

Known Higher-Order Ignorance

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Abstract

You are *higher-order ignorant* when you fail to know whether you know something. I show that ordinary people sometimes say that they are higher-order ignorant, and use this to undermine the claim that knowing implies knowing that one knows ($KK: \Box p \supset \Box \Box p$), and the thought that taking seriously the possibility that one does not know something raises the standards for the application of the word “know” to the point where one fails to satisfy them.

Kewywords: KK, assertion, knowledge, contextualism

1 Infelicitous assertion and the logic of knowledge

To be *higher-order ignorant* is to fail to know whether one knows something. Rudy Giuliani once said that he was higher-order ignorant:¹

(1) I don’t know if I know exactly what the definition of ‘rigged’ means.

In this paper, I will show that ordinary people sometimes assert, presuppose, and implicate that they are higher-order ignorant, and draw out two surprising implications of this observation for the nature of knowledge and the semantics of the word “know”.

The first implication is the falsity of the controversial KK principle:²

$KK. \Box p \supset \Box \Box p$

If you know something, you know that you know it.

Since we can felicitously say only what can be known, the fact that we sometimes felicitously say that we are higher-order ignorant implies that we can sometimes know that we are higher-order ignorant. But if KK were true, such knowledge would be impossible.³ I defend this new objection

¹<https://x.com/CNN/status/792118971180969984>, last accessed 11/23/2024.

²My objection generalizes to Goldstein (2022, 2024)’s Fragility principle $\Box p \supset \Diamond \Box \Box p$, and any principle of the form $\Box p \supset \Diamond^n \Box \Box p$ for $n \geq 0$.

³Fine (2018, thm.4) in effect observed that KK is inconsistent with known higher-order ignorance. However, he considered this a corollary rather than an objection to KK .

to *KK*, and explain how it is more general than existing objections while avoiding their most controversial assumptions.⁴

But those who reject *KK* can learn something from avowals of higher-order ignorance, too. The standard objection to *KK*, due to Williamson (2000), is non-constructive, proving the existence of counterexamples to *KK* without constructing an instance. One might suspect, then, that higher-order ignorance is “elusive”: the word “know” is context-sensitive or vague in ways that prevent us from self-ascribing instances of higher-order ignorance. Some have hypothesized that taking seriously the possibility that one does not know something raises the standards for the application of the word “know” to the point where one fails to satisfy them.⁵ Others have suggested that we cannot provide examples of higher-order ignorance because that would require resolving the vagueness of “know” in too particular a way.⁶ Whatever we make of these explanations, one might suspect that we cannot *say* that we are higher-order ignorant.⁷ The second implication of my data is that higher-order ignorance is *not* elusive.

I will begin by arguing that people sometimes felicitously say that they are higher-order ignorant (§2), and so higher-order ignorance is neither elusive (§3) nor unknowable (§4). But if *KK* were true, it would be unknowable, and so *KK* is false (§§5-6). I consider an objection to the effect that my data involve non-standard interpretations of the word “know” (§7), and conclude with some lessons regarding so-called dubious conjunctions of the form $\lceil p \text{ and I don't know that I know that } p \rceil$ (§8).

2 Felicity

When we have some reason to think that we know, and some reason to think that we do not, we sometimes find ourselves unable to tell whether we know. Sometimes we inform others of our predicament, like *Game of Thrones* actor John Bradley on *The Ellen Show*:⁸

(2) Ellen DeGeneres: Do you know how it [i.e. the show] ends?

John Bradley: Well I thought I knew how it was going to end, and we definitely shot an ending. But I read an interview with our show

⁴Williamson (2000) argues that *KK* can fail for *boundary knowledge*: beliefs that do constitute knowledge but are very similar to beliefs that do not. My argument avoids the controversial margin-for-error premise of Williamson's, according to which one cannot know a belief to constitute knowledge when it is very similar to a belief that does not. Nor does my argument rely on conjunctive syllogism, or other bits of classical logic that could play the same role in Williamson's argument but are rejected by Fine (2020, ms).

⁵See Salow (2019) and Fraser (2022, 581-3).

⁶See DeRose (2002, 184). Goodman (2024, 4) gives a parallel explanation why we could never *assert* counterexamples to the closure of knowledge under conjunction.

⁷Dorr & Hawthorne (2013, 889) assume that we expect people not to assert or implicate ignorance about what they know.

⁸<https://www.youtube.com/watch?v=qPH1aLti3g0>, last accessed 2/19/2024.

runners not too long ago when they said, "The actors think they know how it's going to end." So, you know, actor's paranoia instantly kicked in and you think, "What does that mean?"

E. D.: So you don't know?

J. B.: I don't know what I know, is true.

E. D.: Because he may be just saying that to make you think you don't know, but you may know?

J. B.: I may ultimately know.

E. D.: But you may not know.

J. B.: But I'm not going to know that I know until I know that I know.

E. D.: Right, exactly. I'm with you.

The degree to which John Bradley scrutinises his epistemic situation may be unusual, but the situation itself is not. When we have a source that may or may not be a reliable source on some question, we can felicitously assert that we don't know whether we know. Uncertain memories give rise to assertions of higher-order ignorance in the same way:

- (3) Like, one of our producers will come up to me and say, do you know that blah, blah, blah, blah, blah, blah, blah? And then I'll say, in all honesty, that sounds familiar. I don't know if I know that.⁹

Appendix A provides many more examples of this phenomenon, which I omit here to streamline discussion.

Higher-order ignorance can not only be felicitously asserted, but also felicitously presupposed. $\lceil S \text{ wonders whether } p \rceil$ and $\lceil S \text{ hopes that } p \rceil$ both presuppose that S does not know whether p .¹⁰ The following examples then suggest that one can presuppose higher-order ignorance:

- (4) I wonder, said the Lord I wonder if I know the answer any more.¹¹
- (5) I hope that I know when to put down my phone and actually watch her life unfold, instead of frantically trying to capture it for posterity.¹²

A survey from the WHO's question bank concerning the symptoms of Covid-19 suggests a similar conclusion:

⁹Terry Gross on NPR: <https://www.npr.org/2016/08/02/488238350/a-neuroscientist-explores-the-illogical-behaviors-of-the-mind-in-idiot-brain>.

¹⁰Heim (1992, 198) and von Fintel (1998, 117) posit that 'want' presupposes ignorance. There may be counterexamples for 'want' (such as Heim (1992, 199)'s own 'I want this weekend to last forever'), but they seem bad for 'hope' (see Blumberg & Hawthorne, 2022). Horn (1972, 29) observes that 'wonder' presupposes ignorance, see Uegaki (2015, 65) and Ciardelli & Roelofsen (2015, 1659) for recent explanations. It is not so clear whether ignorance is a presupposition or an entailment, but my examples support known higher-order ignorance either way (see Blumberg & Hawthorne, 2022, §3.2 for discussion).

¹¹Norman Mailer, *Deaths For The Ladies*: <https://www.goodreads.com/quotes/852464-i-wonder-said-the-lord-i-wonder-if-i-know>.

¹²<https://www.nytimes.com/2020/04/17/parenting/children-social-media.html>.

- (6) Do you feel you know the signs and symptoms of coronavirus?
 a. Definitely know b. Probably know c. Unsure if I know d. Probably do not know e. Definitely do not know.¹³

Clearly, whoever wrote this survey meant to leave open that one may know that one doesn't know whether one knows—else there would be little point in including the answer 'Unsure if I know'.

