Chapter Vol 2 Ch 6
Gauss's Law

Physics 2426
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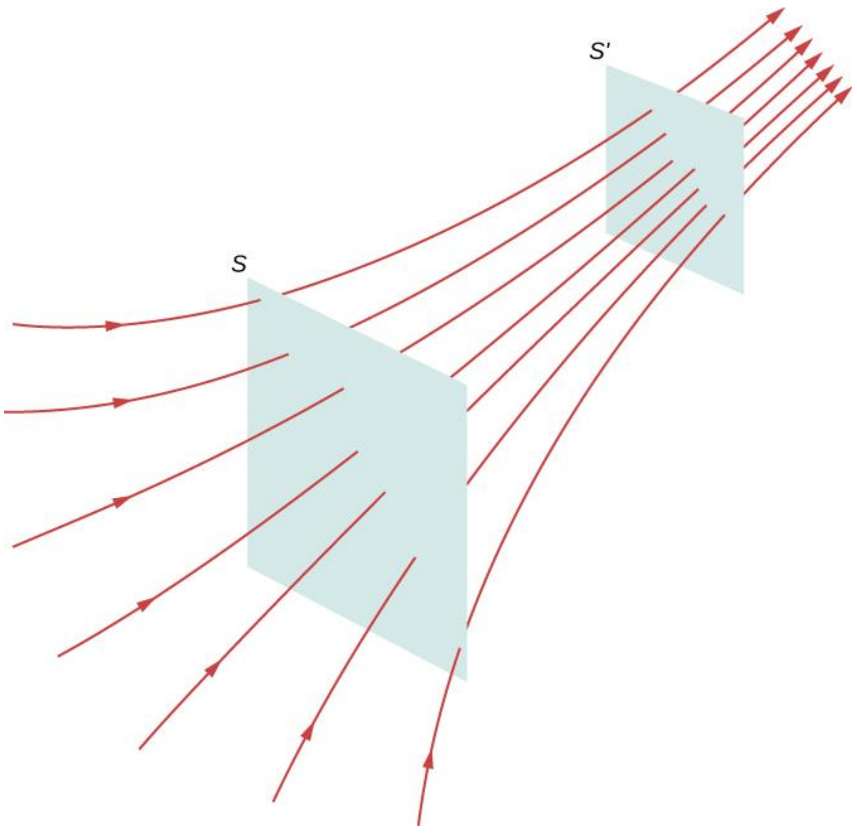
Gauss’s Law

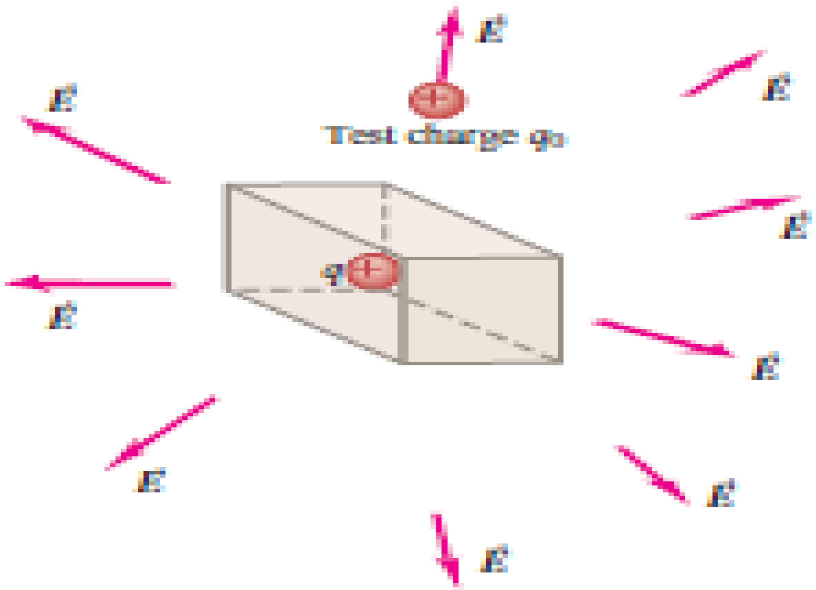
- Gauss’s Law provides and alternative method of calculating the field around a distribution of charges.
- It also provides additional insights into the relationship between electric fields and charges.
- As we will see later, it became the first of Maxwell’s equations which describe all electromagnetic phenomena.

Electric Flux

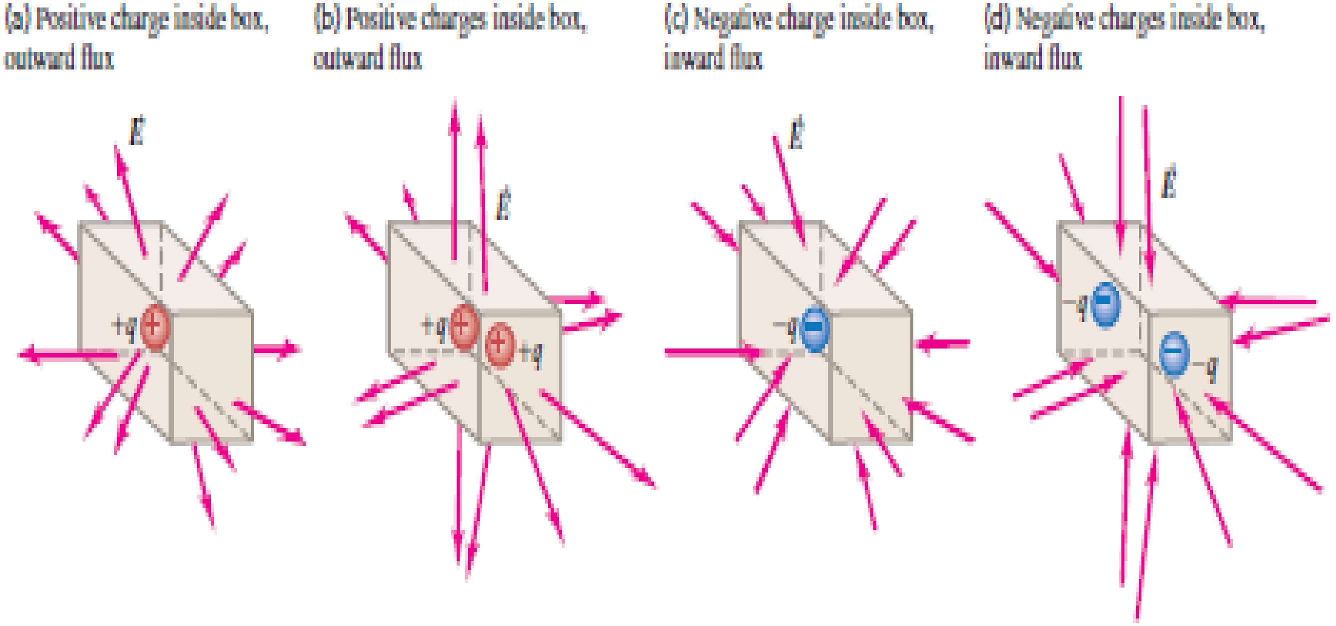
- To understand Gauss's Law we need to define the electric flux.
- Flux refers to flow and in the the case of electrical flux, it is a measure of the number of electric field lines passing through a given surface.



(b) Using a test charge outside the box to probe the amount of charge inside the box



22.2 The electric field on the surface of boxes containing (a) a single positive point charge, (b) two positive point charges, (c) a single negative point charge, or (d) two negative point charges.

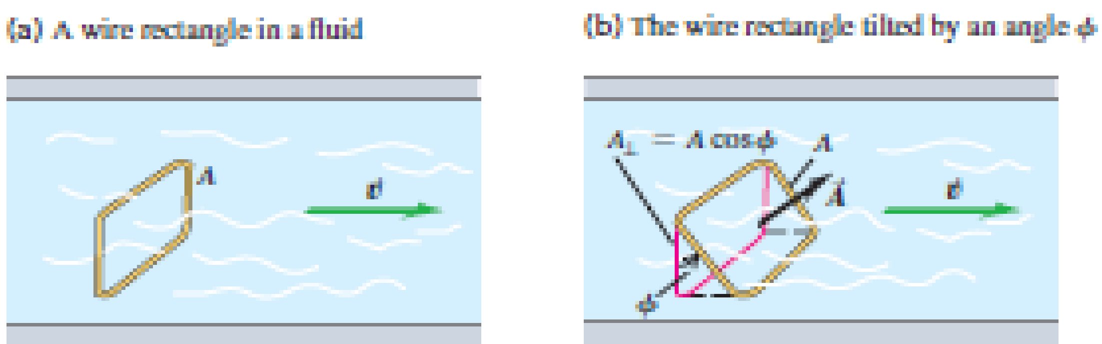


Flux: Fluid-Flow Analogy

Figure 22.5 shows a fluid flowing steadily from left to right. Let's examine the volume flow rate dV/dt (in, say, cubic meters per second) through the wire rectangle with area A. When the area is perpendicular to the flow velocity $\vec{v}$ (Fig. 22.5a) and the flow velocity is the same at all points in the fluid, the volume flow rate dV/dt is the area A multiplied by the flow speed v:

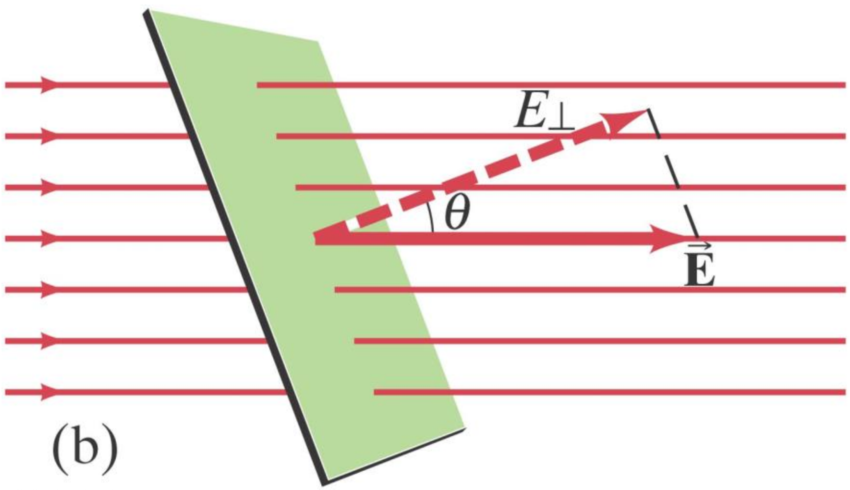
$$\frac{dV}{dt} = vA$$

When the rectangle is tilted at an angle ϕ (Fig. 22.5b) so that its face is not perpendicular to $\vec{v}$, the area that counts is the silhouette area that we see when we look in the direction of $\vec{v}$. This area, which is outlined in red and labeled $A_\perp$ in Fig. 22.5b, is the *projection* of the area A onto a surface perpendicular to $\vec{v}$. Two sides of the projected rectangle have the same length as the original one, but the



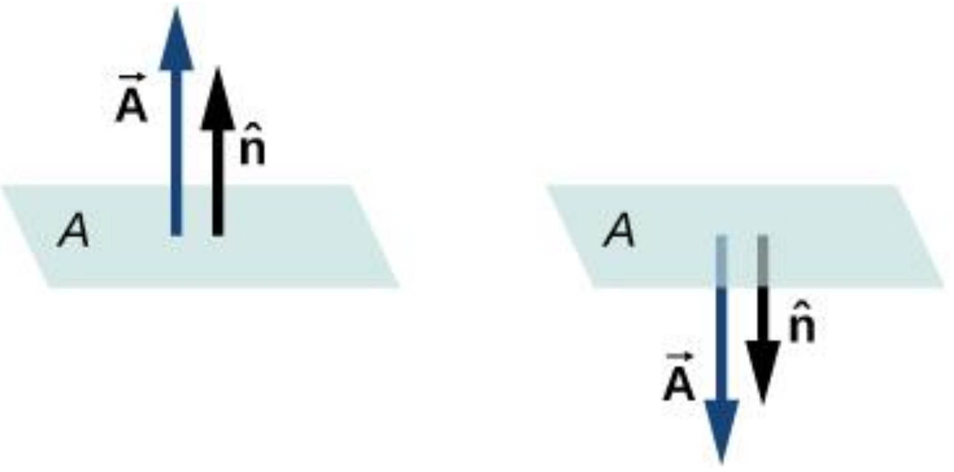
Electric Flux

- For a uniform electric field, E, passing through an area A, the value for the flux is
- $\Phi_E = E \cdot A \cdot \cos\theta$
- $E \cdot \cos\theta$ is the magnitude of the field perpendicular to the area.



Electric Flux

- The standard way of expressing this relationship is to define an area vector, $\vec{A}$, which has a magnitude of Area and a direction normal (perpendicular) to the surface of A

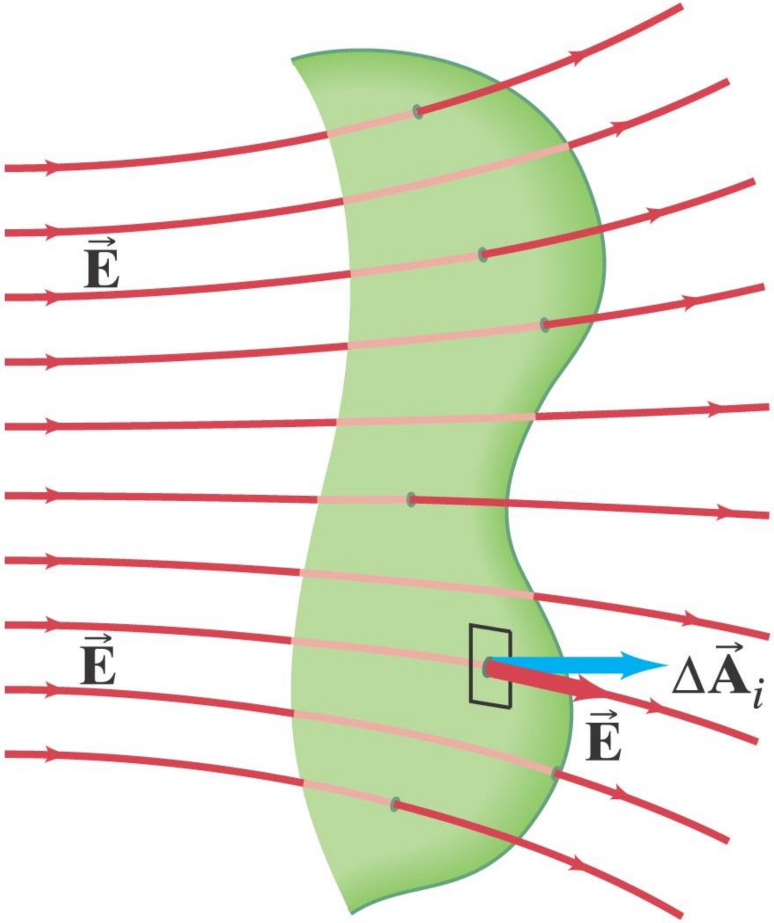


- Flux has the form:

$$\Phi_E = \vec{E} \bullet \vec{A}$$

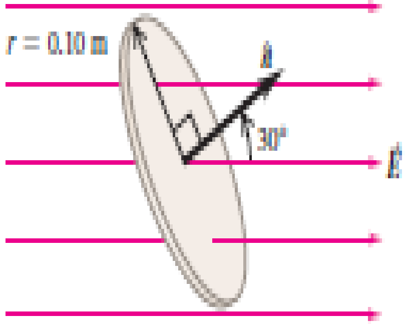
Electric Flux

- For a non-uniform field or an non-planar surface, we use a differential form and integrate over the surface.
- $d\Phi_E = \vec{E} \bullet d\vec{A}$



A disk of radius 0.10 m is oriented with its normal unit vector $\hat{n}$ at 30° to a uniform electric field $\vec{E}$ of magnitude 2.0×10^3 N/C (**Fig. 22.7**). (Since this isn't a closed surface, it has no "inside" or "outside." That's why we have to specify the direction of $\hat{n}$ in the figure.) (a) What is the electric flux through the disk? (b) What is the flux through the disk if it is turned so that $\hat{n}$ is perpendicular to $\vec{E}$? (c) What is the flux through the disk if $\hat{n}$ is parallel to $\vec{E}$?

