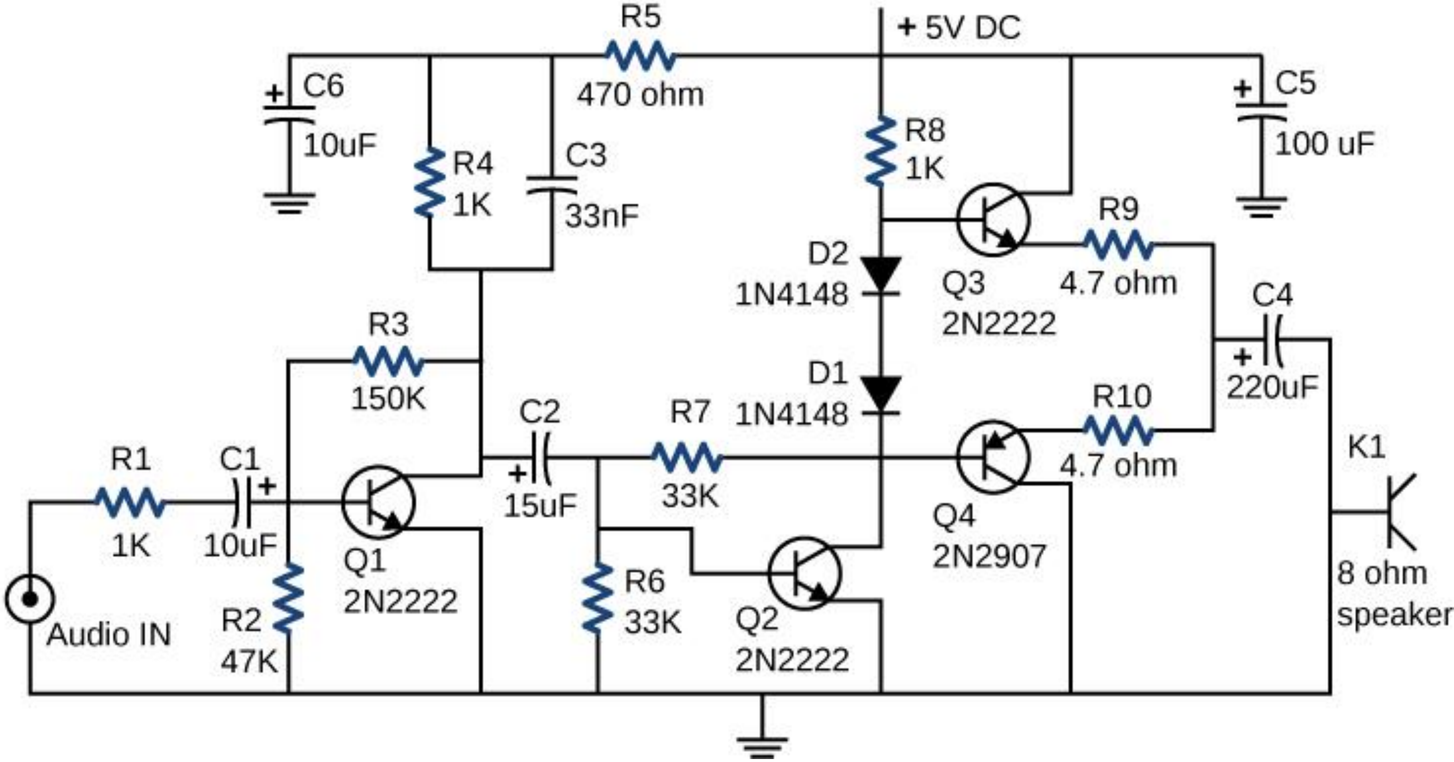
Volume 2 Chapter 10 - Basic Electric Circuits

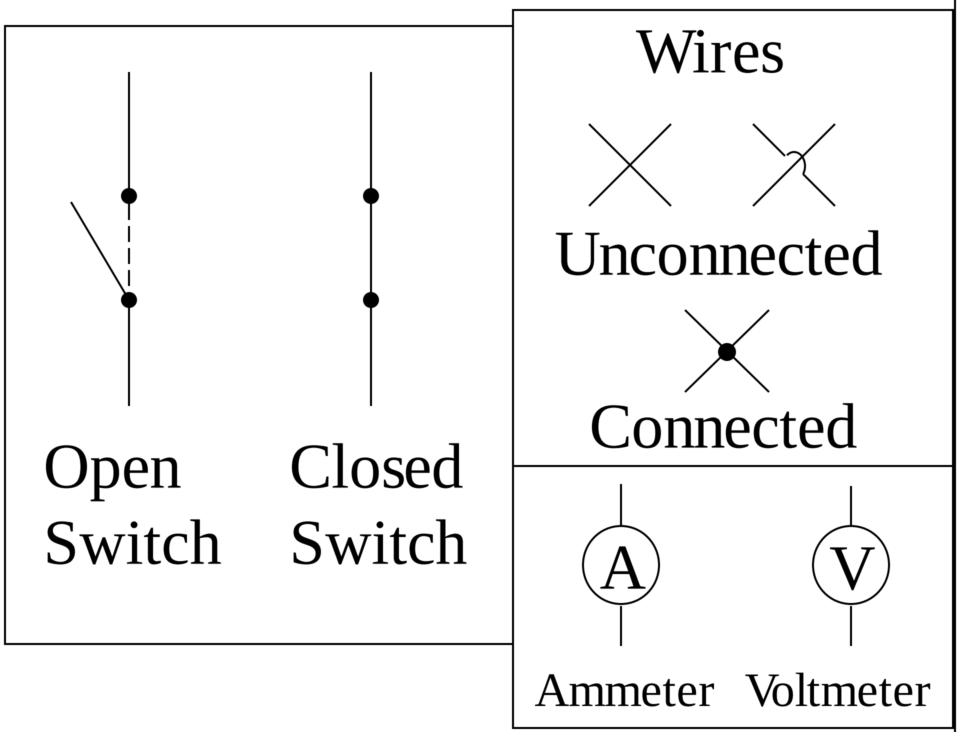


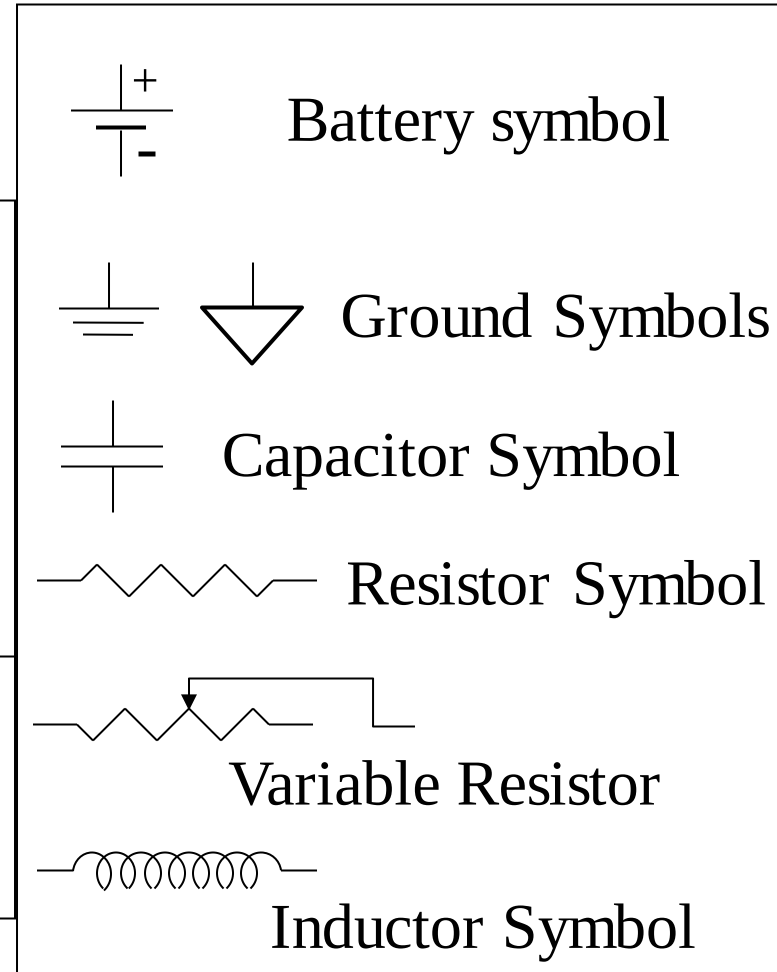
Physics 2426

Ashok Kumar

Schematic Diagrams

- Schematic Symbol list





It's like lifting a book from the floor to a high shelf at constant speed. The increase in potential energy is just equal to the nonelectrostatic work W_n, so $q\mathcal{E} = qV_{ab}$, or

$$V_{ab} = \mathcal{E} \qquad \text{(ideal source of emf)} \tag{25.13}$$

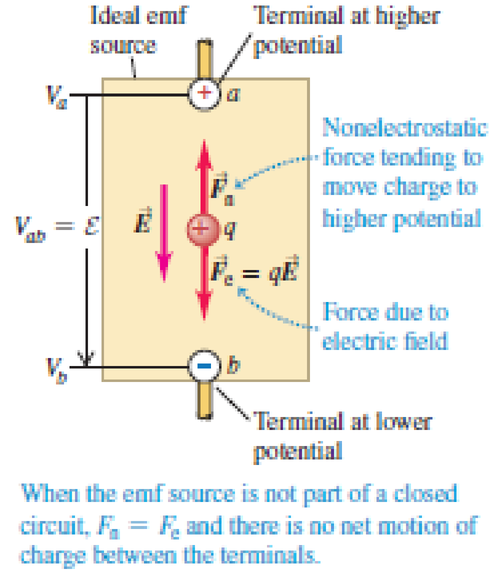
Now let's make a complete circuit by connecting a wire with resistance R to the terminals of a source (**Fig. 25.14**). The potential difference between terminals a and b sets up an electric field within the wire; this causes current to flow around the loop from a toward b, from higher to lower potential. Where the wire bends, equal amounts of positive and negative charge persist on the "inside" and "outside" of the bend. These charges exert the forces that cause the current to follow the bends in the wire.

From Eq. (25.11) the potential difference between the ends of the wire in Fig. 25.14 is given by $V_{ab} = IR$. Combining with Eq. (25.13), we have

$$\mathcal{E} = V_{ab} = IR \qquad \text{(ideal source of emf)} \tag{25.14}$$

That is, when a positive charge q flows around the circuit, the potential *rise* $\mathcal{E}$ as it passes through the ideal source is numerically equal to the potential *drop* $V_{ab} = IR$ as it passes through the remainder of the circuit. Once $\mathcal{E}$ and R are known, this relationship determines the current in the circuit.

25.13 Schematic diagram of a source of emf in an "open-circuit" situation. The electric-field force $\vec{F}_e = q\vec{E}$ and the nonelectrostatic force $\vec{F}_n$ are shown for a positive charge q.



BIO Application Danger: Electric Ray! Electric rays deliver electric shocks to stun their prey and to discourage predators. (In ancient Rome, physicians practiced a primitive form of electroconvulsive therapy by placing electric rays on their patients to cure headaches and gout.) The shocks are produced by specialized flattened cells called electroplaques. Such a cell moves ions across membranes to produce an emf of about 0.05 V. Thousands of electroplaques are stacked on top of each other, so their emfs add to a total of as much as 200 V. These stacks make up more than half of an electric ray's body mass. A ray can use these to deliver an impressive current of up to 30 A for a few milliseconds.

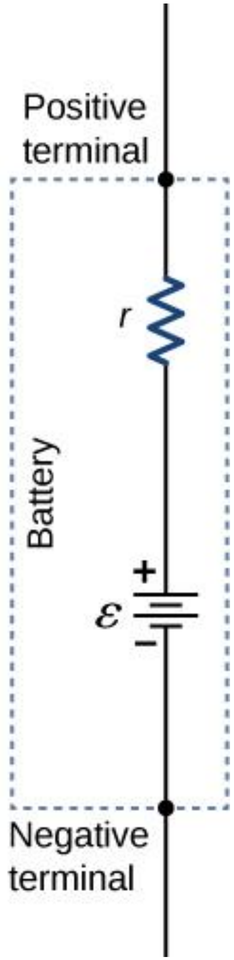
Internal Resistance

Real sources of emf in a circuit don't behave in exactly the way we have described; the potential difference across a real source in a circuit is *not* equal to the emf as in Eq. (25.14). The reason is that charge moving through the material of any real source encounters *resistance*. We call this the **internal resistance** of the source, denoted by r. If this resistance behaves according to Ohm's law, r is constant and independent of the current I. As the current moves through r, it experiences an associated drop in potential equal to Ir. Thus, when a current is flowing through a source from the negative terminal b to the positive terminal a, the potential difference V_{ab} between the terminals is

$$\underbrace{V_{ab}}_{\text{Terminal voltage, source with internal resistance}} = \overbrace{\mathcal{E}}^{\text{emf of source}} - \overbrace{I}^{\text{Current through source}}\underbrace{r}_{\text{Internal resistance of source}} \qquad (25.15)$$

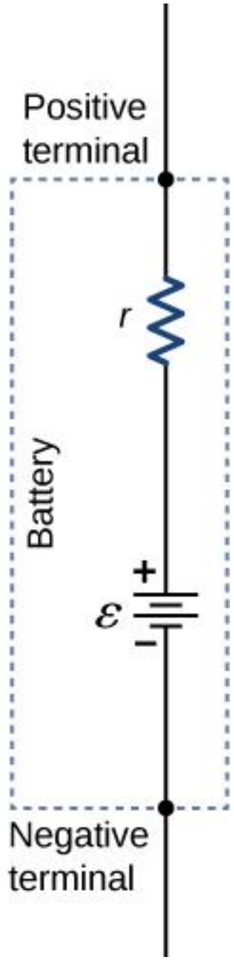
The potential V_{ab}, called the **terminal voltage,** is less than the emf $\mathcal{E}$ because of the term Ir representing the potential drop across the internal resistance r. Hence the increase in potential energy qV_{ab} as a charge q moves from b to a within the source is less than the work $q\mathcal{E}$ done by the nonelectrostatic force $\vec{F}_n$, since some potential energy is lost in traversing the internal resistance.

EMF & Terminal Voltage



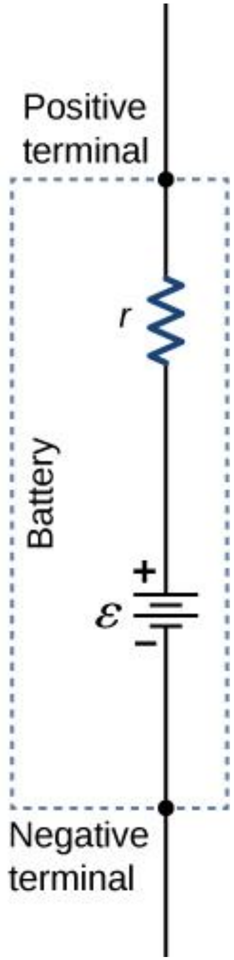
- A source of voltage or electromotive force (EMF) is needed to act as a pump for the electrons or other charges in the circuit.
- This may be a battery, a generator, other source of electrical potential.

EMF & Terminal Voltage



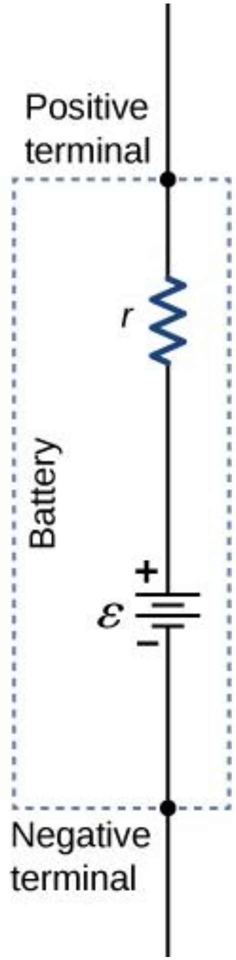
- Since mobile devices are so common today, lets look at how batteries actually work.
- The rated voltage of a battery is referred to as the EMF of the battery.
- However, due to internal resistance, the actual output or terminal voltage will be less.

EMF & Terminal Voltage



- We represent this as a perfect chemical source having voltage, $\mathcal{E}$, and an internal resistor, r.
- When the battery is used to generate a current, I, the current flowing through r reduces the output voltage to
- $V_{output} = \mathcal{E} - I \cdot r$

EMF & Terminal Voltage



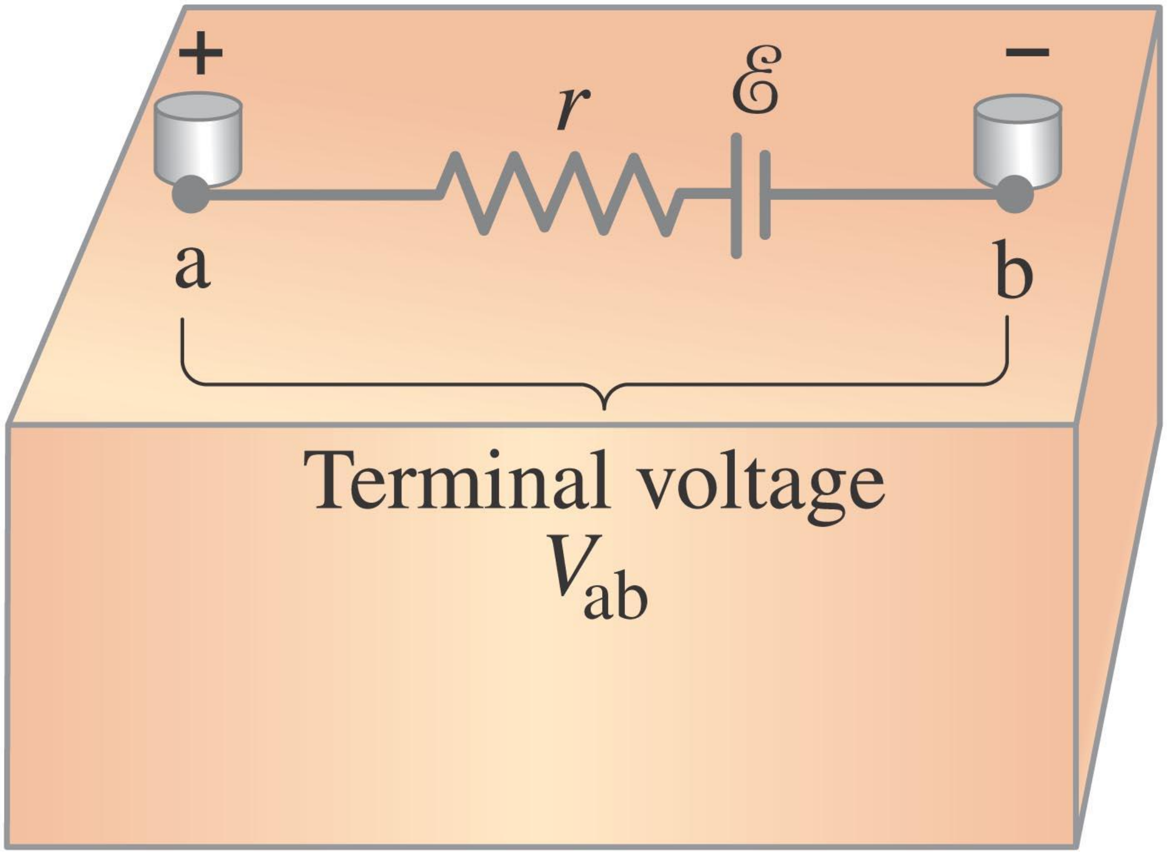
- Note: As a battery discharges, EMF will fall somewhat.
- Also, as the electrolyte is depleted, internal resistance increases.
- Modern ‘power management’ chips use these characteristics to track the battery’s charge.

Electric circuit needs battery or generator to produce current – these are called sources of emf.

Battery is a nearly constant voltage source, but does have a small internal resistance, which reduces the actual voltage from the ideal emf:

$$V_{\mathrm{ab}} = \mathcal{E} - Ir.$$

This resistance behaves as though it were in series with the emf.



Example 26-1: Battery with internal resistance.

A 65.0-Ω resistor is connected to the terminals of a battery whose emf is 12.0 V and whose internal resistance is 0.5 Ω. Calculate (a) the current in the circuit, (b) the terminal voltage of the battery, V_{ab}, and (c) the power dissipated in the resistor R and in the battery's internal resistance r.

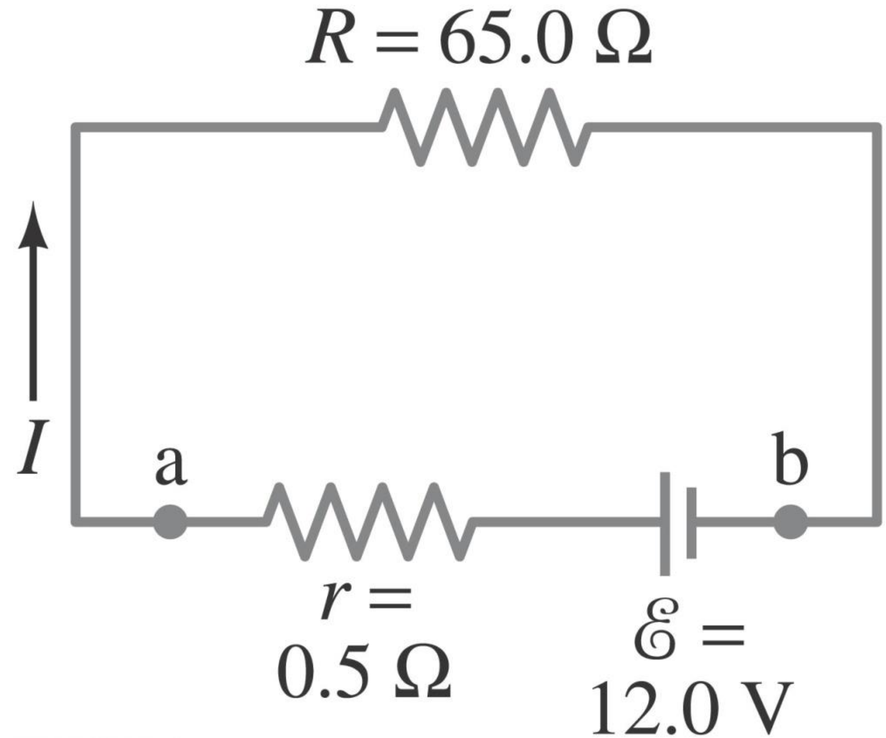
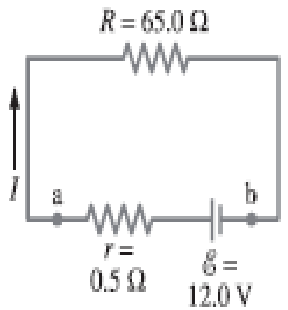


FIGURE 26–2 Example 26–1.



EXAMPLE 26–1 **Battery with internal resistance.** A 65.0-Ω resistor is connected to the terminals of a battery whose emf is 12.0 V and whose internal resistance is 0.5 Ω, Fig. 26–2. Calculate (*a*) the current in the circuit, (*b*) the terminal voltage of the battery, V_{ab}, and (*c*) the power dissipated in the resistor R and in the battery's internal resistance r.

APPROACH We first consider the battery as a whole, which is shown in Fig. 26–2 as an emf $\mathscr{E}$ and internal resistance r between points a and b. Then we apply $V = IR$ to the circuit itself.

[†]When a battery is being charged, a current is forced to pass through it; we then have to write

$$V_{ab} = \mathscr{E} + Ir.$$

See Section 26–4 or Problem 28 and Fig. 26–46.

SOLUTION (*a*) From Eq. 26–1, we have

$$V_{ab} = \mathscr{E} - Ir.$$

We apply Ohm's law (Eqs. 25–2) to this battery and the resistance R of the circuit: $V_{ab} = IR$. Hence $IR = \mathscr{E} - Ir$ or $\mathscr{E} = I(R + r)$, and so

$$I = \frac{\mathscr{E}}{R + r} = \frac{12.0\text{ V}}{65.0\,\Omega + 0.5\,\Omega} = \frac{12.0\text{ V}}{65.5\,\Omega} = 0.183\text{ A}.$$

(*b*) The terminal voltage is

$$V_{ab} = \mathscr{E} - Ir = 12.0\text{ V} - (0.183\text{ A})(0.5\,\Omega) = 11.9\text{ V}.$$

(*c*) The power dissipated (Eq. 25–7) in R is

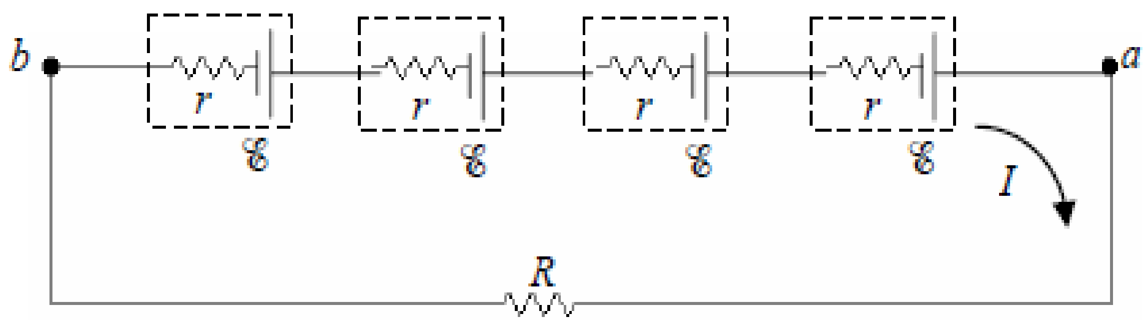
$$P_R = I^2R = (0.183\text{ A})^2(65.0\,\Omega) = 2.18\text{ W},$$

and in r is

$$P_r = I^2r = (0.183\text{ A})^2(0.5\,\Omega) = 0.02\text{ W}.$$

2. (I) Four 1.50-V cells are connected in series to a 12-Ω light-bulb. If the resulting current is 0.45 A, what is the internal resistance of each cell, assuming they are identical and neglecting the resistance of the wires?

2. See the circuit diagram below. The current in the circuit is I. The voltage V_{ab} is given by Ohm's law to be $V_{ab} = IR$. That same voltage is the terminal voltage of the series EMF.



$$V_{ab} = (\mathcal{E} - Ir) + (\mathcal{E} - Ir) + (\mathcal{E} - Ir) + (\mathcal{E} - Ir) = 4(\mathcal{E} - Ir) \quad \text{and} \quad V_{ab} = IR$$

$$4(\mathcal{E} - Ir) = IR \quad \rightarrow \quad r = \frac{\mathcal{E} - \frac{1}{4}IR}{I} = \frac{(1.5\,\text{V}) - \frac{1}{4}(0.45\,\text{A})(12\,\Omega)}{0.45\,\text{A}} = 0.333\,\Omega \approx \boxed{0.3\,\Omega}$$

4. (II) What is the internal resistance of a 12.0-V car battery whose terminal voltage drops to 8.4 V when the starter draws 95 A? What is the resistance of the starter?