Finally, one can also implicate higher-order ignorance, or at least felicitously utter sentences whose combined asserted and implicated content amounts to higher-order ignorance. I assume that 'might p ' implicates 'not must p ', 'perhaps p ' implicates 'not certainly p ', and 'S thinks that p ' implicates 'S does not know that p '.¹⁴ With these in mind, consider:

- (7) I think I know why everyone's lost their minds over Taylor Swift and Travis Kelce.¹⁵
 ~> I don't know that I know why everyone's lost their minds over Taylor Swift and Travis Kelce.
- (8) I don't even know where Little Debbie lives. I might know where we can find Sara Lee, though.¹⁶
 ~> I might not know where we can find Sara Lee.
- (9) Perhaps I know the answer to my question, really. I just need someone else to tell me the right thing to do.¹⁷
 ~> Perhaps I don't know the answer to my question.

In all these examples, it follows from what the speaker says that they don't know that they don't know something, and it follows from what the speaker implicates that they don't know that they know it. (I'm assuming here that 'perhaps' and 'might', when unembedded, are roughly equivalent to 'for all I know'.¹⁸) Taking what's asserted and what's implicated together, we get that the speaker doesn't know whether they know.

In conclusion, we have *Felicity*: One can felicitously assert, presuppose, and implicate that one is higher-order ignorant.

¹³Covid-19 PULSE survey: https://cdn.who.int/media/docs/default-source/documents/ddi/data-collection-tools/covid-19-question-bank_91ef4cc6-85c4-4865-8db9-b17851693f85.xlsx?sfvrsn=be51ef9c_4

¹⁴All three are standard scalar implicatures. Gazdar (1979, 56, 61, and 50) assumes that 'might' and 'must', 'think' and 'know', and 'perhaps' and 'necessarily' are on a scale. See also Horn (1972, 120f. and 273).

¹⁵<https://www.adn.com/opinions/national-opinions/2024/02/07/opinion-i-think-i-know-why-everyones-lost-their-minds-over-taylor-swift-and-travis-kelce/>.

¹⁶<https://www.instagram.com/p/CfErGPqs1F1/>.

¹⁷<https://www.mirror.co.uk/lifestyle/sex-relationships/relationships/should-gay-affair-married-man-677967>.

¹⁸Embedded 'might' and 'perhaps' are *not* equivalent to 'for all I know' (see Yalcin, 2007; Mandelkern, 2019), but for unembedded occurrences something in the vicinity is plausible (see DeRose, 1991; von Stechow & Gillies, 2011; Hacking, 1967; Kratzer, 1981, 1991).

3 Bounding the meaning of “know”

A familiar theme from contextualist accounts of scepticism is the idea that some of our knowledge may be “elusive”. Roughly, the thought is that although we do know the propositions in question, we cannot felicitously say that we do because that would raise the standards for the application of the word “know” to the point where we fail to satisfy them.¹⁹

A few authors have recently proposed that higher-order ignorance may be elusive in a similar way. Their idea is that the word “know” is context-sensitive or vague in ways that prevent us from truly saying that we are higher-order ignorant. The purpose of this section is to argue that asser-tions of higher-order ignorance put pressure on their generalizations.

Salow (2019) and Fraser (2022, 581-3) assume that bringing up or tak-ing seriously the possibility that one does not know something raises the standards for the application of the word “know” to the point where one clearly fails to satisfy them. This hypothesis is meant to explain why we cannot accept (or must reject) certain sentences attributing higher-order ignorance to ourselves. But it cannot be right: To say that one does not know whether one knows seems like a paradigmatic way of bringing up the possibility that one does not know.²⁰ And yet it fails to raise the stan-dards for the application of the word “know”, or at least does not raise them to the point where one clearly fails to satisfy them.

Of course, our examples do not show that there is *no* connection be-tween bringing up the question whether one knows something and raising the standards for the interpretation of the word “know”, but only that the connection is not as rigid as Salow (2019) and Fraser (2022) claim it to be. This conclusion is nevertheless interesting, since Salow and Fraser really do need the strong version of the claim for their theories.²¹

Similar data also put pressure on the thought that the meaning of the word “know” is rigidly tied to *stakes* (DeRose, 1992). Suppose you need to access your bank account to make an important payment, and you have tried to log in twice but failed. You are not sure whether this is because you are mistaken or because of a typo or technical error. You might then say:

¹⁹See e.g. Lewis (1996) on anti-skeptical and Greco (2017) on anti-defeat knowledge.

²⁰It would not help if the possibility that I do not know whether *p* was raised but somehow still properly ignored. First, standard contextualist theories in the tradition of Lewis (1996) would then predict “I know that I don’t know whether *p*” to be trivially true, and hence I should be *unable* to say its negation. Second, if proper ignoring is pragmatic presupposition (Blome-Tillmann, 2009), I should be forced to say “I don’t know whether I know *that p*” rather than “I don’t know whether I know *whether p*” by the constraint that we must make the strongest presupposition possible (see Heim, 1991, 515).

²¹Fraser (2022, 581-3) uses the hypothesis to explain the infelicity of conditionals such as “(Even) if I don’t know that Torino is in Italy, it is.” Salow (2019) takes the hypothesis to explain why rational agents must *reject* self-ascriptions of higher-order ignorance, such as “I don’t know that Torino is in Italy, but I also don’t know that I don’t know that.” These explanations falsely predict that “I don’t know whether I know that Torino is in Italy” should be equally infelicitous, and something rational agents must reject.

“I don’t know whether I know my password.” If high stakes *always* made knowledge itself hard to come by, or *forced us* to interpret the word “know” in a very strict way, one would expect such assertions to be infelicitous. But they aren’t! (Again, that’s compatible with the weaker idea that “know” *tends to* be receive demanding interpretations in high-stakes contexts.)

A related but different thought is the idea that the vagueness of the word “know” prevents us from diagnosing instances of higher-order ignorance. Prototypical *KK*-failures involve *boundary knowledge*, that is beliefs that do constitute knowledge but are very similar to beliefs that do not. Since the word “know” is vague, different beliefs will be boundary cases on different interpretations of “know”. Following DeRose (2002, 183-5), one might then expect that we cannot say particular instances of *KK*-failure, since that would involve making a “close call” — resolving the vagueness of the word “know” in too particular a way.²² At least when higher-order ignorance arises because one’s belief is a boundary case, one may then expect that we cannot say (or know) that it arises.²³ But again, avowals of higher-order ignorance rule out this natural thought, since they show that we *can* self-ascribe higher-order ignorance. At least some uses of the word “know” are not as vague as hypothesized, that is some beliefs count as boundary cases on *all* interpretations admissible in context.²⁴

Finally, our data also put some pressure on Dorr & Hawthorne (2013, 889)’s weaker claim that we (ordinarily) expect one another not to assert or implicate ignorance about what we know. For it is not just *in principle* possible to say or suggest that one is higher-order ignorant — ordinary speakers make a wide range of perfectly hum-drum speeches to this effect.

This concludes my discussion of the implications of higher-order ignorance for the semantics of the word “know”. I will now turn to implications for the logic of knowledge.

4 Knowability

In this section, I argue from *Felicity* — the claim that we can felicitously assert, presuppose, and implicate that we are higher-order ignorant — to *Knowability* — the claim that we can sometimes know that we are higher-order ignorant. My argument builds on the following premise:

Felicity-Knowability. If a sentence can be felicitously uttered, then

²²See also Goodman (2024, 4) for a parallel explanation why we could never *assert* counterexamples to the closure of knowledge under conjunction.

