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Effect of Altruism on Segregation Models

Project Report

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Abstract

This project investigates the role of altruism and relocation policies in agent-based segregation models. Building on the Schelling model, we extend previous work by integrating diverse relocation policies and introducing altruistic agents, who prioritize collective happiness over individual satisfaction. Simulations reveal that altruism influences segregation patterns and convergence times in a policy-dependent manner. Specifically, policies emphasizing similarity exacerbate segregation as altruism increases, while random and equity-focused policies remain stable, and intermediate patterns are observed for others. The "giant catalytic effect" of altruism on segregation noticed in [1] with an altruism percentage of 0.2 was not observed in this broader framework.

Contents

Chapter 1
Introduction

Since the publication of Schelling's model in 1971 [2], the study of segregation dynamics among different populations with varying characteristics has become widespread in sociological and computational research. The model, which demonstrated how individual preferences for similar neighbors can lead to large-scale segregation, has inspired a great number of extensions and variations, as summarized in [3]. These adaptations address a wide range of factors, including income disparities [4], housing policies [5], thermodynamic principles [6], and different definitions of satisfaction [1] to better reflect real-world complexities.

In addition to theoretical extensions, previous works have also attempted to empirically validate the Schelling model using real-world population data [7, 8]. A particularly interesting extension by Jensen et al. [1] explored the concept of altruism, introducing agents who prioritize collective happiness over individual satisfaction. Their approach considered a happiness function defined by neighborhood density and demonstrated how altruistic behavior could influence segregation patterns.

In our project, we extend the segregation model framework by incorporating altruism into a setting with various relocation policies[1]. Specifically, we consider a two-population Schelling model where agents are classified as altruists or egoists (see Section 1.2.3), with their behaviors adapting accordingly. This extension allows us to analyze how altruistic behavior interacts with different policies and decision-making frameworks, offering deeper insights into the segregation process.

Our hypothesis is that altruism, in this broader context, influences the model's convergence time in a policy-dependent manner. While altruism may slow down the segregation process, its effects depend heavily on the specific relocation policy.

This work merges two previous areas of investigation: the role of altruistic behavior explored by Jensen et al. [1] and the impact of relocation policies on agents' satisfaction and neighborhood composition explored by Mauro and Pappalardo [5]. Our contribution lies in integrating these concepts into a unified, generalized

[1]Following the definition of [5], a relocation policy is a strategy or guideline that directly influences the relocation of unhappy agents in a segregation model (cfr. Section 1.2.1).

framework, enabling the study of altruism across a broader range of scenarios. Through this, we aim to deepen our understanding of the mechanisms driving segregation.

1.1 Schelling's Model of Segregation

The original Schelling model [2] is a foundational framework for studying segregation dynamics. Simulates the behavior of agents on a two-dimensional grid, where each cell can be occupied by an agent from one of two groups or remain vacant. The original model operates on the basis of the following parameters:

- **Agent Type:** Each agent belongs to one of two groups (e.g., Group A and Group B).
- **Neighborhood:** Defined as the set of adjacent cells surrounding an agent, typically including up to eight neighbors in a grid.[2]
- **Homophily Threshold:** A parameter that determines the minimum proportion of similar neighbors required for an agent to be satisfied.
- **Relocation Rule:** Dissatisfied agents (those below the homophily threshold) relocate to a vacant random cell.

Over time, the model demonstrates how individual preferences, even for low homophily, can lead to emergent patterns of segregation at the macro-level.

1.2 Model Implementation

This study extends the classic Schelling model of segregation by integrating altruistic agents and diverse relocation policies to explore their combined effects on segregation dynamics. In this work, we introduce two main parameters in the model: different relocation policies and altruism.

1.2.1 Relocation Policies

Relocation policies guide how agents select new locations when they decide to move. These policies strictly apply to egoistic agents, who follow the rules to maximize their individual happiness. For altruistic agents, on the other hand, altruism overwrites the standard relocation policies: they make decisions based on whether the relocation leads to an overall increase in collective happiness, such that $\Delta U > 0$.

[2] Also called "Moore neighborhood".

