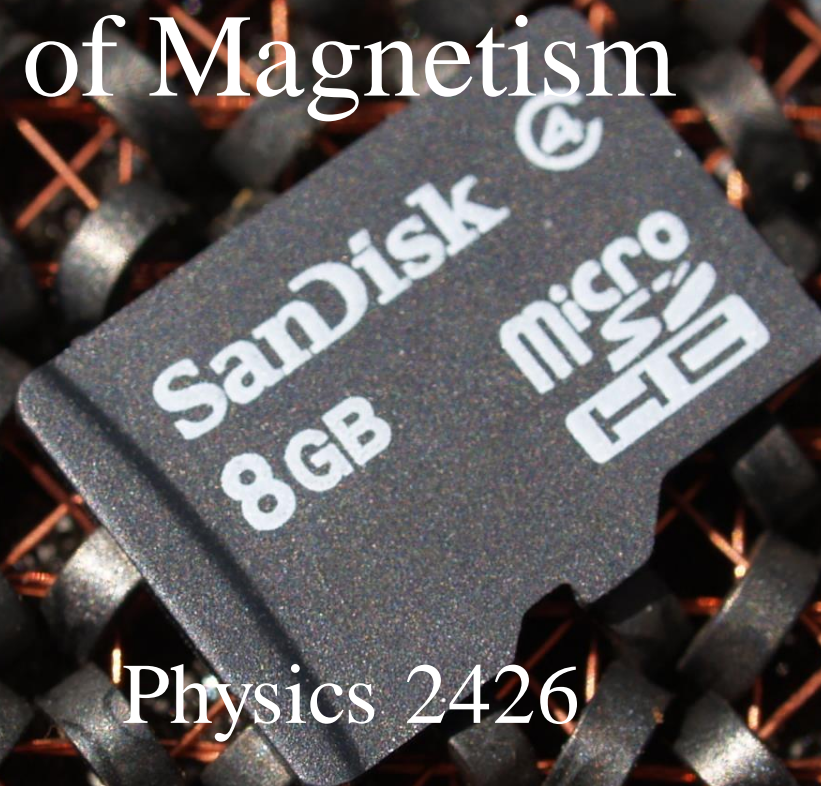


# Volume 2 Chapter 12 – Sources of Magnetism

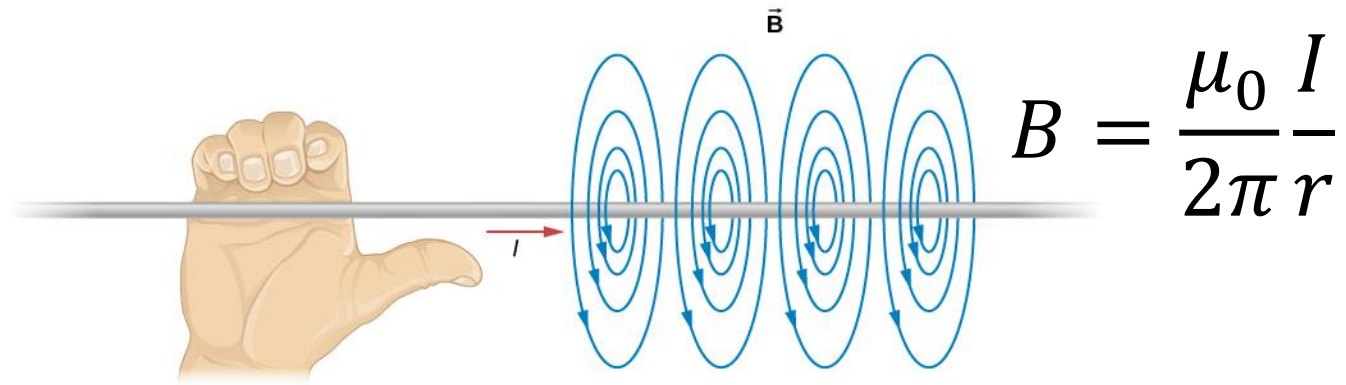


Physics 2426  
Ashok kumar

# Magnetism

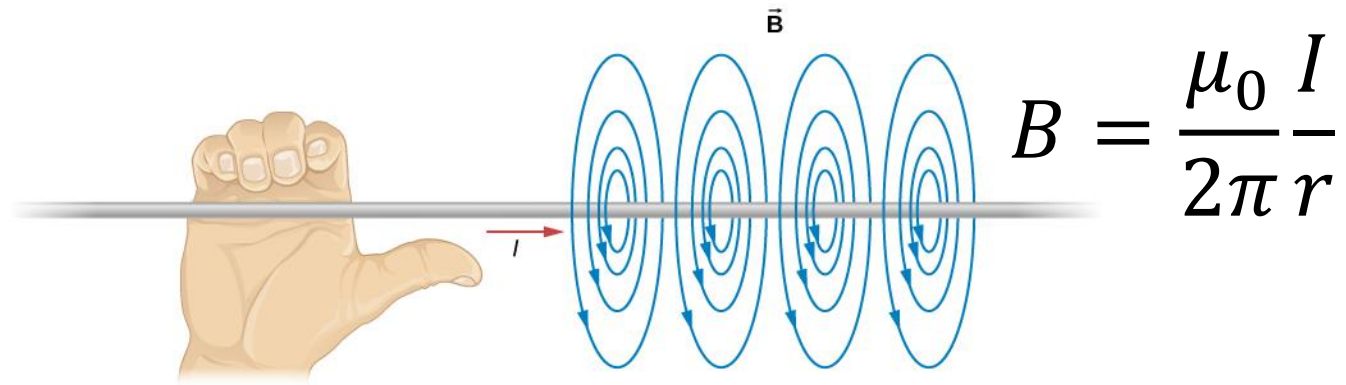
- Quantum mechanics seems to predict that individual N or S magnetic poles, called monopoles should exist just as  $\pm$  electrical charges do.
- Unfortunately, despite several attempts to find magnetic monopoles, none have ever been detected.
- Therefore, the current theory of magnetism assumes that all magnetic fields are generated by moving electric charges.

# Magnetic Field around a Long Wire



- The magnitude of the field around a long wire is proportional to the current,  $I$ , and inversely proportional to the distance,  $r$ , from the wire.
- It has become standard to write the proportionality constant as  $\mu_0/2\pi$ ,
- Where  $\mu_0$  is the magnetic permeability of free space.

# Magnetic Field around a Long Wire

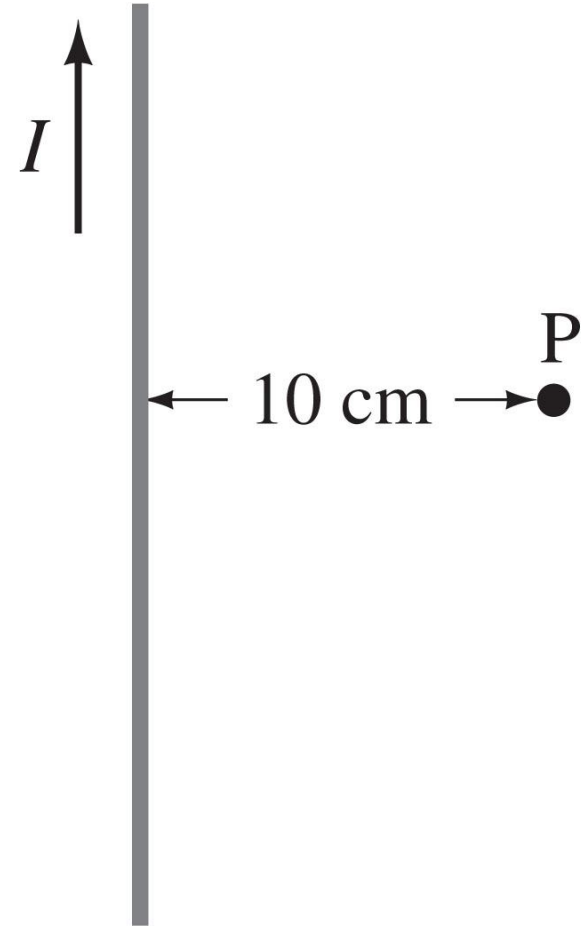


- Using  $\frac{\mu_0}{2\pi}$ , here provides a more elegant form for Ampere's Law and subsequent equations and yields a value for  $\mu_0$  of
- $\mu_0 = 4\pi \times 10^{-7} \frac{\text{Tesla} \cdot \text{meter}}{\text{Amp}}$
- Example: What is  $B$ , 10.0 cm from a wire carrying 25 A of current?

# 28-1 Magnetic Field Due to a Straight Wire

**Example 28-1: Calculation of  $\vec{B}$  near a wire.**

**An electric wire in the wall of a building carries a dc current of 25 A vertically upward. What is the magnetic field due to this current at a point P 10 cm due north of the wire?**



**EXAMPLE 28-1** **Calculation of  $\vec{B}$  near a wire.** An electric wire in the wall of a building carries a dc current of 25 A vertically upward. What is the magnetic field due to this current at a point P, 10 cm due north of the wire (Fig. 28-2)?

**APPROACH** We assume the wire is much longer than the 10-cm distance to the point P so we can apply Eq. 28-1.

**SOLUTION** According to Eq. 28-1:

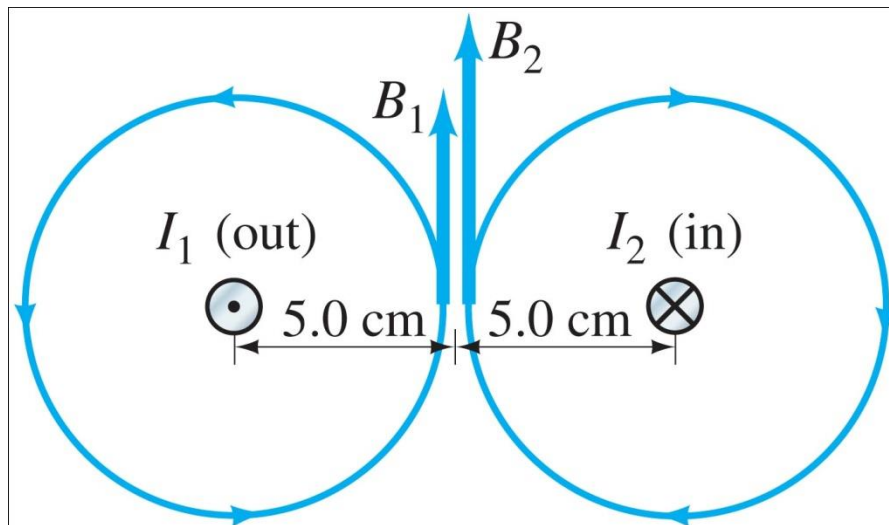
$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(25 \text{ A})}{(2\pi)(0.10 \text{ m})} = 5.0 \times 10^{-5} \text{ T},$$

or 0.50 G. By the right-hand rule (Table 27-1, page 716), the field due to the current points to the west (into the page in Fig. 28-2) at point P.

**NOTE** The wire's field has about the same magnitude as Earth's magnetic field, so a compass at P would not point north but in a northwesterly direction.

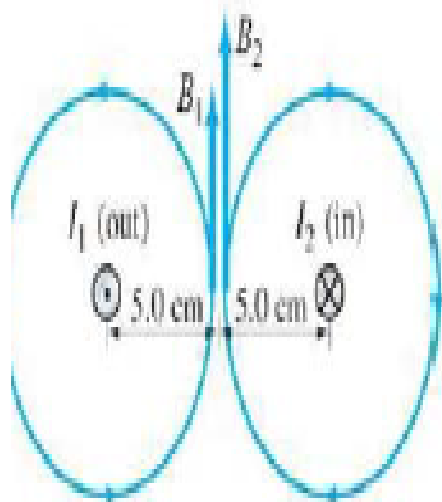
## Example 28-2: Magnetic field midway between two currents.

Two parallel straight wires 10.0 cm apart carry currents in opposite directions. Current  $I_1 = 5.0$  A is out of the page, and  $I_2 = 7.0$  A is into the page. Determine the magnitude and direction of the magnetic field halfway between the two wires.



**FIGURE 28-3** Example 28-2.

Wire 1 carrying current  $I_1$  out towards us, and wire 2 carrying current  $I_2$  into the page, produce magnetic fields whose lines are circles around their respective wires.



significant effect on a compass:

**EXAMPLE 28-2** **Magnetic field midway between two currents.** Two parallel straight wires 10.0 cm apart carry currents in opposite directions (Fig. 28-3). Current  $I_1 = 5.0$  A is out of the page, and  $I_2 = 7.0$  A is into the page. Determine the magnitude and direction of the magnetic field halfway between the two wires.

**APPROACH** The magnitude of the field produced by each wire is calculated from Eq. 28-1. The direction of *each* wire's field is determined with the right-hand rule. The total field is the vector sum of the two fields at the midway point.

**SOLUTION** The magnetic field lines due to current  $I_1$  form circles around the wire of  $I_1$ , and right-hand-rule-1 (Fig. 27-8c) tells us they point counterclockwise around the wire. The field lines due to  $I_2$  form circles around the wire of  $I_2$  and point clockwise, Fig. 28-3. At the midpoint, both fields point upward as shown, and so add together.

<sup>†</sup>The constant is chosen in this complicated way so that Ampère's law (Section 28-4), which is considered more fundamental, will have a simple and elegant form.

The midpoint is 0.050 m from each wire, and from Eq. 28-1 the magnitudes of  $B_1$  and  $B_2$  are

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(5.0 \text{ A})}{2\pi(0.050 \text{ m})} = 2.0 \times 10^{-5} \text{ T};$$

$$B_2 = \frac{\mu_0 I_2}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(7.0 \text{ A})}{2\pi(0.050 \text{ m})} = 2.8 \times 10^{-5} \text{ T}.$$

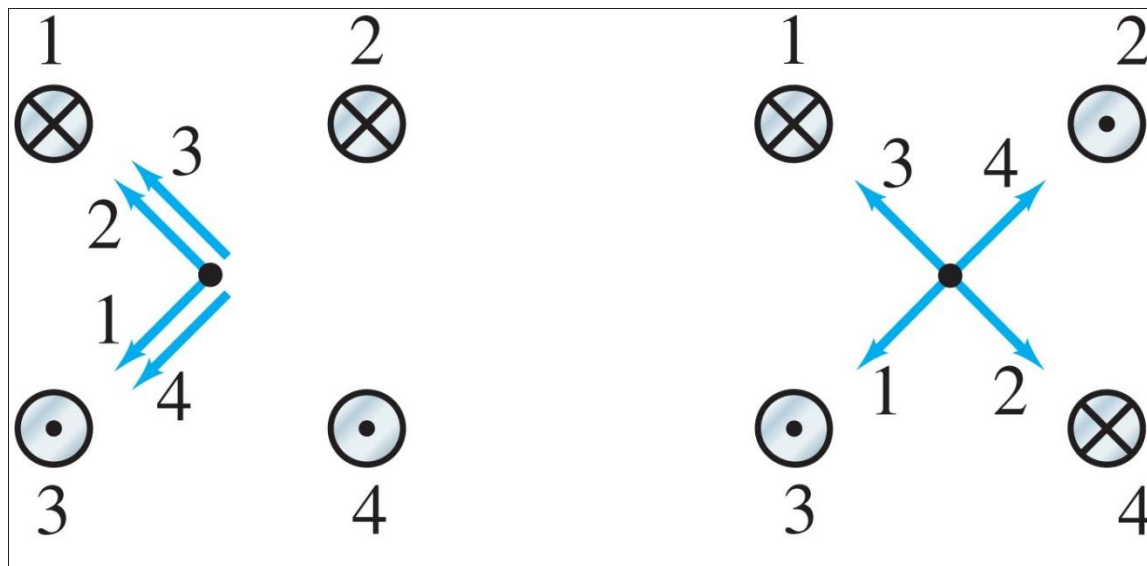
The total field is *up* with a magnitude of

$$B = B_1 + B_2 = 4.8 \times 10^{-5} \text{ T}.$$

# 28-1 Magnetic Field Due to a Straight Wire

**Conceptual Example 28-3: Magnetic field due to four wires.**

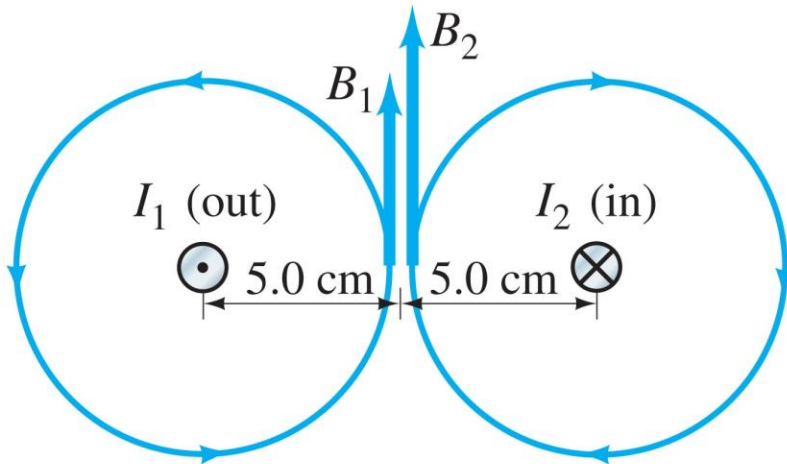
**This figure shows four long parallel wires which carry equal currents into or out of the page. In which configuration, (a) or (b), is the magnetic field greater at the center of the square?**



**CONCEPTUAL EXAMPLE 28-3** **Magnetic field due to four wires.** Figure 28-4 shows four long parallel wires which carry equal currents into or out of the page as shown. In which configuration, (a) or (b), is the magnetic field greater at the center of the square?

**RESPONSE** It is greater in (a). The arrows illustrate the directions of the field produced by each wire; check it out, using the right-hand rule to confirm these results. The net field at the center is the superposition of the four fields, which will point to the left in (a) and is zero in (b).

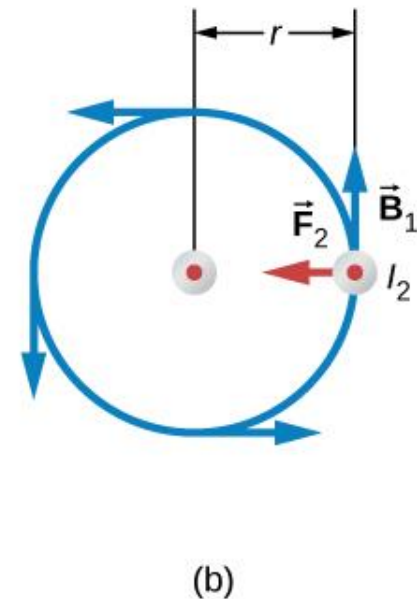
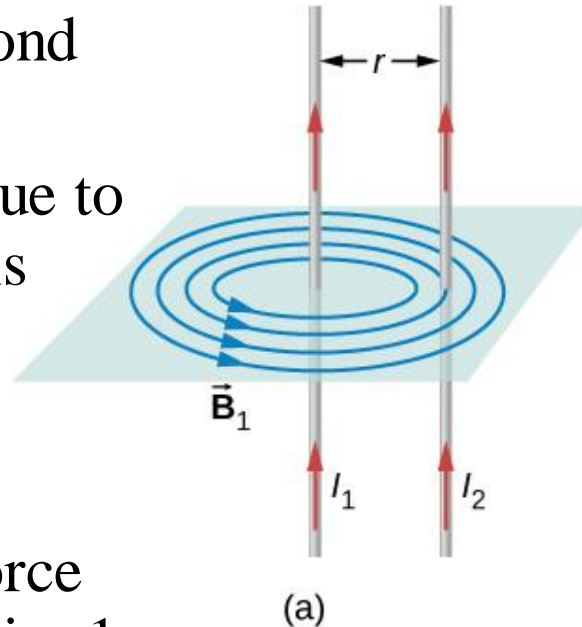
# Field Midway between 2 Wires



- Compute  $B$  at the midpoint between two straight wires 10.0 cm apart.
- $I_1 = 5.0$  A out of page.
- $I_2 = 70$  A in to page.
- From the diagram we see that we add the two fields.

