Spring Semester CAPE Laboratory 2025 - 2026

Assignment - 3

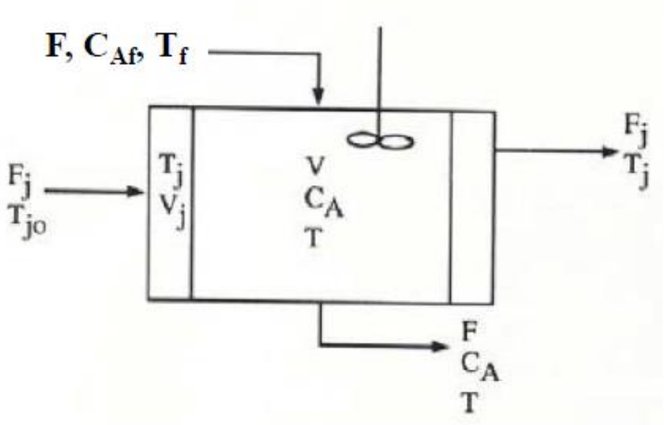
This Assignment contains <u>two problems</u>. Solve both the problems.

<u>Problem-1A</u>

<u>Objective</u>: Solution of Ordinary Differential Equations: Initial Value Problems

Consider the perfectly mixed CSTR where a first-order exothermic irreversible reaction takes place (r = rate of reaction). Heat generated by reaction is being removed by the jacket fluid. The reactor volume (V) is constant.

$$A \rightarrow B, \quad r = k_0 e^{\left(\frac{-E}{RT}\right)} C_A$$



<u>Governing Equations</u>:

(Subscript j indicates parameters related to jacket. Symbols carry their usual significance. Refer to the figure.)

$$V\frac{dC_A}{dt} = FC_{Af} - FC_A - rV$$

$$\rho C_p V\frac{dT}{dt} = \rho C_p F\left(T_f - T\right) + \left(-\Delta H\right)Vr - UA\left(T - T_j\right)$$

$$\rho_j C_j V_j \frac{dT_j}{dt} = \rho_j C_j F_j\left(T_{j0} - T_j\right) + UA\left(T - T_j\right)$$

Cont'd

Model Parameter Values:

Parameter	Value	Parameter	Value
F (m^3/h)	1	C_{Af} (kgmol/m^3)	10
V (m^3)	1	UA (kcal/°C h)	150
k_0 (h^{-1})	36×10^6	T_{j0} (K)	298
$(-\Delta H)$ (kcal/kgmol)	6500	$(\rho_j C_j)$ (kcal/m^3 °C)	600
E (kcal/kgmol)	12000	F_j (m^3/h)	1.25
(ρC_p) (kcal/m^3 °C)	500	V_j (m^3)	0.25
T_f (K)	298		

There are three steady states for this system and you should have identified all the three steady states in Assignment 2. Now study the dynamic behaviour of the system by solving the above ODEs as follows.

First consider any one steady state that you have obtained. Now obtain 3 different initial conditions by perturbing the selected steady state by 1%, 5%, and 25%. Simulate the system with these three different initial conditions for time t = 0 to t = 50 and plot the three state variables vs time. Does the system go to new steady state? Or does it return to the same steady state? How much time it takes to reach the steady state? Comment on the stability of each steady state. **Repeat for other two steady states.**

Write your own code implementing **4th order Runge Kutta method** and compare your results using MATLAB function `ode45`. Analyse the effect of step size of your RK-4 method on the accuracy of your solution.

Problem-1B **(Stiff Differential Equations)**

Solve the following system of ODEs using both ode45 and ode15s. Comment on their efficiency.

$$\frac{dy_1}{dt} = -5y_1 + 3y_2$$

$$\frac{dy_2}{dt} = 100y_1 - 301y_2$$

$$y_1(0) = 52.29 \text{ and } y_2(0) = 83.82,$$

Cont'd

Problem-2

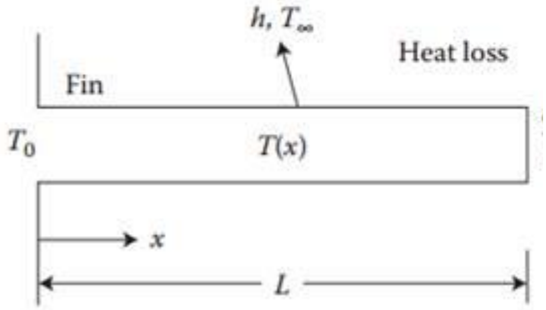
Objective: Numerical solution of Ordinary Differential Equation: Boundary Value Problem

Consider the steady-state heat transfer in a fin of uniform cross-section as shown below. The thermo-physical properties of the fin material are constant. Find the temperature along the length of the fin $T(x)$ using

(a) Finite Difference Method (write your own code)

(b) Shooting Method (write your own code)

(c) MATLAB function `bvp4c`



The following BVP represents the governing equation for the fin.

$$\frac{d^2T}{dx^2} - \alpha(T - T_\infty) - \beta(T^4 - T_\infty^4) = 0, \; T(x=0) = T_0, \; T(x=L) = T_L$$

Given: $T_0 = 300$, $T_L = 400$, $T_\infty = 200$, $L = 10$, $\alpha = 0.05$, $\beta = 2.7\times 10^{-9}$ (in appropriate units)

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