a) Muestra que $r_{xy}(t) = r_{yx}(-t)$

$$r_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)\, y(\tau)\, d\tau$$

$$r_{yx}(t) = \int_{-\infty}^{\infty} y(t+\tau)\, x(\tau)\, d\tau$$

$$r_{yx}(-t) = \int_{-\infty}^{\infty} y(-t+\tau)\, x(\tau)\, d\tau \qquad \text{con } u = -t+\tau \;\Rightarrow\; \tau = u+t$$

$$= \int_{-\infty}^{\infty} y(u)\, x(u+t)\, du$$

$\therefore \quad r_{xy}(t) = r_{yx}(-t)$

b) Muestre que $r_{xx}(t) = r_{xx}(-t)$.

$$r_{xx}(t) = \int_{-\infty}^{\infty} x(t+\tau)\, x(t)\, d\tau$$

$$r_{xx}(-t) = \int_{-\infty}^{\infty} x(-t+\tau)\, x(\tau)\, d\tau \;, \qquad \text{haciendo } u = -t+\tau,\; \tau = t+u$$

$$= \int_{-\infty}^{\infty} x(u)\, x(t+u)\, du$$

$\therefore \; r_{xx}(t) = r_{xx}(-t)$ y la autocorrelación es par.

c) Sean las señales, $x(t)$ y $y = x(t+T)$. Expresa $r_{xy}(t)$ y $r_{yy}(t)$ en términos de $r_{xx}(t)$

$$r_{xx}(t) = \int_{-\infty}^{\infty} x(t+\tau)\, x(\tau)\, d\tau$$

$$r_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)\, y(\tau)\, d\tau = \int_{-\infty}^{\infty} x(t+\tau)\, x(\tau+T)\, d\tau$$

$$r_{yy}(t) = \int_{-\infty}^{\infty} y(t+\tau)\, y(\tau)\, d\tau = \int_{-\infty}^{\infty} x(t+\tau+T)\, x(\tau+T)\, d\tau$$

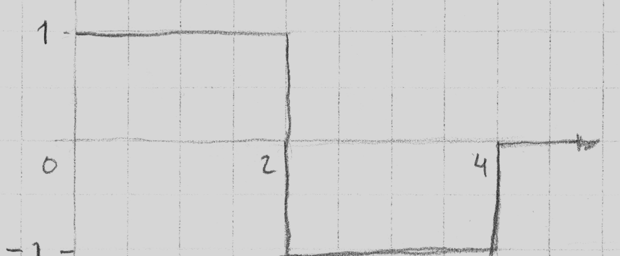
$$r_{yx}(t) = r_{xx}(t+T) = \int_{-\infty}^{\infty} x(t+T+\tau)\, x(\tau)\, d\tau$$

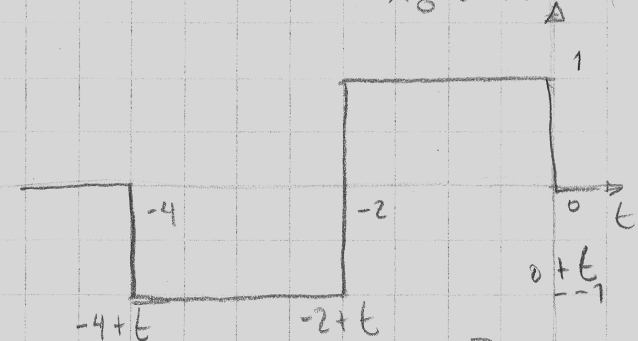
$$r_{yy}(t) = \int_{-\infty}^{\infty} x(t+\tau+T)\, x(\tau+T)\, d\tau$$

d) Realiza la autocorrelación de $x_0 = u(t) - 2u(t-2) + u(t-4)$

$x_0(t) = [u(t) - u(t-2)] - [u(t-2) - u(t-4)]$ $\quad r_{x_0 x_0}(t) = x_0(t) * x_0(-t)$