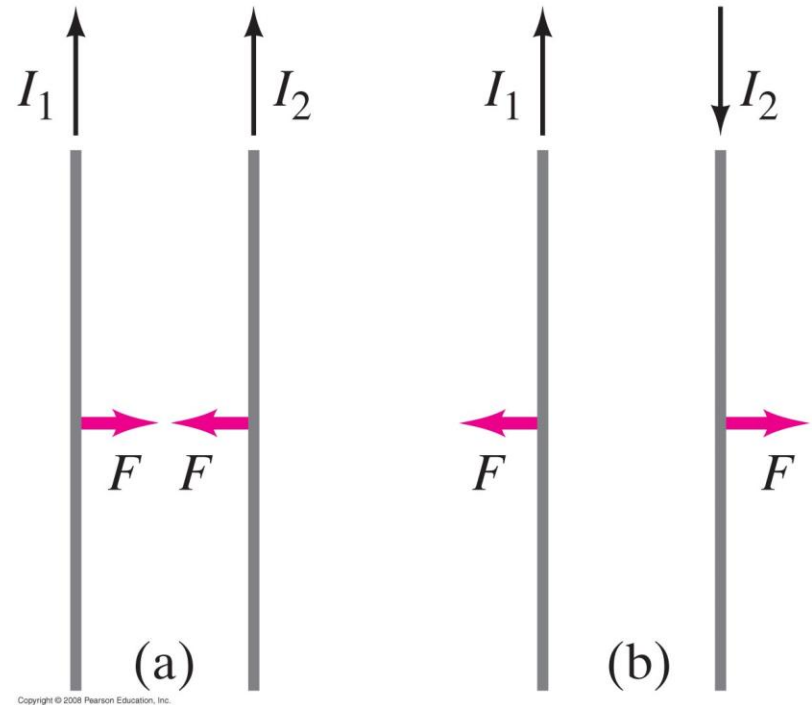
# Ampere's Force Law for two Parallel Wires

- If we have another wire carrying current parallel to our first wire, the second wire will feel a force.
- The force on wire 2 due to the B-field of wire 1 is
- $F_2 = I_2 l_2 B_1$  or
- $F_2 = I_2 l_2 \frac{\mu_0 I_1}{2\pi r}$
- From the RHR, the force on wire 2 is toward wire 1.



# Ampere's Force Law for two Parallel Wires

- Of course there is an equal and opposite force on wire 1 due to the magnetic field around wire 2.
- A separate calculation would confirm this.
- However, we know this from Newton's Third Law.
- If both currents flow in the opposite direction, the force is repulsive.

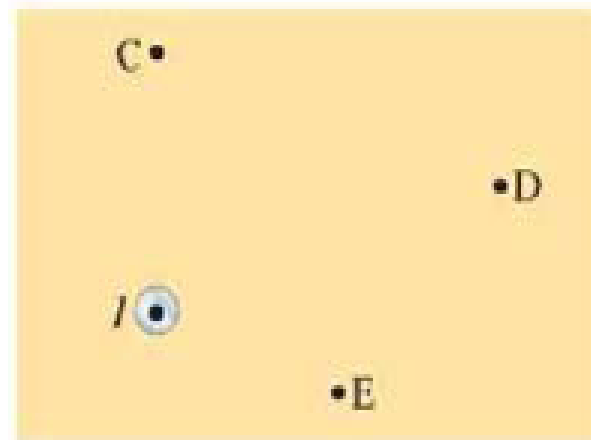


3. (I) Determine the magnitude and direction of the force between two parallel wires 25 m long and 4.0 cm apart, each carrying 35 A in the same direction.

Since the currents are parallel, the force on each wire will be attractive, toward the other wire. Use Eq. 28-2 to calculate the magnitude of the force.

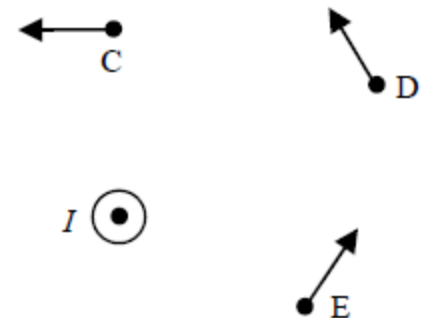
$$F_2 = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} \ell_2 = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})}{2\pi} \frac{(35 \text{ A})^2}{(0.040 \text{ m})} (25 \text{ m}) = \boxed{0.15 \text{ N, attractive}}$$

5. (I) In Fig. 28–33, a long straight wire carries current  $I$  out of the page toward the viewer. Indicate, with appropriate arrows, the direction of  $\vec{B}$  at each of the points C, D, and E in the plane of the page.

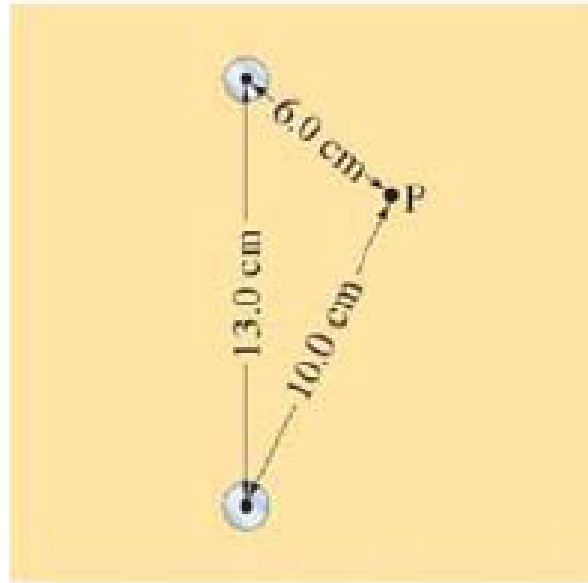


**FIGURE 28–33**  
Problem 5.

5. To find the direction, draw a radius line from the wire to the field point. Then at the field point, draw a perpendicular to the radius line, directed so that the perpendicular line would be part of a counterclockwise circle.



7. (II) Two long thin parallel wires 13.0 cm apart carry 35-A currents in the same direction. Determine the magnetic field vector at a point 10.0 cm from one wire and 6.0 cm from the other (Fig. 28–34).



**FIGURE 28–34**  
Problem 7.

7.

Since the magnetic field from a current carrying wire circles the wire, the individual field at point P from each wire is perpendicular to the radial line from that wire to point P. We define  $\vec{B}_1$  as the field from the top wire, and  $\vec{B}_2$  as the field from the bottom wire. We use Eq. 28-1 to calculate the magnitude of each individual field.

$$B_1 = \frac{\mu_0 I}{2\pi r_1} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(35 \text{ A})}{2\pi(0.060 \text{ m})} = 1.17 \times 10^{-4} \text{ T}$$

$$B_2 = \frac{\mu_0 I}{2\pi r_2} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(35 \text{ A})}{2\pi(0.100 \text{ m})} = 7.00 \times 10^{-5} \text{ T}$$

We use the law of cosines to determine the angle that the radial line from each wire to point P makes with the vertical. Since the field is perpendicular to the radial line, this is the same angle that the magnetic fields make with the horizontal.

$$\theta_1 = \cos^{-1} \left( \frac{(0.060 \text{ m})^2 + (0.130 \text{ m})^2 - (0.100 \text{ m})^2}{2(0.060 \text{ m})(0.130 \text{ m})} \right) = 47.7^\circ$$

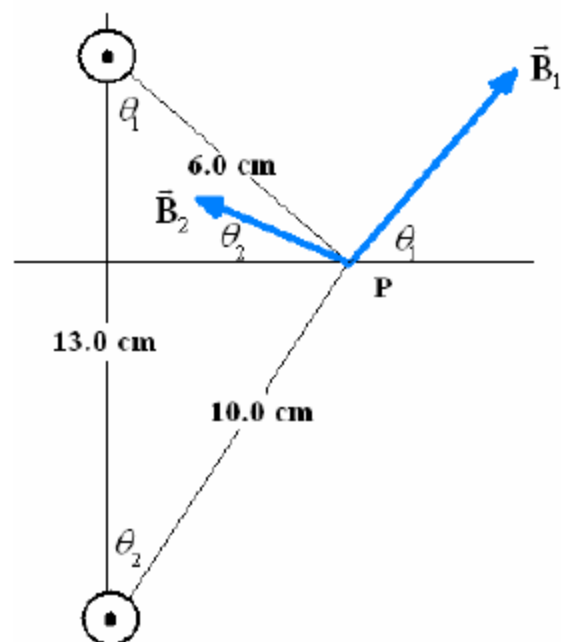
$$\theta_2 = \cos^{-1} \left( \frac{(0.100 \text{ m})^2 + (0.130 \text{ m})^2 - (0.060 \text{ m})^2}{2(0.100 \text{ m})(0.130 \text{ m})} \right) = 26.3^\circ$$

Using the magnitudes and angles of each magnetic field we calculate the horizontal and vertical components, add the vectors, and calculate the resultant magnetic field and angle.

$$B_{\text{net } x} = B_1 \cos(\theta_1) - B_2 \cos \theta_2 = (1.174 \times 10^{-4} \text{ T}) \cos 47.7^\circ - (7.00 \times 10^{-5} \text{ T}) \cos 26.3^\circ = 1.626 \times 10^{-5} \text{ T}$$

$$B_{\text{net } y} = B_1 \sin(\theta_1) + B_2 \sin \theta_1 = (1.17 \times 10^{-4} \text{ T}) \sin 47.7^\circ + (7.00 \times 10^{-5} \text{ T}) \sin 26.3^\circ = 1.18 \times 10^{-4} \text{ T}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(1.626 \times 10^{-5} \text{ T})^2 + (1.18 \times 10^{-4} \text{ T})^2} = 1.19 \times 10^{-4} \text{ T}$$



11. (II) Determine the magnetic field midway between two long straight wires 2.0 cm apart in terms of the current  $I$  in one when the other carries 25 A. Assume these currents are (a) in the same direction, and (b) in opposite directions.

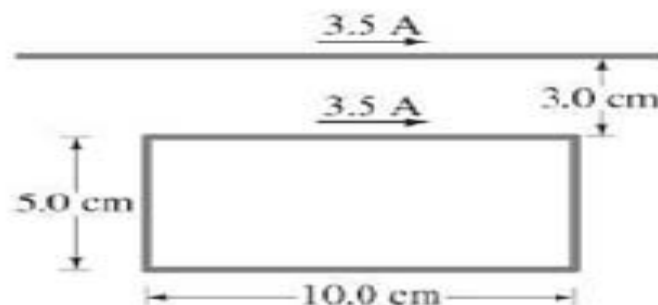
11. (a) If the currents are in the same direction, the magnetic fields at the midpoint between the two currents will oppose each other, and so their magnitudes should be subtracted.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})}{2\pi (0.010 \text{ m})} (I - 25 \text{ A}) = \boxed{(2.0 \times 10^{-5} \text{ T/A})(I - 25 \text{ A})}$$

- (b) If the currents are in the opposite direction, the magnetic fields at the midpoint between the two currents will reinforce each other, and so their magnitudes should be added.

$$B_{\text{net}} = \frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})}{2\pi (0.010 \text{ m})} (I + 25 \text{ A}) = \boxed{(2.0 \times 10^{-5} \text{ T/A})(I + 25 \text{ A})}$$

18. (II) A rectangular loop of wire is placed next to a straight wire, as shown in Fig. 28-37. There is a current of 3.5 A in both wires. Determine the magnitude and direction of the net force on the loop.



**FIGURE 28-37**  
Problem 18.

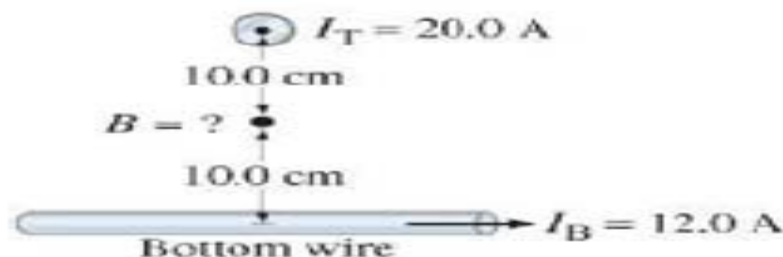
The magnetic field at the loop due to the long wire is into the page, and can be calculated by Eq. 28-1. The force on the segment of the loop closest to the wire is towards the wire, since the currents are in the same direction. The force on the segment of the loop farthest from the wire is away from the wire, since the currents are in the opposite direction.

Because the magnetic field varies with distance, it is more difficult to calculate the total force on the left and right segments of the loop. Using the right hand rule, the force on each small piece of the left segment of wire is to the left, and the force on each small piece of the right segment of wire is to the right. If left and right small pieces are chosen that are equidistant from the long wire, the net force on those two small pieces is zero. Thus the total force on the left and right segments of wire is zero, and so only the parallel segments need to be considered in the calculation. Use Eq. 28-2.

$$\begin{aligned}
 F_{\text{net}} &= F_{\text{near}} - F_{\text{far}} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d_{\text{near}}} \ell_{\text{near}} - \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d_{\text{far}}} \ell_{\text{far}} = \frac{\mu_0}{2\pi} I_1 I_2 \ell \left( \frac{1}{d_{\text{near}}} - \frac{1}{d_{\text{far}}} \right) \\
 &= \frac{4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}}{2\pi} (3.5 \text{ A})^2 (0.100 \text{ m}) \left( \frac{1}{0.030 \text{ m}} - \frac{1}{0.080 \text{ m}} \right) = \boxed{5.1 \times 10^{-6} \text{ N, towards wire}}
 \end{aligned}$$

- 21.** (II) Two long wires are oriented so that they are perpendicular to each other. At their closest, they are 20.0 cm apart (Fig. 28–39). What is the magnitude of the magnetic field at a point midway between them if the top one carries a current of 20.0 A and the bottom one carries 12.0 A?

**FIGURE 28–39**  
Problem 21.



21. The magnetic fields created by the individual currents will be at right angles to each other. The field due to the top wire will be to the right, and the field due to the bottom wire will be out of the page. Since they are at right angles, the net field is the hypotenuse of the two individual fields.

$$\begin{aligned}
 B_{\text{net}} &= \sqrt{\left(\frac{\mu_0 I_{\text{top}}}{2\pi r_{\text{top}}}\right)^2 + \left(\frac{\mu_0 I_{\text{bottom}}}{2\pi r_{\text{bottom}}}\right)^2} = \frac{\mu_0}{2\pi r} \sqrt{I_{\text{top}}^2 + I_{\text{bottom}}^2} = \frac{4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}}{2\pi (0.100 \text{ m})} \sqrt{(20.0 \text{ A})^2 + (12.0 \text{ A})^2} \\
 &= \boxed{4.66 \times 10^{-5} \text{ T}}
 \end{aligned}$$

**Example 28-4. Force between two current-carrying wires.**

**The two wires of a 2.0-m-long appliance cord are 3.0 mm apart and carry a current of 8.0 A dc. Calculate the force one wire exerts on the other.**

**EXAMPLE 28-4** **Force between two current-carrying wires.** The two wires of a 2.0-m-long appliance cord are 3.0 mm apart and carry a current of 8.0 A dc. Calculate the force one wire exerts on the other.

**APPROACH** Each wire is in the magnetic field of the other when the current is on, so we can apply Eq. 28-2.

**SOLUTION** Equation 28-2 gives

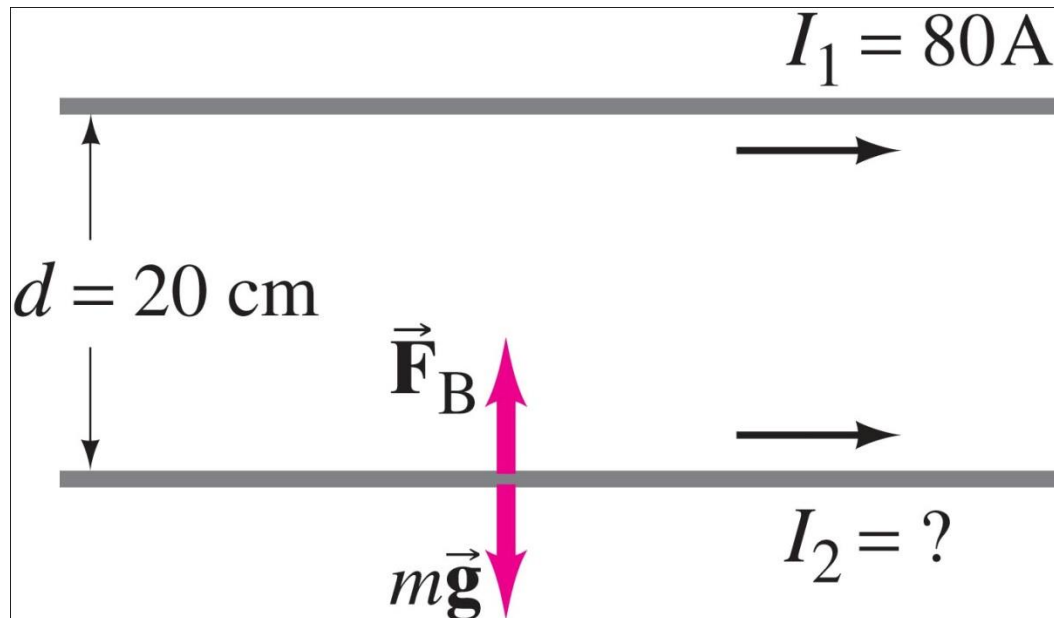
$$F = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(8.0 \text{ A})^2(2.0 \text{ m})}{(2\pi)(3.0 \times 10^{-3} \text{ m})} = 8.5 \times 10^{-3} \text{ N}.$$

The currents are in opposite directions (one toward the appliance, the other away from it), so the force would be repulsive and tend to spread the wires apart.

# 28-2 Force between Two Parallel Wires

## Example 28-5: Suspending a wire with a current.

A horizontal wire carries a current  $I_1 = 80 \text{ A}$  dc. A second parallel wire 20 cm below it must carry how much current  $I_2$  so that it doesn't fall due to gravity? The lower wire has a mass of 0.12 g per meter of length.



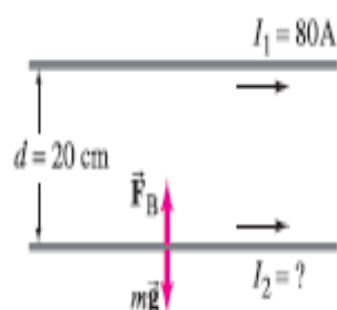


FIGURE 28-7 Example 28-5.

**EXAMPLE 28-5 Suspending a wire with a current.** A horizontal wire carries a current  $I_1 = 80 \text{ A}$  dc. A second parallel wire 20 cm below it (Fig. 28-7) must carry how much current  $I_2$  so that it doesn't fall due to gravity? The lower wire has a mass of 0.12 g per meter of length.

**APPROACH** If wire 2 is not to fall under gravity, which acts downward, the magnetic force on it must be upward. This means that the current in the two wires must be in the same direction (Fig. 28-6). We can find the current  $I_2$  by equating the magnitudes of the magnetic force and the gravitational force on the wire.

**SOLUTION** The force of gravity on wire 2 is downward. For each 1.0 m of wire length, the gravitational force has magnitude

$$F = mg = (0.12 \times 10^{-3} \text{ kg/m})(1.0 \text{ m})(9.8 \text{ m/s}^2) = 1.18 \times 10^{-3} \text{ N}.$$

The magnetic force on wire 2 must be upward, and Eq. 28-2 gives

$$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} \ell$$

where  $d = 0.20 \text{ m}$  and  $I_1 = 80 \text{ A}$ . We solve this for  $I_2$  and set the two force magnitudes equal (letting  $\ell = 1.0 \text{ m}$ ):

$$I_2 = \frac{2\pi d}{\mu_0 I_1} \left( \frac{F}{\ell} \right) = \frac{2\pi(0.20 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(80 \text{ A})} \frac{(1.18 \times 10^{-3} \text{ N/m})}{(1.0 \text{ m})} = 15 \text{ A}.$$

# 28-3 Definitions of the Ampere and the Coulomb

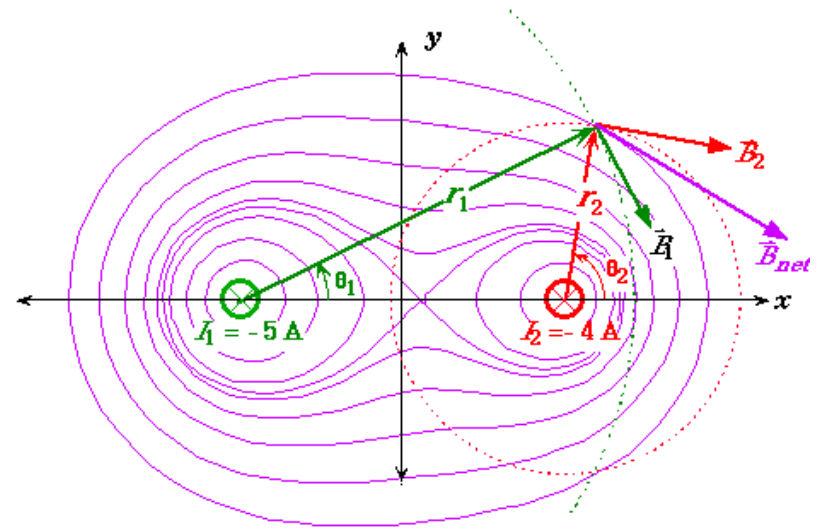
**The ampere is officially defined in terms of the force between two current-carrying wires:**

*One ampere is defined as that current flowing in each of two long parallel wires 1 m apart, which results in a force of exactly  $2 \times 10^{-7}$  N per meter of length of each wire.*

**The coulomb is then defined as exactly one ampere-second.**

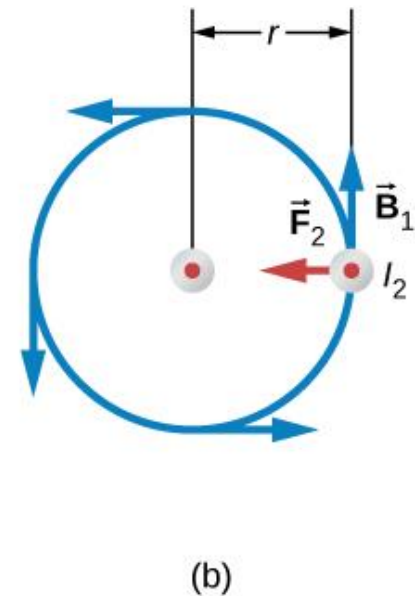
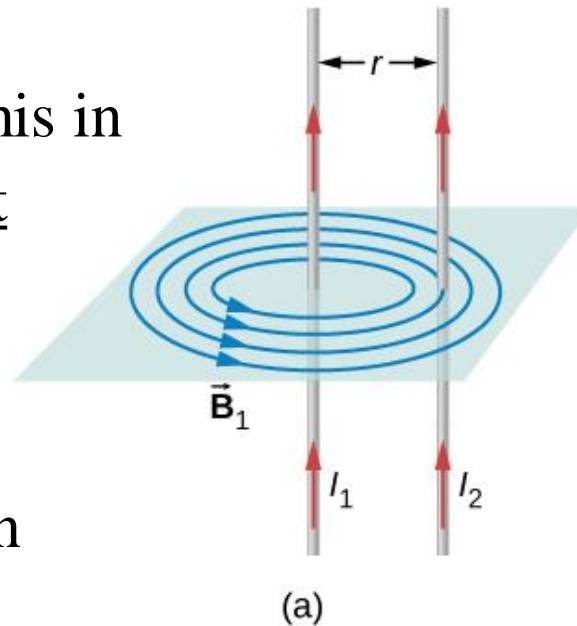
# Ampere's Force Law for two Parallel Wires

- For two wires where the current is flowing in the same direction, the magnetic fields combine and surround both wires.
- The figure at right shows an example.



