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Session Arrows: A Session-Types Based Framework For Parallel Code Generation

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Abstract

Parallel code is notorious for its difficulties in writing, verification and maintenance. However, it is of increasing importance, following the end of Moore's law. Modern programmers are expected to utilize the power of multi-core CPUs and face the challenges brought by parallel programs.

This project builds an embedded framework in Haskell to generate parallel code. Combining the power of multiparty session types with parallel computation, it creates a session typed monadic language as the middle layer and use Arrows, a general interface to computation as an abstraction layer on top of the language. With the help of the Arrow interface, we convert the data flow of the computation to communication and generate parallel code according to the communication pattern between participants involved in the computation. Thanks to the addition of session types, not only the generated code is guaranteed to be deadlock-free, but also we gain a set of local types so that it is possible to reason about the communication structure of the parallel computation.

In order to show the framework is as expressive as usual programming languages, we write a few common parallel computation patterns and three algorithms to benchmark using our framework. It demonstrates that users can express computation similar to traditional sequential code and gain a high-performance parallel code in low-level target languages such as C for free and the use case of the framework is not limited to a standalone tool for parallel computation; we show the framework can act as a code generation backend for other data-flow based high-level parallel languages with an example. Benchmark shows the generated code can have up to 12X speedup on certain input sizes on a 32-core machine.

Acknowledgements

TBD

Contents

1	Introduction	5
1.1	Motivation	5
1.2	Contributions	6
1.3	Report outline	7
2	Background	8
2.1	Arrows	8
2.1.1	Definition	8
2.1.2	Example: Calculate the mean	9
2.1.3	Application in parallel computation	11
2.2	Recursion Schemes	11
2.2.1	Definition	11
2.2.2	Example: Merge sort	12
2.3	Multiparty session types	13
2.3.1	Global types and local types	13
2.3.2	Applications in parallel computing	16
2.4	Message passing concurrency	16
2.4.1	Primitives for message-passing concurrency	17
2.4.2	Concurrency Monads	17
2.5	Free monad	19
2.5.1	Definition	20
2.5.2	Example	20
2.5.3	Applications	22
3	Alg : Algebraic Functional Language	23
3.1	Alg	23
3.1.1	Syntax	23

3.2	ParAlg: Alg + role annotations	25
3.2.1	Syntax	26
3.2.2	Inferring global types	26
3.2.3	Example: Parallel merge sort	27
3.3	Conclusion	27
4	SPar: A session typed free monad EDSL for concurrency	29
4.1	Computation: The Core EDSL	29
4.1.1	Syntax	29
4.1.2	Representation of recursive data structures	30
4.2	Communication: The Proc EDSL	31
4.3	Concurrent computation: A group of Proc	31
4.3.1	Operational semantics	32
4.3.2	Session types and duality checking	34
4.4	Conclusions	35
5	SPar: Implementation	36
5.1	Session types	36
5.1.1	Representations of session types in Haskell	36
5.1.2	Value-level duality check	37
5.1.3	Type-level duality check	37
5.2	SPar interpreter	40
5.2.1	Overview	40
5.2.2	Implementation	40
6	SArrow: An arrow interface for writing SPar expressions	43
6.1	Syntax	43
6.1.1	Arrow interface	44
6.1.2	Example: Parallel programming patterns	44
6.2	Implementation of arrow combinators	47
6.3	Strategies for optimized role allocations	49
6.4	Satisfaction of arrow laws	52
6.5	Conclusions	53
7	Type-safe code generation from SPar	55
7.1	Instr: A low-level EDSL for channel communication	55

7.1.1	Syntax and semantic	55
7.1.2	Representation types	56
7.2	Compilation from SPar to Instr	57
7.2.1	Transformation from Proc to Instr	57
7.2.2	Strategies for channel allocations	59
7.2.3	Monad for code generation	59
7.3	Code generation to C: from Instr to C	60
7.3.1	Representations of Core data type in C	60
7.3.2	Compiling from Core to C	62
7.3.3	The structure of generated C code	63
7.4	Conclusion	63
8	Parallel algorithms and evaluation	65
8.1	Parallel algorithms	65
8.1.1	Four steps to write parallel algorithms in SPar	65
8.1.2	Example: Merge sort	66
8.2	Benchmarks	69
8.2.1	Evaluation	70
9	Conclusions and future works	73
9.1	Conclusions	73
9.2	Future works	73
A	Examples of generated code	77
A.1	Merge sort	77

Chapter 1

Introduction

1.1 Motivation

Writing parallel software is not a trivial task. Parallel code is hard to write because it is usually done in low-level languages with verbose and non-idiomatic decorations, hard to debug because machines, where code is written, are usually different from machines where code is intended to run and hard to maintain and reuse. This is because even though the underlying algorithms are not changed, multiple version of parallel code is needed to tackle various platform and evolution of architectures.

There are many on-going pieces of research aimed at helping programmers to write correct parallel programs smoothly. A common approach is to develop a high-level language and compile programs in this language to parallel code. There are many high-level frameworks for parallel programming (e.g. algorithmic skeletons[1], domain-specific languages for parallelism[2] or the famous MapReduce parallel model[3]). An example is to use arrow terms (Section 2.1) to describe data flow implicitly and hence generate parallel code [4].

The workflow of writing parallel code has evolved from writing it directly in the target platform to writing software in a high-level language designed for parallel computation and then compiling to the target platform. In this project, we present a method to improve the backend of parallel code generation by introducing a monadic domain-specific language: SPAr to act as a bridge between high-level and target low-level parallel languages.

This specific language needs to be general enough so that it supports multiple high-level parallel programming frameworks. It can be used to generate different parallel code such as Message Passing Interface (MPI) and Cuda. Moreover, we developed a simulator to aid debugging parallel programs.

Our framework couples with multiparty session type (MPST) [5]. It is done by inferring session types for computation of each participant. We can take advantages of the collection of local types to enable aggressive optimization but ensuring code correctness and meaningful static analysis; e.g. cost modelling for parallel programming according to properties of MPST.

1.2 Contributions

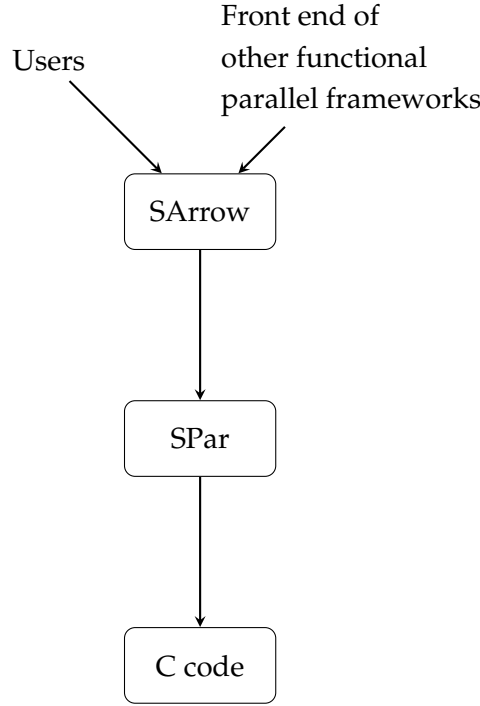


Figure 1.1: Visualization of the workflow

The result of project is an embedded high-level framework in Haskell that is capable of generating low-level parallel code. The major contributions are:

1. **Session-typed intermediate language.** We create an intermediate embedded domain specific language (EDSL): **SPar**: a session typed free monad EDSL for message passing concurrency. SPar is built using free monad technique, and it contains primitive operations for channel communication as well as its representation of computation. This language can be typed by local types, and hence, we can apply multiple results from the multiparty session types to our framework, especially in terms of safety of generated code and reasoning of communication patterns.
2. **Intuitive user interface.** One innovation of this project is that we apply the Arrow interface for users to express parallel computations. Arrow is a general interface for computations (see in Section 2.1). We call our interface **SArrow**: an arrow interface for writing SPar expressions. It is an abstraction layer on top of SPar, which hides communication primitives from users so that users can express parallel algorithms similar to what they would write for sequential programs.
3. **Multiple backends.** We create a backend to generate parallel C code from SPar expressions. The core of the backend is Instr: a low-level EDSL for channel communication as well as operations on variables and resources management. It is independent of target languages. This means that we can support multiple target languages with ease without re-implementing multiple backends. In addition to

the code generation backend, we implement an interpreter backend in Haskell for experimenting and fast verification.

4. **Evaluations.** Finally, we show the expressive power of the framework by implementing several common computation patterns and three algorithms using our interface. We evaluate the performance of the generated code from the algorithms on high-performance computers as well as PCs.

The Figure 1.1 summaries the workflow of the framework visually. The main principle supporting the framework is that **we convert data flow into communication and from the communication pattern, we gain parallel code**. The results of expressing computation in the framework are 1) compilation to efficient deadlock-free low-level parallel programs and 2) a set of local types to reason the structure of the parallel computation.

At the end of the project, we have discovered two use case of the framework. The primary application is a stand-alone tool to generate parallel C code from programs written in Haskell, and another is a backend for other data-flow based parallel frameworks.

1.3 Report outline

Chapter 2 gives an overview of the background and related research. We present the syntax and semantics of SPar in Chapter 4 followed by Chapter 5 introducing the implementation aspect of SPar, like session typing and developing the interpreter. Chapter 6 demonstrates the Arrow interface with examples of parallel patterns formed by the interface and justification of the interface satisfying the arrow laws. The discussion about some implementation specific issues like role allocation is also contained in Chapter 6. In Chapter 7, we show the code generation backend and discuss our solutions to challenges when compiling to C, i.e. the problem of representing polymorphic algebraic data structures in C. Chapter 8 explains our benchmark and shows the performance of the generated code. This chapter can also be regarded as a tutorial on how to use the framework. Finally, we conclude with potential future improvements and remarks on this project. We also include the generated C code in the appendix for the curious readers.

Chapter 2

Background

This section is an overview of techniques that influence the design choices of our monadic language for parallel computation. First of all, we give an overview of techniques applied in the high-level parallel programming framework: arrows (Section 2.1) and recursion schemes (Section 2.2). We then introduce several techniques for message-passing concurrency: multiparty session types (Section 2.3) and monadic languages for concurrency (Section 2.4). In the end, we introduce free monads (Section 2.5), a technique valuable in implementing embedded domain-specific languages (EDSL).

2.1 Arrows

Arrow is a general interface to describe computation. It can ease the process of writing structured code suitable for parallelising. It also demos a common feature of the frameworks: parallelizability is empowered by underlying implicit but precise data-flow. On the other hand, converting to low-level message-passing code, which requires programmers to define communication using message-passing function and primitives, makes the data-flow explicit.

2.1.1 Definition

Listing 1 shows the Arrow definition in Haskell. Intuitively, an arrow type $y \rightarrow a \rightarrow b$ (that is, the application of the parameterised type y to the two parameter types b and c) can be regarded as a computation with input of type b and output of type $b[6]$. Visually, arrows are like pipelines (shown in Figure 2.1). In Haskell, an arrow y is a type that implements the following interface (type classes in Haskell are roughly interfaces). `arr` converts an arbitrary function into an arrow. `>>>` sequences two arrows (illustrated in Figure 2.1b). Taking two input, `first` apply the arrow to the first input while keeping the second untouched (Figure 2.1a). Conversely, `second` modifies the second input and keeps the first one unchanged. `***` applies two arrows to two input side by side (Figure 2.1d). `&&&` takes one input and applies two separate arrows to the input and its duplications (Figure 2.1c).

The simplest instance of arrow class is the function type (shown in Listing 2). It

is worth noticing that only `arr` and `***` need to be implemented. The rest of functions in the arrow type class can be defined in terms of the two functions. For example, `f &&& g = (f *** g) . arr (\b -> (b, b))` and `first = (***) id`

```
1  class Arrow y where
2      arr :: (a -> b) -> y a b
3      first :: y a b -> y (a, c) (b, c)
4      second :: y a b -> y (c, a) (c, b)
5      (***) :: y a c -> y b d -> y (a, b) (c, d)
6      (&&&) :: y a b -> y a c -> y a (b, c)
```

Listing 1: Arrow class in Haskell

```
1  instance Arrow (->) where
2      arr f = f
3      (***) f g ~ (x, y) = (f x, g y)
```

Listing 2: (\rightarrow) instance of Arrow class

2.1.2 Example: Calculate the mean

Consider the function to calculate the mean from a list of floating point numbers, we present the implementation using arrows compared with a point-free Haskell definition. Implementation using arrows can be regarded as point-free programming. Point-free programming is programming paradigm where function definitions only involve combinators and function composition without mentioning variables[8].

```
1  mean :: [Float] -> Float
2  mean xs = sum xs / (fromIntegral . length) xs
3
4  mean' :: [Float] -> Float
5  mean' = (sum &&& (length >>> fromIntegral)) >>> uncurry (/)
```

The arrows implementation can be visualised in Figure 2.2.

```
1  mean'' :: [Float] -> Float
2  mean'' = liftM2 (/) sum (fromIntegral . length)
```

The above code snippet is the more traditional approach form of point-free mean function in Haskell. Arrows are not the only way to form point-free programs. We can argue this form of point-free function is more difficult to understand compared to arrows because it involves knowledge of monads (`liftM2`) and does not map to the intuitive data-flow.

The simple example demonstrates that arrows combinators make writing point-free programs easier. Arrows unites the implementation of algorithm and data-flow in the algorithm.