$x_0(-t) =$

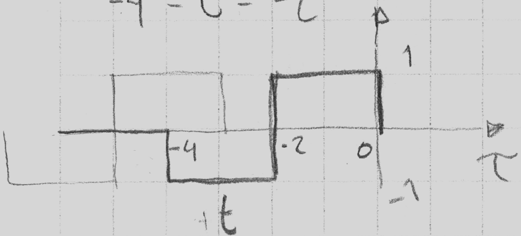




$y(t) = -[u(t+4) - u(t+2)] + [u(t+2) - u(t+4)]$

Caso 1

$-4 \le t \le -2$

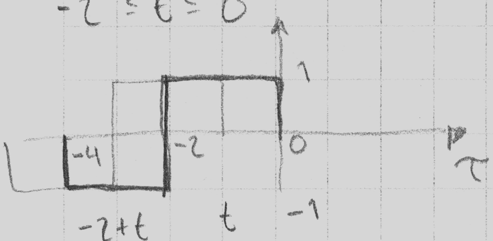


$$\int_{-4}^{t} (1)(-1)\,d\tau = -[\tau]_{-4}^{t}$$

$$= -t - 4$$

Caso 2

$-2 \le t \le 0$



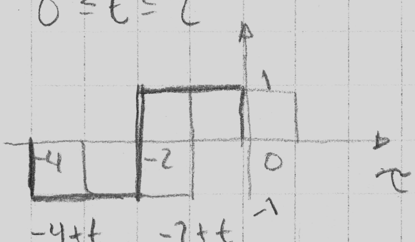
$$\int_{-4}^{-2+t} (-1)(-1)\,d\tau + \int_{-2+t}^{-2} (1)(-1)\,d\tau + \int_{-2}^{t} (1)(1)\,d\tau$$

$$= [\tau]_{-4}^{-2+t} - [\tau]_{-2+t}^{-2} + [\tau]_{-2}^{t}$$

$$= 2 + t + t + t + 2 = 3t + 4$$

Caso 3

$0 \le t \le 2$



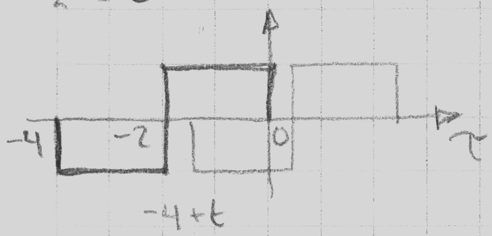
$$\int_{-4+t}^{-2} (-1)(-1)\,d\tau + \int_{-2}^{-2+t} (-1)(1)\,d\tau + \int_{-2+t}^{0} (1)(1)\,d\tau$$