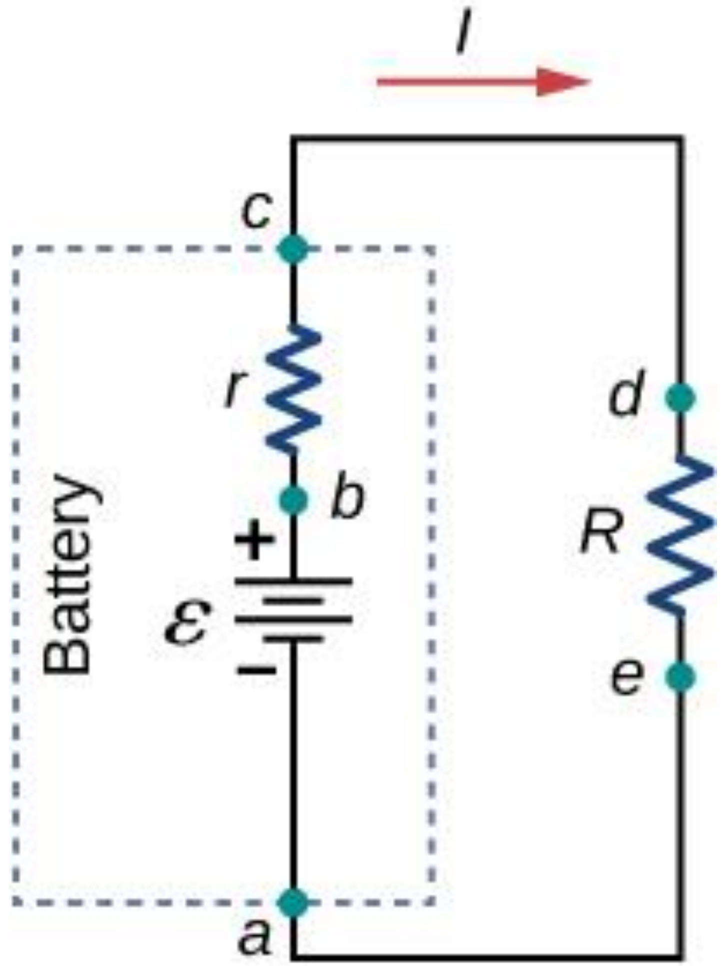
4. See Figure 26-2 for a circuit diagram for this problem. Use Eq. 26-1.

$$V_{ab} = \mathcal{E} - Ir \rightarrow r = \frac{\mathcal{E} - V_{ab}}{I} = \frac{12.0\,\mathrm{V} - 8.4\,\mathrm{V}}{95\,\mathrm{A}} = \boxed{0.038\,\Omega}$$

$$V_{ab} = IR \rightarrow R = \frac{V_{ab}}{I} = \frac{8.4\,\mathrm{V}}{95\,\mathrm{A}} = \boxed{0.088\,\Omega}$$

EMF & Terminal Voltage

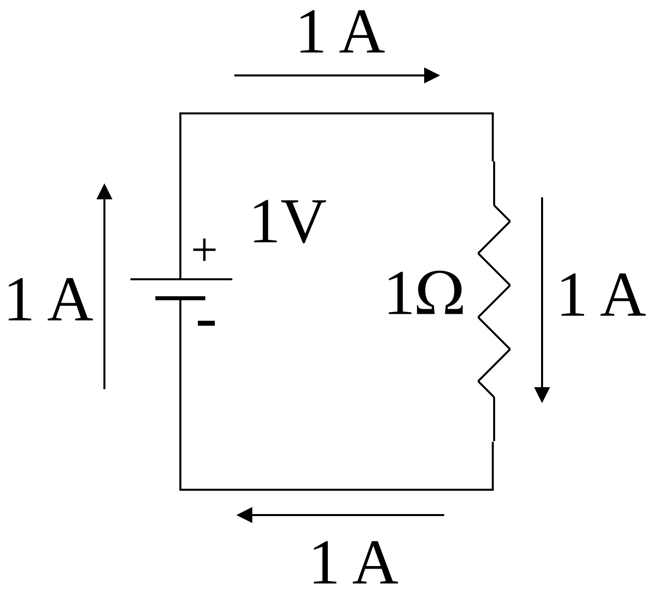
- For $\mathcal{E} = 12v \; and \; r = 0.1\Omega$
- Calculate the output voltage V_{ac} and current for the battery at right for
- R = 10 Ω
- R = 0.5 Ω
- As we deplete the battery's chemicals, r increases.
- Recalculate for $r = 0.5\Omega$



Electrical Current & Resistance

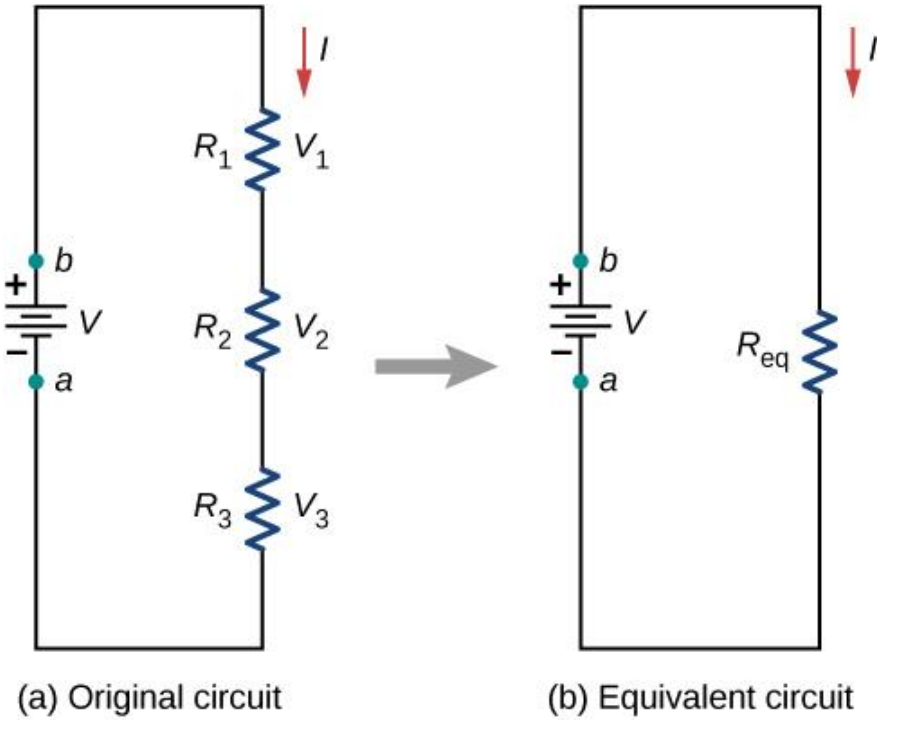
Simple Circuits

- The relationship between voltage, resistance and current is called Ohm's Law.
- $V = I \cdot R$
- $1\ \Omega = 1\text{Volt}/1\text{Amp}$



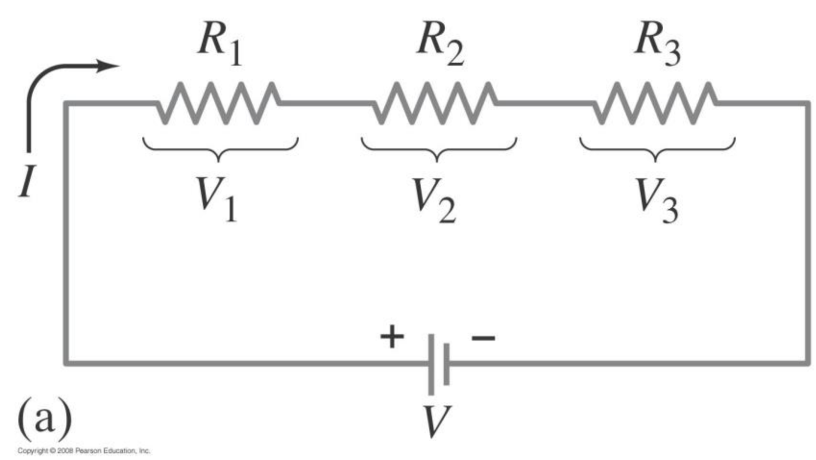
Resistors in Series

- To calculate the effect of several resistors in a series along a circuit, we use the fact that the current does not change as we proceed around the circuit.
- The number of electrons flowing through the battery and through each resistor is the same.



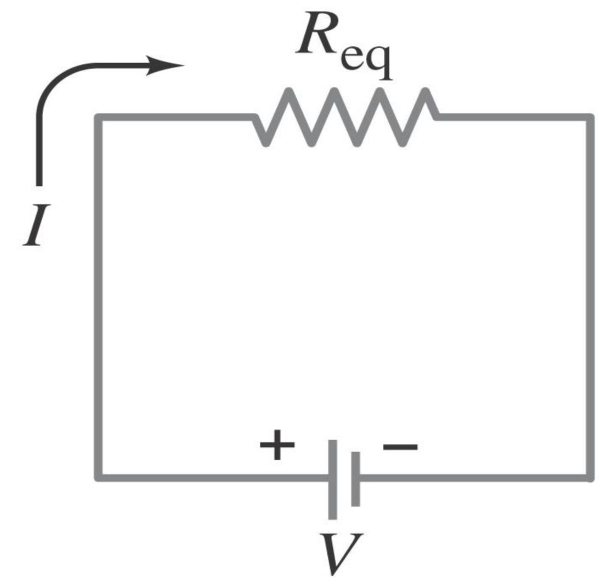
Resistors in Series

- For resistors in series, the same current flows through each resistor.
- Therefore, the voltage drop across each resistor is given by Ohm's Law.
- $V_i = I \cdot Ri$
- And the total voltage drop across the three resistors must equal the voltage output of the battery.
- $V = V_1 + V_2 + V_3$

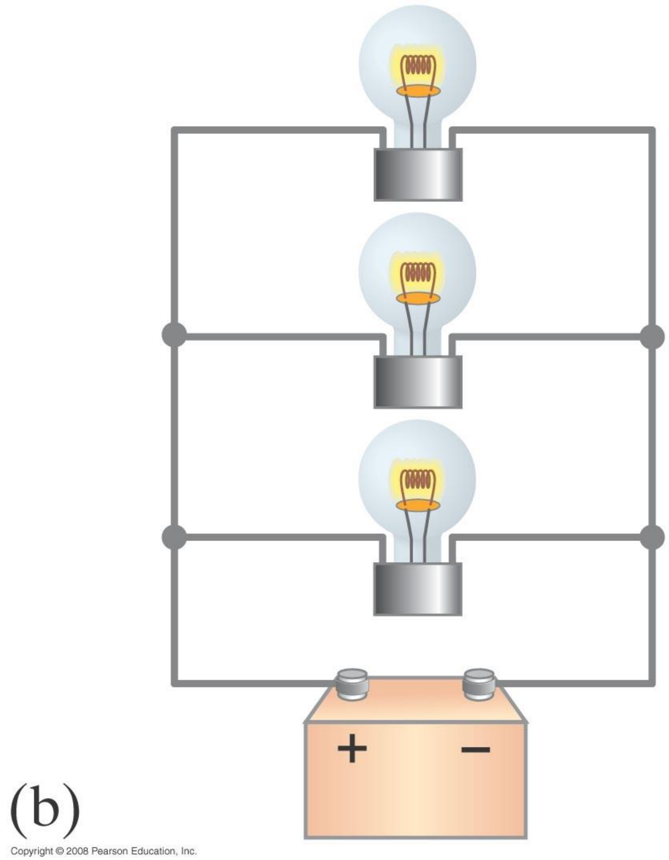


Resistors in Series

- Using Ohm's Law we can write:
- $V = R_1 \cdot I + R_2 \cdot I + R_3 \cdot I$
- To find a resistor value that is equivalent to the combined resistors, we write
- $V = R_{eq} \cdot I = (R_1 + R_2 + R_3) \cdot I$
- And we see that this is simply:
- $R_{eq} = R_1 + R_2 + R_3$
- We could replace these 3 resistors with R_{eq} and the circuit will draw the same current.

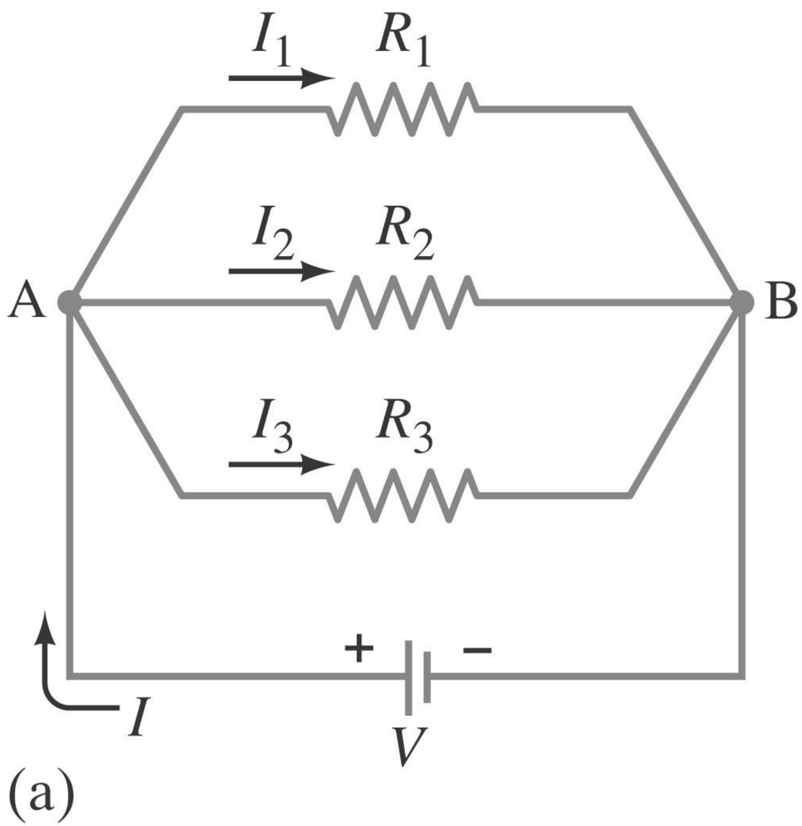


Resistors in Parallel



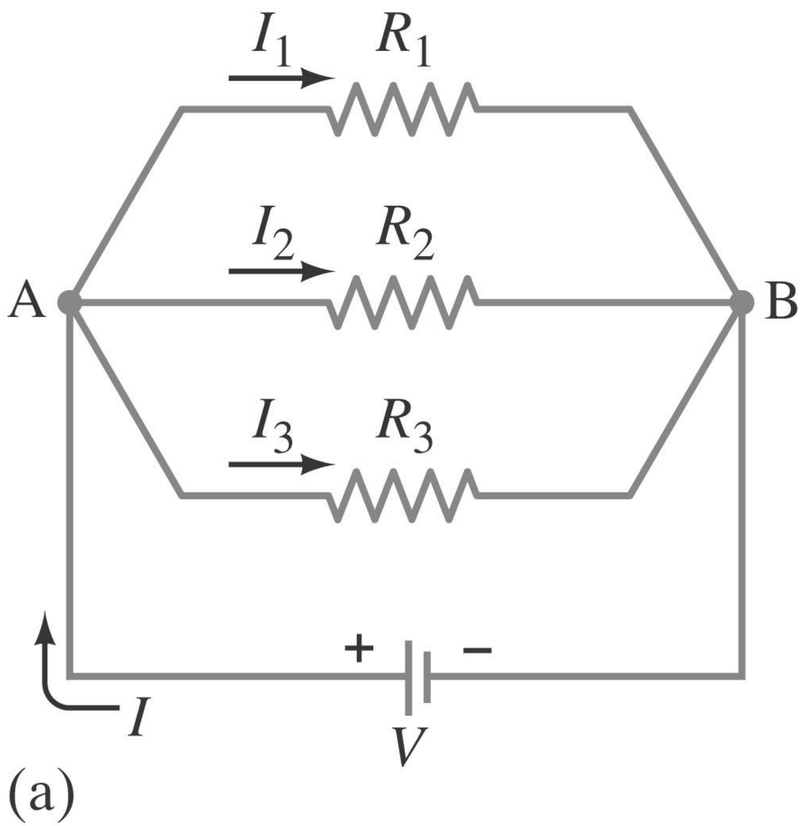
- For resistors in parallel, we can look at the circuit at left and see the the voltage applied to each resistor is the battery voltage, V.
- So, applying Ohm's law to each resistor.
- $V = R_i \cdot I_i$ or $I_i = V/R_i$

Resistors in Parallel



- We can also see that the current that flows through the battery, I, must be equal to the sum of the three currents,
- $I_{total} = I_1 + I_2 + I_3$
- Since this current is larger than the current through any single resistor, the resistance of this parallel combination must be less than any single resistance in the group.

Resistors in Parallel



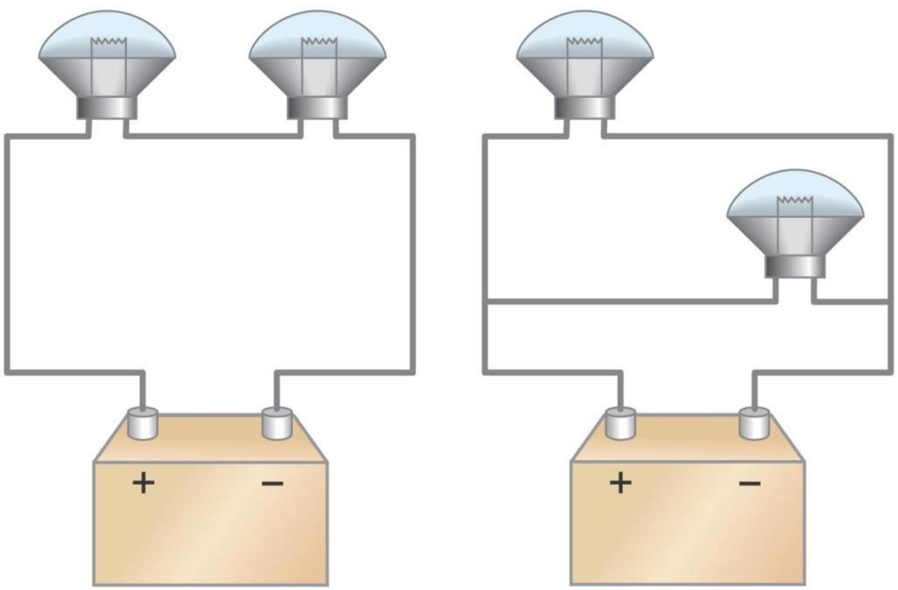
- We calculate the equivalent resistor, R_{eq}, as follows:
- $V = R_1 \cdot I_1 = R_2 \cdot I_2 = R_3 \cdot I_3$
- Find the equivalent resistor where
- $V = Req \cdot (I_1 + I_2 + I_3)$
- $V/R_{eq} = I_1 + I_2 + I_3$
- And using Ohm's Law:
- $V/R_{eq} = (V/R_1) + (V/R_2) + (V/R_3)$
- Which gives us
- $1/Req = 1/R_1 + 1/R_2 + 1/R_3$
- (Inverse addition)

26-2 Resistors in Series and in Parallel

An analogy using water may be helpful in visualizing parallel circuits. The water (current) splits into two streams; each falls the same height, and the total current is the sum of the two currents. With two pipes open, the resistance to water flow is half what it is with one pipe open.

Conceptual Example 26-2: Series or parallel?

(a) The lightbulbs in the figure are identical. Which configuration produces more light? (b) Which way do you think the headlights of a car are wired? Ignore change of filament resistance R with current.

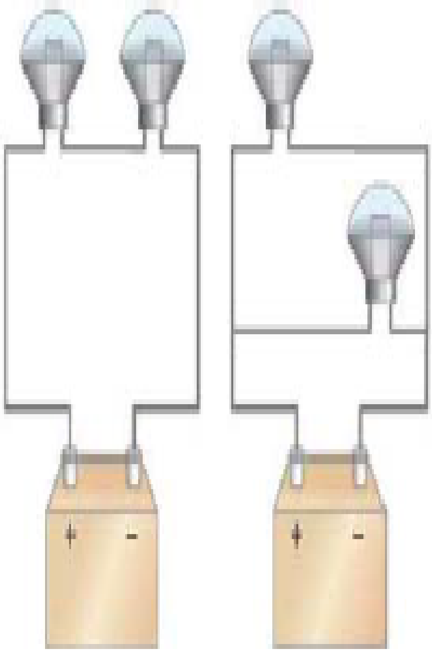


Series Parallel

CONCEPTUAL EXAMPLE 26–2 **Series or parallel?** (*a*) The lightbulbs in Fig. 26–6 are identical. Which configuration produces more light? (*b*) Which way do you think the headlights of a car are wired? Ignore change of filament resistance R with current.

RESPONSE (*a*) The equivalent resistance of the parallel circuit is found from Eq. 26–4, $1/R_{eq} = 1/R + 1/R = 2/R$. Thus $R_{eq} = R/2$. The parallel combination then has lower resistance $(= R/2)$ than the series combination $\left(R_{eq} = R + R = 2R\right)$. There will be more total current in the parallel configuration (2), since $I = V/R_{eq}$ and V is the same for both circuits. The total power transformed, which is related to the light produced, is $P = IV$, so the greater current in (2) means more light produced.
(*b*) Headlights are wired in parallel (2), because if one bulb goes out, the other bulb can stay lit. If they were in series (1), when one bulb burned out (the filament broke), the circuit would be open and no current would flow, so neither bulb would light.

NOTE When you answered the Chapter-Opening Question on page 677, was your answer circuit 2? Can you express any misconceptions you might have had?



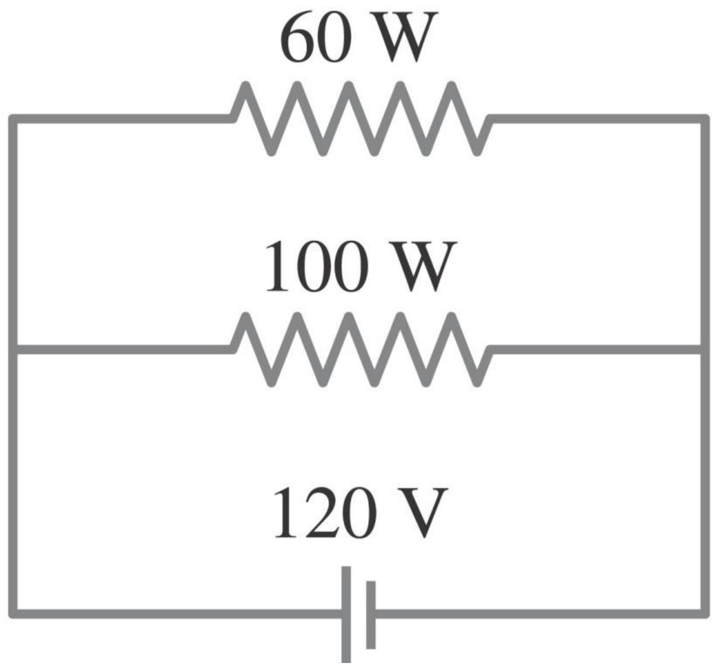
(1) Series (2) Parallel

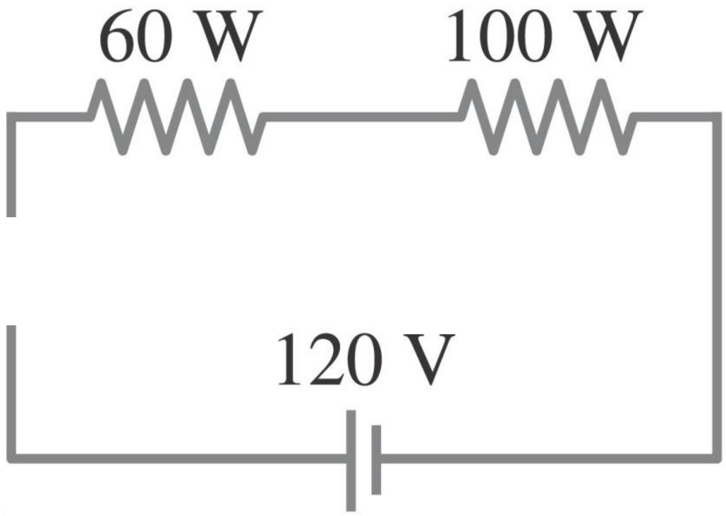
FIGURE 26–6 Example 26–2.

FIGURE 26–7 Example 26–3.

Conceptual Example 26-3: An illuminating surprise.

A 100-W, 120-V lightbulb and a 60-W, 120-V lightbulb are connected in two different ways as shown. In each case, which bulb glows more brightly? Ignore change of filament resistance with current (and temperature).





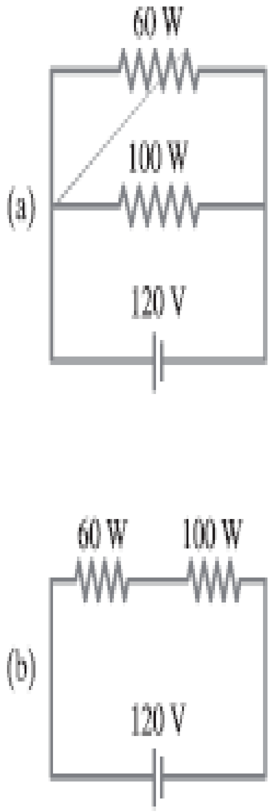
CONCEPTUAL EXAMPLE 26–3 **An illuminating surprise.** A 100-W, 120-V lightbulb and a 60-W, 120-V lightbulb are connected in two different ways as shown in Fig. 26–7. In each case, which bulb glows more brightly? Ignore change of filament resistance with current (and temperature).

RESPONSE (*a*) These are normal lightbulbs with their power rating given for 120 V. They both receive 120 V, so the 100-W bulb is naturally brighter.

(*b*) The resistance of the 100-W bulb is less than that of the 60-W bulb (calculated from $P = V^2/R$ at constant 120 V). Here they are connected in series and receive the same current. Hence, from $P = I^2R$ (I = constant) the higher-resistance "60-W" bulb will transform more power and thus be brighter.

NOTE When connected in series as in (b), the two bulbs do *not* dissipate 60 W and 100 W because neither bulb receives 120 V.

Note that whenever a group of resistors is replaced by the equivalent resistance, current and voltage and power in the rest of the circuit are unaffected.



16. (II) Determine (*a*) the equivalent resistance of the circuit shown in Fig. 26–39, and (*b*) the voltage across each resistor.

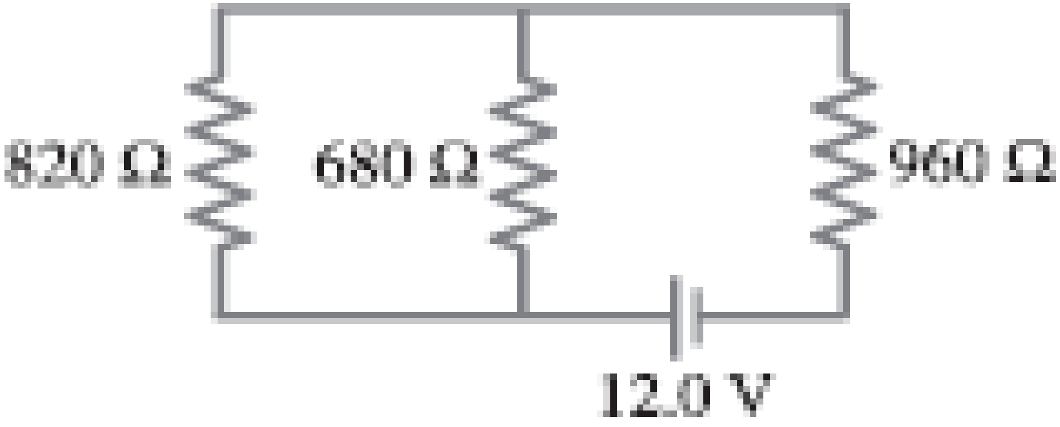


FIGURE 26–39
Problem 16.

. (*a*) The equivalent resistance is found by combining the 820 Ω and 680 Ω resistors in parallel, and then adding the 960 Ω resistor in series with that parallel combination.

$$R_{eq} = \left(\frac{1}{820\,\Omega} + \frac{1}{680\,\Omega} \right)^{-1} + 960\,\Omega = 372\,\Omega + 960\,\Omega = 1332\,\Omega \approx \boxed{1330\,\Omega}$$

(*b*) The current delivered by the battery is $I = \frac{V}{R_{eq}} = \frac{12.0\,\text{V}}{1332\,\Omega} = 9.009\times10^{-3}\,\text{A}$. This is the current in the 960 Ω resistor. The voltage across that resistor can be found by Ohm's law.

$$V_{470} = IR = \left(9.009\times10^{-3}\,\text{A}\right)\left(960\,\Omega\right) = 8.649\,\text{V} \approx \boxed{8.6\,\text{V}}$$

Thus the voltage across the parallel combination must be $12.0\,\text{V} - 8.6\,\text{V} = \boxed{3.4\,\text{V}}$. This is the voltage across both the 820 Ω and 680 Ω resistors, since parallel resistors have the same voltage across them. Note that this voltage value could also be found as follows.

$$V_{parallel} = IR_{parallel} = \left(9.009\times10^{-3}\,\text{A}\right)\left(372\,\Omega\right) = 3.351\,\text{V} \approx 3.4\,\text{V}$$

18. (II) (*a*) Determine the equivalent resistance of the "ladder" of equal 125-Ω resistors shown in Fig. 26–40. In other words, what resistance would an ohmmeter read if connected between points A and B? (*b*) What is the current through each of the three resistors on the left if a 50.0-V battery is connected between points A and B?