In our model, we consider the relocation policies proposed by Mauro and Pappalardo in [5]. These policies[3] provide the strategic guidelines for agent relocation, and take into account various factors such as distance, income, neighborhood similarity, and other criteria:

- **Similar Neighborhood:** Agents prioritize vacant cells with a neighborhood similar to their original one in terms of the agent's average income. This policy represents a preference for maintaining economic homogeneity.
- **Different Neighborhood:** Agents prefer vacant cells with a neighborhood that is economically dissimilar to their original.
- **Minimum Improvement Policy:** Agents relocate to cells where they would be happy, ensuring that the minimum satisfaction threshold is met. Cells are scored inversely to the number of agents of the same class, encouraging agents to move to more diverse neighborhoods.
- **Maximum Improvement Policy:** Agents relocate to cells where they would be happy, prioritizing those with the highest number of agents in the same class. This policy reinforces clustering among similar agents.
- **Recently Emptied:** Agents assign higher scores to recently vacated cells, assuming these are in better condition or more desirable due to their recent occupancy. This policy incentivizes the use of spaces that have been made available.
- **Distance-Relevance Policy:** Agents evaluate vacant cells based on their relevance and travel cost, following the Gravity model. This policy combines the desirability of a location with its proximity, favoring closer and more significant locations.
- **Distance:** Agents prioritize cells closest to their current location to minimize travel cost.
- **Empty Surrounded:** Agents prefer cells with the most vacant neighbors.
- **Historically Emptied:** Agents choose historically empty cells.
- **Poor Neighborhood:** Agents move to lower-income neighborhoods.
- **Random:** Agents select cells randomly.
- **Rich Neighborhood:** Agents prefer higher-income neighborhoods.

[3] For more detailed definitions and further explanations, we refer the reader to [5], as an in-depth discussion of these policies goes beyond the scope of this paper.

The policy adaption rate $p \in [0, 1]$ controls the proportion of agents adhering to these relocation policies. The remaining agents move randomly. This flexibility allows for the exploration of mixed-strategy populations and their effects on segregation dynamics. In the following section we propose two new policies based on the historical occupancy of the cells.

1.2.2 Relocation Policies Based on Historical Occupancy

The idea behind these relocation policies is to incorporate the historical occupancy of cells into the decision-making process of agents. By doing so, we aim to capture the impact of historical preferences on current and future relocation decisions. A real-world analogy can be seen in how agents' preferences shape the development of infrastructure in their neighborhoods, reinforcing patterns over time. To explore this concept, we introduce two new policies: the *Similar History Cell Policy* and the *Similar History Neighborhood Policy.*

Both policies rely on the historical occupancies of a cell or its surrounding neighborhood. Let $H_c = \{h_1, h_2, \ldots, h_n\}$ represent the history of occupancies for a cell c, where each h_i corresponds to the type of agent that occupied the cell during a past time step. Additionally, let a denote the type of the current relocating agent.

The *Similar History Cell Policy* evaluates a cell c based on the number of times agents of type a have previously occupied it, using the following formula:

$$\text{Score}_{\text{cell}}(c, a) = \sum_{i=1}^{n} \delta(a, h_i),$$

where $\delta(a, h_i)$ is the Kronecker delta function:

$$\delta(a, h_i) = \begin{cases} 1 & \text{if } a = h_i, \\ 0 & \text{otherwise.} \end{cases}$$

The *Similar History Neighborhood Policy* extends this concept to consider the neighborhood $N(c)$ of cell c, defined as the set of all cells adjacent to c. The score is calculated as the sum of the scores of all cells in the neighborhood:

$$\text{Score}_{\text{neighborhood}}(c, a) = \sum_{c' \in N(c)} \sum_{i=1}^{n} \delta(a, h_i),$$

where $H_{c'} = \{h_1, h_2, \ldots, h_n\}$ represents the historical occupancies of each cell c' in the neighborhood.

These policies enable agents to prefer locations that align with their type's historical presence, thereby reinforcing patterns shaped by past occupancies. This dynamic mimics real-world scenarios where historical preferences influence urban development and social segregation, providing a nuanced way to study the emergent behaviors in such systems. In the subsection 2.2.1 we present the results from the simulations using these two relocation policies.

1.2.3 Altruism

Altruistic agents are introduced as a new type of agent that prioritizes collective well-being over individual satisfaction. At each iteration of the model, default agents, who we call "egoists" evaluate their status solely based on their personal happiness. On the other hand, altruistic agents care about the collective happiness of the system and consider the impact of their movement on the overall satisfaction of their neighborhood. The proportion of altruistic agents in the population is varied across simulations.