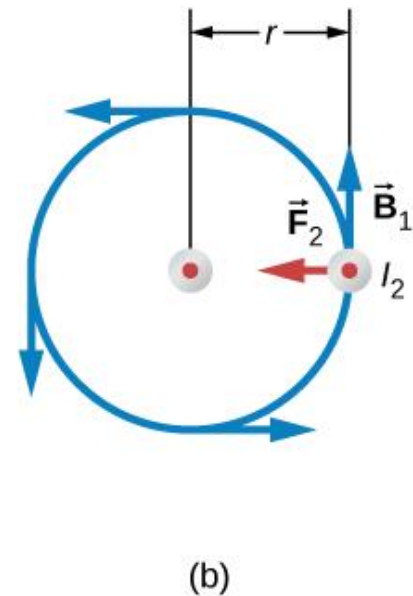
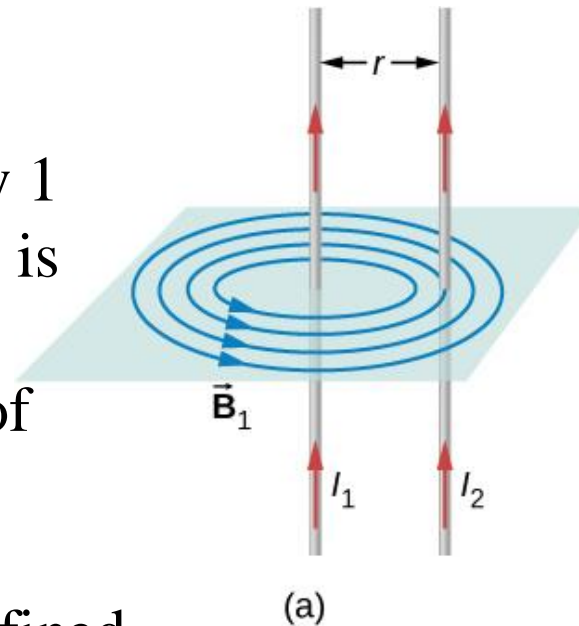
# Ampere's Force Law for two Parallel Wires

- $F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$
- We can also rewrite this in terms of force per unit length
- $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$
- From this form we can now understand the definition of the Ampere.



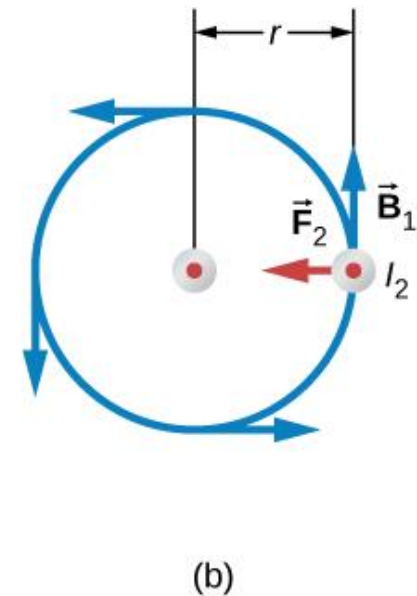
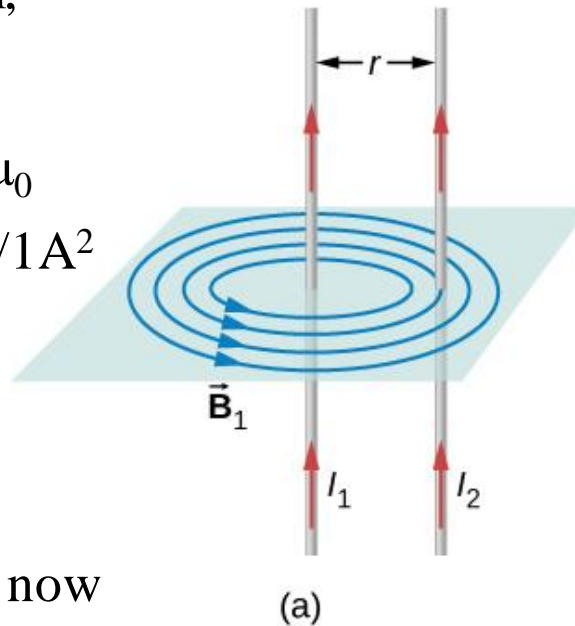
# Ampere's Force Law for two Parallel Wires

- $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$
- If two very long, thin wires are separated by 1 meter, and the current is adjusted to produce a force per unit length of
- $\frac{F}{l} = 2 \times 10^{-7} \text{ N/m}$
- Then the current is defined to be 1 Ampere.



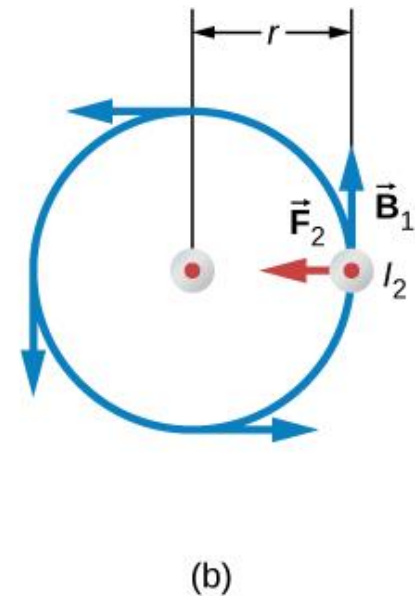
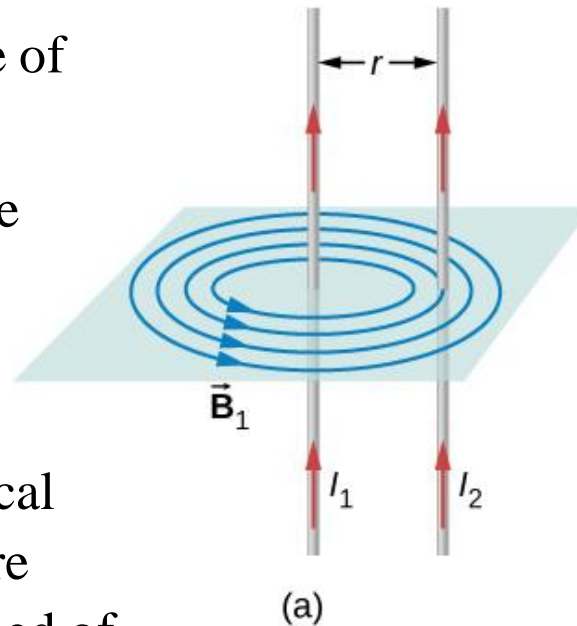
# Ampere's Force Law for two Parallel Wires

- If,  $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d} = 2 \times 10^{-7} \text{ N/m}$ ,
- where  $I_1 = I_2 = 1 \text{ Ampere}$
- Also defines the value of  $\mu_0$
- $\mu_0 = (2 \times 10^{-7} \text{ N/m}) \cdot 2\pi \cdot 1 \text{ m/1 A}^2$
- $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$
- $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$
- In addition, the Ampere is now used to define the Coulomb, Volt, etc.



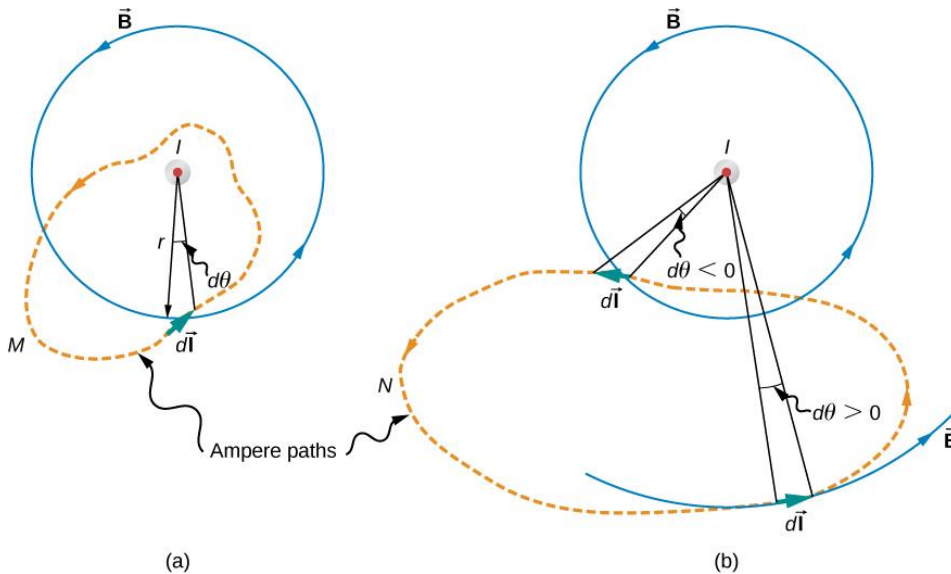
# Ampere's Force Law for two Parallel Wires

- $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$
- This also defines the value of our previous constant
- $\epsilon_0$  - the permittivity of free space which is defined as
- $\epsilon_0 = \frac{1}{c^2 \cdot \mu_0}$
- This means that all electrical and magnetic properties are defined in terms of the speed of light and the ampere.

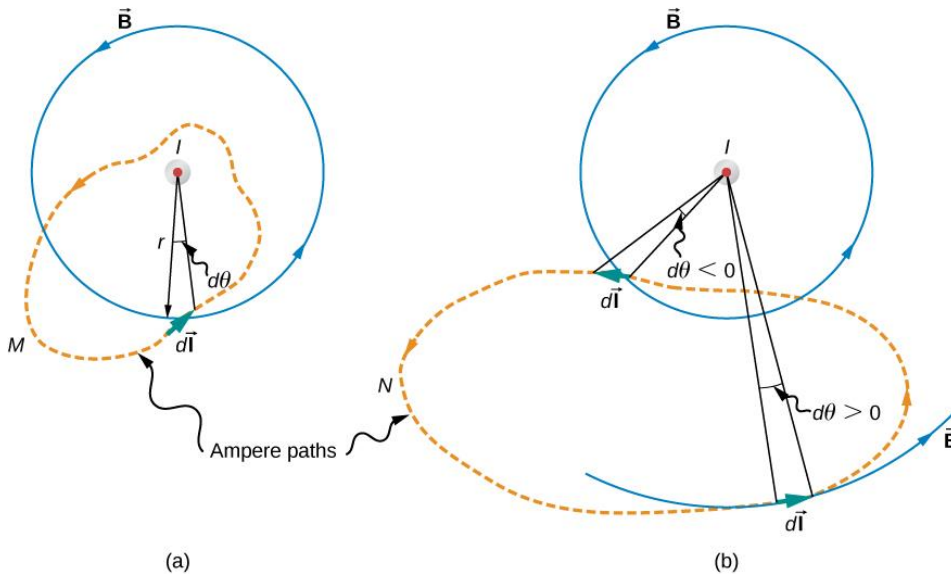


# Ampere's Circuital Law

- Shortly after Oersted discovered that a current in a wire produces a magnetic field, Maxwell developed a general law for determining the magnetic fields produced by a current.

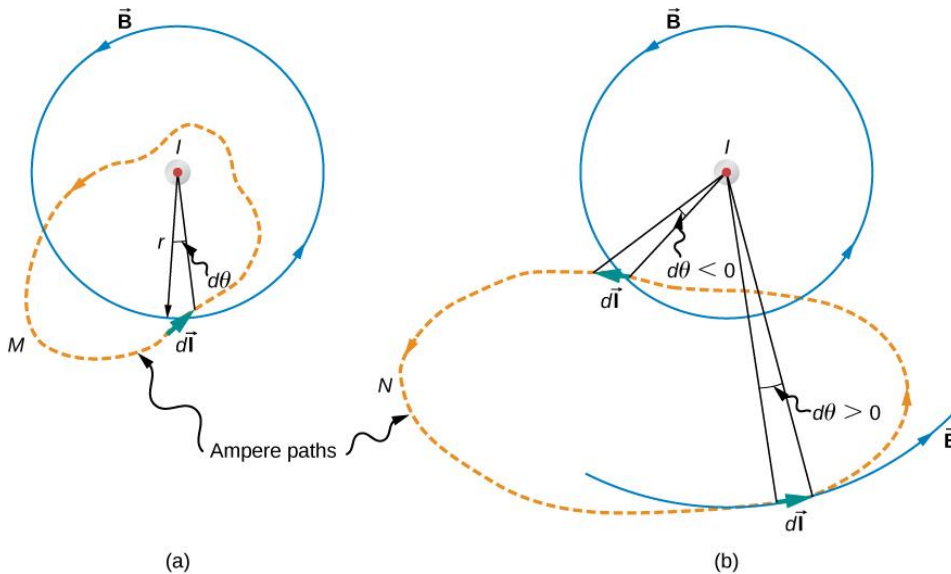


# Ampere's Circuital Law



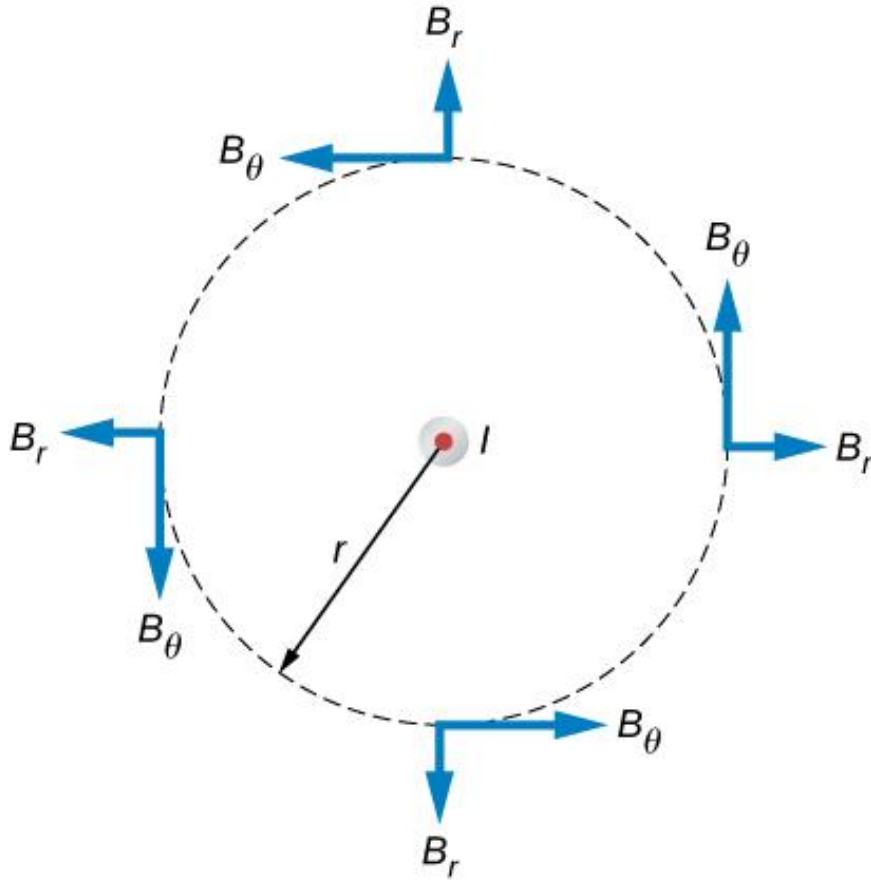
- For any arbitrary closed path around a current,
- $\sum B_{\parallel} \cdot \Delta l = \mu_0 I_{enc}$
- Where  $B_{\parallel}$  is the component in the direction of  $\Delta l$
- The lengths of  $\Delta l$  must be short so that  $B_{\parallel}$  remains nearly constant.

# Ampere's Circuital Law



- If we shrink  $\Delta l$  then the sum becomes the integral.
- $\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{Enc}$
- For example (b), the loop encloses no current and the integral has zero value.

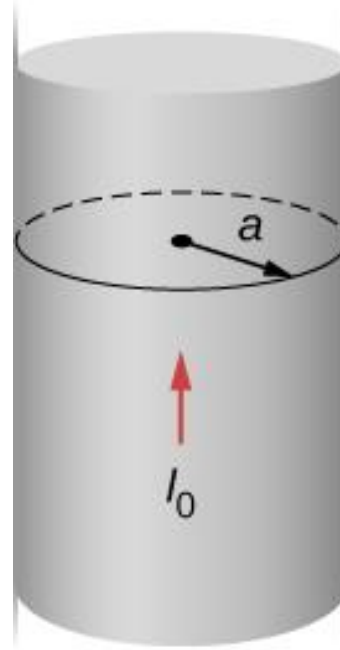
# Ampere's Circuital Law



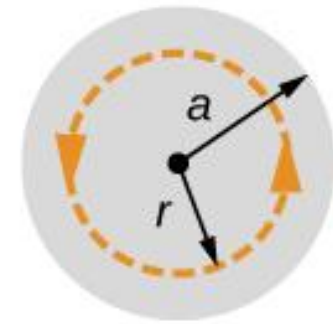
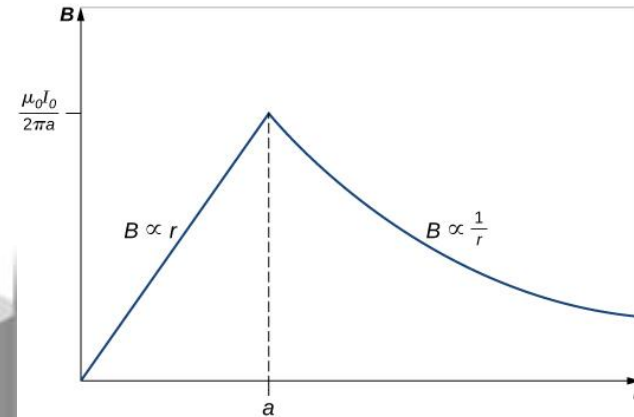
- If we apply Ampere's law to a long straight wire,
- $\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot I_{Enc}$
- If we choose our  $d\vec{l}$ 's to form a circular path, then  $B_{\parallel}$  is simply  $B$  at distance  $r$ .
- The integral of  $d\vec{l}$  around the circle gives
- $2\pi r B = \mu_0 I_{enc}$  or
- $B = \mu_0 I_{enc} / 2\pi r$
- Matching our earlier result.