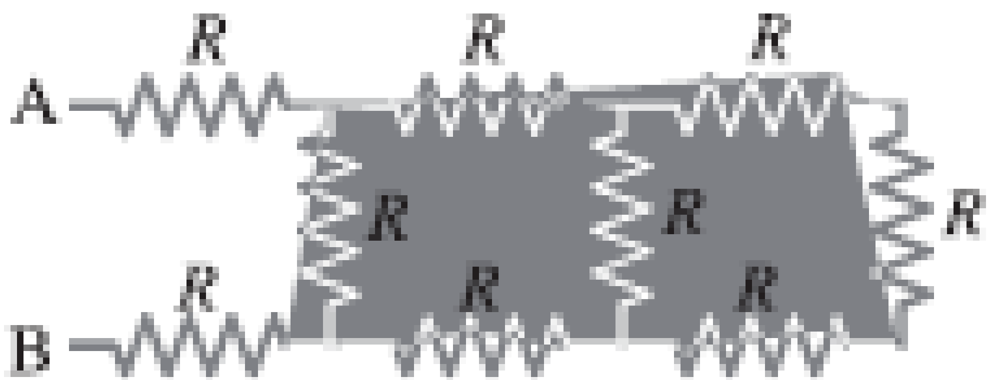
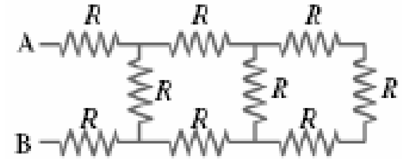
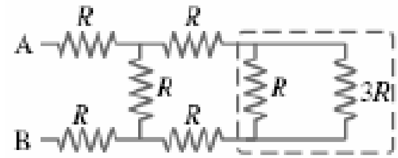


FIGURE 26–40
Problem 18.

18. (*a*) The three resistors on the far right are in series, so their equivalent resistance is $3R$. That combination is in parallel with the next resistor to the left, as shown in the dashed box in the second figure. The equivalent resistance of the dashed box is found as follows.



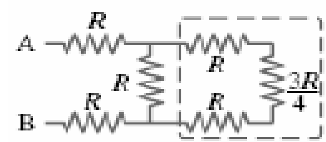
(a)



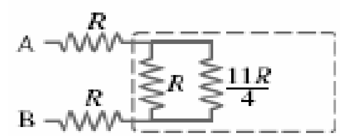
(b)

$$R_{\text{eq1}} = \left(\frac{1}{R} + \frac{1}{3R} \right)^{-1} = \tfrac{3}{4} R$$

This equivalent resistance of $\tfrac{3}{4}R$ is in series with the next two resistors, as shown in the dashed box in the third figure (on the next page). The equivalent resistance of that dashed box is $R_{\text{eq2}} = 2R + \tfrac{3}{4}R = \tfrac{11}{4}R$. This $\tfrac{11}{4}R$ is in parallel with the next resistor to the left, as shown in the fourth figure. The equivalent resistance of that dashed box is found as follows.



(c)



(d)

$$R_{\text{eq2}} = \left(\frac{1}{R} + \frac{4}{11R} \right)^{-1} = \tfrac{11}{15} R.$$

This is in series with the last two resistors, the ones connected directly to A and B. The final equivalent resistance is given below.

$$R_{\text{eq}} = 2R + \tfrac{11}{15}R = \tfrac{41}{15}R = \tfrac{41}{15}(125\,\Omega) = 341.67\,\Omega \approx \boxed{342\,\Omega}$$

(*b*) The current flowing from the battery is found from Ohm's law.

$$I_{\text{total}} = \frac{V}{R_{\text{eq}}} = \frac{50.0\,\text{V}}{341.67\,\Omega} = 0.1463\,\text{A} \approx \boxed{0.146\,\text{A}}$$

This is the current in the top and bottom resistors. There will be less current in the next resistor because the current splits, with some current passing through the resistor in question, and the rest of the current passing through the equivalent resistance of $\tfrac{11}{4}R$, as shown in the last figure. The voltage across R and across $\tfrac{11}{4}R$ must be the same, since they are in parallel. Use this to find the desired current.

$$V_R = V_{\frac{11}{4}R} \;\rightarrow\; I_R R = I_{\frac{11}{4}R}\left(\tfrac{11}{4}R\right) = \left(I_{\text{total}} - I_R\right)\left(\tfrac{11}{4}R\right) \;\rightarrow$$

$$I_R = \tfrac{11}{15} I_{\text{total}} = \tfrac{11}{15}(0.1463\,\text{A})\, I_{\text{total}} = \boxed{0.107\,\text{A}}$$

EXAMPLE 26.1 EQUIVALENT RESISTANCE

Find the equivalent resistance of the network in **Fig. 26.3a** (next page) and the current in each resistor. The source of emf has negligible internal resistance.

SOLUTION

IDENTIFY and SET UP: This network of three resistors is a *combination* of series and parallel resistances, as in Fig. 26.1c. We determine the equivalent resistance of the parallel 6-Ω and 3-Ω resistors, and then that of their series combination with the 4-Ω resistor: This is the equivalent resistance R_{eq} of the network as a whole. We then find the current in the emf, which is the same as that in the 4-Ω resistor. The potential difference is the same across each of the parallel 6-Ω and 3-Ω resistors; we use this to determine how the current is divided between these.

EXECUTE: Figures 26.3b and 26.3c show successive steps in reducing the network to a single equivalent resistance R_{eq}. From Eq. (26.2), the 6-Ω and 3-Ω resistors in parallel in Fig. 26.3a are equivalent to the single 2-Ω resistor in Fig. 26.3b:

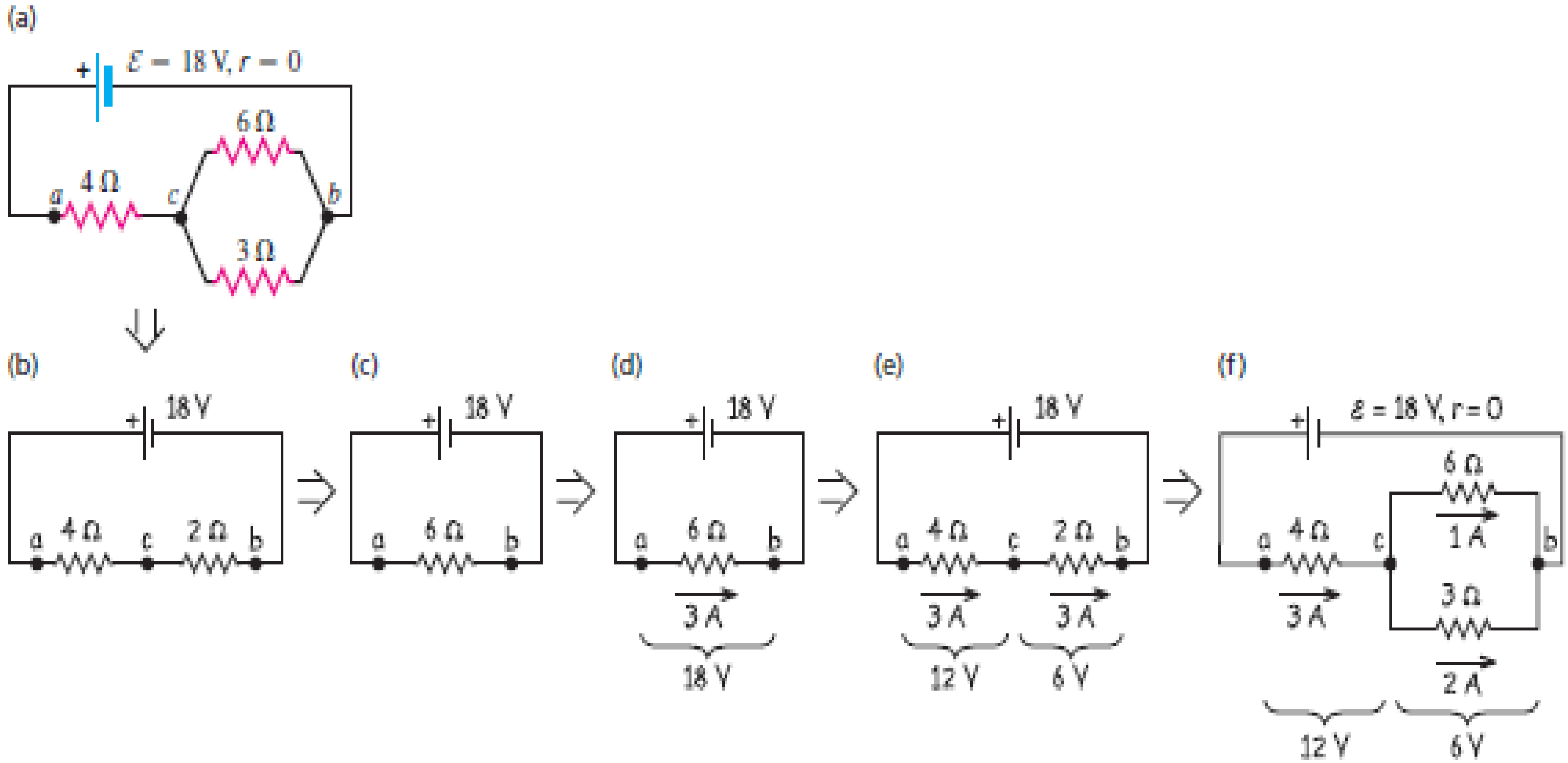
$$\frac{1}{R_{6\,\Omega+3\,\Omega}} = \frac{1}{6\ \Omega} + \frac{1}{3\ \Omega} = \frac{1}{2\ \Omega}$$

[Equation (26.3) gives the same result.] From Eq. (26.1) the series combination of this 2-Ω resistor with the 4-Ω resistor is equivalent to the single 6-Ω resistor in Fig. 26.3c.

We reverse these steps to find the current in each resistor of the original network. In the circuit shown in Fig. 26.3d (identical to Fig. 26.3c), the current is $I = V_{ab}/R = (18\ \text{V})/(6\ \Omega) = 3\ \text{A}$. So the current in the 4-Ω and 2-Ω resistors in Fig. 26.3e (identical

Continued

26.3 Steps in reducing a combination of resistors to a single equivalent resistor and finding the current in each resistor.



to Fig. 26.3b) is also 3 A. The potential difference V_{cb} across the 2-Ω resistor is therefore $V_{cb} = IR = (3\ \text{A})(2\ \Omega) = 6\ \text{V}$. This potential difference must also be 6 V in Fig. 26.3f (identical to Fig. 26.3a). From $I = V_{cb}/R$, the currents in the 6-Ω and 3-Ω resistors in Fig. 26.3f are, respectively, $(6\ \text{V})/(6\ \Omega) = 1\ \text{A}$ and $(6\ \text{V})/(3\ \Omega) = 2\ \text{A}$.

EVALUATE: Note that for the two resistors in parallel between points *c* and *b* in Fig. 26.3f, there is twice as much current through the 3-Ω resistor as through the 6-Ω resistor; more current goes through the path of least resistance, in accordance with Eq. (26.4). Note also that the total current through these two resistors is 3 A, the same as it is through the 4-Ω resistor between points *a* and *c*.

EXAMPLE 26.2 SERIES VERSUS PARALLEL COMBINATIONS

Two identical incandescent light bulbs, each with resistance $R = 2\ \Omega$, are connected to a source with $\mathcal{E} = 8$ V and negligible internal resistance. Find the current through each bulb, the potential difference across each bulb, and the power delivered to each bulb and to the entire network if the bulbs are connected (a) in series and (b) in parallel. (c) Suppose one of the bulbs burns out; that is, its filament breaks and current can no longer flow through it. What happens to the other bulb in the series case? In the parallel case?

SOLUTION

IDENTIFY and SET UP: The light bulbs are just resistors in simple series and parallel connections (**Figs. 26.4a** and 26.4b). Once we find the current I through each bulb, we can find the power delivered to each bulb by using Eq. (25.18), $P = I^2R = V^2/R$.

EXECUTE: (a) From Eq. (26.1) the equivalent resistance of the two bulbs between points a and c in Fig. 26.4a is $R_{eq} = 2R = 2(2\ \Omega) = 4\ \Omega$. In series, the current is the same through each bulb:

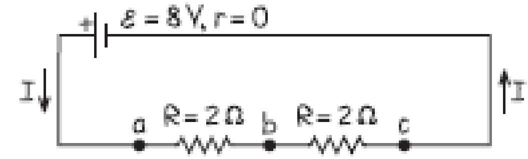
$$I = \frac{V_{ac}}{R_{eq}} = \frac{8\ \text{V}}{4\ \Omega} = 2\ \text{A}$$

Since the bulbs have the same resistance, the potential difference is the same across each bulb:

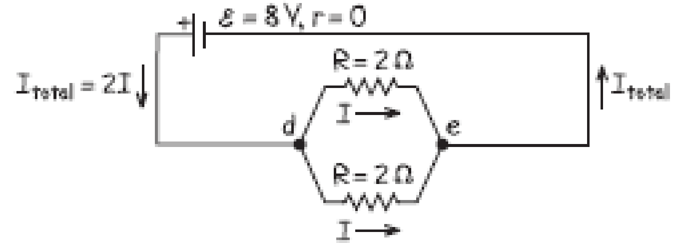
$$V_{ab} = V_{bc} = IR = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$$

26.4 Our sketches for this problem.

(a) Light bulbs in series



(b) Light bulbs in parallel



From Eq. (25.18), the power delivered to each bulb is

$$P = I^2R = (2\ \text{A})^2(2\ \Omega) = 8\ \text{W} \qquad \text{or}$$

$$P = \frac{V_{ab}^2}{R} = \frac{V_{bc}^2}{R} = \frac{(4\ \text{V})^2}{2\ \Omega} = 8\ \text{W}$$

The total power delivered to both bulbs is $P_{tot} = 2P = 16$ W.

(b) If the bulbs are in parallel, as in Fig. 26.4b, the potential difference V_{de} across each bulb is the same and equal to 8 V, the terminal voltage of the source. Hence the current through each light bulb is

$$I = \frac{V_{de}}{R} = \frac{8\ \text{V}}{2\ \Omega} = 4\ \text{A}$$

and the power delivered to each bulb is

$$P = I^2R = (4\ \text{A})^2(2\ \Omega) = 32\ \text{W} \qquad \text{or}$$

$$P = \frac{V_{de}{}^2}{R} = \frac{(8\ \text{V})^2}{2\ \Omega} = 32\ \text{W}$$

Both the potential difference across each bulb and the current through each bulb are twice as great as in the series case. Hence the power delivered to each bulb is *four* times greater, and each bulb is brighter.

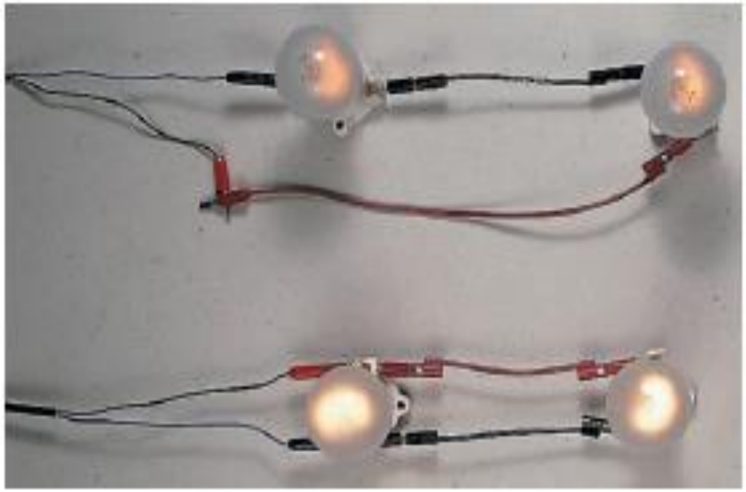
The total power delivered to the parallel network is $P_{total} = 2P = 64$ W, four times greater than in the series case. The increased power compared to the series case isn't obtained "for free"; energy is extracted from the source four times more rapidly in the parallel case than in the series case. If the source is a battery, it will be used up four times as fast.

(c) In the series case the same current flows through both bulbs. If one bulb burns out, there will be no current in the circuit, and neither bulb will glow.

In the parallel case the potential difference across either bulb is unchanged if a bulb burns out. The current through the functional bulb and the power delivered to it are unchanged.

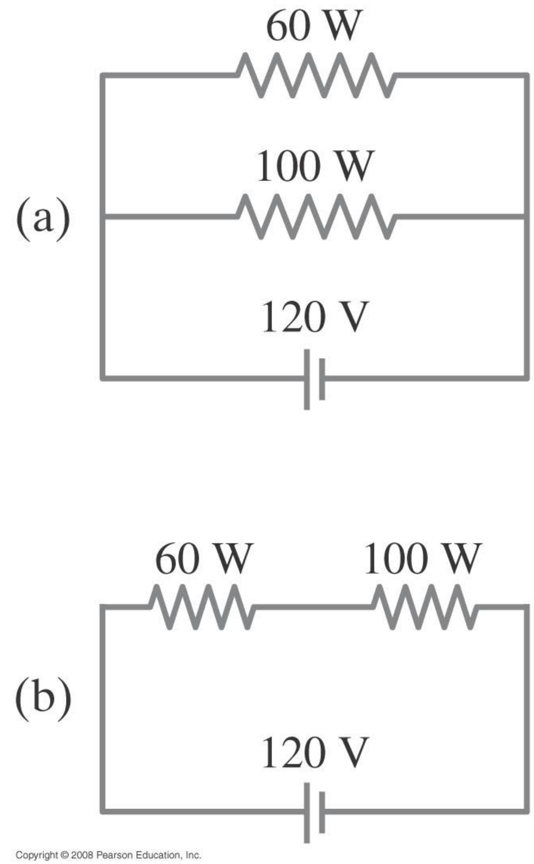
EVALUATE: Our calculation isn't completely accurate, because the resistance $R = V/I$ of real light bulbs depends on the potential difference V across the bulb. That's because the filament resistance increases with increasing operating temperature and therefore with increasing V. But bulbs connected in series across a source do in fact glow less brightly than when connected in parallel across the same source (**Fig. 26.5**).

26.5 When connected to the same source, two incandescent light bulbs in series (shown at top) draw less power and glow less brightly than when they are in parallel (shown at bottom).



Series & Parallel Example

- Calculate the power dissipated in the light bulbs (resistors) at right.
- a) normal parallel circuit
- b) series circuit



19. (II) What is the net resistance of the circuit connected to the battery in Fig. 26–41?

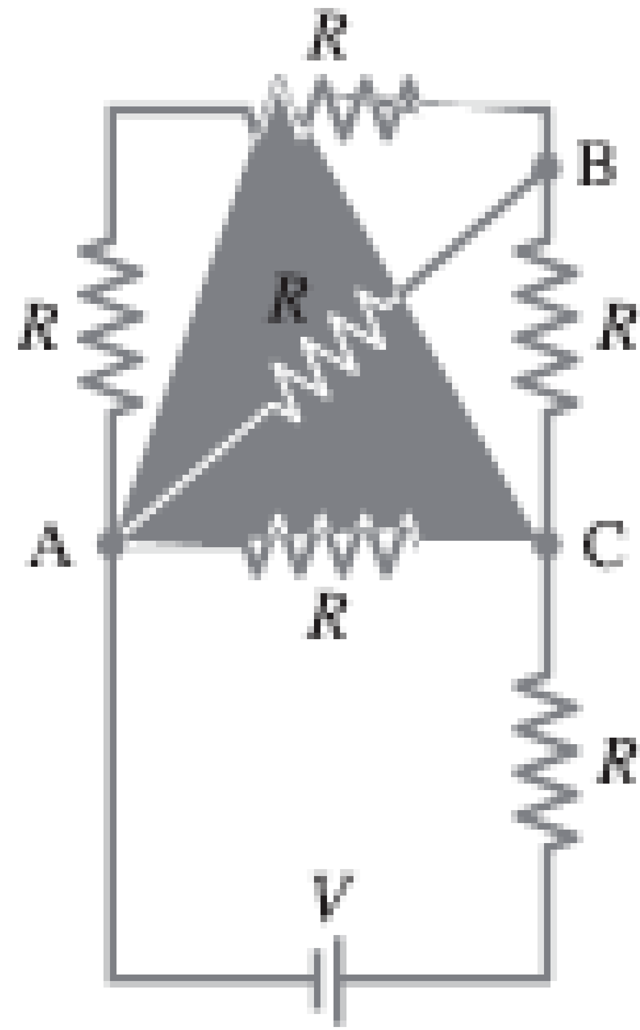
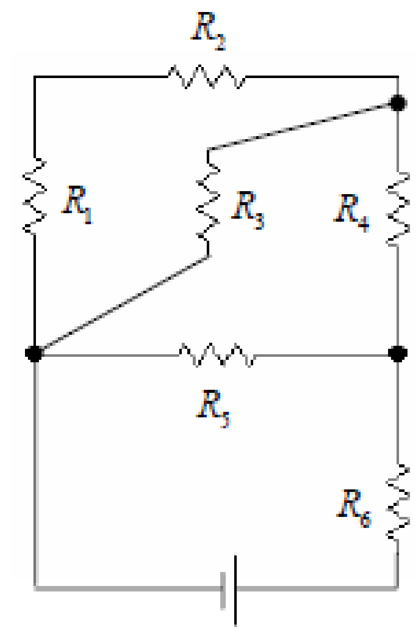


FIGURE 26–41
Problems 19 and 20.

$\boxed{19.}$ The resistors have been numbered in the accompanying diagram to help in the analysis. R_1 and R_2 are in series with an equivalent resistance of $R_{12} = R + R = 2R$. This combination is in parallel with R_3, with an equivalent resistance of $R_{123} = \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = \frac{2}{3}R$. This combination is in series with R_4, with an equivalent resistance of $R_{1234} = \frac{2}{3}R + R = \frac{5}{3}R$. This combination is in parallel with R_5, with an equivalent resistance of $R_{12345} = \left(\frac{1}{R} + \frac{3}{5R}\right)^{-1} = \frac{5}{8}R$. Finally, this combination is in series with R_6, and we calculate the final equivalent resistance.

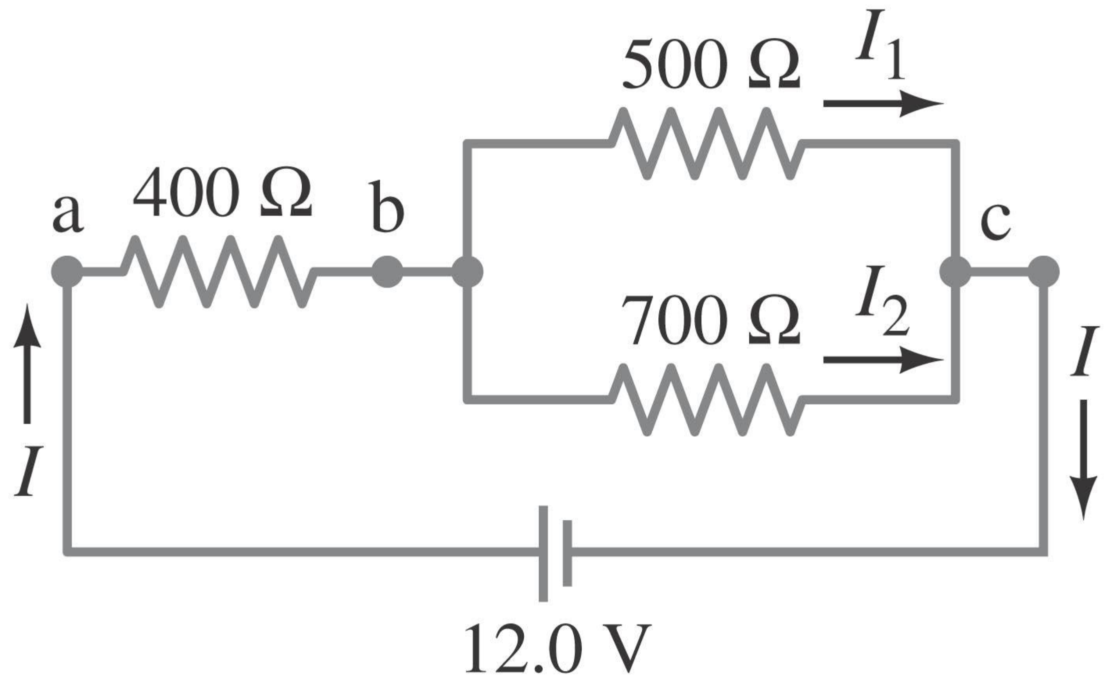


$$R_{eq} = \frac{5}{8}R + R = \boxed{\frac{13}{8}R}$$

26-2 Resistors in Series and in Parallel

Example 26-4: Circuit with series and parallel resistors.

How much current is drawn from the battery shown?



EXAMPLE 26–4 **Circuit with series and parallel resistors.** How much current is drawn from the battery shown in Fig. 26–8a?

APPROACH The current I that flows out of the battery all passes through the 400-Ω resistor, but then it splits into I_1 and I_2 passing through the 500-Ω and 700-Ω resistors. The latter two resistors are in parallel with each other. We look for something that we already know how to treat. So let's start by finding the equivalent resistance, R_P, of the parallel resistors, 500 Ω and 700 Ω. Then we can consider this R_P to be in series with the 400-Ω resistor.

SOLUTION The equivalent resistance, R_P, of the 500-Ω and 700-Ω resistors in parallel is given by

$$\frac{1}{R_P} = \frac{1}{500\,\Omega} + \frac{1}{700\,\Omega} = 0.0020\,\Omega^{-1} + 0.0014\,\Omega^{-1} = 0.0034\,\Omega^{-1}.$$

This is $1/R_P$, so we take the reciprocal to find R_P. It is a common mistake to forget to take this reciprocal. Notice that the units of reciprocal ohms, Ω^{-1}, are a reminder. Thus

$$R_P = \frac{1}{0.0034\,\Omega^{-1}} = 290\,\Omega.$$

This 290 Ω is the equivalent resistance of the two parallel resistors, and is in series with the 400-Ω resistor as shown in the equivalent circuit of Fig. 26–8b. To find the total equivalent resistance R_{eq}, we add the 400-Ω and 290-Ω resistances together, since they are in series, and find

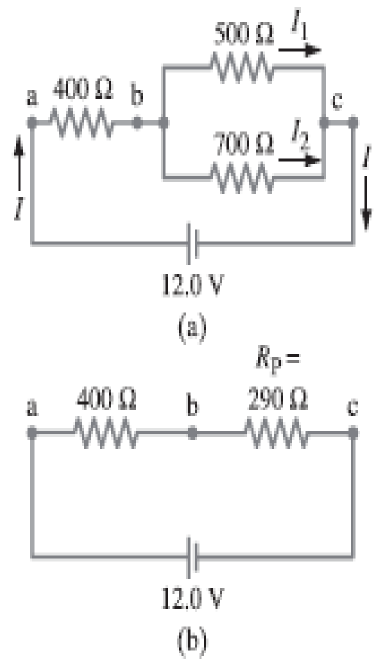
$$R_{eq} = 400\,\Omega + 290\,\Omega = 690\,\Omega.$$

The total current flowing from the battery is then

$$I = \frac{V}{R_{eq}} = \frac{12.0\,\text{V}}{690\,\Omega} = 0.0174\,\text{A} \approx 17\,\text{mA}.$$

NOTE This I is also the current flowing through the 400-Ω resistor, but not through the 500-Ω and 700-Ω resistors (both currents are less—see the next Example).

FIGURE 26–8 (a) Circuit for Examples 26–4 and 26–5. (b) Equivalent circuit, showing the equivalent resistance of 290 Ω for the two parallel resistors in (a).