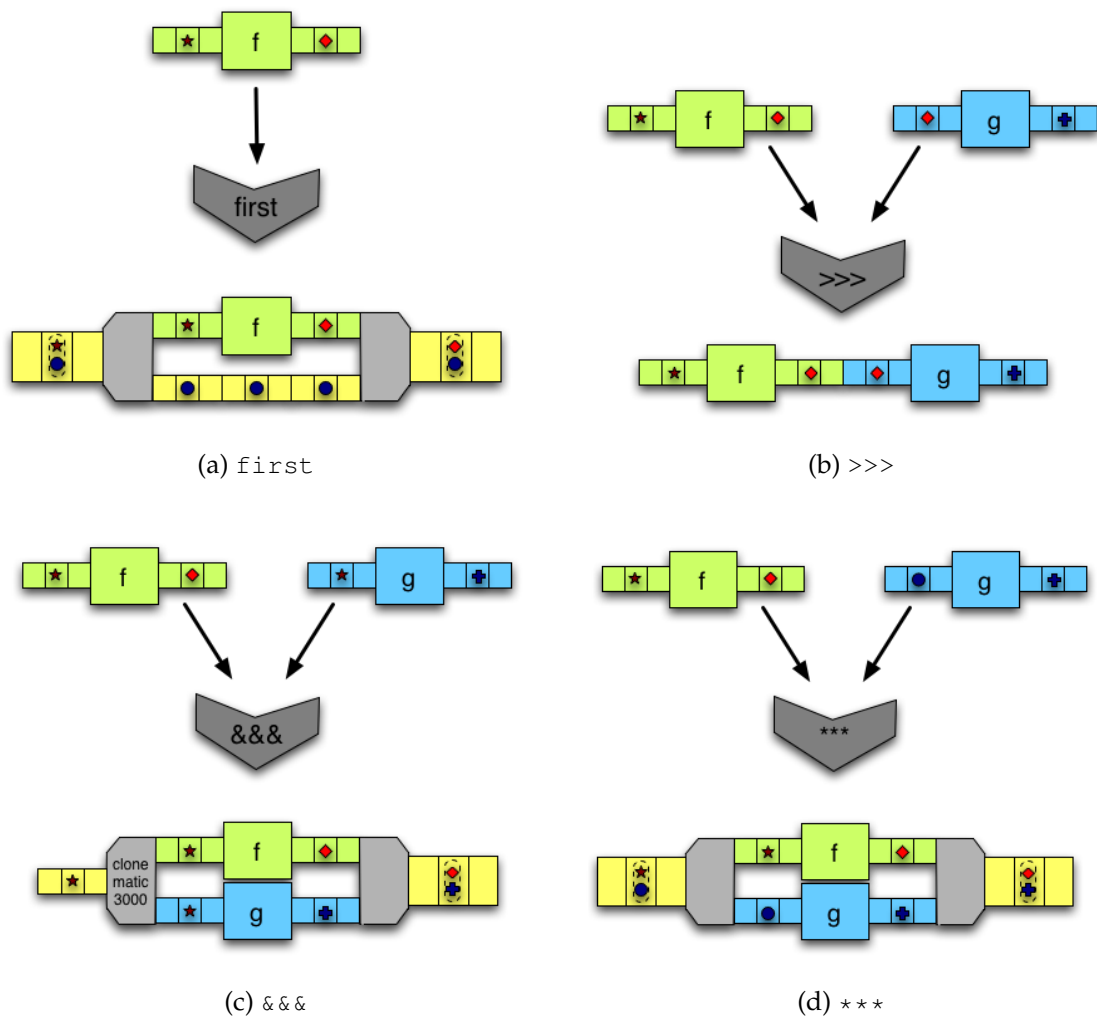


Figure 2.1: The visual representations of arrow combinators[7]

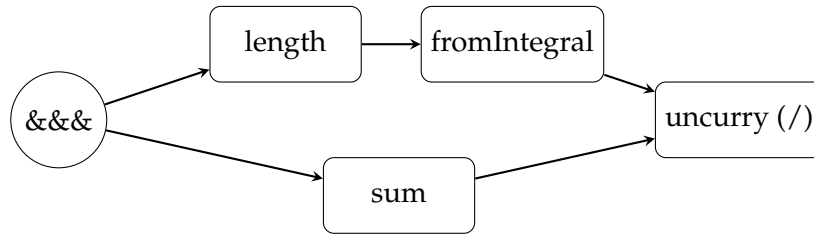
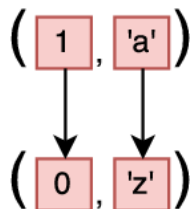


Figure 2.2: Visualization of mean'

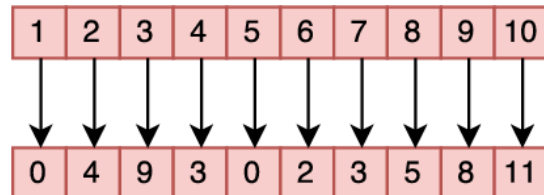
2.1.3 Application in parallel computation

From the previous example, the data flow of programs written using arrow combinators can be easily visualized (shown in Figure 2.1). It is intuitive to recognize that the clean separation between the flow of data and actual computation will be useful in generating parallel code. Indeed, arrow describes data flow implicitly, and it is an example of the so-called algebraic pattern. Many works has been done to generate parallel code from algebraic patterns [4, 9, 10]. In particular, details of [10] are introduced in Chapter 3.

We will use some figures to explain the idea behind arrows as a framework for parallel computation. For example, as shown in the Figure 2.3a, $f \text{ *** } g$ means computations of f and g happen in parallel. Figure 2.3b shows that it can be used to implement parallel map in terms of arrows, taking an arrow computation $\text{arr } a \text{ } b$ and returning a list of computation in parallel ($\text{arr } [a] \text{ } [b]$).



(a) Visualization of parallel ***[4]



(b) Visualization of parMap [4]

2.2 Recursion Schemes

Recursion schemes are patterns for expressing recursive functions. In particular, they are high order function abstracting common patterns of recursion.

2.2.1 Definition

We will introduce three typical recursion schemes: catamorphisms representing folds, anamorphisms representing unfolds and hylomorphisms representing divide-and-conquer algorithm (seen in Listing 4). Recursion schemes express recursion with the help of data structures that mirror the control structure of the recursion.

```
1 newtype Fix f = Fix { unfix :: f (Fix f) }
```

```
2
```

```

3 data TreeF a =
4     Node a a
5   | Leaf int
6   | Empty
7   deriving Functor
8
9 type Tree = Fix TreeF

```

Listing 3: Definition of fix point of data structures

Anamorphisms takes a function from a to $f\ a$ (called the co-algebra) and a value a and return the $\text{Fix}\ f$. Used `Tree` as an example, anamorphisms takes a single value a and applies the co-algebra to the value. It continues to apply itself to the branches of the `TreeF` recursively and finally expands a single value to a complete tree. Intuitively, anamorphism unfolds a single value to a complicated data structure top-down.

Catamorphisms is the reverse of anamorphisms, folding a data structure to a single value bottom-up. It takes a function from $f\ a$ to a (called the algebra) and $\text{Fix}\ f$ to fold and return a single value a . Catamorphisms and anamorphisms describe the process globally (from a to $\text{Fix}\ f$ and from $\text{Fix}\ f$ to a) while co-algebra and algebra capture what happened locally. The elegant part is while co-algebra and algebra do not involve with any recursion data structure (`TreeF` is not recursive), catamorphisms consumes recursive data structure while anamorphism builds them.

Hylomorphisms applies anamorphism followed by catamorphisms. It is the most common pattern to use. We will use an example to illustrate its usefulness. It can be thought of as an abstract divide and conquer algorithm.

```

1 ana :: Functor f => (a -> f a) -> a -> Fix f
2 ana coalg = Fix . fmap (ana coalg) . coalg
3
4 cata :: Functor f => (f a -> a) -> Fix f -> a
5 cata alg = alg . fmap (cata alg) . unfix
6
7 hylo :: (f b -> b) -> (a -> f a) -> b -> a
8 hylo g f = f . fmap (hylo f g) . g

```

Listing 4: Recursion schemes in haskell

2.2.2 Example: Merge sort

We can write merge sort recursively. First of all, we split the list in half and then apply the merge sort recursively to both parts and finally we merge two lists into a single list.

To write merge sort in terms of recursion scheme, we need to define the recursive structure to represent the control structure. By the definition of merge sort, this structure must have a case with two branches, a base case representing a singleton list and a base case representing an empty list hence this structure is the `TreeF` we defined above. Split-

ting a list is a co-algebra while merging is an algebra. We use hylomorphisms to combine them hence getting a sorted list (seen in Listing 5).

```

1 mergeSort :: [Int] -> [Int]
2 mergeSort = hylo merge split where
3   merge Empty      = []
4   merge (Leaf c)   = [c]
5   merge (Node l r) = usualMerge l r
6
7   split [] = Empty
8   split [x] = Leaf x
9   split xs = Node l r where
10      (l, r) = splitAt (length xs `div` 2) xs

```

Listing 5: Merge sort using hylomorphisms

2.3 Multiparty session types

In complicated distributed systems, participants agree on a protocol, specifying the type and direction of data exchanged. Multiparty session types are a branch of behavioral types specifically targeted at describing protocols in distributed systems based on asynchronous communication [5]. They are a type formalism used to model communication-based programming by codifying the structure of the communication. The evolution of computing from the era of data processing to the era of communication witnessed the growth and significance of the theory of session types.

The theory of multiparty session types contains three main elements. Global types (seen in Section 2.3.1), local (session) types and processes. Processes are the concrete descriptions of the behavior of the peers involved in the distributed system [5] using a formal language. The most used and the original language is π -calculus [11]. The coming sections are an intuitive introduction of session types by examples.

2.3.1 Global types and local types

A global type is at the most abstract level, describing a communication protocol from a neutral viewpoint between two or more participants[5]. The syntax of global types is shown in Table 2.1 and an example of global types is shown in Table 2.3.

Local types or session types characterise the same communication protocol as the global type, but from the viewpoint of each peer [5]. Each process is typed by a local type. The syntax of local types is shown in Table 2.2 and an example of local type is shown in Table 2.4.

The relationship between global types and local types are established by the projection operator (seen in the Section 2.3.1.1), and a type system performs syntactic checks, ensuring that processes are typed by their corresponding local types. Hence, at the compile time, three important properties follow [5].

- **communication safety:** Mismatches between the types of sent and expected messages, despite the same communication channel is used for exchanging messages of different types, do not exist [5].
- **protocol fidelity:** The interactions that occur are accounted for by the global type and therefore are allowed by the protocol [5].
- **progress:** Every message sent is eventually received, and every process waiting for a message eventually receives one [5].

We will learn that these properties are valuable not only in the distributed system but also in the domain of parallel computing in Section 2.3.2.

$G ::=$	Global types
$p \rightarrow q : \langle S \rangle . G$	Value exchange
$p \rightarrow q : \langle T \rangle . G$	Channel exchange
$p \rightarrow q : \{l_i : G_i\}_{i \in I}$	Branching
$\mu t . G \mid t \mid \text{end}$	Recursion/End

Table 2.1: Global types

$S ::=$	Sorts	$T ::=$	Session types/local types
$\text{bool} \mid \text{nat} \mid \text{string}$		$! \langle p, S \rangle . T$	Send value
\dots		$! \langle p, T \rangle . T$	Send channel
		$? \langle p, T \rangle . T$	Channel Receive
		$? \langle p, S \rangle . T$	Sorts Receive
		$\oplus \langle p, \{l_i : T_i\}_{i \in I} \rangle$	Selection
		$\& \langle p, \{l_i : T_i\}_{i \in I} \rangle$	Branching
		$\mu t . T \mid t \mid \text{end}$	Recursion/End

Table 2.2: Session types/local types

2.3.1.1 Projection between global types and local types

Projection is the formalization of the relationship between global and local types. It is an operation extracting the local type of each peer from the global type [5]. The definition of projection is shown in Table 2.5.