22.7 The electric flux Φ_E through a disk depends on the angle between its normal $\hat{n}$ and the electric field $\vec{E}$.



SOLUTION

IDENTIFY and SET UP: This problem is about a flat surface in a uniform electric field, so we can apply the ideas of this section. We calculate the electric flux from Eq. (22.1).

EXECUTE: (a) The area is $A = \pi(0.10\text{ m})^2 = 0.0314\text{ m}^2$ and the angle between $\vec{E}$ and $\vec{A} = A\hat{n}$ is $\phi = 30°$, so from Eq. (22.1),

$$\Phi_E = EA\cos\phi = (2.0 \times 10^3\text{ N/C})(0.0314\text{ m}^2)(\cos 30°)$$
$$= 54\text{ N}\cdot\text{m}^2/\text{C}$$

(b) The normal to the disk is now perpendicular to $\vec{E}$, so $\phi = 90°$, $\cos\phi = 0$, and $\Phi_E = 0$.

(c) The normal to the disk is parallel to $\vec{E}$, so $\phi = 0$ and $\cos\phi = 1$:

$$\Phi_E = EA\cos\phi = (2.0 \times 10^3\text{ N/C})(0.0314\text{ m}^2)(1)$$
$$= 63\text{ N}\cdot\text{m}^2/\text{C}$$

EVALUATE: Our answer to part (b) is smaller than that to part (a), which is in turn smaller than that to part (c). Is all this as it should be?

EXAMPLE 22.2 ELECTRIC FLUX THROUGH A CUBE

An imaginary cubical surface of side L is in a region of uniform electric field $\vec{E}$. Find the electric flux through each face of the cube and the total flux through the cube when (a) it is oriented with two of its faces perpendicular to $\vec{E}$ (**Fig. 22.8a**) and (b) the cube is turned by an angle θ about a vertical axis (Fig. 22.8b).

SOLUTION

IDENTIFY and SET UP: Since $\vec{E}$ is uniform and each of the six faces of the cube is flat, we find the flux Φ_{Ei} through each face from Eqs. (22.3) and (22.4). The total flux through the cube is the sum of the six individual fluxes.

EXECUTE: (a) Figure 22.8a shows the unit vectors $\hat{n}_1$ through $\hat{n}_6$ for each face; each unit vector points *outward* from the cube's closed surface. The angle between $\vec{E}$ and $\hat{n}_1$ is 180°, the angle between $\vec{E}$ and $\hat{n}_2$ is 0°, and the angle between $\vec{E}$ and each of the other four unit vectors is 90°. Each face of the cube has area L^2, so the fluxes through the faces are

$$\Phi_{E1} = \vec{E} \cdot \hat{n}_1 A = EL^2 \cos 180° = -EL^2$$
$$\Phi_{E2} = \vec{E} \cdot \hat{n}_2 A = EL^2 \cos 0° = +EL^2$$
$$\Phi_{E3} = \Phi_{E4} = \Phi_{E5} = \Phi_{E6} = EL^2 \cos 90° = 0$$

The flux is negative on face 1, where $\vec{E}$ is directed into the cube, and positive on face 2, where $\vec{E}$ is directed out of the cube. The total flux through the cube is

$$\Phi_E = \Phi_{E1} + \Phi_{E2} + \Phi_{E3} + \Phi_{E4} + \Phi_{E5} + \Phi_{E6}$$
$$= -EL^2 + EL^2 + 0 + 0 + 0 + 0 = 0$$

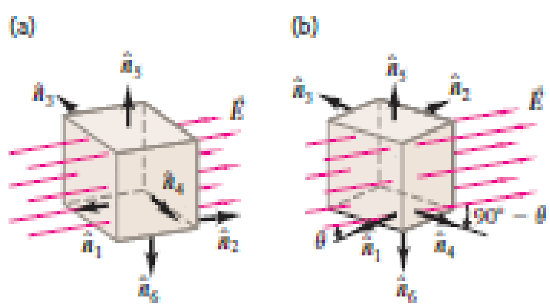
(b) The field $\vec{E}$ is directed into faces 1 and 3, so the fluxes through them are negative; $\vec{E}$ is directed out of faces 2 and 4, so the fluxes through them are positive. We find

$$\Phi_{E1} = \vec{E} \cdot \hat{n}_1 A = EL^2 \cos(180° - \theta) = -EL^2 \cos\theta$$
$$\Phi_{E2} = \vec{E} \cdot \hat{n}_2 A = +EL^2 \cos\theta$$
$$\Phi_{E3} = \vec{E} \cdot \hat{n}_3 A = EL^2 \cos(90° + \theta) = -EL^2 \sin\theta$$
$$\Phi_{E4} = \vec{E} \cdot \hat{n}_4 A = EL^2 \cos(90° - \theta) = +EL^2 \sin\theta$$
$$\Phi_{E5} = \Phi_{E6} = EL^2 \cos 90° = 0$$

The total flux $\Phi_E = \Phi_{E1} + \Phi_{E2} + \Phi_{E3} + \Phi_{E4} + \Phi_{E5} + \Phi_{E6}$ through the surface of the cube is again zero.

EVALUATE: We came to the same conclusion in our discussion of Fig. 22.3c: There is zero net flux of a uniform electric field through a closed surface that contains no electric charge.

22.8 Electric flux of a uniform field $\vec{E}$ through a cubical box of side L in two orientations.



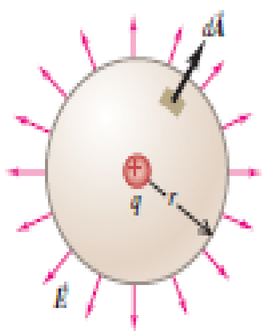
SOLUTION

A point charge $q = +3.0\ \mu C$ is surrounded by an imaginary sphere of radius $r = 0.20$ m centered on the charge (**Fig. 22.9**). Find the resulting electric flux through the sphere.

SOLUTION

IDENTIFY and SET UP: The surface is not flat and the electric field is not uniform, so to calculate the electric flux (our target variable) we must use the general definition, Eq. (22.5). Because the sphere is centered on the point charge, at any point on the spherical surface, $\vec{E}$ is directed out of the sphere perpendicular to the surface. The positive direction for both $\hat{n}$ and $E_\perp$ is outward, so $E_\perp = E$

22.9 Electric flux through a sphere centered on a point charge.



and the flux through a surface element dA is $\vec{E} \cdot d\vec{A} = E\,dA$. This greatly simplifies the integral in Eq. (22.5).

EXECUTE: We must evaluate the integral of Eq. (22.5), $\Phi_E = \int E\,dA$. At any point on the sphere of radius r the electric field has the same magnitude $E = q/4\pi\epsilon_0 r^2$. Hence E can be taken outside the integral, which becomes $\Phi_E = E\int dA = EA$, where A is the area of the spherical surface: $A = 4\pi r^2$. Hence the total flux through the sphere is

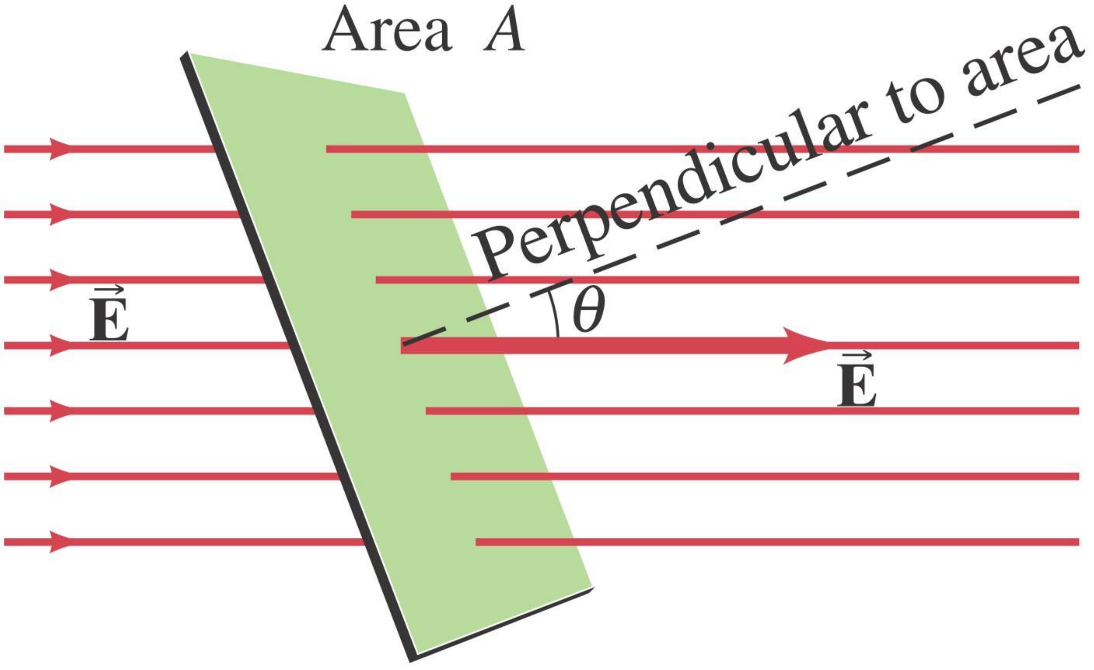
$$\Phi_E = EA = \frac{q}{4\pi\epsilon_0 r^2}4\pi r^2 = \frac{q}{\epsilon_0}$$

$$= \frac{3.0 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 3.4 \times 10^5\ \text{N}\cdot\text{m}^2/\text{C}$$

EVALUATE: The radius r of the sphere cancels out of the result for Φ_E. We would have obtained the same flux with a sphere of radius 2.0 m or 200 m. We came to essentially the same conclusion in our discussion of Fig. 22.4 in Section 22.1, where we considered rectangular closed surfaces of two different sizes enclosing a point charge. There we found that the flux of $\vec{E}$ was independent of the size of the surface; the same result holds true for a spherical surface. Indeed, the flux through *any* surface enclosing a single point charge is independent of the shape or size of the surface, as we'll soon see.

Example 22-1: Electric flux.

Calculate the electric flux through the rectangle shown. The rectangle is 10 cm by 20 cm, the electric field is uniform at 200 N/C, and the angle θ is 30°.



EXAMPLE 22–1 **Electric flux.** Calculate the electric flux through the rectangle shown in Fig. 22–1a. The rectangle is 10 cm by 20 cm, the electric field is uniform at 200 N/C, and the angle θ is 30°.

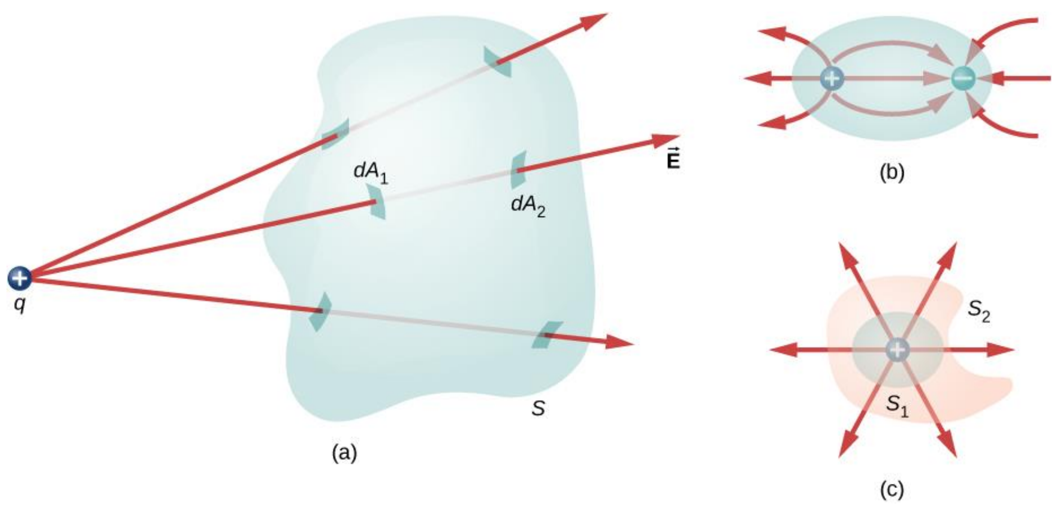
APPROACH We use the definition of flux, $\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA\cos\theta$.

SOLUTION The electric flux is

$$\Phi_E = (200\ \mathrm{N/C})(0.10\ \mathrm{m} \times 0.20\ \mathrm{m})\cos 30^\circ = 3.5\ \mathrm{N \cdot m^2/C}.$$

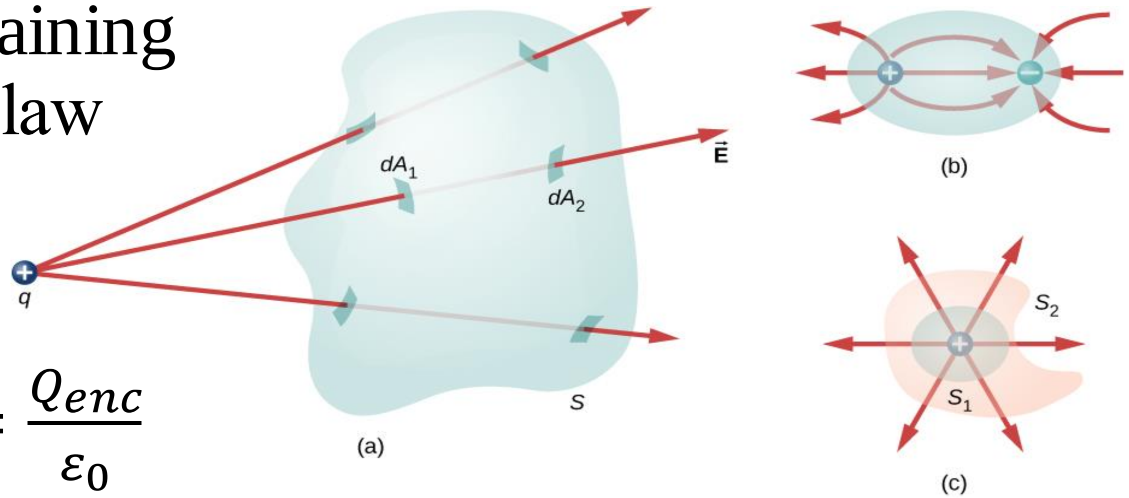
Gauss's Law

- Gauss's Law sets the value of the flux integral when it is carried out over a closed surface.
- $\Phi_E = \oiint \vec{E} \bullet d\vec{A}$
- This flux is proportional to the charge enclosed by the surface.



Gauss's Law

- For a surface containing a charge, Gauss's law has the form:



- $\Phi_E = \oint\!\!\!\!\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

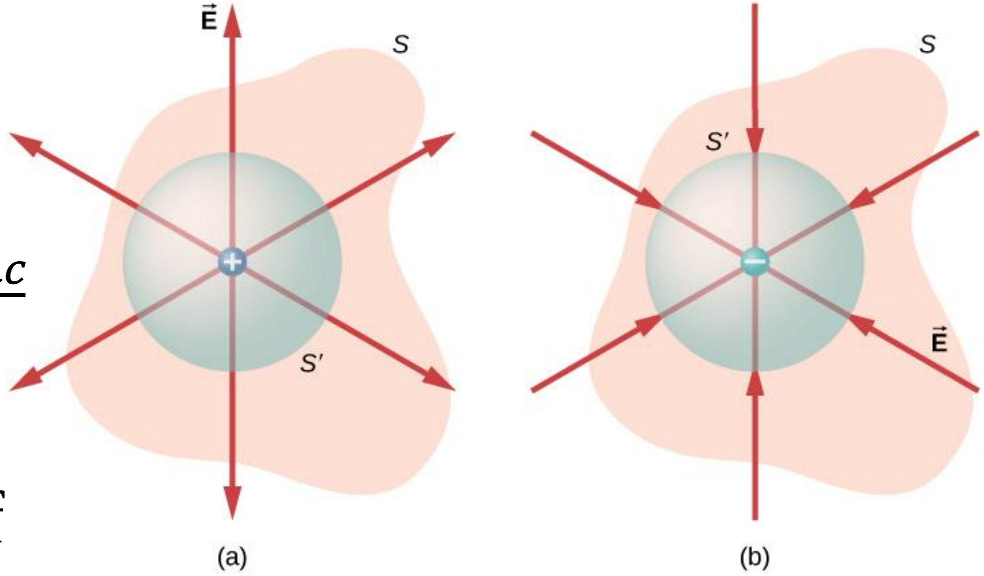
- And is independent of the surface chosen.