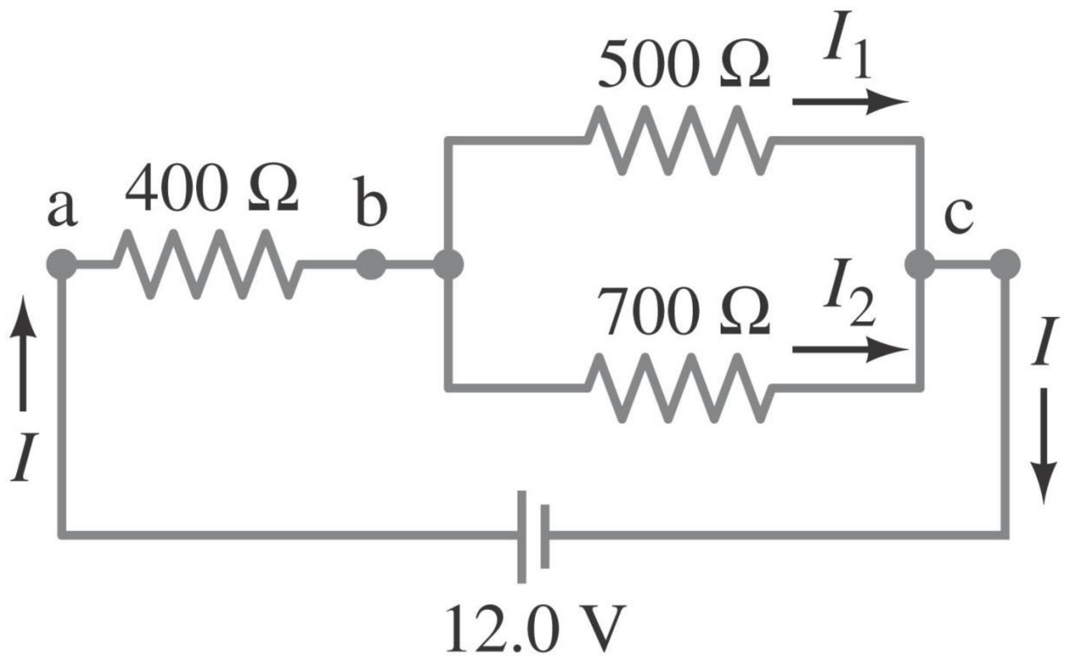
CAUTION

Remember to take the reciprocal

26-2 Resistors in Series and in Parallel

Example 26-5: Current in one branch.

What is the current through the 500-Ω resistor shown? (Note: This is the same circuit as in the previous problem.) The total current in the circuit was found to be 17 mA.



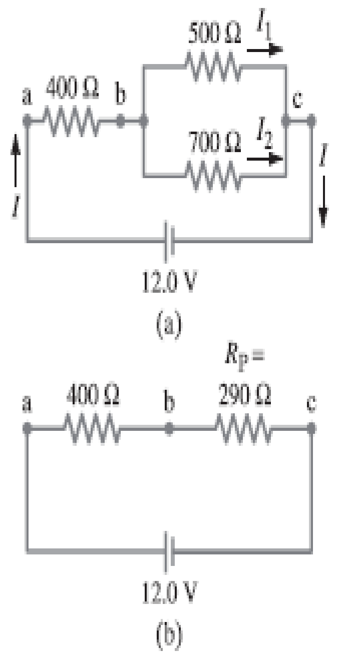


FIGURE 26–8 (repeated)
(a) Circuit for Examples 26–4 and 26–5. (b) Equivalent circuit, showing the equivalent resistance of 290 Ω for the two parallel resistors in (a).

EXAMPLE 26–5 **Current in one branch.** What is the current through the 500-Ω resistor in Fig. 26–8a?

APPROACH We need to find the voltage across the 500-Ω resistor, which is the voltage between points b and c in Fig. 26–8a, and we call it V_{bc}. Once V_{bc} is known, we can apply Ohm's law, $V = IR$, to get the current. First we find the voltage across the 400-Ω resistor, V_{ab}, since we know that 17.4 mA passes through it (Example 26–4).

SOLUTION V_{ab} can be found using $V = IR$:

$$V_{ab} = (0.0174\ \text{A})(400\ \Omega) = 7.0\ \text{V}.$$

Since the total voltage across the network of resistors is $V_{ac} = 12.0\ \text{V}$, then V_{bc} must be $12.0\ \text{V} - 7.0\ \text{V} = 5.0\ \text{V}$. Then Ohm's law applied to the 500-Ω resistor tells us that the current I_1 through that resistor is

$$I_1 = \frac{5.0\ \text{V}}{500\ \Omega} = 1.0 \times 10^{-2}\ \text{A} = 10\ \text{mA}.$$

This is the answer we wanted. We can also calculate the current I_2 through the 700-Ω resistor since the voltage across it is also 5.0 V:

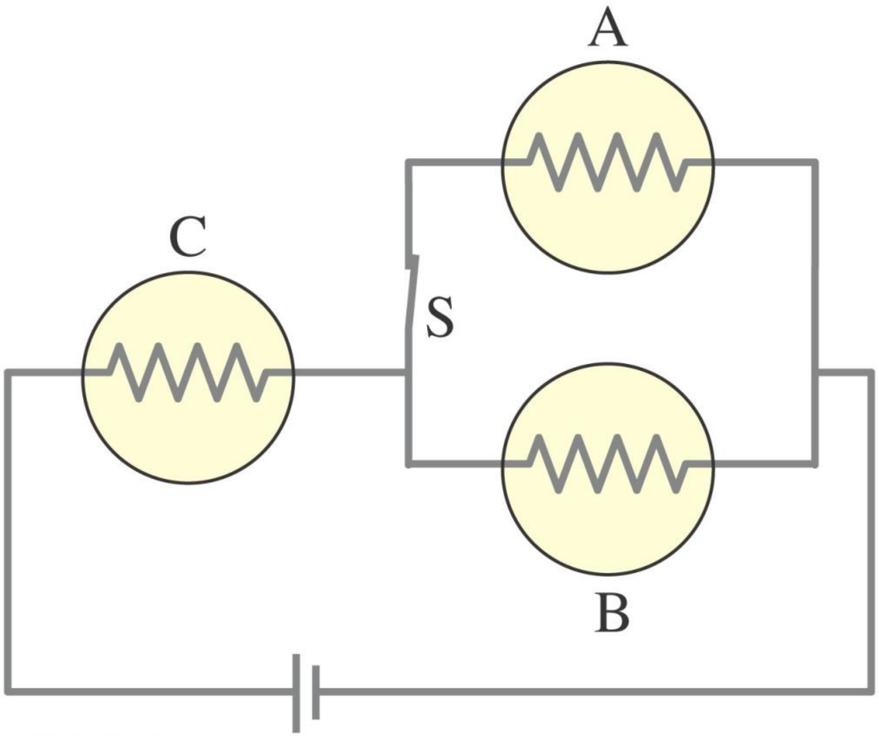
$$I_2 = \frac{5.0\ \text{V}}{700\ \Omega} = 7\ \text{mA}.$$

NOTE When I_1 combines with I_2 to form the total current I (at point c in Fig. 26–8a), their sum is $10\ \text{mA} + 7\ \text{mA} = 17\ \text{mA}$. This equals the total current I as calculated in Example 26–4, as it should.

26-2 Resistors in Series and in Parallel

Conceptual Example 26-6: Bulb brightness in a circuit.

The circuit shown has three identical lightbulbs, each of resistance *R*. (a) When switch S is closed, how will the brightness of bulbs A and B compare with that of bulb C? (b) What happens when switch S is opened? Use a minimum of mathematics in your answers.



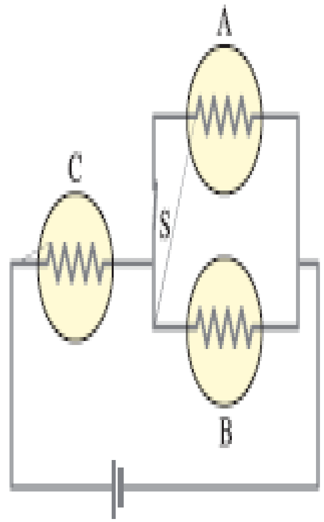


FIGURE 26–9 Example 26–6, three identical lightbulbs. Each yellow circle with -WW- inside represents a lightbulb and its resistance.

CONCEPTUAL EXAMPLE 26–6 **Bulb brightness in a circuit.** The circuit shown in Fig. 26–9 has three identical lightbulbs, each of resistance R. (*a*) When switch S is closed, how will the brightness of bulbs A and B compare with that of bulb C? (*b*) What happens when switch S is opened? Use a minimum of mathematics in your answers.

RESPONSE (*a*) With switch S closed, the current that passes through bulb C must split into two equal parts when it reaches the junction leading to bulbs A and B. It splits into equal parts because the resistance of bulb A equals that of B. Thus, bulbs A and B each receive half of C's current; A and B will be equally bright, but they will be less bright than bulb C ($P = I^2R$). (*b*) When the switch S is open, no current can flow through bulb A, so it will be dark. We now have a simple one-loop series circuit, and we expect bulbs B and C to be equally bright. However, the equivalent resistance of this circuit ($= R + R$) is greater than that of the circuit with the switch closed. When we open the switch, we increase the resistance and reduce the current leaving the battery. Thus, bulb C will be dimmer when we open the switch. Bulb B gets more current when the switch is open (you may have to use some mathematics here), and so it will be brighter than with the switch closed; and B will be as bright as C.

26-2 Resistors in Series and in Parallel

Example 26-7: A two-speed fan.

One way a multiple-speed ventilation fan for a car can be designed is to put resistors in series with the fan motor. The resistors reduce the current through the motor and make it run more slowly. Suppose the current in the motor is 5.0 A when it is connected directly across a 12-V battery. (a) What series resistor should be used to reduce the current to 2.0 A for low-speed operation? (b) What power rating should the resistor have?

EXAMPLE 26–7 ESTIMATE **A two-speed fan.** One way a multiple-speed ventilation fan for a car can be designed is to put resistors in series with the fan motor. The resistors reduce the current through the motor and make it run more slowly. Suppose the current in the motor is 5.0 A when it is connected directly across a 12-V battery. (*a*) What series resistor should be used to reduce the current to 2.0 A for low-speed operation? (*b*) What power rating should the resistor have?

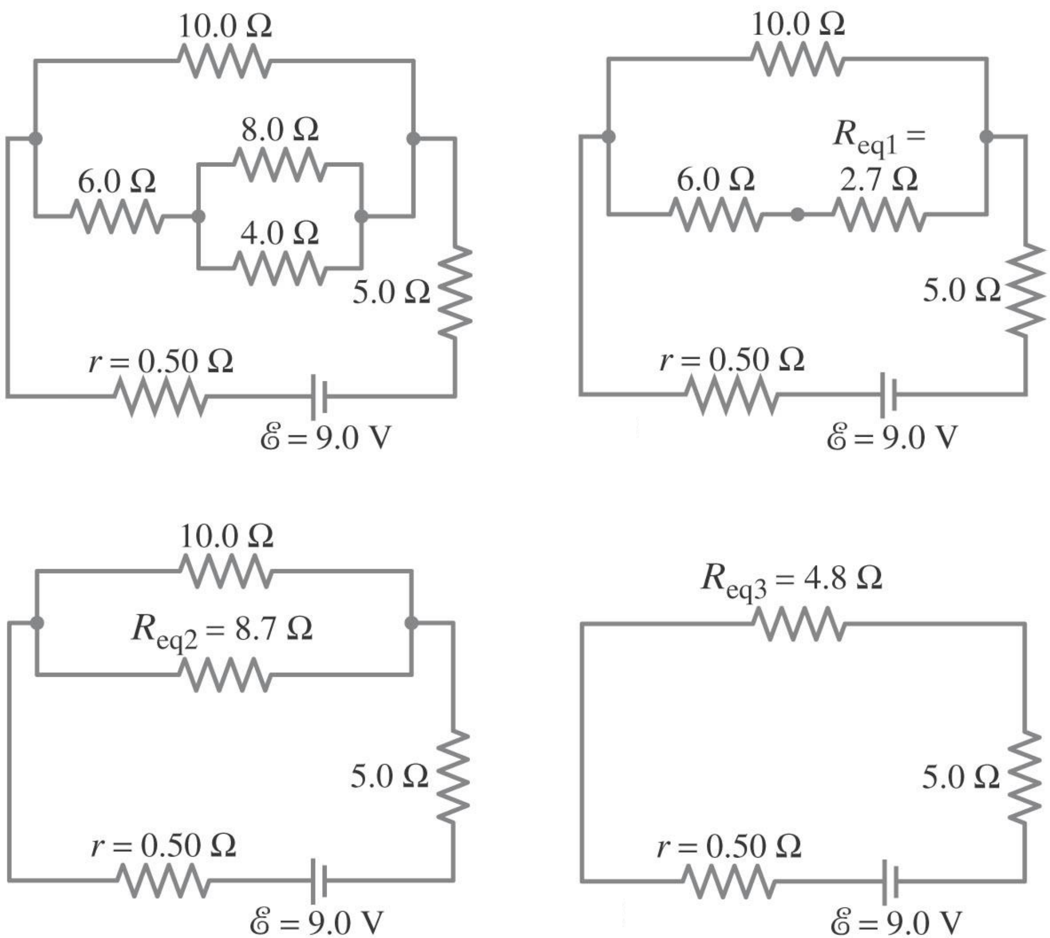
APPROACH An electric motor in series with a resistor can be treated as two resistors in series. The power comes from $P = IV$.

SOLUTION (*a*) When the motor is connected to 12 V and drawing 5.0 A, its resistance is $R = V/I = (12\,\mathrm{V})/(5.0\,\mathrm{A}) = 2.4\,\Omega$. We will assume that this is the motor's resistance for all speeds. (This is an approximation because the current through the motor depends on its speed.) Then, when a current of 2.0 A is flowing, the voltage across the motor is $(2.0\,\mathrm{A})(2.4\,\Omega) = 4.8\,\mathrm{V}$. The remaining $12.0\,\mathrm{V} - 4.8\,\mathrm{V} = 7.2\,\mathrm{V}$ must appear across the series resistor. When 2.0 A flows through the resistor, its resistance must be $R = (7.2\,\mathrm{V})/(2.0\,\mathrm{A}) = 3.6\,\Omega$. (*b*) The power dissipated by the resistor is $P = (7.2\,\mathrm{V})(2.0\,\mathrm{A}) = 14.4\,\mathrm{W}$. To be safe, a power rating of 20 W would be appropriate.

26-2 Resistors in Series and in Parallel

Example 26-8: Analyzing a circuit.

A 9.0-V battery whose internal resistance r is 0.50 Ω is connected in the circuit shown. (a) How much current is drawn from the battery? (b) What is the terminal voltage of the battery? (c) What is the current in the 6.0-Ω resistor?



EXAMPLE 26–8 **Analyzing a circuit.** A 9.0-V battery whose internal resistance r is 0.50 Ω is connected in the circuit shown in Fig. 26–10a. (*a*) How much current is drawn from the battery? (*b*) What is the terminal voltage of the battery? (*c*) What is the current in the 6.0-Ω resistor?

APPROACH To find the current out of the battery, we first need to determine the equivalent resistance R_{eq} of the entire circuit, including r, which we do by identifying and isolating simple series or parallel combinations of resistors. Once we find I from Ohm's law, $I = \mathscr{E}/R_{eq}$, we get the terminal voltage using $V_{ab} = \mathscr{E} - Ir$. For (*c*) we apply Ohm's law to the 6.0-Ω resistor.

SOLUTION (*a*) We want to determine the equivalent resistance of the circuit. But where do we start? We note that the 4.0-Ω and 8.0-Ω resistors are in parallel, and so have an equivalent resistance R_{eq1} given by

$$\frac{1}{R_{eq1}} = \frac{1}{8.0\,\Omega} + \frac{1}{4.0\,\Omega} = \frac{3}{8.0\,\Omega};$$

so $R_{eq1} = 2.7\,\Omega$. This 2.7 Ω is in series with the 6.0-Ω resistor, as shown in the equivalent circuit of Fig. 26–10b. The net resistance of the lower arm of the circuit is then

$$R_{eq2} = 6.0\,\Omega + 2.7\,\Omega = 8.7\,\Omega,$$

as shown in Fig. 26–10c. The equivalent resistance R_{eq3} of the 8.7-Ω and 10.0-Ω resistances in parallel is given by

$$\frac{1}{R_{eq3}} = \frac{1}{10.0\,\Omega} + \frac{1}{8.7\,\Omega} = 0.21\,\Omega^{-1},$$

so $R_{eq3} = \left(1/0.21\,\Omega^{-1}\right) = 4.8\,\Omega$. This 4.8 Ω is in series with the 5.0-Ω resistor and the 0.50-Ω internal resistance of the battery (Fig. 26–10d), so the total equivalent resistance R_{eq} of the circuit is $R_{eq} = 4.8\,\Omega + 5.0\,\Omega + 0.50\,\Omega = 10.3\,\Omega$. Hence the current drawn is

$$I = \frac{\mathscr{E}}{R_{eq}} = \frac{9.0\,\text{V}}{10.3\,\Omega} = 0.87\,\text{A}.$$

(*b*) The terminal voltage of the battery is

$$V_{ab} = \mathscr{E} - Ir = 9.0\,\text{V} - (0.87\,\text{A})(0.50\,\Omega) = 8.6\,\text{V}.$$

(*c*) Now we can work back and get the current in the 6.0-Ω resistor. It must be the same as the current through the 8.7 Ω shown in Fig. 26–10c (why?). The voltage across that 8.7 Ω will be the emf of the battery minus the voltage drops across r and the 5.0-Ω resistor: $V_{8.7} = 9.0\,\text{V} - (0.87\,\text{A})(0.50\,\Omega + 5.0\,\Omega)$. Applying Ohm's law, we get the current (call it I')

$$I' = \frac{9.0\,\text{V} - (0.87\,\text{A})(0.50\,\Omega + 5.0\,\Omega)}{8.7\,\Omega} = 0.48\,\text{A}.$$

This is the current through the 6.0-Ω resistor.

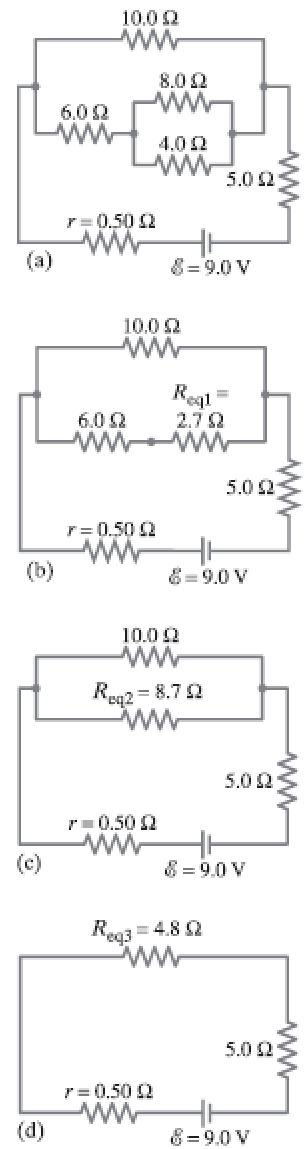
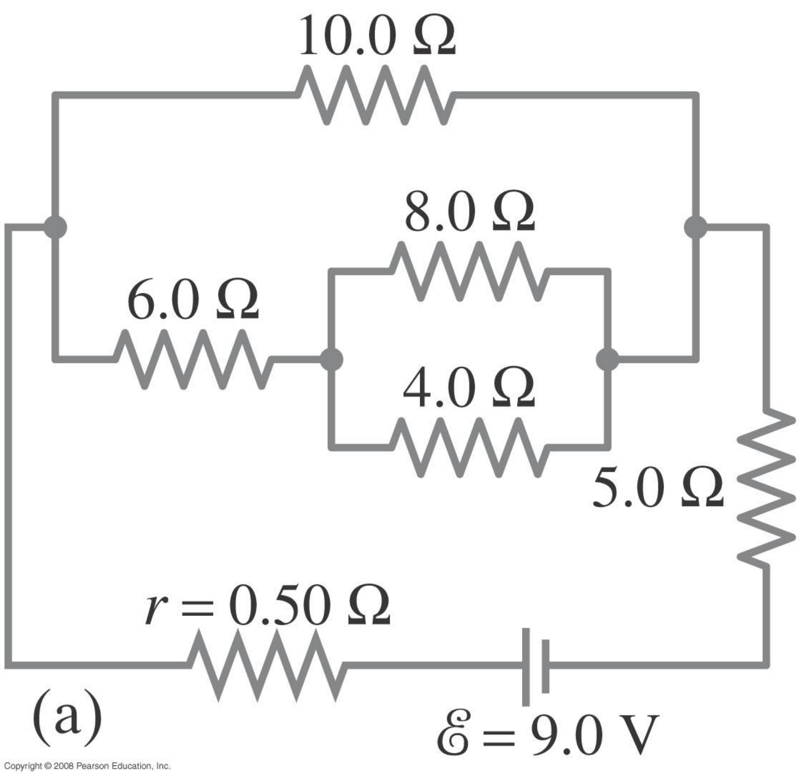


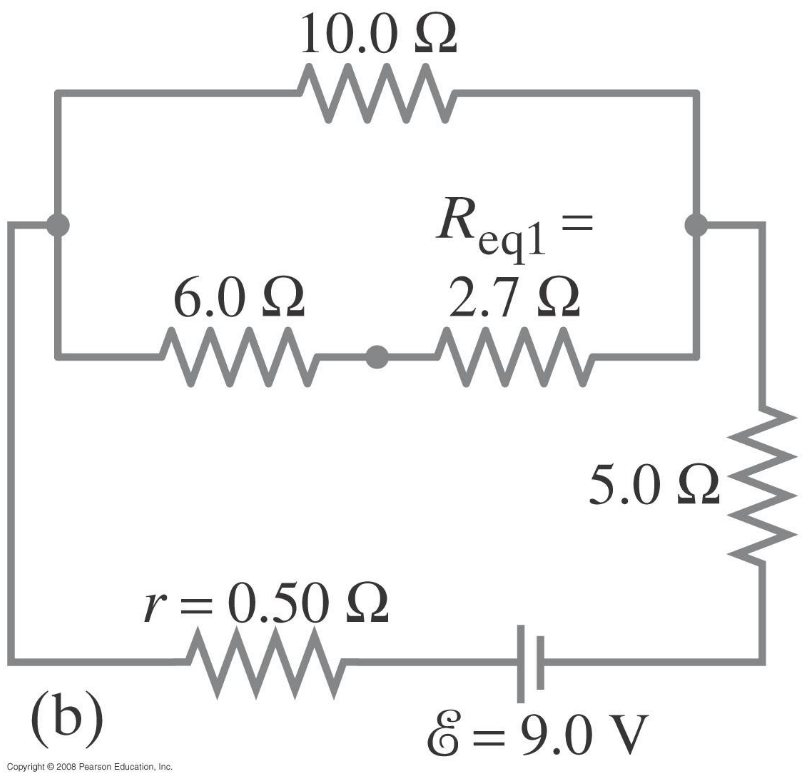
FIGURE 26–10 Circuit for Example 26–8, where r is the internal resistance of the battery.

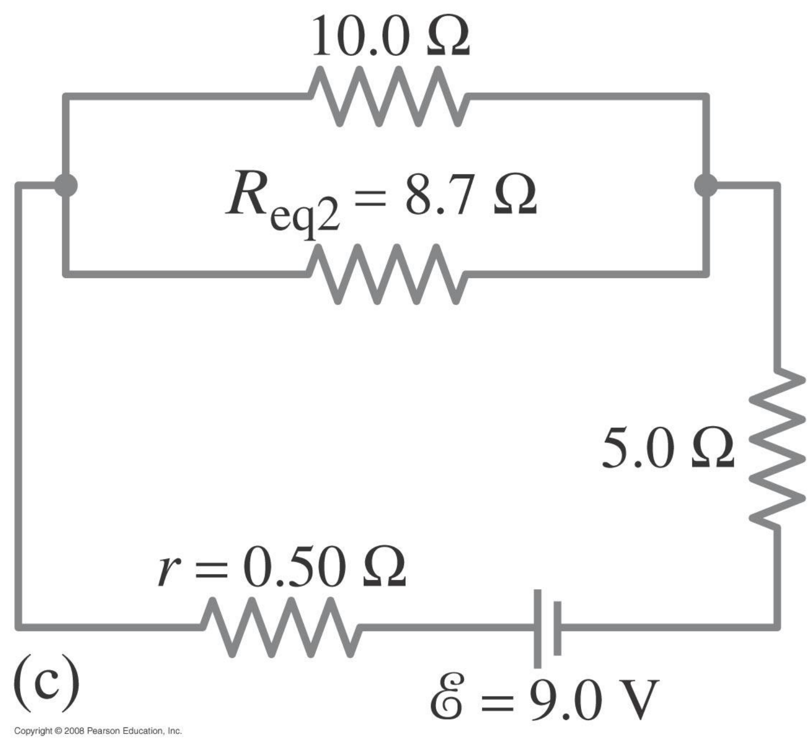
Series & Parallel Example



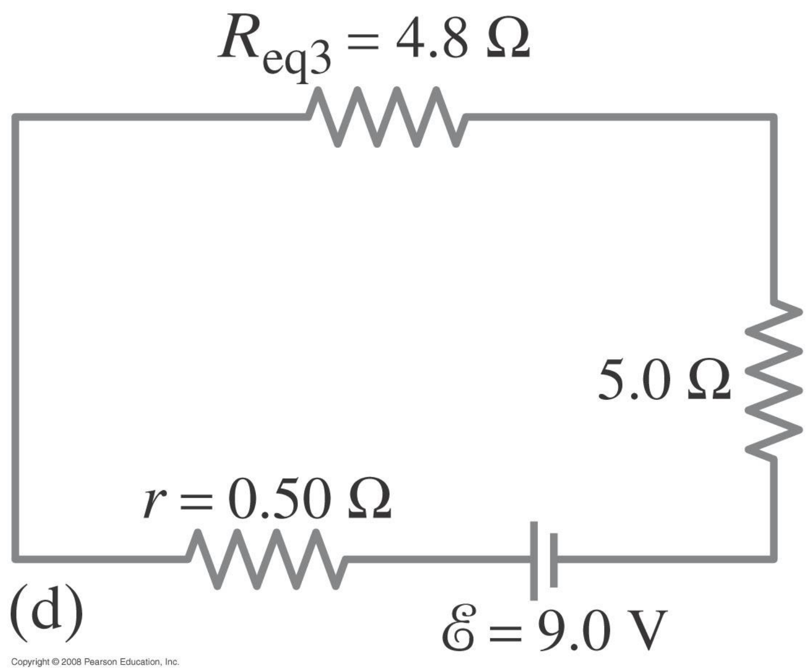
- How do we analyze this circuit to find the current flowing in each resistor?
- We start with the smallest parallel circuit and work through each section.

Series & Parallel Example





Series & Parallel Example



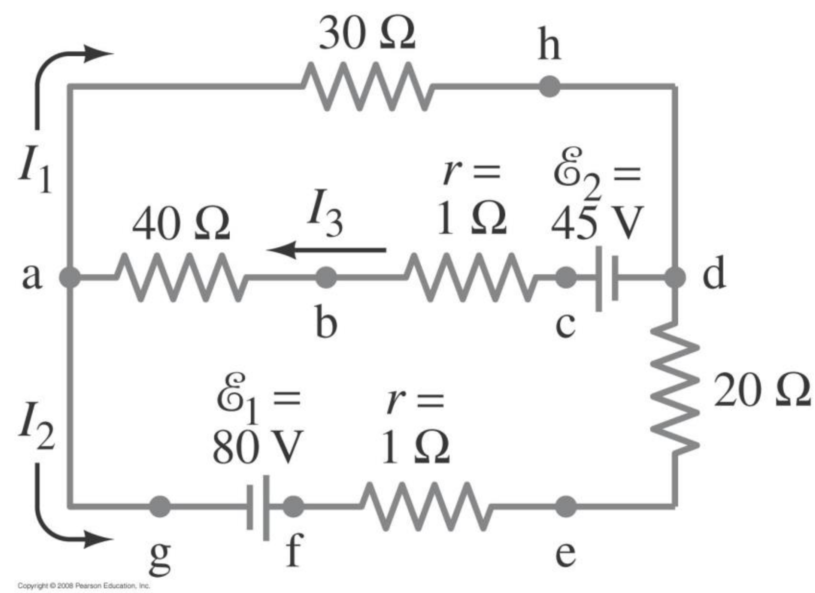
- We now see that the battery sees 10.3 Ω of resistance and produces:
- I = V/R = 9V/10.3Ω
- I = 0.87 A
- We can the work backward to find how this current splits.

