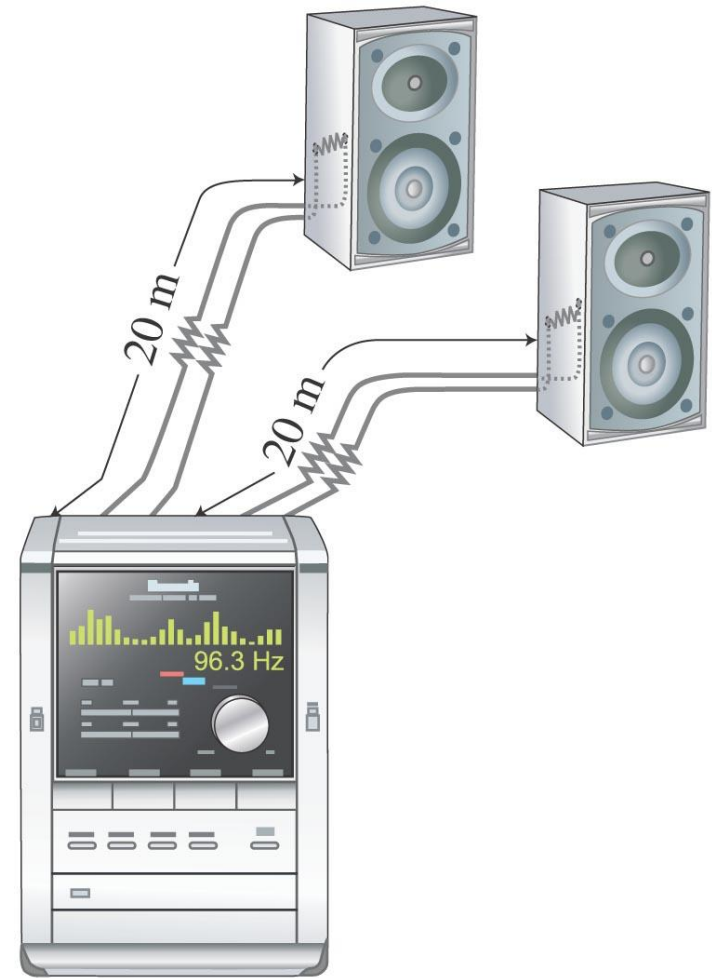
# Electrical Resistance of Materials

- Example 25-5: Speaker Wires:
- We need speaker wires to be 20 m long and to have a resistance of  $< 0.1 \, \Omega$  per wire.
- What diameter (D) of copper wire is needed?
- $R = \rho \cdot \frac{l}{A}$ ,  $A = \pi \cdot \left(\frac{D}{2}\right)^2$



## Example 25-5: Speaker wires.

Suppose you want to connect your stereo to remote speakers. (a) If each wire must be 20 m long, what diameter copper wire should you use to keep the resistance less than  $0.10\ \Omega$  per wire? (b) If the current to each speaker is 4.0 A, what is the potential difference, or voltage drop, across each wire?



**EXAMPLE 25-5 Speaker wires.** Suppose you want to connect your stereo to remote speakers (Fig. 25-14). (a) If each wire must be 20 m long, what diameter copper wire should you use to keep the resistance less than  $0.10\ \Omega$  per wire? (b) If the current to each speaker is  $4.0\ \text{A}$ , what is the potential difference, or voltage drop, across each wire?

**APPROACH** We solve Eq. 25-3 to get the area  $A$ , from which we can calculate the wire's radius using  $A = \pi r^2$ . The diameter is  $2r$ . In (b) we can use Ohm's law,  $V = IR$ .

**SOLUTION** (a) We solve Eq. 25-3 for the area  $A$  and find  $\rho$  for copper in Table 25-1:

$$A = \rho \frac{\ell}{R} = \frac{(1.68 \times 10^{-8}\ \Omega \cdot \text{m})(20\ \text{m})}{(0.10\ \Omega)} = 3.4 \times 10^{-6}\ \text{m}^2.$$

The cross-sectional area  $A$  of a circular wire is  $A = \pi r^2$ . The radius must then be at least

$$r = \sqrt{\frac{A}{\pi}} = 1.04 \times 10^{-3}\ \text{m} = 1.04\ \text{mm}.$$

The diameter is twice the radius and so must be at least  $2r = 2.1\ \text{mm}$ .

(b) From  $V = IR$  we find that the voltage drop across each wire is

$$V = IR = (4.0\ \text{A})(0.10\ \Omega) = 0.40\ \text{V}.$$

**NOTE** The voltage drop across the wires reduces the voltage that reaches the speakers from the stereo amplifier, thus reducing the sound level a bit.



**FIGURE 25-14** Example 25-5.

## **Conceptual Example 25-6: Stretching changes resistance.**

**Suppose a wire of resistance  $R$  could be stretched uniformly until it was twice its original length. What would happen to its resistance?**

**CONCEPTUAL EXAMPLE 25-6** **Stretching changes resistance.** Suppose a wire of resistance  $R$  could be stretched uniformly until it was twice its original length. What would happen to its resistance?

**RESPONSE** If the length  $\ell$  doubles, then the cross-sectional area  $A$  is halved, because the volume ( $V = A\ell$ ) of the wire remains the same. From Eq. 25-3 we see that the resistance would increase by a factor of four ( $2/\frac{1}{2} = 4$ ).

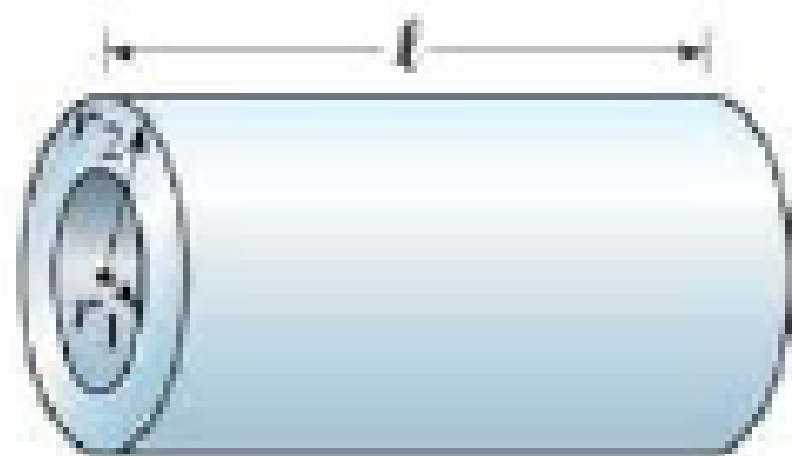
- 30.** (III) A hollow cylindrical resistor with inner radius  $r_1$  and outer radius  $r_2$ , and length  $\ell$ , is made of a material whose resistivity is  $\rho$  (Fig. 25–36). (a) Show that the resistance is given by

$$R = \frac{\rho}{2\pi\ell} \ln \frac{r_2}{r_1}$$

for current that flows radially outward. [*Hint:* Divide the resistor into concentric cylindrical shells and integrate.]

(b) Evaluate the resistance  $R$  for such a resistor made of carbon whose inner and outer radii are 1.0 mm and 1.8 mm and whose length is 2.4 cm. (Choose  $\rho = 15 \times 10^{-5} \Omega \cdot \text{m}$ .)

(c) What is the resistance in part (b) for current flowing *parallel* to the axis?



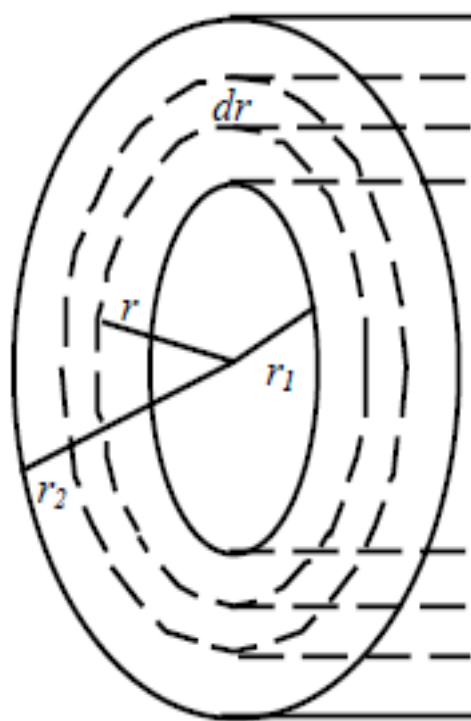
**FIGURE 25–36**  
Problem 30.



30. (a) Divide the cylinder up into concentric cylindrical shells of radius  $r$ , thickness  $dr$ , and length  $\ell$ . See the diagram. The resistance of one of those shells, from Eq. 25-3, is found. Note that the “length” in Eq. 25-3 is in the direction of the current flow, so we must substitute in  $dr$  for the “length” in Eq. 25-3. The area is the surface area of the thin cylindrical shell. Then integrate over the range of radii to find the total resistance.

$$R = \rho \frac{\ell}{A} \rightarrow dR = \rho \frac{dr}{2\pi r \ell} ;$$

$$R = \int dR = \int_{r_1}^{r_2} \rho \frac{dr}{2\pi r \ell} = \boxed{\frac{\rho}{2\pi \ell} \ln \frac{r_2}{r_1}}$$



- (b) Use the data given to calculate the resistance from the above formula.

$$R = \frac{\rho}{2\pi \ell} \ln \frac{r_2}{r_1} = \frac{15 \times 10^{-5} \Omega \cdot \text{m}}{2\pi (0.024 \text{ m})} \ln \left( \frac{1.8 \text{ mm}}{1.0 \text{ mm}} \right) = \boxed{5.8 \times 10^{-4} \Omega}$$

- (c) For resistance along the axis, we again use Eq. 25-3, but the current is flowing in the direction of length  $\ell$ . The area is the cross-sectional area of the face of the hollow cylinder.