Gauss's Law

- For a surface containing a charge, Gauss's law has the form:

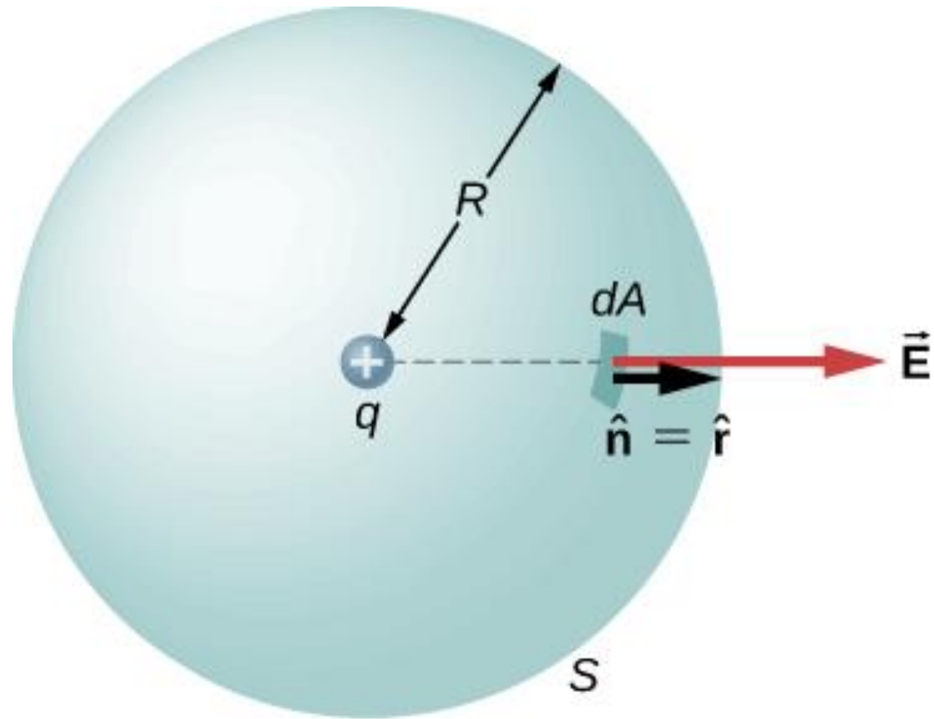
- $\Phi_E = \oiint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

- And is independent of the surface chosen.

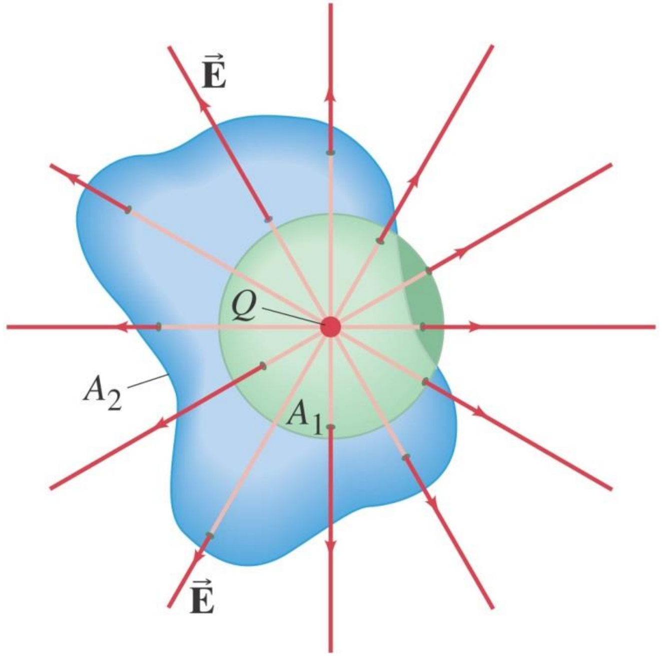


Gauss's Law

- The relationship between Coulomb's Law and Gauss's Law can be shown by placing a point charge, *Q*, at the center of a spherical surface of radius, r.
- $\oiint \vec{E} \bullet d\vec{A} = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$
- Which gives,
- $E = Q/(4\pi\varepsilon_0 r^2)$



Using Coulomb's law to evaluate the integral of the field of a point charge over the surface of a sphere surrounding the charge gives:



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,dA = \frac{Q}{4\pi\epsilon_0 r^2}\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}.$$

Looking at the arbitrarily shaped surface A_2, we see that the same flux passes through it as passes through A_1. Therefore, this result should be valid for any closed surface.

22-2 Gauss's Law

Finally, if a gaussian surface encloses several point charges, the superposition principle shows that:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \left(\Sigma \vec{\mathbf{E}}_i\right) \cdot d\vec{\mathbf{A}} = \Sigma \frac{Q_i}{\epsilon_0} = \frac{Q_{\text{encl}}}{\epsilon_0}.$$

Therefore, Gauss's law is valid for any charge distribution. Note, however, that it only refers to the field due to charges within the gaussian surface – charges outside the surface will also create fields.

Magnitude of electric field $\vec{E}$

Component of $\vec{E}$ perpendicular to surface

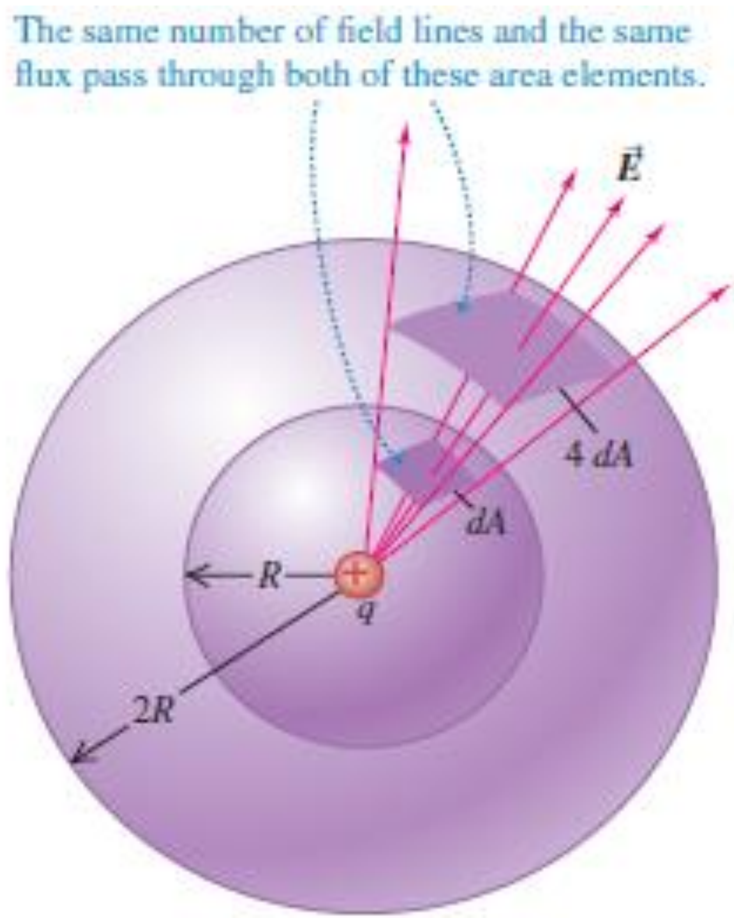
Electric flux through a surface

$$\Phi_E = \int E\cos\phi\, dA = \int E_{\perp}\, dA = \int \vec{E}\cdot d\vec{A}$$

Angle between $\vec{E}$ and normal to surface

Element of surface area

Vector element of surface area


The same number of field lines and the same flux pass through both of these area elements.
E
4 dA
dA
R
q
2R

ONCEPTUAL EXAMPLE 22.4 ELECTRIC FLUX AND ENCLOSED CHARGE

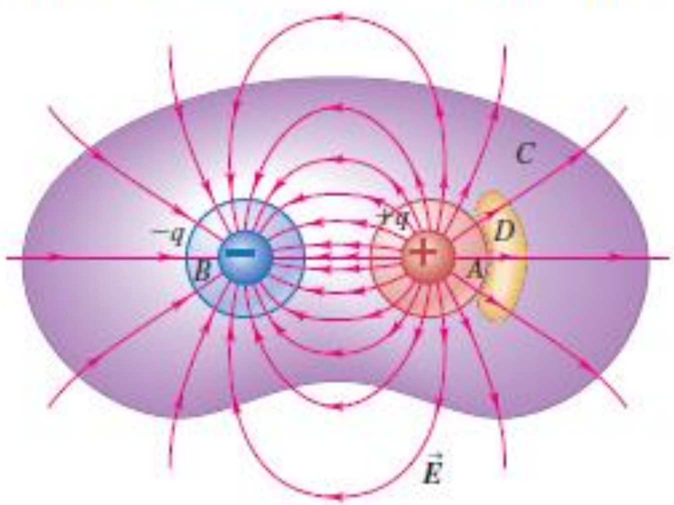
OLUTION

gure 22.15 shows the field produced by two point charges $+q$ d $-q$ (an electric dipole). Find the electric flux through each of closed surfaces A, B, C, and D.

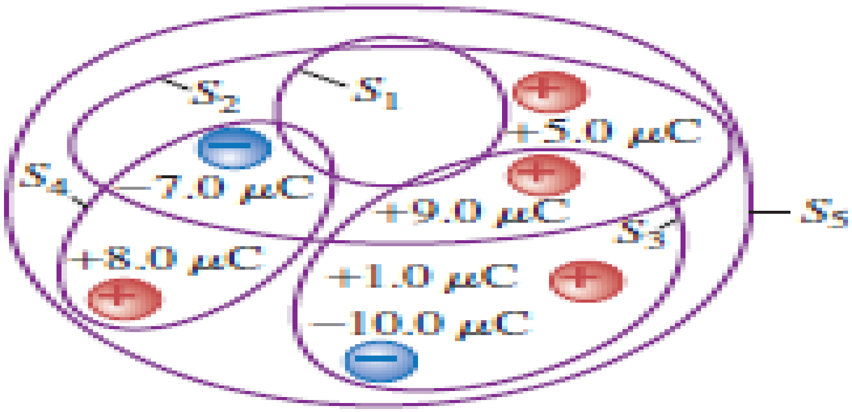
OLUTION

uss's law, Eq. (22.8), says that the total electric flux through a sed surface is equal to the total enclosed charge divided by ϵ_0. In Fig. 22.15, surface A (shown in red) encloses the positive charge, so $Q_{encl} = +q$; surface B (in blue) encloses the negative charge, so $Q_{encl} = -q$; surface C (in purple) encloses *both* charges, so $Q_{encl} = +q + (-q) = 0$; and surface D (in yellow) encloses no charges, so $Q_{encl} = 0$. Hence, without having to do any integration, we have $\Phi_{EA} = +q/\epsilon_0$, $\Phi_{EB} = -q/\epsilon_0$, and $\Phi_{EC} = \Phi_{ED} = 0$. These results depend only on the charges enclosed within each Gaussian surface, not on the precise shapes of the surfaces.

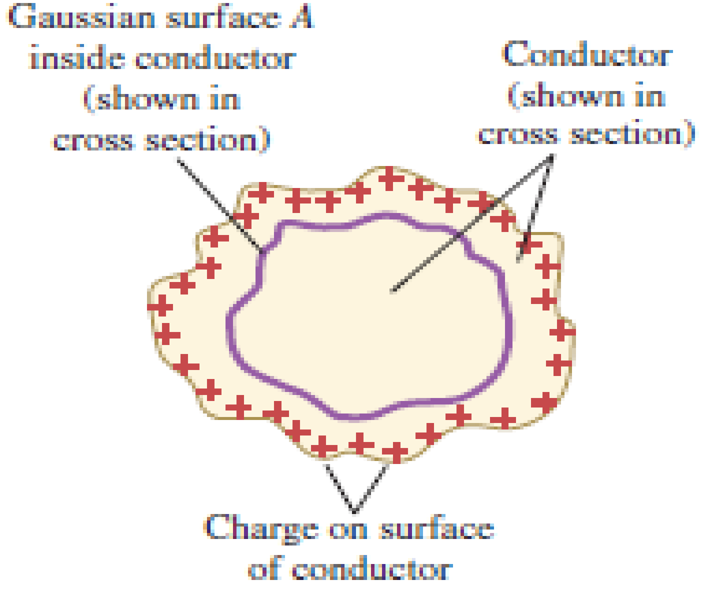
22.15 The net number of field lines leaving a closed surface is proportional to the total charge enclosed by that surface.



22.16 Five Gaussian surfaces and six point charges.

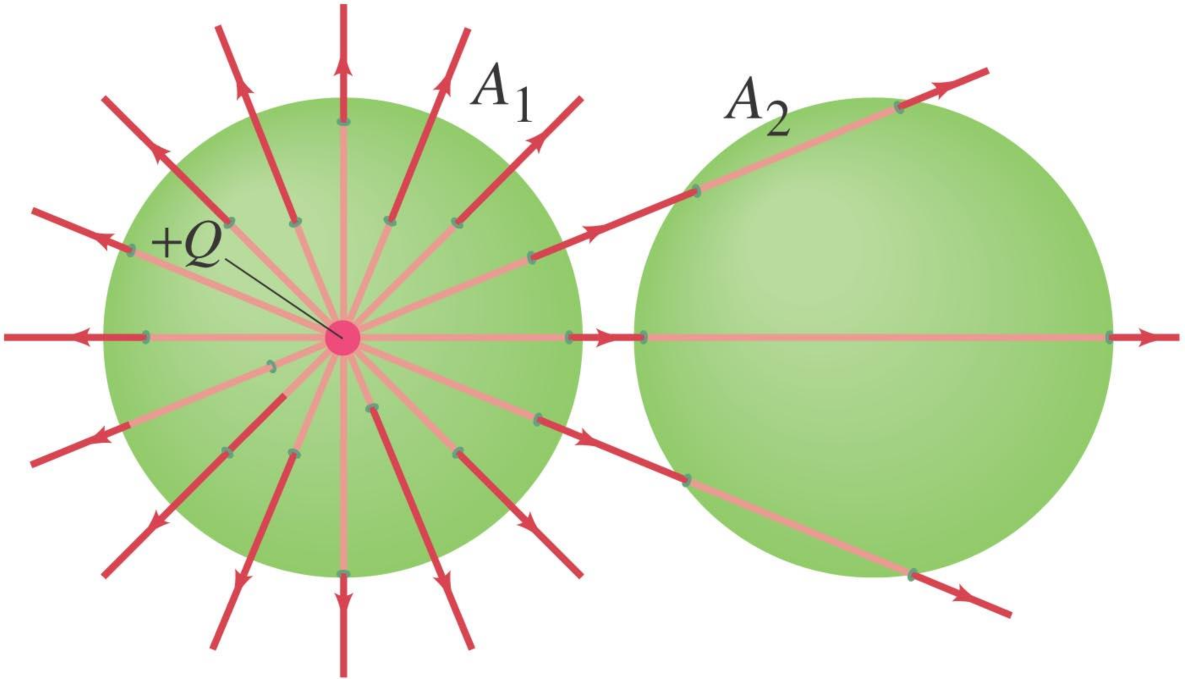


22.17 Under electrostatic conditions (charges not in motion), any excess charge on a solid conductor resides entirely on the conductor's surface.



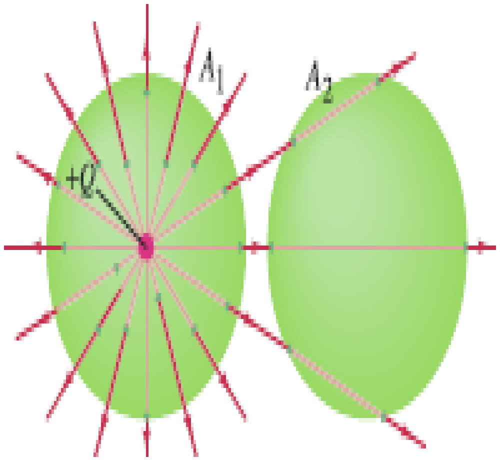
Conceptual Example 22-2: Flux from Gauss's law.