$$= [\tau]_{-4+t}^{-2} - [\tau]_{-2}^{-2+t} + [\tau]_{-2+t}^{0}$$

$$= 2 - t - t + 2 - t = -3t + 4$$

Caso 4

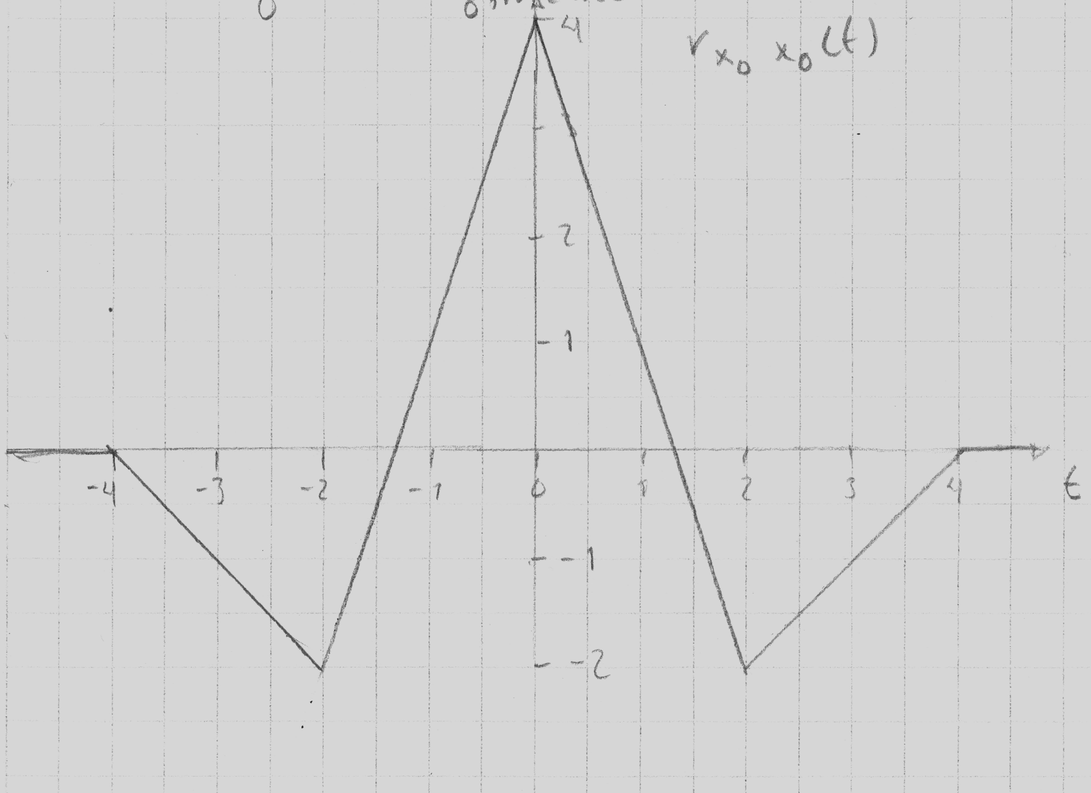
$2 \le t \le 4$



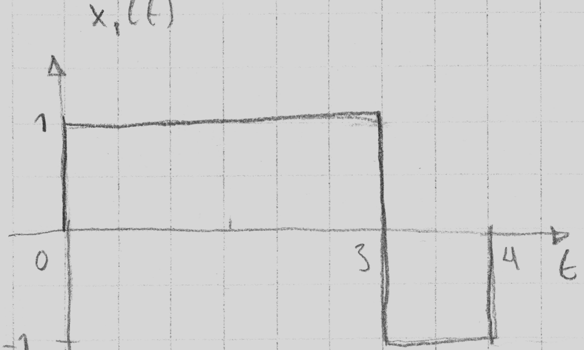
$$\int_{-4+t}^{0} (-1)(1)\,d\tau = -[\tau]_{-4+t}^{0}$$

$$= -4 + t$$

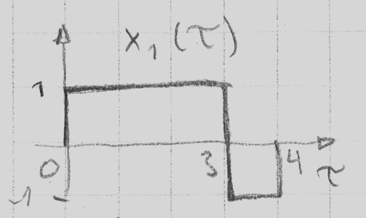
$$r_{x_0 x_0}(t) = \begin{cases} -t-4 & -4 \le t < 2 \\ 3t+4 & -2 \le t \le 0 \\ -3t+4 & 0 \le t \le 2 \\ t-4 & 2 \le t \le 4 \\ 0 & \text{otro caso} \end{cases}$$

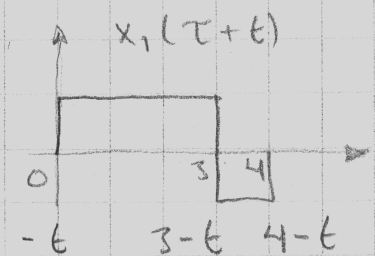


e) Realiza la autocorrelación de $x_1(t) = u(t) - 2u(t-3) + u(t-4)$



$$r_{x_1 x_1} = \int_{-\infty}^{\infty} x_1(t+\tau)\, x_1(\tau)\, d\tau$$

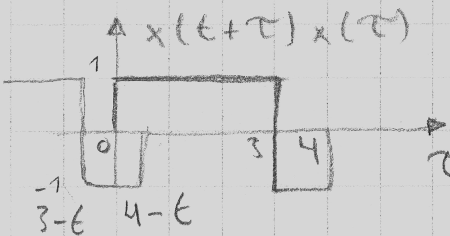




Caso ①

$$-1 \le 3-t \le 0$$
$$-4 \le -t \le -3$$
$$4 \ge t \ge 3$$

$$\int_0^{4-t} (-1)(1)\,d\tau = \Big[-\tau\Big]_0^{4-t}$$



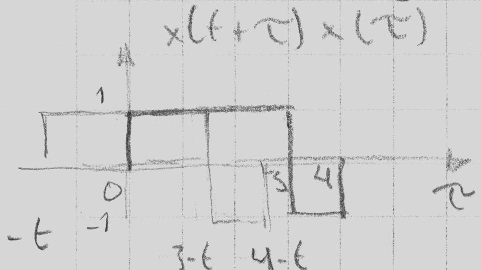
$$= -4+t$$

$$4-t-3+t$$

Caso ②

$$0 \le -3-t \le 2$$
$$-3 \le -t \le -1$$
$$3 \ge t \ge 1$$

$$\int_0^{3-t} (1)(1)\,d\tau + \int_{3+t}^{4-t} (-1)(1)\,d\tau$$



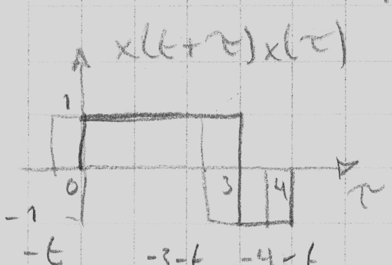
$$= \Big[\tau\Big]_0^{3-t} + \Big[-\tau\Big]_{3-t}^{4-t}$$

$$= 3-t-4+t+3-t = -t+2$$

Caso ③

$$2 \le -3-t \le 3$$
$$-1 \le -t \le 0$$
$$1 \ge t \ge 0$$

$$\int_0^{3-t} (1)(1)\,d\tau + \int_{3-t}^{3} (-1)(1)\,d\tau + \int_3^{4-t} (-1)(-1)\,d\tau$$