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\pi (r_2^2 - r_1^2)} = \frac{(15 \times 10^{-5} \Omega \cdot \text{m})(0.024 \text{ m})}{\pi \left[ (1.8 \times 10^{-3} \text{ m})^2 - (1.0 \times 10^{-3} \text{ m})^2 \right]} = \boxed{0.51 \Omega}$$

The 18-gauge copper wire of Example 25.1 has a cross-sectional area of  $8.20 \times 10^{-7} \text{ m}^2$ . It carries a current of 1.67 A. Find (a) the electric-field magnitude in the wire; (b) the potential difference between two points in the wire 50.0 m apart; (c) the resistance of a 50.0-m length of this wire.

### SOLUTION

**IDENTIFY and SET UP:** We are given the cross-sectional area  $A$  and current  $I$ . Our target variables are the electric-field magnitude  $E$ , potential difference  $V$ , and resistance  $R$ . The current density is  $J = I/A$ . We find  $E$  from Eq. (25.5),  $E = \rho J$  (Table 25.1 gives the resistivity  $\rho$  for copper). The potential difference is then the product of  $E$  and the length of the wire. We can use either Eq. (25.10) or Eq. (25.11) to find  $R$ .

**EXECUTE:** (a) From Table 25.1,  $\rho = 1.72 \times 10^{-8} \Omega \cdot \text{m}$ . Hence, from Eq. (25.5),

$$E = \rho J = \frac{\rho I}{A} = \frac{(1.72 \times 10^{-8} \Omega \cdot \text{m})(1.67 \text{ A})}{8.20 \times 10^{-7} \text{ m}^2} = 0.0350 \text{ V/m}$$

(b) The potential difference is

$$V = EL = (0.0350 \text{ V/m})(50.0 \text{ m}) = 1.75 \text{ V}$$

(c) From Eq. (25.10) the resistance of 50.0 m of this wire is

$$R = \frac{\rho L}{A} = \frac{(1.72 \times 10^{-8} \Omega \cdot \text{m})(50.0 \text{ m})}{8.20 \times 10^{-7} \text{ m}^2} = 1.05 \Omega$$

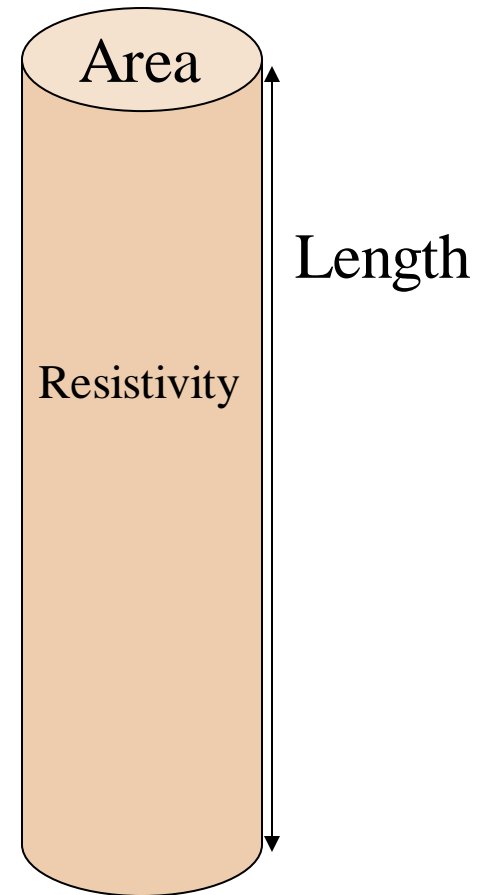
Alternatively, we can find  $R$  from Eq. (25.11):

$$R = \frac{V}{I} = \frac{1.75 \text{ V}}{1.67 \text{ A}} = 1.05 \Omega$$

**EVALUATE:** We emphasize that the resistance of the wire is *defined* to be the ratio of voltage to current. If the wire is made of non-ohmic material, then  $R$  is different for different values of  $V$  but is always given by  $R = V/I$ . Resistance is also always given by  $R = \rho L/A$ ; if the material is nonohmic,  $\rho$  is not constant but depends on  $E$  (or, equivalently, on  $V = EL$ ).

# Temperature Variation of Resistance

- The resistance for many materials, especially metals, have a reasonably linear variation with temperature.
- We can write this in terms of the resistivity as
- $\Delta\rho = \rho_0\alpha\Delta T$  or  $\rho = \rho_0(1+\alpha\Delta T)$
- Since Resistance and resistivity are proportional,
- $\Delta R = R_0\alpha\Delta T$  or  $R = R_0(1+\alpha\Delta T)$
- Values for  $\rho$  and  $\alpha$  can be found in table 25-1.



Suppose the resistance of a copper wire is  $1.05\ \Omega$  at  $20^\circ\text{C}$ . Find the resistance at  $0^\circ\text{C}$  and  $100^\circ\text{C}$ .

### SOLUTION

**IDENTIFY and SET UP:** We are given the resistance  $R_0 = 1.05\ \Omega$  at a reference temperature  $T_0 = 20^\circ\text{C}$ . We use Eq. (25.12) to find the resistances at  $T = 0^\circ\text{C}$  and  $T = 100^\circ\text{C}$  (our target variables), taking the temperature coefficient of resistivity from Table 25.2.

**EXECUTE:** From Table 25.2,  $\alpha = 0.00393\ (\text{C}^\circ)^{-1}$  for copper. Then from Eq. (25.12),

$$\begin{aligned} R &= R_0[1 + \alpha(T - T_0)] \\ &= (1.05\ \Omega) \{1 + [0.00393\ (\text{C}^\circ)^{-1}][0^\circ\text{C} - 20^\circ\text{C}]\} \\ &= 0.97\ \Omega \text{ at } T = 0^\circ\text{C} \\ R &= (1.05\ \Omega) \{1 + [0.00393\ (\text{C}^\circ)^{-1}][100^\circ\text{C} - 20^\circ\text{C}]\} \\ &= 1.38\ \Omega \text{ at } T = 100^\circ\text{C} \end{aligned}$$

## **Example 25-7: Resistance thermometer.**

**The variation in electrical resistance with temperature can be used to make precise temperature measurements. Platinum is commonly used since it is relatively free from corrosive effects and has a high melting point. Suppose at  $20.0^{\circ}\text{C}$  the resistance of a platinum resistance thermometer is  $164.2\ \Omega$ . When placed in a particular solution, the resistance is  $187.4\ \Omega$ . What is the temperature of this solution?**

**EXAMPLE 25-7 Resistance thermometer.** The variation in electrical resistance with temperature can be used to make precise temperature measurements. Platinum is commonly used since it is relatively free from corrosive effects and has a high melting point. Suppose at  $20.0^\circ\text{C}$  the resistance of a platinum resistance thermometer is  $164.2\ \Omega$ . When placed in a particular solution, the resistance is  $187.4\ \Omega$ . What is the temperature of this solution?

**APPROACH** Since the resistance  $R$  is directly proportional to the resistivity  $\rho$ , we can combine Eq. 25-3 with Eq. 25-5 to find  $R$  as a function of temperature  $T$ , and then solve that equation for  $T$ .

**SOLUTION** We multiply Eq. 25-5 by  $(\ell/A)$  to obtain (see also Eq. 25-3)

$$R = R_0[1 + \alpha(T - T_0)].$$

Here  $R_0 = \rho_0 \ell/A$  is the resistance of the wire at  $T_0 = 20.0^\circ\text{C}$ . We solve this equation for  $T$  and find (see Table 25-1 for  $\alpha$ )

$$T = T_0 + \frac{R - R_0}{\alpha R_0} = 20.0^\circ\text{C} + \frac{187.4\ \Omega - 164.2\ \Omega}{(3.927 \times 10^{-3}(\text{C}^\circ)^{-1})(164.2\ \Omega)} = 56.0^\circ\text{C}.$$