Consider the two gaussian surfaces, A_1 and A_2, as shown. The only charge present is the charge Q at the center of surface A_1. What is the net flux through each surface, A_1 and A_2?



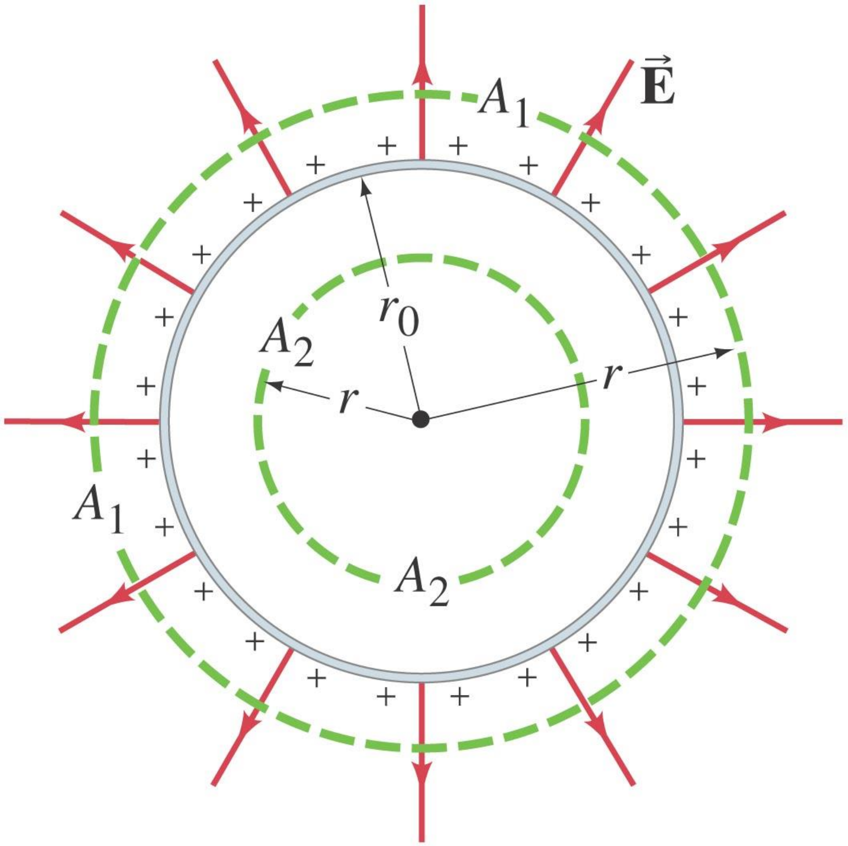
CONCEPTUAL EXAMPLE 22–2 **Flux from Gauss's law.** Consider the two gaussian surfaces, A_1 and A_2, shown in Fig. 22–10. The only charge present is the charge Q at the center of surface A_1. What is the net flux through each surface, A_1 and A_2?

RESPONSE The surface A_1 encloses the charge $+Q$. By Gauss's law, the net flux through A_1 is then Q/ϵ_0. For surface A_2, the charge $+Q$ is outside the surface. Surface A_2 encloses zero net charge, so the net electric flux through A_2 is zero, by Gauss's law. Note that all field lines that enter the volume enclosed by surface A_2 also leave it.



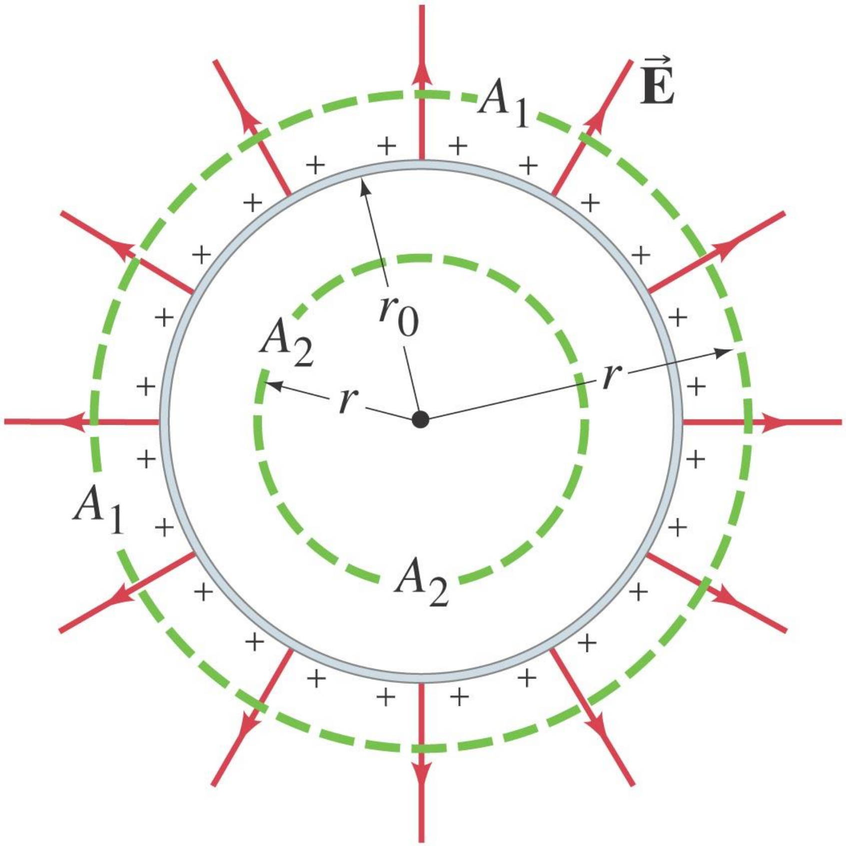
Applications of Gauss's Law

- Gauss's Law allows for simple solutions for many situations.
- We begin with a uniform spherical shell of charge with radius r_0.
- We choose our surface to be a concentric sphere, $A = 4\pi r^2$
- For $r > r_0$, $4\pi r^2 \cdot E = Q/\varepsilon_0$
- Or $E = Q/(4\pi\varepsilon_0 r^2)$



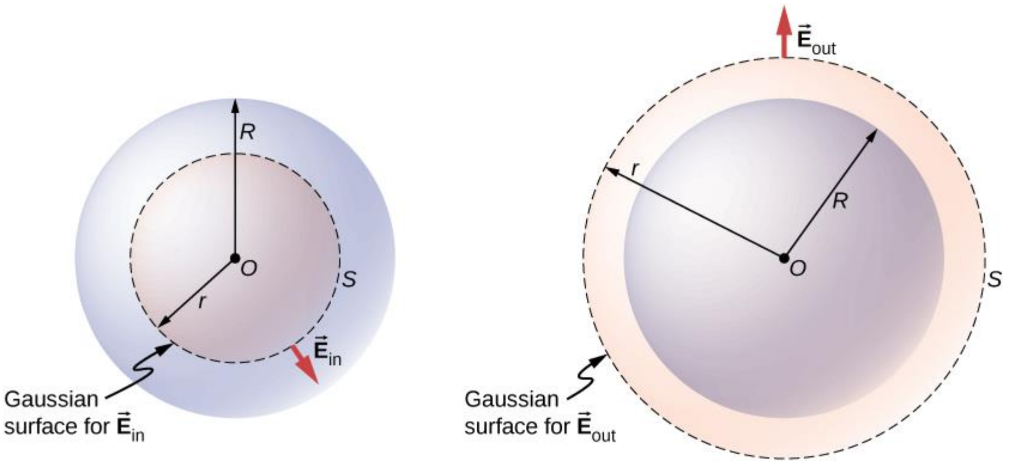
Applications of Gauss's Law

- For $r < r_0$, $4\pi r^2 \cdot E = 0$
- Or $E = 0$
- This agrees with our earlier discussion for solid or hollow conductors, since there the charge will lie on the outer surface.



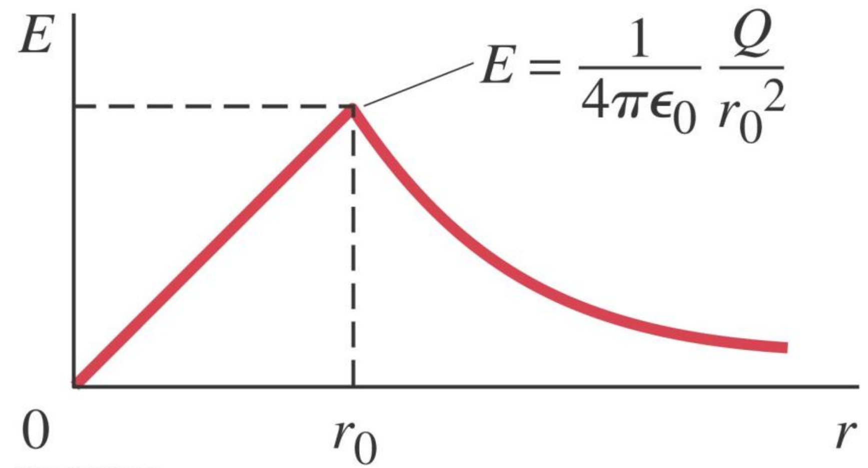
Applications of Gauss's Law

- We can now use our pervious result for a uniform sphere of charge.
- For $r > r_0$
- $E = Q/(4\pi\varepsilon_0 r^2)$
- Which is the same result as a point charge, *Q*.
- At this point we can say that any spherically symmetric charge distribution will give this same result.



Applications of Gauss's Law

- Inside the sphere?
- For $r < r_0$
- The enclosed charge decreases as the volume of our sphere,
- $Q_{enc} = Q(r^3/r_0^3)$
- And $E = Q \cdot r/(4\pi\varepsilon_0 r_0^3)$
- So our field decreases with r.



A thin-walled, hollow sphere of radius 0.250 m has an unknown charge distributed uniformly over its surface. At a distance of 0.300 m from the center of the sphere, the electric field points radially inward and has magnitude 1.80×10^2 N/C. How much charge is on the sphere?

SOLUTION

IDENTIFY and SET UP: The charge distribution is spherically symmetric. As in Examples 22.5 and 22.9, it follows that the electric field is radial everywhere and its magnitude is a function of only the radial distance r from the center of the sphere. We use a spherical Gaussian surface that is concentric with the charge distribution and has radius $r = 0.300$ m. Our target variable is $Q_{encl} = q$.

EXECUTE: The charge distribution is the same as if the charge were on the surface of a 0.250-m-radius conducting sphere. Hence we can borrow the results of Example 22.5. We note that the electric field here is directed toward the sphere, so that q must be *negative*. Furthermore, the electric field is directed into the Gaussian surface, so that $E_\perp = -E$ and $\Phi_E = \oint E_\perp \, dA = -E(4\pi r^2)$.

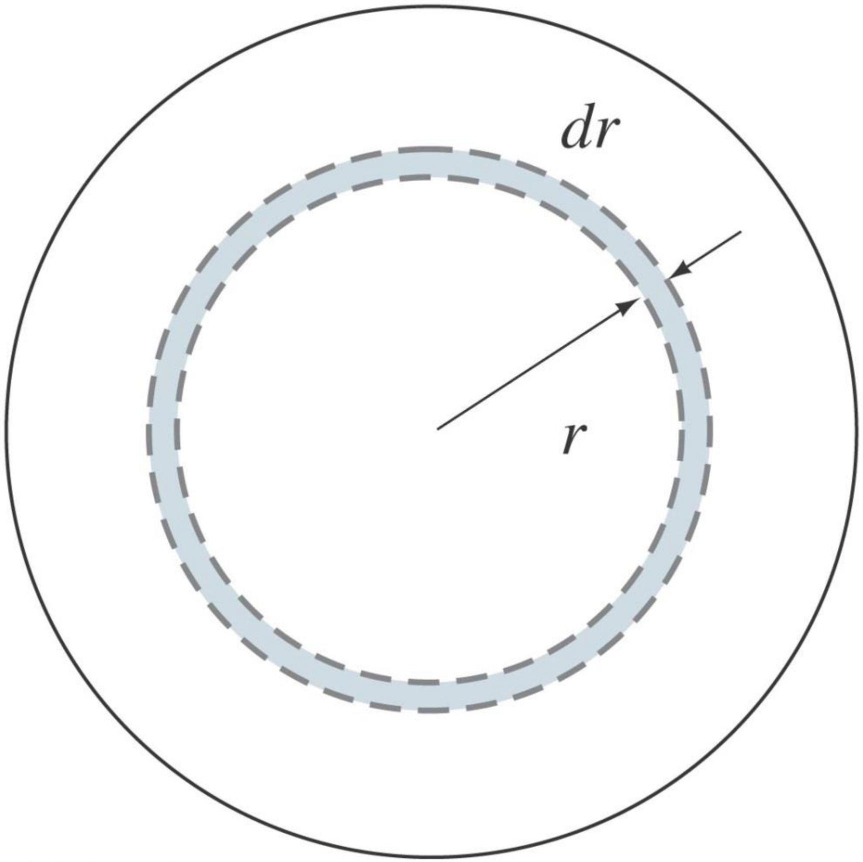
By Gauss's law, the flux is equal to the charge q on the sphere (all of which is enclosed by the Gaussian surface) divided by ϵ_0. Solving for q, we find

$$\begin{aligned} q = -E(4\pi\epsilon_0 r^2) &= -(1.80 \times 10^2 \text{ N/C})(4\pi) \\ &\quad \times (8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(0.300 \text{ m})^2 \\ &= -1.80 \times 10^{-9} \text{ C} = -1.80 \text{ nC} \end{aligned}$$

EVALUATE: To determine the charge, we had to know the electric field at *all* points on the Gaussian surface so that we could calculate the flux integral. This was possible here because the charge distribution is highly symmetric. If the charge distribution is irregular or lacks symmetry, Gauss's law is not very useful for calculating the charge distribution from the field, or vice versa.

Example 22-5: Nonuniformly charged solid sphere.

Suppose the charge density of a solid sphere is given by $\rho_E = \alpha r^2$, where α is a constant. (a) Find α in terms of the total charge Q on the sphere and its radius r_0. (b) Find the electric field as a function of r inside the sphere.



EXAMPLE 22–5 **Nonuniformly charged solid sphere.** Suppose the charge density of the solid sphere in Fig. 22–12, Example 22–4, is given by $\rho_E = \alpha r^2$, where α is a constant. (*a*) Find α in terms of the total charge Q on the sphere and its radius r_0. (*b*) Find the electric field as a function of r inside the sphere.

APPROACH We divide the sphere up into concentric thin shells of thickness dr as shown in Fig. 22–14, and integrate (*a*) setting $Q = \int \rho_E \, dV$ and (*b*) using Gauss's law.

SOLUTION (*a*) A thin shell of radius r and thickness dr (Fig. 22–14) has volume $dV = 4\pi r^2 \, dr$. The total charge is given by

$$Q = \int \rho_E \, dV = \int_0^{r_0} (\alpha r^2)(4\pi r^2 \, dr) = 4\pi\alpha \int_0^{r_0} r^4 \, dr = \frac{4\pi\alpha}{5} r_0^5.$$

Thus $\alpha = 5Q/4\pi r_0^5$.

(*b*) To find E inside the sphere at distance r from its center, we apply Gauss's law to an imaginary sphere of radius r which will enclose a charge

$$Q_{\text{encl}} = \int_0^r \rho_E \, dV = \int_0^r (\alpha r^2) \, 4\pi r^2 \, dr = \int_0^r \left(\frac{5Q}{4\pi r_0^5} r^2 \right) 4\pi r^2 \, dr = Q \frac{r^5}{r_0^5}.$$

By symmetry, E will be the same at all points on the surface of a sphere of radius r, so Gauss's law gives

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

$$(E)(4\pi r^2) = Q \frac{r^5}{\epsilon_0 r_0^5},$$

so

$$E = \frac{Qr^3}{4\pi\epsilon_0 r_0^5}.$$

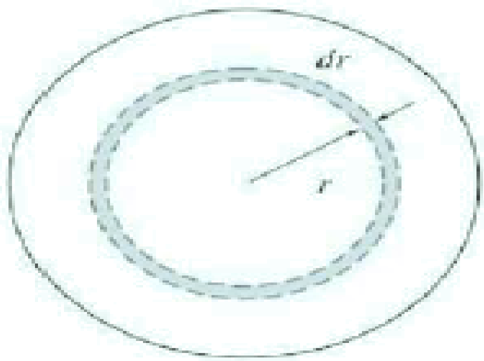


FIGURE 22–14 Example 22–5.