Kirchhoff's Rules

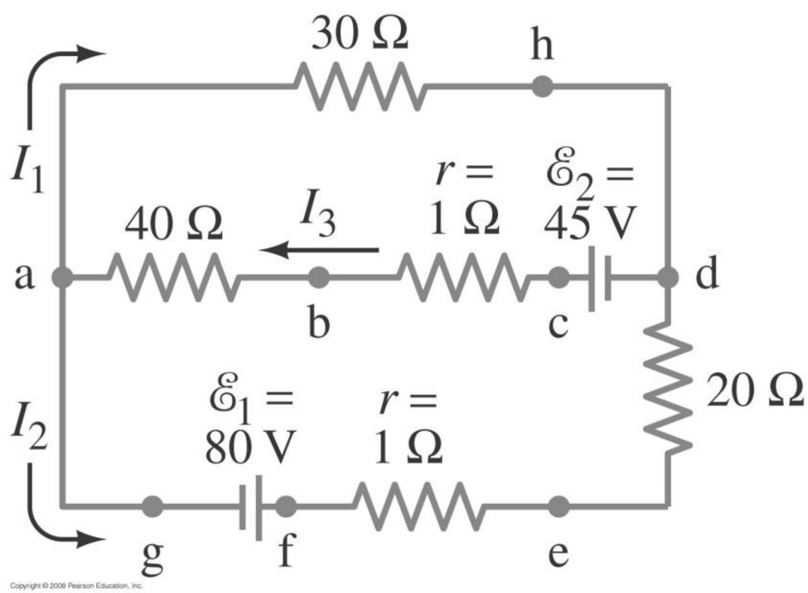
- For more complex circuits with multiple circuits and additional voltage sources, we need a more complex set of rules.
- These are called Kirchhoff’s Rules.

Kirchhoff's Rules

- Suppose we have the circuit of batteries and resistors shown at right.
- Because there are two batteries, we need Kirchhoff's Rules.

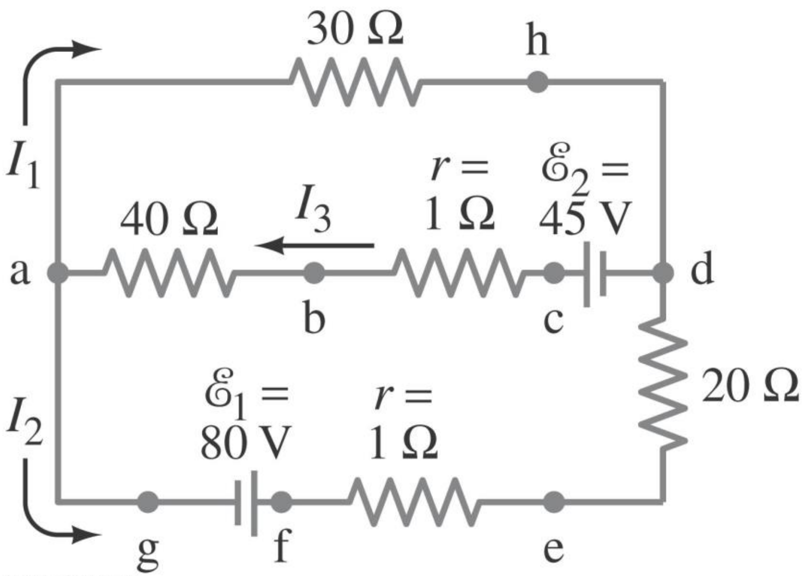


Kirchhoff's First Rule - Junctions



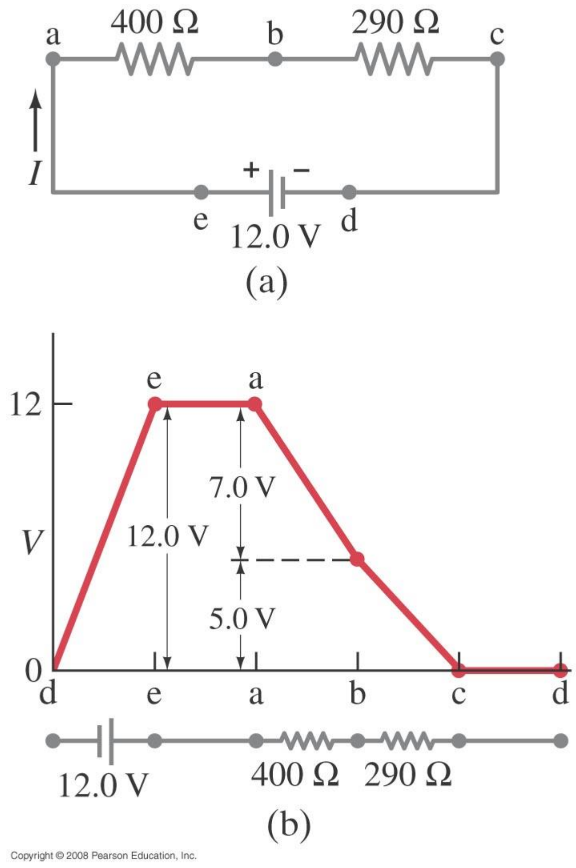
- Junctions or Nodes are places where 3 or more wires meet
- See a & d at left.
- Kirchhoff's first rule for junctions says that the <u>current flowing out of a junction = current flowing into the junction</u>.

Kirchhoff's Second Rule – Loop Rule



- To use the loop rule, we will walk around each complete circuit path.
- Since we return to our starting point at the end of the loop, <u>the sum of all voltages must be zero</u>.

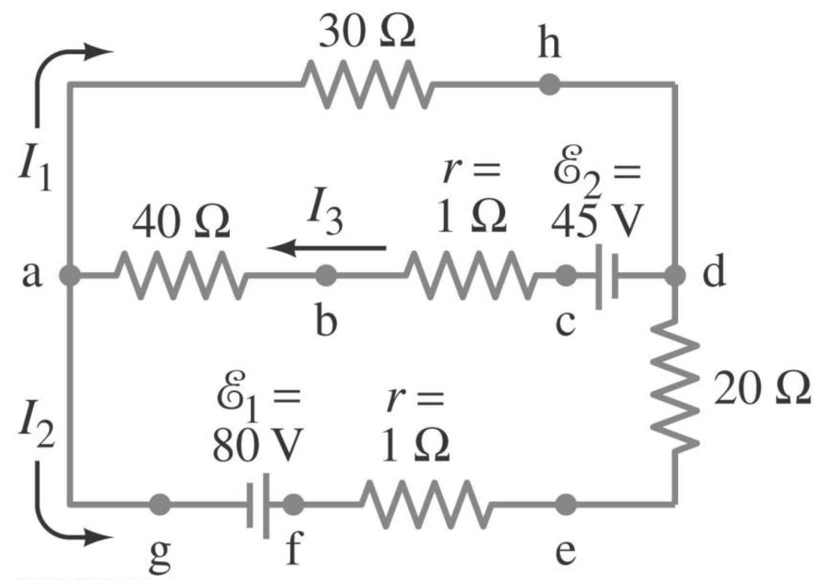
Kirchhoff's Second Rule – Loop Rule



- As an example, look at the loop at left.
- The chart shows the voltage at each point in the loop starting at point d.

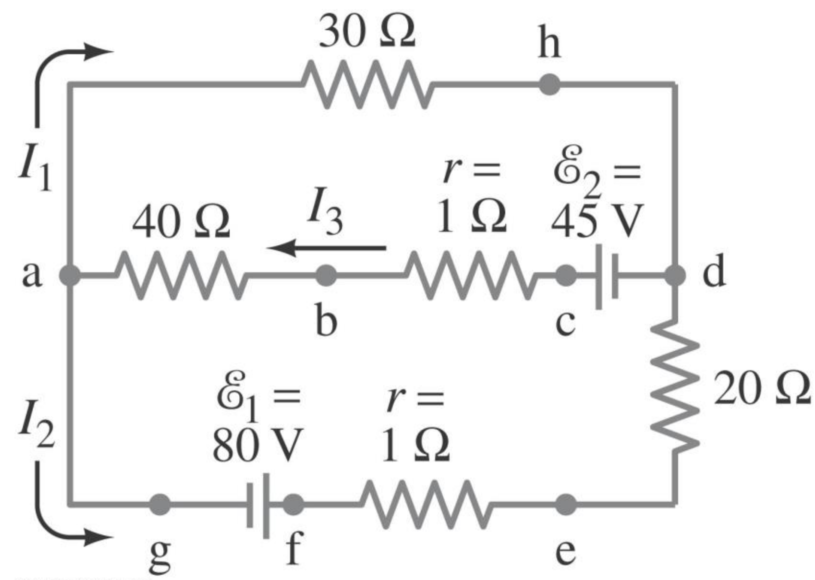
Applying Kirchhoff's Rules

- Lets find the current in each part of the circuit at right.
- Step 1: Label the currents, I_1, I_2, I_3.
- If we choose the wrong direction for any current, we will get a (-) in our final answer.



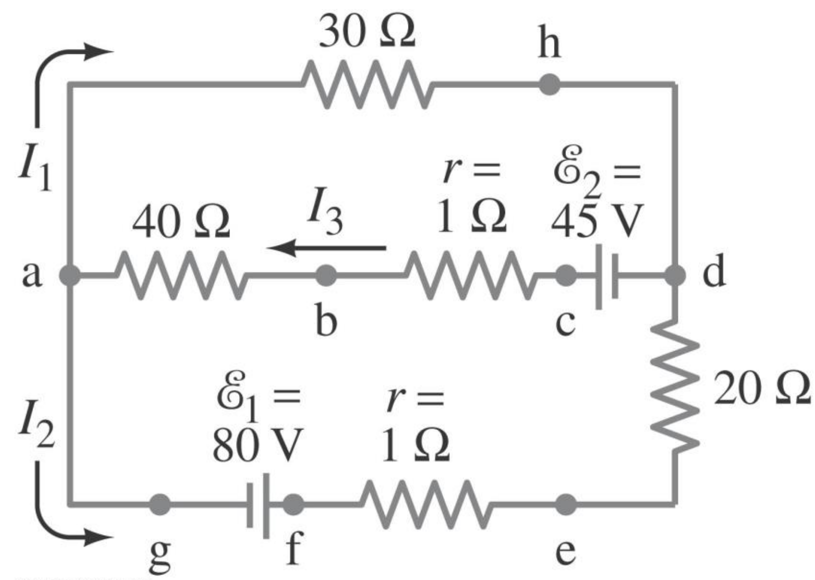
Applying Kirchhoff's Rules

- Step 2: Use the junction rule to get the equations for our currents.
- For this circuit we can show for either junction a or d, that
- $I_1 + I_2 = I_3$.



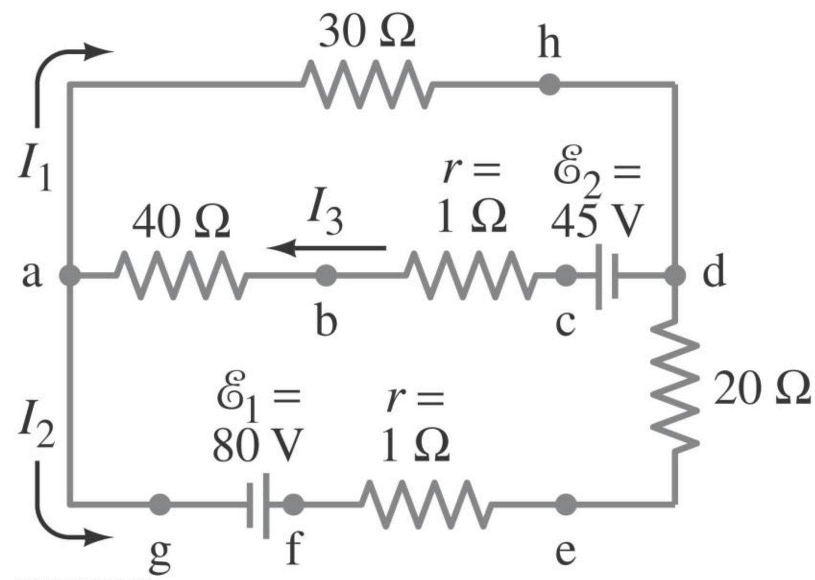
Applying Kirchhoff's Rules

- Step 3: Use the loop rule to add up the voltage changes around each circuit.
- For circuit ahdcba,
- From a to h, the voltage drops, so
- $V_{ha} = -I_1 \cdot 30\Omega$



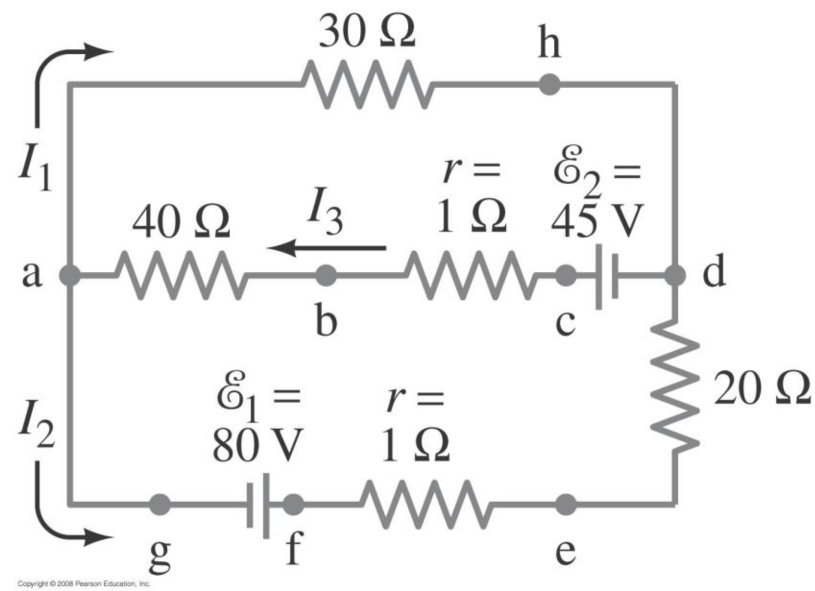
Applying Kirchhoff's Rules

- For circuit ahdcba,
- $V_{ha} = -I_1 \cdot 30\Omega$
- $V_{dh} = 0\ V$
- $V_{cd} = +45V$
- $V_{bc} = -I_3 \cdot 1\Omega$
- $V_{ab} = -I_3 \cdot 40\Omega, so$
- $-I_1 \cdot 30\Omega + 45V - I_3 \cdot 41\Omega = 0\ V$



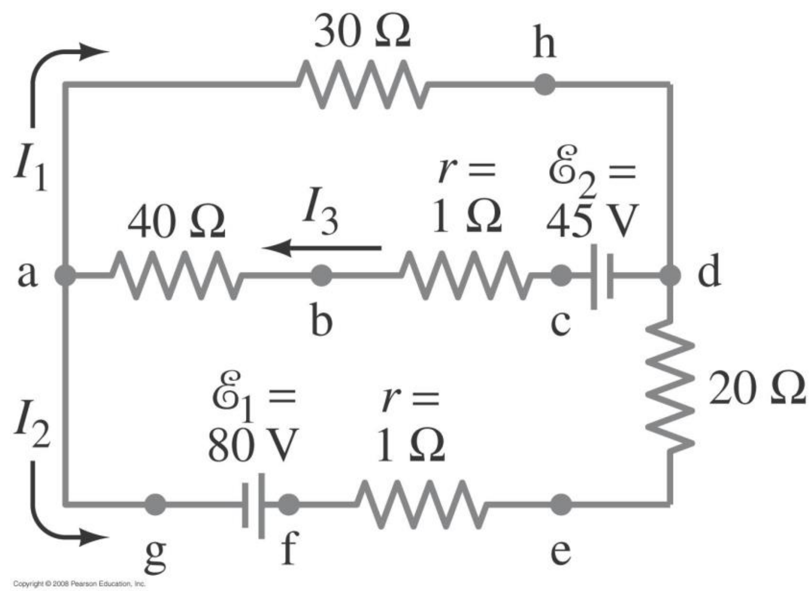
Applying Kirchhoff's Rules

- For circuit ahdefga,
- $V_{ha} = -I_1 \cdot 30\Omega$
- $V_{dh} = 0\ V$
- $V_{ed} = +I_2 \cdot 20\Omega$
- $V_{fe} = +I_2 \cdot 1\Omega$
- $V_{gf} = -80V, so$
- $-I_1 \cdot 30\Omega + I_2 \cdot 21\Omega - 80V = 0\ V$



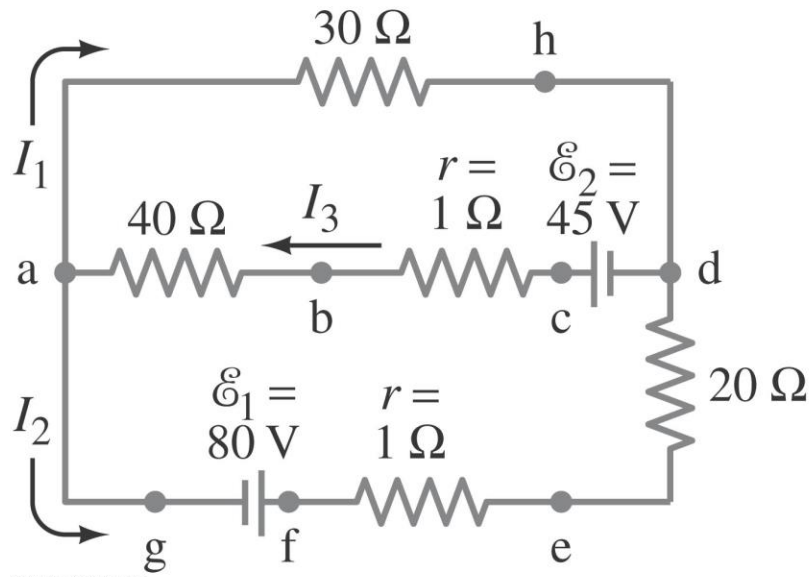
Applying Kirchhoff's Rules

- We now have three equations,
- $I_1 + I_2 = I_3$
- $-I_1 \cdot 30\Omega + 45V - I_3 \cdot 41\Omega = 0\,V$
- $-I_1 \cdot 30\Omega + I_2 \cdot 21\Omega - 80V = 0\,V$
- And can solve for I's.



Applying Kirchhoff's Rules

- The result is,
- $I_1 = -0.87$ A
- $I_2 = +2.6$ A
- $I_3 = +1.7$ A
- The (-) sign on I_1 indicates that we picked the wrong direction originally.



EXAMPLE 26.3 A SINGLE-LOOP CIRCUIT

SOLUTION

The circuit shown in **Fig. 26.10a** (next page) contains two batteries, each with an emf and an internal resistance, and two resistors. Find (a) the current in the circuit, (b) the potential difference V_{ab}, and (c) the power output of the emf of each battery.

SOLUTION

IDENTIFY and SET UP: There are no junctions in this single-loop circuit, so we don't need Kirchhoff's junction rule. To apply Kirchhoff's loop rule, we first assume a direction for the current; let's assume a counterclockwise direction as shown in Fig. 26.10a.

EXECUTE: (a) Starting at a and traveling counterclockwise with the current, we add potential increases and decreases and equate the sum to zero as in Eq. (26.6):

$$-I(4\,\Omega) - 4\text{ V} - I(7\,\Omega) + 12\text{ V} - I(2\,\Omega) - I(3\,\Omega) = 0$$

Collecting like terms and solving for I, we find

$$8\text{ V} = I(16\,\Omega) \qquad \text{and} \qquad I = 0.5\text{ A}$$

The positive result for I shows that our assumed current direction is correct.

(b) To find V_{ab}, the potential at a with respect to b, we start at b and add potential changes as we go toward a. There are two paths from b to a; taking the lower one, we find

$$V_{ab} = (0.5\text{ A})(7\,\Omega) + 4\text{ V} + (0.5\text{ A})(4\,\Omega) = 9.5\text{ V}$$

Point a is at 9.5 V higher potential than b. All the terms in this sum, including the IR terms, are positive because each represents an *increase* in potential as we go from b to a. For the upper path,

$$V_{ab} = 12\text{ V} - (0.5\text{ A})(2\,\Omega) - (0.5\text{ A})(3\,\Omega) = 9.5\text{ V}$$

Here the IR terms are negative because our path goes in the direction of the current, with potential decreases through the resistors. The results for V_{ab} are the same for both paths, as they must be in order for the total potential change around the loop to be zero.

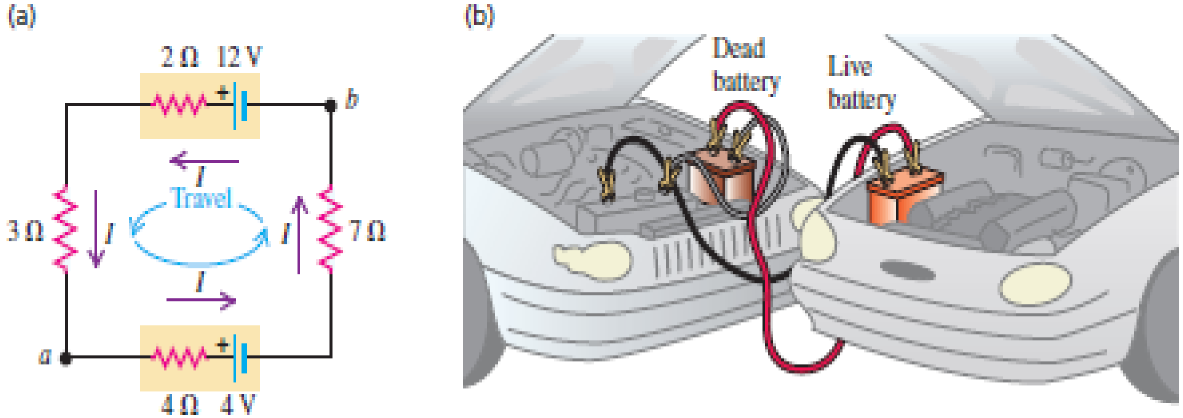
(c) The power outputs of the emf of the 12-V and 4-V batteries are

$$P_{12V} = \mathcal{E}I = (12\text{ V})(0.5\text{ A}) = 6\text{ W}$$

$$P_{4V} = \mathcal{E}I = (-4\text{ V})(0.5\text{ A}) = -2\text{ W}$$

Continued

26.10 (a) In this example we travel around the loop in the same direction as the assumed current, so all the IR terms are negative. The potential decreases as we travel from + to − through the bottom emf but increases as we travel from − to + through the top emf. (b) A real-life example of a circuit of this kind.



The negative sign in $\mathcal{E}$ for the 4-V battery appears because the current actually runs from the higher-potential side of the battery to the lower-potential side. The negative value of P means that we are *storing* energy in that battery; the 12-V battery is *recharging* it (if it is in fact rechargeable; otherwise, we're destroying it).

EVALUATE: By applying the expression $P = I^2R$ to each of the four resistors in Fig. 26.10a, you can show that the total power dissipated in all four resistors is 4 W. Of the 6 W provided by the emf of the 12-V battery, 2 W goes into storing energy in the 4-V battery and 4 W is dissipated in the resistances.

The circuit shown in Fig. 26.10a is much like that used when a fully charged 12-V storage battery (in a car with its engine running) "jump-starts" a car with a run-down battery (Fig. 26.10b). The run-down battery is slightly recharged in the process. The 3-Ω and 7-Ω resistors in Fig. 26.10a represent the resistances of the jumper cables and of the conducting path through the car with the run-down battery. (The values of the resistances in actual automobiles and jumper cables are considerably lower, and the emf of a run-down car battery isn't much less than 12 V.)

29. (II) For the circuit shown in Fig. 26–47, find the potential difference between points a and b. Each resistor has $R = 130\ \Omega$ and each battery is 1.5 V.

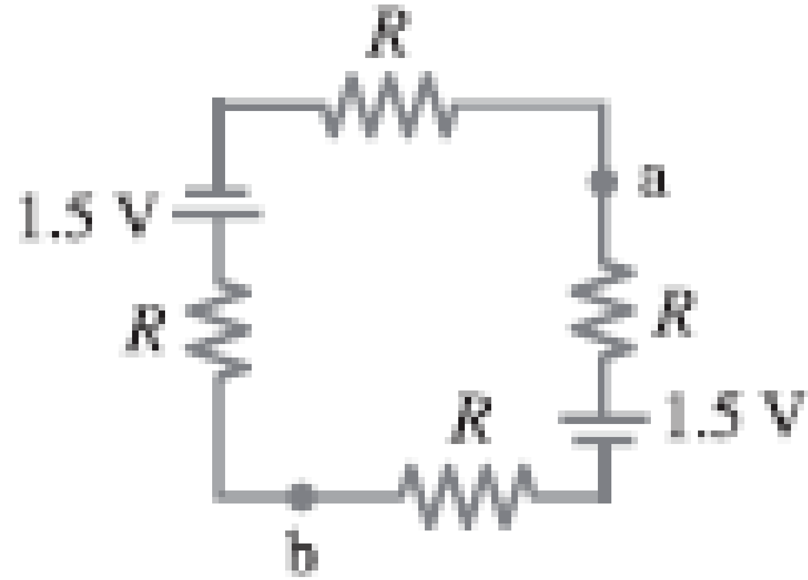


FIGURE 26–47 Problem 29.