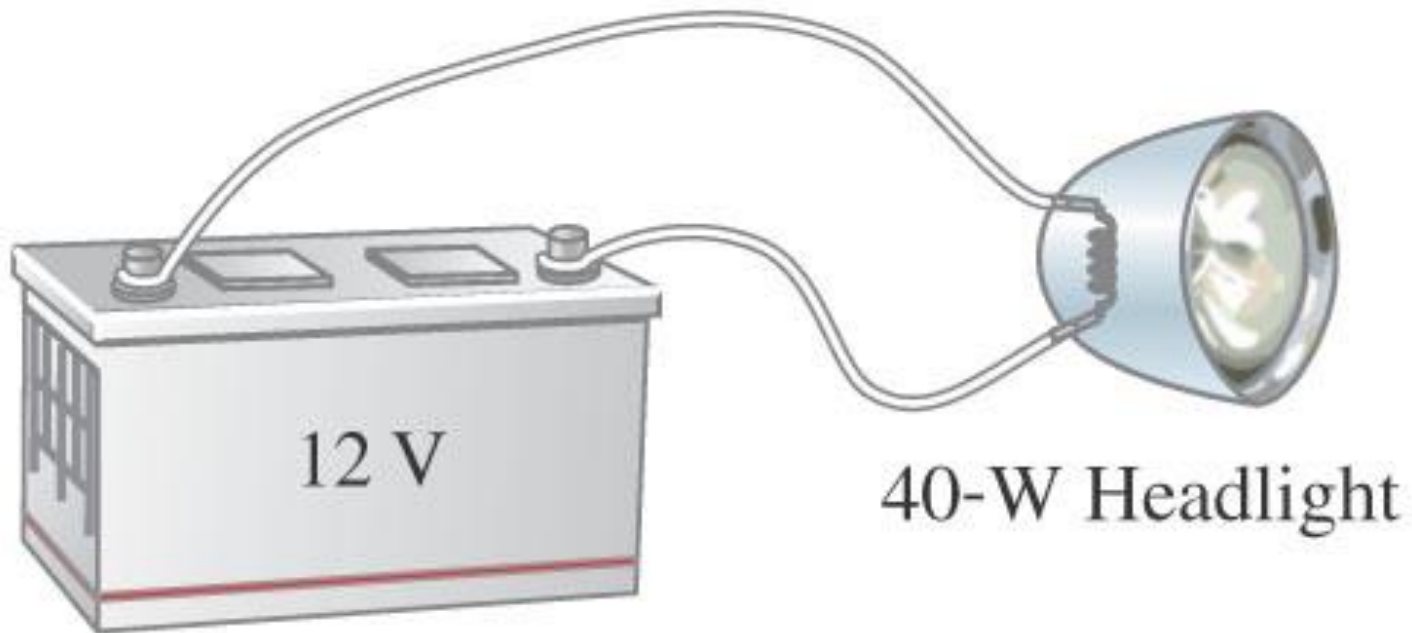
**NOTE** Resistance thermometers have the advantage that they can be used at very high or low temperatures where gas or liquid thermometers would be useless.

# Electrical Power

- From the last chapter, we showed that work done on a charge  $q$  to move it through a voltage  $V$  is just
- $W = q \cdot V$
- Since Power is Work/time (Unit is Watt = J/s),
- $P = \frac{q \cdot V}{t} = V \cdot \frac{q}{t} = V \cdot I$ , since  $I$  is charge per second.
- Using Ohm's Law, we can write the formula as
- $P = V \cdot I = I^2 \cdot R = V^2 / R$

## Example 25-8: Headlights.

Calculate the resistance of a 40-W automobile headlight designed for 12 V.





**EXAMPLE 25-8 Headlights.** Calculate the resistance of a 40-W automobile headlight designed for 12 V (Fig. 25-17).

**APPROACH** We solve Eq. 25-7b for  $R$ .

**SOLUTION** From Eq. 25-7b,

$$R = \frac{V^2}{P} = \frac{(12 \text{ V})^2}{(40 \text{ W})} = 3.6 \Omega.$$

**NOTE** This is the resistance when the bulb is burning brightly at 40 W. When the bulb is cold, the resistance is much lower, as we saw in Eq. 25-5. Since the current is high when the resistance is low, lightbulbs burn out most often when first turned on.



**FIGURE 25-17** Example 25-8.

### **PHYSICS APPLIED**

*Why lightbulbs burn out when first turned on*

- 32.** (I) The heating element of an electric oven is designed to produce 3.3 kW of heat when connected to a 240-V source. What must be the resistance of the element?

32. Use Eq. 25-7b to find the resistance from the voltage and the power.

$$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(240 \text{ V})^2}{3300 \text{ W}} = \boxed{17 \Omega}$$

- 36.** (II) A 120-V hair dryer has two settings: 850 W and 1250 W.  
(a) At which setting do you expect the resistance to be higher? After making a guess, determine the resistance at  
(b) the lower setting; and (c) the higher setting.

36. (a) Since  $P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P}$  says that the resistance is inversely proportional to the power for a constant voltage, we predict that the 850 W setting has the higher resistance.

$$(b) \quad R = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{850 \text{ W}} = \boxed{17 \Omega}$$

$$(c) \quad R = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{1250 \text{ W}} = \boxed{12 \Omega}$$

**38.** (II) You buy a 75-W lightbulb in Europe, where electricity is delivered to homes at 240 V. If you use the lightbulb in the United States at 120 V (assume its resistance does not change), how bright will it be relative to 75-W 120-V bulbs? [*Hint: Assume roughly that brightness is proportional to power consumed.*]

38. The power (and thus the brightness) of the bulb is proportional to the square of the voltage, according to Eq. 25-7b,  $P = \frac{V^2}{R}$ . Since the resistance is assumed to be constant, if the voltage is cut in half from 240 V to 120V, the power will be reduced by a factor of 4. Thus the bulb will appear only about 1/4 as bright in the United States as in Europe.

**45.** (II) A power station delivers 750 kW of power at 12,000 V to a factory through wires with total resistance 3.0  $\Omega$ . How much less power is wasted if the electricity is delivered at 50,000 V rather than 12,000 V?

45. Find the current used to deliver the power in each case, and then find the power dissipated in the resistance at the given current.

$$P = IV \rightarrow I = \frac{P}{V} \quad P_{\text{dissipated}} = I^2 R = \frac{P^2}{V^2} R$$

$$P_{\text{dissipated } 12,000 \text{ V}} = \frac{(7.5 \times 10^5 \text{ W})^2}{(1.2 \times 10^4 \text{ V})^2} (3.0 \Omega) = 11719 \text{ W}$$

$$P_{\text{dissipated } 50,000 \text{ V}} = \frac{(7.5 \times 10^5 \text{ W})^2}{(5 \times 10^4 \text{ V})^2} (3.0 \Omega) = 675 \text{ W} \quad \text{difference} = 11719 \text{ W} - 675 \text{ W} = \boxed{1.1 \times 10^4 \text{ W}}$$

**What you pay for on your electric bill is not power, but energy – the power consumption multiplied by the time.**

**We have been measuring energy in joules, but the electric company measures it in kilowatt-hours, kWh:**

$$1 \text{ kWh} = (1000 \text{ W})(3600 \text{ s}) = 3.60 \times 10^6 \text{ J.}$$

### **Example 25-9: Electric heater.**

**An electric heater draws a steady 15.0 A on a 120-V line. How much power does it require and how much does it cost per month (30 days) if it operates 3.0 h per day and the electric company charges 9.2 cents per kWh?**

**EXAMPLE 25-9 Electric heater.** An electric heater draws a steady 15.0 A on a 120-V line. How much power does it require and how much does it cost per month (30 days) if it operates 3.0 h per day and the electric company charges 9.2 cents per kWh?

**APPROACH** We use Eq. 25-6,  $P = IV$ , to find the power. We multiply the power (in kW) by the time (h) used in a month and by the cost per energy unit, \$0.092 per kWh, to get the cost per month.

**SOLUTION** The power is

$$P = IV = (15.0 \text{ A})(120 \text{ V}) = 1800 \text{ W}$$

or 1.80 kW. The time (in hours) the heater is used per month is  $(3.0 \text{ h/d})(30 \text{ d}) = 90 \text{ h}$ , which at 9.2¢/kWh would cost  $(1.80 \text{ kW})(90 \text{ h})(\$0.092/\text{kWh}) = \$15$ .



## **Example 25-10: Lightning bolt.**

**Lightning is a spectacular example of electric current in a natural phenomenon. There is much variability to lightning bolts, but a typical event can transfer  $10^9$  J of energy across a potential difference of perhaps  $5 \times 10^7$  V during a time interval of about 0.2 s. Use this information to estimate (a) the total amount of charge transferred between cloud and ground, (b) the current in the lightning bolt, and (c) the average power delivered over the 0.2 s.**



**FIGURE 25-18** Example 25-10.  
A lightning bolt.

**EXAMPLE 25-10 ESTIMATE Lightning bolt.** Lightning is a spectacular example of electric current in a natural phenomenon (Fig. 25-18). There is much variability to lightning bolts, but a typical event can transfer  $10^9 \text{ J}$  of energy across a potential difference of perhaps  $5 \times 10^7 \text{ V}$  during a time interval of about  $0.2 \text{ s}$ . Use this information to estimate (a) the total amount of charge transferred between cloud and ground, (b) the current in the lightning bolt, and (c) the average power delivered over the  $0.2 \text{ s}$ .

**APPROACH** We estimate the charge  $Q$ , recalling that potential energy change equals the potential difference  $\Delta V$  times the charge  $Q$ , Eq. 23-3. We equate  $\Delta U$  with the energy transferred,  $\Delta U \approx 10^9 \text{ J}$ . Next, the current  $I$  is  $Q/t$  (Eq. 25-1a) and the power  $P$  is energy/time.