As an example, a projection of global type in Table 2.3 is

$$G \upharpoonright 0 = ! \langle 1, \text{string} \rangle ; ? \langle 1, \text{string} \rangle ; \& \langle 1, \{ \text{accept} : ? \langle 1, \text{string} \rangle ; ! \langle 2, \text{string} \rangle ; ? \langle 2, \text{int} \rangle , \text{reject} : \text{end} \} \rangle$$

1. Customer(0) sends an order number to Agency(1), and Agency sends back a quote to the customer.
2. If Customer is happy with the price then Customer selects accept option and notifies Agency.
3. If Customer thinks the price is too high then Customer terminate the trade by selecting reject.
4. If accept is selected, Agency notify both Customer and Agency2(2).
5. Customer sends an address to Agency2 and Agency2 sends back a delivery date.

$$\begin{aligned}
G = & \\
& 0 \rightarrow 1 : \langle \text{string} \rangle. \\
& 1 \rightarrow 0 : \langle \text{int} \rangle. \\
& 0 \rightarrow 1 : \{ \text{accept} : \\
& \quad 1 \rightarrow \{0, 2\} : \langle \text{string} \rangle. \\
& \quad 0 \rightarrow 2 : \langle \text{string} \rangle. \\
& \quad 2 \rightarrow 0 : \langle \text{int} \rangle. \text{end}, \\
& \text{reject} : \text{end} \}
\end{aligned}$$

Table 2.3: An example of a protocol described by global types G

$$\begin{aligned}
S &\triangleq \mu t. (\&\{\text{balance} : ![\text{nat}]; t, \\
&\quad \text{deposit} : ?[\text{nat}]; ![\text{nat}]; t, \\
&\quad \text{exit} : \text{end}\}) \\
C &\triangleq \oplus\{\text{balance} : ?[\text{nat}]; \text{end}, \\
&\quad \text{deposit} : ![\text{nat}]; ?[\text{nat}]; \text{end}\}
\end{aligned}$$

Table 2.4: Session types of client and server end point of a ATM service

$$\begin{aligned}
(\mathbf{p} \rightarrow \mathbf{p}' : \langle U \rangle. G') \upharpoonright \mathbf{q} &= \begin{cases} !\langle \mathbf{p}', U \rangle. (G' \upharpoonright \mathbf{q}) & \text{if } \mathbf{q} = \mathbf{p}, \\ ?\langle \mathbf{p}, U \rangle. (G' \upharpoonright \mathbf{q}) & \text{if } \mathbf{q} = \mathbf{p}', \\ G' \upharpoonright \mathbf{q} & \text{otherwise.} \end{cases} \\
(\mathbf{p} \rightarrow \mathbf{p}' : \{l_i : G_i\}_{i \in I}) \upharpoonright \mathbf{q} &= \begin{cases} \oplus \langle \mathbf{p}', \{l_i : T_i\}_{i \in I} \rangle & \text{if } \mathbf{q} = \mathbf{p} \\ \&(\mathbf{p}, \{l_i : G_i \upharpoonright \mathbf{q}\}_{i \in I}) & \text{if } \mathbf{q} = \mathbf{p}' \\ G_{i_0} \upharpoonright \mathbf{q} & \text{where } i_0 \in I \text{ if } \mathbf{q} \neq \mathbf{p}, \mathbf{q} \neq \mathbf{p}' \\ & \text{and } G_i \upharpoonright \mathbf{q} = G_j \upharpoonright \mathbf{q} \text{ for all } i, j \in I. \end{cases} \\
(\mu t. G) \upharpoonright \mathbf{q} &= \begin{cases} \mu t. (G \upharpoonright \mathbf{q}) & \text{if } G \upharpoonright \mathbf{q} \neq t, \\ \text{end} & \text{otherwise.} \end{cases} \quad t \upharpoonright \mathbf{q} = t \quad \text{end} \upharpoonright \mathbf{q} = \text{end}.
\end{aligned}$$

Table 2.5: The definition of projection of a global type G onto a participants \mathbf{q} [5]

2.3.1.2 Duality of session types

In binary session types where all protocols are pairwise, duality formalises the relationship between the types of opposite endpoints. For a type T , its dual or co type, written \bar{T} is defined inductively as in Table 2.6.

$$\begin{array}{llll} \overline{?[\tilde{S}]; T} & = & ![\tilde{S}]; \bar{T} & \overline{\oplus\{l_i : T_i\}_{i \in I}} & = & \&\{l_i : \bar{T}_i\}_{i \in I} & \overline{?[T]; T'} & = & ![\bar{T}]; \bar{T}' \\ \overline{![\tilde{S}]; T} & = & ?[\tilde{S}]; \bar{T} & \overline{\&\{l_i : T_i\}_{i \in I}} & = & \oplus\{l_i : \bar{T}_i\}_{i \in I} & \overline{![T]; T'} & = & ?[\bar{T}]; \bar{T}' \\ \overline{\text{end}} & = & \text{end} & \overline{\mu t. T} & = & \mu \bar{t}. \bar{T} & \overline{\bar{t}} & = & t \end{array}$$

Table 2.6: Inductive definition of duality

Duality is essential for checking type compatibility. Compatible types mean that each common channel k is associated with complementary behavior: this ensures that the interactions on k run without errors.

In order to apply duality into multiparty session types in which more than two participants are allowed, the partial projection operation (seen in [5]) from multiparty session type to binary session type was introduced to allow reusing the definition of duality after applying the partial projection.

2.3.2 Applications in parallel computing

Multiparty session types not only have rich applications in distributed systems but also value in the domain of parallel computation.

Existing work[12] has shown how to generate MPI¹ programs using session types. Users describe the communication topology as a skeleton using a protocol language which is type checked by session types. After that, an MPI program is generated by merging the skeleton and user-provided kernels for each peer. The parallel code obtained in this way is guaranteed to be deadlock-free and progressing.

2.4 Message passing concurrency

This section introduces some interfaces for message passing concurrency from the primitive case: channel to more advanced one: monad for message passing concurrency.

For simplicity, they are represented in Haskell, but in general, most languages can implement similar interfaces.

¹Message Passing Interface (MPI) is a standardised and portable message-passing standard designed by a group of researchers from academia and industry to function on a wide variety of parallel computing architectures [13].

2.4.1 Primitives for message-passing concurrency

In Section 2.3, channels are bi-directional and used for communication between two parties. In Haskell, channel primitives are represented in Listing 6. However, just using these primitives cannot guarantee progress or communication safety. For example, a program that has one thread writing channel once combined with another thread reading channel twice is type-correct but will cause deadlock. Many kinds of research to encode MPST using Haskell's type system are presented in [14] so that an (MPST) type-correct Haskell program assures progress, communication safety and session fidelity.

```
1 data Chan a
2   newChan :: IO (Chan a)
3   writeChan :: Chan a -> a -> IO ()
4   readChan :: Chan a -> IO a
5   dupChan :: Chan a -> IO (Chan a)
```

Listing 6: Channel primitives in Haskell

2.4.2 Concurrency Monads

The work done by [15] constructs a monad to express concurrent computation. The definition is in Listing 7. `Action` is the algebraic datatype representing basic concurrency primitives. `Atom`, the atomic unit of computation, is a computation (wrapped in the `IO` monad) followed by an action. `Fork` is two parallel action. `Stop` is the termination of an action. `type C` is a special case of the continuation monad. The continuation monad is an encapsulation of computations in continuation-passing style (CPS)². So `C a` is a CPS computation that produces an intermediate result of type `a` within a CPS computation whose final result type is `Action`. With the help of the monad `C`, sequencing and composing actions can use monadic `bind`.

The idea is using continuation to represent the "future" so that computation can pause and resume as well as expressing sequential computation. `Atom` wraps the actual computation and `Fork` is responsible for spawning threads. In addition, in order to write programs in a monadic way easier, some helper functions are defined (shown in Listing 8). `atom` lifts an `IO` computation to `C`. And `fork` takes a computation in `C` and return a `C` which involves the `Fork` action. Given a `C a`, `action` gives the result of running the CPS computation. We use `_ . Stop` to represent the final continuation (`Stop` action is the last action).

```
1 atom :: IO a -> C a
2 atom m = C $ \k -> Atom $ do
3   r <- m
4   return $ k r
5
```

²In continuation-passing style function result is not returned, but instead is passed to another function, received as a parameter (continuation)[16]

```

1 data Action =
2     Atom (IO Action)
3   | Fork Action Action
4   | Stop
5
6 newtype C a = C { runC :: (a -> Action) -> Action }
7
8 instance Monad C where
9     (>>=) :: C a -> (a -> C b) -> C b
10    m >>= f = C $ \k -> runC m (\v -> runC (f v) k)
11    return :: a -> C a
12    return x = C $ \k -> k x

```

Listing 7: The definition of concurrency monad

```

6 fork :: C () -> C ()
7 fork m = C $ \k -> Fork (runC m (const Stop)) (k ())
8
9 action :: C a -> Action
10 action m = m (\_. Stop)

```

Listing 8: Helper functions

An example of programme written in the concurrency monad is shown below.

```

1 example :: C ()
2 example = do
3     atom $ putStrLn "Hello"
4     name <- atom getLine
5     fork $ atom $ putStrLn "World"
6     atom $ putStrLn name

```

We can easily define a round-robin scheduler for programs in this monad. We can regard a list of action as a queue of threads that are running concurrently. `schedule` will pattern match on the head of the list. If it is `Atom` then the scheduler will run the computation (seen a `<- ioa` at ⑦) and pause its remaining computation and put it at the end of the thread queue (seen at ⑧). If it is `Fork` then the scheduler will spawn the thread and put the new thread and the current thread to the bottom of the queue (seen at ⑨). Finally, If it is `Stop` then it means this thread has finished and the scheduler will resume with the rest of threads in the queue. For example, to run the above example, we call `schedule [action example]`.

```

1 schedule :: [Action] -> IO ()
2 schedule [] = return ()
3 schedule (a:as) = sched as a
4
5 sched :: [Action] -> Action -> IO ()

```

```

6 sched as (Atom ioa) = do
7   a <- ioa ⑦
8   schedule $ as ++ [a] ⑧
9 sched as (Fork a1 a2) = schedule $ as ++ [a2, a1] ⑨
10 sched as Stop = schedule as

```

The concurrency monad can be extended to support many features. For example, work done by [17] modifies the definition of Action as well as implements a work-stealing parallel scheduler (seen in Listing 9) to build a monad for parallel computation.

Besides, extending the concurrency monad to monad for message-passing concurrency can be done by adding channel primitives like newChan, writeChan and readChan into the Action. Since channel primitives are possible to represent in this monad, we naturally think of its prospect in connecting with MPST (will be discussed in Section 4.3.2)

```

1 newtype IVar a = IVar (IORef (IVarContents a)) ①
2 data IVarContents a = Full a | Blocked [a -> Action.]
3
4 data Action . =
5   Fork Action Action
6   | Stop
7   | forall a . Get (IVar a) (a -> Action) ⑦
8   | forall a . Put (IVar a) a Action ⑧
9   | forall a . New (IVar a -> Action)

```

Listing 9: Par Monad

- ① Parent threads and child threads communicate data via IVar
- ⑦ Get operation blocks when the underlying IVarContents is Blocked
- ⑧ Put operation updates the underlying IVarContents to Full with the result a and resume the list of blocking threads by applying a to the continuation.

We draw inspiration from this monad to design our intermediate language.

2.5 Free monad

Free monad[18] is a concept from category theory. Intuitively, a free monad as a programming abstraction is a technique for implementing EDSLs, where a functor represents basic actions of the EDSL and the free monad of this Functor provides a way to sequence and compose actions. Speaking of the advantages, we are particularly interested in its benefits in flexible interpretations which will be illustrated by an example (Section 2.5.2) and discussed further (Section 2.5.3).

2.5.1 Definition

In practice, a free monad in Haskell can be defined as an algebraic data type (ADT) (shown in Listing 10). `Free f` is the monad produced given a functor `f`. `Free` has two type constructors: `Pure` and `Free`. `Monad (Free f)` is the Haskell implementation of the `Monad` interface for `Free f`. Many useful helper functions are derived from the simple definition of the free monad (shown in Listing 11). `liftF` lift the functor to its free monad representations. `freeM` maps a natural transformation of functor (`f a -> g a`) to the natural transformation of their free monad versions. Given `m` is a monad, `freeM` is a special case of interpreting `Free m a`: to the `m` monad itself. Finally, `interpret` shows the power of free monad. We can interpret the free monad version of a functor `f` to any monad `m` given a natural transformation from `f` to `m`.

```
1 data Free f a
2   = Pure a
3   | Free f (Free f a)
4
5 instance Functor f => Monad (Free f) where
6   return = pure
7   (Pure x) >>= fab = fab x
8   (Free fx) >>= fab = Free $ fmap (>>= fab) fx
```

Listing 10: Free monad in Haskell

```
1 liftF :: Functor f => f a -> Free f a
2 liftF = Free . fmap Pure
3
4 freeM :: (Functor f, Functor g) => (f a -> g a) -> (Free f a) -> (Free g a)
5 freeM phi (Pure x) = Pure x
6 freeM phi (Free fa) = Free $ phi (fmap (freeM phi) fa)
7
8 monad :: Monad m => Free m a -> m a
9 monad (Pure x) = pure x
10 monad (Free mfx) = mfx >>= monad
11
12 interpret :: (Functor f, Monad m) => (f a -> m a) -> (Free f a -> m a)
13 interpret phi = monad . freeM phi
```

Listing 11: Helper functions based on free monad

2.5.2 Example

Free monad is useful in interpreting an abstract syntax tree (AST). In order to apply free monad to a given AST, we can follow a routine [18].

1. Create an AST, usually represented as an ADT

2. Implement functor for the ADT
3. Create helper constructors to Free ADT for each type constructor in ADT by liftF
4. Write a monadic program using helper constructors. It is essentially a program written in DSL operations.
5. Build interpreters for Free ADT by interpreting
6. Interpret the program by the interpreter.