# Magnetic Field of a Thick Wire

- For a thick wire, the field outside the wire is the same as for a thin wire.
- However, inside the wire,  $I_{enc} = \frac{I_0}{\pi a^2} \cdot \pi r^2$
- And  $\oint dl = 2\pi r$ , so
- $B = \frac{I_0 \cdot r}{2\pi \cdot a^2}$



(a)

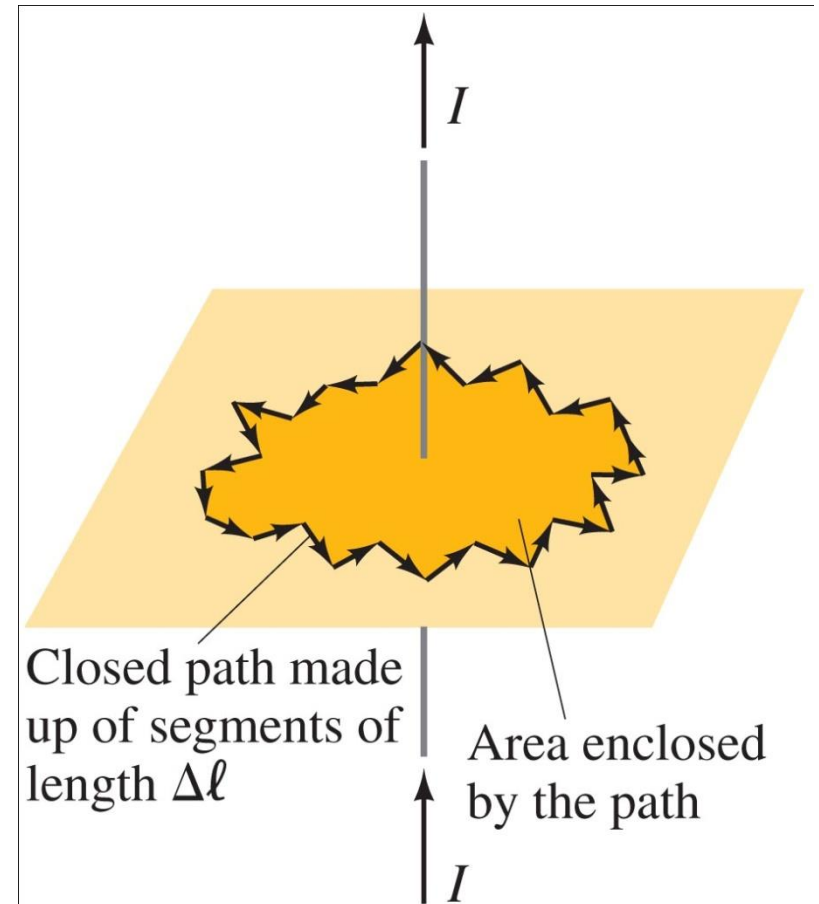


(b)

**Ampère's law relates the magnetic field around a closed loop to the total current flowing through the loop:**

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl.}}$$

**This integral is taken around the edge of the closed loop.**

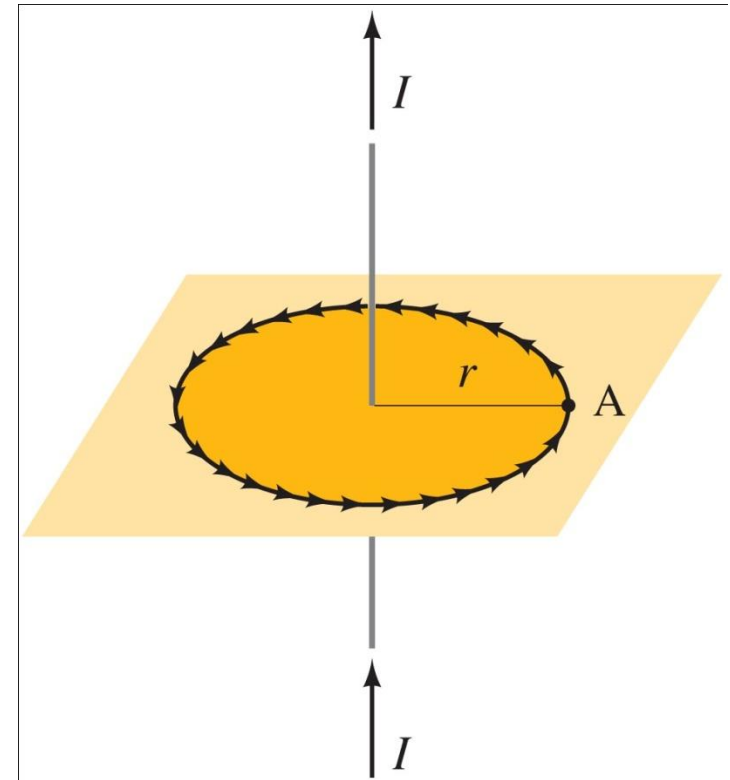


Using Ampère's law to find the field around a long straight wire:

Use a circular path with the wire at the center; then  $\vec{B}$  is tangent to  $d\vec{\ell}$  at every point. The integral then gives

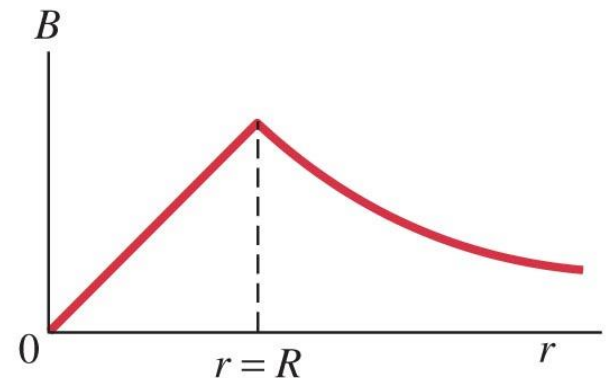
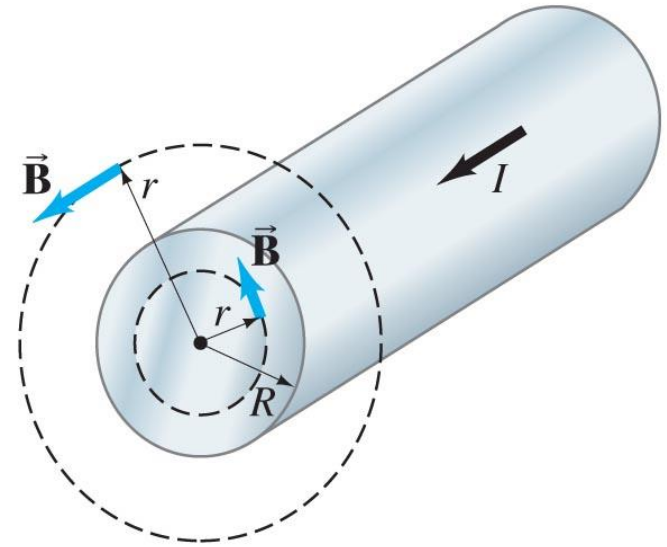
$$\begin{aligned}\mu_0 I &= \oint \vec{B} \cdot d\vec{\ell} \\ &= \oint B \, d\ell = B \oint d\ell = B(2\pi r).\end{aligned}$$

so  $B = \mu_0 I / 2\pi r$ , as before.



## Example 28-6: Field inside and outside a wire.

A long straight cylindrical wire conductor of radius  $R$  carries a current  $I$  of uniform current density in the conductor. Determine the magnetic field due to this current at (a) points outside the conductor ( $r > R$ ) and (b) points inside the conductor ( $r < R$ ). Assume that  $r$ , the radial distance from the axis, is much less than the length of the wire. (c) If  $R = 2.0$  mm and  $I = 60$  A, what is  $B$  at  $r = 1.0$  mm,  $r = 2.0$  mm, and  $r = 3.0$  mm?



**EXAMPLE 28-6 Field inside and outside a wire.** A long straight cylindrical wire conductor of radius  $R$  carries a current  $I$  of uniform current density in the conductor. Determine the magnetic field due to this current at (a) points outside the conductor ( $r > R$ ), and (b) points inside the conductor ( $r < R$ ). See Fig. 28-11. Assume that  $r$ , the radial distance from the axis, is much less than the length of the wire. (c) If  $R = 2.0 \text{ mm}$  and  $I = 60 \text{ A}$ , what is  $B$  at  $r = 1.0 \text{ mm}$ ,  $r = 2.0 \text{ mm}$ , and  $r = 3.0 \text{ mm}$ ?

**APPROACH** We can use symmetry: Because the wire is long, straight, and cylindrical, we expect from symmetry that the magnetic field must be the same at all points that are the same distance from the center of the conductor. There is no reason why any such point should have preference over others at the same distance from the wire (they are physically equivalent). So  $B$  must have the same value at all points the same distance from the center. We also expect  $\vec{B}$  to be tangent to circles around the wire (Fig. 28-1), so we choose a circular path of integration as we did in Fig. 28-9.

**SOLUTION** (a) We apply Ampère's law, integrating around a circle ( $r > R$ ) centered on the wire (Fig. 28-11a), and then  $I_{\text{encl}} = I$ :

$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_{\text{encl}}$$

or

$$B = \frac{\mu_0 I}{2\pi r}, \quad [r > R]$$

which is the same result as for a thin wire.

(b) Inside the wire ( $r < R$ ), we again choose a circular path concentric with the cylinder; we expect  $\vec{B}$  to be tangential to this path, and again, because of the symmetry, it will have the same magnitude at all points on the circle. The current enclosed in this case is less than  $I$  by a factor of the ratio of the areas:

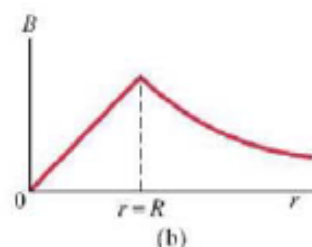
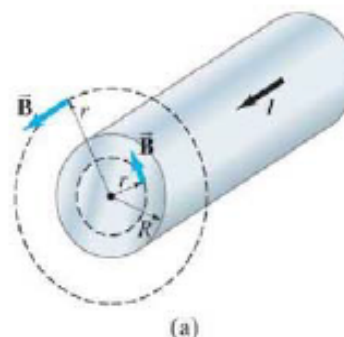
$$I_{\text{encl}} = I \frac{\pi r^2}{\pi R^2}.$$

So Ampère's law gives

$$\begin{aligned} \oint \vec{B} \cdot d\vec{\ell} &= \mu_0 I_{\text{encl}} \\ B(2\pi r) &= \mu_0 I \left( \frac{\pi r^2}{\pi R^2} \right) \end{aligned}$$

so

$$B = \frac{\mu_0 I r}{2\pi R^2}. \quad [r < R]$$



**FIGURE 28-11** Magnetic field inside and outside a cylindrical conductor (Example 28-6).

The field is zero at the center of the conductor and increases linearly with  $r$  until  $r = R$ ; beyond  $r = R$ ,  $B$  decreases as  $1/r$ . This is shown in Fig. 28-11b. Note that these results are valid only for points close to the center of the conductor as compared to its length. For a current to flow, there must be connecting wires (to a battery, say), and the field due to these conducting wires, if not very far away, will destroy the assumed symmetry.



C  
as

(c) At  $r = 2.0$  mm, the surface of the wire,  $r = R$ , so

$$B = \frac{\mu_0 I}{2\pi R} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(60 \text{ A})}{(2\pi)(2.0 \times 10^{-3} \text{ m})} = 6.0 \times 10^{-3} \text{ T}.$$

We saw in (b) that inside the wire  $B$  is linear in  $r$ . So at  $r = 1.0$  mm,  $B$  will be half what it is at  $r = 2.0$  mm or  $3.0 \times 10^{-3}$  T. Outside the wire,  $B$  falls off as  $1/r$ , so at  $r = 3.0$  mm it will be two-thirds as great as at  $r = 2.0$  mm, or  $B = 4.0 \times 10^{-3}$  T. To check, we use our result in (a),  $B = \mu_0 I / 2\pi r$ , which gives the same result.

- 27.** (I) A 2.5-mm-diameter copper wire carries a 33-A current (uniform across its cross section). Determine the magnetic field: (a) at the surface of the wire; (b) inside the wire, 0.50 mm below the surface; (c) outside the wire 2.5 mm from the surface.

27. (a) We use Eq. 28-1, with  $r$  equal to the radius of the wire.

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(33 \text{ A})}{2\pi(1.25 \times 10^{-3} \text{ m})} = \boxed{5.3 \text{ mT}}$$

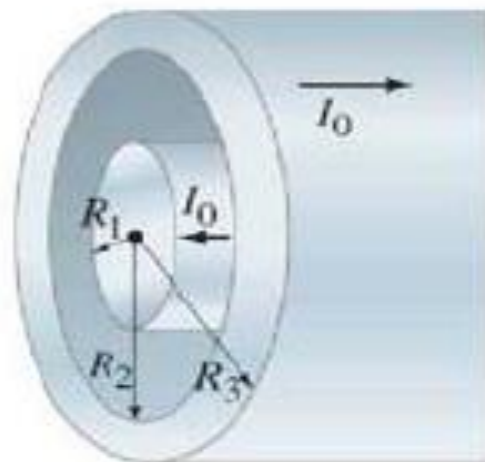
(b) We use the results of Example 28-6, for points inside the wire. Note that  $r = (1.25 - 0.50) \text{ mm} = 0.75 \text{ mm}$ .

$$B = \frac{\mu_0 I r}{2\pi R^2} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(33 \text{ A})(0.75 \times 10^{-3} \text{ m})}{2\pi(1.25 \times 10^{-3} \text{ m})^2} = \boxed{3.2 \text{ mT}}$$

(c) We use Eq. 28-1, with  $r$  equal to the distance from the center of the wire.

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(33 \text{ A})}{2\pi(1.25 \times 10^{-3} \text{ m} + 2.5 \times 10^{-3} \text{ m})} = \boxed{1.8 \text{ mT}}$$

- 31.** (II) A coaxial cable consists of a solid inner conductor of radius  $R_1$ , surrounded by a concentric cylindrical tube of inner radius  $R_2$  and outer radius  $R_3$  (Fig. 28-42). The conductors carry equal and opposite currents  $I_0$  distributed uniformly across their cross sections. Determine the magnetic field at a distance  $R$  from the axis for:
- (a)  $R < R_1$ ; (b)  $R_1 < R < R_2$ ;
  - (c)  $R_2 < R < R_3$ ; (d)  $R > R_3$ .
  - (e) Let  $I_0 = 1.50$  A,  $R_1 = 1.00$  cm,  $R_2 = 2.00$  cm, and  $R_3 = 2.50$  cm. Graph  $B$  from  $R = 0$  to  $R = 3.00$  cm.



**FIGURE 28-42**  
Problems 31 and 32.

31. Because of the cylindrical symmetry, the magnetic fields will be circular. In each case, we can determine the magnetic field using Ampere's law with concentric loops. The current densities in the wires are given by the total current divided by the cross-sectional area.

$$J_{\text{inner}} = \frac{I_0}{\pi R_1^2} \quad J_{\text{outer}} = -\frac{I_0}{\pi(R_3^2 - R_2^2)}$$

- (a) Inside the inner wire the enclosed current is determined by the current density of the inner wire.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0 (J_{\text{inner}} \pi R^2)$$

$$B(2\pi R) = \mu_0 \frac{I_0 \pi R^2}{\pi R_1^2} \rightarrow \boxed{B = \frac{\mu_0 I_0 R}{2\pi R_1^2}}$$

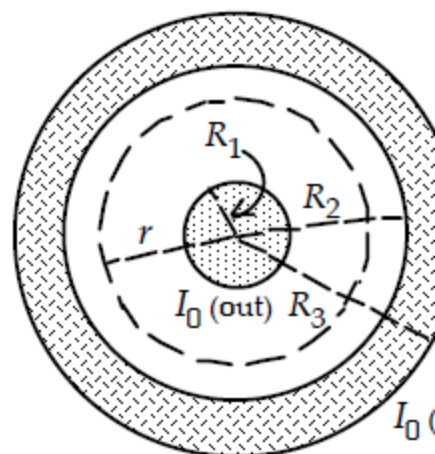
- (b) Between the wires the current enclosed is the current on the inner wire.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encl}} \rightarrow B(2\pi R) = \mu_0 I_0 \rightarrow \boxed{B = \frac{\mu_0 I_0}{2\pi R}}$$

- (c) Inside the outer wire the current enclosed is the current from the inner wire and a portion of the current from the outer wire.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = \mu_0 \left[ I_0 + J_{\text{outer}} \pi (R^2 - R_2^2) \right]$$

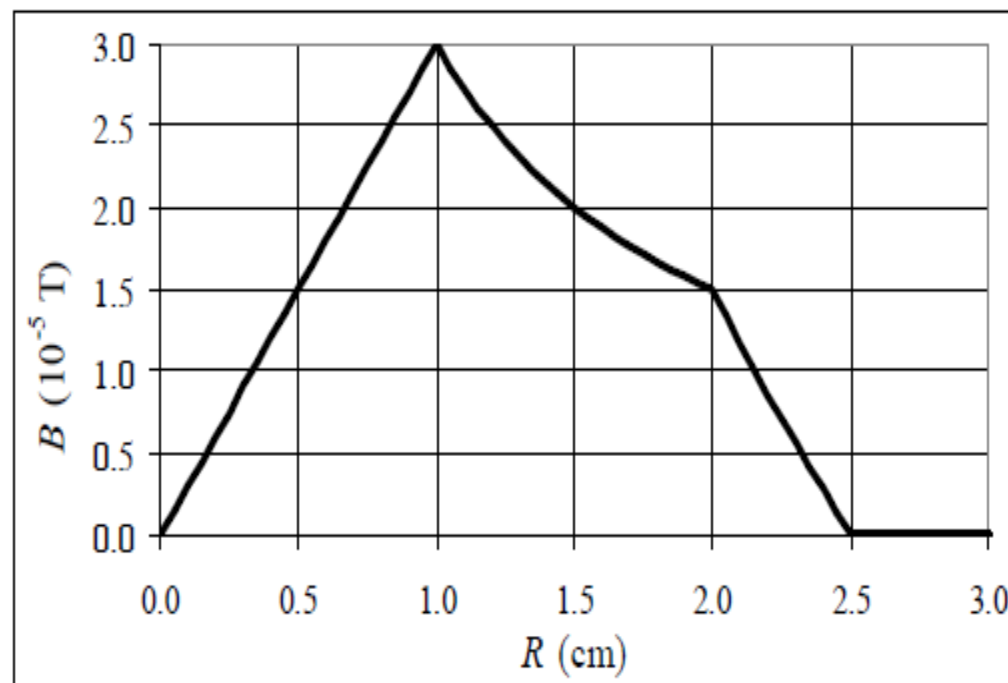
$$B(2\pi r) = \mu_0 \left[ I_0 - I_0 \frac{\pi (R^2 - R_2^2)}{\pi (R_3^2 - R_2^2)} \right] \rightarrow \boxed{B = \frac{\mu_0 I_0 (R_3^2 - R^2)}{2\pi R (R_3^2 - R_2^2)}}$$



(d) Outside the outer wire the net current enclosed is zero.

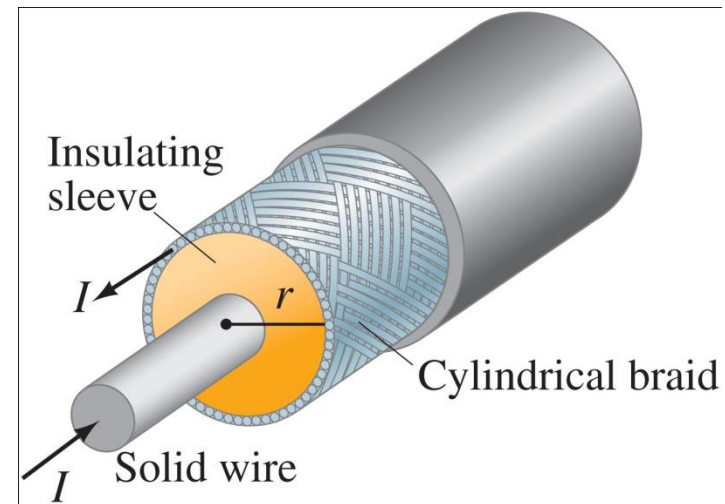
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encl}} = 0 \rightarrow B(2\pi R) = 0 \rightarrow \boxed{B = 0}$$

(e) See the adjacent graph. The spreadsheet used for this problem can be found on the Media Manager, with filename “PSE4\_ISM\_CH28.XLS,” on tab “Problem 28.31e.”



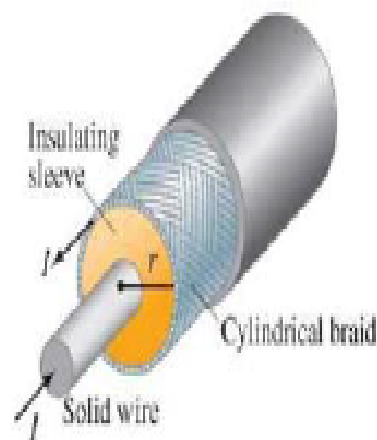
## Conceptual Example 28-7: Coaxial cable.

A coaxial cable is a single wire surrounded by a cylindrical metallic braid. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (a) in the space between the conductors, and (b) outside the cable.



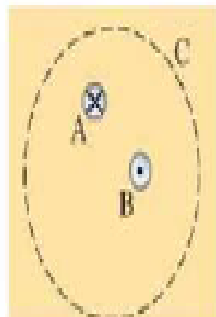
## PHYSICS APPLIED

Coaxial cable  
(shielding)



**FIGURE 28-12** Coaxial cable.  
Example 28-7.

**FIGURE 28-13** Exercise C.



**CONCEPTUAL EXAMPLE 28-7 Coaxial cable.** A *coaxial cable* is a single wire surrounded by a cylindrical metallic braid, as shown in Fig. 28-12. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (*a*) in the space between the conductors, and (*b*) outside the cable.

**RESPONSE** (*a*) In the space between the conductors, we can apply Ampère's law for a circular path around the center wire, just as we did for the case shown in Figs. 28-9 and 28-11. The magnetic field lines will be concentric circles centered on the center of the wire, and the magnitude is given by Eq. 28-1. The current in the outer conductor has no bearing on this result. (Ampère's law uses only the current enclosed *inside* the path; as long as the currents outside the path don't affect the symmetry of the field, they do not contribute to the field along the path at all). (*b*) Outside the cable, we can draw a similar circular path, for we expect the field to have the same cylindrical symmetry. Now, however, there are two currents enclosed by the path, and they add up to zero. The field outside the cable is zero.