Egoists' and Altruists' Happiness Functions

Let $u(a)$ denote the happiness function of an agent a, quantifying their satisfaction, which is determined by their personal neighborhood satisfaction:

$$u(a) = \frac{n_s(a)}{n_t(a)},$$

where:

- $n_s(a)$ is the number of similar agents in the neighborhood of a,
- $n_t(a)$ is the total number of agents in the neighborhood of a.

The collective happiness U of the system is defined as:

$$U = \sum_{a \in A} u(a),$$

where A is the set of all agents. Egoists follow predefined relocation policies to maximize their individual happiness. The happiness change for an egoistic agent a is given by:

$$\Delta u(a) = u_{\text{new}}(a) - u_{\text{current}}(a).$$

Altruistic agents evaluate their relocation based on the change in collective happiness:

$$\Delta U = U_{\text{new}} - U_{\text{current}},$$

where altruistic agents proceed with relocation only if $\Delta U > 0$. Otherwise, they remain in their current location, "sacrificing" for the benefit of the community.

Relocation Decision Rule

At each step:

1. Egoistic agents follow the relocation policy and aim to maximize their individual happiness $\Delta u(a)$.
2. Altruistic agents evaluate the effect of their relocation on the collective happiness ΔU and relocate if and only if $\Delta U > 0$.

This model enables the analysis of how altruistic behavior and relocation policies interact to influence segregation dynamics.

Chapter 2

Simulation & Results

2.1 Simulation Setup

We run our model on a 50×50-grid, with a minority agent proportion of 40% and a grid density of 75%. Following the original Schelling model, we fix the homophily threshold at 3, meaning agents are considered unhappy if fewer than 3 neighbors belong to the same class[1]. We run our model with varying policy acceptance and altruism rates. To ensure robustness, each simulation is run 50 times with random initializations. We present averages and standard deviations of the setups. The parameters and the procedure are chosen in alignment with [5] to enable comparison.

2.2 Results

2.2.1 History Policy Results

To evaluate the performance of the newly introduced *Similar History Cell Policy* and *Similar History Neighborhood Policy*, we conduct simulations comparing them against the established *Maximum Improvement Policy* and *Minimum Improvement Policy*. For each policy, we perform 10 batch runs and measured three key metrics: the number of steps required for the system to converge, the percentage of agents that were happy at the end, and the segregation measure at the final step. Convergence is defined as reaching a stable state within a maximum of 100 steps.

The results, summarized in Table 2.1, reveal interesting patterns. The *Maximum Improvement Policy* consistently achieved full convergence in an average of 4.2 steps, with all agents ending in a happy state and a segregation measure of 0.746. Conversely, the *Minimum Improvement Policy* required significantly more steps to converge, with an average of 7.6 steps, and produced a lower segregation measure of 0.548, despite also achieving 100% happiness.

[1] Schelling [2] showed that this low value is enough to observe significant segregation.

The *Similar History Neighborhood Policy* shows promising results, achieving convergence within an average of 9.2 steps. While it does not result in universal happiness, with only 69.3% of agents happy at the end, it produces a relatively moderate segregation measure of 0.693. This indicates that the policy promotes some degree of integration but struggles to satisfy all agents fully. In contrast, the *Similar History Cell Policy* fails to converge within the allotted 100 steps across all runs. With only 66.1% of agents happy at the end and a low segregation measure of 0.397, this policy demonstrates limited effectiveness under the given parameters.

Policy	Steps to Converge		Segregation		Percentage Happy	
	Mean	Std	Mean	Std	Mean	Std
Maximum Improvement	4.2	0.837	0.746	0.006	1.000	0.000
Minimum Improvement	7.6	4.037	0.548	0.002	1.000	0.000
Similar History Cell	100.0	0.000	0.397	0.006	0.661	0.009
Similar History Neighborhood	9.2	1.643	0.693	0.008	0.693	0.000

