CS446: Machine Learning, Fall 2017, Homework 1

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Worked individually

Multivariate Poisson Naive Bayes

Problem 1

Solution:

$$P(y = A) = \frac{3}{7},\ P(y = B) = \frac{4}{7}$$

Using MLE to compute λ's:

$$P(x_1 = x|y = A) = \frac{e^{-\lambda_1^A}(\lambda_1^A)^x}{x!}$$

$$\Longrightarrow P(D_n|y = A) = \prod_{i=1}^{3} P(x_1 = x|y = A) = \frac{e^{-3\lambda_1^A}(\lambda_1^A)^{\sum x_{1i}}}{x_{11}! \cdot ... \cdot x_{13}!}$$

$$\Longrightarrow logP(D_n|y = A) = -3\lambda_1^A + \sum x_{1i}log(\lambda_1^A) - log(\prod x_{1i}!)$$

$$\Longrightarrow \frac{\partial logP(D_n|y = A)}{\partial \lambda_1^A} = -3 + \frac{\sum x_{1i}}{\lambda_1^A}$$

$$\Longrightarrow \lambda_1^A = \frac{\sum x_{1i}}{3} = \frac{1+5+3}{3} = 3$$

Similarly, $\lambda_1^B = 5,\ \lambda_2^A = 6,\ \lambda_2^B = 4.$

Problem 2

Solution:

We have

$$P[x_1 = x|y = A] = \frac{e^{-3} \cdot 3^x}{x!},\ P[x_1 = x|y = B] = \frac{e^{-5} \cdot 5^x}{x!},$$

$$P[x_2 = x|y = A] = \frac{e^{-6} \cdot 6^x}{x!},\ P[x_2 = x|y = B] = \frac{e^{-4} \cdot 4^x}{x!}$$

Then

$$P[x_1 = 2|y = A] = 0.61,\ P[x_2 = 3|y = A] = 0.09$$
$$P[x_1 = 2|y = B] = 0.08,\ P[x_2 = 3|y = B] = 0.20$$

Thus,

$$\frac{P(x_1 = 2, x_2 = 3|y = A)}{P(x_1 = 2, x_2 = 3|y = B)} = 1.215$$

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Problem 3

Derive an algebraic expression for the Poisson Naive Bayes predictior for Y in terms of the parameters estimated from the data.
Solution:

$$\begin{aligned} P(y = A|x_1, x_2) &= \frac{P(y = A) \cdot P(x_1, x_2|y = A)}{P(x_1, x_2)} \\ &\propto P(y = A) \cdot P(x_1|y = A) \cdot P(x_2|y = A) \\ &\propto \frac{3}{7}\frac{e^{-3} \cdot 3^{x_1}}{x_1!} \cdot \frac{e^{-6} \cdot 6^{x_2}}{x_2!} \end{aligned}$$

Similarly,

$$\begin{aligned} P(y = B|x_1, x_2) &= \frac{P(y = B) \cdot P(x_1, x_2|y = B)}{P(x_1, x_2)} \\ &\propto P(y = B) \cdot P(x_1|y = B) \cdot P(x_2|y = B) \\ &\propto \frac{4}{7}\frac{e^{-6} \cdot 6^{x_1}}{x_1!} \cdot \frac{e^{-4} \cdot 4^{x_2}}{x_2!} \end{aligned}$$

Thus

$$\frac{P(y = A|\boldsymbol{x})}{P(y = B|\boldsymbol{x})} = \frac{3}{4}(\frac{3}{5})^{x_1}(\frac{3}{2})^{x_2}$$

Problem 4

Solution:
For $\boldsymbol{x} = [x_1, x_2] = [2, 3]$, we have

$$\frac{P(y = A|\boldsymbol{x})}{P(y = B|\boldsymbol{x})} = \frac{3}{4}(\frac{3}{5})^2(\frac{3}{2})^3 = 0.91125 < 1,$$

So we have $y_{pred} = B$.