The nice feature of coaxial cables is that they are self-shielding: no stray magnetic fields exist outside the cable. The outer cylindrical conductor also shields external electric fields from coming in (see also Example 21-14). This makes them ideal for carrying signals near sensitive equipment. Audiophiles use coaxial cables between stereo equipment components and even to the loudspeakers.

**EXERCISE C** In Fig. 28-13, the dashed circle  $C$  is a circular path of radius  $R$ . The current in the wire at  $A$  is  $I_A$  and the current in the wire at  $B$  is  $I_B$ . (a) What is the magnetic field at the center of the path?

## Example 28-8: A nice use for Ampère's law.

### 28-4 Ampère's Law

Use Ampère's law to show that in any region of space where there are no currents the magnetic field cannot be both unidirectional and nonuniform as shown in the figure.

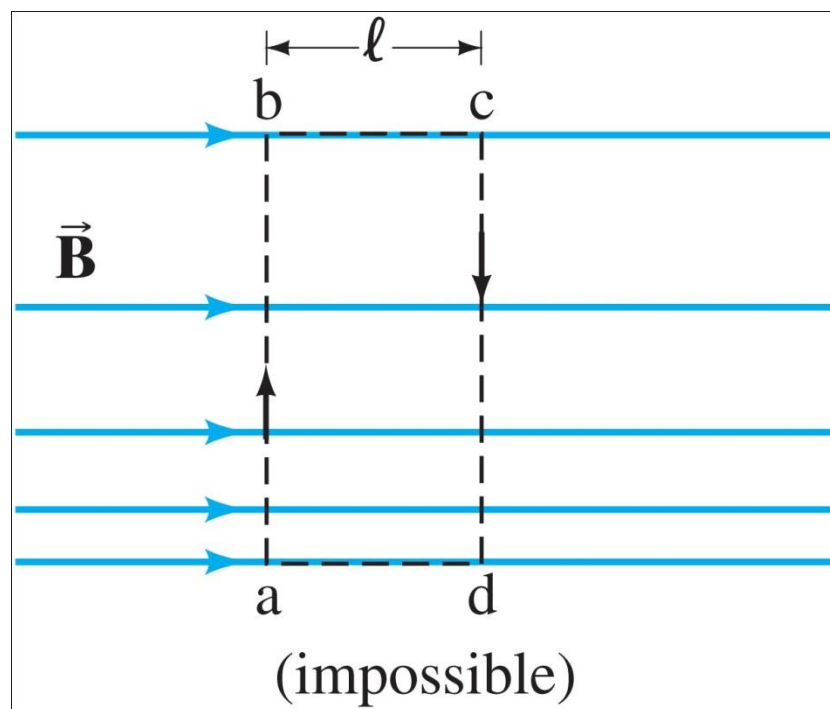
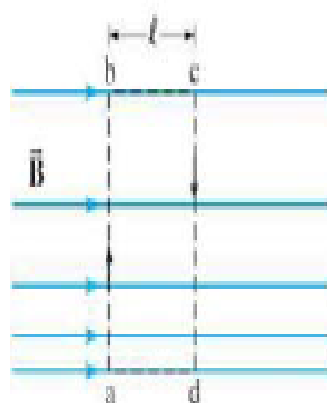
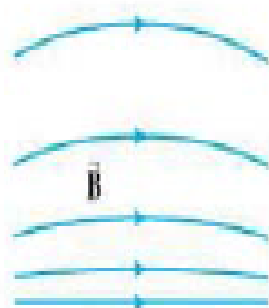


FIGURE 28-14 Example 28-8.



(a) (impossible)



(b) (possible)

WORKING  
EXAMPLE

**EXAMPLE 28-8 A nice use for Ampère's law.** Use Ampère's law to show that in any region of space where there are no currents the magnetic field cannot be both unidirectional and nonuniform as shown in Fig. 28-14a.

**APPROACH** The wider spacing of lines near the top of Fig. 28-14a indicates the field  $\vec{B}$  has a smaller magnitude at the top than it does lower down. We apply Ampère's law to the rectangular path abcd shown dashed in Fig. 28-14a.

**SOLUTION** Because no current is enclosed by the chosen path, Ampère's law gives

$$\oint \vec{B} \cdot d\vec{\ell} = 0.$$

The integral along sections ab and cd is zero, since  $\vec{B} \perp d\vec{\ell}$ . Thus

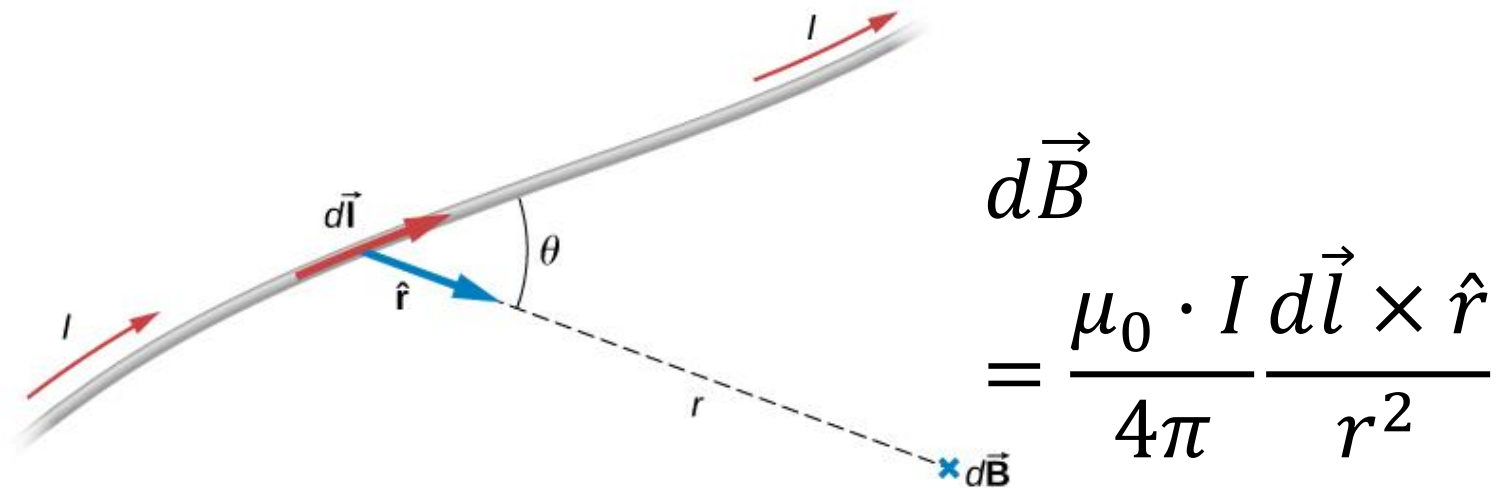
$$\oint \vec{B} \cdot d\vec{\ell} = B_{bc}\ell - B_{da}\ell = (B_{bc} - B_{da})\ell,$$

which is not zero since the field  $B_{bc}$  along the path bc is less than the field  $B_{da}$  along path da. Hence we have a contradiction:  $\oint \vec{B} \cdot d\vec{\ell}$  cannot be both zero (since  $I = 0$ ) and nonzero. Thus we have shown that a nonuniform unidirectional field is not consistent with Ampère's law. A nonuniform field whose direction also changes, as in Fig. 28-14b, is consistent with Ampère's law (convince yourself this is so), and possible. The fringing of a permanent magnet's field (Fig. 27-7) has this shape.

## **Solving problems using Ampère's law:**

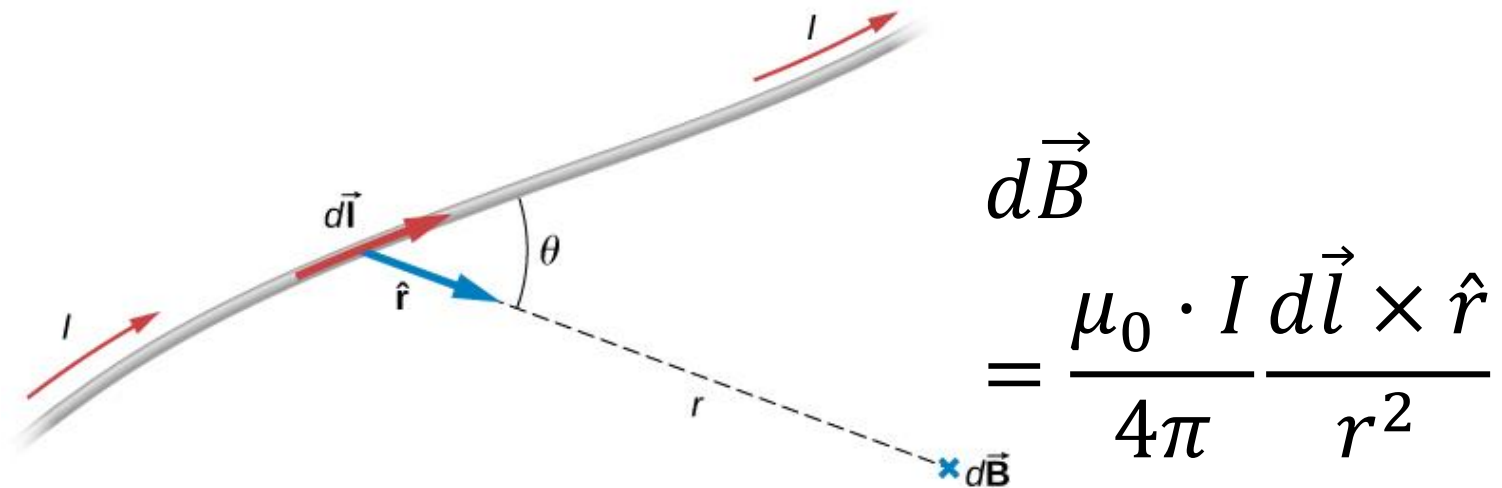
- Ampère's law is only useful for solving problems when there is a great deal of symmetry. Identify the symmetry.**
- Choose an integration path that reflects the symmetry (typically, the path is along lines where the field is constant and perpendicular to the field where it is changing).**
- Use the symmetry to determine the direction of the field.**
- Determine the enclosed current.**

# Biot–Savart law



- Unfortunately, Ampere's law can be difficult to use for general magnetic field calculations.
- Of more general use is the Biot–Savart law

# Biot–Savart law

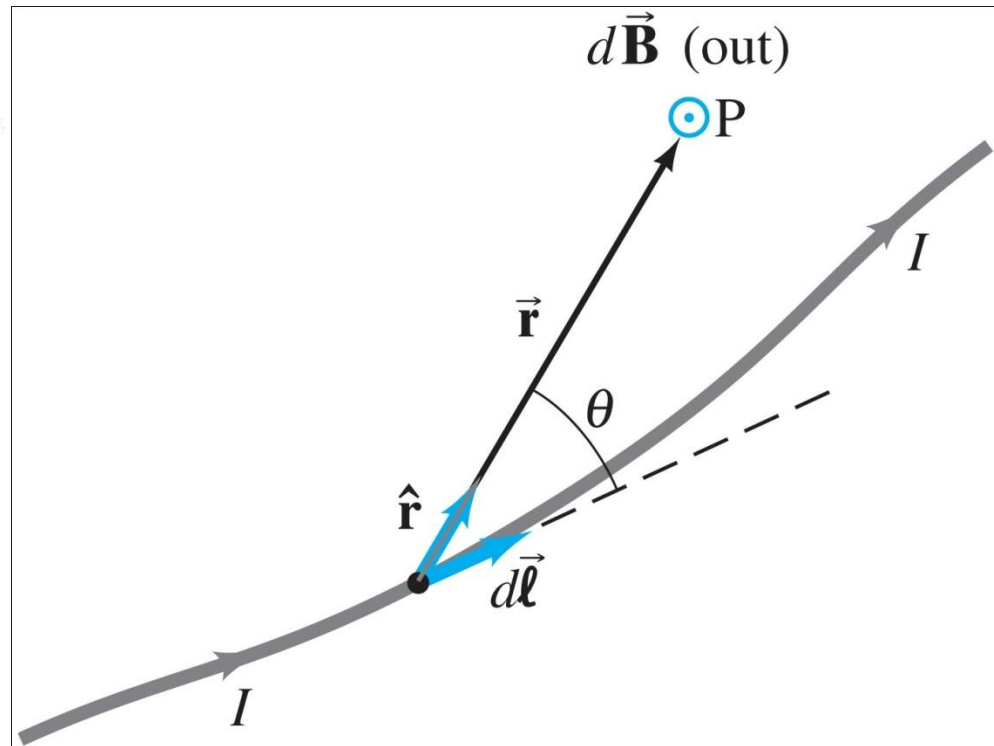


- For the diagram above the magnitude field at point x due the the current  $I$  in wire segment  $dl$  is
- And using the RHR, the field points into the paper.

**28-6 Biot-Savart Law**

The Biot-Savart law gives the magnetic field due to an infinitesimal length of current; the total field can then be found by integrating over the total length of all currents:

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{\mathbf{r}}}{r^2}.$$



# Biot–Savart law

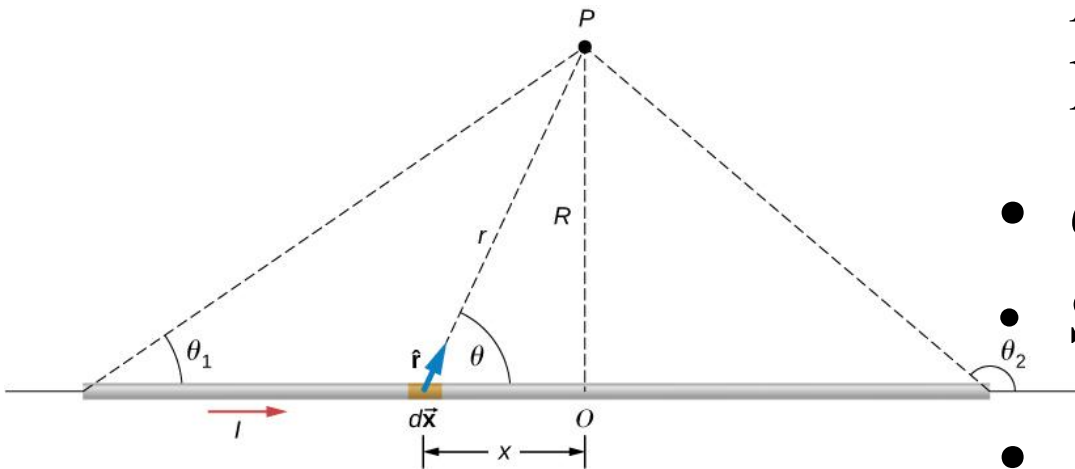
$$d\vec{B} = \frac{\mu_0 \cdot I d\vec{l} \times \hat{r}}{4\pi r^2}$$

- To find the field of a long straight wire, we integrate dB along the length of the wire.

- $d\vec{l} \times \hat{r} = dx \cdot \sin \theta$

- So,

- $B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{\sin \theta \cdot dx}{r^2}$



# Biot–Savart law

$$d\vec{B} = \frac{\mu_0 \cdot I d\vec{l} \times \hat{r}}{4\pi r^2}$$

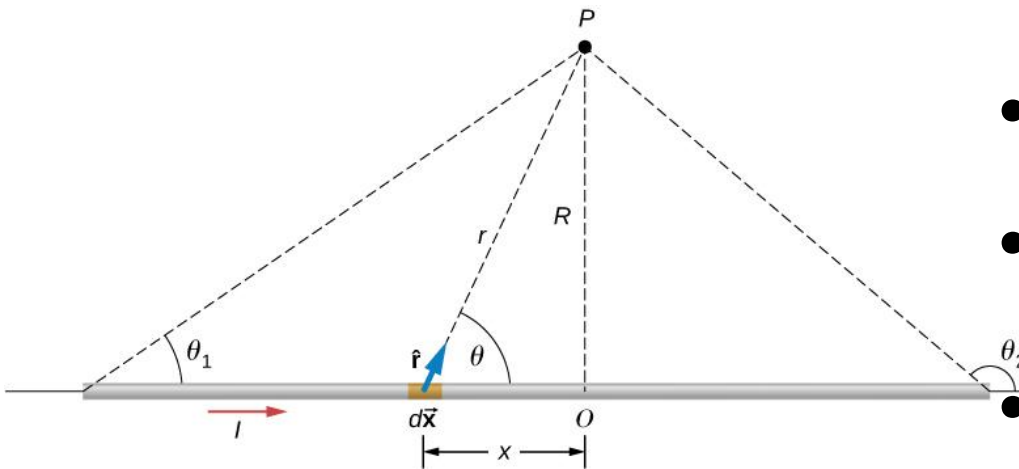
- $B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{R \cdot dx}{(x^2 + R^2)^{3/2}}$

- From the diagram,

- $r = \sqrt{x^2 + R^2}$ , and

- $\sin \theta = \frac{R}{\sqrt{x^2 + R^2}}$ , so

- $B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{R \cdot dx}{(x^2 + R^2)^{3/2}}$



# Biot–Savart law

$$d\vec{B} = \frac{\mu_0 \cdot I d\vec{l} \times \hat{r}}{4\pi r^2}$$

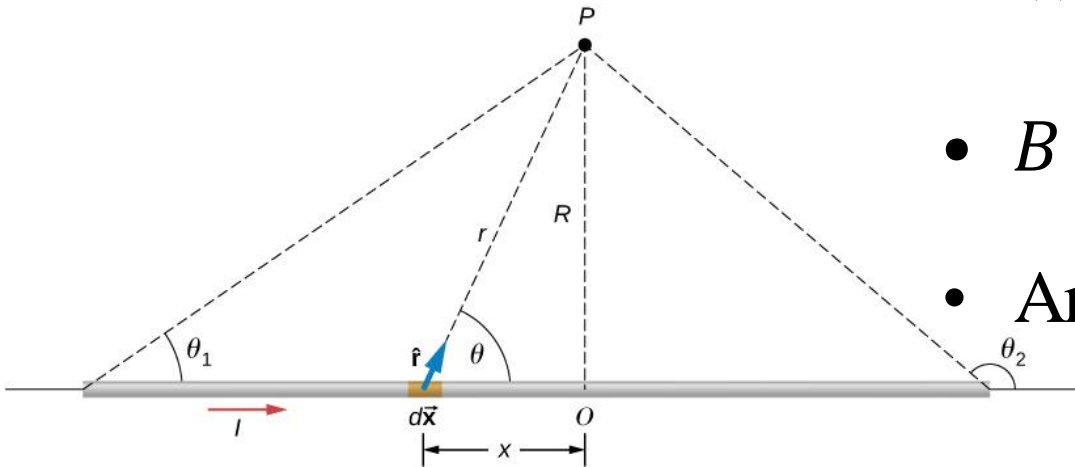
- $B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{R \cdot dx}{(x^2 + R^2)^{3/2}}$

- Which integrates to

- $B = \frac{\mu_0 I \cdot R}{4\pi} \left[ \frac{x}{R^2 (x^2 + R^2)^{1/2}} \right]_{-\infty}^{\infty}$

- And, finally,

- $B = \frac{\mu_0 I}{2\pi \cdot R}$



# Field along the Axis of a Loop

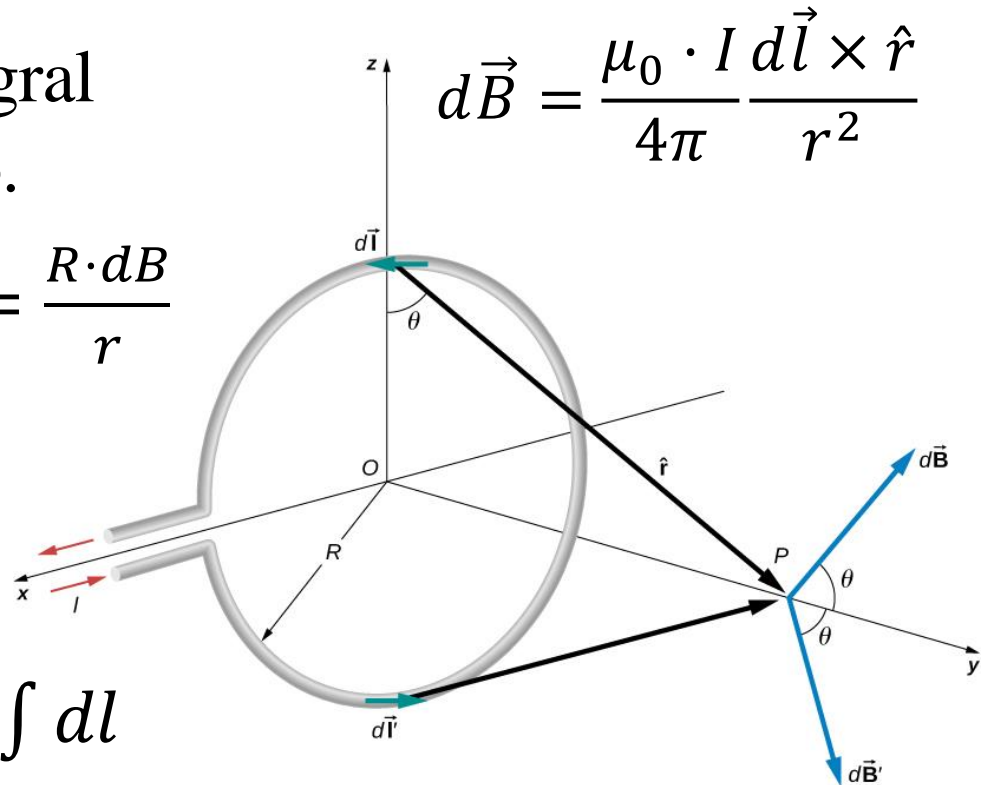
- For a loop, the integral of  $dB_{\perp}$  will be zero.