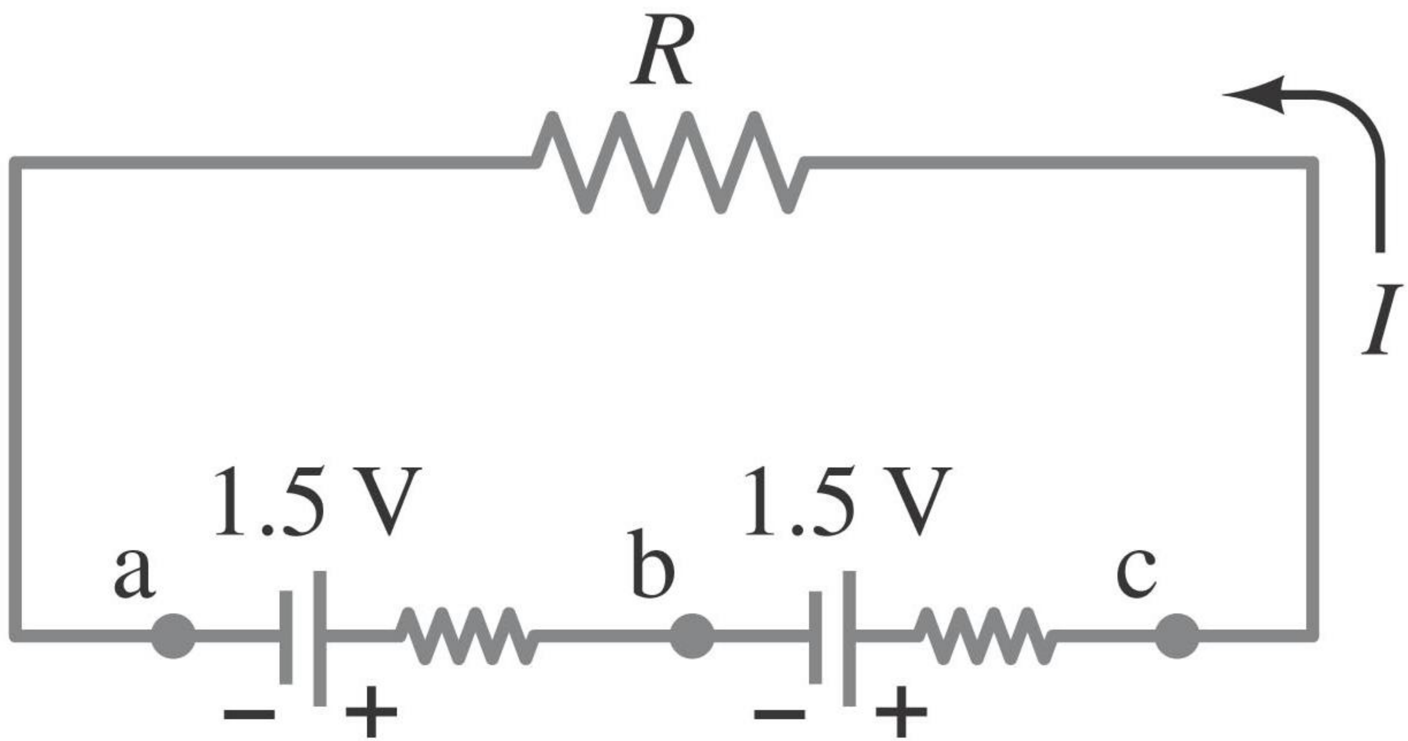
29. To find the potential difference between points a and b, the current must be found from Kirchhoff's loop law. Start at point a and go counterclockwise around the entire circuit, taking the current to be counterclockwise.

$$-IR + \mathcal{E} - IR - IR + \mathcal{E} - IR = 0 \quad \rightarrow \quad I = \frac{\mathcal{E}}{2R}$$

$$V_{\text{ab}} = V_{\text{a}} - V_{\text{b}} = -IR + \mathcal{E} - IR = \mathcal{E} - 2IR = \mathcal{E} - 2\frac{\mathcal{E}}{2R}R = \boxed{0\,\text{V}}$$

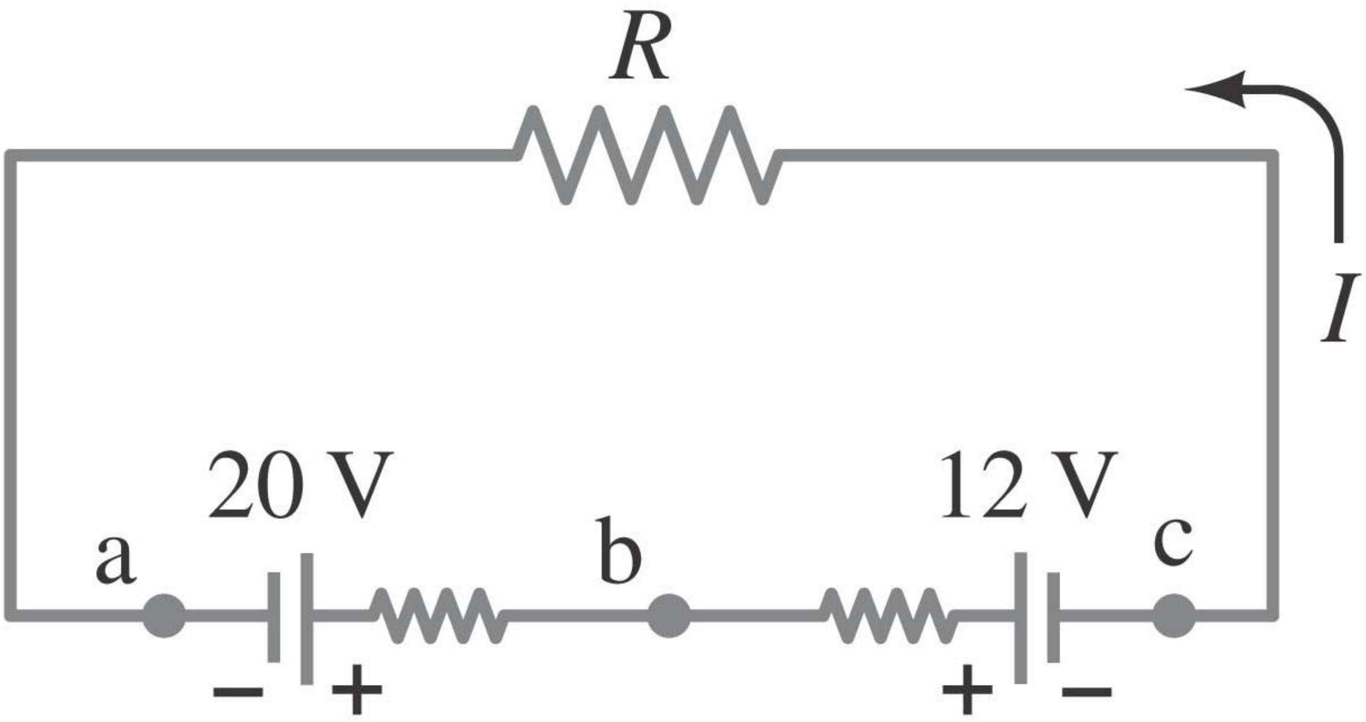
26-4 Series and Parallel EMFs; Battery Charging

EMFs in series in the same direction: total voltage is the sum of the separate voltages.



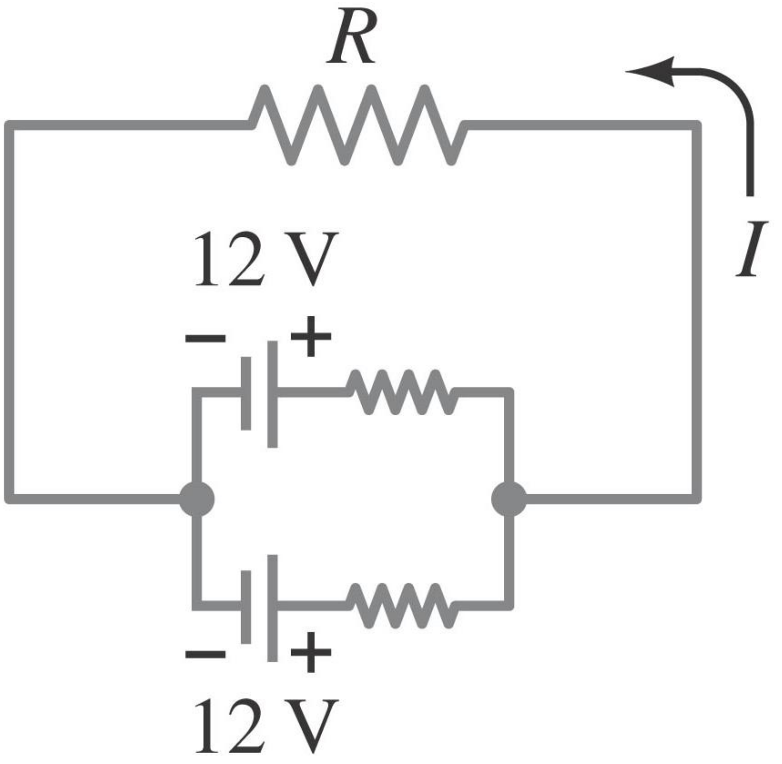
26-4 Series and Parallel EMFs; Battery Charging

EMFs in series, opposite direction: total voltage is the difference, but the lower-voltage battery is charged.



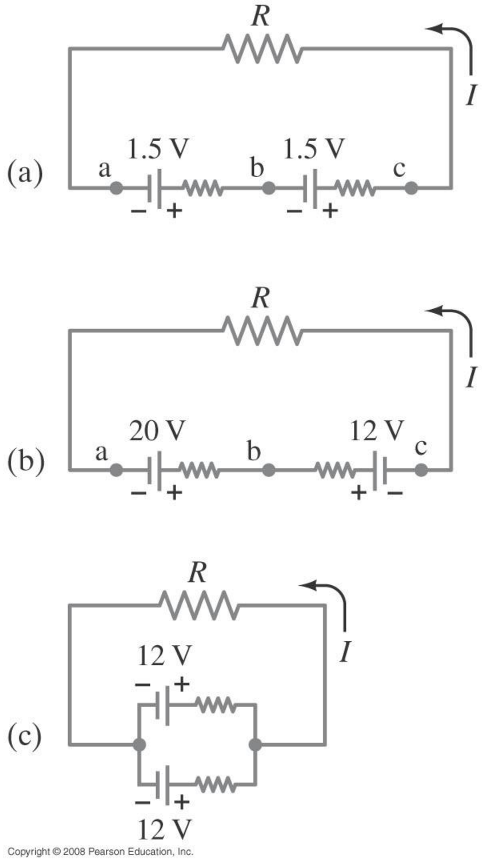
26-4 Series and Parallel EMFs; Battery Charging

EMFs in parallel only make sense if the voltages are the same; this arrangement can produce more current than a single emf.

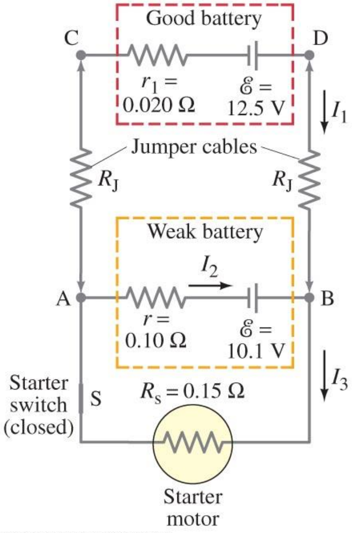


Series & Parallel EMFs

- Supplying more voltage (a) by connecting EMFs in series.
- Charging a battery with another source (b).
- Supplying more current (c) by connecting EMFs in parrallel.

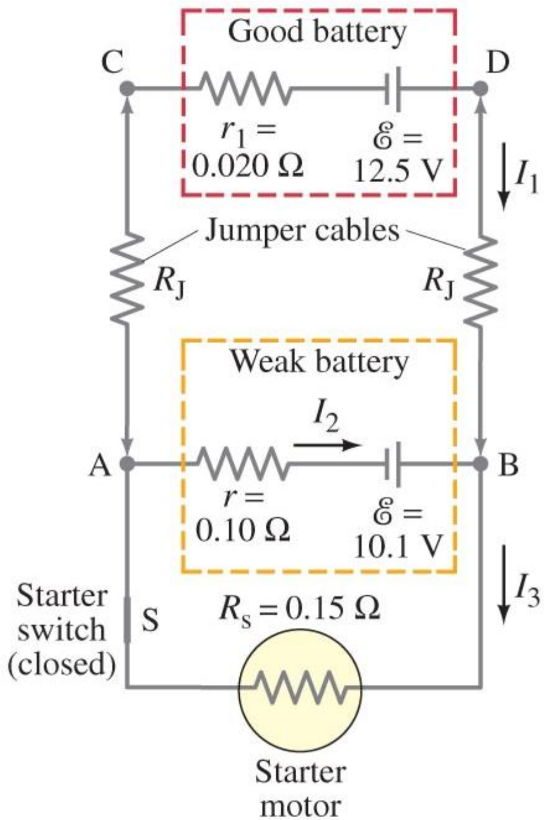


Series & Parallel EMFs



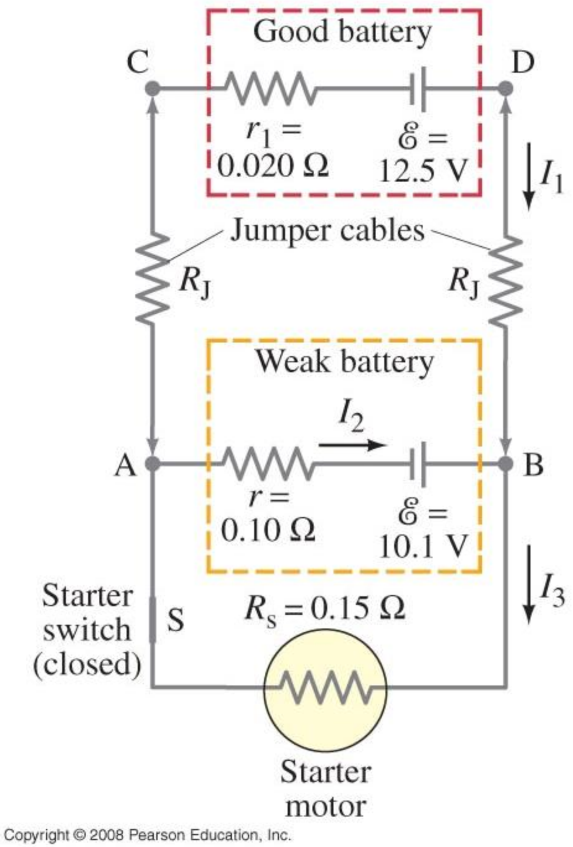
- Jump Starting a Car:
- We jump start a car by connecting a charged battery in parallel with the battery of the car we want to start.
- This provides greater power to the starter motor to turn the engine.

Series & Parallel EMFs



- Without the good battery, the current is
- $I = 10.1V/(0.10\Omega + 0.15\Omega) = 40.4\ A$
- The main problem is the high internal resistance of the weak battery.

Series & Parallel EMFs



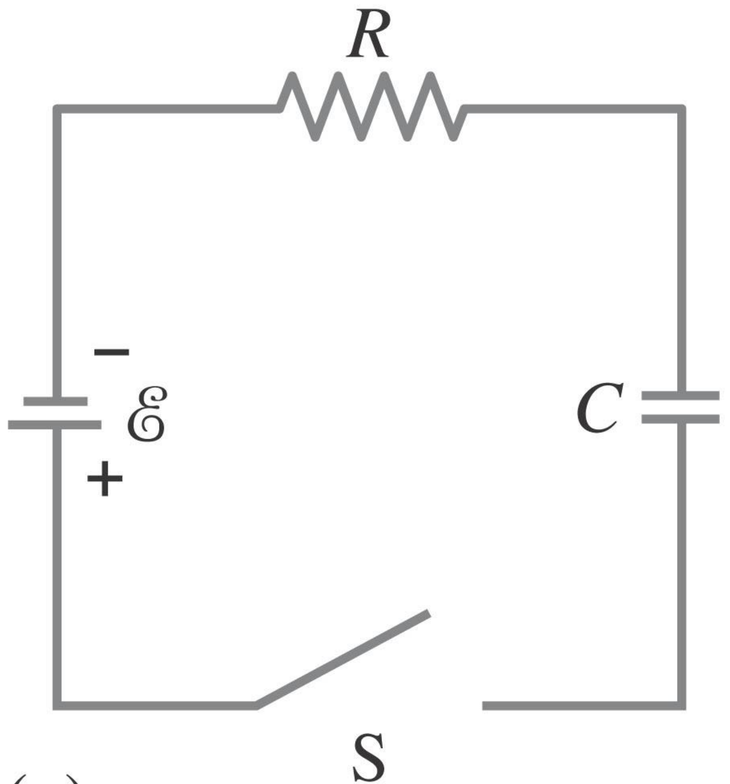
- With the good battery, we must use Kirchhoff's rules
- Given that $R_j = 0.0026\Omega$
- $I_1 + I_2 = I_3$
- $12.5\,V - 0.025\Omega \cdot I_1 - 0.15\Omega \cdot I_3 = 0$
- $10.1V - I_2 \cdot 0.10\Omega - I_3 \cdot 0.15\Omega = 0$

- Yields $I_3 = 71A$
- Nearly doubling the current in the starter.

- The other currents are
- $I_1 = 76\ A, and\ I_2 = -5\ A$

RC Circuits – Time Variation

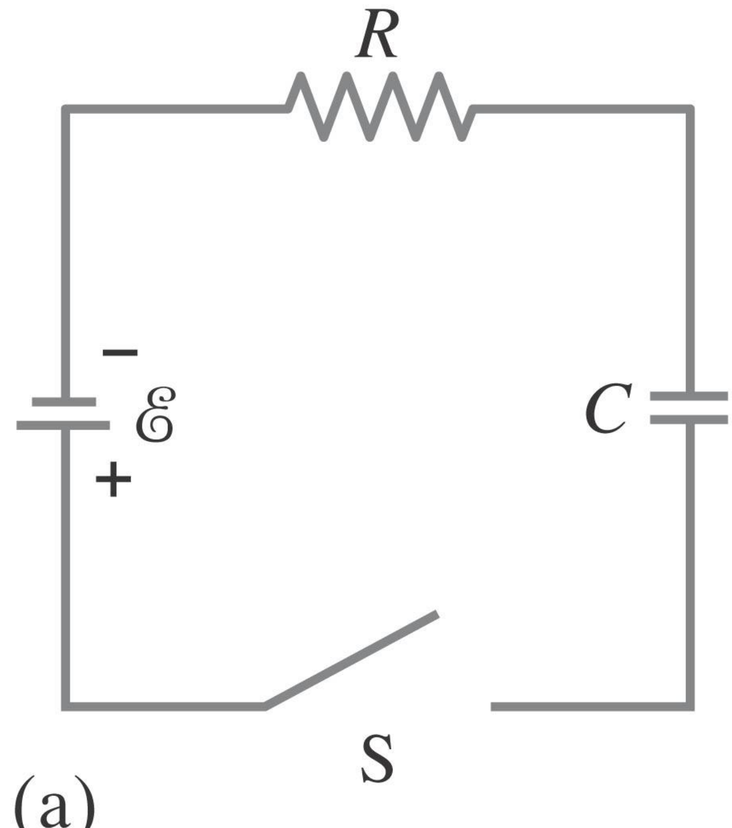
- An important circuit is the Resistor-Capacitor or RC circuit.
- These are used in many timing circuits.
- If we start with an uncharged capacitor, the initial current will be $I = \mathcal{E}/R$



(a)

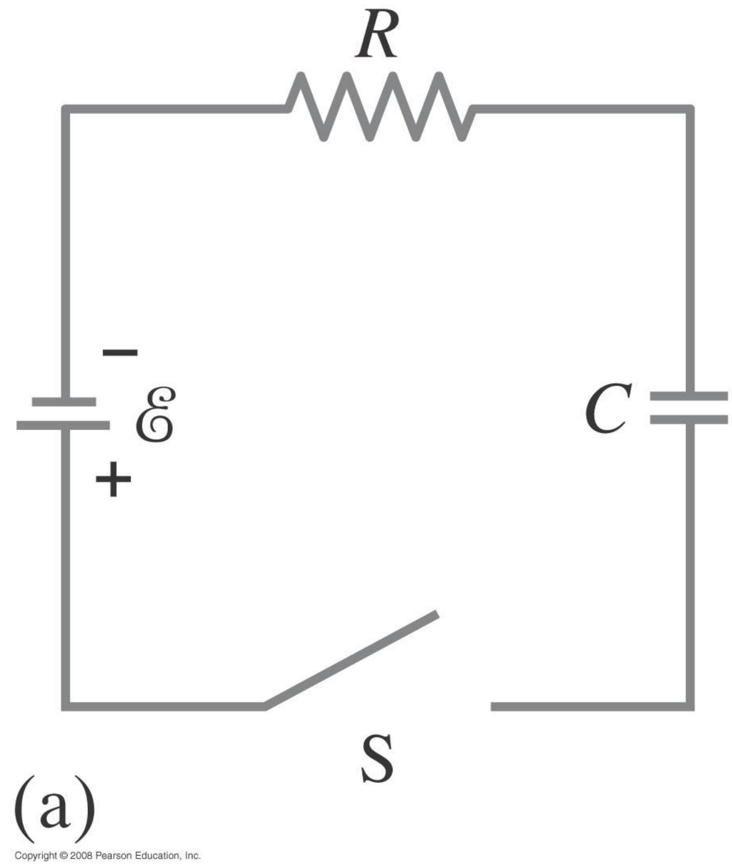
RC Circuits – Time Variation

- However as the capacitor charges, its voltage will balance the battery so the net voltage across the resistor will be
- $I \cdot R = \mathcal{E} - {}^{Q}/_{C}$
- Since $I = dQ/dt$ we can write the equation
- $\mathcal{E} = R \cdot {}^{dQ}/_{dt} + {}^{Q}/_{C}$



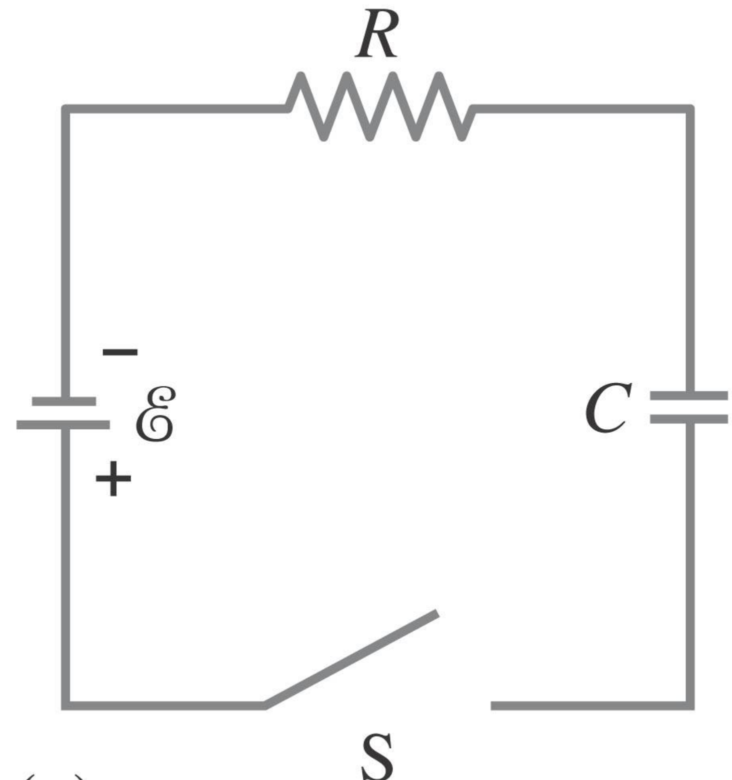
RC Circuits – Time Variation

- This can be written as
- $\frac{dQ}{(C\cdot\mathcal{E}-Q)} = \frac{dt}{R\cdot C}$
- Which can be integrated to yield
- $Q = C\cdot\mathcal{E}\left(1 - e^{-t/_{R\cdot C}}\right)$
- So the voltage across the capacitor is just
- $V_c = \mathcal{E}\left(1 - e^{-t/_{R\cdot C}}\right)$



RC Circuits – Displacement Current

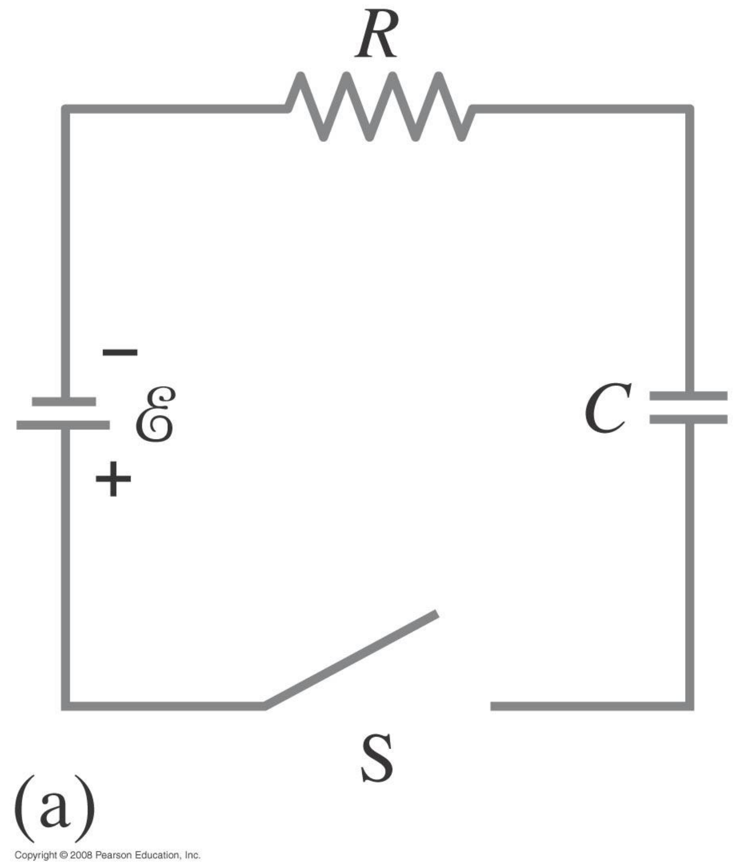
- The current which flows through the rest of the circuit does not flow through the capacitor.
- Instead the electrons are 'displaced' from one side to the other.
- This concept of a <u>displacement current</u> will be useful later.



(a)

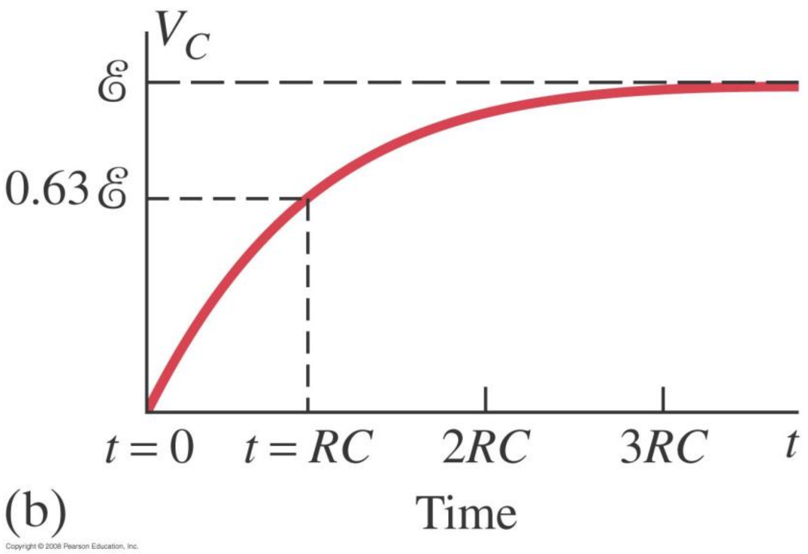
RC Circuits – Time Variation

- Note: The units of RC is Seconds.
- Ohm = Volt/Amp
- Farad = Coulomb/Volt
- Ohm·Farad = Coulomb/Amp
- Ohm·Farad = Seconds
- $\tau = R \cdot C$ is used



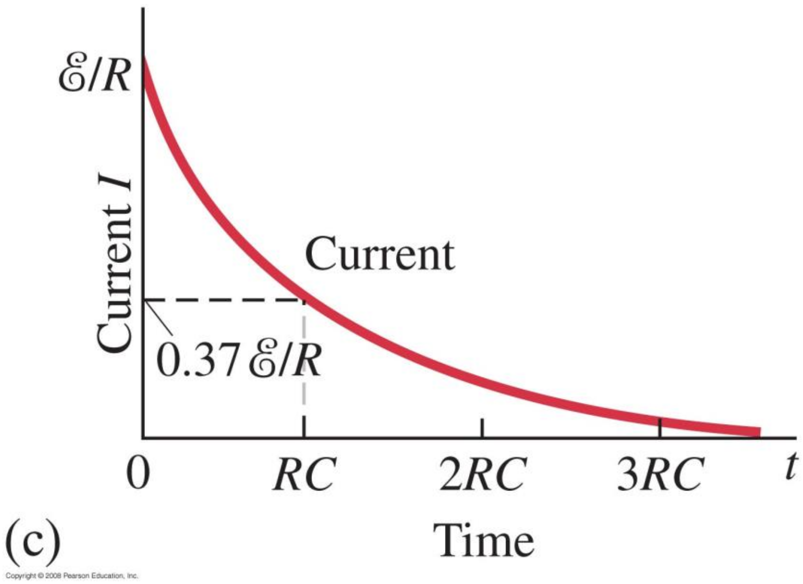
Charging an RC Circuit

$$V_c = \mathcal{E}\left(1 - e^{-t/_{R\cdot C}}\right)$$



- The time to charge an RC circuit is shown at left, with the time axis labeled in units of t = RC.
- Note: This curve approaches $\mathcal{E}$ asymptotically.
- How long till
- $V_c = 0.99 \cdot \mathcal{E}$?