22.53 •• CALC A nonuniform, but spherically symmetric, distribution of charge has a charge density $\rho(r)$ given as follows:

$$\rho(r) = \rho_0\left(1 - \frac{r}{R}\right) \qquad \text{for } r \le R$$

$$\rho(r) = 0 \qquad \text{for } r \ge R$$

where $\rho_0 = 3Q/\pi R^3$ is a positive constant. (a) Show that the total charge contained in the charge distribution is Q. (b) Show that the electric field in the region $r \ge R$ is identical to that produced by a point charge Q at $r = 0$. (c) Obtain an expression for the electric field in the region $r \le R$. (d) Graph the electric-field magnitude E as a function of r. (e) Find the value of r at which the electric field is maximum, and find the value of that maximum field.

(a) **IDENTIFY:** The charge density varies with r inside the spherical volume. Divide the volume up into thin concentric shells, of radius r and thickness dr. Find the charge dq in each shell and integrate to find the total charge.

SET UP: $\rho(r) = \rho_0(1 - r/R)$ for $r \le R$ where $\rho_0 = 3Q/\pi R^3$. $\rho(r) = 0$ for $r \ge R$. The thin shell is sketched in Figure 22.53a.

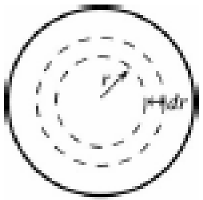


Figure 22.53a

EXECUTE: The volume of such a shell is $dV = 4\pi r^2 dr$.
The charge contained within the shell is
$dq = \rho(r)dV = 4\pi r^2 \rho_0(1 - r/R)dr$.

The total charge Q_{tot} in the charge distribution is obtained by integrating dq over all such shells into which the sphere can be subdivided:

$$Q_{\text{tot}} = \int dq = \int_0^R 4\pi r^2 \rho_0(1 - r/R)dr = 4\pi\rho_0 \int_0^R (r^2 - r^3/R)dr$$

$$Q_{\text{tot}} = 4\pi\rho_0 \left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^R = 4\pi\rho_0\left(\frac{R^3}{3} - \frac{R^4}{4R}\right) = 4\pi\rho_0(R^3/12) = 4\pi(3Q/\pi R^3)(R^3/12) = Q,$$ as was to be shown.

(b) **IDENTIFY:** Apply Gauss's law to a spherical surface of radius r, where $r > R$.

Figure 22.53b

$E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$; same as for point charge of charge Q.

(c) **Identify:** Apply Gauss's law to a spherical surface of radius r, where $r < R$.
Set Up: The Gaussian surface is shown in Figure 22.53c.

Execute: $\Phi_E = \dfrac{Q_{\text{encl}}}{\varepsilon_0}$.

$\Phi_E = E(4\pi r^2)$.

Figure 22.53c

To calculate the enclosed charge Q_{encl} use the same technique as in part (a), except integrate dq out to r rather than R. (We want the charge that is inside radius r.)

$$Q_{\text{encl}} = \int_0^r 4\pi r'^2 \rho_0 \left(1 - \frac{r'}{R}\right) dr' = 4\pi\rho_0 \int_0^r \left(r'^2 - \frac{r'^3}{R}\right) dr'.$$

$$Q_{\text{encl}} = 4\pi\rho_0 \left[\frac{r'^3}{3} - \frac{r'^4}{4R}\right]_0^r = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) = 4\pi\rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R}\right).$$

$$\rho_0 = \frac{3Q}{\pi R^3} \text{ so } Q_{\text{encl}} = 12Q\frac{r^3}{R^3}\left(\frac{1}{3} - \frac{r}{4R}\right) = Q\left(\frac{r^3}{R^3}\right)\left(4 - 3\frac{r}{R}\right).$$

Thus Gauss's law gives $E(4\pi r^2) = \dfrac{Q}{\varepsilon_0}\left(\dfrac{r^3}{R^3}\right)\left(4 - 3\dfrac{r}{R}\right)$.

(d) The graph of E versus r is sketched in Figure 22.53d.

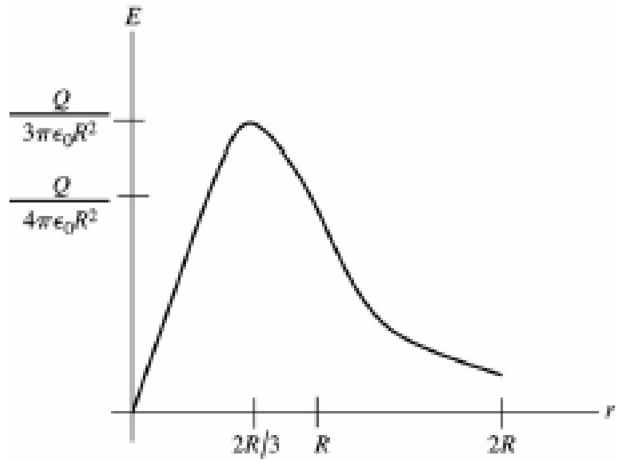


Figure 22.53d

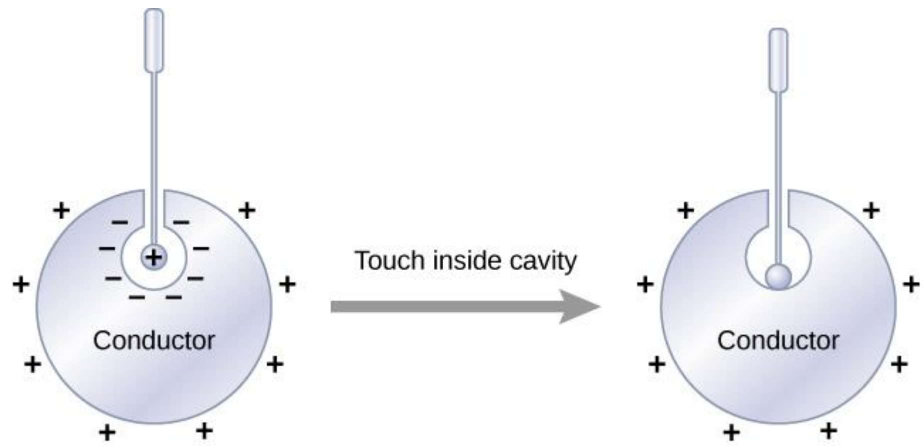
(e) Where the electric field is a maximum, $\frac{dE}{dr} = 0$. Thus $\frac{d}{dr}\left(4r - \frac{3r^2}{R}\right) = 0$ so $4 - 6r/R = 0$ and $r = 2R/3$.

At this value of r, $E = \frac{Q}{4\pi\varepsilon_0 R^3}\left(\frac{2R}{3}\right)\left(4 - \frac{3}{R}\frac{2R}{3}\right) = \frac{Q}{3\pi\varepsilon_0 R^2}$.

EVALUATE: Our expressions for $E(r)$ for $r < R$ and for $r > R$ agree at $r = R$. The results of part (e) for the value of r where $E(r)$ is a maximum agrees with the graph in part (d).

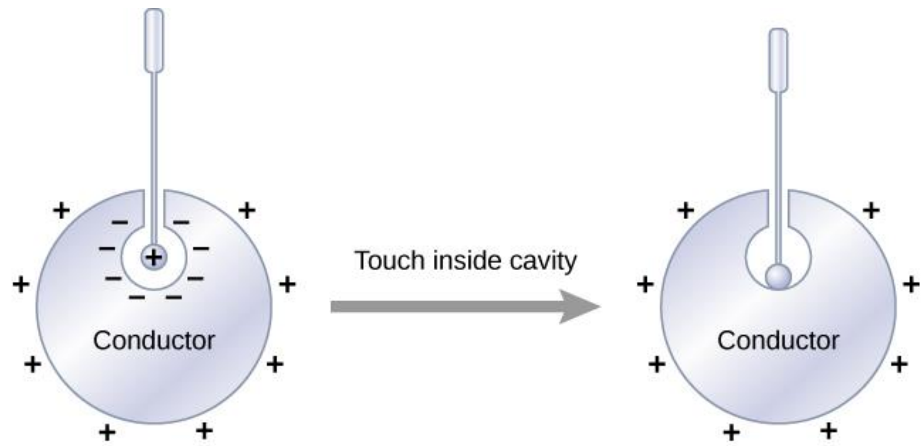
Electric fields around a charged conductor

- Let us suppose that we insert a charge +*Q* inside a conducting object.
- Since the electrons are free to move, -*Q* electrons will move to cancel the induced field.
- This leaves +Q charges on the surface.

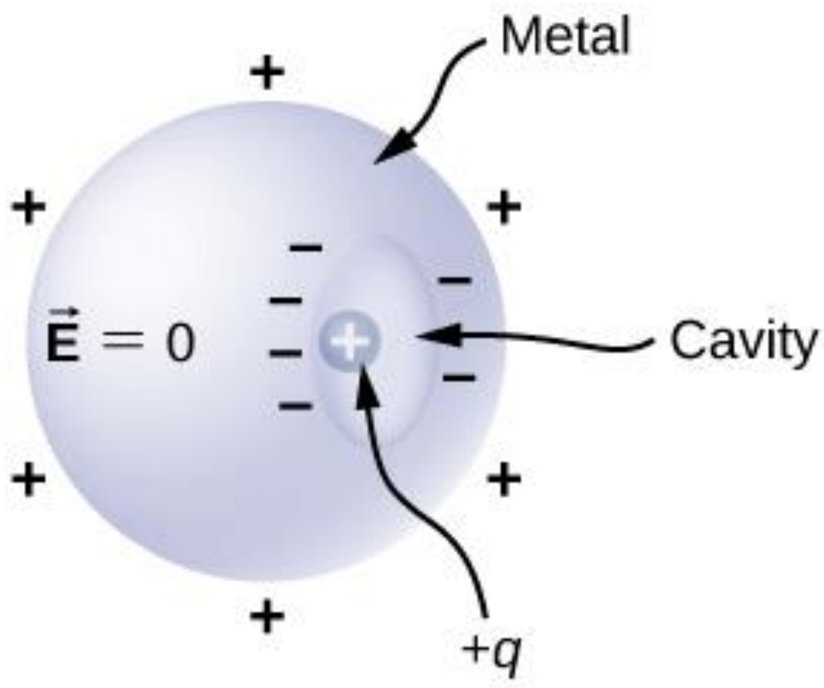


Electric fields around a charged conductor

- Touching the inside cavity and the internal charges combine and neutralize one another.
- The $+Q$ charges remain on the surface.
- So the inserted $+Q$ charge effectively transfers to the surface.



Electric fields around a charged conductor

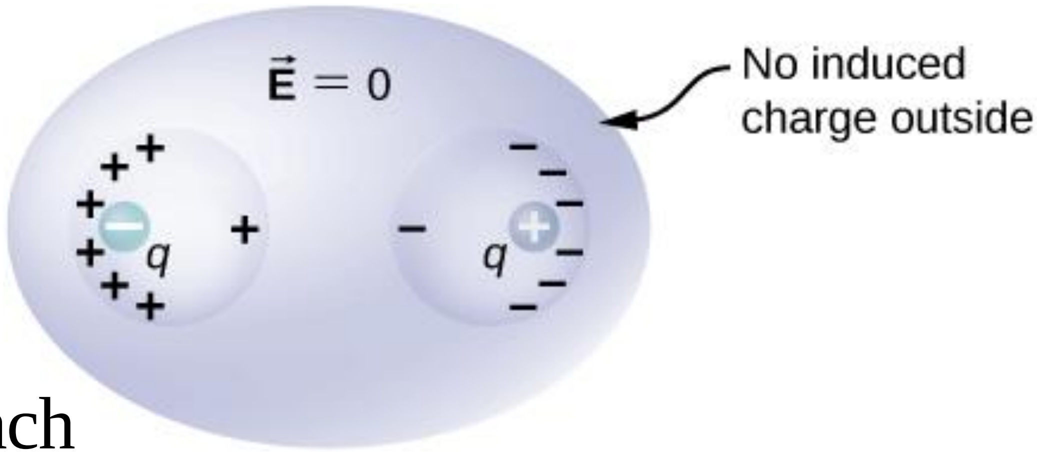


(a)

- Electrons will flow until the electric fields inside the conductor are neutralized.
- INSIDE A CONDUCTOR:
- E = 0
- This is true for a charged metal sphere or a charge in a cavity inside the sphere.

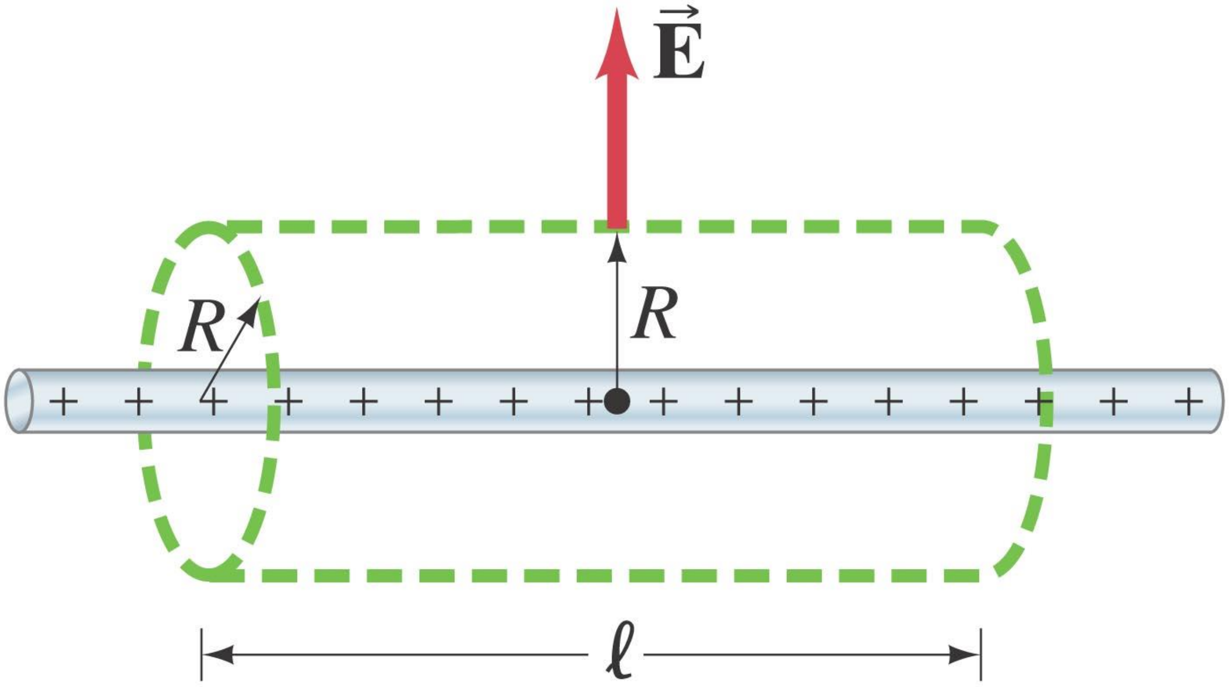
Opposite Charges Inside a Conductor

- One more example:
- Suppose we have two cavities and we place equal but opposite charges in them.
- The new charge distribution around each cavity will cancel the field at the surface of the cavity, leaving a neutral outer surface.



Example 22-6: Long uniform line of charge.

A very long straight wire possesses a uniform positive charge per unit length, λ. Calculate the electric field at points near (but outside) the wire, far from the ends.



MPLE 22–6 **Long uniform line of charge.** A very long straight wire
esses a uniform positive charge per unit length, λ. Calculate the electric field
oints near (but outside) the wire, far from the ends.

ROACH Because of the symmetry, we expect the field to be directed radially
vard and to depend only on the perpendicular distance, R, from the wire.
ause of the cylindrical symmetry, the field will be the same at all points on a
sian surface that is a cylinder with the wire along its axis, Fig. 22–15. $\vec{\mathbf{E}}$ is
endicular to this surface at all points. For Gauss's law, we need a closed
ace, so we include the flat ends of the cylinder. Since $\vec{\mathbf{E}}$ is parallel to the ends,
e is no flux through the ends (the cosine of the angle between $\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$ on
ends is $\cos 90° = 0$).