²³DeRose (2002) recognizes another source of higher-order ignorance, resulting from cases where the subject makes a reasonable mistake, thinking that they know although they do not. But of course such cases do not give rise to *known* higher-order ignorance.

²⁴So either “know” expresses fewer relations, or which relations *these uses* of “know” express is sufficiently narrowed down by speakers’ beliefs and purposes (see Dorr, 2024).

one can (simultaneously) know the propositions asserted, presupposed, and implicated by one's utterance of it.

Why accept **Felicity-Knowability**? First, it fits the data: natural candidates for unknowability cannot be asserted, presupposed, or implicated felicitously. This includes contradictions, Moorean conjunctions, and outcomes of future or counterfactual random events:

- (10) a. #It's raining but it isn't.
b. #It's raining but you're aware it isn't.
c. #It's raining but I wouldn't say it is.
- (11) a. #It's raining but I don't know that it is.
b. #It's raining but I {hope that/wonder whether} it is.
c. #It's raining and perhaps I know that it is.
- (12) a. #If I flip this fair coin, it will come up heads.
b. #You're aware that if I flip this fair coin, it will come up heads.
c. #All that I'm permitted to tell you is that if I flip this fair coin, the people who bet on tails shouldn't expect to win anything.

Asserting, presupposing, or implicating jointly unknowable combinations of propositions sounds weird.

Second, **Felicity-Knowability** follows from popular theories of assertion, presupposition, and implicature. For assertion, it follows from the norm that you must assert only what you know,²⁵ or *pretend* you know,²⁶ or the idea that successful assertion adds to the common ground, where the common ground is information treated as known.²⁷ For presupposition, it follows from the idea that felicitous presupposition requires what's presupposed to already follow from the common ground, again assuming that the common ground is treated as commonly (or mutually) known.^{28,29}

²⁵See Williamson (2000). The principle still follows from variations, such as the norm that you must assert only what you reasonably believe you know (Brown, 2008), what you are in a position to know (Willard-Kyle, 2020), what you omega know (Goldstein, 2024, forthcomingb), or what your audience will thereby be in a position to know (García-Carpintero, 2004), or packages of a belief/surety norm of assertion with a knowledge norm for belief/surety (Bach, 2008; Goodman & Holguín, 2022).

²⁶See Mandelkern & Dorst (2022), and the idea that assertion involves implying that you know (Gazdar, 1979; Moore, 1962, 277), or representing yourself as knowing (Unger, 1975).

²⁷See Stalnaker (1978, 84). Stalnaker (1973, 448f.) and Yalcin (2007, 1008) suggest that we can take something to be known for the purposes of a conversation that is in fact known to be unknown. Whatever we make of this, we cannot allow the *unknowable* to be treated as known, or else Moorean conjunctions could be presupposed and asserted. (One thought: Perhaps we cannot pretend the inconceivable, and knowing Moorean conjunctions is inconceivable. We *can* pretend that some metaphysical impossibilities are true, for example that you are my parent, but only when they are conceivable (see Chalmers, 2002, §3).)

²⁸See (Stalnaker, 1973; Karttunen, 1974) for this account of presupposition. The simpler idea that presuppositions must be common knowledge would suffice (García-Carpintero, 2020). Zakkou (forthcoming)'s counterexamples are very different from my data.

²⁹**Felicity-Knowability** for presupposition also follows from the principle for assertion, given that presupposition is at least as committal as assertion (Hawthorne, 2012, §VIII).

Felicity-Knowability is least secure for implicatures.³⁰ For *scalar* implicatures like (13), the principle falls out of their classic Gricean derivation.

- (13) I ate some of the cookies.
 \leadsto I didn't eat all of the cookies.

When I assert that I ate some of the cookies, hearers assume that I didn't eat all of them. Griceans take the implicature to arise via three steps.³¹ First, the cooperativeness presumption: Hearers assume that if I had known that I had eaten all of the cookies, I'd have said so (because 'some' and 'all' lie on a scale). Second, the informedness presumption: Hearers also assume that I know whether I ate all of the cookies, and hence know I didn't eat all of the cookies (so knowledge of the scalar implicature is ensured). The third step exploits factivity: Hearers infer from the fact that *I know* I didn't eat all of the cookies that I didn't eat all of them (see Gazdar, 1979, 59). My data involve scalar implicatures, so securing knowability of scalar implicatures would suffice.³²

If you have doubts about the implicature version, note that accepting **Felicity-Knowability** for assertion (or presupposition) only would suffice for most of my data — the implicature version is only needed for the argument from sentences such as 'I might know that it is raining.'

Upshot: **Felicity-Knowability** is empirically and theoretically plausible. The fact that we can felicitously assert, presuppose, and implicate that we are higher-order ignorant is good reason to think that we can know that we are higher-order ignorant. Felicity implies Knowability. In the next two sections, I will show that the knowability of higher-order ignorance undermines some popular principles about knowledge.

5 Unknowability

Recall that *KK* is the claim that whenever you know something, you know that you know it ($\Box p \supset \Box \Box p$). Whether *KK* is true is a controversial question in recent epistemology.³³ We will now show that if *KK* or certain weakenings were true, one could not know that one is higher-order ignorant. This gives us a novel objection to *KK* and some of its weakenings.

In this section, I will focus on failing to know whether one knows-*that*:

- (14) I don't know whether I know that p . ($\nabla \Box p$)

³⁰Though Haziza (2021) and Hewson & Lewerentz (2020) defend a knowledge norm.

³¹See Horn (1972); Gazdar (1979, 55ff.).

³²On exhaustification theories of scalar implicature, whether knowledge of the falsity of stronger scalar alternatives is derived depends on whether the exhaust-operator scopes over or under the matrix-*K* operator posited by such theories (e.g. whether we parse as $\text{EXH } K \Diamond \Box p$ or $K \text{ EXH } \Diamond \Box p$). See Meyer (2013, ch. 2).

³³See Williamson (2000) and Stalnaker (2006), and Greco (2015a,b) for an overview.

I will write ' \Box ' to express knowledge-that and ' Δ ' to express knowing-whether. ' ∇ ' will be shorthand for ' $\neg\Delta$ ', that is *not* knowing whether.

Intuitions first: If *KK* were true, then the only way you could fail to know whether you know is if you didn't. But then, if you *knew* that you fail to know whether you know, you could go through this reasoning and infer that you don't know, and so you wouldn't fail to know whether you know after all. Upshot: If *KK* were right, you could not know that you fail to know whether you know that *p*. Let's prove this intuition rigorously!

I will make four assumptions about knowledge: What's known is true (**T**. $\Box p \supset p$), logical truths are known (**N**. $\Box \top$), single-premise consequences of what's known are in turn known (**RM**. If $p \vdash q$, then $\Box p \vdash \Box q$), and knowing whether *p* amounts to either knowing *p* or knowing not *p* (**?**. $\Delta p \equiv (\Box p \vee \Box \neg p)$).³⁴ I will assume a classical background logic, though this assumption could be weakened. For now we need *not* assume that knowledge is multi-premise closed (**C**. $\Box p \wedge \Box q \supset \Box(p \wedge q)$).