We will demo the above procedure by a made-up example. We would like to build a simple EDSL for getting customers' name and greeting customers. First of all, we build a functor `GreetingF` to represent the basic operations: getting the name and greeting. Then we wrap the functor with `Free` constructor so that a program written in our EDSL can be regarded as a Haskell expression with type `Free GreetingF a`.

```

1 data GreetingF next
2   = Getname (String -> next)
3   | Greet String next
4   deriving Functor
5
6 type Greeting = Free GreetingF

```

Then we create helper functions of `Greeting` using `liftF`.

```

1 getName = liftF $ Getname id
2 greet str = liftF $ Greet str ()

```

Then we can write a simple program using operations provided by `Greeting`.

```

1 exampleProgram :: Greeting ()
2 exampleProgram = do
3     a <- getName
4     greet a
5     b <- getName
6     greet b

```

Then we can easily implement an interpreter for the example program

```

1 goodMorningInterpreter :: Greeting a -> IO a
2 goodMorningInterpreter = interpret helper
3   where
4       helper (Getname next) =
5           fmap next getLine
6       helper (Greet str next) =
7           putStrLn ("Good morning " ++ str) >> return next

```

Finally, execute the program.


```
ghci:> goodMorningInterpreter exampleProgram
Tom
Good morning Tom
Mary
Good morning Mary
```

2.5.3 Applications

As illustrated by the example (Section 2.5.2), free monad decouple the abstract syntax tree of domain specific language (DSL) and the interpreter. Interpreters with different purposes can be implemented without changing the syntax.

In the project, we apply free monad to the intermediate language so not only we make the languages monadic for free but also benefits from decoupling the interpreter and the syntax to implement different interpreters, e.g. Simulator, code generators to different platforms easily.

Chapter 3

Alg : Algebraic Functional Language

Algebraic Functional Language (Alg) is an example of the high-level languages to generate parallel code, proposed in the paper [10]. The work done by [10] also proposes a method to do the code generation. Part of this project is about implementing an alternative code generation backend for the PAL. In the evaluation section, we will compare the speed of generated code of our method against the original method. We will give an overview of the language in this section.

3.1 Alg

3.1.1 Syntax

		$t_1, t_2 ::=$	Type
$F_1, F_2 ::=$		$() \mid \text{int} \mid \dots$	Primitive types
I	Identity functor	$a \rightarrow b$	Function types
Kt	Constant functor	$a + b$	Sum types
$F_1 + F_2$	Sum functor	$a \times b$	Product types
$F_1 \times F_2$	Product functor	$F t_1$	Functor types
		$\mu.F$	Recursive types

		$e_1, e_2 ::=$	Expression
		$f \mid v \mid \text{const } e \mid e_1 \circ e_2 \mid \pi_i \mid e_1 \Delta e_2 \mid e_1 \nabla e_2 \mid l_i \mid F e \mid \text{in}_F \mid \text{out}_F \mid \text{rec}_F e_1 e_2$	

Table 3.1: Syntax of Alg language

```
newtype L = K () + K Int * I
type List = Rec L
```

Listing 12: Type of integer list in PAL

The syntax of Alg is shown in Table 3.1. In terms of the syntax of expressions, f represents atomic functions which are functions of which we only know their types [10]. The

presence of atomic function is important in the code generation which we will discuss in the later chapter. v is the primitive value like integer 1. F represents functor and a, b are types. Besides primitives type, function types and recursive type constructor μ , Alg use four functors to form more types hence representing complicated data structures by composing them. For example, a list of integer is expressed in Listing 12. One thing we should learn from the syntax is that the Alg and its combinators avoid mentioning variables explicitly and hence Alg programs are known as point-free programs.

$$\begin{array}{c}
\frac{f : a \rightarrow b \in \Gamma}{\vdash f : a \rightarrow b} \\
\\
\frac{\vdash e : a}{\vdash \text{cons } e : b \rightarrow a} \\
\\
\frac{}{\vdash \text{id} : a \rightarrow a} \\
\\
\frac{}{\vdash \text{in}_F : F \mu F \rightarrow \mu F} \\
\\
\frac{}{\vdash \text{out}_F : \mu F \rightarrow F \mu F} \\
\\
\frac{\vdash e_1 : b \rightarrow c, \quad \vdash e_2 : a \rightarrow b}{e_1 \circ e_2 : a \rightarrow c} \\
\\
\frac{i \in [1, 2]}{\pi_i : a_1 \times a_2 \rightarrow a_i} \\
\\
\frac{i \in [1, 2]}{l_i : a_i \rightarrow a_1 + a_2} \\
\\
\frac{\vdash e_1 : a \rightarrow b, \quad \vdash e_2 : a \rightarrow b}{e_1 \Delta e_2 : a \rightarrow b \times b} \\
\\
\frac{\vdash e_1 : a \rightarrow c, \quad \vdash e_2 : b \rightarrow c}{e_1 \nabla e_2 : a + b \rightarrow c} \\
\\
\frac{\vdash e : a \rightarrow b}{F e : F a \rightarrow F b} \\
\\
\frac{\vdash e_1 : F b \rightarrow b, \quad \vdash e_2 : a \rightarrow F a}{\text{rec}_F e_1 e_2 : a \rightarrow b}
\end{array}$$

Table 3.2: Typing rules for Alg

The typing rule for alg is expressed in Table 3.2 and the semantic for alg is shown in Table 3.3.

An important feature of Alg is the lack of usual control flow like if branch or while loop to build algorithms, instead, it uses the flow of transformation of data structures to replace conventional control flow. For example, ∇ combinator is the case operation whose types is $a + b \rightarrow c$. It can be seen as analogy of if branch in normal programming languages. Δ combinator represents split operation. More importantly, the combinator rec_F uses the idea of recursion schemes (explained in Section 2.2) to build divided-and-conquer algorithms.

To summary, algorithms in Alg are represented as a series of transformations of data, making it easy to transform the Alg programs to programs in arrows mechanically since arrows also express flow of data and their transformations naturally. This property al-

Constant, Identity and Composition

Products

$\pi_i = \lambda(x_1, x_2). x_i \text{ if } i \in [1, 2]$

Coproducts

$\iota_i = \lambda x. \text{inj}_i x$

Functors

$\text{I } a = a$
 $\text{I } e = e$

Recursion

$\text{const } e = \lambda x. e \quad \text{id} = \lambda x. x \quad e_1 \circ e_2 = \lambda x. e_1 (e_2 x)$

$e_1 \Delta e_2 = \lambda x. (e_1 x, e_2 x) \quad e_1 \times e_2 = (e_1 \circ \pi_1) \Delta (e_2 \circ \pi_2)$

$e_1 \nabla e_2 = \lambda x. e_i y \text{ if } x = \text{inj}_i y \quad e_1 + e_2 = (\iota_1 \circ e_1) \nabla (\iota_2 \circ e_2)$

$(F_1 \times F_2) a = F_1 a \times F_2 a \quad (F_1 + F_2) a = F_1 a + F_2 a$
 $(F_1 \times F_2) e = F_1 e \times F_2 e \quad (F_1 + F_2) e = F_1 e + F_2 e$

$\text{in}_F = \lambda x. \text{in}_F x \quad \text{out}_F = \lambda x. y \text{ where } \text{in}_F y = x \quad \text{rec}_F e_1 e_2 = f \text{ where } f = e_1 \circ F f \circ e_2$

Table 3.3: Semantics of Alg expression[10]

lows us to generate parallel code without burdens.

$$\begin{aligned}
\mathbf{ms} &= \text{rec}_T \text{ mrg spl} = \text{mrg} \circ T (\text{rec}_T \text{ mrg spl}) \circ \text{spl} \\
&= \text{mrg} \circ (\text{id} + \text{id} + (\text{rec}_T \text{ mrg spl}) \times (\text{rec}_T \text{ mrg spl})) \circ \text{spl} \\
&= \text{mrg} \circ (\text{id} + \text{id} + \mathbf{ms} \times \mathbf{ms}) \circ \text{spl}
\end{aligned}$$

Listing 13: Merge sort in Alg

By making the hylomorphism as a built-in combinator rec_F , Alg can express merge sort similarly as the example shown in Section 2.2.2, We use the functor $T = K() + Ka + I \times I$ to be substitute the functor F in rec_F and the type Ls to represent a list of elements whose type is a . We also have two atomic function $\text{spl} : Ls \rightarrow T Ls$ and $\text{mrg} : T Ls \rightarrow Ls$. Finally we express merge sort as $\mathbf{ms} = \text{rec}_T \text{ mrg spl}$. It is shown in Listing 13. From the example, we observe that \mathbf{ms} can expanded infinitely. Later, we will exploit this property to generate parallel code.

3.2 ParAlg: Alg + role annotations

Alg programs are point-free programs which has an implicit but precise data-flow. For example, $e_1 \circ e_2$ is the function composition so the output of applying e_2 will be used as the input of e_1 . We can interpret this as e_2 sends a message to e_1 . Parallel Algebraic Language is the formalization of the above idea. In essence, it is Alg with roles annotations, converting implicit data-flow to explicit role communication. Due to the limited contexts, in this section, we will only present the main results and use examples to build intuitions about its principles. More details and proof can be found in [10]. We will briefly mention results that will be useful in our project.

$$\begin{array}{ll}
\rho_1, \rho_2 ::= r \mid \oplus_k^r[\rho] \mid \text{id} \mid \rho_1 \circ \rho_2 \mid \pi_i \mid \rho_1 \triangle \rho_2 \mid \iota_i \mid \rho_1 \nabla \rho_2 & R_1, R_2 ::= r \mid \iota_i R \mid R_1 \times R_2 \\
p_1, p_2 ::= e @ r \mid \oplus_k^r[p] \mid \text{id} \mid p_1 \circ p_2 \mid \pi_i \mid p_1 \triangle p_2 \mid \iota_i \mid p_1 \nabla p_2 & A, B ::= a @ r \mid \iota_i A \mid A \times B \\
x \in X \implies x \in \mathcal{T}_X & T_x \in \mathcal{T}_X \wedge T_y \in \mathcal{T}_X \implies T_x \cup^k T_y \in \mathcal{T}_X
\end{array}$$

Table 3.4: Syntax of ParAlg

3.2.1 Syntax

The syntax of ParAlg is shown in Table 3.4. r is the role identifier representing a unit of computation. $R_1 \times R_2$ specifies that the input is split across roles R_1 and R_2 [10]. More explanation of different constructor can be seen in [10]. In short, ParAlg is just Alg with role annotations at certain points.

Notice that rec_F in Alg does not belongs to constructs of ParAlg. So we need to specify the number of unroll beforehand and unroll the expression in rec_F by the corresponding number and then apply the role annotations to the unrolled expression. The unroll process is shown in Listing 13 and in this example, the number of unroll is 1. The unroll-and-annotate operation will parallelize the recursive functions.

3.2.2 Inferring global types

ParAlg turns implicit data-flow into communication and hence we should be able to describe its communication protocols from a valid ParAlg program. In [10], global types (explained in Section 2.3) are used to represent communication protocols for ParAlg which means for any valid ParAlg program, we can infer its corresponding global types. The inferring rules and typing for ParAlg can be seen in [10].

```

G =
  r0 → r1 : Rec L
  r1 → {r2, r3}
  {l0 :
    r1 → r3 : () + int.
    end;
  l1 :
    r1 → r2 : Rec L.
    r1 → r3 : Rec L.
    r2 → r3 : Rec L.
    end};
  r3 → r0 : Rec L.
end

```

Listing 14: Global types for merge sort

An example of the global types is seen in Listing 14. It is the inferred global types of `recT mrg spl` with the number of unroll equal to two. The global types tell us r_1 receives input list from r_0 , based on the length of input list, r_1 either send it to r_3 directly when the list is a singleton or empty or split the list into halves and send them to r_2 and r_3 respectively. In the second case, r_2 will process the received list and send it to r_3 , r_3 will process its received part and waiting for another half of list to be received from r_2 . After both has done, r_3 will process the combined results. Finally r_3 will send the data back to r_0 .

3.2.3 Example: Parallel merge sort

```

par_fun msp
  : (Rec L)@r0 -> (Rec L)@r0
  = id@r0
    . (merge@r3
      . (inj[0]
        ||| (inj[1]
          . ((merge
            . (inj[0]
              ||| (inj[1]
                . ((ms. proj[0])&&& (ms. proj[1, 2]))))
            . split)@r2
            . proj[0]@r1)
          &&& (merge
            . (inj[0]
              ||| (inj[1]
                . ((ms. proj[0])&&& (ms. proj[1]))))
            . split)@r3
            . proj[1]@r1))))
        . split@r1)

```

Listing 15: ParAlg for merge sort

The ParAlg expression for merge sort unrolling twice is shown in Listing 15. We use arrow combinator `&&&` and `|||` to replace ∇ and Δ in the actual ParAlg expression. They are equivalent while arrow combinator is well-known and easier to be expressed in plain texts. Similar reason also applies to `inj`, the replacement of l_i and `proj`, the replacement of π_i . Its inferred global types is shown in the last subsection.

3.3 Conclusion

A visualization of the pipeline can be seen in the Figure 3.1. The content of the section is a skim of Alg and ParAlg. Curious readers should read about [10] for more details.

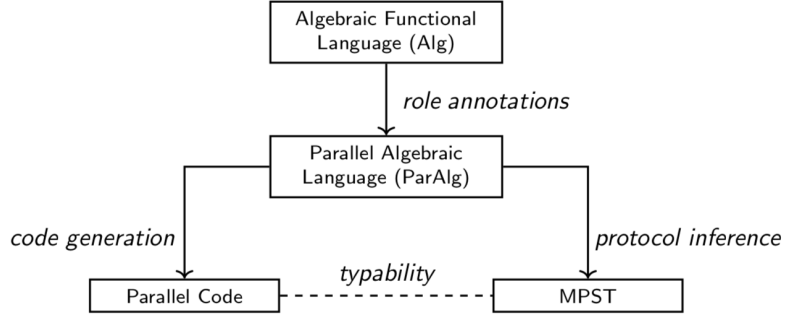


Figure 3.1: Overview of code generating pipeline from Alg[10]

Combinators and the syntax of Alg makes every Alg program to be point-free. Without the use of explicit variables, point-free programs express the underlying data-flow of the computation clearly. Adding role annotations transforms Alg programs to ParAlg programs converting the implicit data-flow to explicit communication. Communication will aid us generate parallel code using message-passing concurrency. We will introduce the our method from next chapter.