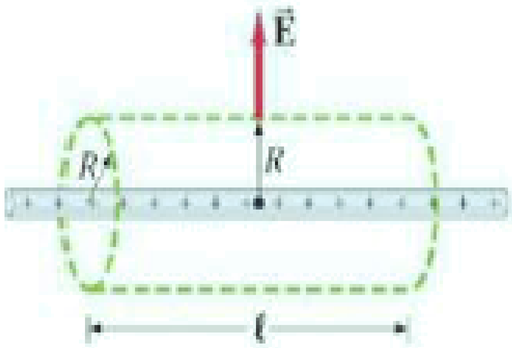
UTION For our chosen gaussian surface Gauss's law gives

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\lambda\ell}{\epsilon_0},$$

re ℓ is the length of our chosen gaussian surface ($\ell \ll$ length of wire), and
ℓ is its circumference. Hence

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{R}.$$

FIGURE 22–15 Calculation of $\vec{\mathbf{E}}$ due to a very long line of charge. Example 22–6.



Use Gauss's law to find the electric field caused by a thin, flat, infinite sheet with a uniform positive surface charge density σ.

SOLUTION

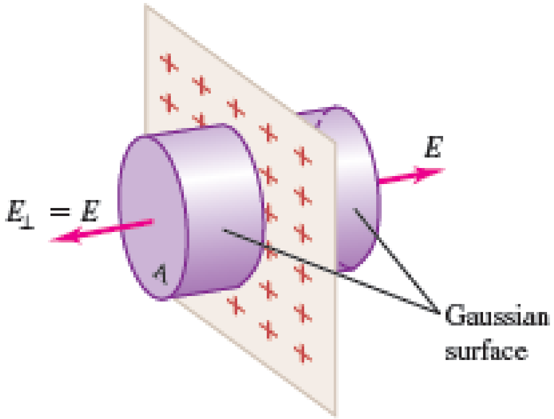
IDENTIFY and SET UP: In Example 21.11 (Section 21.5) we found that the field $\vec{E}$ of a uniformly charged infinite sheet is normal to the sheet, and that its magnitude is independent of the distance from the sheet. To take advantage of these symmetry properties, we use a cylindrical Gaussian surface with ends of area A and with its axis perpendicular to the sheet of charge (Fig. 22.20).

EXECUTE: The flux through the cylindrical part of our Gaussian surface is zero because $\vec{E} \cdot \hat{n} = 0$ everywhere. The flux through each flat end of the surface is $+EA$ because $\vec{E} \cdot \hat{n} = E_{\perp} = E$ everywhere, so the total flux through both ends—and hence the total flux Φ_E through the Gaussian surface—is $+2EA$. The total enclosed charge is $Q_{\text{encl}} = \sigma A$, and so from Gauss's law,

$$2EA = \frac{\sigma A}{\epsilon_0} \quad \text{and}$$

$$E = \frac{\sigma}{2\epsilon_0} \qquad \text{(field of an infinite sheet of charge)}$$

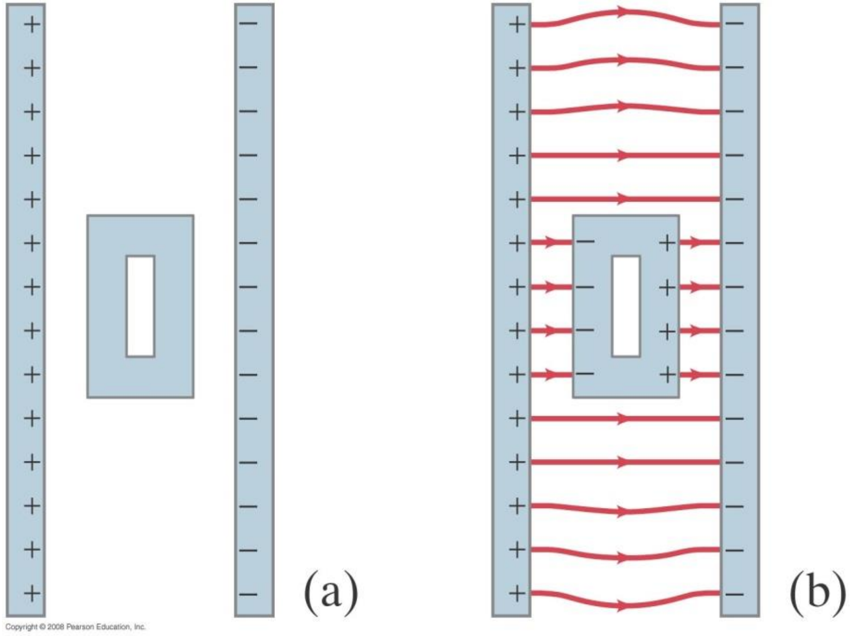
22.20 A cylindrical Gaussian surface is used to find the field of an infinite plane sheet of charge.

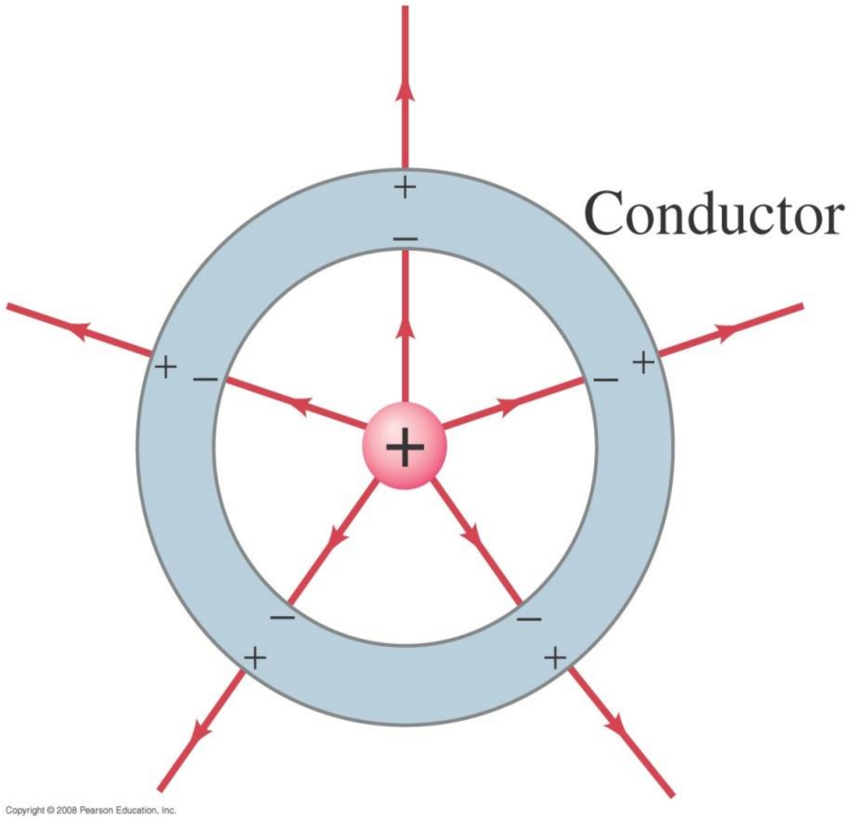


If σ is negative, $\vec{E}$ is directed *toward* the sheet, the flux through the Gaussian surface in Fig. 22.20 is negative, and σ in the expression $E = \sigma/2\epsilon_0$ denotes the magnitude (absolute value) of the charge density.

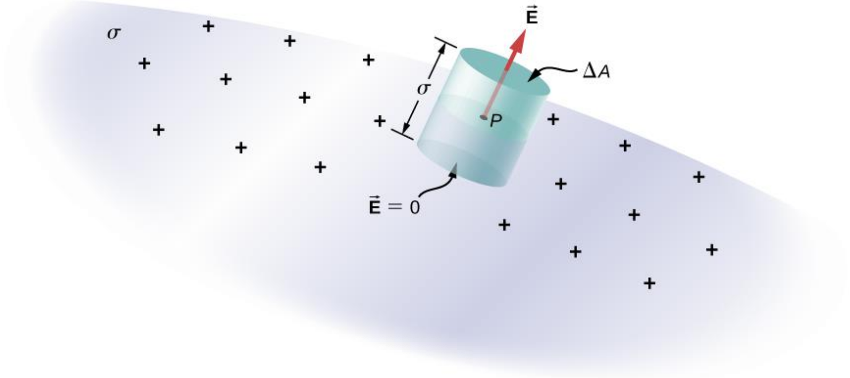
EVALUATE: We got the same result for the field of an infinite sheet of charge in Example 21.11 (Section 21.5). That calculation was much more complex and involved a fairly challenging integral. Thanks to the favorable symmetry, Gauss's law makes it much easier to solve this problem.

Electric fields around a charged conductor





Field near a Metal Surface



- For the local field near a conducting surface, we choose the same cylinder with 0 height.
- Now however, there is no field pointing inward; only outward, so
- $\oint E \bullet A = AE = \sigma A/\varepsilon_0$
- $E = \sigma/(\varepsilon_0)$
- Note: for this case, σ will vary locally if the curvature varies.

The difference between the electric field outside a conducting plane of charge and outside a nonconducting plane of charge can be thought of in two ways:

1. The field inside the conductor is zero, so the flux is all through one end of the cylinder.

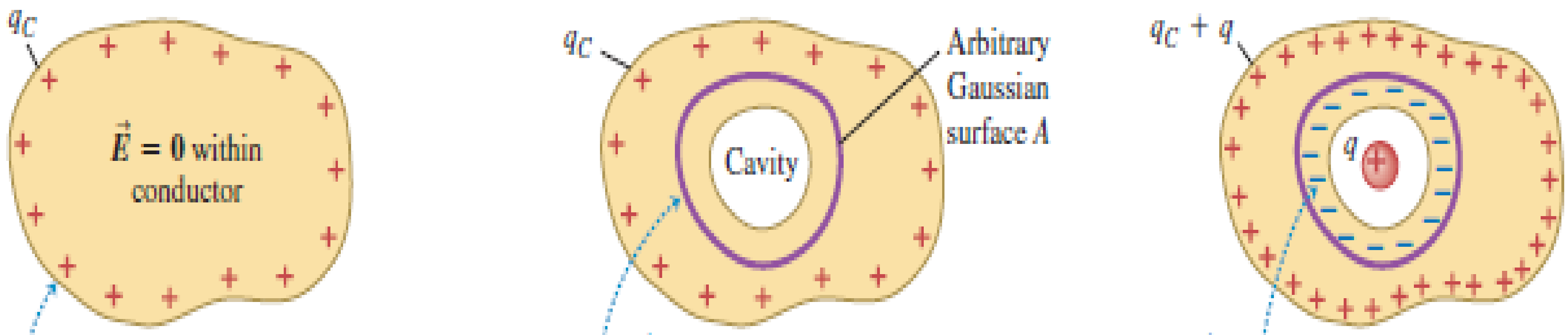
2. The nonconducting plane has a total charge density σ, whereas the conducting plane has a charge density σ on each side, effectively giving it twice the charge density.

22.23 Finding the electric field within a charged conductor.

(a) Solid conductor with charge q_C

(b) The same conductor with an internal cavity

(c) An isolated charge q placed in the cavity



(a) The charge q_C resides entirely on the surface of the conductor. The situation is electrostatic, so $\vec{E} = \mathbf{0}$ within the conductor.

(b) Because $\vec{E} = \mathbf{0}$ at all points within the conductor, the electric field at all points on the Gaussian surface must be zero.

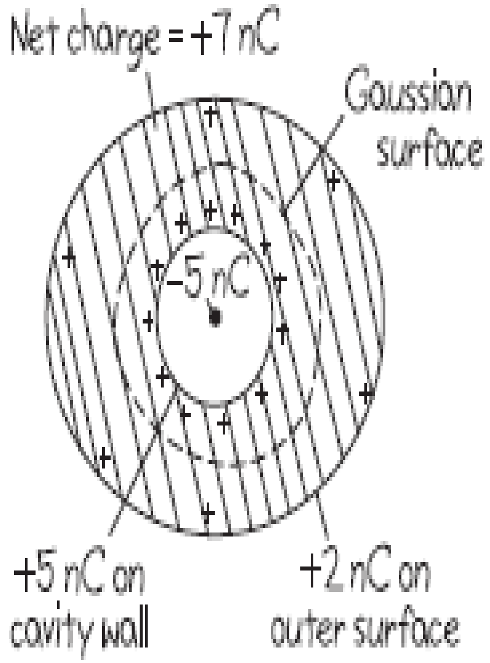
(c) For $\vec{E}$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$.

A conductor with a cavity carries a total charge of +7 nC. Within the cavity, insulated from the conductor, is a point charge of −5 nC. How much charge is on each surface (inner and outer) of the conductor?

SOLUTION

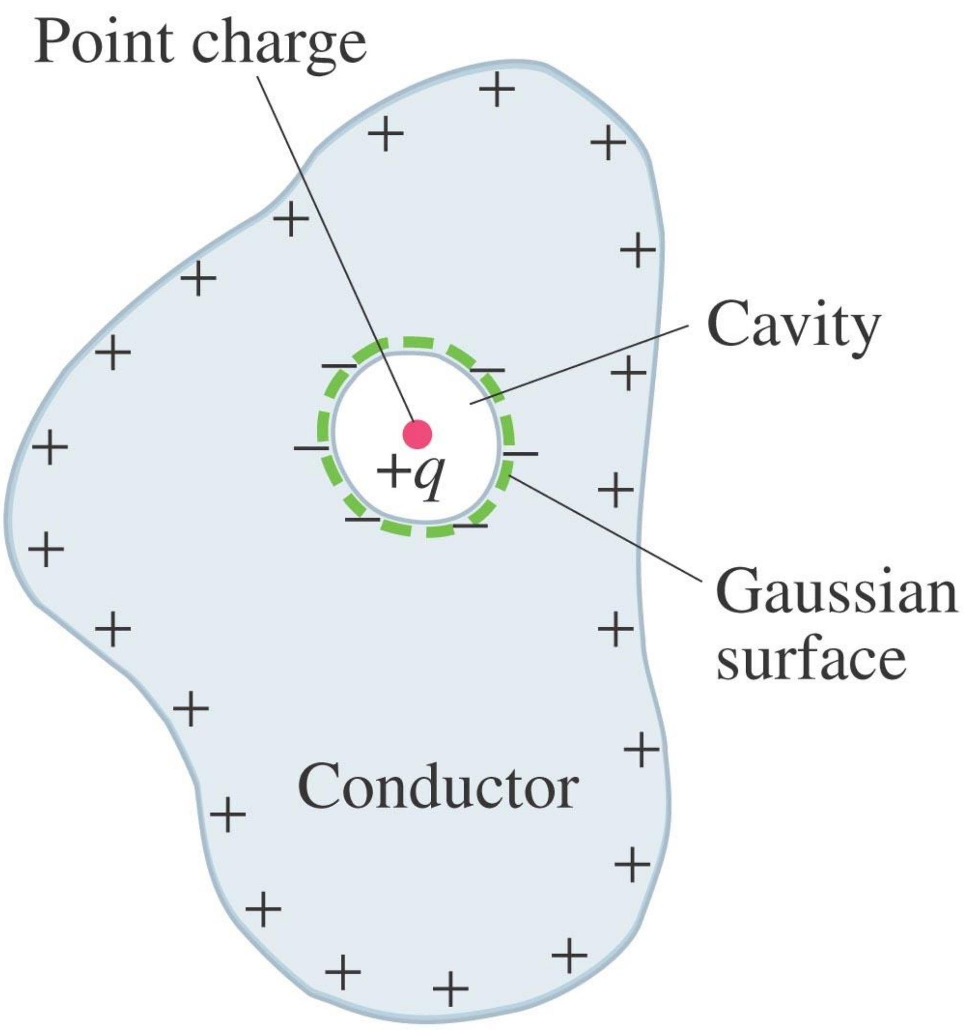
Figure 22.24 shows the situation. If the charge in the cavity is $q = -5$ nC, the charge on the inner cavity surface must be $-q = -(-5\text{ nC}) = +5$ nC. The conductor carries a *total* charge of +7 nC, none of which is in the interior of the material. If +5 nC is on the inner surface of the cavity, then there must be $(+7\text{ nC}) - (+5\text{ nC}) = +2$ nC on the outer surface of the conductor.