We now show that higher-order ignorance involving knowledge-that, as expressed in (14), is unknowable given *KK*:

- | | |
|--|---------------------------|
| 1. $\neg\Box\Box p \supset \neg\Box p$ | <i>KK</i> |
| 2. $\Box\neg\Box\Box p \supset \Box\neg\Box p$ | RM , 1 |
| 3. $\Box(\neg\Box\neg\Box p \wedge \neg\Box\Box p) \supset \Box\neg\Box p$ | PC + RM , 2 |
| 4. $\Box(\neg\Box\neg\Box p \wedge \neg\Box\Box p) \supset \neg\Box\neg\Box p$ | PC + T |
| 5. $\neg\Box(\neg\Box\neg\Box p \wedge \neg\Box\Box p)$ | PC , 3, 4 |
| 6. $\neg\Box\nabla\Box p$ | ? + RM , 5 |

We thus have to choose between *KK* and the possibility of known higher-order ignorance. Since my examples suggest that higher-order ignorance can be known, I think we should reject *KK*.³⁵

Of course, I am not the first to object to *KK*. Williamson (2000) has prominently objected to *KK* based on so-called *margin-for-error* principles. My argument differs in three interesting ways from Williamson's. First, my argument generalizes to a family of weakenings of *KK*:

$$KK_n^\Diamond. \Box p \supset \Diamond^n \Box p$$

If you know something, then for all you know to level *n*, you know that you know it.

³⁴See Hintikka (1962, 12).

³⁵This result follows from Fine (2018)'s observation that *KK* collapses higher-order ignorance. One is *first-order ignorant* whether *p* iff one does not know whether *p* (∇p), and that one is *n+1th order ignorant* whether *p* if one is ignorant whether one is *n*-th order ignorant whether *p* ($\nabla^{n+1} p = \nabla \nabla^n p$). Fine shows *m*-th level and *n*-th level ignorance are equivalent given *KK*. Fine and Fano & Graziani (2020) view the unknowability and collapse of higher-order ignorance as surprising corollaries of *KK*, whereas I use them to object to *KK*.

Goldstein (2022, forthcomingb) has recently defended the instance of KK_n^\diamond for $n = 1$, which says that knowing requires that one doesn't know that one fails to know that one knows ($\Box p \supset \Diamond \Box \Box p$), partly on the grounds that it is compatible with margin-for-error principles when the margins are variable.³⁶ We have the resources to object to such weakenings, too: Not just KK , but any principle of the form KK_n^\diamond is incompatible with known higher-order ignorance.³⁷

Second, Williamson's argument is non-constructive: while it shows that KK must fail for *boundary knowledge* — beliefs that do constitute knowledge but are very similar to beliefs that do not — it leaves open that we can never know of a particular belief that it is boundary knowledge. Could it be that there are KK -failures, but we cannot specify an instance — just like there are numbers that will never be considered, but we cannot specify an instance? If it is successful, my argument suggests that the answer is no. Suppose that John Bradley really knows that he is higher-order ignorant, and imagine that the script he read was in fact reliable. Then presumably he in fact knows how the show ends from the script. (If he failed to know even in this knowledge-conducive setting, he could know that he doesn't know!) So we have a plausible instance of KK -failure: John Bradley knows how the show ends, but he doesn't know that he knows this. If my argument is any good, we can construct particular KK -failures.

Finally, my argument does not rely essentially on conjunctive syllogism, or any other bits of classical logic that could play the same role in Williamson's argument but are rejected by Fine (2020, ms). It is thereby immune to a range of non-classical responses to his argument.

Upshot: If KK (or KK_n^\diamond) is true, one cannot know that one is higher-order ignorant in the case of knowledge-*that*. The next section generalizes these considerations to knowledge-*wh*.

6 Extension to knowledge-*wh*

Most of my examples involve people saying that they don't know whether they know-*what*, know-*why*, know-*when*, or know-*how*. Could it be that we

³⁶See §5 of Goldstein (2022) and §5.4 of Goldstein (forthcominga).

³⁷The proof is slightly more complex, but essentially analogous:

- | | |
|--|---------------------------|
| 1. $\Box^n \neg \Box \Box p \supset \neg \Box p$ | KK_n^\diamond |
| 2. $\Box \Box^n \neg \Box \Box p \supset \Box \neg \Box p$ | RM , 1 |
| 3. $\Box \Box^n (\neg \Box \neg \Box p \wedge \neg \Box \Box p) \supset \Box \neg \Box p$ | PC + RM , 2 |
| 4. $\Box \Box^n (\neg \Box \neg \Box p \wedge \neg \Box \Box p) \supset \neg \Box \neg \Box p$ | PC + T |
| 5. $\neg \Box \Box^n (\neg \Box \neg \Box p \wedge \neg \Box \Box p)$ | PC , 3, 4 |
| 6. $\Box^{n \times n} \neg \Box \Box^n (\neg \Box \neg \Box p \wedge \neg \Box \Box p)$ | N + RM , 5 |
| 7. $\neg \Box (\neg \Box \neg \Box p \wedge \neg \Box \Box p)$ | KK_n^\diamond , 6 |
| 8. $\neg \Box \nabla \Box p$ | ? + RM , 7 |

cannot know whether we know-*that*, but can know whether we know-*wh*?

No, I will argue. If KK holds for knowledge-*that* ($\Box p \supset \Box \Box p$), then a parallel principle KK^Q holds for knowledge-*wh* ($\Box Q \supset \Box \Box Q$). (We use upper-case letters like ‘ Q ’ to denote questions.) But then, by a parallel proof to that for knowledge-*that*, KK is incompatible with knowing that you fail to know whether you know-*wh*.

To derive KK^Q from KK , I need two assumptions:

Bridge. $\Box Q \equiv \exists q(AA(q, Q) \wedge \Box q)$

You know-*wh* Q iff you know some admissible answer to Q .³⁸

Admissibility Knowledge. $\forall q(AA(q, Q) \supset \Box AA(q, Q))$

If q counts as an admissible answer to Q , then you know that.

The first assumption identifies knowledge-*wh* with knowing some admissible answer, while the second says that you know which propositions count as admissible answers. Given these assumptions, we can intuitively derive **Q4** from KK : If one knows-*wh*, one must know some admissible answer by **Bridge**. By KK , one knows that one knows that answer, and by **Admissibility Knowledge** one knows that it counts as an answer. Combining both and using **Bridge**, one can infer that one knows-*wh*.

To make this argument precise, we assume standard quantifier rules \exists -Elim and \exists -Intro, and one application of the agglomeration axiom **C** ($\Box p \wedge \Box q \supset \Box(p \wedge q)$). We now show that KK for knowledge-*that* ($\Box p \supset \Box \Box p$) implies KK^Q , the parallel principle for knowledge-*wh* ($\Box Q \supset \Box \Box Q$):³⁹

- | | | |
|----|--|-----------------------------------|
| 1. | $\Box Q \supset \exists q(AA(q, Q) \wedge \Box q)$ | Bridge |
| 2. | $\Box Q \supset \exists q(AA(q, Q) \wedge \Box \Box q)$ | $KK, 1$ |
| 3. | $\Box Q \supset \exists q(\Box AA(q, Q) \wedge \Box \Box q)$ | Admissibility Knowledge, 2 |
| 4. | $\Box Q \supset \exists q \Box(AA(q, Q) \wedge \Box q)$ | C, 3 |
| 5. | $\Box Q \supset \Box(AA(r, Q) \wedge \Box r)$ | \exists -Elim, 4 |
| 6. | $\Box Q \supset \Box(\exists q(AA(q, Q) \wedge \Box q))$ | \exists -Intro, 5 |
| 7. | $\Box Q \supset \Box \Box Q$ | Bridge + RM, 6 |

The reasoning for KK_n^\diamond is parallel, and will be relegated to a footnote.⁴⁰

³⁸Remark: Our results would still go through if we replaced the existential with a universal conception of knowledge-*wh* on which it requires knowing *every* admissible answer. That is, we could just as well run with the principle $\Box Q \equiv \forall q(AA(q, Q) \supset \Box q)$.

