hw6

April 16, 2023

[ ]:
```
import random
from pathlib import Path
from typing import List
from IPython.display import display, Image

import numpy as np
import matplotlib.pyplot as plt

plt.rcParams['text.usetex'] = True
```

1 Exercise 1:

[ ]:
```
display(Image(filename="./images/hw6_p1.png", height=400, width=500))
```

Exercise #1

a) $g = [\bullet\ \bullet\ \bullet\ \bullet\ \bullet\ \bullet\ \bullet\ \bullet] = h_1$

b/c this matches y_n the best

3-matches = 1, 2-matches = 3,

1-matches = 3, 0-matches = 1

b) $g = [\circ\ \circ\ \circ\ \circ\ \circ\ \circ\ \circ\ \circ] = h_2$

3-matches = 1, 2-matches = 3, 1-matches = 3

0-matches = 1

c) $g = [\circ\ \bullet\ \bullet\ \circ\ \bullet\ \circ\ \circ\ \bullet]$

3-matches = 1, 2-matches = 3, 1-matches = 3

0-matches = 1

2 Exercise 2:

2.0.1 Problem 2a:

Since the coins are independent and fair coins,

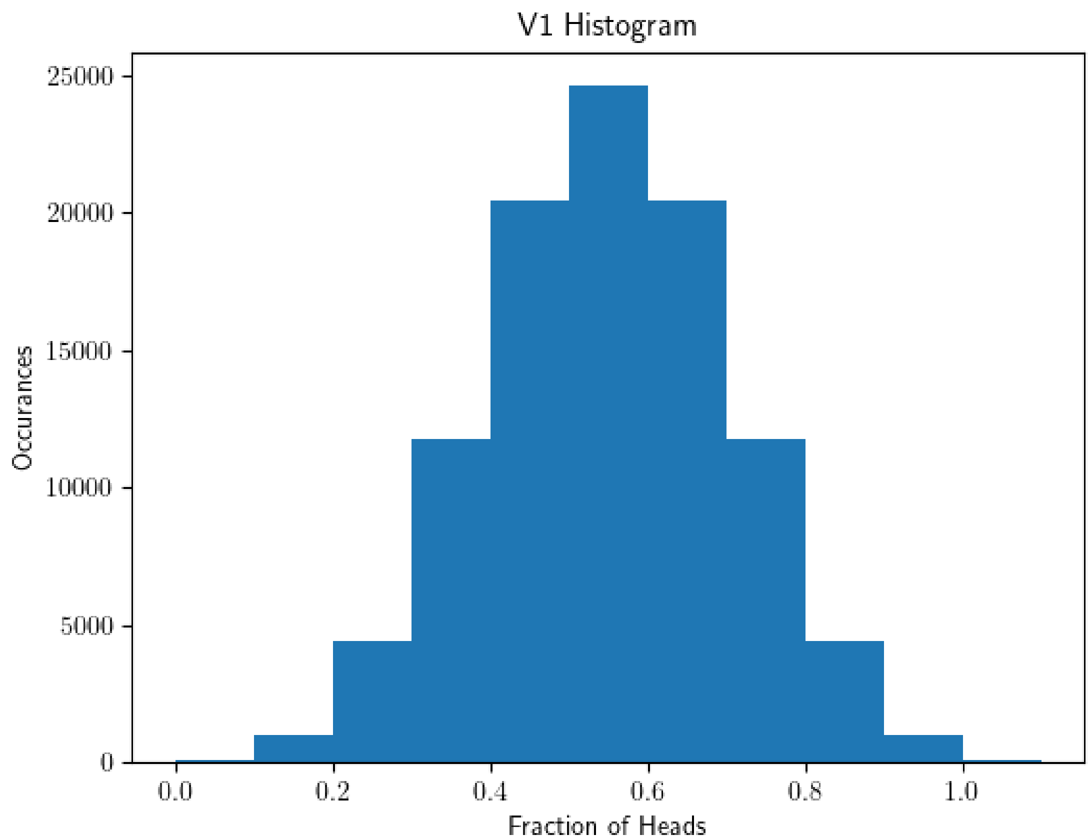
$$\mu_1 = \mu_{rand} = \mu_{min} = 0.5$$