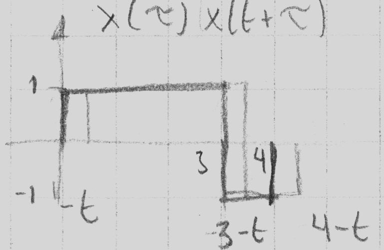
$$= \Big[\tau\Big]_0^{3-t} - \Big[\tau\Big]_{3-t}^{3} + \Big[\tau\Big]_3^{4-t}$$

$$= 3-t-\Big[3-3+t\Big]+4-t-3 = -t+4$$

Caso 4

$3 \le -3-t \le 4$
$0 \le -t \le 1$
$0 \ge t \ge -1$

$$\int_{-t}^{3} (1)(1)\,d\tau + \int_{3}^{3-t} (-1)(1)\,d\tau + \int_{3-t}^{4} (-1)(-1)\,d\tau$$



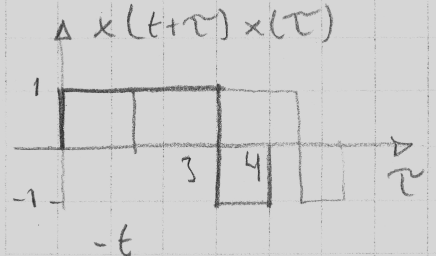
$$= [\tau]_{-t}^{3} - [\tau]_{3}^{3-t} + [\tau]_{3-t}^{4}$$