- $dB_{\parallel} = dB \cos \theta = \frac{R \cdot dB}{r}$

- $r = \sqrt{R^2 + y^2}$

- $d\vec{l} \times \hat{r} = \hat{j} dl$ , so

- $\vec{B} = \hat{j} \frac{\mu_0}{4\pi} \frac{I \cdot R}{(\sqrt{R^2 + y^2})^3} \int dl$



# Field along the Axis of a Loop

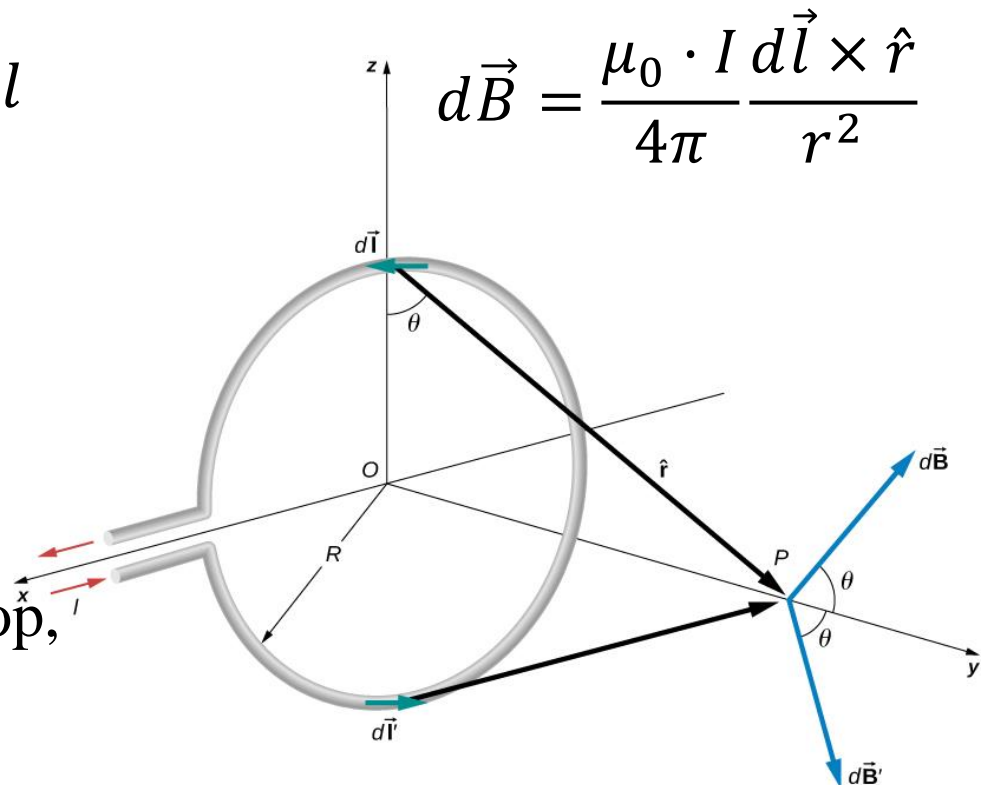
- $\vec{B} = \hat{j} \frac{\mu_0}{4\pi} \frac{I \cdot R}{(\sqrt{R^2 + y^2})^3} \int dl$

- $\int dl = 2\pi R$ , so

- $\vec{B} = \frac{\mu_0}{2} \frac{I \cdot R^2}{(\sqrt{R^2 + y^2})^3} \hat{j}$

- At the center of the loop, where  $x=0$ ,

- $\vec{B} = \frac{\mu_0 I}{2R} \hat{j}$

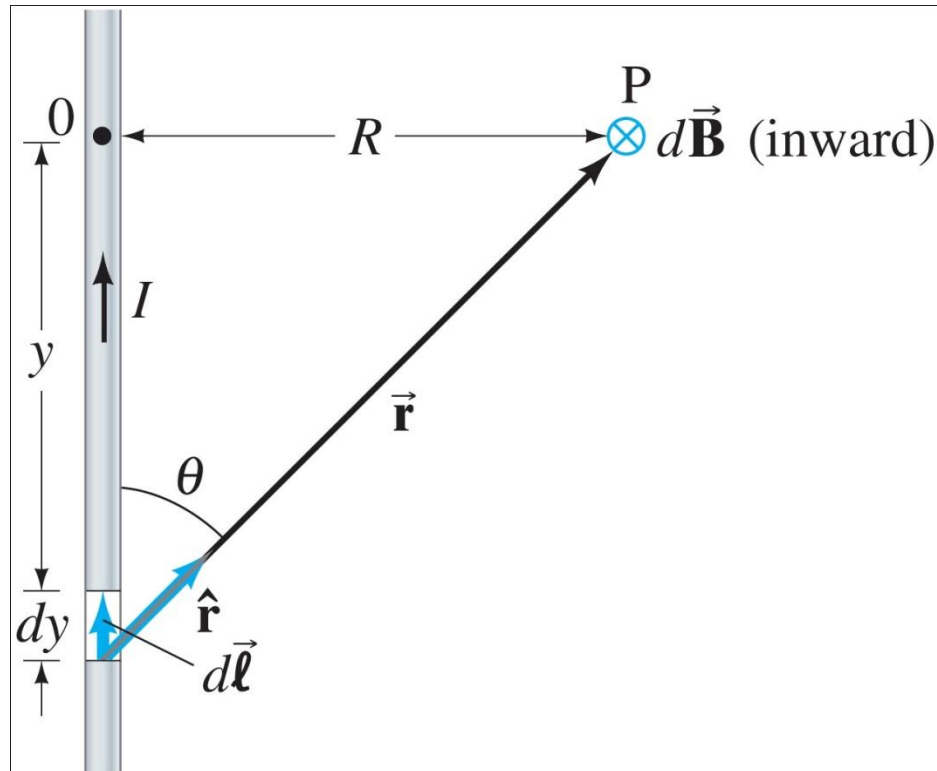


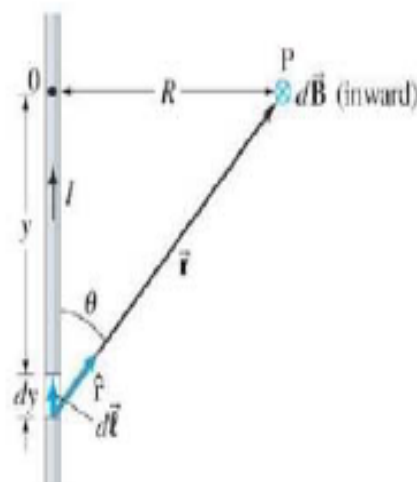
## Example 28-11: $\vec{B}$ due to current $I$ in straight wire.

### 28-6 Biot-Savart Law

For the field near a long straight wire carrying a current  $I$ , show that the Biot-Savart law gives

$$B = \mu_0 I / 2\pi r.$$





**FIGURE 28-19** Determining  $\vec{B}$  due to a long straight wire using the Biot-Savart law.

**EXAMPLE 28-11**  $\vec{B}$  due to current  $I$  in straight wire. For the field near a long straight wire carrying a current  $I$ , show that the Biot-Savart law gives the same result as Eq. 28-1,  $B = \mu_0 I / 2\pi r$ .

**APPROACH** We calculate the magnetic field in Fig. 28-19 at point P, which is a perpendicular distance  $R$  from an infinitely long wire. The current is moving upwards, and both  $d\vec{\ell}$  and  $\hat{r}$ , which appear in the cross product of Eq. 28-5, are in the plane of the page. Hence the direction of the field  $d\vec{B}$  due to each element of current must be directed into the plane of the page as shown (right-hand rule for the cross product  $d\vec{\ell} \times \hat{r}$ ). Thus all the  $d\vec{B}$  have the same direction at point P, and add up to give  $\vec{B}$  the same direction consistent with our previous results (Figs. 28-1 and 28-11).

**SOLUTION** The magnitude of  $\vec{B}$  will be

$$B = \frac{\mu_0 I}{4\pi} \int_{y=-\infty}^{+\infty} \frac{dy \sin \theta}{r^2},$$

where  $dy = d\ell$  and  $r^2 = R^2 + y^2$ . Note that we are integrating over  $y$  (the length of the wire) so  $R$  is considered constant. Both  $y$  and  $\theta$  are variables, but they are not independent. In fact,  $y = -R/\tan \theta$ . Note that we measure  $y$  as positive upward from point 0, so for the current element we are considering  $y < 0$ . Then

$$dy = +R \csc^2 \theta d\theta = \frac{R d\theta}{\sin^2 \theta} = \frac{R d\theta}{(R/r)^2} = \frac{r^2 d\theta}{R}.$$

From Fig. 28-19 we can see that  $y = -\infty$  corresponds to  $\theta = 0$  and that  $y = +\infty$  corresponds to  $\theta = \pi$  radians. So our integral becomes

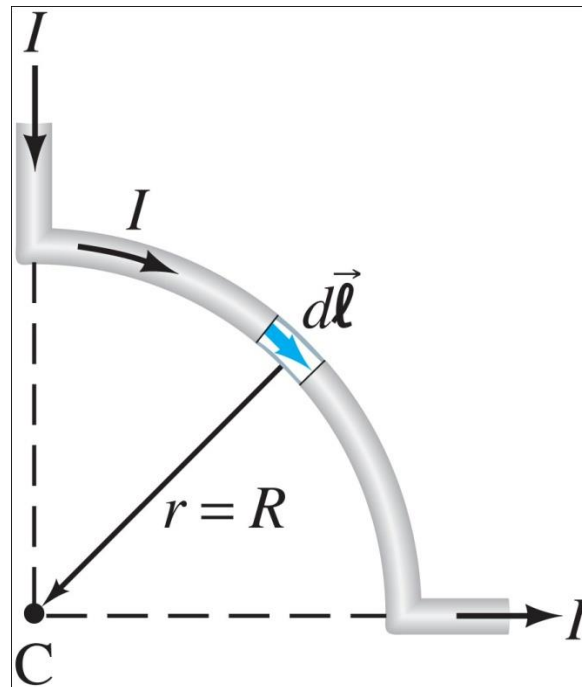
$$B = \frac{\mu_0 I}{4\pi} \frac{1}{R} \int_{\theta=0}^{\pi} \sin \theta d\theta = -\frac{\mu_0 I}{4\pi R} \cos \theta \Big|_0^{\pi} = \frac{\mu_0 I}{2\pi R}.$$

This is just Eq. 28-1 for the field near a long wire, where  $R$  has been used instead of  $r$ .

Example 28-13:  $\vec{B}$  due to a wire segment.

## 28-6 Biot-Savart Law

One quarter of a circular loop of wire carries a current  $I$ . The current  $I$  enters and leaves on straight segments of wire, as shown; the straight wires are along the radial direction from the center  $C$  of the circular portion. Find the magnetic field at point  $C$ .



**EXAMPLE 28-13**  $\vec{B}$  due to a wire segment. One quarter of a circular loop of wire carries a current  $I$  as shown in Fig. 28-22. The current  $I$  enters and leaves on straight segments of wire, as shown; the straight wires are along the radial direction from the center  $C$  of the circular portion. Find the magnetic field at point  $C$ .

**APPROACH** The current in the straight sections produces no magnetic field at point  $C$  because  $d\vec{\ell}$  and  $\hat{r}$  in the Biot-Savart law (Eq. 28-5) are parallel and therefore  $d\vec{\ell} \times \hat{r} = 0$ . Each piece  $d\vec{\ell}$  of the curved section of wire produces a field  $d\vec{B}$  that points into the page at  $C$  (right-hand rule).

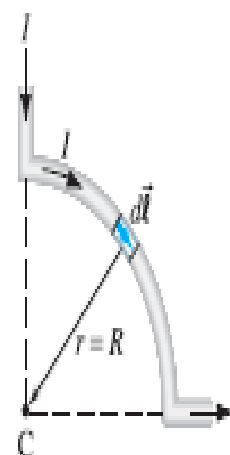
**SOLUTION** The magnitude of each  $d\vec{B}$  due to each  $d\vec{\ell}$  of the circular portion of wire is (Eq. 28-6)

$$dB = \frac{\mu_0 I d\ell}{4\pi R^2}$$

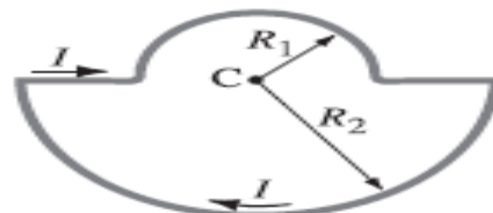
where  $r = R$  is the radius of the curved section, and  $\sin \theta$  in Eq. 28-6 is  $\sin 90^\circ = 1$ . With  $r = R$  for all pieces  $d\vec{\ell}$ , we integrate over a quarter of a circle.

$$B = \int dB = \frac{\mu_0 I}{4\pi R^2} \int d\ell = \frac{\mu_0 I}{4\pi R^2} \left( \frac{1}{4} 2\pi R \right) = \frac{\mu_0 I}{8R}.$$

**FIGURE 28-22** Example 28-13.

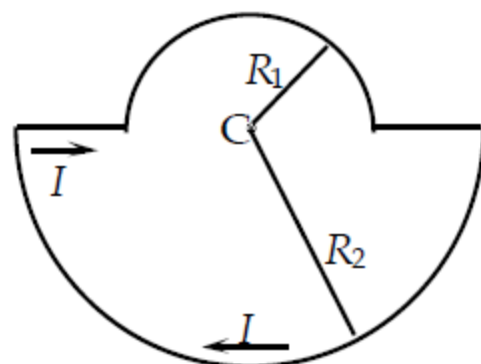


- 37.** (II) A wire is formed into the shape of two half circles connected by equal-length straight sections as shown in Fig. 28–45. A current  $I$  flows in the circuit clockwise as shown. Determine (a) the magnitude and direction of the magnetic field at the center,  $C$ , and (b) the magnetic dipole moment of the circuit.



**FIGURE 28–45**  
Problem 37.

- 37.** (a) The magnetic field at point  $C$  can be obtained using the Biot-Savart law (Eq. 28-5, integrated over the current). First break the loop into four sections: 1) the upper semi-circle, 2) the lower semi-circle, 3) the right straight segment, and 4) the left straight segment. The two straight segments do not contribute to the magnetic field as the point  $C$  is in the same direction that the



current is flowing. Therefore, along these segments  $\hat{r}$  and  $d\hat{\ell}$  are parallel and  $d\hat{\ell} \times \hat{r} = 0$ . For the upper segment, each infinitesimal line segment is perpendicular to the constant magnitude radial vector, so the magnetic field points downward with constant magnitude.

$$\bar{\mathbf{B}}_{\text{upper}} = \int \frac{\mu_0 I}{4\pi} \frac{d\hat{\ell} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{-\hat{k}}{R_1^2} (\pi R_1^2) = -\frac{\mu_0 I}{4R_1} \hat{k}.$$

Along the lower segment, each infinitesimal line segment is also perpendicular to the constant radial vector.

$$\bar{\mathbf{B}}_{\text{lower}} = \int \frac{\mu_0 I}{4\pi} \frac{d\hat{\ell} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{-\hat{k}}{R_2^2} (\pi R_2^2) = -\frac{\mu_0 I}{4R_2} \hat{k}$$

Adding the two contributions yields the total magnetic field.

$$\bar{\mathbf{B}} = \bar{\mathbf{B}}_{\text{upper}} + \bar{\mathbf{B}}_{\text{lower}} = -\frac{\mu_0 I}{4R_1} \hat{k} - \frac{\mu_0 I}{4R_2} \hat{k} = \boxed{-\frac{\mu_0 I}{4} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \hat{k}}$$

- (b) The magnetic moment is the product of the area and the current. The area is the sum of the two half circles. By the right-hand-rule, curling your fingers in the direction of the current, the thumb points into the page, so the magnetic moment is in the  $-\hat{k}$  direction.

$$\bar{\mu} = -\left( \frac{\pi R_1^2}{2} + \frac{\pi R_2^2}{2} \right) I \hat{k} = \boxed{\frac{-\pi I}{2} (R_1^2 + R_2^2) \hat{k}}$$

A long, straight conductor carries a 1.0-A current. At what distance from the axis of the conductor does the resulting magnetic field have magnitude  $B = 0.5 \times 10^{-4} \text{ T}$  (about that of the earth's magnetic field in Pittsburgh)?

**EXECUTE:** We solve Eq. (28.9) for  $r$ :

$$r = \frac{\mu_0 I}{2\pi B} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.0 \text{ A})}{(2\pi)(0.5 \times 10^{-4} \text{ T})}$$

$$= 4 \times 10^{-3} \text{ m} = 4 \text{ mm}$$

### SOLUTION

**IDENTIFY and SET UP:** The length of a “long” conductor is much greater than the distance from the conductor to the field point. Hence we can use the ideas of this section. The geometry is the same as that of Fig. 28.6, so we use Eq. (28.9). All of the quantities in this equation are known except the target variable, the distance  $r$ .

**EVALUATE:** As we saw in Example 26.14, currents of an ampere or more are typical of those found in the wiring of home appliances. This example shows that the magnetic fields produced by these appliances are very weak even very close to the wire; the fields are proportional to  $1/r$ , so they become even weaker at greater distances.

**28.11** This electromagnet contains a current-carrying coil with numerous turns of wire. The resulting magnetic field can pick up large quantities of steel bars and other iron-bearing items.



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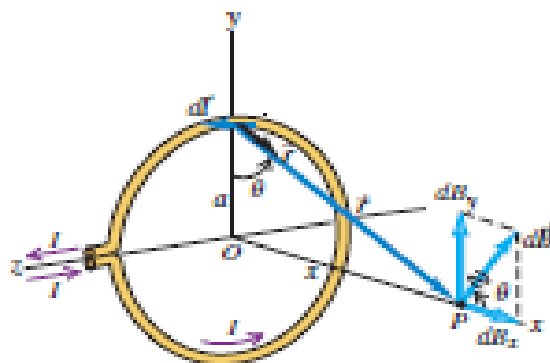
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If you look inside a doorbell, a transformer, an electric motor, or an electromagnet (Fig. 28.11), you will find coils of wire with a large number of turns, spaced so closely that each turn is very nearly a planar circular loop. A current in such a coil is used to establish a magnetic field. In Section 27.7 we considered the force and torque on such a current loop placed in an external magnetic field produced by other currents; we are now about to find the magnetic field produced by such a loop or by a collection of closely spaced loops forming a coil.

**28.12** Magnetic field on the axis of a circular loop. The current in the segment  $d\vec{l}$  causes the field  $d\vec{B}$ , which lies in the  $xy$ -plane. The currents in other  $d\vec{l}$ 's cause  $d\vec{B}$ 's with different components perpendicular to the  $x$ -axis; these components add to zero. The  $x$ -components of the  $d\vec{B}$ 's combine to give the total  $\vec{B}$  field at point  $P$ .



$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)}$$

The components of the vector  $d\vec{B}$  are

$$dB_x = dB \cos \theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{a}{(x^2 + a^2)^{1/2}}$$

$$dB_y = dB \sin \theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{x}{(x^2 + a^2)^{1/2}}$$

To obtain the  $x$ -component of the total field  $\vec{B}$ , we integrate Eq. (28.13), including all the  $d\vec{l}$ 's around the loop. Everything in this expression except  $dl$  is constant and can be taken outside the integral, and we have

$$B_x = \int \frac{\mu_0 I}{4\pi} \frac{a dl}{(x^2 + a^2)^{3/2}} = \frac{\mu_0 I a}{4\pi(x^2 + a^2)^{3/2}} \int dl$$

## Magnetic Field on the Axis of a Coil

Now suppose that instead of the single loop in Fig. 28.12 we have a coil consisting of  $N$  loops, all with the same radius. The loops are closely spaced so that the plane of each loop is essentially the same distance  $x$  from the field point  $P$ . Then the total field is  $N$  times the field of a single loop:

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}} \quad (\text{on the axis of } N \text{ circular loops}) \quad (28.16)$$

Magnetic field at center of  $N$  circular current-carrying loops

$$B_x = \frac{\mu_0 N I}{2a}$$

Magnetic constant

Number of loops

Current

Radius of loop

Magnetic field on axis of a circular current-carrying loop

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

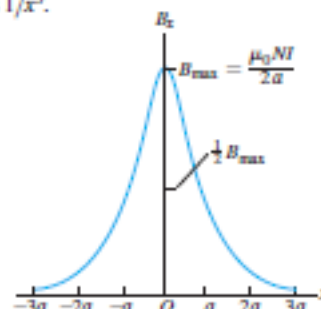
Magnetic constant

Current

Radius of loop

Distance along axis from center of loop to field point

**28.14** Graph of the magnetic field along the axis of a circular coil with  $N$  turns. When  $x$  is much larger than  $a$ , the field magnitude decreases approximately as  $1/x^3$ .