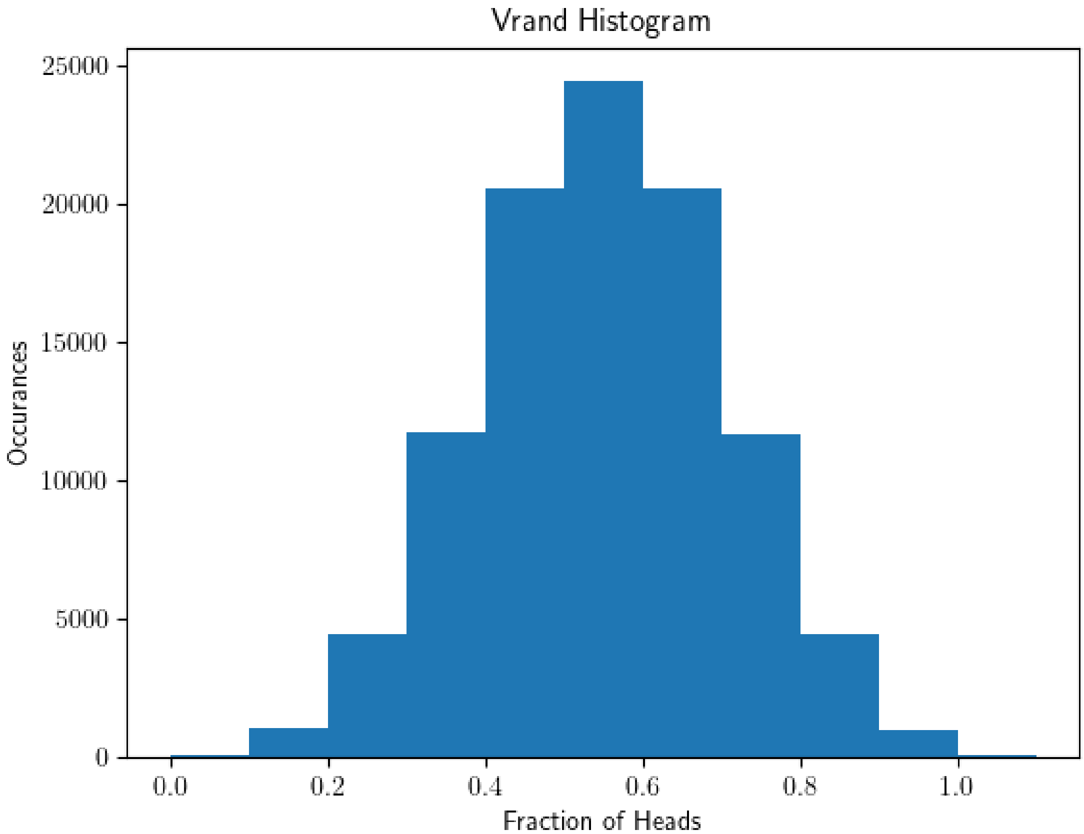
2.0.2 Problem 2b:

[ ]:
```python
def run_experiment():
    # Heads == 1, Tails == 0
    n_coins = 1000
    num_flips = 10
    vals = np.random.choice([0, 1], size=(n_coins, num_flips), p=[.5, .5])
    sums = vals.sum(axis=1)
    v_1 = sums[0] / num_flips
    v_rand = sums[random.randint(0, n_coins-1)] / num_flips
    v_min = min(sums) / num_flips
    return (v_1, v_rand, v_min)

num_experiments = 100_000
v_1s = []
v_rands = []
v_mins = []
for i in range(num_experiments):
    v_1, v_rand, v_min = run_experiment()
    v_1s.append(v_1)
    v_rands.append(v_rand)
    v_mins.append(v_min)
```