³⁹See also Fine (2018)’s discussion of *general ignorance*, and his theorem 7. Fine (2018) shows that KK makes not just $\nabla \nabla p$ a unknowable, but also $\nabla(\nabla p_1 \vee \dots \vee \nabla p_n)$ and $\nabla(\nabla p_1 \wedge \dots \wedge \nabla p_n)$. Our result is similar, except we set things up in terms of quantifiers.

⁴⁰Note that $\Box p \wedge \Diamond q \vdash \Diamond(p \wedge q)$ given **C** and **RM**. We argue:

- | | | |
|----|--|---------------------------------------|
| 1. | $\Box Q \supset \exists q(AA(q, Q) \wedge \Box q)$ | Bridge |
| 2. | $\Box Q \supset \exists q(AA(q, Q) \wedge \Diamond^n \Box q)$ | $KK_n^\diamond, 1$ |
| 3. | $\Box Q \supset \exists q(\Box^n AA(q, Q) \wedge \Diamond^n \Box q)$ | Admissibility Knowledge + N, 2 |

How plausible are **Bridge** and **Admissibility Knowledge**? **Bridge** seems totally unassailable to me, and is also the cornerstone of the orthodox semantics of questions.⁴¹ I will not defend it further.

The status of **Admissibility Knowledge** is more complex, since there is contextual variation in what counts as an admissible answer. Work on questions has brought out three notions of *answer admissibility*, which are best illustrated with an example:

- (15) Situation: Only Tez and Milo left the party. Question: Who left?
 (a) Strongly-exhaustive answer: ‘Tez and Milo left, and nobody else.’
 (b) Weakly-exhaustive answer: ‘Tez and Milo left.’
 (c) Mention-some answers: ‘Tez left’, and ‘Milo left.’

Strongly-exhaustive (and mention-some) answers are *transparent*, in that knowing such an answer suffices for knowing that it is a strongly-exhaustive (or mention-some) answer. That’s because these notions of answerhood are a purely logical matter. Weakly-exhaustive answers are not transparent in this way. You might know that Tez and Milo left, but fail to know that nobody else did. You would then know the weakly-exhaustive answer without knowing that it is the weakly-exhaustive answer.

Where does this leave us with respect to **Admissibility Knowledge**? On the one hand, the principle can admittedly fail when answers have to be weakly-exhaustive to be admissible. On the other hand, such contexts are known to be rare, especially with verbs like “know”.⁴² And in any case there are decisive reasons for thinking this isn’t happening in our data. First, most of our data presuppose that there is only one true mention-some answer: constructions like ‘how *Game of Thrones* ends’, ‘the answer’, ‘why everyone’s lost their minds over Taylor Swift and Travis Kelce’, and ‘where Little Debbie lives’ strongly suggest that there is only one way the show ends, only one answer, only one reason why everyone lost their minds over Taylor and Travis, and only one place Little Debbie lives. In this case, all three notions of admissibility coincide, and are transparent.

Second, if our examples were felicitous only because the subjects in question fail to know what counts as an admissible answer, conjunctions such as (16a) ought to be assertable — but they are not:⁴³

-
- | | |
|--|---------------------|
| 4. $\Box Q \supset \exists q \Diamond \Box (AA(q, Q) \wedge \Box q)$ | C + RM, 3 |
| 5. $\Box Q \supset \Diamond \Box (AA(r, Q) \wedge \Box r)$ | \exists -Elim, 4 |
| 6. $\Box Q \supset \Diamond \Box (\exists q (AA(q, Q) \wedge \Box q))$ | \exists -Intro, 5 |
| 7. $\Box Q \supset \Diamond \Box \Box Q$ | Bridge + RM, 6 |

⁴¹See Karttunen (1977), Groenendijk & Stokhof (1982), and Heim (1994).

⁴²See Groenendijk & Stokhof (1982), Heim (1994). Weakly-exhaustive readings seem to arise more with verbs such as ‘realise’ or ‘surprise’, if at all (Guerzoni, 2007).

⁴³See Holguín (2021b, 281) for a related argument.

- (16) a. I don't know whether I know how *Game of Thrones* ends.
 b. #*Game of Thrones* ends with Daenerys' death, but I don't know whether I know how *Game of Thrones* ends.

If (16a) were acceptable for me because I don't know what counts as an answer to the question 'How does *Game of Thrones* end?', though I do know (and know that I know) that it ends with Daenerys' death, then I ought to be able to assert (16b) also. But I cannot assert (16b), so the problem must be with *KK*, not with **Admissibility Knowledge**.

We started this section with the observation that when higher-order ignorance is asserted in the wild, it is ignorance of what one knows-*wh*.⁴⁴ We have found decisive reasons for thinking that being ignorant whether one knows-*wh* entails being ignorant whether one knows-*that*. Let's grant that conclusion. What else explains our observation, then? I think the answer comes from the presuppositions of the word 'know'. The construction 'I know that p ' tends to presuppose the truth of p .⁴⁵ Asserting 'I don't know whether I know that p ' thus is tantamount to saying 'I know that (p but I don't know whether I know that p)', which is clearly infelicitous.

The factivity presupposition of 'know' is softer than the presuppositions of classic presupposition triggers (such as 'stop'). If it can be cancelled, why isn't it cancelled in the (otherwise infelicitous) conjunction 'I don't know whether I know that p '? I suspect that 'knows that' competes with 'knows whether': since it is an option to say 'I don't know whether I know whether p ', which carries no factivity presupposition, we avoid saying 'I don't know whether I know that p ' by competition.⁴⁶

Upshot: *KK* (or KK_n^\diamond) preclude not just knowing that one fails to know whether one knows-*that*, but also knowing that one fails to know whether one knows-*wh*. Since we say that we don't know whether we know-*wh*, we can know that we don't know whether we know-*wh*. So *KK* (and KK_n^\diamond) are false. The next section considers an objection to this kind of argument.

7 Non-Univocality of 'know'?

Defenders of *KK* and KK_n^\diamond might endorse these principles only when all occurrences of 'know' are interpreted the same way. After all, to make any interesting logical generalizations, we in general have to focus on "uniform" interpretations (see Dorr, 2014). This opens up a response strategy to my arguments: Perhaps the two occurrences of 'know' in assertions of

⁴⁴Compare Woozley (1953, 164)'s observation that there "is undeniably something odd about 'I think I know that...', whereas there's nothing at all odd about 'I think I know...' with any of the interrogatives." Holguín (2021b) similarly observes that knowledge-that variants are less natural in his *History Exam* case, quoted in §7.2 below.

⁴⁵See Kiparsky & Kiparsky (1971), Morgan (1969), and also Hintikka (1962, 13ff.).

⁴⁶Egré (2008) explains why 'Aisha doesn't know whether it's raining' suggests 'I don't know whether it's raining' through competition with 'Aisha doesn't know that it's raining.'

higher-order ignorance mean different things. This section considers a few ways of cashing out this idea, one building on fragmentation, the other on restricted readings of attitude verbs.