22.24 Our sketch for this problem. There is zero electric field inside the bulk conductor and hence zero flux through the Gaussian surface shown, so the charge on the cavity wall must be the opposite of the point charge.



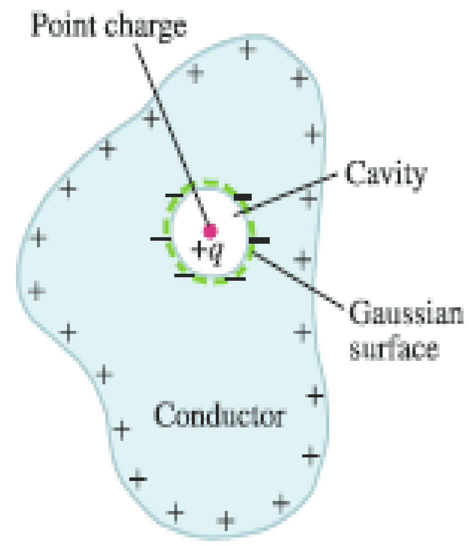
Conceptual Example 22-9: Conductor with charge inside a cavity.

Suppose a conductor carries a net charge +Q and contains a cavity, inside of which resides a point charge +q. What can you say about the charges on the inner and outer surfaces of the conductor?



CONCEPTUAL EXAMPLE 22–9 **Conductor with charge inside a cavity.** Suppose a conductor carries a net charge $+Q$ and contains a cavity, inside of which resides a point charge $+q$. What can you say about the charges on the inner and outer surfaces of the conductor?

RESPONSE As shown in Fig. 22–21, a gaussian surface just inside the conductor surrounding the cavity must contain zero net charge ($E = 0$ in a conductor). Thus a net charge of $-q$ must exist on the cavity surface. The conductor itself carries a net charge $+Q$, so its outer surface must carry a charge equal to $Q + q$. These results apply to a cavity of any shape.



Verify Eq. (22.10) for a conducting sphere with radius R and total charge q.

SOLUTION

In Example 22.5 (Section 22.4) we showed that the electric field just outside the surface is

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$$

The surface charge density is uniform and equal to q divided by the surface area of the sphere:

$$\sigma = \frac{q}{4\pi R^2}$$

Comparing these two expressions, we see that $E = \sigma/\epsilon_0$, which verifies Eq. (22.10).

The earth (a conductor) has a net electric charge. The resulting electric field near the surface has an average value of about 150 N/C, directed toward the center of the earth. (a) What is the corresponding surface charge density? (b) What is the *total* surface charge of the earth?

SOLUTION

IDENTIFY and SET UP: We are given the electric-field magnitude at the surface of the conducting earth. We can calculate the surface charge density σ from Eq. (22.10). The total charge Q on the earth's surface is then the product of σ and the earth's surface area.

EXECUTE: (a) The direction of the field means that σ is negative (corresponding to $\vec{E}$ being directed *into* the surface, so $E_\perp$ is negative). From Eq. (22.10),

$$\sigma = \epsilon_0 E_\perp = (8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(-150\ \mathrm{N/C})$$
$$= -1.33 \times 10^{-9}\ \mathrm{C/m^2} = -1.33\ \mathrm{nC/m^2}$$

(b) The earth's surface area is $4\pi R_E^2$, where $R_E = 6.38 \times 10^6$ m is the radius of the earth (see Appendix F). The total charge Q is the product $4\pi R_E^2 \sigma$, or

$$Q = 4\pi(6.38 \times 10^6\ \mathrm{m})^2(-1.33 \times 10^{-9}\ \mathrm{C/m^2})$$
$$= -6.8 \times 10^5\ \mathrm{C} = -680\ \mathrm{kC}$$

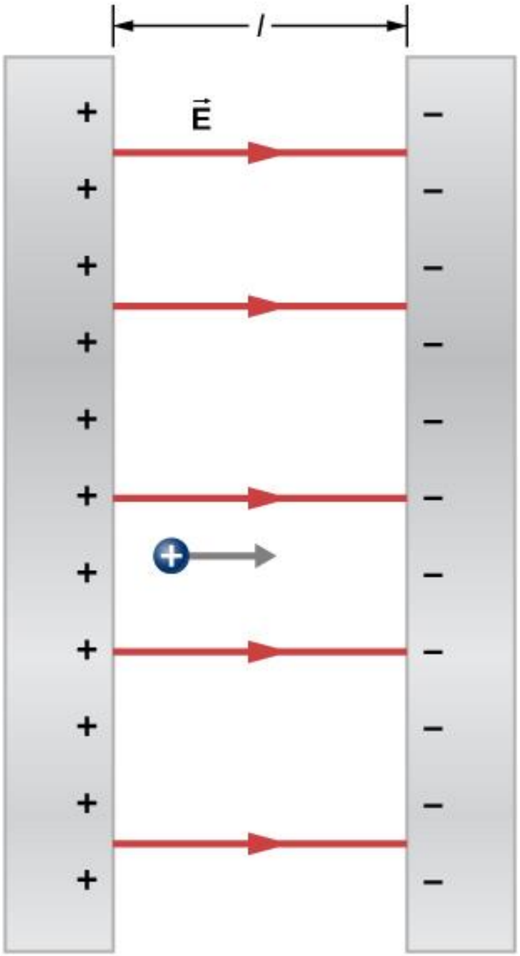
EVALUATE: You can check our result in part (b) by using the result of Example 22.5. Solving for Q, we find

$$Q = 4\pi\epsilon_0 R^2 E_\perp$$
$$= \frac{1}{9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}}(6.38 \times 10^6\ \mathrm{m})^2(-150\ \mathrm{N/C})$$
$$= -6.8 \times 10^5\ \mathrm{C}$$

One electron has a charge of -1.60×10^{-19} C. Hence this much excess negative electric charge corresponds to there being $(-6.8 \times 10^5\ \mathrm{C})/(-1.60 \times 10^{-19}\ \mathrm{C}) = 4.2 \times 10^{24}$ excess electrons on the earth, or about 7 moles of excess electrons. This is compensated by an equal *deficiency* of electrons in the earth's upper atmosphere, so the combination of the earth and its atmosphere is electrically neutral.

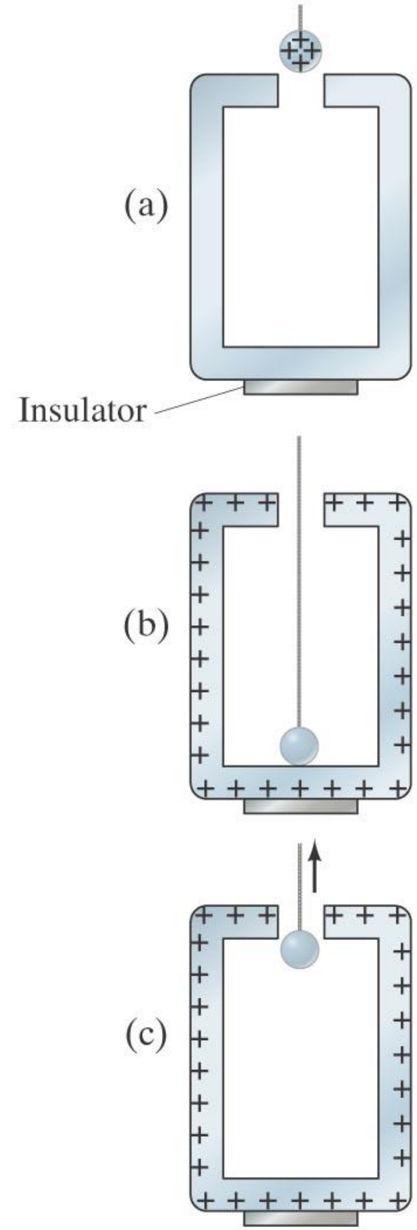
Reminder: Electric Field of A Capacitor

- Note that the field at the surface agrees with our previous calculation of the field of a capacitor.
- We showed that the magnitude of the electric field between the plates is:
- $E = \frac{\sigma}{\varepsilon_0}$



Verification of Gauss’s Law

- An elegant proof of Gauss’s law is shown at right.
- A small charged metal ball is lowered into a metal can through a small hole and allowed to touch the bottom of the can.
- When the ball is removed, it has no charge.



1\. (I) A uniform electric field of magnitude 5.8×10^2 N/C passes through a circle of radius 13 cm. What is the electric flux through the circle when its face is (*a*) perpendicular to the field lines, (*b*) at 45° to the field lines, and (*c*) parallel to the field lines?

$\boxed{1.}$ The electric flux of a uniform field is given by Eq. 22-1b.

(*a*) $\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA\cos\theta = (580\,\mathrm{N/C})\pi(0.13\,\mathrm{m})^2\cos 0 = \boxed{31\,\mathrm{N{\cdot}m^2/C}}$

(*b*) $\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA\cos\theta = (580\,\mathrm{N/C})\pi(0.13\,\mathrm{m})^2\cos 45^\circ = \boxed{22\,\mathrm{N{\cdot}m^2/C}}$

(*c*) $\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA\cos\theta = (580\,\mathrm{N/C})\pi(0.13\,\mathrm{m})^2\cos 90^\circ = \boxed{0}$

4. (II) A uniform field $\vec{E}$ is parallel to the axis of a hollow hemisphere of radius r, Fig. 22–25. (*a*) What is the electric flux through the hemispherical surface? (*b*) What is the result if $\vec{E}$ is instead perpendicular to the axis?

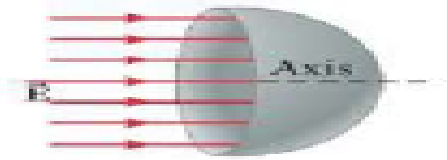


FIGURE 22–25 Problem 4.

4. (*a*) From the diagram in the textbook, we see that the flux outward through the hemispherical surface is the same as the flux inward through the circular surface base of the hemisphere. On that surface all of the flux is perpendicular to the surface. Or, we say that on the circular base, $\vec{\mathbf{E}} \parallel \vec{\mathbf{A}}$. Thus $\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = \boxed{\pi r^2 E}$.

(b) $\vec{\mathbf{E}}$ is perpendicular to the axis, then every field line would both enter through the hemispherical surface and leave through the hemispherical surface, and so $\Phi_E = \boxed{0}$.

8. (II) A ring of charge with uniform charge density is completely enclosed in a hollow donut shape. An exact copy of the ring is completely enclosed in a hollow sphere. What is the ratio of the flux out of the donut shape to that out of the sphere?

9. (II) In a certain region of space, the electric field is constant in direction (say horizontal, in the x direction), but its magnitude decreases from $E = 560\,\text{N/C}$ at $x = 0$ to $E = 410\,\text{N/C}$ at $x = 25\,\text{m}$. Determine the charge within a cubical box of side $\ell = 25\,\text{m}$, where the box is oriented so that four of its sides are parallel to the field lines (Fig. 22–28).

FIGURE 22–28 Problem 9.

8. The net flux is only dependent on the charge enclosed by the surface. Since both surfaces enclose the same amount of charge, the flux through both surfaces is the same. Thus the ratio is $\boxed{1:1}$.

9. The only contributions to the flux are from the faces perpendicular to the electric field. Over each of these two surfaces, the magnitude of the field is constant, so the flux is just $\vec{\mathbf{E}} \cdot \vec{\mathbf{A}}$ on each of these two surfaces.

$$\Phi_E = \left(\vec{\mathbf{E}} \cdot \vec{\mathbf{A}}\right)_{\text{right}} + \left(\vec{\mathbf{E}} \cdot \vec{\mathbf{A}}\right)_{\text{left}} = E_{\text{right}}\ell^2 - E_{\text{left}}\ell^2 = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$Q_{\text{encl}} = \left(E_{\text{right}} - E_{\text{left}}\right)\ell^2\varepsilon_0 = \left(410\,\text{N/C} - 560\,\text{N/C}\right)\left(25\,\text{m}\right)^2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{-8.3\times10^{-7}\,\text{C}}$$

15. (I) A long thin wire, hundreds of meters long, carries a uniformly distributed charge of $-7.2\,\mu\text{C}$ per meter of length. Estimate the magnitude and direction of the electric field at points (*a*) 5.0 m and (*b*) 1.5 m perpendicular from the center of the wire.

15. The electric field due to a long thin wire is given in Example 22-6 as $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R}$.

(*a*) $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{R} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{2\left(-7.2\times10^{-6}\ \text{C/m}\right)}{(5.0\,\text{m})} = \boxed{-2.6\times10^4\ \text{N/C}}$

The negative sign indicates the electric field is pointed $\boxed{\text{towards the wire}}$.

(*b*) $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{R} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{2\left(-7.2\times10^{-6}\ \text{C/m}\right)}{(1.5\,\text{m})} = \boxed{-8.6\times10^4\ \text{N/C}}$

The negative sign indicates the electric field is pointed $\boxed{\text{towards the wire}}$.

17. (II) A nonconducting sphere is made of two layers. The innermost section has a radius of 6.0 cm and a uniform charge density of $-5.0\ \mathrm{C/m^3}$. The outer layer has a uniform charge density of $+8.0\ \mathrm{C/m^3}$ and extends from an inner radius of 6.0 cm to an outer radius of 12.0 cm. Determine the electric field for (*a*) $0 < r < 6.0$ cm, (*b*) $6.0\ \mathrm{cm} < r < 12.0$ cm, and (*c*) $12.0\ \mathrm{cm} < r < 50.0$ cm. (*d*) Plot the magnitude of the electric field for $0 < r < 50.0$ cm. Is the field continuous at the edges of the layers?

18. (II) A solid metal sphere of radius 3.00 m carries a total charge of $-5.50\ \mu\mathrm{C}$. What is the magnitude of the electric field at a distance from the sphere's center of (*a*) 0.250 m, (*b*) 2.90 m, (*c*) 3.10 m, and (*d*) 8.00 m? How would the answers differ if the sphere were (*e*) a thin shell, or (*f*) a solid nonconductor uniformly charged throughout?

17. Due to the spherical symmetry of the problem, the electric field can be evaluated using Gauss's law and the charge enclosed by a spherical Gaussian surface of radius r.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2}$$

Since the charge densities are constant, the charge enclosed is found by multiplying the appropriate charge density times the volume of charge enclosed by the Gaussian sphere. Let $r_1 = 6.0\,\text{cm}$ and $r_2 = 12.0\,\text{cm}$.