$$= 3+t + t + 4 - 3 + t = 4 + 3t$$

Caso 5

$1 \le -t \le 3$
$-1 \ge t \ge -3$

$$\int_{-t}^{3} (1)(1)\,d\tau + \int_{3}^{4} (1)(-1)\,d\tau$$

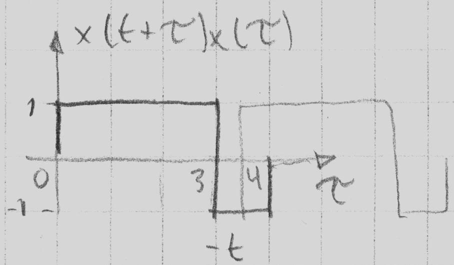


$$= [\tau]_{-t}^{3} - [\tau]_{3}^{4} = 3+t-1 = t+2$$

Caso 6

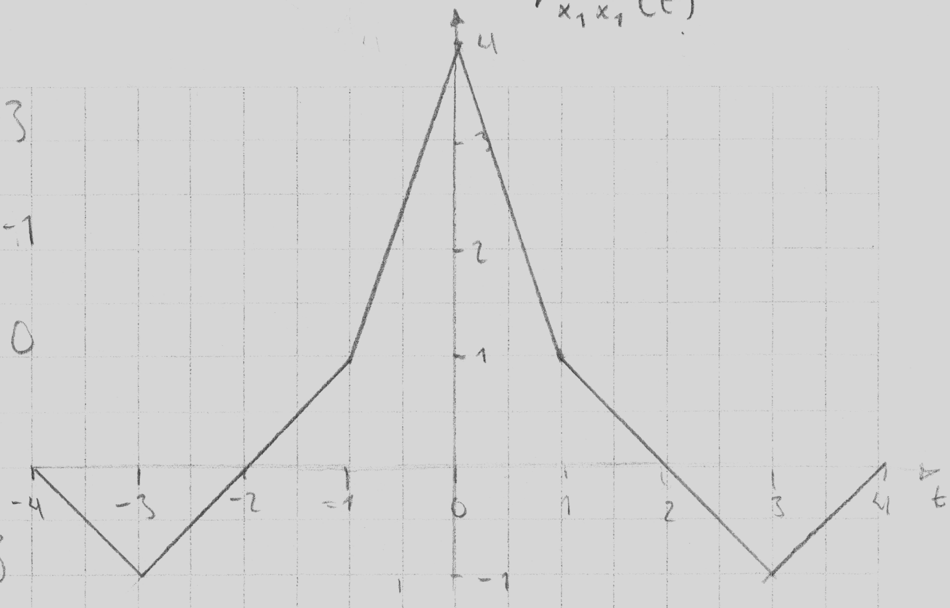
$3 \le -t \le 4$
$-3 \ge t \ge -4$

$$\int_{-t}^{4} (-1)(1)\,d\tau = -[\tau]_{-t}^{4}$$



$$= -t-4$$

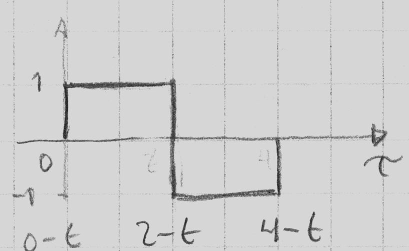
$$r_{x_1x_1}(t)=\begin{cases}-4-t, & -4\le t\le -3\\ t+2, & -3\le t\le -1\\ 4+3t, & -1\le t\le 0\\ -t+4, & 0\le t\le 1\\ -t+2, & 1\le t\le 3\\ -4+t, & 3\le t\le 4\\ 0 & \text{otro caso}\end{cases}$$



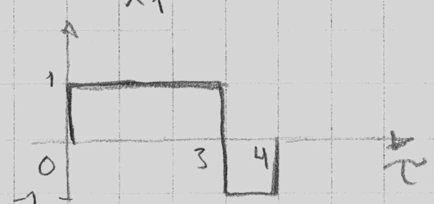
f) Realiza la correlación de $r_{x_0x_1}(t)$ $\cancel{r_{x_0x_1}(t)}$, con las señales definidas en los ejercicios anteriores

- $$r_{x_0x_1}(t)=\int_{-\infty}^{\infty}x_0(t+\tau)\,x_1(\tau)\,d\tau$$

$x_0(t+\tau)$



$x_1(\tau)$



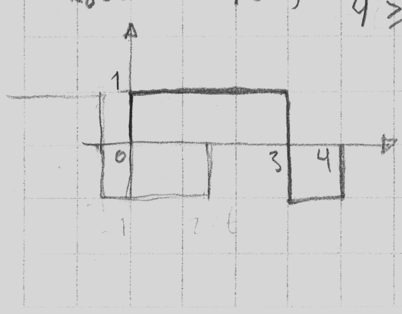
Caso 1

$0\le 4-t\le 2$

$-4\le -t\le -2$

$4\ge t\ge 2$

$x_0(t+\tau)\,x_1(\tau)$



$$\int_0^{4-t}(-1)(1)\,d\tau=-\left[\tau\right]_0^{4-t}$$

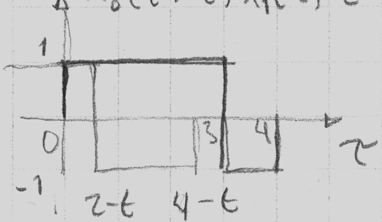
$$=-4+t$$

Caso 2

$2 \le 4-t \le 3$

$-2 \le -t \le -1$

$x_0(t+\tau)\,x_1(\tau)$ $\quad 2 \ge t \ge 1$



$$\int_0^{2-t} (1)(1)\,d\tau + \int_{2-t}^{4-t} (-1)(1)\,d\tau$$

$$= [\tau]_0^{2-t} - [\tau]_{2-t}^{4-t} = 2-t-2 = -t$$

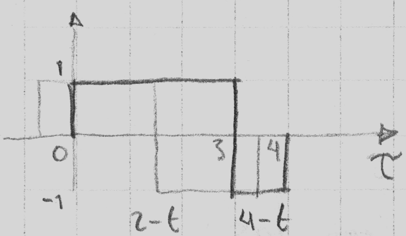
Caso 3

$3 \le 4-t \le 4$

$-1 \le -t \le 0$

$1 \ge t \ge 0$

$x_0(t+\tau)\,x_1(\tau)$



$$\int_0^{2-t} (1)(1)\,d\tau + \int_{2-t}^{3} (-1)(1)\,d\tau + \int_3^{4-t} (-1)(-1)\,d\tau$$