7.1 Fragmentation

Fragmentationists hold that concepts like ‘knowledge’ are too crude for us to expect a once-and-for-all answer to the question whether you know something. Rather, your epistemic state is fragmented, and your different fragments know different things. In different contexts, we can use the word ‘know’ to talk about different fragments.

Greco (2014, 2023) points out that fragmentation gives defenders of *KK* an extra resource: when we say that ‘You know, but don’t know that you know’, we may really mean that one of your fragments knows something, but a second fragment fails to know whether the first fragment knows. Applying this to assertions of higher-order ignorance, might

(17) I don’t know whether I know that it is raining.

perhaps mean that I don’t know₁ whether I know₂ that it is raining?

My main worry about such fragmentationist views is twofold. Without further constraints, the fragmentationist massively over-generates felicitous sentences. But once suitable constraints are in place, they may well rule out the proposed interpretation of (17). A simple example of over-generation are Moorean conjunctions like

(18) #It’s raining but I don’t know that it’s raining

Presumably, one fragment can know₁ that (it’s raining, but another fragment doesn’t know₂ that it’s raining)—just like I can know that (there is climate change, but Trump doesn’t know that there is). What prevents me from interpreting (18) in this perfectly felicitous way? We are generally great at finding felicitous interpretations for utterances when they are available. The fragmentationist needs to add constraints to their theory to explain why I cannot interpret (18) in this way.⁴⁷ Once such constraints are in place, they may rule out the proposed interpretation of (17).

The issue arises especially acutely for fragmentationists like Greco (2014, 2023) who argue for *KK* from conjunctions such as

(21) #It’s raining but I don’t know whether I know that it’s raining.

⁴⁷Problematic examples can be multiplied very easily here. Presumably one fragment can know₁ that another knows₂ whether *p* without itself knowing₁ whether *p*. In light of this, why can’t we access a good interpretation of (19) and (20):

(19) #Either I know that it is raining, or I know that it isn’t, but I don’t know which.

(20) #I know the answer but I don’t know what it is.

Greco (2014, 2023) suggests that we need *KK* to explain why (21) sounds weird. This argument is undermined if we explain the felicity of (17) by appeal to the fact that the two different occurrences of ‘know’ can be interpreted different ways. For then I should be able to interpret the two occurrences of ‘know’ in (21) in different ways, too. Fragmentationists have to make up their mind: if they appeal to fragmentation to explain why we *can* say (17), they lose their explanation of why we *cannot* say (21).

Upshot: Fragmentationists need constraints to prevent Moorean conjunctions like (18) and dubious conjunctions like (21) from receiving knowable interpretations. But there is at least some danger that such extra constraints will require ‘know’ to be interpreted univocally in assertions of higher-order ignorance, too, forcing them to give up *KK* and KK_n^\diamond after all. The next section explores on recent proposal for constraining context-sensitivity of attitude verbs like ‘know’ a bit more.

7.2 Question-sensitivity

Attitude verbs can arguably receive interpretations which are constrained by the answer to a relevant question.⁴⁸ Say that you know *p* relative to a question *Q* iff you know *p* conditional on the true complete answer to *Q*. To motivate that ‘know’ can express such question-relative knowledge, consider an example:

History Exam. Two students, Peggy and Pete, have purchased and subsequently memorized answer sheets from their corrupt teaching assistant, Roger. However, moments before the exam Roger shares an unfortunate discovery: one of Peggy or Pete was given answers to the wrong exam, but he doesn’t know who it was. All Roger knows is that the two answersheets give different answers to every question.

The first question of the exam reads: “In what year did the Berlin Wall fall?”. Peggy’s answersheet says b: 1989, Pete’s says c: 1991. Peggy is thus the one with the good answers. (Holguín, 2021b, 275)

Intuitively, Peggy might say the following things:

- (22) a. If I have the good answer sheet, then I know the answer.
b. If I have the bad answer sheet, then I don’t know the answer.
- (23) I don’t know whether I know the answer.

Suppose ‘know’ can express knowing relative to the question question *Q* = *Does Peggy have the good answer sheet?*. Then we can make sense of what Peggy says: she (unrestrictedly) fails to know what she knows relative to

⁴⁸See Blumberg & Holguín (2019); Blumberg & Lederman (2020); Holguín (2021b), building on the theory of epistemic modals by Dorr & Hawthorne (2013).

Q. There is one answer to Q compatible with Peggy's knowledge (*yes!*) conditional on which she clearly *does* know the exam answer, and another answer (*no!*) conditional on which she is ignorant of the exam answer.

(23) already is an expression of higher-order ignorance. Can this account be extended to our examples? For some of our examples, this seems possible. For example, in the case of John Bradley, the question could be *Is the ending we shot the real ending?*. Generically, the question could be *Is what I believe to be the answer really the answer?*.

In contrast to the fragmentationist view, the question-sensitive theory also has a natural way of ruling out assertions of Moorean and dubious conjunctions, by assuming the following constraints:

Assertion. Felicitous assertion requires *unconstrained* knowledge of what one says.

Monotonicity. If one knows something unconstrainedly, one knows it relative to every question.

Together, **Assertion** and **Monotonicity** entail that Moorean conjunctions like (18) are unassertable.⁴⁹ Moreover, dubious conjunctions like (21) are predicated unassertable given the analogue of KK (or KK_n^\diamond) for question-sensitive knowledge.⁵⁰ And if we accept KK (or KK_n^\diamond) for unconstrained knowledge, it is natural to accept analogues for question-relative knowledge.⁵¹ Even with all these constraints in place, the question-relative theory still predicts that assertions of higher-order ignorance can have a knowable interpretation.⁵²

This question-sensitive theory is an internally stable theory of Moorean conjunctions, dubious conjunctions, and assertions of higher-order ignorance, and I recommend it to those who are already inclined to accept KK

⁴⁹We assume that knowledge relative to a question satisfies analogues of **RM** (**QRM**: if $p \vdash r$ then $\Box^Q p \vdash \Box^Q r$), **N** (**QN**: if $\vdash p$ then $\vdash \Box^Q p$), and **T** (**QT**: $\Box^Q p \supset p$). Suppose you felicitously assert the Moorean conjunction $(p \wedge \neg \Box^Q p)$. By **Assertion**, we have $\Box(p \wedge \neg \Box^Q p)$, which entails $\Box^Q(p \wedge \neg \Box^Q p)$ by **Monotonicity**. By **QRM** we get $\Box^Q p \wedge \Box^Q \neg \Box^Q p$, from which the contradiction $\Box^Q p \wedge \neg \Box^Q p$ follows by **QT**.

⁵⁰Felicitously asserting $(p \wedge \neg \Box^Q \Box^Q p)$ by **Assertion** requires $\Box(p \wedge \neg \Box^Q \Box^Q p)$, which entails $\Box^Q(p \wedge \neg \Box^Q \Box^Q p)$ by **Monotonicity**. Reasoning from §5 shows this to be inconsistent with QKK ($\Box^Q p \supset \Box^Q \Box^Q p$) or even QKK_n^\diamond ($\Box^Q p \supset \Box^Q \Box^Q \Box^Q p$).