**SOLUTION** (a) From Eq. 23-3, the energy transformed is  $\Delta U = Q \Delta V$ . We solve for  $Q$ :

$$Q = \frac{\Delta U}{\Delta V} \approx \frac{10^9 \text{ J}}{5 \times 10^7 \text{ V}} = 20 \text{ coulombs.}$$

(b) The current during the  $0.2 \text{ s}$  is about

$$I = \frac{Q}{t} \approx \frac{20 \text{ C}}{0.2 \text{ s}} = 100 \text{ A.}$$

(c) The average power delivered is

$$P = \frac{\text{energy}}{\text{time}} = \frac{10^9 \text{ J}}{0.2 \text{ s}} = 5 \times 10^9 \text{ W} = 5 \text{ GW.}$$

We can also use Eq. 25-6:

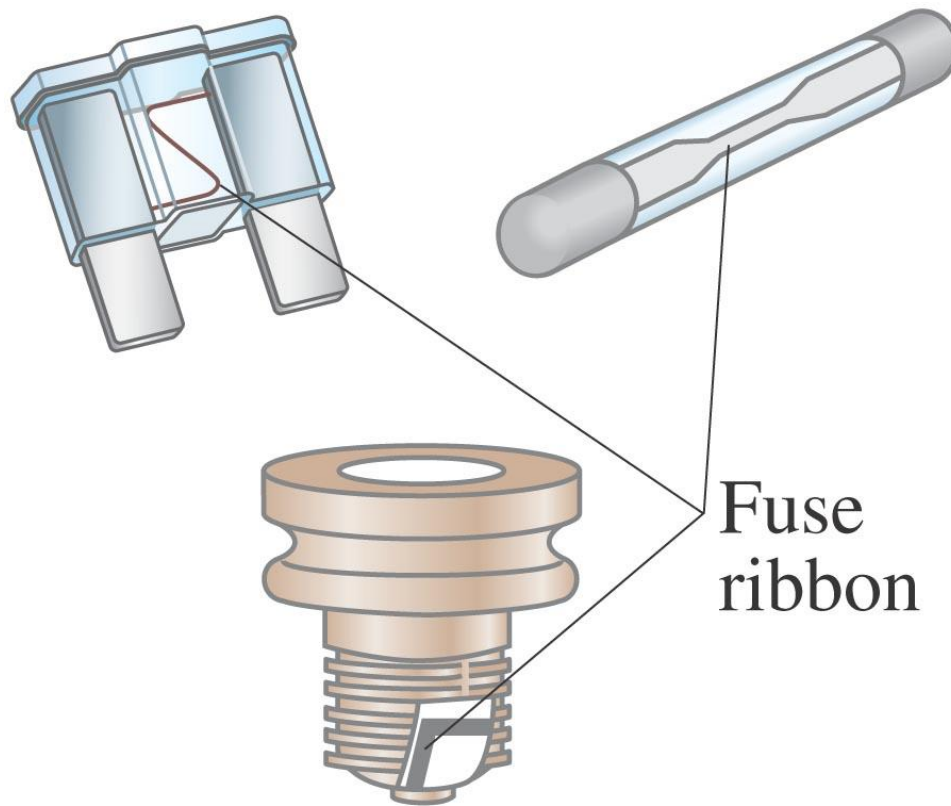
$$P = IV = (100 \text{ A})(5 \times 10^7 \text{ V}) = 5 \text{ GW.}$$

**NOTE** Since most lightning bolts consist of several stages, it is possible that individual parts could carry currents much higher than the  $100 \text{ A}$  calculated above.

**The wires used in homes to carry electricity have very low resistance. However, if the current is high enough, the power will increase and the wires can become hot enough to start a fire.**

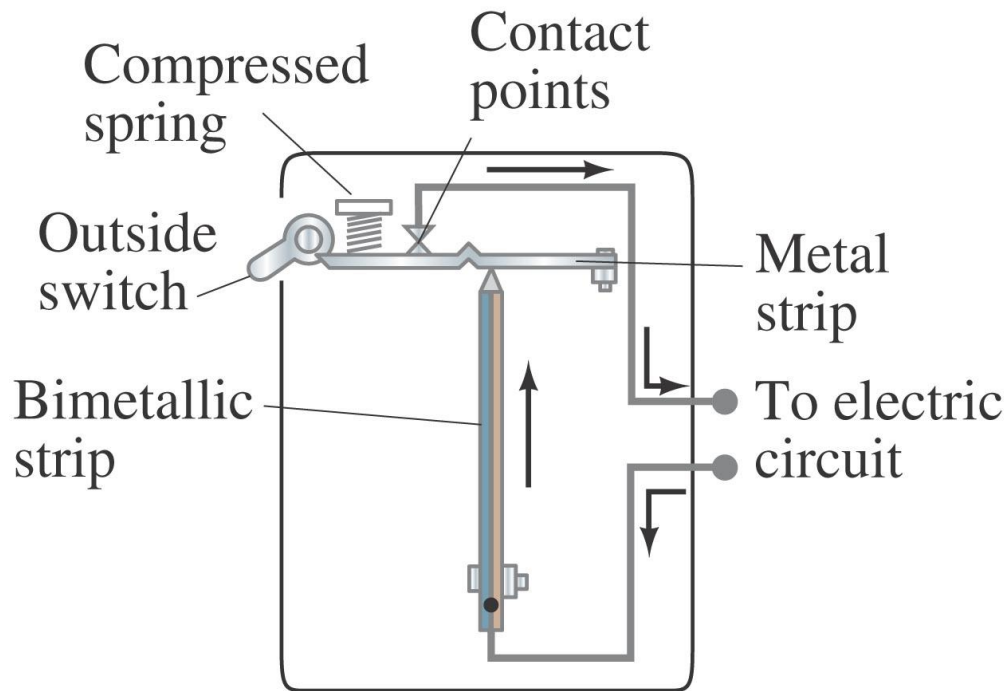
**To avoid this, we use fuses or circuit breakers, which disconnect when the current goes above a predetermined value.**

**Fuses are one-use items – if they blow, the fuse is destroyed and must be replaced.**

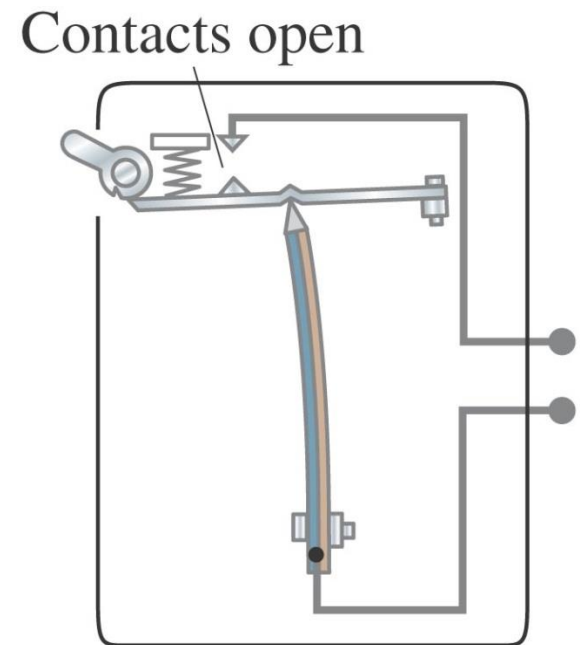


Types of fuses

**Circuit breakers, which are now much more common in homes than they once were, are switches that will open if the current is too high; they can then be reset.**



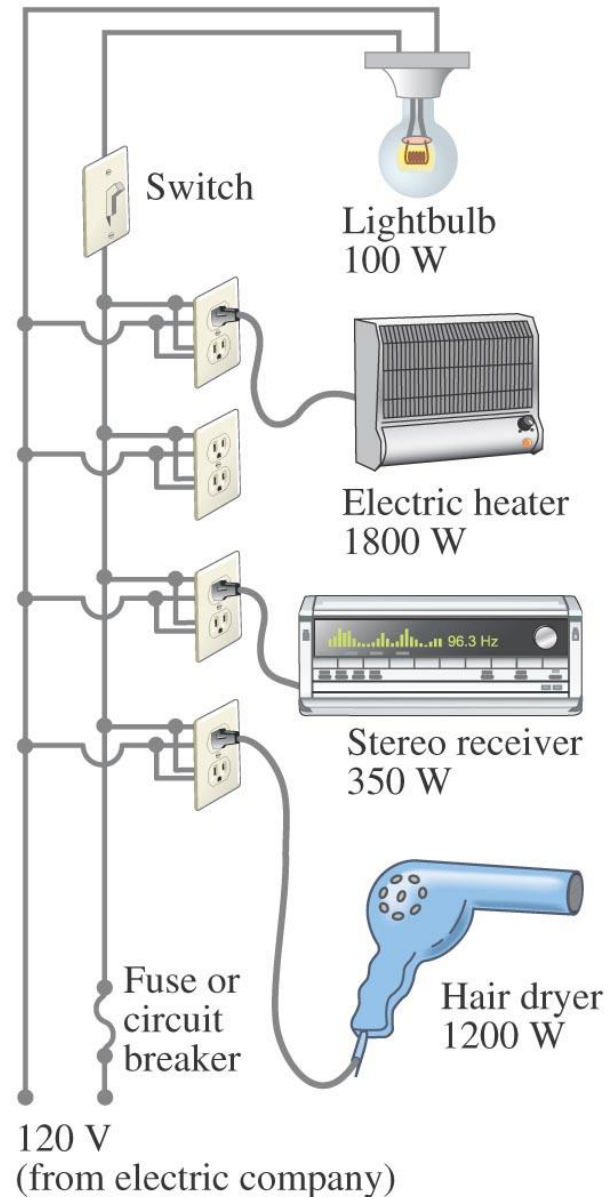
Circuit breaker  
(closed)



Circuit breaker  
(open)

## Example 25-11: Will a fuse blow?

Determine the total current drawn by all the devices in the circuit shown.