### BIO Application Magnetic Fields for MRI

Magnetic resonance imaging (see Section 27.7), requires a magnetic field of about 1.5 T. In a typical MRI scan, the patient lies inside a coil that produces the intense field. The currents required are very high, so the coils are bathed in liquid helium at a temperature of 4.2 K to keep them from overheating.



A coil consisting of 100 circular loops with radius 0.60 m carries a 5.0-A current. (a) Find the magnetic field at a point along the axis of the coil, 0.80 m from the center. (b) Along the axis, at what distance from the center of the coil is the field magnitude  $\frac{1}{8}$  as great as it is at the center?

#### SOLUTION

**IDENTIFY and SET UP:** This problem concerns the magnetic field magnitude  $B$  along the axis of a current-carrying coil, so we can use Eq. (28.16). We are given  $N = 100$ ,  $I = 5.0$  A, and  $a = 0.60$  m. In part (a) our target variable is  $B_x$  at a given value of  $x$ . In part (b) the target variable is the value of  $x$  at which the field has  $\frac{1}{8}$  of the magnitude that it has at the origin.

**EXECUTE:** (a) Using  $x = 0.80$  m, from Eq. (28.16) we have

$$\begin{aligned} B_x &= \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(100)(5.0 \text{ A})(0.60 \text{ m})^2}{2[(0.80 \text{ m})^2 + (0.60 \text{ m})^2]^{3/2}} \\ &= 1.1 \times 10^{-4} \text{ T} \end{aligned}$$

the loop. If there are  $N$  loops, the total magnetic moment is  $NIA$ . The circular loop in Fig. 28.12 has area  $A = \pi a^2$ , so the magnetic moment of a single loop is  $\mu = I\pi a^2$ ; for  $N$  loops,  $\mu = NI\pi a^2$ . Substituting these results into Eqs. (28.15) and (28.16), we find

$$B_x = \frac{\mu_0 \mu}{2\pi(x^2 + a^2)^{3/2}} \quad (\text{on the axis of any number of circular loops}) \quad (28.18)$$

(b) Considering Eq. (28.16), we seek a value of  $x$  such that

$$\frac{1}{(x^2 + a^2)^{3/2}} = \frac{1}{8} \frac{1}{(0^2 + a^2)^{3/2}}$$

To solve this for  $x$ , we take the reciprocal of the whole thing and then take the  $2/3$  power of both sides; the result is

$$x = \pm \sqrt{3}a = \pm 1.04 \text{ m}$$

**EVALUATE:** We check our answer in part (a) by finding the coil's magnetic moment and substituting the result into Eq. (28.18):

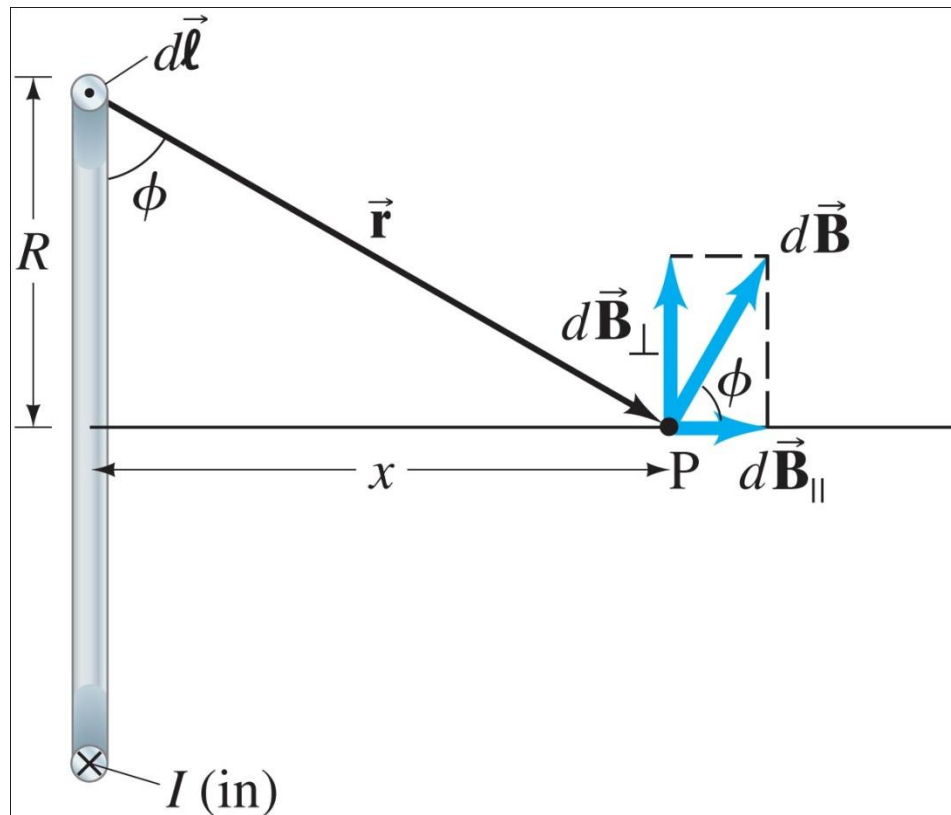
$$\begin{aligned} \mu &= NI\pi a^2 = (100)(5.0 \text{ A})\pi(0.60 \text{ m})^2 = 5.7 \times 10^2 \text{ A} \cdot \text{m}^2 \\ B_x &= \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(5.7 \times 10^2 \text{ A} \cdot \text{m}^2)}{2\pi[(0.80 \text{ m})^2 + (0.60 \text{ m})^2]^{3/2}} = 1.1 \times 10^{-4} \text{ T} \end{aligned}$$

The magnetic moment  $\mu$  is relatively large, yet it produces a rather small field, comparable to that of the earth. This illustrates how difficult it is to produce strong fields of 1 T or more.

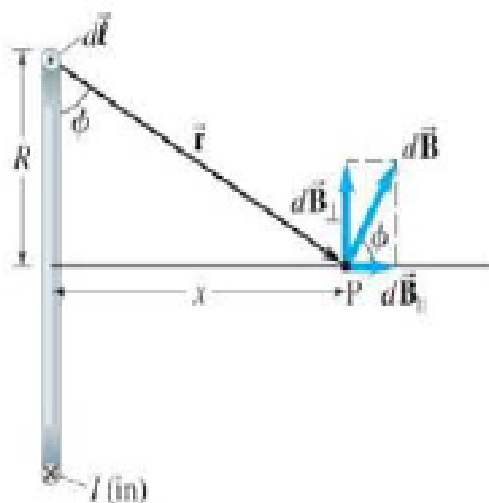
## Example 28-12: Current loop.

### 28-6 Biot-Savart Law

Determine  $\vec{B}$  for points on the axis of a circular loop of wire of radius  $R$  carrying a current  $I$ .



**FIGURE 28–20** Determining  $\vec{B}$  due to a current loop.



**EXAMPLE 28–12** **Current loop.** Determine  $\vec{B}$  for points on the axis of a circular loop of wire of radius  $R$  carrying a current  $I$ , Fig. 28–20.

**APPROACH** For an element of current at the top of the loop, the magnetic field  $d\vec{B}$  at point P on the axis is perpendicular to  $\vec{r}$  as shown, and has magnitude (Eq. 28–5)

$$dB = \frac{\mu_0 I d\ell}{4\pi r^2}$$

since  $d\vec{\ell}$  is perpendicular to  $\vec{r}$  so  $|d\vec{\ell} \times \hat{r}| = d\ell$ . We can break  $d\vec{B}$  down into components  $dB_\parallel$  and  $dB_\perp$ , which are parallel and perpendicular to the axis as shown.

**SOLUTION** When we sum over all the elements of the loop, symmetry tells us that the perpendicular components will cancel on opposite sides, so  $B_\perp = 0$ . Hence, the total  $\vec{B}$  will point along the axis, and will have magnitude

$$B = B_\parallel = \int dB \cos \phi = \int dB \frac{R}{r} = \int dB \frac{R}{(R^2 + x^2)^{3/2}},$$

where  $x$  is the distance of P from the center of the ring, and  $r^2 = R^2 + x^2$ . Now we put in  $dB$  from the equation above and integrate around the current loop, noting that all segments  $d\vec{\ell}$  of current are the same distance,  $(R^2 + x^2)^{1/2}$ , from point P:

$$B = \frac{\mu_0 I}{4\pi} \frac{R}{(R^2 + x^2)^{3/2}} \int d\ell = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

since  $\int d\ell = 2\pi R$ , the circumference of the loop.

**NOTE** At the very center of the loop (where  $x = 0$ ) the field has its maximum value

$$B = \frac{\mu_0 I}{2R}. \quad \text{[at center of current loop]}$$

Recall from Section 27-5 that a current loop, such as that just discussed (Fig. 28-20), is considered a **magnetic dipole**. We saw there that a current loop has a magnetic dipole moment

$$\mu = NIA,$$

where  $A$  is the area of the loop and  $N$  is the number of coils in the loop, each carrying current  $I$ . We also saw in Chapter 27 that a magnetic dipole placed in an external magnetic field experiences a torque and possesses potential energy, just like an electric dipole. In Example 28-12, we looked at another aspect of a magnetic dipole: the magnetic field *produced* by a magnetic dipole has magnitude, along the dipole axis, of

$$B = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}.$$

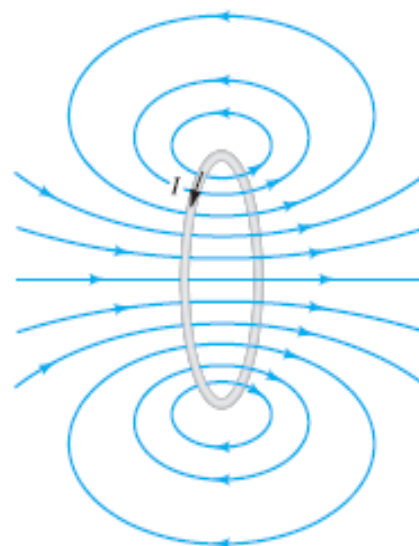
We can write this in terms of the magnetic dipole moment  $\mu = IA = I\pi R^2$  (for a single loop  $N = 1$ ):

$$B = \frac{\mu_0}{2\pi} \frac{\mu}{(R^2 + x^2)^{3/2}}. \quad [\text{magnetic dipole}] \quad (28-7a)$$

(Be careful to distinguish  $\mu$  for dipole moment from  $\mu_0$ , the magnetic permeability constant.) For distances far from the loop,  $x \gg R$ , this becomes

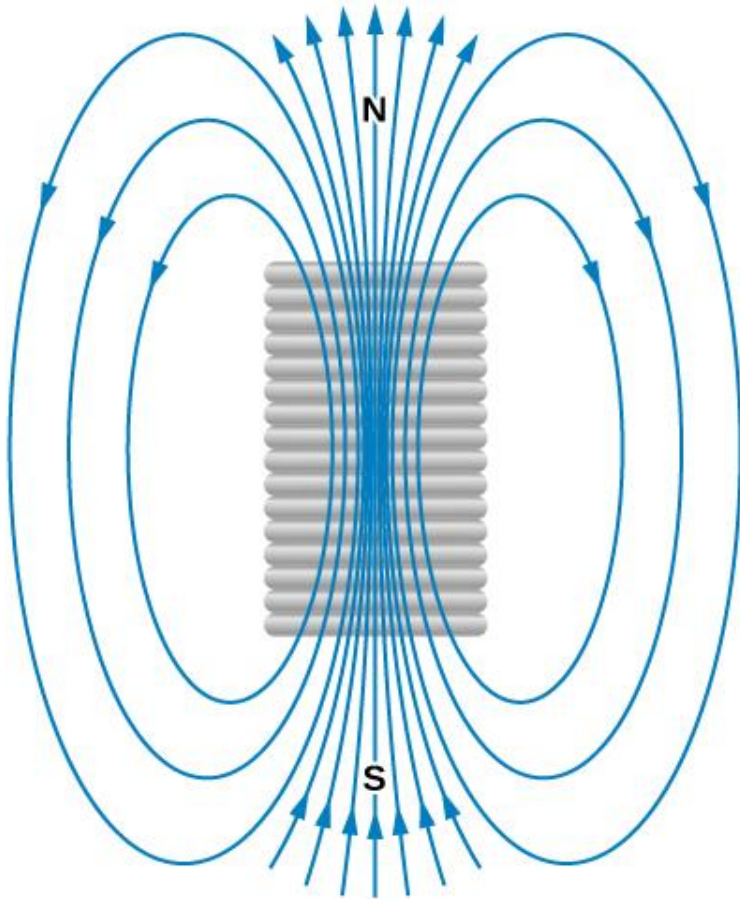
$$B \approx \frac{\mu_0}{2\pi} \frac{\mu}{x^3}. \quad \left[ \begin{array}{c} \text{on axis,} \\ \text{magnetic dipole, } x \gg R \end{array} \right] \quad (28-7b)$$

The magnetic field on the axis of a magnetic dipole decreases with the cube of the distance, just as the electric field does for an electric dipole.  $B$  decreases as the cube of the distance also for points not on the axis, although the multiplying factor is not the same. The magnetic field due to a current loop can be determined at various points using the Biot-Savart law and the results are in accord with experiment. The field lines around a current loop are shown in Fig. 28-21.



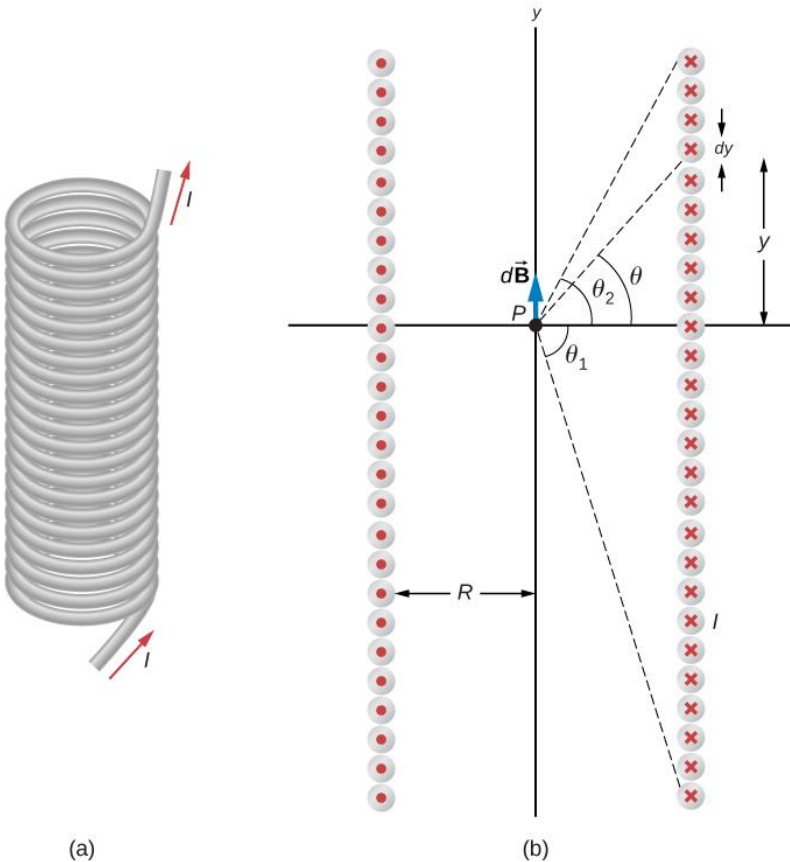
**FIGURE 28-21** Magnetic field due to a circular loop of wire. (Same as Fig. 27-9.)

# Magnetic Field of a Solenoid



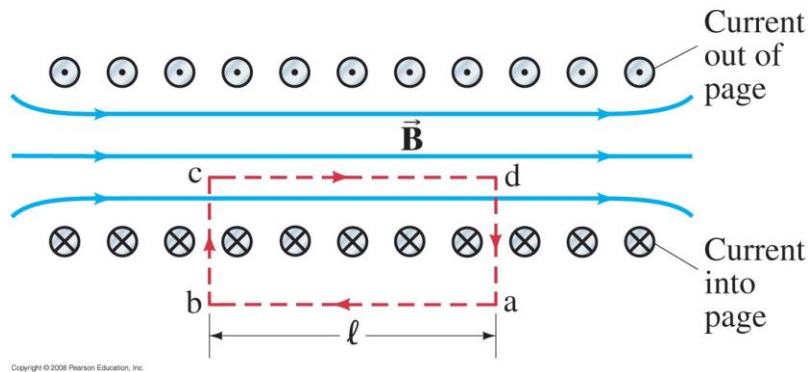
- Many practical uses of magnetic fields are based on magnetic coils.
- When the coil is long compared to its diameter, it is called a solenoid.

# Magnetic Field of a Solenoid



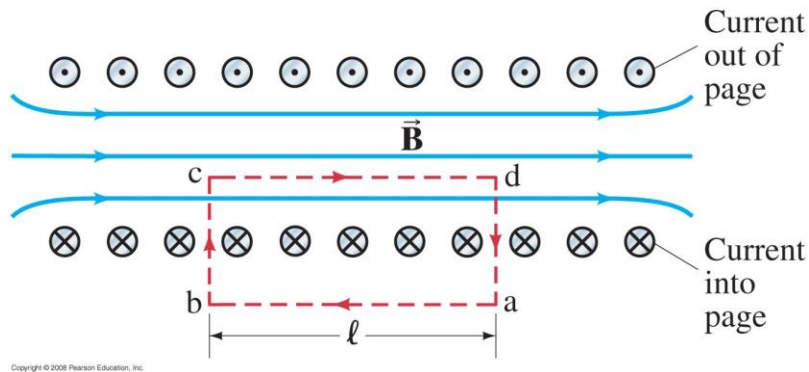
- Using the Biot-Savart law we could calculate the complete field of a solenoid by summing over the fields produced by individual wires.
- However, here Ampere's law works to simplify the calculation.

# Magnetic Field of a Solenoid



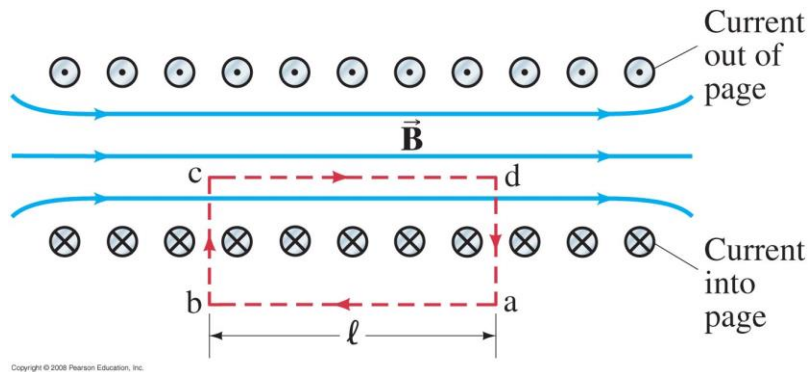
- Using Ampere's law to find the field of a solenoid.
- $\vec{B} \cdot d\vec{l}$  is zero for segments bc & da, since  $B$  is zero between the wires.
- $B$  outside the coil is effectively zero, so the only non-zero contribution to our path integral is cd.

# Magnetic Field of a Solenoid



- $\oint \vec{B} \cdot d\vec{l}$  becomes  $Bl$
- So, if the wire is carrying current  $I$ ,
- $Bl = \mu_0 I_{Enc} = N \cdot \mu_0 \cdot I$
- We typically will define the number of turns per length,  $n = N/l$ , so that
- $B = \frac{N}{l} \mu_0 \cdot I = n \cdot \mu_0 \cdot I$

# Magnetic Field of a Solenoid



- Calculate the field in a solenoid carrying 2 amps of current, if it has 400 turns of wire and is 10 cm long.

- $B = \frac{N}{l} \mu_0 I_{Enc}, \text{ or}$

- $B = n \mu_0 I_{Enc}$

- 25.** (I) A 40.0-cm-long solenoid 1.35 cm in diameter is to produce a field of 0.385 mT at its center. How much current should the solenoid carry if it has 765 turns of wire?

25. Use Eq. 28-4 for the field inside a solenoid.

$$B = \frac{\mu_0 IN}{\ell} \rightarrow I = \frac{B\ell}{\mu_0 N} = \frac{(0.385 \times 10^{-3} \text{ T})(0.400 \text{ m})}{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(765)} = \boxed{0.160 \text{ A}}$$

# 28-5 Magnetic Field of a Solenoid and a Toroid

## **Example 28-9: Field inside a solenoid.**

**A thin 10-cm-long solenoid used for fast electromechanical switching has a total of 400 turns of wire and carries a current of 2.0 A. Calculate the field inside near the center.**

**EXAMPLE 28-9** **Field inside a solenoid.** A thin 10-cm-long solenoid used for fast electromechanical switching has a total of 400 turns of wire and carries a current of 2.0 A. Calculate the field inside near the center.