(*a*) Negative charge is enclosed for $r < r_1$.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_{(-)}\left(\frac{4}{3}\pi r^3\right)}{r^2} = \frac{\rho_{(-)}r}{3\varepsilon_0} = \frac{\left(-5.0\,\text{C/m}^3\right)r}{3\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}$$

$$= \boxed{\left(-1.9\times10^{11}\,\text{N/C}\cdot\text{m}\right)r}$$

(*b*) In the region $r_1 < r < r_2$, all of the negative charge and part of the positive charge is enclosed.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_{(-)}\left(\frac{4}{3}\pi r_1^3\right) + \rho_{(+)}\left[\frac{4}{3}\pi\left(r^3 - r_1^3\right)\right]}{r^2} = \frac{\left(\rho_{(-)} - \rho_{(+)}\right)\left(r_1^3\right)}{3\varepsilon_0 r^2} + \frac{\rho_{(+)}r}{3\varepsilon_0}$$

$$= \frac{\left[\left(-5.0\,\text{C/m}^3\right) - \left(8.0\,\text{C/m}^3\right)\right]\left(0.060\,\text{m}\right)^3}{3\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)r^2} + \frac{\left(8.0\,\text{C/m}^3\right)r}{3\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}$$

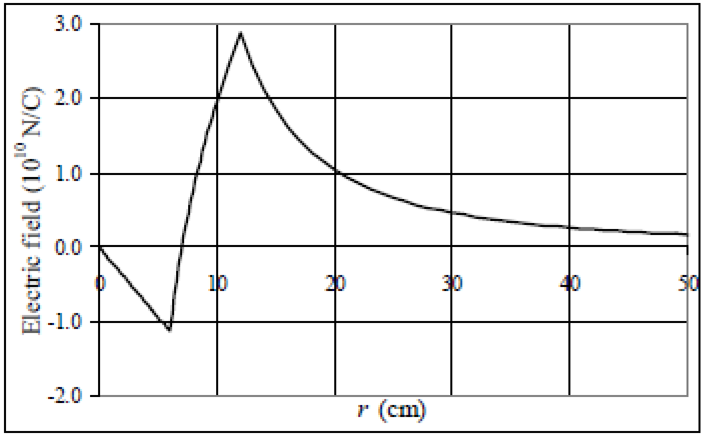
$$= \boxed{\frac{\left(-1.1\times10^{8}\,\text{N}\cdot\text{m}^2/\text{C}\right)}{r^2} + \left(3.0\times10^{11}\,\text{N/C}\cdot\text{m}\right)r}$$

(*c*) In the region $r_2 < r$, all of the charge is enclosed.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{\text{encl}}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho_{(-)}\left(\frac{4}{3}\pi r_1^3\right) + \rho_{(+)}\left[\frac{4}{3}\pi\left(r_2^3 - r_1^3\right)\right]}{r^2} = \frac{\left(\rho_{(-)} - \rho_{(+)}\right)\left(r_1^3\right) + \rho_{(+)}\left(r_2^3\right)}{3\varepsilon_0 r^2} =$$

$$= \frac{\left[\left(-5.0\,\text{C/m}^3\right) - \left(8.0\,\text{C/m}^3\right)\right]\left(0.060\,\text{m}\right)^3 + \left(8.0\,\text{C/m}^3\right)\left(0.120\,\text{m}\right)^3}{3\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)r^2} = \boxed{\frac{\left(4.1\times10^{8}\,\text{N}\cdot\text{m}^2/\text{C}\right)}{r^2}}$$

(*d*) See the adjacent plot. The field is continuous at the edges of the layers. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.17d."



18. See Example 22-3 for a detailed discussion related to this problem.

(*a*) Inside a solid metal sphere the electric field is $\boxed{0}$.

(*b*) Inside a solid metal sphere the electric field is $\boxed{0}$.

(*c*) Outside a solid metal sphere the electric field is the same as if all the charge were concentrated at the center as a point charge.

$$|E| = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(5.50\times10^{-6}\,\mathrm{C}\right)}{\left(3.10\,\mathrm{m}\right)^2} = \boxed{5140\,\mathrm{N/C}}$$

The field would point towards the center of the sphere.

(*d*) Same reasoning as in part (*c*).

$$|E| = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r^2} = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(5.50\times10^{-6}\,\mathrm{C}\right)}{\left(8.00\,\mathrm{m}\right)^2} = \boxed{772\,\mathrm{N/C}}$$

The field would point towards the center of the sphere.

(*e*) The answers would be $\boxed{\text{no different}}$ for a thin metal shell.

(*f*) The solid sphere of charge is dealt with in Example 22-4. We see from that Example that the field inside the sphere is given by $|E| = \frac{1}{4\pi\varepsilon_0}\frac{|Q|}{r_0^3}r$. Outside the sphere the field is no different.

So we have these results for the solid sphere.

$$|E(r = 0.250\,\mathrm{m})| = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{5.50\times10^{-6}\,\mathrm{C}}{\left(3.00\,\mathrm{m}\right)^3}\left(0.250\,\mathrm{m}\right) = \boxed{458\,\mathrm{N/C}}$$

$$|E(r = 2.90\,\mathrm{m})| = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{5.50\times10^{-6}\,\mathrm{C}}{\left(3.00\,\mathrm{m}\right)^3}\left(2.90\,\mathrm{m}\right) = \boxed{5310\,\mathrm{N/C}}$$

$$|E(r = 3.10\,\mathrm{m})| = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{5.50\times10^{-6}\,\mathrm{C}}{\left(3.10\,\mathrm{m}\right)^2} = \boxed{5140\,\mathrm{N/C}}$$

$$|E(r = 8.00\,\mathrm{m})| = \left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\frac{5.50\times10^{-6}\,\mathrm{C}}{\left(3.10\,\mathrm{m}\right)^2} = \boxed{772\,\mathrm{N/C}}$$

All point towards the center of the sphere.

22. (II) A point charge Q rests at the center of an uncharged thin spherical conducting shell. What is the electric field E as a function of r (*a*) for r less than the radius of the shell, (*b*) inside the shell, and (*c*) beyond the shell? (*d*) Does the shell affect the field due to Q alone? Does the charge Q affect the shell?

22. (*a*) Inside the shell, the field is that of the point charge, $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$.

(*b*) There is no field inside the conducting material: $\boxed{E = 0}$.

(*c*) Outside the shell, the field is that of the point charge, $\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$.

(*d*) The shell does not affect the field due to Q alone, except in the shell material, where the field is 0. The charge Q does affect the shell – it polarizes it. There will be an induced charge of $-Q$ uniformly distributed over the inside surface of the shell, and an induced charge of $+Q$ uniformly distributed over the outside surface of the shell.

24. (II) Two large, flat metal plates are separated by a distance that is very small compared to their height and width. The conductors are given equal but opposite uniform surface charge densities $\pm\sigma$. Ignore edge effects and use Gauss's law to show (*a*) that for points far from the edges, the electric field between the plates is $E = \sigma/\epsilon_0$ and (*b*) that outside the plates on either side the field is zero. (*c*) How would your results be altered if the two plates were nonconductors? (See Fig. 22–30).

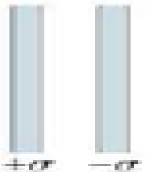
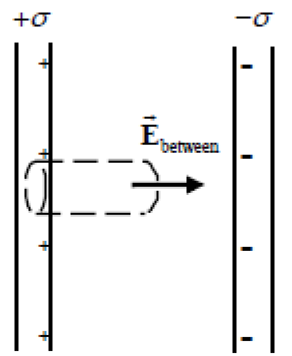


FIGURE 22–30
Problems 24, 25, and 26.

24. Since the charges are of opposite sign, and since the charges are free to move since they are on conductors, the charges will attract each other and move to the inside or facing edges of the plates. There will be no charge on the outside edges of the plates. And there cannot be charge in the plates themselves, since they are conductors. All of the charge must reside on surfaces. Due to the symmetry of the problem, all field lines must be perpendicular to the plates, as discussed in Example 22-7.

(*a*) To find the field between the plates, we choose a gaussian cylinder, perpendicular to the plates, with area A for the ends of the cylinder. We place one end inside the left plate (where the field must be zero), and the other end between the plates. No flux passes through the curved surface of the cylinder.

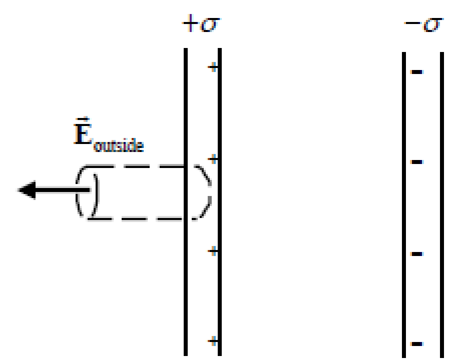


$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\substack{\text{right}\\ \text{end}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$E_{\text{between}} A = \frac{\sigma A}{\varepsilon_0} \rightarrow \boxed{E_{\text{between}} = \frac{\sigma}{\varepsilon_0}}$$

The field lines between the plates leave the inside surface of the left plate, and terminate on the inside surface of the right plate. A similar derivation could have been done with the right end of the cylinder inside of the right plate, and the left end of the cylinder in the space between the plates.

(*b*) If we now put the cylinder from above so that the right end is inside the conducting material, and the left end is to the left of the left plate, the only possible location for flux is through the left end of the cylinder. Note that there is NO charge enclosed by the Gaussian cylinder.



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\text{ends}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_{\text{side}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_{\substack{\text{left}\\ \text{end}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} \rightarrow$$

$$E_{\text{outside}}A = \frac{0}{\varepsilon_0} \quad \rightarrow \quad \boxed{E_{\text{outside}} = \frac{0}{\varepsilon_0}}$$

(*c*) If the two plates were nonconductors, the results would not change. The charge would be distributed over the two plates in a different fashion, and the field inside of the plates would not be zero, but the charge in the empty regions of space would be the same as when the plates are conductors.

27. (II) Two thin concentric spherical shells of radii r_1 and r_2 ($r_1 < r_2$) contain uniform surface charge densities σ_1 and σ_2, respectively (see Fig. 22–31). Determine the electric field for (*a*) $0 < r < r_1$, (*b*) $r_1 < r < r_2$, and (*c*) $r > r_2$. (*d*) Under what conditions will $E = 0$ for $r > r_2$? (*e*) Under what conditions will $E = 0$ for $r_1 < r < r_2$? Neglect the thickness of the shells.

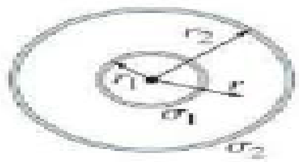


FIGURE 22–31 Two spherical shells (Problem 27).

27. (*a*) In the region $0 < r < r_1$, a gaussian surface would enclose no charge. Thus, due to the spherical symmetry, we have the following.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} = 0 \rightarrow E = \boxed{0}$$

(*b*) In the region $r_1 < r < r_2$, only the charge on the inner shell will be enclosed.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma_1 4\pi r_1^2}{\varepsilon_0} \rightarrow E = \boxed{\frac{\sigma_1 r_1^2}{\varepsilon_0 r^2}}$$

(*c*) In the region $r_2 < r$, the charge on both shells will be enclosed.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma_1 4\pi r_1^2 + \sigma_2 4\pi r_2^2}{\varepsilon_0} \rightarrow E = \boxed{\frac{\sigma_1 r_1^2 + \sigma_2 r_2^2}{\varepsilon_0 r^2}}$$

(*d*) To make $E = 0$ for $r_2 < r$, we must have $\boxed{\sigma_1 r_1^2 + \sigma_2 r_2^2 = 0}$. This implies that the shells are of opposite charge.

(*e*) To make $E = 0$ for $r_1 < r < r_2$, we must have $\boxed{\sigma_1 = 0}$. Or, if a charge $Q = -4\pi\sigma_1 r_1^2$ were placed at the center of the shells, that would also make $E = 0$.

34. (II) A very long solid nonconducting cylinder of radius R_0 and length ℓ ($R_0 \ll \ell$) possesses a uniform volume charge density ρ_E (C/m^3), Fig. 22–34. Determine the electric field at points (*a*) outside the cylinder ($R > R_0$) and (*b*) inside the cylinder ($R < R_0$). Do only for points far from the ends and for which $R \ll \ell$.

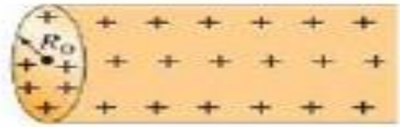
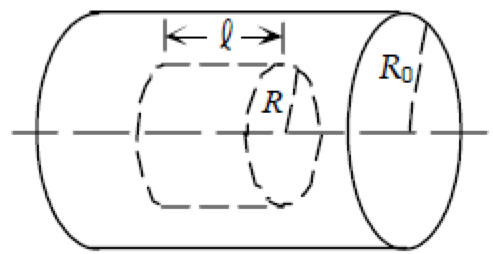


FIGURE 22–34 Problem 34.

34. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(2\pi R\ell\right) = \frac{Q_{\rm encl}}{\varepsilon_0} = \frac{\rho_E V_{\rm encl}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{\rho_E V_{\rm encl}}{2\pi\varepsilon_0 R\ell}$$

(*a*) For $R > R_0$, the enclosed volume of the shell is

$V_{\rm encl} = \pi R_0^2 \ell$.

$$E = \frac{\rho_E V_{\rm encl}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R_0^2}{2\varepsilon_0 R}, \text{ radially outward}}$$

(*b*) For $R < R_0$, the enclosed volume of the shell is $V_{\rm encl} = \pi R^2 \ell$.

$$E = \frac{\rho_E V_{\rm encl}}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R}{2\varepsilon_0}, \text{ radially outward}}$$

38. (II) A very long solid nonconducting cylinder of radius R_1 is uniformly charged with a charge density ρ_E. It is surrounded by a concentric cylindrical tube of inner radius R_2 and outer radius R_3 as shown in Fig. 22–36, and it too carries a uniform charge density ρ_E. Determine the electric field as a function of the distance R from the center of the cylinders for (*a*) $0 < R < R_1$, (*b*) $R_1 < R < R_2$, (*c*) $R_2 < R < R_3$, and (*d*) $R > R_3$. (*e*) If $\rho_E = 15\ \mu C/m^3$ and $R_1 = \frac{1}{2}R_2 = \frac{1}{3}R_3 = 5.0$ cm, plot E as a function of R from $R = 0$ to $R = 20.0$ cm. Assume the cylinders are very long compared to R_3.

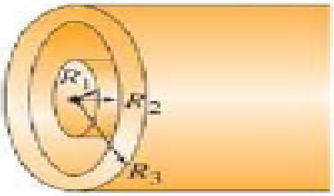
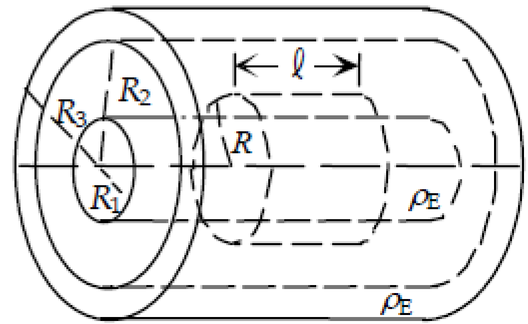


FIGURE 22–36
Problem 38.

38. The geometry of this problem is similar to Problem 33, and so we use the same development, following Example 22-6. See the solution of Problem 33 for details.



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(2\pi R\ell) = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \rightarrow \quad E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell}$$

(*a*) For $0 < R < R_1$, the enclosed charge is the volume of charge enclosed, times the charge density.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R^2 \ell}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R}{2\varepsilon_0}}$$

(*b*) For $R_1 < R < R_2$, the enclosed charge is all of the charge on the inner cylinder.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E R_1^2}{2\varepsilon_0 R}}$$

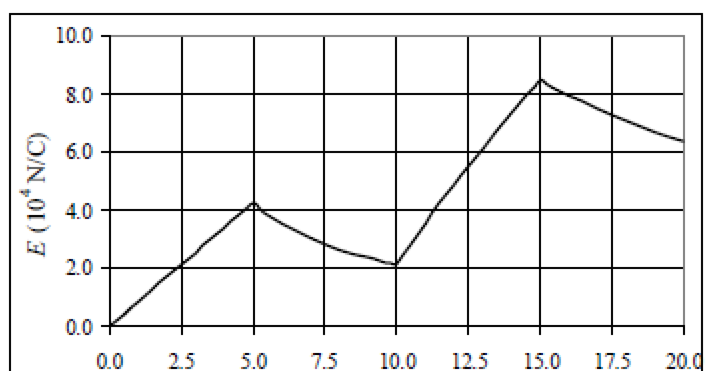
(*c*) For $R_2 < R < R_3$, the enclosed charge is all of the charge on the inner cylinder, and the part of the charge on the shell that is enclosed by the gaussian cylinder.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell + \rho_E\left(\pi R^2 \ell - \pi R_2^2 \ell\right)}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E\left(R_1^2 + R^2 - R_2^2\right)}{2\varepsilon_0 R}}$$

(*d*) For $R > R_3$, the enclosed charge is all of the charge on both the inner cylinder and the shell.

$$E = \frac{Q_{\text{encl}}}{2\pi\varepsilon_0 R\ell} = \frac{\rho_E \pi R_1^2 \ell + \rho_E\left(\pi R_3^2 \ell - \pi R_2^2 \ell\right)}{2\pi\varepsilon_0 R\ell} = \boxed{\frac{\rho_E\left(R_1^2 + R_3^2 - R_2^2\right)}{2\varepsilon_0 R}}$$

(*e*) See the graph. The spreadsheet used for this problem can be found on the Media Manager, with filename "PSE4_ISM_CH22.XLS," on tab "Problem 22.38e."



End of Chapter 22

- Review the other examples of applications of Gauss's Law
- Go through Chapter 22 Summary -
- Complete Homework for Ch 22 (Vol 2 Ch 6)