**EXAMPLE 25–11 Will a fuse blow?** Determine the total current drawn by all the devices in the circuit of Fig. 25–20.

**APPROACH** Each device has the same 120-V voltage across it. The current each draws from the source is found from  $I = P/V$ , Eq. 25–6.

**SOLUTION** The circuit in Fig. 25–20 draws the following currents: the lightbulb draws  $I = P/V = 100 \text{ W}/120 \text{ V} = 0.8 \text{ A}$ ; the heater draws  $1800 \text{ W}/120 \text{ V} = 15.0 \text{ A}$ ; the stereo draws a maximum of  $350 \text{ W}/120 \text{ V} = 2.9 \text{ A}$ ; and the hair dryer draws  $1200 \text{ W}/120 \text{ V} = 10.0 \text{ A}$ . The total current drawn, if all devices are used at the same time, is

$$0.8 \text{ A} + 15.0 \text{ A} + 2.9 \text{ A} + 10.0 \text{ A} = 28.7 \text{ A}.$$

**NOTE** The heater draws as much current as 18 100-W lightbulbs. For safety, the heater should probably be on a circuit by itself.

## **Conceptual Example 25-12: A dangerous extension cord.**

**Your 1800-W portable electric heater is too far from your desk to warm your feet. Its cord is too short, so you plug it into an extension cord rated at 11 A. Why is this dangerous?**



**CONCEPTUAL EXAMPLE 25-12**

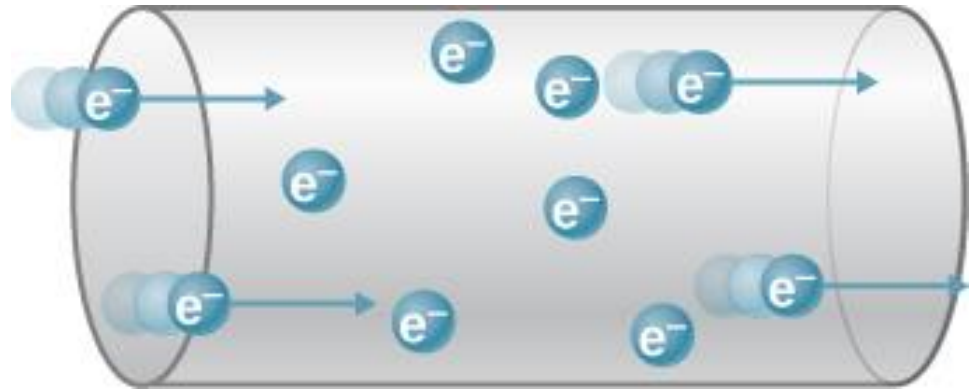
**A dangerous extension cord.** Your 1800-W portable electric heater is too far from your desk to warm your feet. Its cord is too short, so you plug it into an extension cord rated at 11 A. Why is this dangerous?

**RESPONSE** 1800 W at 120 V draws a 15-A current. The wires in the extension cord rated at 11 A could become hot enough to melt the insulation and cause a fire.



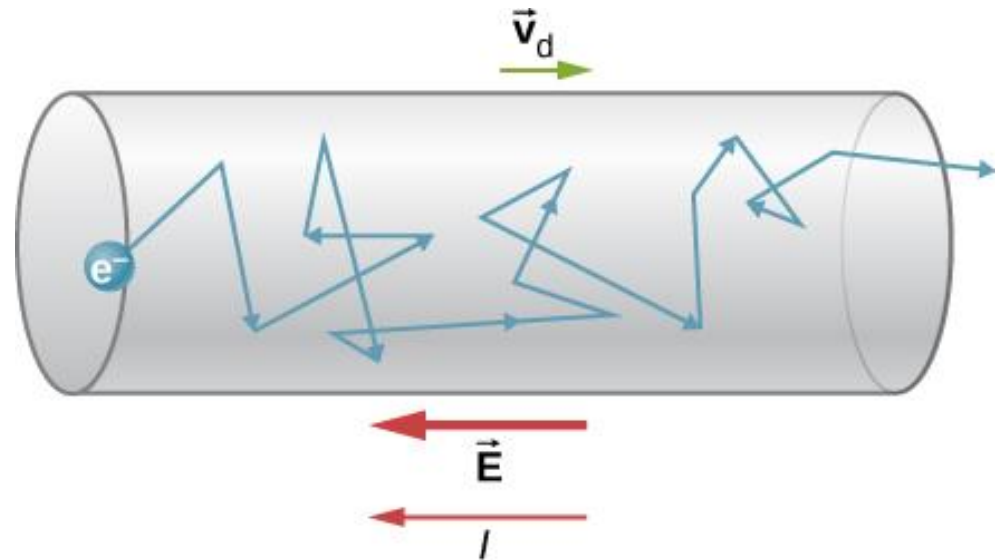
# Drift Velocity & Current Density

- We typically visualize the current flow in a wire like a smooth flow or water, but in fact the thermal velocity of the electrons is much faster than their velocity along the wire.



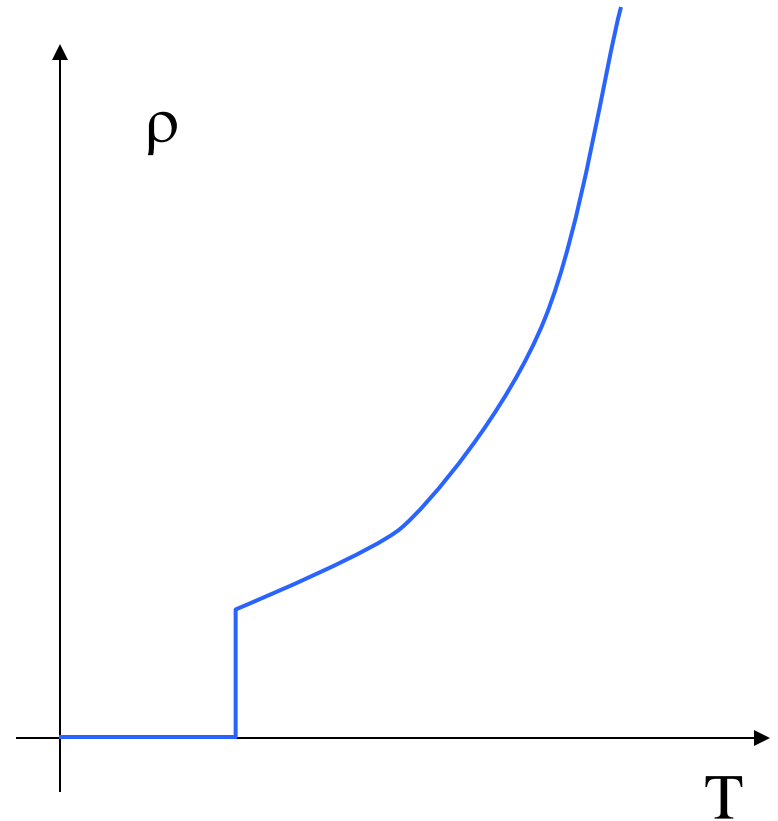
# Drift Velocity & Current Density

- When voltage is applied to a wire, it establishes an electric field ( $E$ ) which causes the electrons to start to move with a small average speed ( $\sim 1\text{mm/sec}$ ).
- The signal ( $E$ -field) propagates through the wire a near light speed.



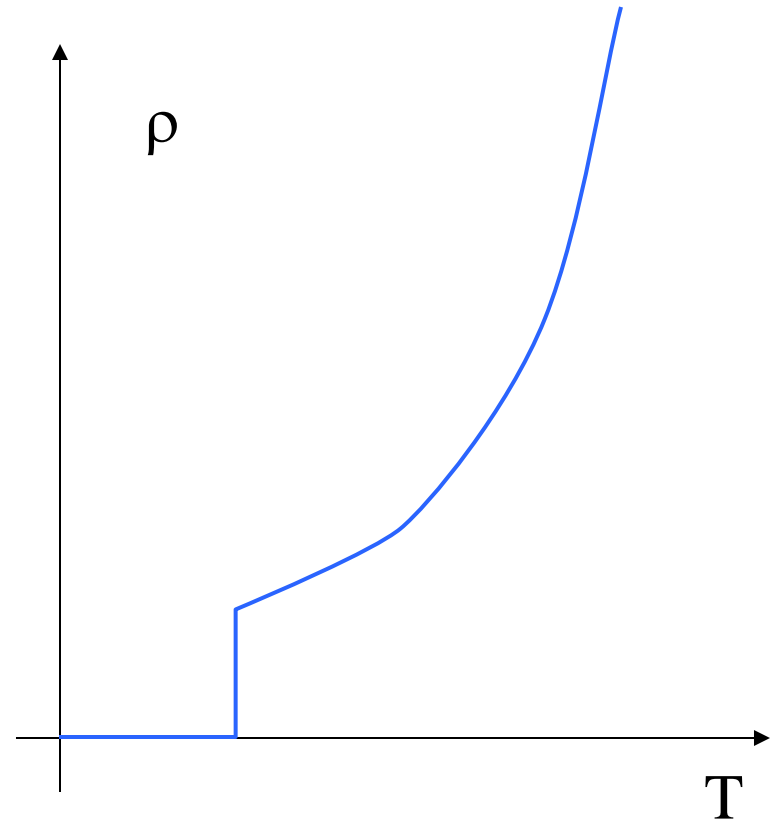
# Superconductivity

- In 1911, it was discovered that at  $\sim 4$  Kelvin, mercury suddenly lost ALL resistance. Other superconductors were later discovered.
- Today 'Yttrium barium copper oxide' compounds are superconducting at 92K.