Chapter 4

SPar: A session typed free monad EDSL for concurrency

In order to generate parallel code from ParAlg, we first introduce the syntax of our intermediate language, the session-typed free monad EDSL for concurrency hosted in Haskell (SPar). SPar are compromised of two components: Core and Proc. Core is the language expressing sequential computation while Proc is a monadic language with message-passing primitives, communicating Core expression between different roles. We use a group of Proc interacting with each other to represent parallel computations. In addition, session typing the group of Proc ensure the computation is deadlock-free.

4.1 Computation: The Core EDSL

Core is the elemental computation. The syntax of Core is mostly inspired by Alg [10] and FunC, a demo DSL defined in the work done by [19]. For this project, we choose to implement Core syntax as small as possible without sacrificing expressibility.

4.1.1 Syntax

The syntax of Core is shown in Listing 16. Inl and Inr are for the construction of sum type. Pair is responsible for constructing value of product type while Fst and Snd extract value from product type. Supporting sum type, product type and inductive type (see next section) is enough to express any data structure in any computation. In addition to these actions manipulating basic data structure, we have Lit which is used in the Haskell interpreter (see Section 5.2) and benchmarking (see Section 8) and Var, a constructor which is useful when what we do not actually evaluation the Core expression but inspect its static structure. It is used in the code generation (see Section 7.3.2) and session typing (see Section 5.1). Id is similar to identity function and Const is similar to the const function in Haskell. Prim representing user defined functions takes two field name and the haskell implementation. The first field will be useful in the code generation (see Section 7.3.2) when applying user-defined function calls. The second field will be used in the interpreter implementation.


```

data Core a where
  Lit  :: a -> Core a
  Var  :: Int -> Core a
  Prim  :: String
         -> a
         -> Core a

  Ap  :: Core (a -> b) -> Core a -> Core b
  Id  :: Core (a -> a)
  Const :: Core a -> Core (b -> a)

  Fst  :: Core ((a, b) -> a)
  Snd  :: Core ((a, b) -> b)
  Pair :: Core a -> Core b -> Core (a, b)

  Inl  :: Core (a -> Either a b)
  Inr  :: Core (b -> Either a b)

```

Listing 16: The syntax of Core

4.1.2 Representation of recursive data structures

Core has primitives to operate on sum and product types. Representing recursive type like $\mu\text{list}(). + \text{Int} \times \text{list}$ will be covered in this section. The method are taken from the implementation of Alg language in [10]. First of all, we extend the core with the following two operation. `In` represents the fold operation on iso-recursive types and `Out` represents the unfold operation on iso-recursive types. Consider a recursive type $\mu\alpha.\tau$, the type parameter t is equivalent to α , $f : @ : t$ is equivalent to τ and the typeclass `Data f t` associate two types which is equivalent to $\mu\alpha.\tau$. $:@ :$ is a type family which is a function acting on types instead of values. $:@ :$ converts f to sum and product type in Haskell which are `(,)` and `Either`.

```

data Core a where
  In  :: Data f t => Core (f :@: t -> t)
  Out :: Data f t => Core (t -> f :@: t)

type family (:@:) (a :: Poly Type) (b :: Type) :: Type where
  'PId :@: x = x
  'PK y :@: _ = y
  'PProd f g :@: x = (f :@: x, g :@: x)
  'PSum f g :@: x = Either (f :@: x) (g :@: x)

class Data (f :: Poly Type) t | t -> f where
  roll  :: f :@: t -> t
  unroll :: t -> f :@: t

```

A concrete example is a list shown below. We know a list has recursive type: $\mu\alpha.() + a \times \alpha$. So the `f` is `('PSum ('PK ()) ('PProd ('PK a) 'PId))` and we use haskell list type `[a]` to present α (equivalent to `t` in `f :: t`). `f :: t` is evaluated to the type `Either () (a, [a])`.

```
instance Data ('PSum ('PK ()) ('PProd ('PK a) 'PId)) [a] where
  roll (Left _) = []
  roll (Right (a, b)) = a : b

  unroll [] = Left ()
  unroll (x:xs) = Right (x, xs)
```

However, even though this way of representing recursive types is very smart but later we found out they are seldomly used in the code generation of parallel algorithms. Because functions processing complicated recursive data structure is usually represented by the `Prim` constructor hiding the implementation details so `In` and `Out` are too low-level operations and hence user are difficult to write complicated function by them. So in the conclusion, `Core` provides basic functionality, i.e `Fst` or `Inl` to process data and more importantly, `Prim`, a black box constructor hiding the implementation details of the sequential computation. `Prim` turn out to be a very useful abstraction in terms of code generation. More details will be discussed in the Section 7.3.2.

4.2 Communication: The Proc EDSL

`Proc` will be a free monad EDSL for message passing. As introduced in the free monad section in the background, the first thing to do it to define the algebra of message-passing concurrency: `ProcF` and `Proc` is defined using free monad constructor and `ProcF`. The definition is shown in the Listing 17. Careful reader might notice that `Proc` and `ProcF` are defined mutually with each others in `Branch`, `Select` and `Broadcast`.

The semantics will be defined in the next subsection in terms of a group of `Proc` programs. A single `Proc` program is either sequential or deadlock. Operational semantics is only worth discussing when given a group of `Proc` programs interacting with each others.

4.3 Concurrent computation: A group of Proc

We have introduced syntax for computation and communication. We also know that a single `Proc` expression is meaningless since there does not exist another party to interact with this `Proc` hence the computation has no progress. Naturally, we use a group of `Proc` to represent concurrent computations. To be more precise, a collection of `Proc` with their own role identifiers can be treated as a system of roles executing their own programs concurrently. In most of the cases, in order to make the group of `Proc` meaningful, we will allocate a start role in the system acting as the original data provider and a end role whose `Proc` program will receive data from others, process and output the final computation which is wrapped by the `Pure` constructor at the end of the `Proc` program.

```

data ProcF next where
  Send :: Nat -> Core a -> next -> ProcF next

  Recv :: Nat -> (Core a -> next) -> ProcF next

  Select :: Nat
    -> Core (Either a b)
    -> (Core a -> Proc c)
    -> (Core b -> Proc c)
    -> next
    -> ProcF next

  Branch :: Nat
    -> Proc c
    -> Proc c
    -> (Core c -> next)
    -> ProcF next

  Broadcast :: [Nat]
    -> Core (Either a b)
    -> (Core a -> Proc c)
    -> (Core b -> Proc c)
    -> next
    -> ProcF next

type Proc a = Free ProcF (Core a)

```

Listing 17: The algebra for message-passing

Readers might find it easy to visualize the group of Proc as a computation graph. The start role is the source node and the end role is the sink node. A pair of nodes are connected if they communicate data with each others.

4.3.1 Operational semantics

Due to the similarities between Proc and multiparty session calculus introduced in [5]. We borrow some syntax and operational semantics rule from multiparty session calculus to define the operational semantics of Proc. P, Q denote Proc programs. A message queue is h which contains messages (q, p, v) meaning that the sender q sends the receiver p with value v . $h \cdot m$ is a message queue whose bottom element is message m . h are runtime syntax to model the asynchronous message communication where the order of the messages are retained [5]. $e \downarrow v$ means the evaluation of the Core expression to the value v .

Table 4.1 shows the small step semantics for Proc. Rule (Init) describes the initializa-

$(P_1, r_1) \mid (P_2, r_2) \mid \dots \mid (P_n, r_n) \rightarrow$	(Init)
$(P_1, r_1) \mid (P_2, r_2) \mid \dots \mid (P_n, r_n) \mid \emptyset$	
$(\text{Free } (\text{Send } r_j \ e \ \text{next}), r_i) \mid \dots \mid h \rightarrow$	(Send)
$(\text{next}, r_i) \mid \dots \mid h \cdot (r_i, r_j, v) \quad (e \downarrow v)$	
$(\text{Free } (\text{Recv } r_i \ \text{cont}), r_j) \mid \dots \mid (r_i, r_j, v) \cdot h \rightarrow$	(Recv)
$(p, r_j) \mid \dots \mid h \quad (\text{cont } v \downarrow p)$	
$(\text{Free } (\text{Select } r_j \ e \ \text{cont1} \ \text{cont2} \ \text{next}), r_i) \mid \dots \mid h \rightarrow$	(Sel-Left)
$(\text{cont1 } v \gg \text{next}, r_i) \mid \dots \mid h \cdot (r_i, r_j, L) \quad (e \downarrow v, \text{label}(v) \downarrow L)$	
$(\text{Free } (\text{Select } r_j \ e \ \text{cont1} \ \text{cont2} \ \text{next}), r_i) \mid \dots \mid h \rightarrow$	(Sel-Right)
$(\text{cont2 } v \gg \text{next}, r_i) \mid \dots \mid h \cdot (r_i, r_j, R) \quad (e \downarrow v, \text{label}(v) \downarrow R)$	
$(\text{Free } (\text{Branch } r_i \ \text{next1} \ \text{next2} \ \text{cont}), r_j) \mid \dots \mid (r_i, r_j, L) \cdot h \rightarrow$	(Branch-Left)
$(\text{next1} \gg \text{cont}, r_j) \mid \dots \mid h$	
$(\text{Free } (\text{Branch } r_i \ \text{next1} \ \text{next2} \ \text{cont}), r_j) \mid \dots \mid (r_i, r_j, R) \cdot h \rightarrow$	(Branch-Right)
$(\text{next2} \gg \text{cont}, r_j) \mid \dots \mid h$	
$(\text{Free } (\text{Broadcast } [r_{k_1}, \dots, r_{k_n}] \ e \ \text{cont1} \ \text{cont2} \ \text{next}), r_i) \mid \dots \mid h \rightarrow$	(Broadcast-1)
$(\text{Free } (\text{Select } r_{k_1} \ v \ c \ c \ (\text{Pure } ())), r_i) \mid \dots \mid h$	
where $(e \downarrow v, c = \text{Free } (\text{Broadcast } [r_{k_2}, \dots, r_{k_n}] \ \text{cont1} \ \text{cont2} \ \text{next}))$	
$(\text{Free } (\text{Broadcast } [r_{k_1}] \ e \ \text{cont1} \ \text{cont2} \ \text{next}), r_i) \mid \dots \mid h \rightarrow$	(Broadcast-2)
$(\text{Free } (\text{Select } r_{k_1} \ v \ \text{cont1} \ \text{cont2} \ \text{next}), r_i) \mid \dots \mid h \quad (e \downarrow v)$	
$(\text{Pure } v, r_i) \mid (P_i, r_i) \mid \dots \mid (P_j, r_j) \mid h \rightarrow$	(Pure)
$(P_i, r_i) \mid \dots \mid (P_j, r_j) \mid h$	

Table 4.1: Small step semantics for Proc

tion of a group of Proc programs with an empty message queue at the beginning. Rule (Send) append the value to the message queue. Its complementary rule: (Recv) will recv the value at the top of the message queue. Rule (Branch) is also the complementary rule of the rule (Sel). You might notice that the Rule (Broadcast) is kind of special. Its rule states that Broadcast is expressed in terms of a series of Select operations. This operation broadcasts the label to a group of receivers. Even though its semantics can be expressed in terms of Select, the reason why we still treat broadcast an independent operations in SPar is because 1) this operation is very common in communication, including Broadcast as a primitive operation is beneficial for user to write code 2) In the code generation stages, we can generate more efficient code for Broadcast operation instead of generating a series of Select operations. Broadcast operation can be treated as a syntax sugar in the Proc language.

4.3.2 Session types and duality checking

Immediately, we notice that a Proc program can be typed by session types. `Send` operation in ProcF corresponds to the type $!\langle p, S \rangle.T$, `Select` corresponds to $\oplus\langle p, \{l_i : T_i\}_{i \in I} \rangle$ and so on. One exception is the `broadcast` operation which does not mapped to any type of the session types. We will discuss how to handle `Broadcast` in Section 5.1.