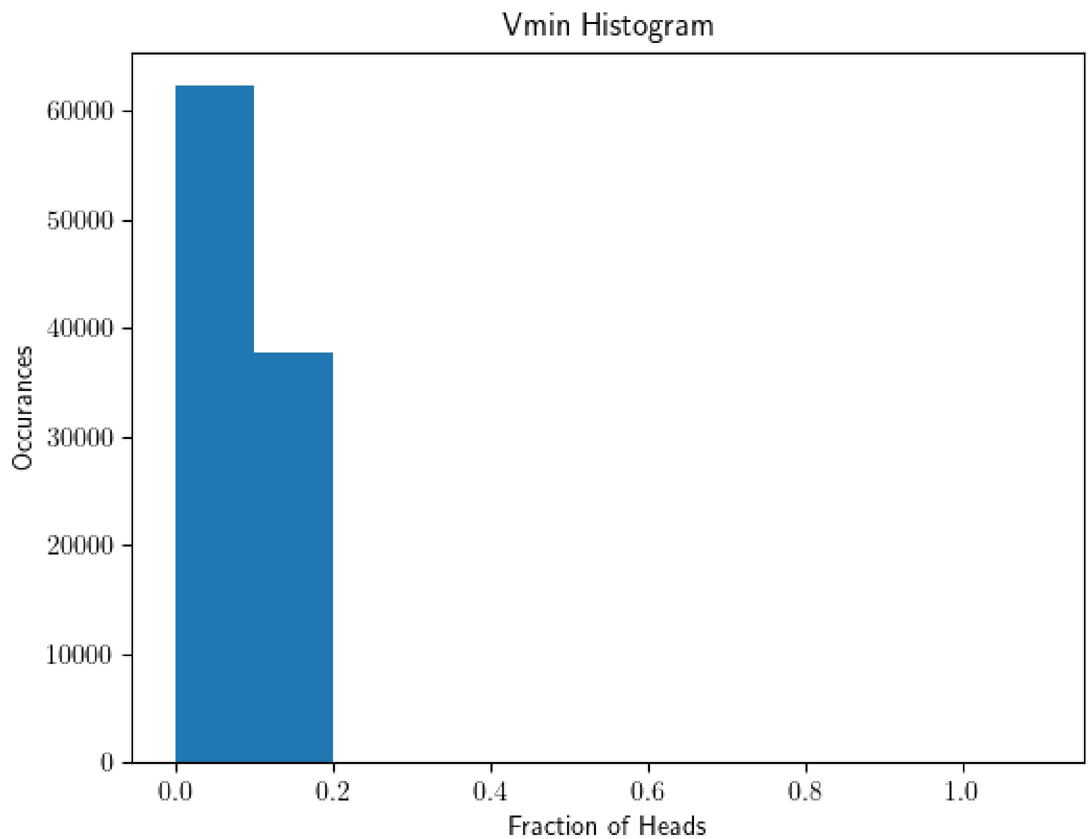
[ ]:
```python
# Generate Histogram Plots
bins_sequence = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]
bins_sequence = [val / 10 for val in bins_sequence]

fig = plt.figure()
plt.hist(v_1s, bins=bins_sequence)
plt.title("V1 Histogram")
plt.xlabel("Fraction of Heads")
plt.ylabel("Occurances")

fig = plt.figure()
plt.hist(v_rands, bins=bins_sequence)
plt.title("Vrand Histogram")
plt.xlabel("Fraction of Heads")
plt.ylabel("Occurances")

fig = plt.figure()
plt.hist(v_mins, bins=bins_sequence)
plt.title("Vmin Histogram")
plt.xlabel("Fraction of Heads")
plt.ylabel("Occurances")
```

[ ]:
```
Text(0, 0.5, 'Occurances')
```


V1 Histogram
25000
20000
15000
10000
5000
0
Occurances
0.0
0.2
0.4
0.6
0.8
1.0
Fraction of Heads



Vrand Histogram
25000
20000
15000
10000
5000
0
Occurances
0.0
0.2
0.4
0.6
0.8
1.0
Fraction of Heads