# Superconductivity

- The primary use of superconductors today is high power magnets (MRI systems, particle accelerators, etc.)
- However, an economical high-temperature ( $\sim$  room temp?) superconductor could save millions of watts of power each year that are lost to resistance of power lines.



Electrons in a conductor have large, random speeds just due to their temperature. When a potential difference is applied, the electrons also acquire an average drift velocity, which is generally considerably smaller than the thermal velocity.



# 25-8 Microscopic View of Electric Current: Current Density and Drift Velocity

**We define the current density (current per unit area) – this is a convenient concept for relating the microscopic motions of electrons to the macroscopic current:**

$$j = \frac{I}{A} \quad \text{or} \quad I = jA.$$

**If the current is not uniform:**

$$I = \int \vec{j} \cdot d\vec{A}.$$

# 25-8 Microscopic View of Electric Current: Current Density and Drift Velocity

**This drift speed is related to the current in the wire, and also to the number of electrons per unit volume:**

$$\begin{aligned}\Delta Q &= (\text{no. of charges, } N) \times (\text{charge per particle}) \\ &= (nV)(-e) = -(nAv_d \Delta t)(e)\end{aligned}$$

**and**

$$I = \frac{\Delta Q}{\Delta t} = -neAv_d.$$



# 25-8 Microscopic View of Electric Current: Current Density and Drift Velocity

## **Example 25-14: Electron speeds in a wire.**

**A copper wire 3.2 mm in diameter carries a 5.0-A current. Determine (a) the current density in the wire, and (b) the drift velocity of the free electrons. (c) Estimate the rms speed of electrons assuming they behave like an ideal gas at 20° C. Assume that one electron per Cu atom is free to move (the others remain bound to the atom).**

**EXAMPLE 25-14 Electron speeds in a wire.** A copper wire 3.2 mm in diameter carries a 5.0-A current. Determine (a) the current density in the wire, and (b) the drift velocity of the free electrons. (c) Estimate the rms speed of electrons assuming they behave like an ideal gas at 20°C. Assume that one electron per Cu atom is free to move (the others remain bound to the atom).

**APPROACH** For (a)  $j = I/A = I/\pi r^2$ . For (b) we can apply Eq. 25-14 to find  $v_d$  if we can determine the number  $n$  of free electrons per unit volume. Since we assume there is one free electron per atom, the density of free electrons,  $n$ , is the same as the density of Cu atoms. The atomic mass of Cu is 63.5 u (see Periodic Table inside the back cover), so 63.5 g of Cu contains one mole or  $6.02 \times 10^{23}$  free electrons. The mass density of copper (Table 13-1) is  $\rho_D = 8.9 \times 10^3 \text{ kg/m}^3$ , where  $\rho_D = m/V$ . (We use  $\rho_D$  to distinguish it here from  $\rho$  for resistivity.) In (c) we use  $K = \frac{3}{2}kT$ , Eq. 18-4. (Do not confuse  $V$  for volume with  $V$  for voltage.)

**SOLUTION** (a) The current density is (with  $r = \frac{1}{2}(3.2 \text{ mm}) = 1.6 \times 10^{-3} \text{ m}$ )

$$j = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{5.0 \text{ A}}{\pi(1.6 \times 10^{-3} \text{ m})^2} = 6.2 \times 10^5 \text{ A/m}^2.$$

(b) The number of free electrons per unit volume,  $n = N/V$  (where  $V = m/\rho_D$ ), is

$$n = \frac{N}{V} = \frac{N}{m/\rho_D} = \frac{N(1 \text{ mole})}{m(1 \text{ mole})} \rho_D$$

$$n = \left( \frac{6.02 \times 10^{23} \text{ electrons}}{63.5 \times 10^{-3} \text{ kg}} \right) (8.9 \times 10^3 \text{ kg/m}^3) = 8.4 \times 10^{28} \text{ m}^{-3}.$$

Then, by Eq. 25-14, the drift velocity has magnitude

$$v_d = \frac{j}{ne} = \frac{6.2 \times 10^5 \text{ A/m}^2}{(8.4 \times 10^{28} \text{ m}^{-3})(1.6 \times 10^{-19} \text{ C})} = 4.6 \times 10^{-5} \text{ m/s} \approx 0.05 \text{ mm/s}.$$

(c) If we model the free electrons as an ideal gas (a rather rough approximation), we use Eq. 18-5 to estimate the random rms speed of an electron as it darts around:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{9.11 \times 10^{-31} \text{ kg}}} = 1.2 \times 10^5 \text{ m/s}.$$

The drift velocity (average speed in the direction of the current) is very much less than the rms thermal speed of the electrons, by a factor of about  $10^9$ .

**NOTE** The result in (c) is an underestimate. Quantum theory calculations, and experiments, give the rms speed in copper to be about  $1.6 \times 10^6 \text{ m/s}$ .

## 25-8 Microscopic View of Electric Current: Current Density and Drift Velocity

The electric field inside a current-carrying wire can be found from the relationship between the current, voltage, and resistance. Writing  $R = \rho l / A$ ,  $I = jA$ , and  $V = E l$ , and substituting in Ohm's law gives:

$$j = \frac{1}{\rho} E = \sigma E.$$

## **Example 25-15: Electric field inside a wire.**

**What is the electric field inside the wire of Example 25–14? (The current density was found to be  $6.2 \times 10^5 \text{ A/m}^2$ .)**

**EXAMPLE 25-15** **Electric field inside a wire.** What is the electric field inside the wire of Example 25-14?

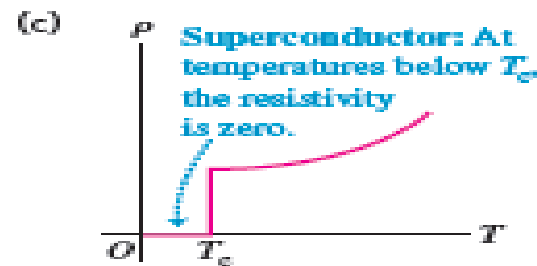
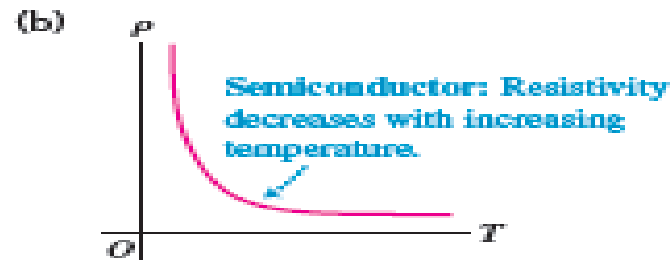
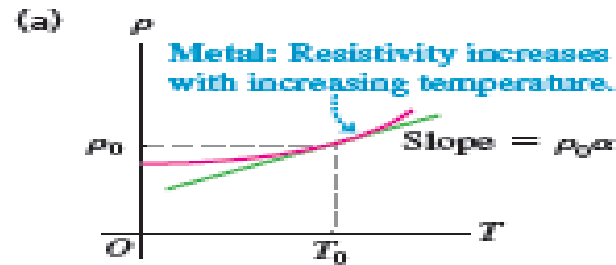
**APPROACH** We use Eq. 25-17 and  $\rho = 1.68 \times 10^{-8} \Omega \cdot \text{m}$  for copper.

**SOLUTION** Example 25-14 gives  $j = 6.2 \times 10^5 \text{ A/m}^2$ , so

$$E = \rho j = (1.68 \times 10^{-8} \Omega \cdot \text{m})(6.2 \times 10^5 \text{ A/m}^2) = 1.0 \times 10^{-2} \text{ V/m}.$$

**NOTE** For comparison, the electric field between the plates of a capacitor is often much larger; in Example 24-1, for example,  $E$  is on the order of  $10^4 \text{ V/m}$ . Thus we see that only a modest electric field is needed for current flow in practical cases.

**25.6** Variation of resistivity  $\rho$  with absolute temperature  $T$  for (a) a normal metal, (b) a semiconductor, and (c) a superconductor. In (a) the linear approximation to  $\rho$  as a function of  $T$  is shown as a green line; the approximation agrees exactly at  $T = T_0$ , where  $\rho = \rho_0$ .