**APPROACH** We use Eq. 28-4, where the number of turns per unit length is  $n = 400/0.10 \text{ m} = 4.0 \times 10^3 \text{ m}^{-1}$ .

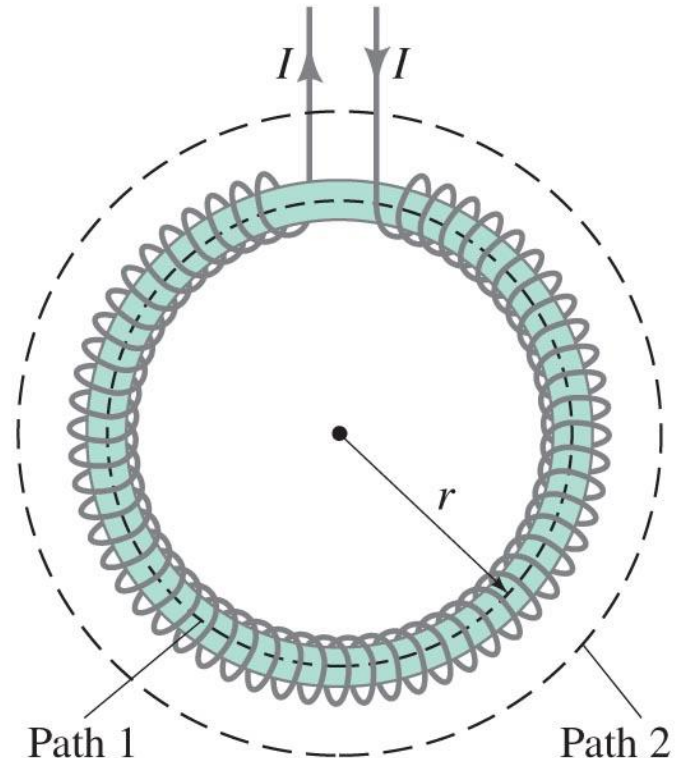
**SOLUTION**  $B = \mu_0 nI = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(4.0 \times 10^3 \text{ m}^{-1})(2.0 \text{ A}) = 1.0 \times 10^{-2} \text{ T}.$

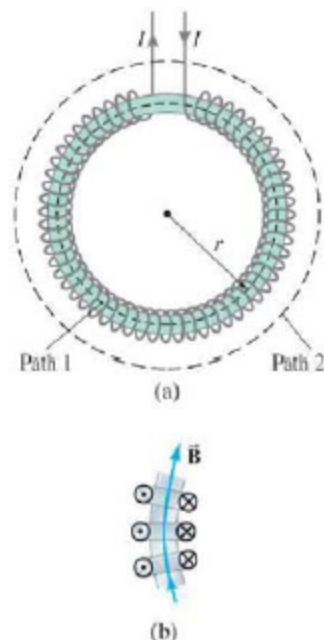
A close look at Fig. 28-15 shows that the field outside of a solenoid is much like that of a bar magnet (Fig. 27-4). Indeed, a solenoid acts like a magnet with one end acting as a north pole and the other as south pole, depending on the direction of the current in the loops. Since magnetic field lines leave the north pole of a magnet, the north poles of the solenoids in Fig. 28-15 are on the right.

# 28-5 Magnetic Field of a Solenoid and a Toroid

## Example 28-10: Toroid.

Use Ampère's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle as shown.





**FIGURE 28-17** (a) A toroid. (b) A section of the toroid showing direction of the current for three loops:  $\odot$  means current toward you,  $\otimes$  means current away from you.

**EXAMPLE 28-10 Toroid.** Use Ampère's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle as shown in Fig. 28-17a.

**APPROACH** The magnetic field lines inside the toroid will be circles concentric with the toroid. (If you think of the toroid as a solenoid bent into a circle, the field lines bend along with the solenoid.) The direction of  $\vec{B}$  is clockwise. We choose as our path of integration one of these field lines of radius  $r$  inside the toroid as shown by the dashed line labeled "path 1" in Fig. 28-17a. We make this choice to use the symmetry of the situation, so  $B$  will be tangent to the path and will have the same magnitude at all points along the path (although it is not necessarily the same across the whole cross section of the toroid). This chosen path encloses *all* the coils; if there are  $N$  coils, each carrying current  $I$ , then  $I_{\text{encl}} = NI$ .

**SOLUTION** (a) Ampère's law applied along this path gives

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$$

$$B(2\pi r) = \mu_0 NI,$$

where  $N$  is the total number of coils and  $I$  is the current in each of the coils. Thus

$$B = \frac{\mu_0 NI}{2\pi r}.$$

The magnetic field  $B$  is not uniform within the toroid: it is largest along the inner edge (where  $r$  is smallest) and smallest at the outer edge. However, if the toroid is large, but thin (so that the difference between the inner and outer radii is small compared to the average radius), the field will be essentially uniform within the toroid. In this case, the formula for  $B$  reduces to that for a straight solenoid  $B = \mu_0 nI$  where  $n = N/(2\pi r)$  is the number of coils per unit length. (b) Outside the toroid, we choose as our path of integration a circle concentric with the toroid, "path 2" in Fig. 28-17a. This path encloses  $N$  loops carrying current  $I$  in one direction and  $N$  loops carrying the same current in the opposite direction. (Figure 28-17b shows the directions of the current for the parts of the loop on the inside and outside of the toroid.) Thus the net current enclosed by path 2 is zero. For a very tightly packed toroid, all points on path 2 are equidistant from the toroid and equivalent, so we expect  $B$  to be the same at all points along the path. Hence, Ampère's law gives

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$$

$$B(2\pi r) = 0$$

or

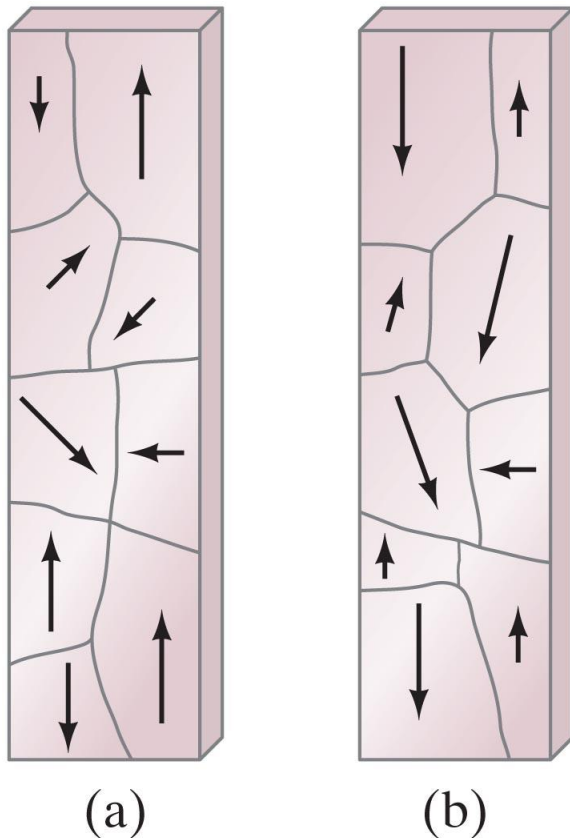
$$B = 0.$$

The same is true for a path taken at a radius smaller than that of the toroid. So there is no field exterior to a very tightly wound toroid. It is all inside the loops.

# Natural Magnetism of Materials

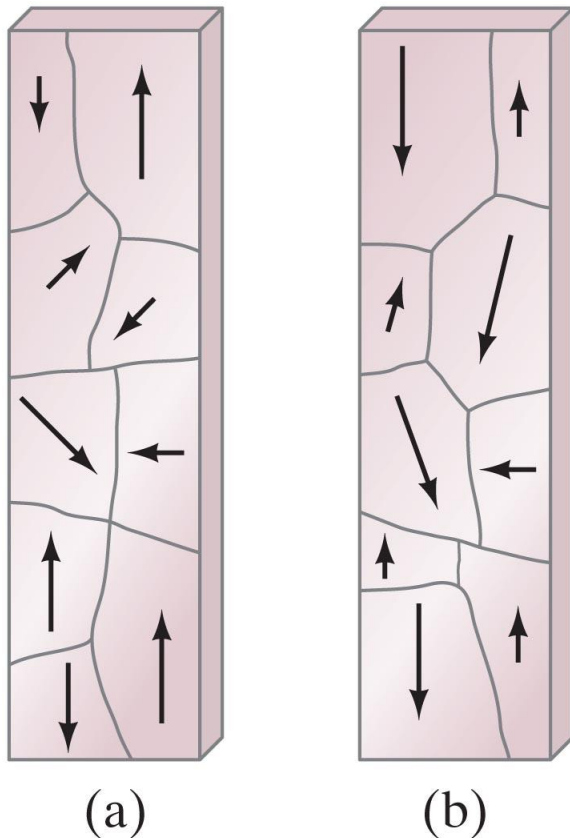
- The protons and electrons in atoms have intrinsic magnetic fields.
- Also the orbiting electrons produce a magnetic field.
- Depending upon how these fields combine in different kinds of atoms, the atoms may have also have an intrinsic magnetic field.
- The behavior of these atomic magnets in the presence of an external magnetic field fall largely into 3 classes

# Natural Magnetism of Materials



- Iron and other materials can be made into strong magnets. These materials are called ferromagnetic.
- The atoms of a ferromagnetic material have strong dipole magnetic fields and can line up to form magnetic regions called domains.

# Natural Magnetism of Materials

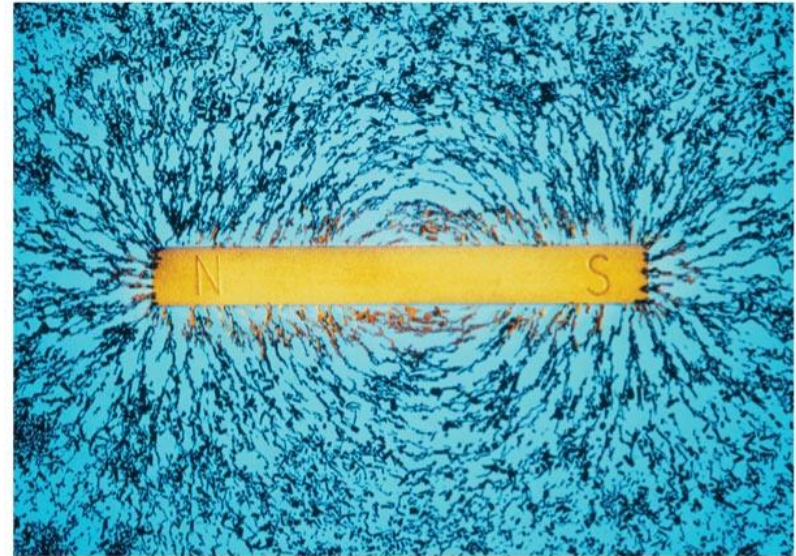


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- When a ferromagnetic material is not magnetized, it means that the domains are small and randomly oriented.
- In an external field, the dipoles turn to align themselves with the external field. This causes local domains to grow and merge, forming a strong overall magnetic field.

# Natural Magnetism of Materials

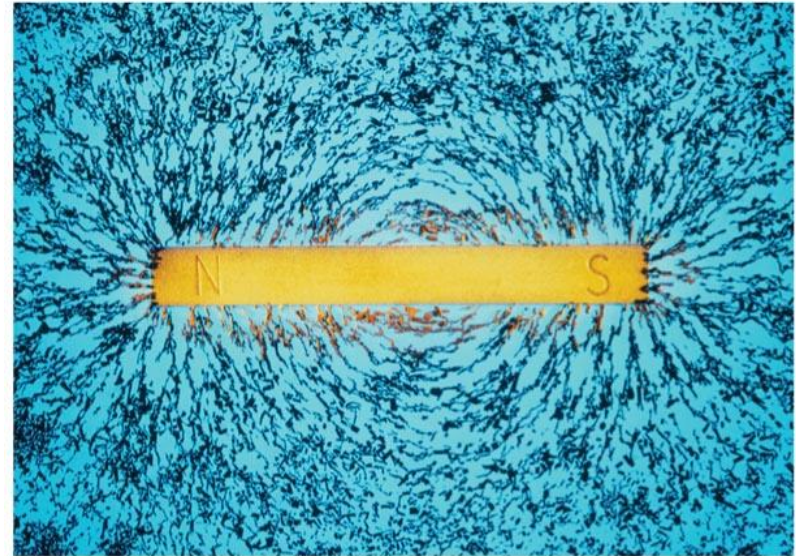
- Permanent magnets:
- Permanent magnets are formed by heating a ferromagnetic material to a temperature where the thermal motion of the atoms causes them to become randomly oriented (no net magnetic field).
- This is called the Curie temperature for the material.



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# Natural Magnetism of Materials

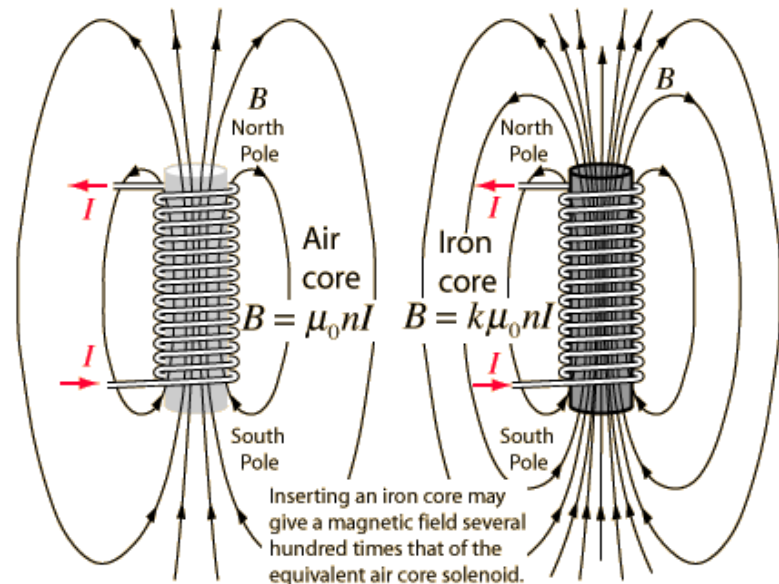
- Permanent magnets:
- A large magnetic field is applied to align the atoms and the material is cooled.
- Once below the Curie Temperature, the atoms cannot realign and the magnetic alignment becomes permanent (unless reheated or otherwise disturbed.)



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# Natural Magnetism of Materials

- For a solenoid wound around a material core, we replace  $\mu_0$  with  $\mu$  for the material.
- $B = \mu nI$
- Since  $\mu$  for iron can be as large as  $10,000 \cdot \mu_0$ . The increase in the magnetic field can be extreme.

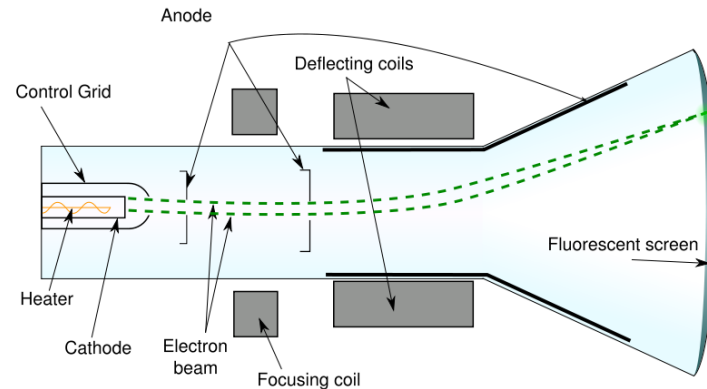


# End of Chapter 12

- Read Chapter 12 Summary and
- Complete the homework for Ch 28 (Vol 2 Ch 12)

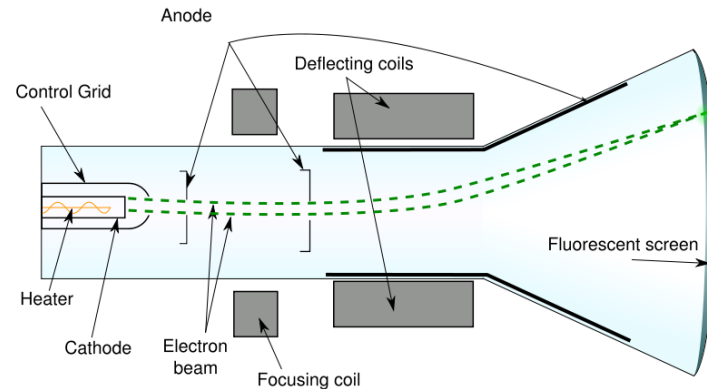
# Backup slides

# Electromagnets in your TV or Computer



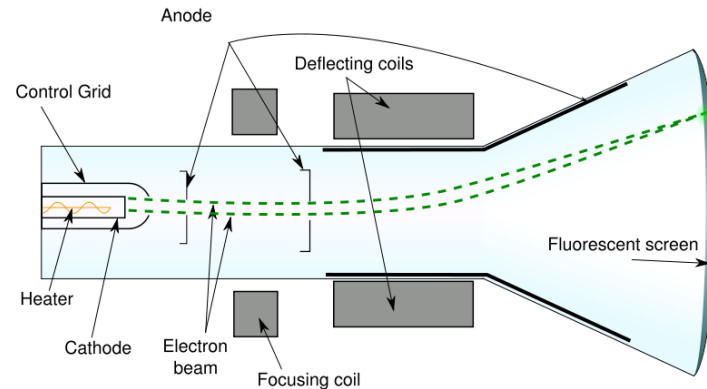
- The cathode ray tube (CRT) is the backbone of older TVs, computer monitors and oscilloscopes.
- An electron beam is emitted by the heated cathode (-). Its strength is controlled by the control grid.

# Electromagnets in your TV or Computer



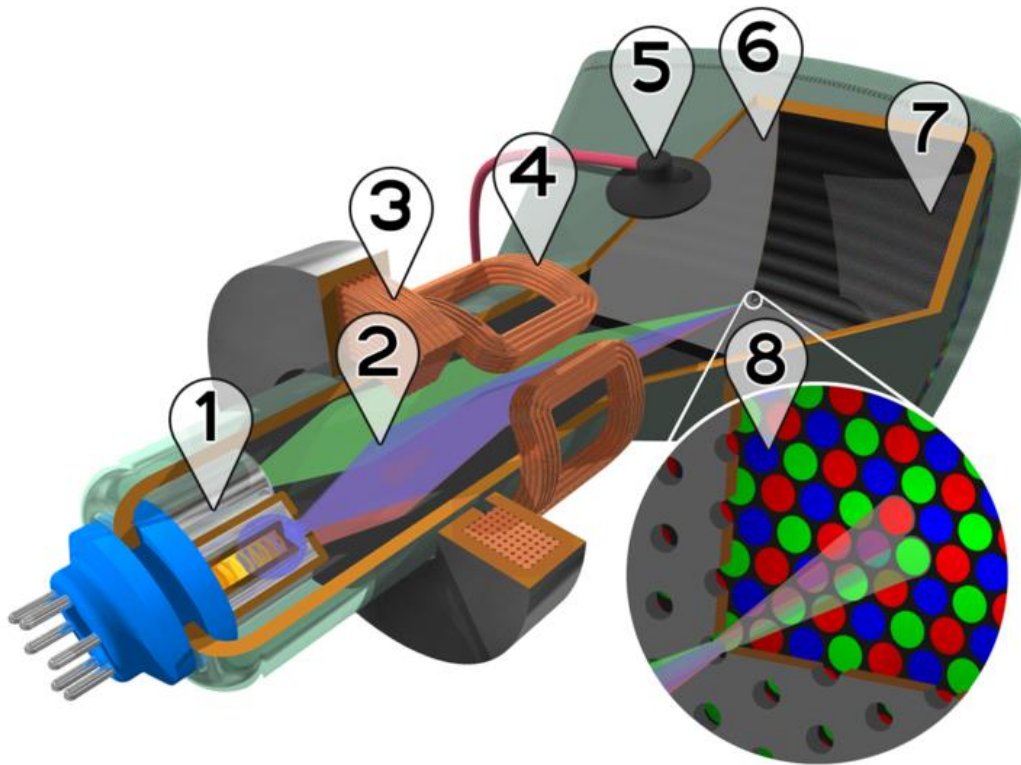
- The beam is accelerated by the first anode (+), focused by the first electromagnet coil, and accelerated more by the second anode.
- It then reaches a set of 4 coils that produce magnetic fields that steer it to a specific spot on the screen.

# Electromagnets in your TV or Computer



- The screen is coated with a fluorescent material that glows when hit by the electron beam.
- The strength of the beam determines the brightness.

# Electromagnets in your TV or Computer



- A color CRT works the same way, but has three electron guns.
- One lights a red dot or line, one blue, and one green.
- The % of each color causes the eye to see the complete picture in full color.