$$= [\tau]_0^{2-t} - [\tau]_{2-t}^{3} + [\tau]_3^{4-t}$$

$$= 2-t-1-t+1-t = -3t+2$$

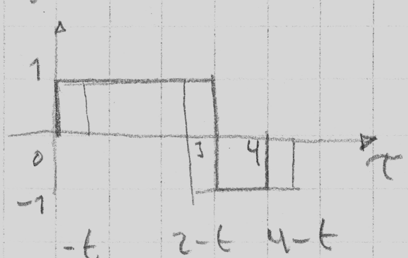
Caso 4

$4 \le 4-t \le 5$

$0 \le -t \le 1$

$0 \ge t \ge -1$

$x_0(t+\tau)\,x_1(\tau)$



$$\int_{-t}^{2-t} (1)(1)\,d\tau + \int_{2-t}^{3} (-1)(1)\,d\tau + \int_3^{4} (-1)(-1)\,d\tau$$

$$= [\tau]_{-t}^{2-t} - [\tau]_{2-t}^{3} + [\tau]_3^{4}$$

$$= 2-1-t+1 = -t+2$$

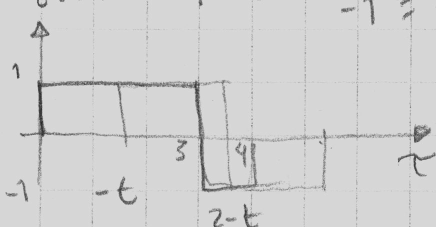
Caso 5

$3 \le 2-t \le 4$

$1 \le -t \le 2$

$-1 \ge t \ge -2$

$x_0(t+\tau)\,x_1(\tau)$



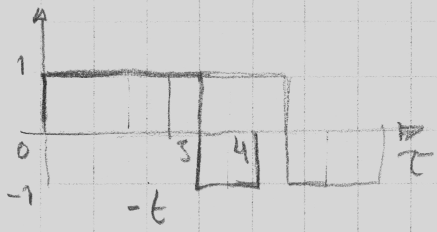
$$\int_{-t}^{3} (1)(1)\,d\tau + \int_{3}^{2-t} (1)(-1)\,d\tau + \int_{2-t}^{4} (-1)(-1)\,d\tau$$

$$= [\tau]_{-t}^{3} - [\tau]_{3}^{2-t} + [\tau]_{2-t}^{4}$$

$$= 3+t+1+t+2+t = 3t+6$$

Caso 6 $\quad 2 \le -t \le 3$

$x_0(t+\tau)\,x_1(\tau) \quad -2 \ge t \ge -3$



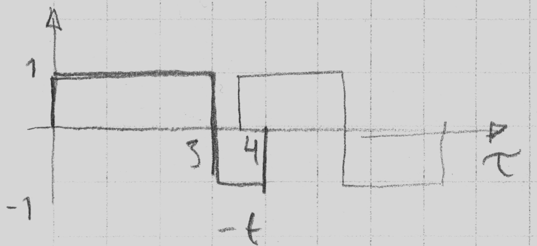
$$\int_{-t}^{3}(1)(1)\,d\tau + \int_{3}^{4}(1)(-1)\,d\tau$$

$$= [\tau]_{-t}^{3} - [\tau]_{3}^{4} = 3+t-1 = t+2$$

$3+t$

Caso 7 $\quad 3 \le -t \le 4$

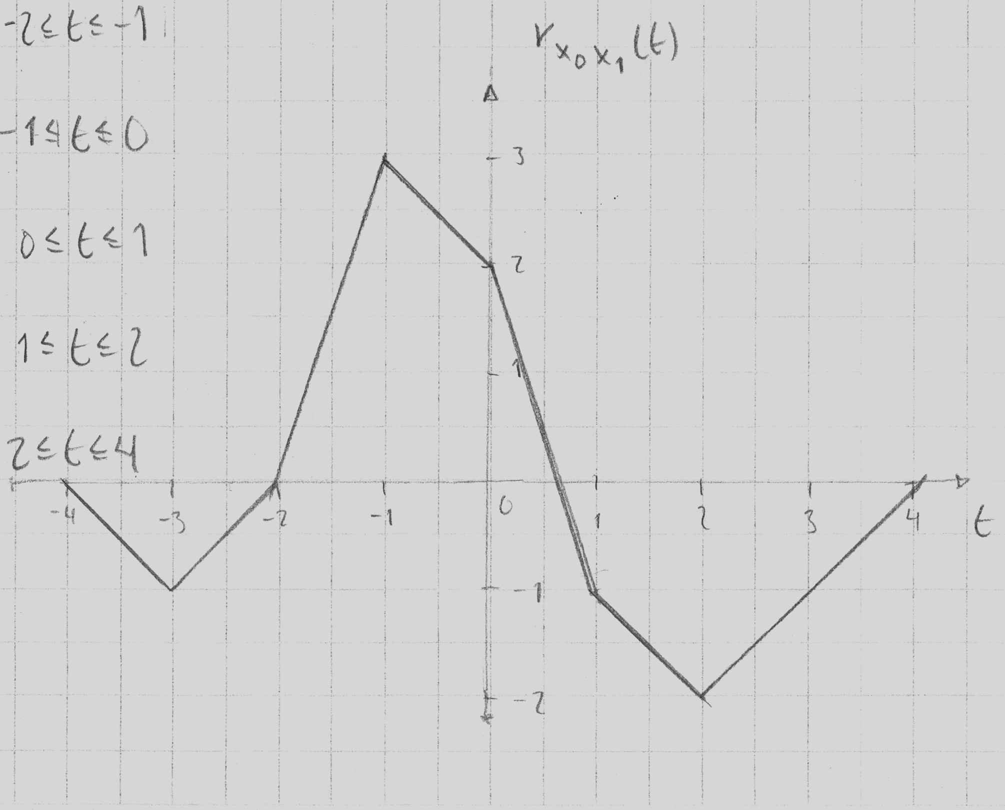
$x_0(t+\tau)\,x_1(\tau) \quad -3 \ge t \ge -4$