# End of Chapter 9

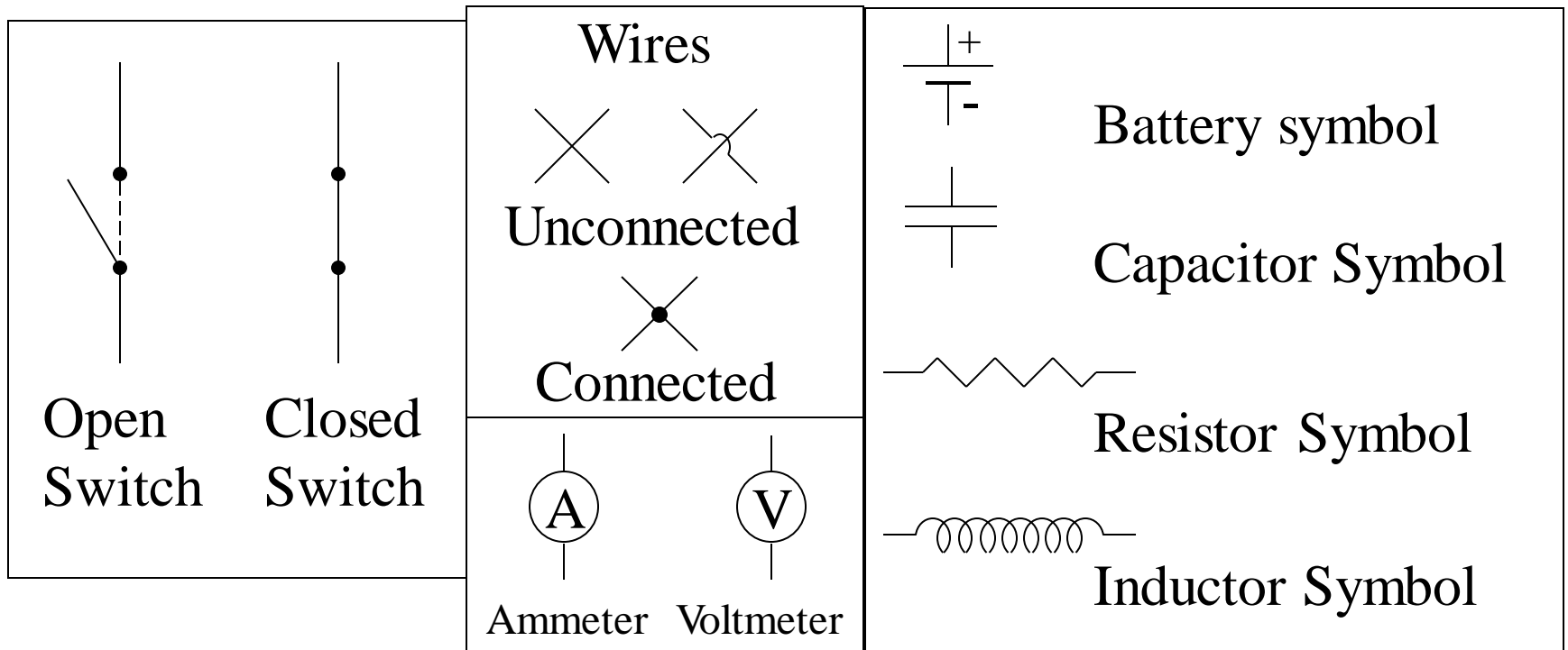
- Read Chapter 9 Summary -
- Complete Homework for Vol 2 Ch 9

# Backup slides



# Schematic Diagram Symbols

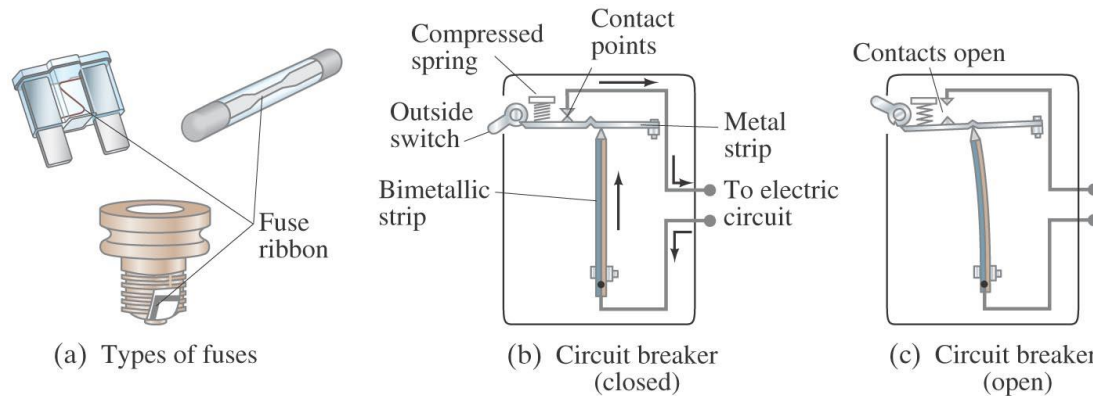
- Schematic Symbol list



# Household Electrical Power

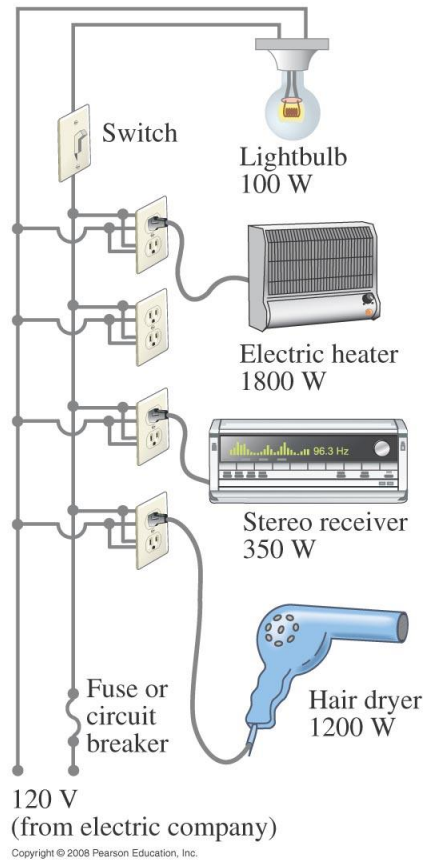
- We use a lot of electrical power in modern houses.
- That electrical power is dissipated as heat.
- To prevent fires, household and automobile electrical circuits are limited in the amount of power they can deliver.
- Some individual appliances and systems are also limited.

# Household Electrical Power



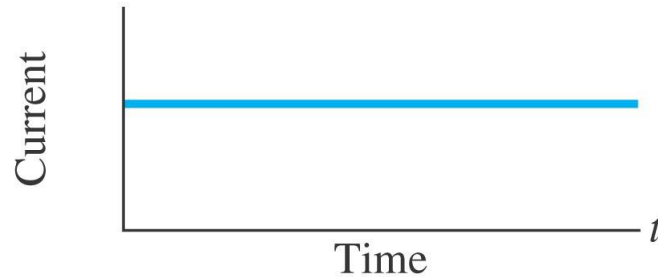
- Fuses (melting wires) and circuit breakers (thermal or magnetic) are used to limit the current and therefore power in a circuit.
- The typical household circuit breaker is 20A.

# Will the Circuit Breaker Trip

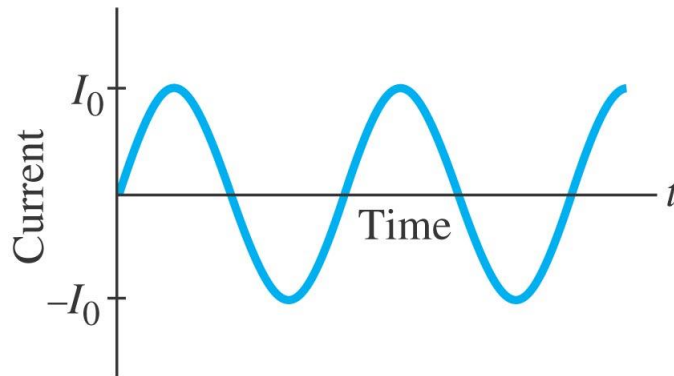


- If all of these items are plugged into one household circuit, will it work or trip the 20A circuit breaker?

# Alternating Current (AC)



(a) DC

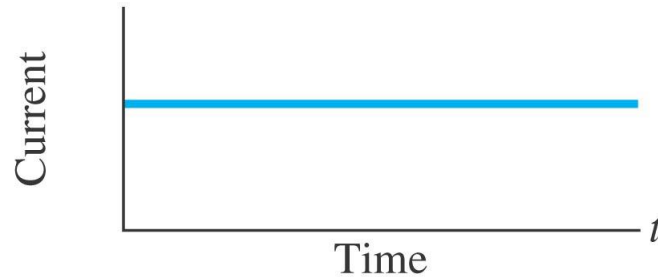


(b) AC

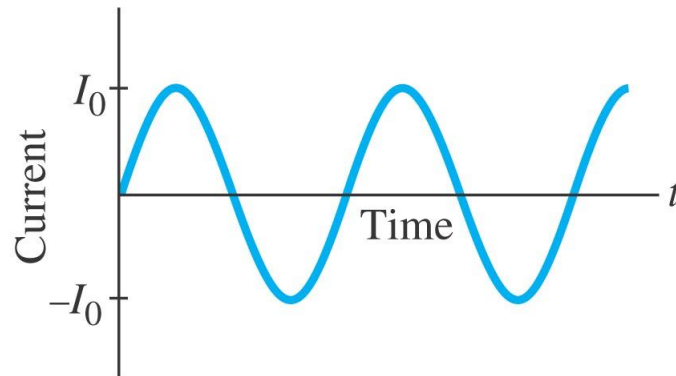
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- The current produced by a battery is basically constant and is referred to as direct current (DC).
- The electrical generators that provide household current produce alternating current or AC.