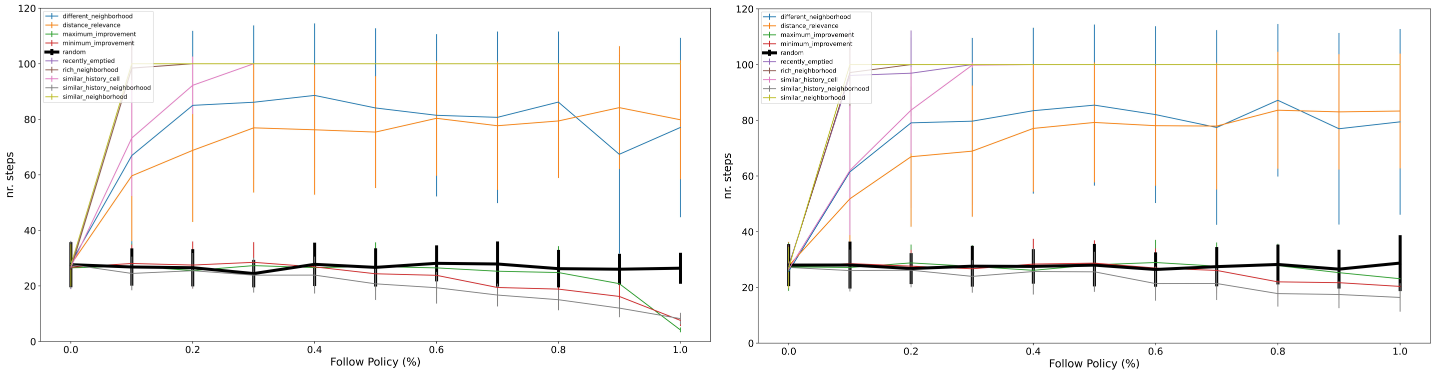
Table 2.1: Performance metrics for different relocation policies.

These results highlight the trade-offs inherent in the different policies. The *Similar History Neighborhood Policy* achieves a balance between segregation and agent satisfaction, albeit requiring more steps to converge. On the other hand, the *Similar History Cell Policy* struggles to guide the system to convergence, likely due to its localized focus on individual cells rather than broader neighborhood patterns.

2.2.2 Comparison of Relocation Policies for Small Degrees of Altruism

To find out whether the "giant" effect of altruism on segregation discovered in [1] also shows in a more general framework, we set the proportion of altruistic agents to 20 percent.

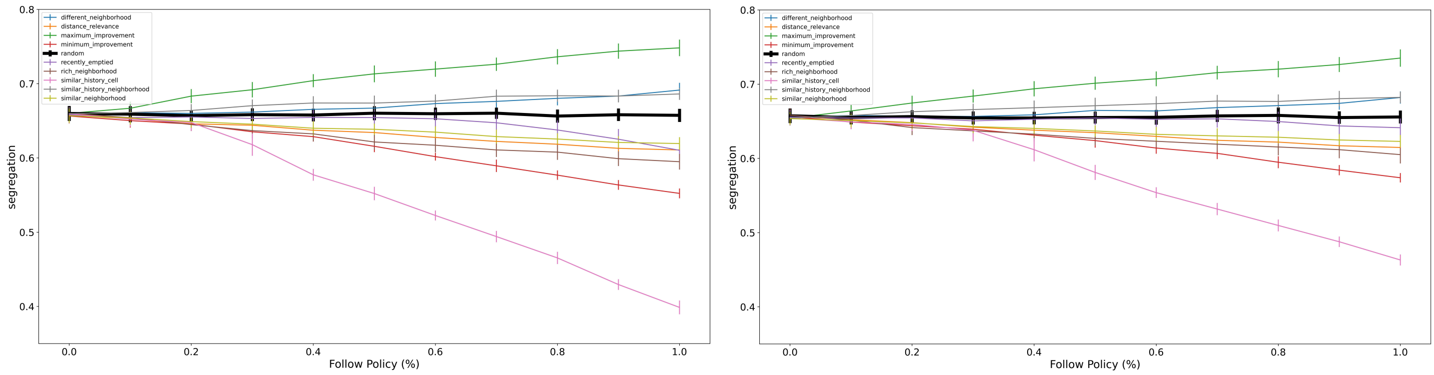
Figure 2.2a and Figure 2.2b show the number of steps needed to converge for different policy acceptance rates. First, we see that the graphs are very similar, although the standard deviations for 20% altruism are slightly higher. The policies can be clustered into three different subgroups. The first subgroup behaves similar to the random policy and is formed by *Minimum Improvement, Maximum Improvement* and *Similar History Cell.* Only for high policy acceptance rates ($p \geq 0.7$), there is a significant delay in convergence time. The second cluster consists of *Distance Relevance* and *Different Neighborhood.* They still



(a) 0% altruism (b) 20% altruism

Figure 2.1: Number of Steps Before Convergence

converge, but at a lower rate. On average, they converge faster for small policy acceptance rates ($p \leq 0.5$) and are more or less the same from there on. However, because of their high standard deviation, we cannot infer a significant change. The remaining policies do not converge before 100 steps for ($p \geq 0.1$). However, with 20% altruism, the *Recently Emptied* policy does converge for ($p \leq 0.2$). This seems to be an indicator that altruism can influence convergence time in that specific case, although further computational resources would be required for validation.