$$\int_{-t}^{4}(1)(-1)\,d\tau = -[\tau]_{-t}^{4}$$

$$= -4-t$$

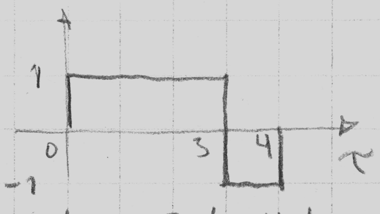
$$r_{x_0x_1}(t) = \begin{cases} -4-t, & -4 \le t \le 3 \\ t+2, & -3 \le t \le -2 \\ 3t+6, & -2 \le t \le -1 \\ -t+2, & -1 \le t \le 0 \\ -3t+2, & 0 \le t \le 1 \\ -t, & 1 \le t \le 2 \\ -4+t & 2 \le t \le 4 \\ 0 & \text{otro caso} \end{cases}$$



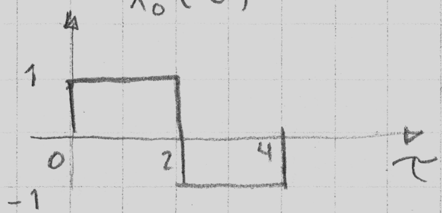
g) Realiza la correlación $r_{x_1 x_0}(t)$ de las señales definidas en los ejercicios anteriores

$$r_{x_1 x_0} = \int_{-\infty}^{\infty} x_1(t+\tau)\, x_0(\tau)\, d\tau$$

$x_1(t+\tau)$



$x_0(\tau)$



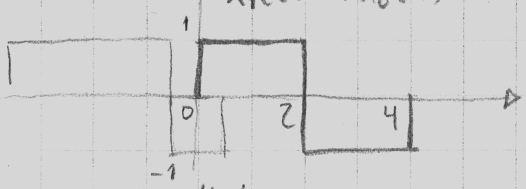
Caso ①

$0 \le 4-t \le 1$
$-4 \le -t \le -3$
$4 \ge t \ge 3$

$$\int_0^{4-t} (-1)(1)\, d\tau = -[\tau]_0^{4-t}$$

$$= -4+t$$

$x_1(t+\tau)\, x_0(\tau)$

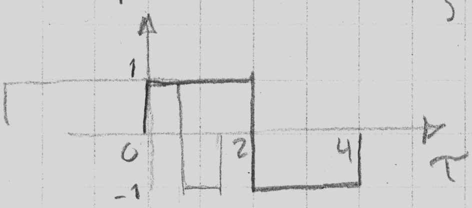


Caso ②

$1 \le 4-t \le 2$
$-3 \le -t \le -2$
$3 \ge t \ge 2$

$$\int_0^{3-t} (1)(1)\, d\tau + \int_{3-t}^{4-t} (-1)(1)\, d\tau$$

$x_1(t+\tau)\, x_0(\tau)$



$$= [\tau]_0^{3-t} - [\tau]_{3-t}^{4-t}$$

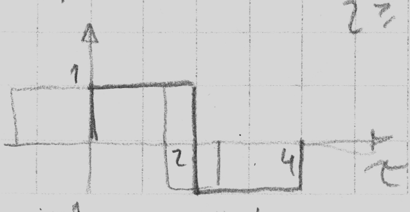
$$= 3-t-1 = 2-t$$

Caso ③

$2 \le 4-t \le 3$
$-2 \le -t \le -1$
$2 \ge t \ge 1$

$x_1(t+\tau)\, x_0(\tau)$

$$\int_0^{3-t} (1)(1)\, d\tau + \int_{3-t}^{2} (-1)(1)\, d\tau + \int_2^{4-t} (-1)(-1)\, d\tau$$