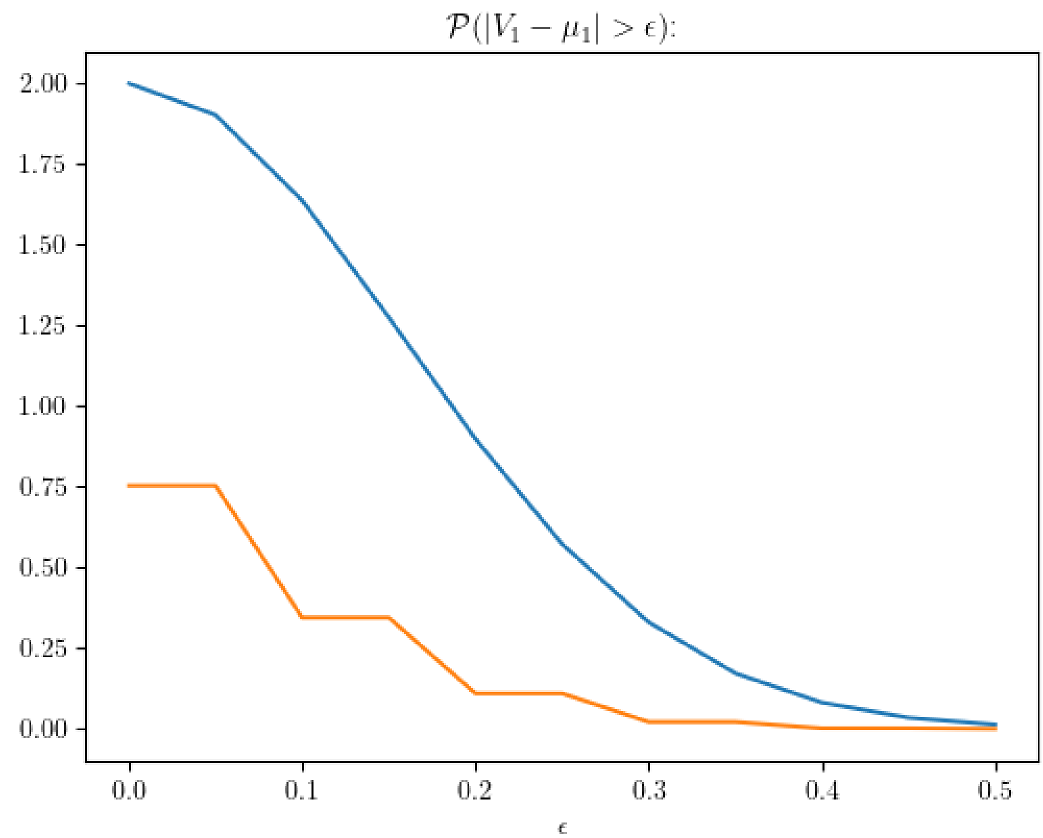


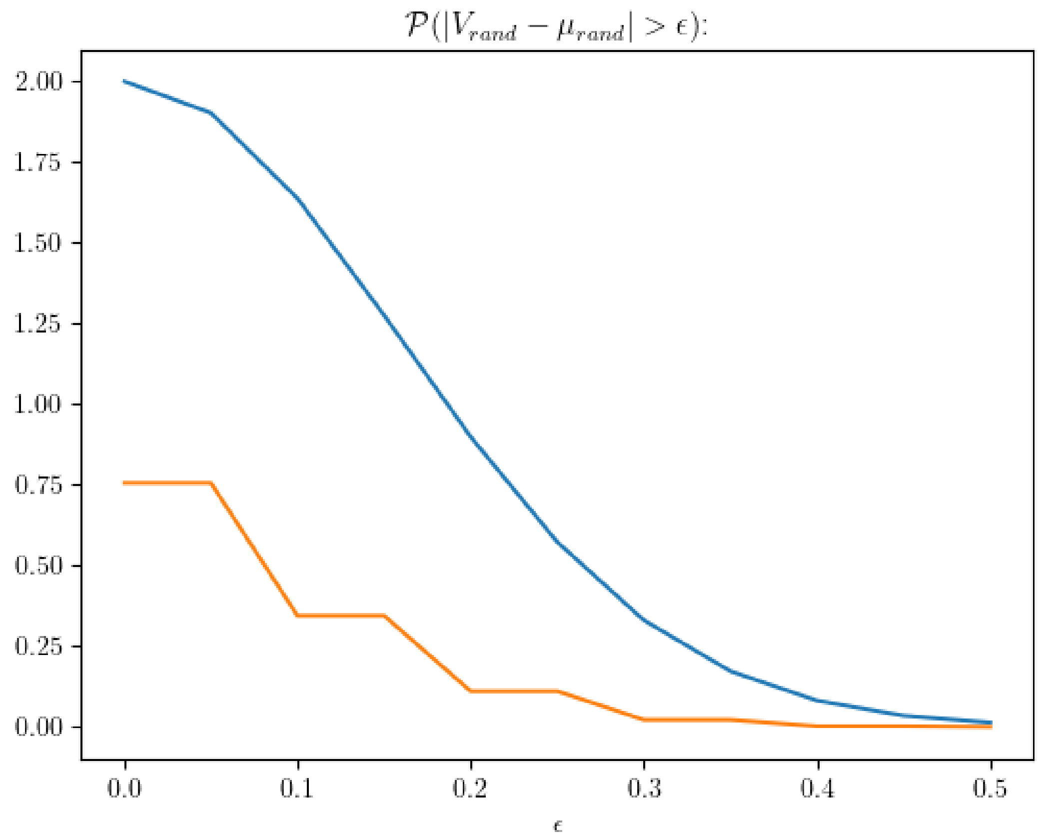
2.0.3 Problem 2c:

[ ]:
```
def hoeffding_bound(epsilon: np.array, N):
    return 2 * np.exp(-2 * np.square(epsilon) * N)
epsilon = np.linspace(0.0, 0.5, int((0.5 - 0.0) / 0.05) + 1)
hoeffding = hoeffding_bound(epsilon, 10)

mu = 0.5
v1_delta = np.abs(np.array(v_1s) - mu)
p_v_1 = [np.count_nonzero(v1_delta > eps) / num_experiments for eps in epsilon]
fig = plt.figure()
plt.plot(epsilon, hoeffding)
plt.plot(epsilon, p_v_1)
plt.title(r"$\mathcal{P}(|V_1 - \mu_1| > \epsilon)$:")
plt.xlabel(r"$\epsilon$")

vrand_delta = np.abs(np.array(v_rands) - mu)
p_v_rand = [np.count_nonzero(vrand_delta > eps) / num_experiments for eps in
 ↪epsilon]
fig = plt.figure()
```