(a) 0% altruism (b) 20% altruism

Figure 2.2: Final Segregation Rate

However, the final segregation rate is smoothed out by altruism, which means that an increasing degree of policy acceptance increases or decreases the final segregation rate more slowly. Moreover, the kink in the *Similar History Cell* policy is delayed by 0.1. In general, the results look similar for 10% and 30 % altruism.

2.2.3 Altruism Results

The results presented are based on multiple runs of our Schelling model, comparing different relocation policies under varying levels of altruism. Each simulation employs a specific policy that determines how agents choose their relocation sites, considering factors such as similarity, wealth, or randomness. The outcomes are evaluated using two key metrics: the degree of segregation achieved and the number of steps required for the system to reach equilibrium. To ensure computational feasibility, the number of steps per simulation is capped at a maximum of 100. These simulations provide insights into how altruism interacts with relocation policies to influence patterns of segregation and equilibrium dynamics.

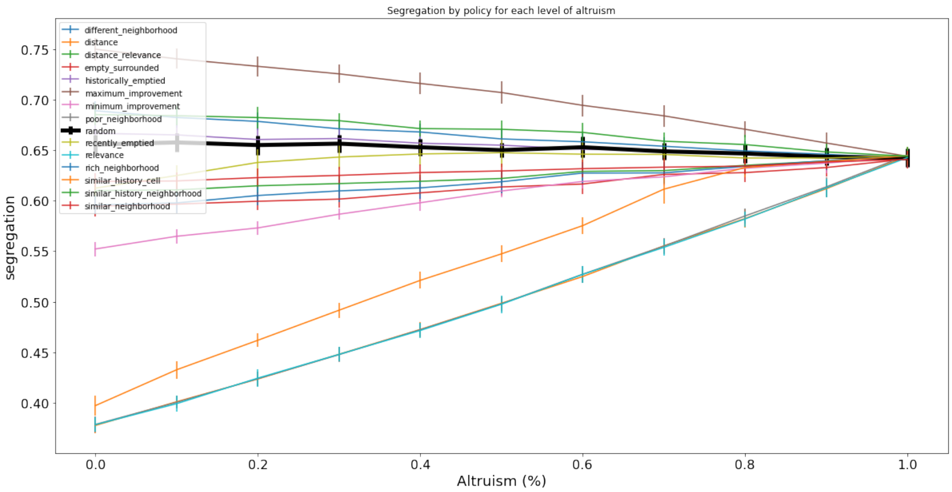


Figure 2.3: The impact of different relocation policies on segregation across various levels of altruism, ranging from 0% to 100%.

As shown in Figure 2.3 policies emphasizing similarity (*similar_neighborhood*, *similar_history_cell*) exhibit a strong positive correlation between altruism and segregation. As individuals become more altruistic, they tend to cluster into homogenous groups, exacerbating segregation. Conversely, policies like *random* and *poor_neighborhood* remain largely unaffected by altruism, maintaining relatively stable segregation levels. Policies such as *distance* and *rich_neighborhood* exhibit intermediate patterns, indicating that altruism partially influences segregation but not as dramatically as similarity-based approaches.