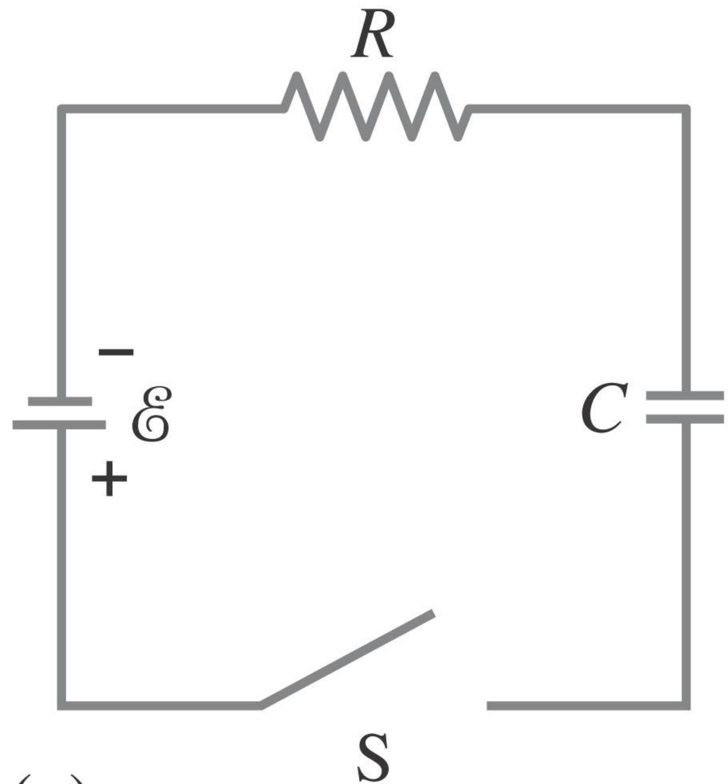
Charging an RC Circuit



- The current in the circuit can be found by differentiating
- $Q = C \cdot \mathcal{E}\left(1 - e^{-t/_{R \cdot C}}\right)$
- To get
- $I = \frac{\mathcal{E}}{R} \cdot e^{-t/_{R \cdot C}}$

RC Circuit

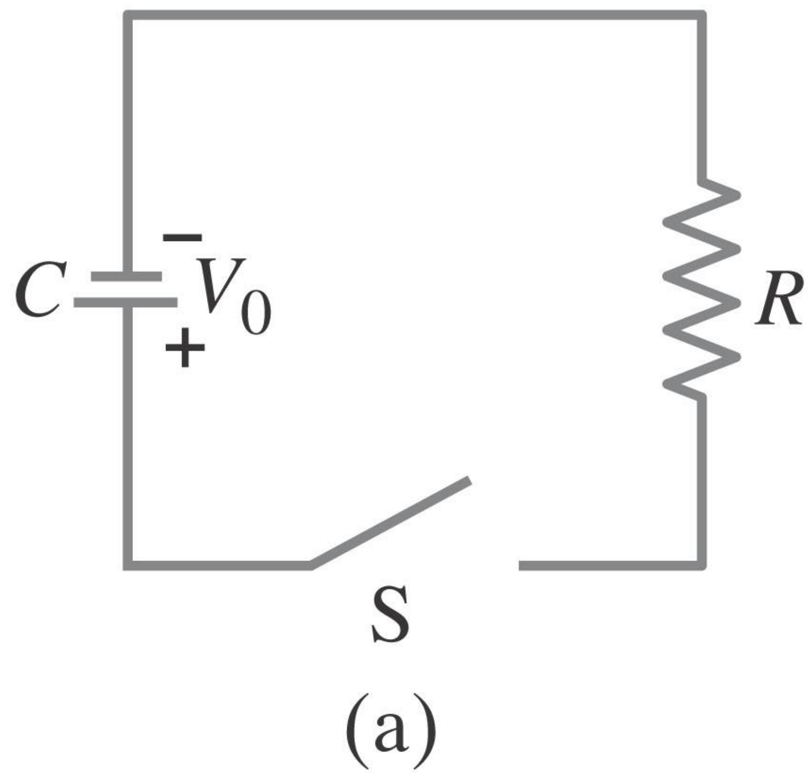
- For: $R = 20k\Omega$,
- $C = 0.3$ mF,
- $\mathcal{E} = 12$ V,
- Find τ,
- Q_{max},
- t at $Q_{99\%}$,
- I at $Q_{½}$,
- Imax, Q at $I_{20\%\ \text{of max}}$



(a)

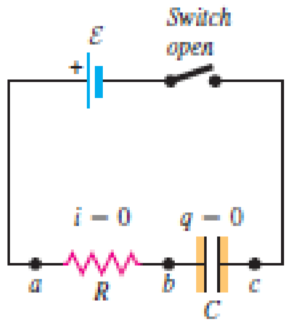
RC Discharge

- When we discharge a capacitor, the charge, Q, and the voltage, V, both decrease exponentially
- $Q = Q_0 \cdot e^{-t/_{R\cdot C}}$
- $V_c = V_{c0} \cdot e^{-t/_{R\cdot C}}$

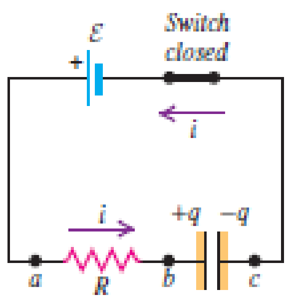


current i approaches zero.

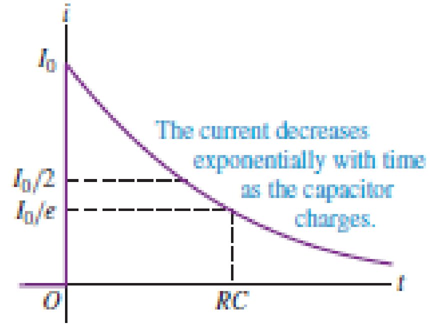
(a) Capacitor initially uncharged



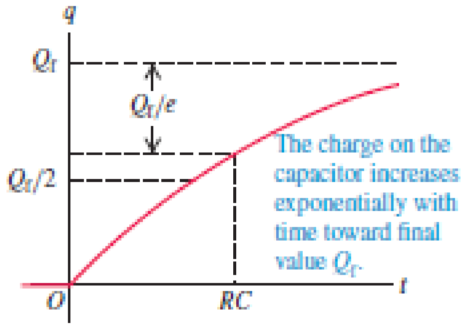
(b) Charging the capacitor



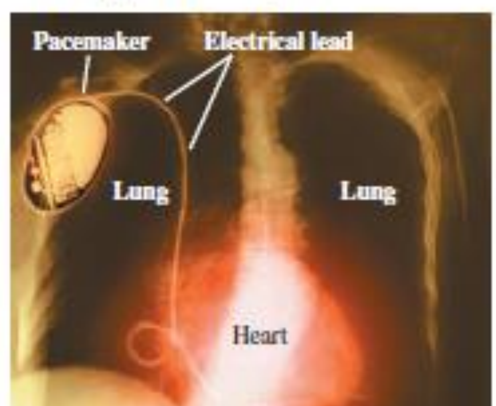
(a) Graph of current versus time for a charging capacitor



(b) Graph of capacitor charge versus time for a charging capacitor



BIO Application Pacemakers and Capacitors This x-ray image shows a pacemaker implanted in a patient with a malfunctioning sinoatrial node, the part of the heart that generates the electrical signal to trigger heartbeats. The pacemaker circuit contains a battery, a capacitor, and a computer-controlled switch. To maintain regular beating, once per second the switch discharges the capacitor and sends an electrical pulse along the lead to the heart. The switch then flips to allow the capacitor to recharge for the next pulse.



$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\mathcal{E})$$

We can rearrange this to

$$\frac{dq}{q - C\mathcal{E}} = -\frac{dt}{RC}$$

and then integrate both sides. We change the integration variables to q' and t' so that we can use q and t for the upper limits. The lower limits are $q' = 0$ and $t' = 0$:

$$\int_0^q \frac{dq'}{q' - C\mathcal{E}} = -\int_0^t \frac{dt'}{RC}$$

When we carry out the integration, we get

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

Exponentiating both sides (that is, taking the inverse logarithm) and solving for q, we find

$$\frac{q - C\mathcal{E}}{-C\mathcal{E}} = e^{-t/RC}$$

R-C circuit, charging capacitor:

Capacitor charge — Capacitance — Final capacitor charge $= C\mathcal{E}$

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC}) \qquad (26.12)$$

Battery emf — Time since switch closed — Resistance

The instantaneous current i is just the time derivative of Eq. (26.12):

R-C circuit, charging capacitor:

Current — Battery emf — Time since switch closed — Initial current $= \mathcal{E}/R$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \qquad (26.13)$$

Rate of change of capacitor charge — Resistance — Capacitance

Discharging a Capacitor

Now suppose that after the capacitor in Fig. 26.21b has acquired a charge Q_0, we remove the battery from our R-C circuit and connect points a and c to an open switch (**Fig. 26.22a**). We then close the switch and at the same instant reset our stopwatch to $t = 0$; at that time, $q = Q_0$. The capacitor then *discharges* through the resistor, and its charge eventually decreases to zero.

Again let i and q represent the time-varying current and charge at some instant after the connection is made. In Fig. 26.22b we make the same choice of the positive direction for current as in Fig. 26.20b. Then Kirchhoff's loop rule gives Eq. (26.10) but with $\mathcal{E} = 0$; that is,

$$i = \frac{dq}{dt} = -\frac{q}{RC} \qquad (26.15)$$

The current i is now negative; this is because positive charge q is leaving the left-hand capacitor plate in Fig. 26.22b, so the current is in the direction opposite to that shown. At time $t = 0$, when $q = Q_0$, the initial current is $I_0 = -Q_0/RC$.

To find q as a function of time, we rearrange Eq. (26.15), again change the variables to q' and t', and integrate. This time the limits for q' are Q_0 to q:

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\frac{1}{RC}\int_0^t dt'$$

$$\ln\frac{q}{Q_0} = -\frac{t}{RC}$$

***R*-*C* circuit, discharging capacitor:**

Capacitor charge; Initial capacitor charge; Capacitance; Time since switch closed; Resistance

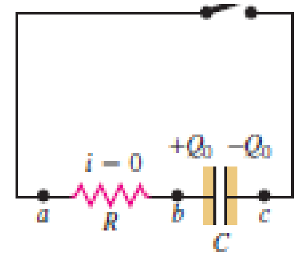
$$q = Q_0 e^{-t/RC} \qquad (26.16)$$

The instantaneous current i is the derivative of this with respect to time:

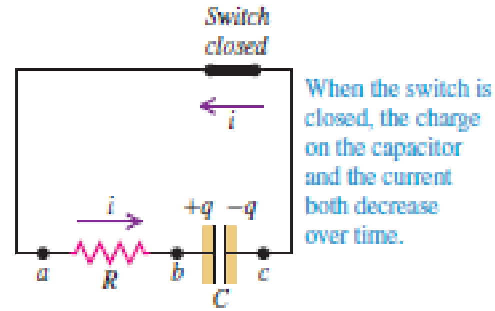
***R*-*C* circuit, discharging capacitor:**

Current; Initial capacitor charge; Capacitance; Rate of change of capacitor charge; Resistance; Initial current $= -Q_0/RC$; Time since switch closed

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC} \qquad (26.17)$$

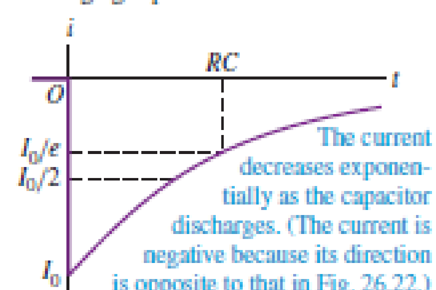


(b) Discharging the capacitor



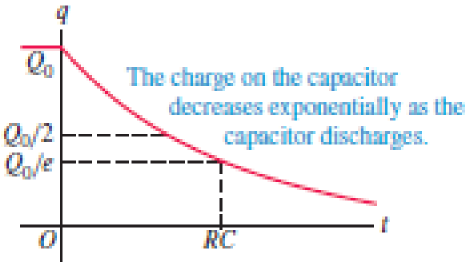
26.23 Current i and capacitor charge q as functions of time for the circuit of Fig. 26.22. The initial current is I_0 and the initial capacitor charge is Q_0. Both i and q asymptotically approach zero.

(a) Graph of current versus time for a discharging capacitor



(b) Graph of capacitor charge versus time

(b) Graph of capacitor charge versus time for a discharging capacitor



EXAMPLE 26.12 CHARGING A CAPACITOR

SOLUTION

A 10-MΩ resistor is connected in series with a 1.0-μF capacitor and a battery with emf 12.0 V. Before the switch is closed at time $t = 0$, the capacitor is uncharged. (a) What is the time constant? (b) What fraction of the final charge Q_f is on the capacitor at $t = 46$ s? (c) What fraction of the initial current I_0 is still flowing at $t = 46$ s?

SOLUTION

IDENTIFY and SET UP: This is the situation shown in Fig. 26.20, with $R = 10\ \text{M}\Omega$, $C = 1.0\ \mu\text{F}$, and $\mathcal{E} = 12.0$ V. The charge q and current i vary with time as shown in Fig. 26.21. Our target variables are (a) the time constant τ, (b) the ratio q/Q_f at $t = 46$ s, and (c) the ratio i/I_0 at $t = 46$ s. Equation (26.14) gives τ. For a capacitor being charged, Eq. (26.12) gives q and Eq. (26.13) gives i.

EXECUTE: (a) From Eq. (26.14),

$$\tau = RC = (10 \times 10^6\ \Omega)(1.0 \times 10^{-6}\ \text{F}) = 10\ \text{s}$$

(b) From Eq. (26.12),

$$\frac{q}{Q_f} = 1 - e^{-t/RC} = 1 - e^{-(46\ \text{s})/(10\ \text{s})} = 0.99$$

(c) From Eq. (26.13),

$$\frac{i}{I_0} = e^{-t/RC} = e^{-(46\ \text{s})/(10\ \text{s})} = 0.010$$

EVALUATE: After 4.6 time constants the capacitor is 99% charged and the charging current has decreased to 1.0% of its initial value. The circuit would charge more rapidly if we reduced the time constant by using a smaller resistance.

EXAMPLE 26.13 DISCHARGING A CAPACITOR

SOLUTION

The resistor and capacitor of Example 26.12 are reconnected as shown in Fig. 26.22. The capacitor has an initial charge of 5.0 μC and is discharged by closing the switch at $t = 0$. (a) At what time will the charge be 0.50 μC? (b) What is the current at this time?

SOLUTION

IDENTIFY and SET UP: Now the capacitor is being discharged, so q and i vary with time as in Fig. 26.23, with $Q_0 = 5.0 \times 10^{-6}$ C. Again we have $RC = \tau = 10$ s. Our target variables are (a) the value of t at which $q = 0.50\ \mu$C and (b) the value of i at this time. We first solve Eq. (26.16) for t, and then solve Eq. (26.17) for i.

EXECUTE: (a) Solving Eq. (26.16) for the time t gives

$$t = -RC \ln\frac{q}{Q_0} = -(10\ \text{s}) \ln\frac{0.50\ \mu\text{C}}{5.0\ \mu\text{C}} = 23\ \text{s} = 2.3\tau$$

(b) From Eq. (26.17), with $Q_0 = 5.0\ \mu\text{C} = 5.0 \times 10^{-6}$ C,

$$i = -\frac{Q_0}{RC}e^{-t/RC} = -\frac{5.0 \times 10^{-6}\ \text{C}}{10\ \text{s}}e^{-2.3} = -5.0 \times 10^{-8}\ \text{A}$$

EVALUATE: The current in part (b) is negative because i has the opposite sign when the capacitor is discharging than when it is charging. Note that we could have avoided evaluating $e^{-t/RC}$ by noticing that at the time in question, $q = 0.10Q_0$; from Eq. (26.16) this means that $e^{-t/RC} = 0.10$.

26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

Example 26-12: Discharging *RC* circuit.

In the *RC* circuit shown, the battery has fully charged the capacitor, so $Q_0 = C\mathcal{E}$. Then at $t = 0$ the switch is thrown from position a to b. The battery emf is 20.0 V, and the capacitance $C = 1.02\ \mu F$. The current I is observed to decrease to 0.50 of its initial value in 40 μs. (a) What is the value of Q, the charge on the capacitor, at $t = 0$? (b) What is the value of R? (c) What is Q at $t = 60\ \mu s$?

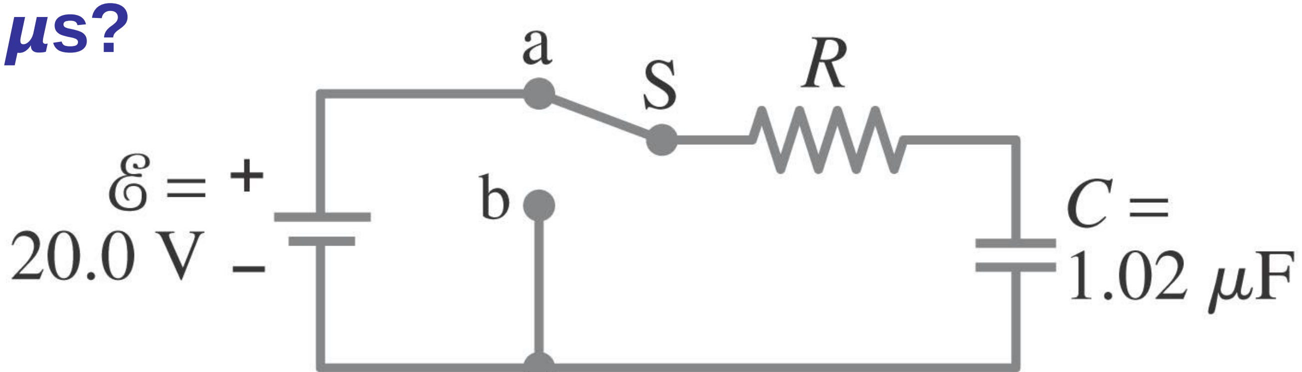
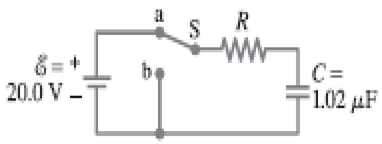


FIGURE 26–19 Example 26–12.



EXAMPLE 26–12 **Discharging *RC* circuit.** In the *RC* circuit shown in Fig. 26–19, the battery has fully charged the capacitor, so $Q_0 = C\mathscr{E}$. Then at $t = 0$ the switch is thrown from position a to b. The battery emf is 20.0 V, and the capacitance $C = 1.02\,\mu\text{F}$. The current I is observed to decrease to 0.50 of its initial value in $40\,\mu\text{s}$. (*a*) What is the value of Q, the charge on the capacitor, at $t = 0$? (*b*) What is the value of R? (*c*) What is Q at $t = 60\,\mu\text{s}$?

APPROACH At $t = 0$, the capacitor has charge $Q_0 = C\mathscr{E}$, and then the battery is removed from the circuit and the capacitor begins discharging through the resistor, as in Fig. 26–18. At any time t later (Eq. 26–9a) we have

$$Q = Q_0 e^{-t/RC} = C\mathscr{E}e^{-t/RC}.$$

SOLUTION (*a*) At $t = 0$,

$$Q = Q_0 = C\mathscr{E} = (1.02 \times 10^{-6}\,\text{F})(20.0\,\text{V}) = 2.04 \times 10^{-5}\,\text{C} = 20.4\,\mu\text{C}.$$

(*b*) To find R, we are given that at $t = 40\,\mu\text{s}$, $I = 0.50 I_0$. Hence

$$0.50 I_0 = I_0 e^{-t/RC}.$$

Taking natural logs on both sides $(\ln 0.50 = -0.693)$:

$$0.693 = \frac{t}{RC}$$

so

$$R = \frac{t}{(0.693)C} = \frac{(40 \times 10^{-6}\,\text{s})}{(0.693)(1.02 \times 10^{-6}\,\text{F})} = 57\,\Omega.$$

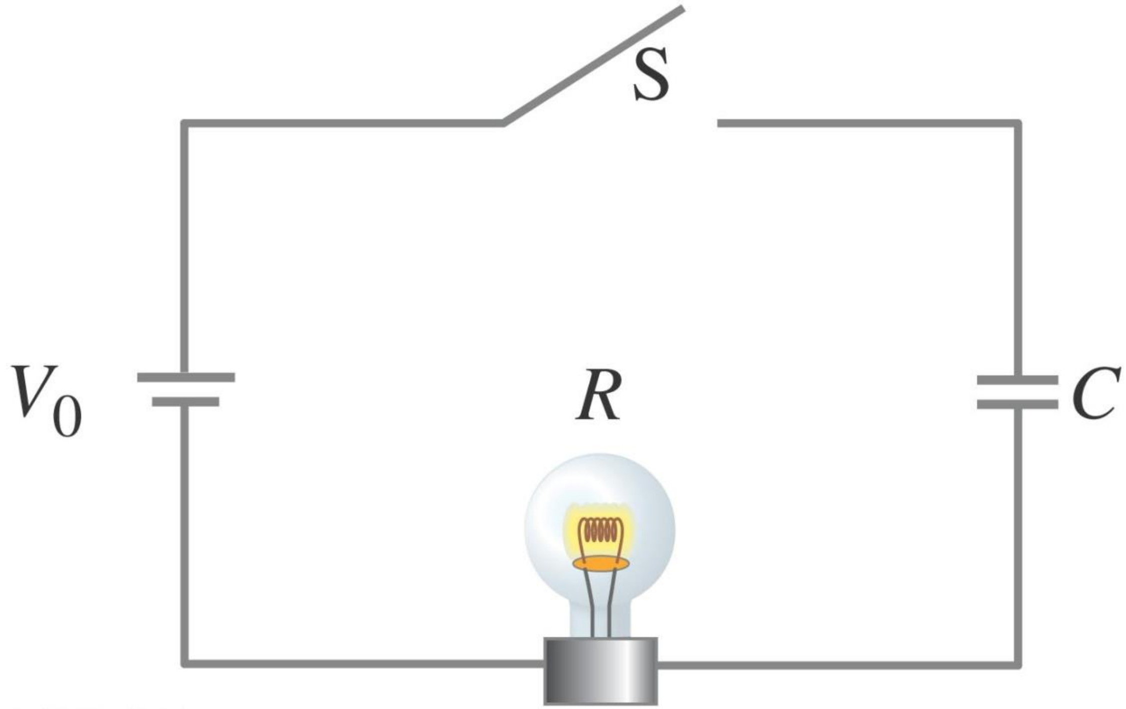
(*c*) At $t = 60\,\mu\text{s}$,

$$Q = Q_0 e^{-t/RC} = (20.4 \times 10^{-6}\,\text{C})e^{-\frac{60\times10^{-6}\,\text{s}}{(57\,\Omega)(1.02\times10^{-6}\,\text{F})}} = 7.3\,\mu\text{C}.$$

26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

Conceptual Example 26-13: Bulb in *RC* circuit.

In the circuit shown, the capacitor is originally uncharged. Describe the behavior of the lightbulb from the instant switch S is closed until a long time later.



CONCEPTUAL EXAMPLE 26–13 **Bulb in *RC* circuit.** In the circuit of Fig. 26–20, the capacitor is originally uncharged. Describe the behavior of the lightbulb from the instant switch S is closed until a long time later.

RESPONSE When the switch is first closed, the current in the circuit is high and the lightbulb burns brightly. As the capacitor charges, the voltage across the capacitor increases causing the current to be reduced, and the lightbulb dims. As the potential difference across the capacitor approaches the same voltage as the battery, the current decreases toward zero and the lightbulb goes out.

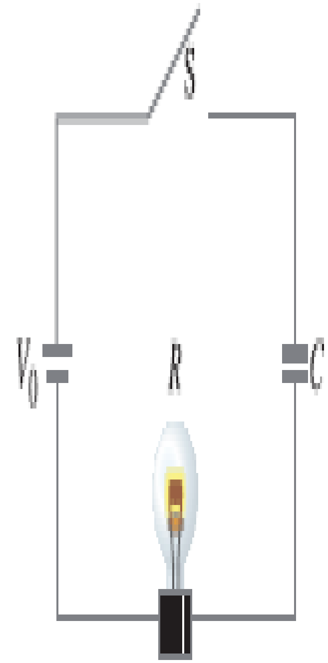
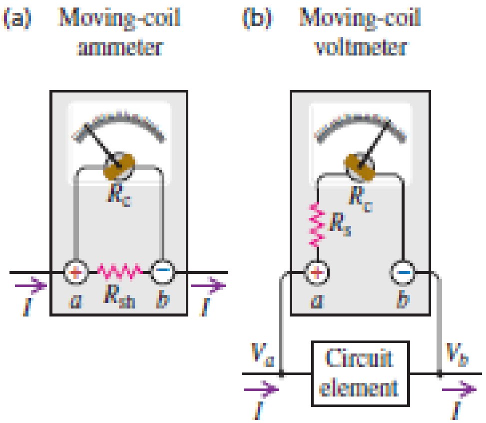


FIGURE 26–20 Example 26–13.

26.15 Using the same meter to measure (a) current and (b) voltage.



$$I_{fs}R_c = (I_a - I_{fs})R_{sh} \qquad \text{(for an ammeter)}$$

What shunt resistance is required to make the 1.00-mA, 20.0-Ω meter described above into an ammeter with a range of 0 to 50.0 mA?

SOLUTION

IDENTIFY and SET UP: Since the meter is being used as an ammeter, its internal connections are as shown in Fig. 26.15a. Our target variable is the shunt resistance R_{sh}, which we will find from Eq. (26.7). The ammeter must handle a maximum current $I_a = 50.0 \times 10^{-3}$ A. The coil resistance is $R_c = 20.0\ \Omega$, and the meter shows full-scale deflection when the current through the coil is $I_{fs} = 1.00 \times 10^{-3}$ A.

EXECUTE: Solving Eq. (26.7) for R_{sh}, we find

$$R_{sh} = \frac{I_{fs}R_c}{I_a - I_{fs}} = \frac{(1.00 \times 10^{-3}\ \text{A})(20.0\ \Omega)}{50.0 \times 10^{-3}\ \text{A} - 1.00 \times 10^{-3}\ \text{A}}$$
$$= 0.408\ \Omega$$

EVALUATE: It's useful to consider the equivalent resistance R_{eq} of the ammeter as a whole. From Eq. (26.2),

$$R_{eq} = \left(\frac{1}{R_c} + \frac{1}{R_{sh}}\right)^{-1} = \left(\frac{1}{20.0\ \Omega} + \frac{1}{0.408\ \Omega}\right)^{-1}$$
$$= 0.400\ \Omega$$

The shunt resistance is so small in comparison to the coil resistance that the equivalent resistance is very nearly equal to the shunt resistance. The result is an ammeter with a low equivalent resistance and the desired 0–50.0-mA range. At full-scale deflection, $I = I_a = 50.0$ mA, the current through the galvanometer is 1.00 mA, the current through the shunt resistor is 49.0 mA, and $V_{ab} = 0.0200$ V. If the current I is *less* than 50.0 mA, the coil current and the deflection are proportionally less.

BIO Application Electromyography
A fine needle containing two electrodes is being inserted into a muscle in this patient's hand. By using a sensitive voltmeter to measure the potential difference between these electrodes, a physician can probe the muscle's electrical activity. This is an important technique for diagnosing neurological and neuromuscular diseases.

For the meter of Example 26.8, the voltage across the meter coil at full-scale deflection is only $I_{fs}R_c = (1.00 \times 10^{-3}\ \text{A})(20.0\ \Omega) = 0.0200\ \text{V}$. We can extend this range by connecting a resistor R_s in *series* with the coil (Fig. 26.15b). Then only a fraction of the total potential difference appears across the coil itself, and the remainder appears across R_s. For a voltmeter with full-scale reading V_V, we need a series resistor R_s in Fig. 26.15b such that

$$V_V = I_{fs}(R_c + R_s) \qquad \text{(for a voltmeter)} \qquad (26.8)$$

What series resistance is required to make the 1.00-mA, 20.0-Ω meter described above into a voltmeter with a range of 0 to 10.0 V?

SOLUTION

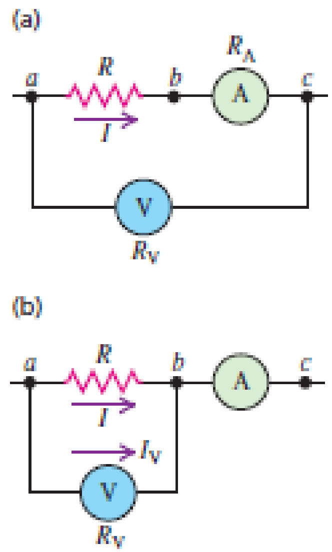
IDENTIFY and SET UP: Since this meter is being used as a voltmeter, its internal connections are as shown in Fig. 26.15b. The maximum allowable voltage across the voltmeter is $V_V = 10.0$ V. We want this to occur when the current through the coil is $I_{fs} = 1.00 \times 10^{-3}$ A. Our target variable is the series resistance R_s, which we find from Eq. (26.8).