⁵¹Blumberg & Lederman (2020, fn. 32) suggest, crediting Jeremy Goodman and Matt Mandelkern, that question-sensitive knowledge may be radically introspectively inaccessible in a way that unconstrained knowledge is not. They observe that one can fail to know whether one knows p relative to Q because one does not know what the true answer to Q is. Their observation suggests only that one may lack *unconstrained* knowledge of what one knows relative to a question Q , not that one may lack knowledge *relative to question* Q of what one knows relative to question Q . Their example only motivates widespread failures of the principle $\Box^Q p \supset \Box \Box^Q p$, not failures of QKK ($\Box^Q p \supset \Box^Q \Box^Q p$).

⁵²*Proof.* Let $W = \{w_1, w_2\}$ with $R(w_1) = R(w_2) = W$ and $R^Q(w) = R(w) \cap [Q]_w$, for $[Q]_w$ the true answer to Q at w . Consider $V(p) = \{w_1\}$ and $V(Q) = \{\{w_1\}, \{w_2\}\}$. Then $\Box(\neg \Box^Q p \wedge \neg \Box \neg \Box^Q p)$ is true at both w_1 and w_2 .

and KK_n^\diamond . I am also independently sympathetic to the idea that ‘know’ can have question-relative interpretations. This being said, I think the question-sensitive theory still is on balance not all that attractive.

First, just because question-relative interpretations of ‘know’ are *sometimes* accessible when a question is very salient, they need not *always* be accessible. Simply rejecting KK (or KK_n^\diamond) also allows assertions of higher-order ignorance to be known, and does not commit us to the idea that question-relative interpretations are so freely available.

Second, the question-sensitive theory still over-generates acceptable readings. Imagine Peggy says the following:

(24) #I know the answer, I just don’t know whether I know that it’s 1989 or that it’s 1991.

(25) #Either I know that the answer is 1989 or I know that it is 1991.

Recall that $Q = \text{Do I have the good answer sheet?}$. Peggy can know that she either knows relative to Q that the answer is 1989, or else that she knows relative to Q that the answer is 1991 ($\Box(\Box^Q p \vee \Box^Q \neg p)$). But even so, she cannot felicitously say (24) and (25), suggesting that question-relative readings are not as freely available as the theory hypothesises.⁵³ Of course, one could attempt to constrain the question-sensitive theory further to rule out the predicted acceptable interpretations of (24) and (25). But again, it is not clear that this can be done in a way that does not rule out the knowable interpretation of expressions of higher-order ignorance.

A third problem concerns anaphora. I can felicitously say:⁵⁴

(26) John doesn’t know the answer, and I don’t know whether I’m in the same situation.

Presumably due to the parallel sentence structure, the first and second occurrence of ‘know’ have to be interpreted in a coordinated way. And presumably due to anaphora the interpretation of ‘in the same position’ has to be coordinated with the interpretation of ‘doesn’t know the answer’. But on a reading where all three occurrences of ‘know’ mean the same thing, the sentence should be unknowable because its second conjunct is, assuming that KK or KK_n^\diamond hold.⁵⁵

Finally, there is a controversial, but to my mind convincing, independent objections to KK , due to Williamson (2000). I will not press the objection here because it is unlikely to be dialectically helpful.

⁵³See Holguín (2021b, §4.2 and 4.3) for interesting discussion. Holguín suggests that constrained readings are only possible when they do not trivialise the knowledge-ascription. Anti-triviality appears to be a general feature of quantifier domain restrictions. (At a department party, ‘Everyone is happy’ is easily interpreted as ‘Everyone at the party is happy’, but ‘Everyone is at the party’ is not.) But it doesn’t help with our examples.

⁵⁴Thanks to *Anonymized* for suggesting this argument.

⁵⁵ $\Box(\neg\Box^Q\Box^Q p \wedge \neg\Box^Q\neg\Box^Q p)$ entails $\Box^Q(\neg\Box^Q\Box^Q p \wedge \neg\Box^Q\neg\Box^Q p)$ by **Monotonicity**, which is inconsistent with QKK and QKK_n^\diamond by the reasoning from §5.

7.3 Metalinguistic knowledge

If you ask me if my friend Patrick is tall, I will say that I don't know, despite knowing Patrick's height to the centimetre. This phenomenon has been explained in all sorts of ways, but one interpretation of my answer may be meta-linguistic: perhaps what I'm saying is that I don't know how the word 'tall' is interpreted in the context of our conversation.⁵⁶

Could meta-linguistic interpretations of attitudes explain why we can say that we are higher-order ignorant? For example, when Rudy Giuliani said he doesn't know if he knows what the definition of 'rigged' means, might he have meant that he didn't know how stringently the word 'know' was being interpreted in the context? Perhaps. I don't mean to deny that meta-linguistic readings of knowledge-ascriptions are possible. Even so, meta-linguistic interpretations are not plausible for many other examples above. When the speaker of (31) says that they don't know whether they know their password, they are not ignorant of the operative interpretation of 'know'. They know perfectly well what it would *take* to count as 'knowing' their password, they just don't know if they satisfy that bar. When Marian Schembari writes that she hopes she knows when to just watch her daughter's life unfold, rather than trying to capture it by filming it, she is not hoping that she is interpreting the word 'know' correctly.⁵⁷

The meta-linguistic interpretation may well be right for some expressions of higher-order ignorance, but the sheer variety of examples suggests that it is not a good way to save KK and KK_n^\diamond from my objections.

8 Infelicity without Ignorance?

If we reject KK and KK_n^\diamond , we predict higher-order ignorance can be known, and hence felicitously asserted. But we will also predict that dubious conjunctions like (21) can be known, which are not felicitous.

(21) #It's raining but I don't know whether I know that it's raining.

If (21) can be known, why can't we say it? Everything I have said is compatible with a wide range of pragmatic explanations of the infelicity of dubious conjunctions. Let me mention three possible accounts.

First, we could take on board the idea that norms such as *Assert only what you know!* tend to generate a secondary norm roughly of the form *Assert only if you know that you know!* parasitic on the primary norm.⁵⁸ Even if an assertion is conforming to the primary norm, it may be infelicitous if it clearly falls short of the secondary norm. Second-level knowledge of

⁵⁶For very different implementations of this idea, see Barker (2002) and Stalnaker (1978).

⁵⁷Though of course there is a meta-linguistic interpretation of 'I hope that I'm tall', too.

⁵⁸See Benton (2013), DeRose (2002, 180f.), Goodman & Holguín (2022, 639f.), and Williamson (2000, 256; 2005, 231-4; 2013, 82f.).

dubious conjunctions is impossible, a secondary norm of this form suffices to explain why dubious conjunctions sound weird.

Second, perhaps felicitously asserted propositions become common ground, and the common ground must be *treated* as commonly known, or at least mutually known to be known (Stalnaker, 1978, 84). Since dubious conjunctions cannot be known to be known, they cannot very well be *treated as if* they were known to be known.⁵⁹

Third, building on Mandelkern & Dorst (2022)’s idea that assertion requires posturing as if one is absolutely certain of what one says, we might argue that in asserting something, we must posture not only as if we know what we assert, but also as if we know that we know what we assert, and so on.⁶⁰ Posturing comes cheap: one can posture as if one is absolutely certain even if it is totally obvious that one is not. But posturing is still constrained: one cannot posture as if one knows structural unknowabilities like Moorean conjunctions. Similarly, one cannot posture as if one knows that one knows dubious conjunctions.⁶¹

Whatever the explanation of the infelicity of dubious conjunctions, assertions of higher-order ignorance strongly suggest that it does not involve KK or KK_n^\diamond . This is a significant result, since dubious conjunctions are one of the most popular arguments for KK and KK_n^\diamond .⁶²

Let me conclude by repeating the two central lessons of this paper. First, the word “know” is not as context-sensitive or vague as is sometimes hypothesized. Since we can self-ascribe higher-order ignorance, taking seriously the possibility that one does not know something cannot in general raise the standards for the application of the word “know” to the point where one fails to satisfy them. Higher-order ignorance is not “elusive”. Second, knowing does not imply knowing that one knows. If it did, higher-order ignorance would be unknowable — but it isn’t.