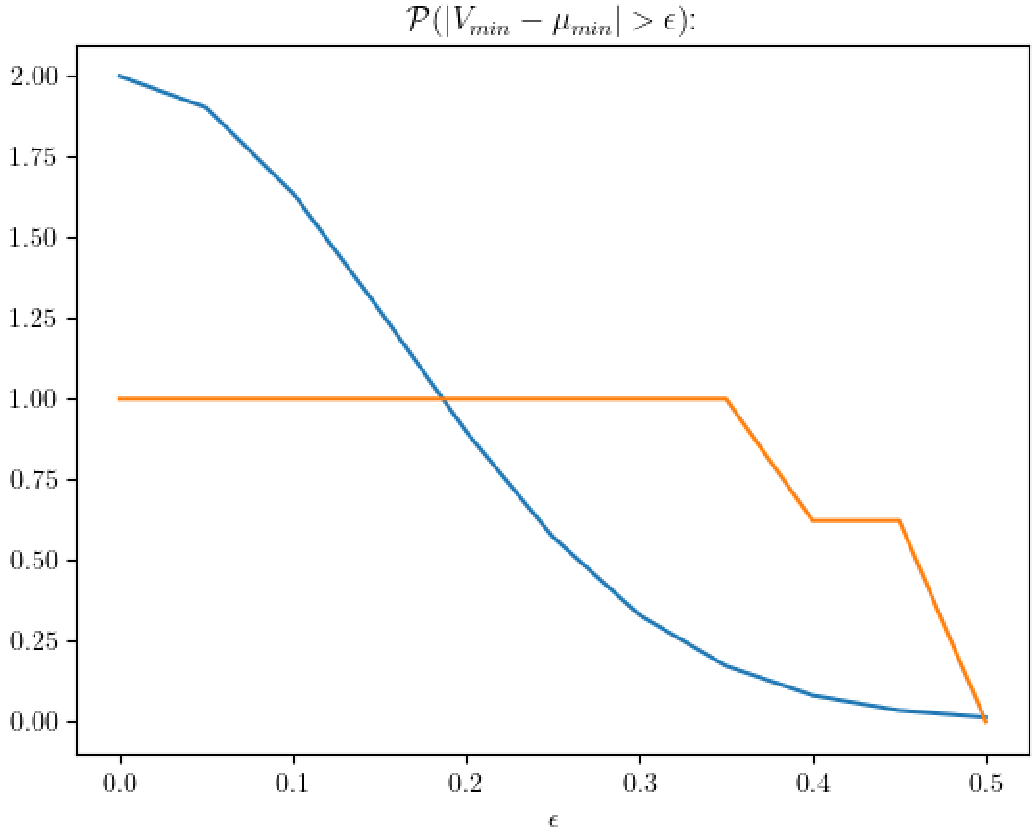
```
plt.plot(epsilon, hoeffding)
plt.plot(epsilon, p_v_rand)
plt.title(r"$\mathcal{P}(|V_{rand} - \mu_{rand}| > \epsilon)$:")
plt.xlabel(r"$\epsilon$")

vmin_delta = np.abs(np.array(v_mins) - mu)
p_v_min = [np.count_nonzero(vmin_delta > eps) / num_experiments for eps in␣
 ↪epsilon]
fig = plt.figure()
plt.plot(epsilon, hoeffding)
plt.plot(epsilon, p_v_min)
plt.title(r"$\mathcal{P}(|V_{min} - \mu_{min}| > \epsilon)$:")
plt.xlabel(r"$\epsilon$")
```

```
[ ]: Text(0.5, 0, '$\\epsilon$')
```





$\mathcal{P}(|V_{rand} - \mu_{rand}| > \epsilon)$:
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.0
0.1
0.2
0.3
0.4
0.5
ϵ



2.0.4 Problem 2d:

Coins V_1 and V_{min} obey Hoeffding's bound; however, V_{min} does not. This is because we are NOT allowed to consider the input/training data when applying the inequality, let alone manipulate it. When we always choose the minimum coin, we are manipulating the underlying distribution of V_{min} which voids Hoeffding's bound.