EXECUTE: From Eq. (26.8),

$$R_s = \frac{V_V}{I_{fs}} - R_c = \frac{10.0\ \text{V}}{0.00100\ \text{A}} - 20.0\ \Omega = 9980\ \Omega$$

EVALUATE: At full-scale deflection, $V_{ab} = 10.0$ V, the voltage across the meter is 0.0200 V, the voltage across R_s is 9.98 V, and the current through the voltmeter is 0.00100 A. Most of the voltage appears across the series resistor. The meter's equivalent resistance is a desirably high $R_{eq} = 20.0\ \Omega + 9980\ \Omega = 10{,}000\ \Omega$. Such a meter is called a "1000 ohms-per-volt" meter, referring to the ratio of resistance to full-scale deflection. In normal operation the current through the circuit element being measured (I in Fig. 26.15b) is much greater than 0.00100 A, and the resistance between points a and b in the circuit is much less than 10,000 Ω. The voltmeter draws off only a small fraction of the current and thus disturbs the circuit being measured only slightly.

26.16 Ammeter–voltmeter method for measuring resistance.



The voltmeter in the circuit of Fig. 26.16a reads 12.0 V and the ammeter reads 0.100 A. The meter resistances are $R_V = 10{,}000\ \Omega$ (for the voltmeter) and $R_A = 2.00\ \Omega$ (for the ammeter). What are the resistance R and the power dissipated in the resistor?

SOLUTION

IDENTIFY and SET UP: The ammeter reads the current $I = 0.100$ A through the resistor, and the voltmeter reads the potential difference between a and c. If the ammeter were *ideal* (that is, if $R_A = 0$), there would be zero potential difference between b and c, the voltmeter reading $V = 12.0$ V would be equal to the potential difference V_{ab} across the resistor, and the resistance would be equal to $R = V/I = (12.0\ \text{V})/(0.100\ \text{A}) = 120\ \Omega$. The ammeter is *not* ideal, however (its resistance is $R_A = 2.00\ \Omega$), so the voltmeter reading V is actually the sum of the potential differences V_{bc} (across the ammeter) and V_{ab} (across the resistor). We use Ohm's law to find the voltage V_{bc} from the known current and ammeter resistance. Then we solve for V_{ab} and R. Given these, we are able to calculate the power P into the resistor.

EXECUTE: From Ohm's law, $V_{bc} = IR_A = (0.100\ \text{A})(2.00\ \Omega) = 0.200$ V and $V_{ab} = IR$. The sum of these is $V = 12.0$ V, so the potential difference across the resistor is $V_{ab} = V - V_{bc} = (12.0\ \text{V}) - (0.200\ \text{V}) = 11.8$ V. Hence the resistance is

$$R = \frac{V_{ab}}{I} = \frac{11.8\ \text{V}}{0.100\ \text{A}} = 118\ \Omega$$

The power dissipated in this resistor is

$$P = V_{ab}I = (11.8\ \text{V})(0.100\ \text{A}) = 1.18\ \text{W}$$

EVALUATE: You can confirm this result for the power by using the alternative formula $P = I^2R$. Do you get the same answer?

59. (II) A 45-V battery of negligible internal resistance is connected to a 44-kΩ and a 27-kΩ resistor in series. What reading will a voltmeter, of internal resistance 95 kΩ, give when used to measure the voltage across each resistor? What is the percent inaccuracy due to meter resistance for each case?

59. If the voltmeter were ideal, then the only resistance in the circuit would be the series combination of the two resistors. The current can be found from the battery and the equivalent resistance, and then the voltage across each resistor can be found.

$$R_{tot} = R_1 + R_2 = 44\,\mathrm{k\Omega} + 27\,\mathrm{k\Omega} = 71\,\mathrm{k\Omega} \;;\; I = \frac{V}{R_{tot}} = \frac{45\,\mathrm{V}}{71\times10^3\,\Omega} = 6.338\times10^{-4}\,\mathrm{A}$$

$$V_{44} = IR_1 = \left(6.338\times10^{-4}\,\mathrm{A}\right)\left(44\times10^3\,\Omega\right) = 27.89\,\mathrm{V}$$

$$V_{27} = IR_2 = \left(6.338\times10^{-4}\,\mathrm{A}\right)\left(27\times10^3\,\Omega\right) = 17.11\,\mathrm{V}$$

Now put the voltmeter in parallel with the $44\,\mathrm{k\Omega}$ resistor. Find its equivalent resistance, and then follow the same analysis as above.

$$R_{eq} = \left(\frac{1}{44\,\mathrm{k\Omega}} + \frac{1}{95\,\mathrm{k\Omega}}\right)^{-1} = 30.07\,\mathrm{k\Omega}$$

$$R_{tot} = R_{eq} + R_2 = 30.07\,\mathrm{k\Omega} + 27\mathrm{k\Omega} = 57.07\,\mathrm{k\Omega} \qquad I = \frac{V}{R_{tot}} = \frac{45\,\mathrm{V}}{57.07\times10^3\,\Omega} = 7.885\times10^{-4}\,\mathrm{A}$$

$$V_{44} = V_{eq} = IR_{eq} = \left(7.885\times10^{-4}\,\mathrm{A}\right)\left(30.07\times10^3\,\Omega\right) = 23.71\,\mathrm{V} \approx \boxed{24\,\mathrm{V}}$$

$$\%\ \text{error} = \frac{23.71\,\mathrm{V} - 27.89\mathrm{V}}{27.89\,\mathrm{V}}\times100 = \boxed{-15\%\left(\text{reading too low}\right)}$$

And now put the voltmeter in parallel with the $27\,\mathrm{k\Omega}$ resistor, and repeat the process.

$$R_{eq} = \left(\frac{1}{27\,\mathrm{k\Omega}} + \frac{1}{95\,\mathrm{k\Omega}}\right)^{-1} = 21.02\,\mathrm{k\Omega}$$

$$R_{tot} = R_{eq} + R_1 = 21.02\,\mathrm{k\Omega} + 44\,\mathrm{k\Omega} = 65.02\,\mathrm{k\Omega} \qquad I = \frac{V}{R_{tot}} = \frac{45\,\mathrm{V}}{65.02\times10^3\,\Omega} = 6.921\times10^{-4}\,\mathrm{A}$$

$$V_{27} = V_{eq} = IR_{eq} = \left(6.921\times10^{-4}\,\mathrm{A}\right)\left(21.02\times10^3\,\Omega\right) = 14.55\mathrm{V} \approx \boxed{15\,\mathrm{V}}$$

$$\%\ \text{error} = \frac{14.55\,\mathrm{V} - 17.11\mathrm{V}}{17.11\,\mathrm{V}}\times100 = \boxed{-15\%\ \ \left(\text{reading too low}\right)}$$

•60. (II) An ammeter whose internal resistance is 53 Ω reads 5.25 mA when connected in a circuit containing a battery and two resistors in series whose values are 650 Ω and 480 Ω. What is the actual current when the ammeter is absent?

60. The total resistance with the ammeter present is $R_{eq} = 650\,\Omega + 480\,\Omega + 53\,\Omega = 1183\,\Omega$. The voltage supplied by the battery is found from Ohm's law to be $V_{\text{battery}} = IR_{eq} = \left(5.25\times10^{-3}\,\text{A}\right)\left(1183\,\Omega\right)$ $= 6.211\,\text{V}$. When the ammeter is removed, we assume that the battery voltage does not change. The equivalent resistance changes to $R'_{eq} = 1130\,\Omega$, and the new current is again found from Ohm's law.

$$I = \frac{V_{\text{battery}}}{R'_{eq}} = \frac{6.211\,\text{V}}{1130\,\Omega} = \boxed{5.50\times10^{-3}\,\text{A}}$$

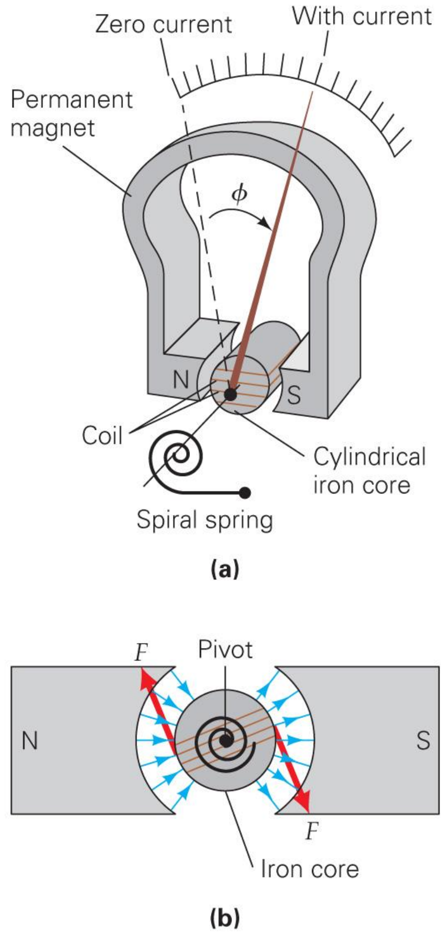
End of Volume 2 Chapter 10

- Read Chapter 10 Summary -
- Complete homework for Chapter 10.

Backup slides

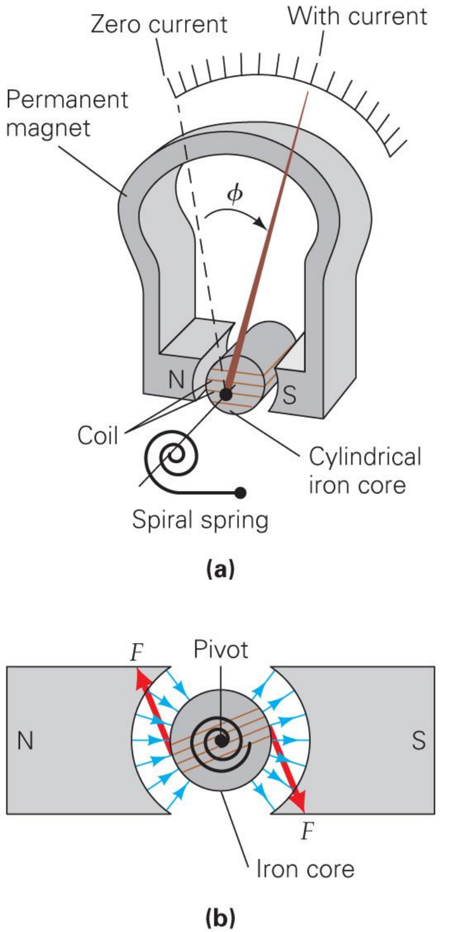
Electrical Meters

- The basic analogue electrical meter is the galvanometer
- A small current flowing through the coil of wire around the iron core forms an electromagnet.
- The torque from the electromagnet works against the spring and turns the needle.



Electrical Meters

- The balance between coil and spring determines the sensitivity of the meter.
- By using a very light spring and fine wires for the coil the meter can measure a few μA of current.

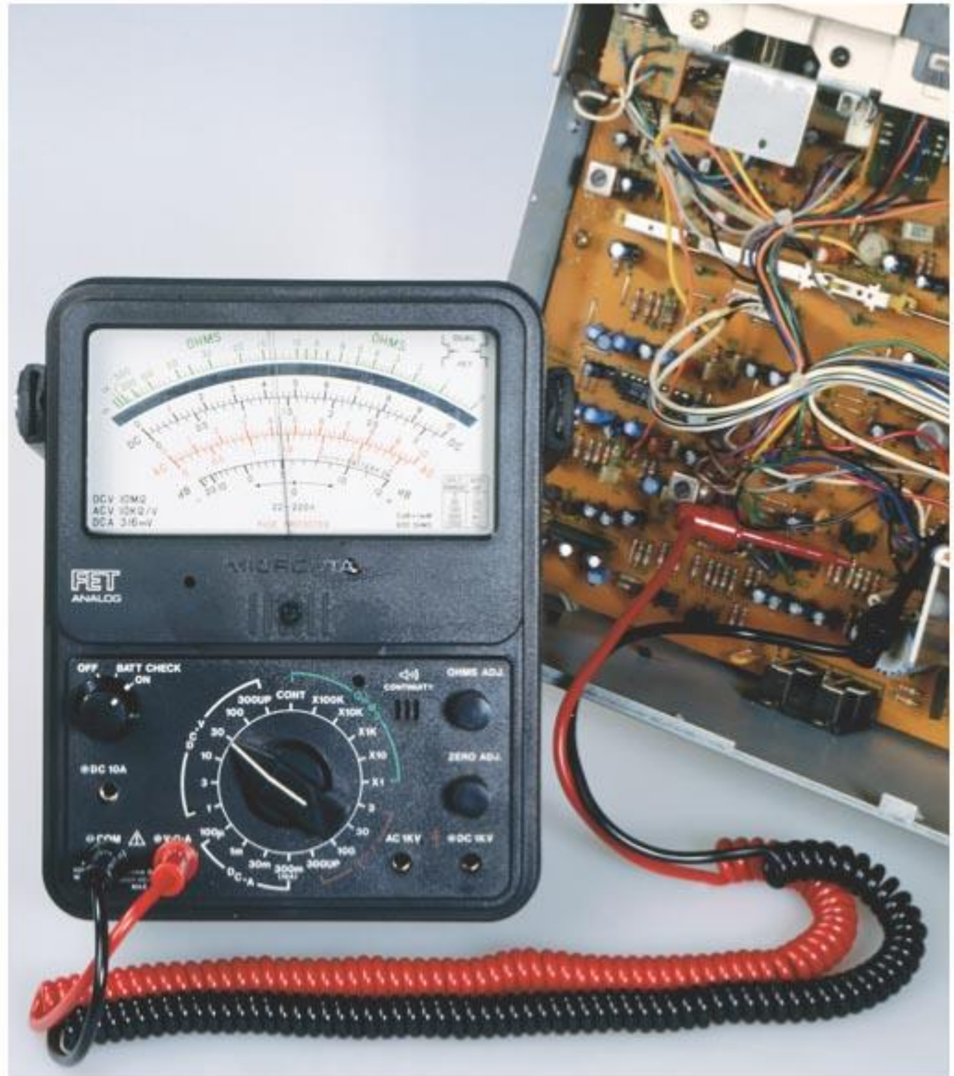


The Galvanometer

The wire coil of the galvanometer does add some resistance to the circuit being measured.

We represent the galvanometer using the symbols at the right.

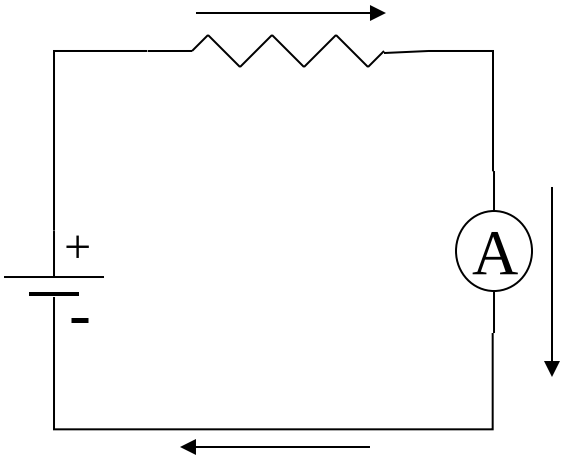
G for the meter itself and R_i for the internal resistance of the coil.



(a)

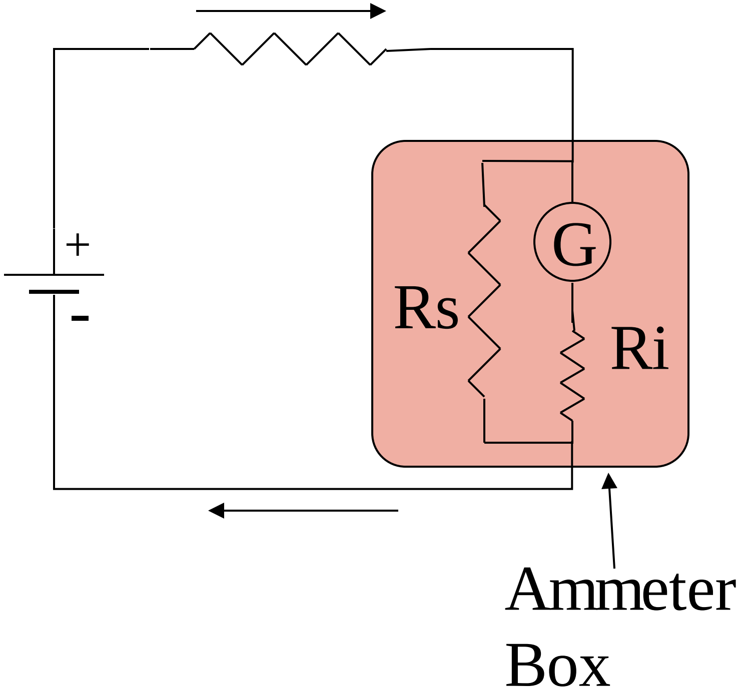
Ammeters - Ampere Meters

- How do we use this meter to measure current in a circuit?
- To measure the current in a circuit, that <u>current must pass through the meter</u>, ie. the meter must be in series with the circuit.
- Galvanometers have very low resistance, which is good for current measurements, but they are also very sensitive and can be damaged by large currents.



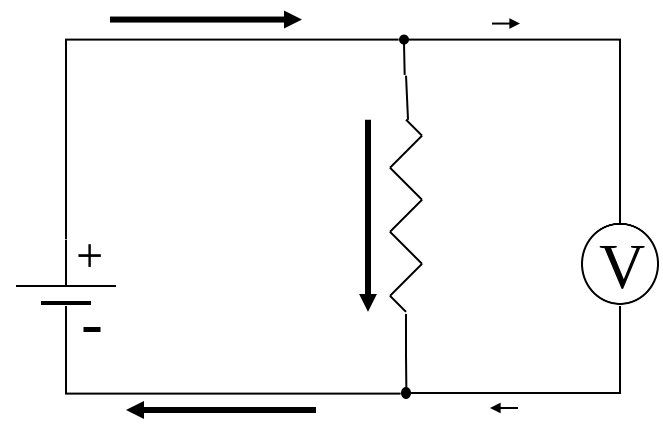
Ammeters

- We can protect the meter and extend its range by adding a low resistance, parallel resistor, called a shunt resistor, R_s.
- Now most of the current by-passes the meter.
- The scale on the meter is then calibrated to show the total current flowing through both the meter and the shunt.



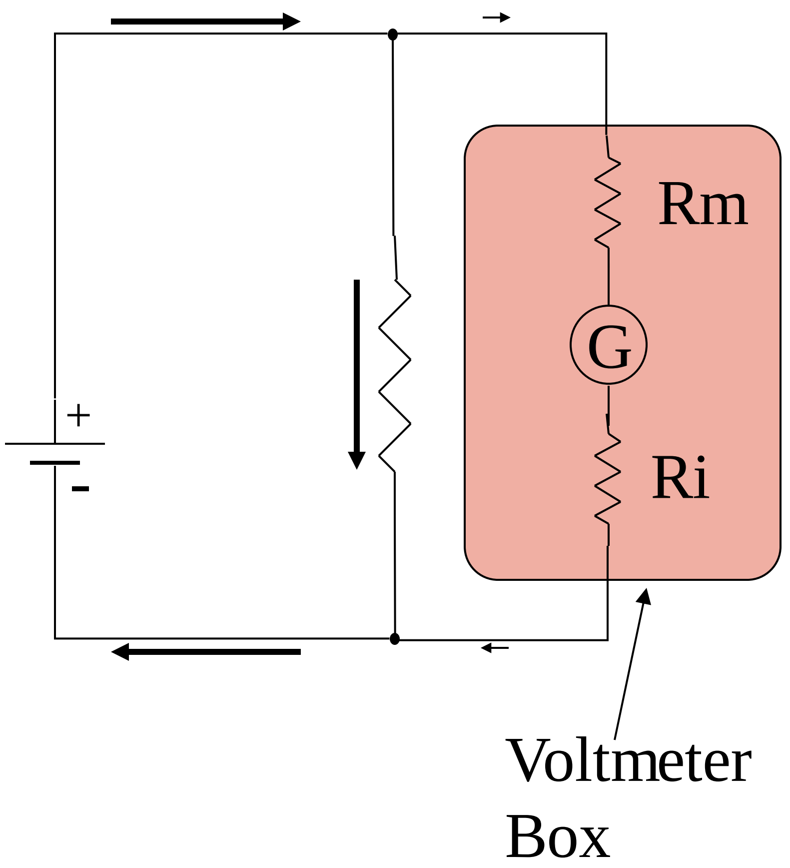
Voltmeters

- How do we use this meter to measure voltage?
- To measure the ΔV across a part of a circuit, the meter must be in parallel with the circuit.
- If the voltmeter is not going to change the current flowing in rest of the circuit that it is measuring, it must have very <u>high resistance</u>.



Voltmeters

- By adding a high resistance, series resistor, called a multiplier resistor, Rm, very little current flows through the meter.
- The total voltage needed to make a current I flow through the meter is
- V = I•(Rm + Ri)
- The scale on the meter is then calibrated to show this voltage.



Digital Meters

Today, circuits called analog-to-digital converters can be used to read voltages directly.

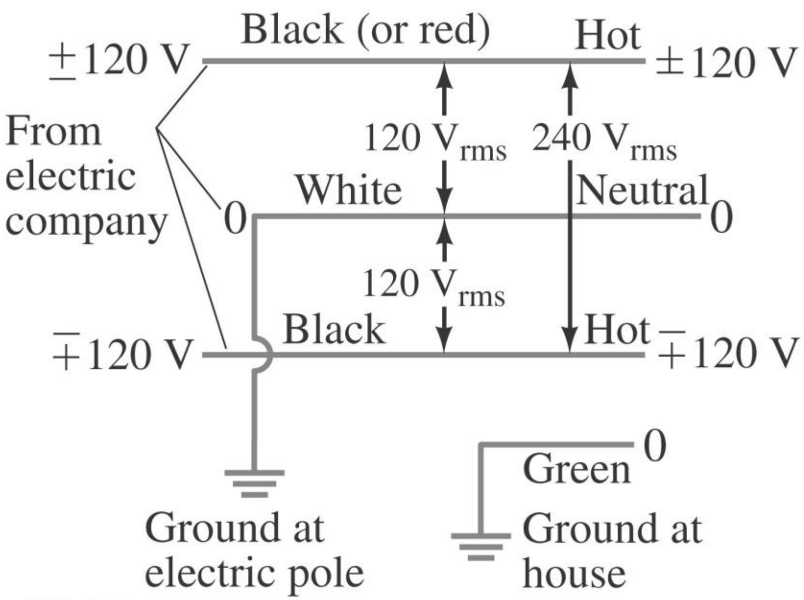
Using a voltage divider, the effective range of the A-to-D converter can be changed.

Reading the voltage across a shunt resistor produces converts the A-to-D converter to an ammeter.

The output of the A-to-D converter then goes to a processor, that controls the digital readout.

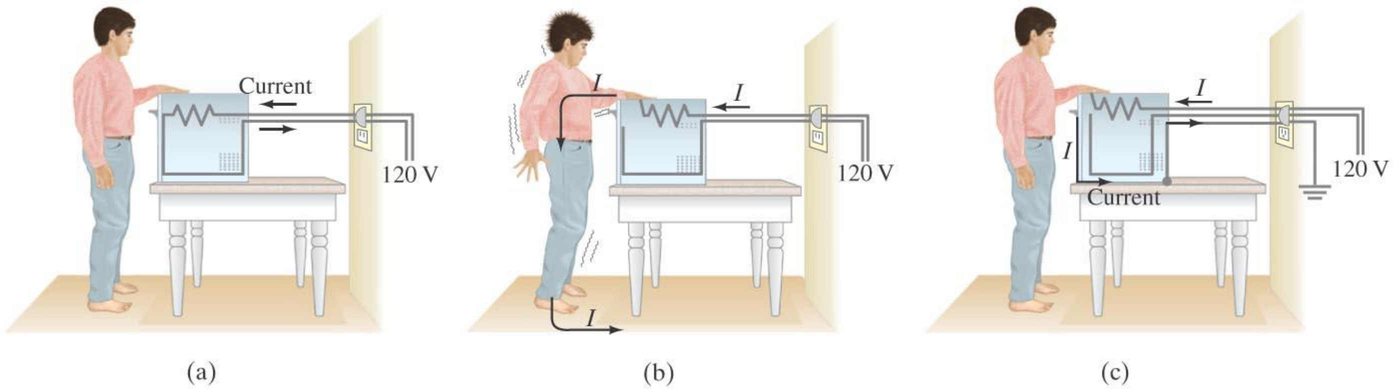
(b)

Household Electrical Circuits



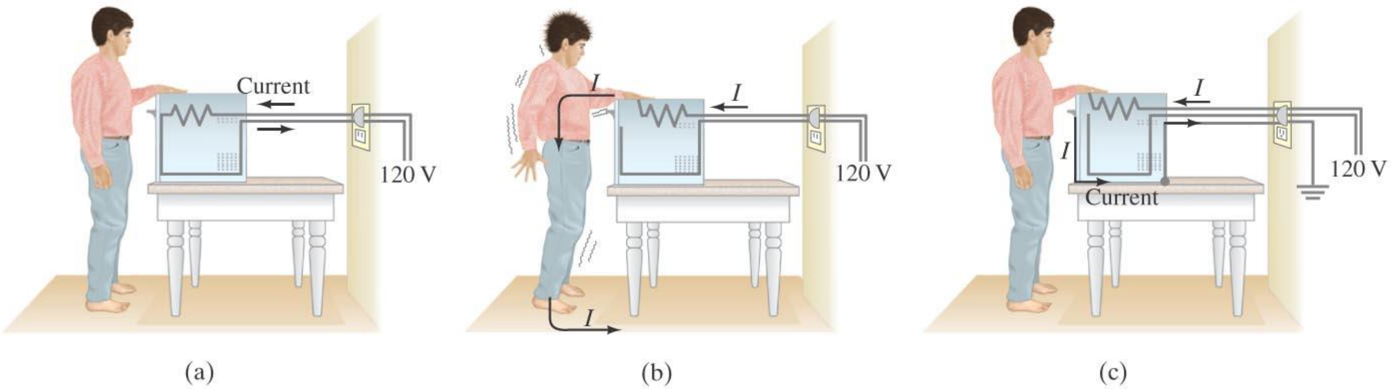
- Two ‘hot’ wires come to your house from the transformer and carry 240V AC (60Hz).
- A third wire serves as the ground.
- Standard appliances are connected between one of the hot wires and ground, providing 120 V AC.
- High current appliances are connected across the two hot lines for 240V.

Household Hazards



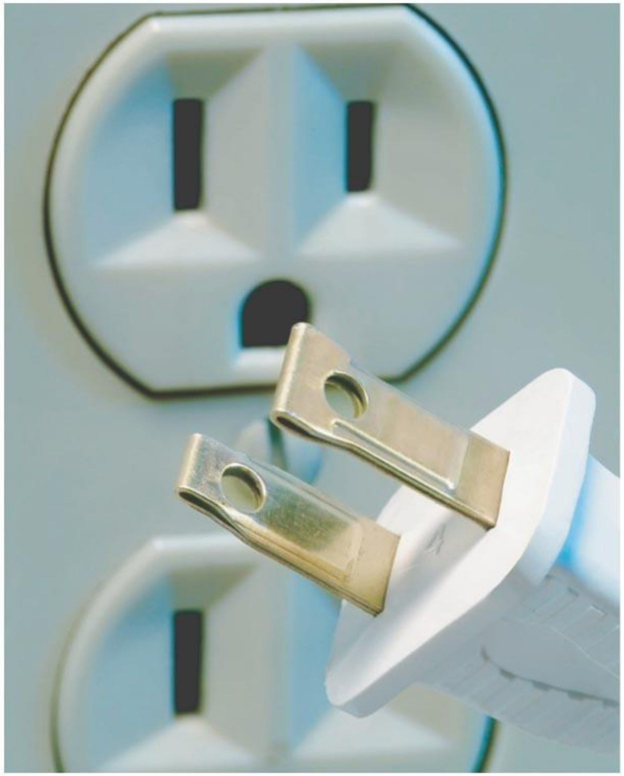
- Since the peak voltage in a household circuit is 170V, electrical appliances pose a significant shock hazard.
- The pictures above show such a hazard and how modern appliances avoid it.

Household Hazards



- For appliances with exposed metal parts, a 3-wire ground can insure that there is no shock hazard even if there is an internal short to the exposed metal.

Household Hazards



(c)

- A polarized 2-wire plug can provide similar protection if the internal electrical circuits are isolated (double insulation) from the case.