# Alternating Current (AC)



(a) DC

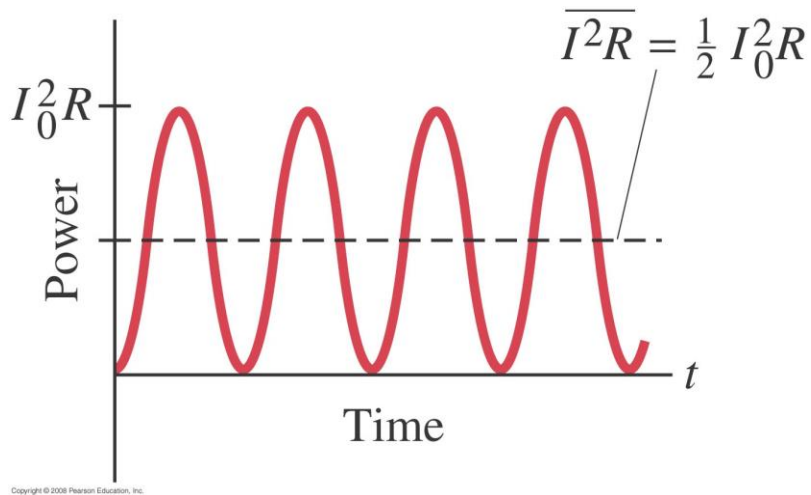


(b) AC

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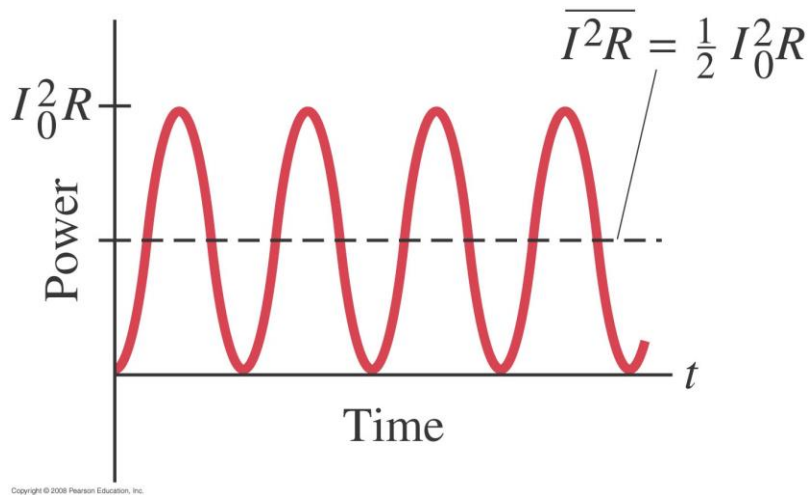
- The voltage and current are time varying
- $V = V_0 \sin(2\pi ft)$
- Where  $f = 60$  Hz in the US
- Current in a resistor will be
- $I = V/R = I_0 \sin(2\pi ft)$

# Alternating Current (AC)



- Power dissipation will be
- $P = I^2 R = I_0^2 R \sin^2(2\pi ft)$
- Note that this result is always positive and the average power dissipated over time
- $P_{\text{avg}} = \frac{1}{2} I_0^2 R = \frac{1}{2} V_0^2 / R$

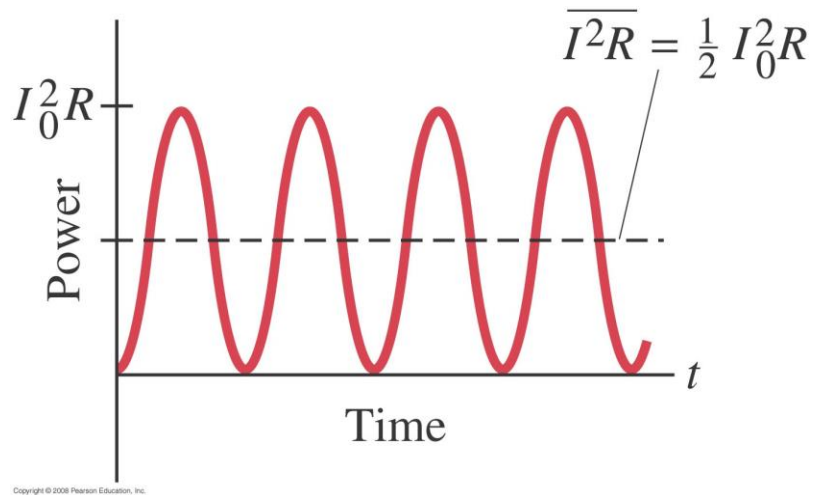
# Alternating Current (AC)



- $P_{\text{avg}} = \frac{1}{2} I_0^2 R = \frac{1}{2} V_0^2 / R$
- Since we averaged the square of the current, we refer to this power average as the RMS average.
- The voltages and power ratings of the appliances in your house are also RMS.



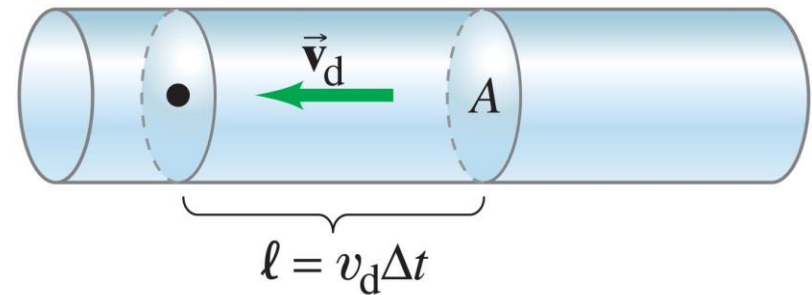
# Alternating Current (AC)



- $P_{\text{avg}} = \frac{1}{2} I_0^2 R = \frac{1}{2} V_0^2 / R$
- What is  $V_0$  for US household systems?
- $V_{\text{rms}} = \sqrt{\overline{V^2}} = \sqrt{1/2} V_0$
- $V_{\text{rms}}$  is 120 V, so the peak Voltage in the circuits in your house is 170V.

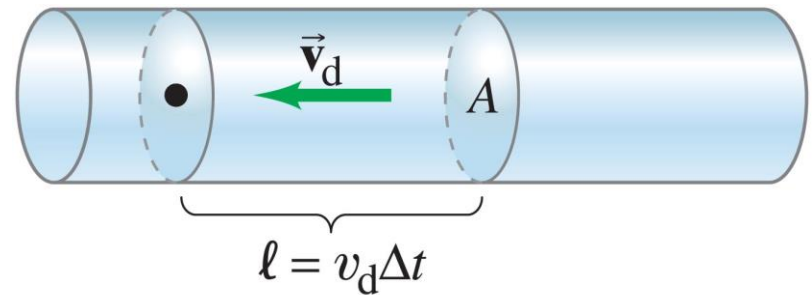
# Drift Velocity & Current Density

- The current through the wire is
- $I = dq/dt = -Ne/dt$
- Where  $N$  is the number of electrons which pass by.
- If the electron density is  $n = N/V$ , we can write
- $I = -neV/dt$



# Drift Velocity & Current Density

- $I = -neV/dt$
- From the diagram at right, the volume of electrons which drift past in time  $dt$  are
- $Vol = A\ell = A \cdot v_d \cdot dt$
- $I = -ne \cdot A \cdot v_d \cdot dt/dt$
- $I = -ne \cdot v_d \cdot A$



- 13.** (II) Calculate the ratio of the resistance of 10.0 m of aluminum wire 2.0 mm in diameter, to 20.0 m of copper wire 1.8 mm in diameter.

**13.** Use Eq. 25-3 to calculate the resistances, with the area as  $A = \pi r^2 = \pi d^2/4$ .

$$R = \rho \frac{\ell}{A} = \rho \frac{4\ell}{\pi d^2}.$$

$$\frac{R_{\text{Al}}}{R_{\text{Cu}}} = \frac{\rho_{\text{Al}} \frac{4\ell_{\text{Al}}}{\pi d_{\text{Al}}^2}}{\rho_{\text{Cu}} \frac{4\ell_{\text{Cu}}}{\pi d_{\text{Cu}}^2}} = \frac{\rho_{\text{Al}} \ell_{\text{Al}} d_{\text{Cu}}^2}{\rho_{\text{Cu}} \ell_{\text{Cu}} d_{\text{Al}}^2} = \frac{(2.65 \times 10^{-8} \Omega \cdot \text{m})(10.0 \text{ m})(1.8 \text{ mm})^2}{(1.68 \times 10^{-8} \Omega \cdot \text{m})(20.0 \text{ m})(2.0 \text{ mm})^2} = \boxed{0.64}$$

19. (II) A 100-W lightbulb has a resistance of about  $12\ \Omega$  when cold ( $20^\circ\text{C}$ ) and  $140\ \Omega$  when on (hot). Estimate the temperature of the filament when hot assuming an average temperature coefficient of resistivity  $\alpha = 0.0045\ (\text{C}^\circ)^{-1}$ .

19. Use Eq. 25-5 multiplied by  $\ell/A$  so that it expresses resistances instead of resistivity.

$$R = R_0 [1 + \alpha(T - T_0)] \rightarrow$$

$$T = T_0 + \frac{1}{\alpha} \left( \frac{R}{R_0} - 1 \right) = 20^\circ\text{C} + \frac{1}{0.0045(\text{C}^\circ)^{-1}} \left( \frac{140\ \Omega}{12\ \Omega} - 1 \right) = 2390^\circ\text{C} \approx \boxed{2400^\circ\text{C}}$$