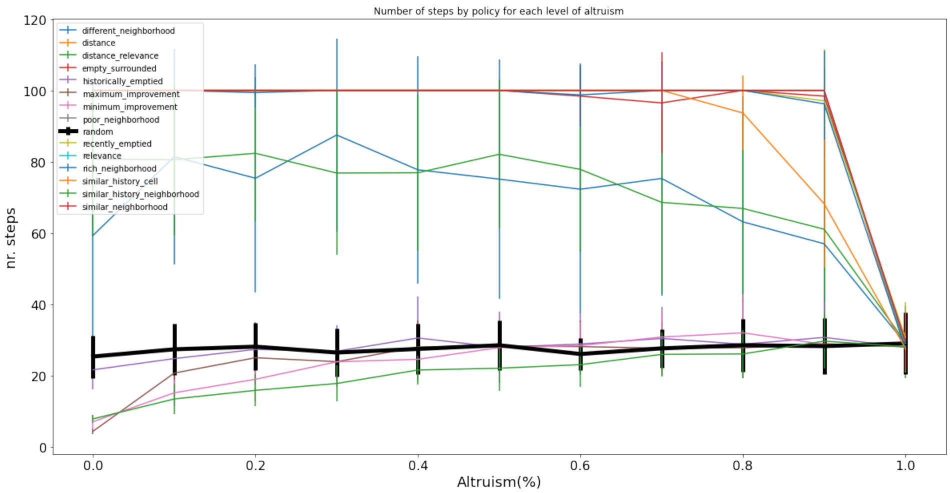


Figure 2.4: The number of simulation steps required to achieve equilibrium under different relocation policies across varying levels of altruism. The simulation was capped at 100 steps, meaning that if equilibrium was not reached within this limit, the simulation stopped regardless.

As shown in Figure 2.4 the number of steps required to reach equilibrium varies widely across policies. Policies like *maximum_ improvement* consistently require a high number of steps. In contrast, *random* and *recently_ emptied* policies settle quickly, suggesting less structured or constrained relocation decisions. Interestingly, the variance in steps for many policies increases with altruism, indicating greater unpredictability in equilibrium dynamics at higher altruism levels.

Chapter 3

Conclusions

In this chapter, we summarize our results that urge us to reject the research hypothesis. We discuss the behavior of history policies and make a remark on the the effect of altruism on the final segregation rate.

3.1 Result Discussion

History Policies

The results highlight the contrasting outcomes of the Similar History Cell and Similar History Neighborhood Policies. The Similar History Neighborhood Policy performed well, converging in an average of 9.2 steps with full happiness (100%) among agents. In contrast, the Similar History Cell Policy failed to converge within the 100-step limit across all simulations. These results indicate that the Similar History Neighborhood Policy is effective at achieving both high agent satisfaction and moderate segregation, though it requires more time to converge. On the other hand, the Similar History Cell Policy's limitations highlight the challenge of focusing on small-scale interactions, which may prevent the system from reaching an optimal state. Future work could explore adjustments to these policies to improve their effectiveness and convergence behavior.

The Influence of Altruism

The results from [1] could not be reproduced or generalized with our model. We argue that this is due to their tailor-made definition of happiness and altruism, where agents want to live with some but not too many other agents, regardless of their type. According to our work, it can at least not easily be applied in a broader framework. We did not observe any "giant catalytic effect" on segregation with a small percentage of altruistic agents. Neither with respect to figure 2.4, we can infer that altruism influences the model's convergence time in a policy-dependent manner. Therefore, we need to reject our research hypothesis. However, in figure 2.3, we found a policy-dependent affine-linear relationship between altruism and final segregation if the policy acceptance rate is 1. Future works might deepen the understanding of this phenomenon.

3.2 Model Limitations and Extensions

While the presented work provides insights into the role of altruism and relocation policies in segregation dynamics, our results have several limitations.

As first, we define altruism simplistically, focusing only on collective happiness, but real-world altruism is often context-dependent, influenced by social, cultural, or psychological factors. Future work could explore more complex definitions, such as conditional altruism or reciprocity. The abstractness of our model also limits its applicability to real-world urban planning, where socio-economic constraints, heterogeneous preferences, and institutional interventions are significant. Incorporating empirical data and testing in realistic settings could improve relevance. Currently, agents adhere to single relocation policies, but introducing mixed strategies influenced by personal circumstances or policy incentives could add depth. Additional policies incorporating environmental factors, access to services, fairness, or inequality reduction could enhance the model. Lastly, addressing over-simplistic assumptions, such as static grids and uniform agents, by incorporating economic constraints, policy interventions, and networked neighborhoods, could improve the model's applicability.

We propose expanding the definition of altruism to include social and cultural dimensions, testing the model with real-world data to evaluate its predictive validity, and introducing mixed-policy strategies within the agent population. Additionally, we propose developing new relocation policies that incorporate fairness, sustainability, and long-term goals, exploring the use of machine learning for agent decision-making, and extending the model to include dynamic grids, economic constraints, and network effects.

By addressing these limitations, the model can be extended to better capture the complexity of real-world segregation dynamics and provide more actionable insights for urban planners and policymakers.

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