Inst	$\frac{\Gamma \vdash e : \forall l. \text{Proc } L \ a}{\Gamma \vdash e : \text{Proc } ([\text{end}/l]L) \ a}$
Gen	$\frac{\Gamma \vdash e : \forall l. \text{Proc } L \ a, \quad \text{fresh } l}{\Gamma \vdash e : \text{Proc } ([l/\text{end}]L) \ a}$
Ret	$\frac{\Gamma \vdash v : a}{\Gamma \vdash \text{Pure } v : \forall l. \text{Proc } l \ a}$
Abs	$\frac{\Gamma, x : a \vdash e : \forall l. \text{Proc } L \ b}{\Gamma \vdash \lambda x. e : a \rightarrow \forall l. \text{Proc } L \ b}$
Bind	$\frac{\Gamma \vdash m : \forall l_1. \text{Proc } L_1 \ a, \quad \Gamma \vdash f : a \rightarrow \forall l_2. \text{Proc } L_2 \ b}{\Gamma \vdash m \gg f : \forall l_2. \text{Proc } [L_2/L_1]L_1 \ b}$
Send	$\frac{\Gamma \vdash v : a}{\Gamma \vdash \text{Free } (\text{Send } r \ v \ (\text{Pure } ())) : \forall l. \text{Proc } (r! \langle a \rangle. l) \ ()}$
Recv	$\frac{}{\Gamma \vdash \text{Free } (\text{Recv } r \ (\backslash v \rightarrow \text{Pure } v)) : \forall l. \text{Proc } (r?(a). l) \ a}$
Select	$\frac{\Gamma \vdash v : a + b, \quad \Gamma \vdash f_1 : a \rightarrow \forall l_1. \text{Proc } L_1 \ c, \quad \Gamma \vdash f_2 : b \rightarrow \forall l_2. \text{Proc } L_2 \ c}{\Gamma \vdash \text{Free } (\text{Select } r \ v \ f_1 \ f_2 \ (\text{Pure } ())) : \forall l. \text{Proc } r \oplus \{L : [l/l_1]L_1, R : [l/l_2]L_2\} \ ()}$
Branch	$\frac{\Gamma \vdash \text{next1} : \forall l_1. \text{Proc } L_1 \ c, \quad \Gamma \vdash \text{next2} : \forall l_2. \text{Proc } L_2 \ c}{\Gamma \vdash \text{Free } (\text{Branch } r \ \text{next1} \ \text{next2} \ (\backslash x \rightarrow \text{Pure } x)) : \forall l. \text{Proc } r \ \& \{L : [l/l_1]L_1, R : [l/l_2]L_2\} \ ()}$

Table 4.2: Typing rules for Proc expressions

The Table 4.2 shows the session typing rule for a Proc expression. We borrowed some notation from the work done [10]. $\Gamma \vdash e : \text{Proc } L \ a$ is the type of a Proc expression that follows protocol L and returns a value of type a . The types are parameterized by a variable l representing the continuation of a local type L . The typing rules contains all the operations except `Broadcast`. This is because `Broadcast` can be expressed in terms of a series of `Select` as explained in the previous subsection. So its session type $\oplus\{r_j\}_{j \in [1, n]} L : L_1, R : L_2$ can be expanded to $r_1 \oplus \{l : r_2 \oplus \{\dots \oplus \{r_n \dots\}\}, \dots\}_{l \in [L, R]}$.

We have argued that a Proc program can be typed by multi-party session types. In order to utilize this property, we should check the duality of the each pair of Proc in the group. In short, duality check examines whether any pair of Proc in the system are complement with each other. If the duality properties is satisfied, the computation is guaranteed to be deadlock-free. This safety guarantee is useful and powerful in the application of parallel code generation. In this domain, SPar is considered to be the intermediate languages. Passing duality check for the intermediate representation means that as long as we preserve types and communication pattern carefully in later stages of code generation pipeline, the generated code obtained will share the same non-trivial properties: communication safety, protocol fidelity and deadlock-free.

The work done by [5] constructed the theoretic foundation of algorithms for checking

dualities and we will give an overview of the implementation in the Section 5.1.

4.4 Conclusions

In this section, we have introduced our intermediate language. It is friendly to use thanks to the monadic interface. In addition, communication and computation are independent in SPar. We can make Proc to parameterized by the type represent sequential computation so users can simply use their construction for sequential computation if they found Core is limited. More importantly, our strategy for parallelism is clear now. In a nutshell, we achieve parallelism by message-passing concurrency: spawning a group of threads on a multi-core CPU where each thread executes its corresponding Proc program.

Before jumping into the code generation, we will use next chapter to give an overview of some implementation challenges related to SPar first.

Chapter 5

SPar: Implementation

5.1 Session types

Haskell does not support session type natively. Other encodings of session types in Haskell is an overkill to this project since Proc does not support all actions that can be typed by session types i.e. channel delegation. We decide to create our representation of session types in Haskell corresponding to set of actions supported by Proc. We will introduce two method for session typing the group of Proc programs followed by duality checking. One is at the value level and another is at the type level.

Essentially, what we do is to infer a collection of local types from a set of SPar expressions. The collection not only can be used to check the duality compatibility of the overall computation but also can work with external tool [20] for additional analysis and verification

5.1.1 Representations of session types in Haskell

Session types belong to the family of behavior types. So the type looks very similar to the value that will be typed. We exploit this similarity to defined our session types in terms of free monad, the same method we have used in defining Proc.

```
data STypeF a next where
  S :: Nat -> a -> next -> STypeF a next
  R :: Nat -> a -> next -> STypeF a next
  B :: Nat -> SType a c -> SType a c -> next -> STypeF a next
  Se :: Nat -> SType a c -> SType a c -> next -> STypeF a next

type SType a next = Free (STypeF a) next
```

Listing 18: Session types in Haskell

The Listing 18 shows the definition in Haskell. `SType` is the session type in Haskell. It is parameterized by a type variable `a` so that the value-level session types and the type-

level session types can share the same basic definition of session types. S is mapped to $!(p, S).T$. R is mapped to $?(p, S).T$. B is mapped to Branch type and Se is mapped to Select type.

5.1.2 Value-level duality check

For the value-level session types, the type variable a is instantiated with type `TypeRep`. `TypeRep` reifies types to some extent by associating type representations to types [21]. Due to session types are represented as value expressions in Haskell, session typing a `Proc` program is the same as writing an interpreter which can be easily done since a `Proc` program is based on free monad.

We traverse `Proc` programs converting each operation to its corresponding type in `STypeF` and convert the value to its `typeRep`. For output actions, we recursively call the substructure to build the rest of session types. For input actions like `Recv`, we will apply the continuation with the `Core` value constructed by `Var` and recursively call the function on the result. The trick to apply `Var` to the continuation make it possible to inspect the static structure of every `Proc` programs because `b -> Proc a` is not inspectable i.e, we cannot pattern match on `i`, while `Proc a` can be inspected. We will also use this trick in the code generation which will be introduced in later chapter.

5.1.3 Type-level duality check

The value-level duality check works well in checking duality at runtime but as programmers, we aim to eliminate problems earlier. Hence, we proposed a solution that makes use of Haskell's powerful type systems to check the duality of the system at the compile time. In addition, we will introduce some combinators to help us build a group of `Proc` to form parallel computation and this mechanism can act as an extra safety guard to make sure the correctness of these combinators.

The general approach of type-level duality checks can be summarized as the following steps.

1. Create a type-level representation of session types.
2. Modify the algebra of `Proc` to make it indexed by session types so that we can session type a `proc` while building it at the same time. Unlike the above method, we can only session type a `Proc` by interpretation after it has been constructed.
3. Gather the indexed session types of each `Proc` in the system and check the duality pair-wise at the type level.

The first step is achieved by reusing the definition in Listing 18 and use Haskell `DataKind` extension to promote data constructors of `STypeF` and data constructors of Free monad to type constructors. At this stage, type parameter a has been promoted to kind parameter and it will be instantiated with kind `*` representing the kind of all types that have values i.e. `Int`, `List of float`. In addition, we should also create type level function that is


```

type family (>*>) (a :: SType * c) (b :: SType * c) where
  'Free ('S r v n) >*> b = 'Free ('S r v (n >*> b))
  'Free ('R r v n) >*> b = 'Free ('R r v (n >*> b))
  'Free ('B r n1 n2 n3) >*> b = 'Free ('B r n1 n2 (n3 >*> b))
  'Free ('Se r n1 n2 n3) >*> b = 'Free ('Se r n1 n2 (n3 >*> b))
  'Pure _ >*> b = b

```

Listing 19: Implementations of type level bind

equivalent to bind in Free monad to help us compose session types. The implementation of type level bind >*> can be seen in Listing 19. It is similar to bind in Free monad but defined in type family.

```

data ProcF (i :: SType * *) (j :: SType * *) next where
  Send :: Sing (n :: Nat) -> Core a -> next
    -> ProcF ('Free ('S n a j)) j next
  Recv :: Sing (n :: Nat) -> (Core a -> next)
    -> ProcF ('Free ('R n a j)) j next
  Branch :: Sing (n :: Nat) ->
    Proc' left ('Pure ()) c ->
    Proc' right ('Pure ()) c ->
    next ->
    ProcF ('Free ('B n left right j)) j next
  Select :: Sing (n :: Nat) ->
    Core (Either a b) ->
    (Core a -> Proc' left ('Pure ()) c) ->
    (Core b -> Proc' right ('Pure ()) c) ->
    next ->
    ProcF ('Free ('Se n left right j)) j next

type Proc (i :: SType * *) a =
  forall j . IxFree ProcF (i >*> j) j (Core a)

```

Listing 20: The algebra of Proc indexed by session types and the definition of indexed Proc

The second step is challenging since we need to find a way to make the Proc indexed by our free monad. Obviously, the original definition of Free monad and Proc does not provide any extra type parameters to be indexed by session types. Hence we use the indexed free monad. It is indexed by two parameter *i* and *j*. In the context of this project, you can treat *i* as the session type for the current proc and *j* as the continuation of session types. Accordingly, we will modify the definition of the algebra of Proc as well as Proc (see in Listing 20). The main operations remained unchanged and the main difference is 1) makes the algebra of Proc indexed by its corresponding session type (*i*) and continuation (*haskj*) 2) For the role identifier, we use the type level identifier: the type

whose kind is `Nat` instead of values because in the later stage, we have to check duality at the type level. The definition of `Proc` use Haskell's `RankNType` and type family `>*>` to extract its corresponding indexed session types `i` from the continuation. By the definition of `>*>`, session type `i` must end with the `Pure` type constructor which is mapped to end in the session type. The basic helper functions for constructing `Proc` expression are also indexed by session types. Some example can be in seen in Listing 21. We will omit the implementation for some helpers functions. Observing the function signatures is crucial in understanding how it works.

```
liftF :: ProcF i j a -> IxFree ProcF i j a

(>=>) :: IxFree ProcF i j a
      -> (a -> IxFree ProcF j k b)
      -> IxFree ProcF i k b

send
  Sing n
  -> Core a
  -> Proc ( 'Free ( 'S n a ( 'Pure ())) ) a
send role value = liftF $ Send role value value

recv :: Sing n -> Proc ( 'Free ( 'R n a ( 'Pure ())) ) a
recv role = liftF (Recv role id)
```

Listing 21: Implementations of helper functions

```
example = do
  x :: Core Int <- recv zero
  send one x

ghci> :type example
example
  :: Proc
    ( 'Free
      ( 'R 0 Int
        ( 'Free ( 'S 1 Int (Pure ()))) )
      Int
```

Listing 22: An example of session type

We will conclude the implementation of second step with an example. After all, dependent types in Haskell is not an easy topic. An example is worth thousand words to build understanding (see in Listing 22). It represents a simple proc that receives an int from role zero and send the int to role one. Haskell type system infers its session type is `'Free ('R 0 Int ('Free ('S 1 Int (Pure ())))` which corresponds to the

behavior of the example. Session typing can be done simultaneously and automatically while users are building the Proc processes thanks to Haskell's type inference.

For the third step, we can assume we have already gathered a type level list of session types paired with its role identifier. The duality check algorithm is still the same. We match different proc pair-wise, and check whether the projection of both session types are complementary. The algorithm is easy to implement as the Haskell function but lifting the computation into type level is tricky. One of the obstacles is that type family does not support higher order type-level functions. We divide the problem into sub-problems. We need to implement 1) a type family that converts a list of Session type to a list of Session type pair 2) a type family that maps a function to list and combine the result 3) a type family that includes projection, checking whether a pair of session types are complementary. We combine the solutions to these problems and encapsulate them into type class constraint. The constraint is satisfied only if the duality checked is passed at the compile time.

5.2 SPar interpreter

5.2.1 Overview

SPar interpreter is an interpreter that interprets a group of Proc programs in Haskell. It can be considered as the simplest backend for evaluating SPar expressions. It records traces of the executions and the final output values of each Proc programs in the system.

It focused on providing a reference implementation explaining the semantics of SPar expressions not on performance. In addition, SPar interpreter served as a very useful tool in the development of this project. We use it as prototype to quickly verify whether the computation produces the expected results. This feature is useful especially in the early stage of an implementation or during debugging.

5.2.2 Implementation

The implementation of SPar interpreter is standard. It is similar to the implementation of the scheduler we explained in the free monad section of the background chapter. In essence, it is a round robin scheduler for a group of Proc programs. A partial implementation can be seen in Listing 23. It takes a list of Proc as a parameter and it maintain a state which is the combination of message queue, the trace and the list of output values. For the base case, the list is empty which means all processes has exited, it returns the trace and the list of output values. For the recursive case, it pattern matches on the first process at the beginning of the list. If the operation is Pure, it will update the list of output values, then abandon this process, and call itself recursively on the tail of the list. If the operation is an output action i.e, Send, Select or Broadcast, it will update the message queue with the corresponding message containing the sender, the receiver and the value, then pop the head proc and append its next step to the end of list of Procs, and call the list recursively. If the operation is an input action i.e, Recv or Branch, it will

```

data InterpState = InterpState
{
    outputValues :: [(Nat, String)],
    trace :: [String],
    messageQueue :: [(Nat, Nat, String)]
}

interpret :: [(Proc (), Nat)] -> State InterpState ()
interpret [] = return ()
interpret (x : xs) = interp x xs

interp :: (Proc (), Nat) -> [(Proc (), Nat)] -> State InterpState ()
interp (Pure value, role) xs = do
    updateOutputValue role value
    interpret xs
interp (Free (Send receiver value next), role) xs = do
    updateMessageQueue (role, receiver, show $ interpCore value)
    updateTrace "send"
    interpret $ xs ++ [(next, role)]
interp p@(Free (Recv sender cont), role) xs = do
    (x, y, v) <- getTopMessage
    if x == sender && y == role
    then do
        popMessageQueue
        updateTrace "recv"
        let value = Lit (read v)
        interpret $ xs ++ [(cont value, role)]
    else
        interpret $ xs ++ [p]

```

Listing 23: Partial implementation of the SPar interpreter

first examine whether sender and receiver pairs match the pair from the message at the top of the message queue. If so, it applies the continuation with the value in the message, removes the message from the queue, removes the head of list, moves the result of applying continuation to the end of the list and calls the list recursively. If not, it simply moves the head of list to the end of list without changing the value of head process and call the list recursively. Because we have checked the duality of the processes in the system, it guarantee that for any input action, the required message will eventually appear at the head of the message queue in finite steps.