⁵⁹Moorean conjunctions cannot be felicitously asserted, so we better say they cannot be treated as if they were known because they are obviously unknowable. But then it should be similarly hard to treat other structural unknowabilities as if they were known.

⁶⁰See also Goldstein (2024, forthcomingb)’s omega knowledge norm.

⁶¹One plausible explanation of this fact is that posturing is constrained by conceivability. You *can* posture as if you are my parent (even though that’s impossible), because such a scenario is at least conceivable.

⁶²Explaining the badness of dubious conjunctions is “widely regarded as a non-trivial challenge for the those who reject” KK (Holguín, 2021a, 572), and “arguably the leading quick and easy motivation for self-knowledge requirements if anything is.” (Barnett, 2021, 234). Goldstein (2022, forthcominga) uses dubious conjunctions to argue for KK_n^\diamond . For arguments from dubious conjunctions to KK or similar principles for justification, see Cohen & Comesaña (2013, 24f.), Das & Salow (2018, 3), DeRose (2002, §2.3), Greco (2014, §6; 2015a, §2.2; 2023, ch. 6), Hamburger (1987, 251f.), McHugh (2010, 244), Smithies (2012, §§3-4).

A Data

In this appendix, I provide data showing that higher-order ignorance can be felicitously asserted, presupposed, and implicated.

A.1 Assertion

Ignorance whether one knows

- (27) I don't know whether I know the answer, basically.⁶³
- (28) You do know what I mean, don't you Chris?—I don't know whether I know or not.⁶⁴
- (29) I don't know if I know what I'm doing with this race mod.⁶⁵
- (30) I still don't know whether I know how to write a sentence.⁶⁶

Uncertainty about whether one knows

- (31) What if I'm not sure whether I know the password?⁶⁷
- (32) Sometimes I have moments where I'm not sure whether I know what I'm doing as a parent.⁶⁸
- (33) [I]t's only been three months and I'm not sure if I know that I'm ready.⁶⁹
- (34) I am not sure whether I know what a common cultural heritage is, but I suspect that it is grounded in the ideas and values of the Renaissance, in the Enlightenment Project and in liberal political thought.⁷⁰

A.2 Presupposition

- (35) But I sometimes wonder whether I know what this word means.⁷¹
- (36) How Do I Know If I Know?⁷²

⁶³<https://academic.oup.com/humrep/article/20/3/810/2356635>.

⁶⁴Anna Chapin, *Kitty Love*, p. 12, New York, Dodd, Mead and Company, 1912: <https://www.loc.gov/item/12021325/>.

⁶⁵<https://community.playstarbound.com/threads/i-dont-know-if-i-know-what-im-doing-with-this-race-mod-151874/>.

⁶⁶Apparently by Ethan Canin, see <https://www.allgreatquotes.com/quote-133768/>.

⁶⁷<https://support.tigertech.net/lost-email-password>.

⁶⁸<https://www.watchingyoutgrow.co.uk/2015/10/5-reasons-why-i-dont-want-your.html>.

⁶⁹https://www.washingtonpost.com/lifestyle/style/ask-amy-pot-smoking-husband-wants-to-toke-freely/2018/11/19/c7daee0a-e85f-11e8-a939-9469f1166f9d_story.html.

⁷⁰Mrs. Clwyd, House of Commons Jan. 1993: <https://publications.parliament.uk/pa/cm199293/cmhansrd/1993-01-19/Debate-2.html>.

⁷¹Heather Love: <https://muse.jhu.edu/article/421589/summary>.

⁷²Book on religion: <https://www.amazon.com/How-Do-Know-If/dp/1609079213>.

- (37) How do I know if I know what I'm doing? Narratives of Self Confidence and Competence Among Elderly Men Caregivers.⁷³
- (38) From your comments I hope that I know where you have problem with the implementation.⁷⁴
- (39) Well, I'm like a drug addict, I'm always saying I'm going to stop, and then I don't, what I've said consistently is that I hope I know when to stop: when it starts to get repetitive.⁷⁵
- (40) I hope I know when, when people are kind of tired of me.⁷⁶
- (41) I hope I know when it's time quit the business.⁷⁷
- (42) As a historian/biographer, I hope that I know how to filter for embellishment, for bragging, for self-flattery, and for lying.⁷⁸

A.3 Implicature

- (43) I think I know why you're toying with me everyday
 [...] I think I know what you're saying with your heart on your sleeve
 [...] I think I know that you are the queen
 [...] I think I know how you got in my head.⁷⁹
- (44) My Girlfriend Wears Too Much Makeup and Now I Think I Know Why.⁸⁰
- (45) Logan: Don't worry, gorgeous. I'm just using your phone. I'm not going to murder you. — Grace: Oh, I know that. Or at least I think I know that. I mean, you seem like a decent guy, but then again, lots of serial killers probably seem decent too when you first meet them.⁸¹
- (46) I think I know who it is. I saw his picture in the paper.⁸²
- (47) I might know who the phantom is and it may be surprising.⁸³

⁷³Title of an abstract by R. Russell in Abstracts, *The Gerontologist*, Vol. 49, Iss. suppl. 2, 2009, p. 360: <https://doi.org/10.1093/geront/gnp147>.

⁷⁴<https://stackoverflow.com/questions/7067439/reload-of-jqgrid-not-happening-with-loadonctrue>.

⁷⁵Stephen King: <https://www.theguardian.com/books/2000/sep/14/stephenking>. fiction.

⁷⁶Ellen DeGeneres: <https://www.youtube.com/watch?v=xrCLlg31L4A>.

⁷⁷https://twitter.com/Jonathan_Katz/status/79006652694663168.

⁷⁸Mark Harris: <https://twitter.com/MarkHarrisNYC/status/1665784399063928835>.

⁷⁹Lyrics from a song called *I think I know*: <https://www.boomplay.com/lyrics/152977299>.

⁸⁰https://www.reddit.com/r/nosleep/comments/uv4sfv/my_girlfriend_wears_too_much_makeup_and_now_i/.

⁸¹<https://www.theramblingbooknerd.com/post/the-mistake-by-elle-kennedy>.

⁸²<https://trailers.getyarn.io/yarn-clip/95ee7f13-e889-4060-a77f-a4502eaf3998/gif>.

⁸³https://www.reddit.com/r/IEExpectYouToDie/comments/15wljra/i_might_know_who_the_phantom_is_and_it_may_be/

- (48) I might know where the First Elemental Masters got their Powers from.⁸⁴
- (49) I might know a way in.⁸⁵
- (50) I think I know what's causing the pain in your shins (shin splints).⁸⁶

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⁸⁴<https://ninjago.fandom.com/f/p/3235013524980032818>.

⁸⁵Line in the movie 'Kickboxer 4: The Aggressor'. See <https://www.quotes.net/mquote/1027708>.

⁸⁶<https://www.runandbecome.com/running-product-advice/know-foot-strike>.

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