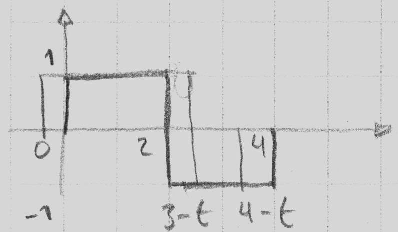
$$= [\tau]_0^{3-t} - [\tau]_{3-t}^{2} + [\tau]_2^{4-t}$$

$$= 3-t+1-t+2-t = -3t+6$$

Caso (4)

$3 \le 4-t \le 4$
$-1 \le -t \le 0$
$1 \ge t \ge 0$

$x_1(t+\tau)\,x_0(\tau)$



$$\int_0^2 (1)(1)\,d\tau + \int_2^{3-t} (1)(-1)\,d\tau + \int_{3-t}^{4-t} (-1)(-1)\,d\tau$$

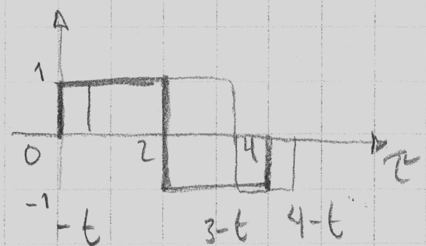
$$= [\tau]_0^2 - [\tau]_2^{3-t} + [\tau]_{3-t}^{4-t}$$

$$= 2 - 1 + t + 1 = 2 + t$$

Caso (5)

$4 \le 4-t \le 5$
$0 \le -t \le 1$
$0 \ge t \ge -1$

$x_1(t+\tau)\,x_0(\tau)$



$$\int_{-t}^2 (1)(1)\,d\tau + \int_2^{3-t} (1)(-1)\,d\tau + \int_{3-t}^{4} (-1)(-1)\,d\tau$$

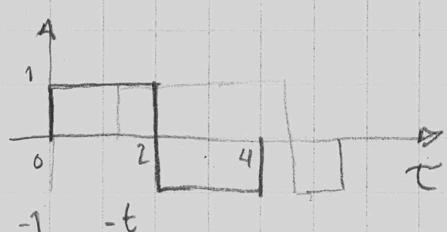
$$= [\tau]_{-t}^2 - [\tau]_2^{3-t} + [\tau]_{3-t}^{4}$$

$$= 2 + t - 1 + t + 1 + t = 3t + 2$$

Caso (6)

$1 \le -t \le 2$
$-1 \ge t \ge -2$

$x_1(t+\tau)\,x_0(\tau)$



$$\int_{-t}^2 (1)(1)\,d\tau + \int_2^{4} (1)(-1)\,d\tau$$

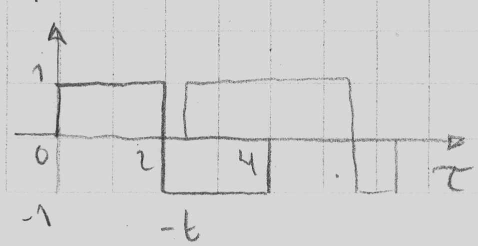
$$= [\tau]_{-t}^2 - [\tau]_2^4$$

$$= 2 + t - 2 = t$$

Caso (7)

$2 \le -t \le 4$
$-2 \ge t \ge -4$

$x_1(t+\tau)\,x_0(\tau)$



$$\int_{-t}^4 (1)(-1)\,d\tau = -[\tau]_{-t}^4$$

$$= -4 - t$$

$$r_{x_1 x_0}(t) = \begin{cases} -4-t, & -4 \le t \le -2 \\ t, & -2 \le t \le -1 \\ 3t+2, & -1 \le t \le 0 \\ 2+t, & 0 \le t \le 1 \\ -3t+6, & 1 \le t \le 2 \\ 2-t, & 2 \le t \le 3 \\ -4+t, & 3 \le t \le 4 \\ 0 & \text{otro caso} \end{cases}$$