Also, the interpreter has a helper function that interpret Core expression into haskell value. This helper function is easy to write because operation on product or sum type has its own mapping function in Haskell and for the `Prim` and `Lit`, we can get their underlying Haskell implementation directly. As for `Var`, this constructor is not intended

to externally so we will not encounter.

Chapter 6

SArrow: An arrow interface for writing SPar expressions

When trying to express more complicated and interesting parallel patterns, e.g map or reduce, We realize SPar is too low-level so that it is difficult to express simple computation because of overheads of expressing communication patterns by hand. In addition, compilation from Par-Alg to SPar is hard since they are very different domain specific languages.

To solve both issues, we draw inspirations from the Arrow interface (in particular, work done by [4] where they use arrow interface to express parallel computation) and introduce SArrow.

SArrow is an arrow interface for writing SPar expressions. Withe the help from SArrow, Users can use canonical arrow combinators to write algorithms in Arrow without writing any explicit communication and gain parallelized algorithms for free. Similarly, SArrow makes hassle-free compilation from Par-Alg to SPar possible because Par-Alg is also an arrow expression and simply interpreting arrow combinators by the SArrow implementations fills the gap between Par-Alg and SPar.

6.1 Syntax

The simplified syntax of SArrow can be found in Listing 24. SArrow is a type synonym of `Nat -> Pipe a b`. It consumes `Nat` which means the identifier of a process and output `Pipe a b`. The reson why we use `Nat` as the only parameter is to ensuring no duplication of processes name since in most of the time, duplication is bad for parallelization. It will be explained more thoroughly in Section 6.3.

`Pipe a b` data structures is the essential component of SArrow. It regards computation as a pipe where data with type `a` goes into the pipe and data with type `b` get out of the pipe. Internally, it's a record type of four fields. `start` field identifies the process where the input data is received. `cont` field has the type `a -> Proc` which is a continuation waiting for the input data produced by the last pipe. `env` represents a group of Procs interacting inside the pipe to produce the output data, in other words, it is the

```

data Pipe a b = Pipe
  { start :: Nat
  , cont  :: a -> Proc
  , env   :: Map Nat Proc
  , end   :: Nat
  }

type SArrow a b = Nat -> Pipe a b

instance Arrow SArrow where
instance ArrowChoice SArrow where

```

Listing 24: Definition of SArrow

parallel computation. `end` indicates the process that produces the output data in the end. We can retrieve the corresponding process by a look up on the `env` with the key `end`. The returned `Proc` returns a data with type `b`.

6.1.1 Arrow interface

`SArrow` is an instance of `Arrow` typeclass as well as `ArrowChoice` type class. For example, the type signature of the combinators `>>>`, `|||`, `&&&` and `arr` are shown below. The main difference between their type signatures and the usual arrow interface is that in the `arr`, the function is wrapped with `Core`. In general, it captures the same meaning as the usual arrow interfaces. Implementation details of these combinators will be explained in Section 6.2.

```

(>>>) :: SArrow a b -> SArrow b c -> SArrow a c
arr :: Core (a -> b) -> SArrow a b
(|||) :: SArrow a c -> SArrow b c -> SArrow (Either a b) c
(&&&) :: SArrow b c -> SArrow b c' -> SArrow b (c, c')
(***) :: SArrow b c -> SArrow b' c' -> SArrow (b, b') (c, c')

```

6.1.2 Example: Parallel programming patterns

As an example, we will illustrate some typical computation patterns used in parallel computing.

First of all, the branching pattern illustrated by Figure 6.1 is equivalent to an expression formed by `|||` combinators, where the data constructor `Left` leads to one computation path and the data constructor `Right` leads to another computation path. It seems to be a simple pattern but it is useful when composed with other complicated patterns. We will use an example to illustrate at the end of this section. Secondly, the fundamental building block of parallel pattern, the fork-join pattern illustrated by Figure 6.2a can be expressed by `&&&` combinator. The `SArrow` produced by `&&&` has the two-ary tuple as the output type collecting the computation result of the main thread and the forked thread

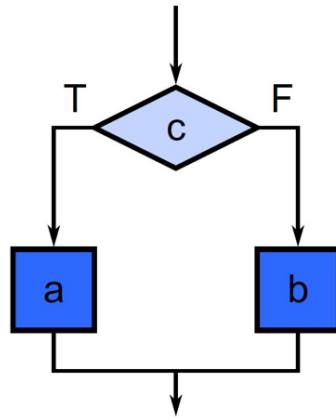


Figure 6.1: Visualization of the branching pattern [22]

and also acts as a synchronization point. Thirdly, the familiar parallel map pattern illus-

```
pmap :: SArrow a b
      -> SArrow a (a, (a, (a, a)))
      -> SArrow (b, (b, (b, b))) b
      -> SArrow a b
pmap f s c = s >>> (f *** (f *** (f *** f))) >>> c
```

Listing 25: Parallel map in SArrow

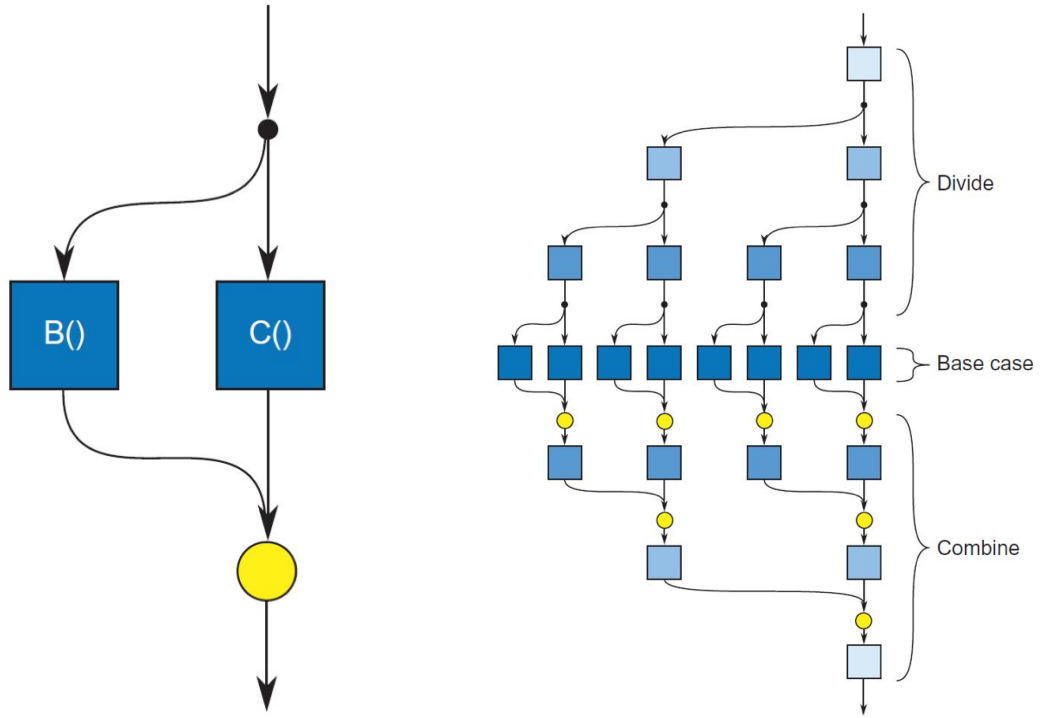
trated in Figure 6.3 is also a candidate to be expressed in SArrow. The code sample is in Listing 25. `pmap` splits the input `a` into 4 chunks using the splitting function `s`, applied the elemental function `f` and the arrow combinator `***` in parallel and finally use the collecting function `c` to collect the results. Usually, the input `a` is a list and `s` splits the list into 4 equal chunks. The number of times where function is `f` applied decides the number of ways of parallelism.

```
preduc :: SArrow (a, a) a -> SArrow (a, (a, (a, a))) a
preduc r = helper >>> (r *** r) >>> r
where
  helper = (arr Id *** arr Fst) &&& (arr Snd >>> arr Snd)
```

Listing 26: Parallel reduce in SArrow

Fourthly, we can apply the similar logic to express parallel reduce pattern shown in Figure 6.4. The code sample is in Listing 26. The result of parallel reduce has similar type signature as the collecting function in `pmap` so it is often used with the `pmap` function. We use nested tuple `(a, (a, (a, a)))` to represent a sized array of data. The `helper` function transforms the array representation of data into a form so that we apply the reduce function `r` to the elements pair-wise and parallel.

Finally, more complicated pattern can be expressed compositely from simpler pattern expressed in SArrow. We can use a typical divide-and-conquer algorithms implemented with fork-join as an example. Figure 6.2b shows a divide-and-conquer algorithms with



(a) Visualization of the fork-join pattern [22] (b) Fork-Join Pattern for Divide-Conquer [22]

Figure 6.2: Fork-join pattern and divide-and-conquer algorithms

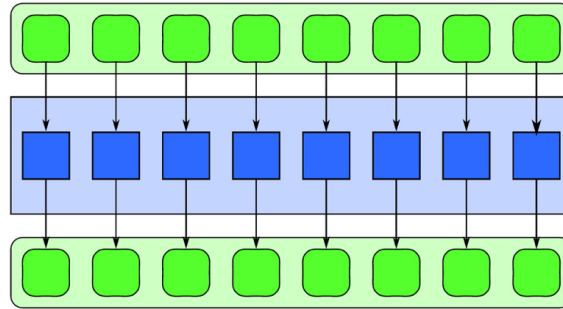


Figure 6.3: Visualization of parallel map [22]

2-ways and 3-levels of fork-join. The algorithm can be expressed in SArrow shown in Listing 27. The divide-and-conquer pattern can be built recursively in Haskell. For the base case, we simply apply the basic computation. Otherwise, we first call split and then call the function recursively with the level decremented by one and, in the end, call the merge to combine the results. Every expressions in the function definition are connected using arrow combinators. A 3-level divided-and-conquer algorithm is constructed by passing 3 to the function resulting a algorithm with $2^3 = 8$ -way parallelism.

In addition, the divide-and-conquer parallel pattern can be optimized when combining with branch pattern. The branch pattern allows us to add shortcuts to the pattern (illustrated in the Figure 6.5, redlines represent the alternative computation path provided by branching patterns). The shortcut give us the ability to decide whether to do local computation or split into multiple subtasks depending on the input size. When the input size is small, the overhead of the latter usually outweighs its parallelism. Adding

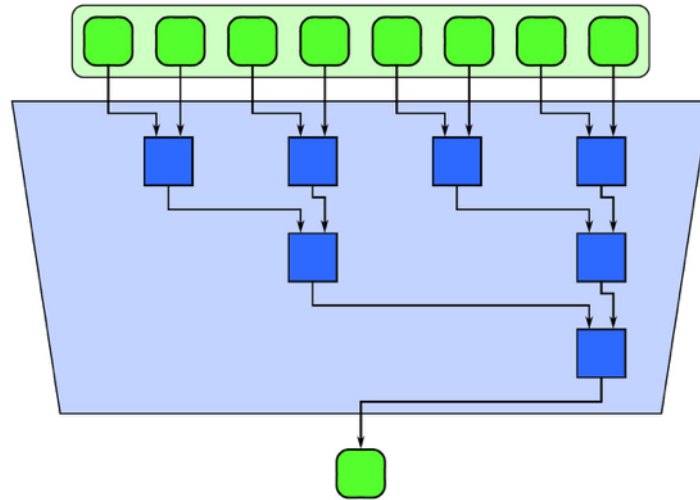


Figure 6.4: Visualization of parallel reduce in SArrow [22]

the simple branching pattern results in a pattern that is adaptive to various input size with better performance.

The implementation demos the power of implementing SArrow as a domain specific language embedded in Haskell. We make full use of Haskell features, i.e high order functions and polymorphic functions to construct expressive, composable and generic computation patterns.

More examples of algorithms formed by SArrow, e.g. dot product or merge sort are shown in the Section 8.

6.2 Implementation of arrow combinators

In this chapter, we will present naive implementation and the optimized solution is introduced in the next section.

The intuition why SArrow is an instance of Arrow comes from the Kleisli arrow of a monad is an instance of Arrow class (shown in Listing 28). The `cont` field in the Pipe has similar type signatures as the `runKleisli` field in the Kleisli arrow. From the previous section, we shown that Proc is a monad so Pipe is just an extended version of Kleisli arrow where computations in Pipe usually finish in one of the process stored in `env` instead of finishing at `cont` like Kleisli arrow. Intuitively, SArrow, a function from the role to the Pipe, should be an instance of Arrow since Pipe looks like an arrow instance and functions are composite.

The essential issue when implementing arrow combinators is how to connect one Pipe by another Pipe. The first problem we need to address is how to deal with the `cont` in the tail Pipe. We know that only one `cont` field exists in the resulting Pipe and it must be that from the head Pipe. Hence the only option is to convert it to a Proc expression and store the converted expression in the updated `env` in the resulting Pipe. A right way to do it bind the `cont` with the action: receive from end in the first Pipe. Also, extend the proc related to the end in the head Porc with action: send to start in the second Pipe.

```

divConquer
  :: Int
  -> SArrow a b
  -> SArrow a (a, a)
  -> SArrow (b, b) b
  -> SArrow a b
divConquer 0 baseFunc _split _merge = baseFunc
divConquer level baseFunc split merge =
  split
    >>> (   divConquer (level - 1) baseFunc split merge
          *** divConquer (level - 1) baseFunc split merge
        )
    >>> merge

twoWayThreeLevelDq = divConquer 3

```

Listing 27: 2-ways and 3-levels divided-and-conquer algorithm in SArrow

```

newtype Kleisli m a b = Kleisli { runKleisli :: a -> m b }

instance Monad m => Arrow (Kleisli m) where
  id = Kleisli return
  (Kleisli f) . (Kleisli g) = Kleisli (\b -> g b >>= f)
  arr f = Kleisli (return . f)
  first (Kleisli f) =
    Kleisli (\ ~(b,d) -> f b >>= \c -> return (c,d))
  second (Kleisli f) =
    Kleisli (\ ~(d,b) -> f b >>= \c -> return (d,c))

```

Listing 28: The implementation of arrow instance for Kleisli arrow of a monad

Besides addition of the converted `cont` expression, the new env is formed by merging the env from the head Pipe and the env from the tail Pipe. Merging two envs is trivial. When there are duplication, we simply use monadic bind to combine them so that the actions belonging to head Pipe followed by the actions belonging to the tail Pipe. The `start` field in the resulting Pipe is the same as that from the head Pipe and the `end` field will be set the same as that from the tail Pipe. We can apply the Pipe composing function to implement arrow combinators for SArrow. The implementation is just apply the first SArrow with the input role (don't forget SArrow is a function) to get the Pipe and apply the second SArrow with a new role to get the second Pipe (usually in order to avoid duplication of roles, the new role is set to be maximum role in the first Pipe + 1) and finally apply the Pipe composing functions to both Pipe. A simplified code explanation can be seen in Listing 29. The rest of combinators can be implemented in a similar fashion.

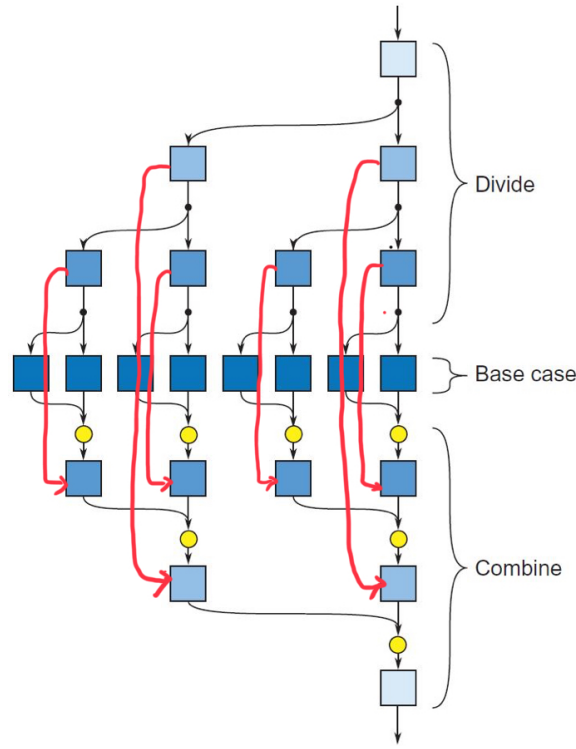


Figure 6.5: Combination of branching pattern and divide-and-conquer pattern

```
(>>>) :: (SArrow a b) -> (SArrow b c) -> (SArrow a c)
(>>>) leftArrow rightArrow start = compose firstPipe secondPipe
where
  firstPipe  = leftArrow start
  secondPipe = rightArrow (end leftP + 1)
```

Listing 29: The simplified implementation of >>>

6.3 Strategies for optimized role allocations

From the last section, we know the number of roles in the system is directly related to the number of processes in the final generated code. Hence, role allocating is an essential part in generating efficient parallel programs.

In this section, we propose strategies for optimizing role allocations. We have two goals in mind when optimizing; The first one is we would like to reduce the number of roles (processes) in the computation since the overhead of thread creation and data transmission has negative impact on performance. The second one is we do not want roles duplication when we try to compose SArrows since role duplications means the different computation must be merged in the same role and computations in the same thread is sequential hence role duplications has negative impact on degree of parallelization.

If we only put the first goal in mind, an easy solution will be setup an upper bound of the number roles, and then we cycle through a fixed bound when allocating new roles. Processes corresponding to duplicated roles can be simply merged using binds since Proc

is a monadic DSL and duality check ensures binding will not cause deadlocks. However, this strategies is not ideal since duplications of roles will decrease the degree of parallelization in the system.

If we only consider the second goal, naive strategies used in the previous section will satisfy the goal. However, the number of channels required and the number of roles in the system will grow exponentially. In a divided-and-conquer algorithm, the number of channels increases from 10 to 120 and the number of roles increases from 6 to 36 when the level is increased from 1 to 3.

$$\text{id} \frac{x : \text{Role}, \quad a : \text{Type}}{id : \text{SArrow } a \ a, x \Rightarrow x}$$

Table 6.1: Role allocation for id

For the purpose of illustration, we use inference rules to explain our proposed strategies for optimized role allocations when composing SArrows. Please see Table 6.1 as an example. $x \Rightarrow x$ means the computation start with role x and end with role x .

$$\text{compose} \frac{e1 : \text{SArrow } a \ b, x \Rightarrow y, \quad e2 : \text{SArrow } b \ c, y \Rightarrow z}{e1 >>> e2 : \text{SArrow } a \ c, x \Rightarrow z}$$

$$\text{arr} \frac{f : \text{Core } (a \rightarrow b), \quad x : \text{Role}}{\text{arr } f : \text{SArrow } a \ b, x \Rightarrow x}$$

$$\text{arrow choice: } ||| \frac{e1 : \text{SArrow } a \ c, x \Rightarrow y, \quad e2 : \text{SArrow } b \ c, x \Rightarrow z}{e1 ||| e2 : \text{SArrow } (\text{Either } a \ b) \ c, x \Rightarrow \max(y, z)}$$

$$\text{arrow choice: } +++ \frac{e1 : \text{SArrow } a \ c, x \Rightarrow y, \quad e2 : \text{SArrow } b \ d, x \Rightarrow z}{e1 +++ e2 : \text{SArrow } (\text{Either } a \ b) \ (\text{Either } c \ d), x \Rightarrow \max(y, z)}$$

$$\text{arrow: } \&\&\& \frac{e1 : \text{SArrow } a \ b, x \Rightarrow y, \quad e2 : \text{SArrow } a \ c, (y + 1) \Rightarrow z}{e1 \&\&\& e2 : \text{SArrow } a \ (b, c), x \Rightarrow z}$$

$$\text{arrow: } *** \frac{e1 : \text{SArrow } a \ b, x \Rightarrow y, \quad e2 : \text{SArrow } a' \ c, (y + 1) \Rightarrow z}{e1 *** e2 : \text{SArrow } (a, a') \ (b, c), x \Rightarrow z}$$

Table 6.2: Rules fo role allocations of different combinators

The rule for the rest of combinators are shown in Table 6.2. Notice that for compose, id, arr and ArrowChoice we do not introduce any new roles, in other words, there is no parallelization for these combinators. Reader may find it strange that we do not intent to parallelize arr combinator which lifts a sequential computation represented by Core $(a \rightarrow b)$ into SArrow. It makes sense to introduce a new role to execute the computation

and hence parallelize computational heavy tasks. We use this strategy in the first place but later we found a more suitable strategy exists which will be introduced in the later paragraph. Also, another reason not to introduce new role when encountering arr combinators is that we gained function fusion for free. Simple function i.e. `fst`, `inject left` or `snd` are automatically fused into more complicated user defined functions. Introducing new roles for these simple functions will damage performance.

For the class of combinators belonging to arrow choice, we do not introduce any new role. The expressions at the lhs and at the rhs starts with the same role x because when only one code path will be executed as the name choice suggested so we should not use separate roles for two expressions that will never be executed simultaneously. In the end, we decided the computation end in the role $\max(y, z)$. `Max` guarantees that there will not be role duplications when we compose expressions formed by `ArrowChoice` combinators with other combinators. For the implementation, all process in both left and right `SArrow` expression are wrapped inside a branch operation separately. Assume $\max(y, z) = y$, the process at the role y will be extended with actions that receive data from $\min(y, z) = z$ role at its right branch. Finally, applying `inject left` and `inject right` at left and right branches gives us `Either` type as the output.

Finally, we discovered the right place to allocate new roles is `&&&` combinator. As shown in type signature, product types mean computation at both branches will both be executed and they are independent. In order to make sure both computation are executed simultaneously, we constraint that the right `SArrow` expression must start with a role greater than the end role of the left `SArrow` expression. This ensures no role duplications hence maximize parallelism. The combined expression ends in the end role of the right `SArrow` expression instead of introducing a unnecessary new role. For the implementations, the process corresponded to end role z are extended with actions that receive data from the end role y of the left process and store the computation of `SArrow` expression at the right side and finally output a pair.

Even though from the implementation point of view, `SArrow` composition with the optimized role allocations is ad-hoc and less elegant to implement because we need to consider composition by send-and-receive and composition by local monadic bind and more edge cases to be dealt with compared to the naive solution in the last section where composition is done solely by send-and-receive and role allocations is mindless. We believe the effort is worthy because for a n -level divided-and-conquer algorithms, the optimized role allocation strategies allocate 2^n roles in total which is the same as the number of way of parallelization in theory. All the roles are used to maximize parallelism instead of wasting the valuable resources to create roles that merely transmit data.

In conclusion, the optimized strategy presented in this chapter is not the only solution. For example, there is a strategy that only introduce new roles into the computation graph when encountering a specific atomic functions. Different strategies result in different communication structures hence different kind of parallelisms. We should choose the right strategy depending on the specific task.

6.4 Satisfaction of arrow laws

We have provided the implementation of SArrow combinators that is similar to the arrow interface. However, the implementation is not enough to state that SArrow is an arrow interface. We will present a justification on why the arrow laws are satisfied. We will focus on arrow laws. The justification for arrow-choice law can be reasoned in a similar way.

$$arr(Id) \ggg a = a \quad (1)$$

$$a \ggg arr(Id) = a \quad (2)$$

$$(a \ggg b) \ggg c = a \ggg (b \ggg c) \quad (3)$$

$$arr(f; g) = arr(f) \ggg arr(g) \quad (4)$$

$$first(a) \ggg arr(Fst) = arr(Fst) \ggg a \quad (5)$$

$$first(a) \ggg arr(\alpha) = arr(\alpha) \ggg first(first(a)) \quad (6)$$

$$first(a \ggg b) = first(a) \ggg first(b) \quad (7)$$

Table 6.3: Arrow laws [23]

Table 6.3 shows the rules of arrow laws. We includes a subset of law that contains the `first` combinator. The other half of laws that contains the `second` combinator can be proved by symmetry. `first` combinator is implemented as `first = (***) id` and $\alpha :: ((x, y), z) \rightarrow (x, (y, z))$.

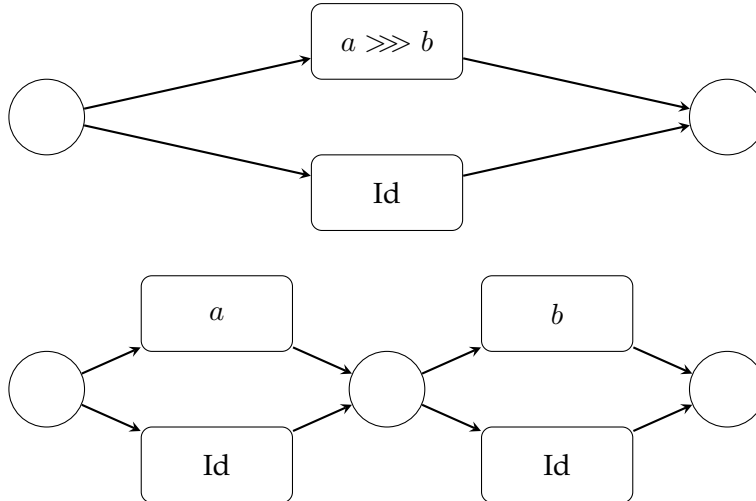


Figure 6.6: Graphical representation of equation (7)

We argue above equation holds if both sides of equation express the same computation. Equation (1), (2) holds because lifting `Id` from Core to SArrow will not modify the result of computation due to the semantics of `Id`. Equation (3) is the associativity rule of `>>>`. Left side of the equation sends the output from `a >>> b` to the input of `c`



Figure 6.7: Graphical representation of equation (6)

while right side sends the output of a to the input $b \ggg c$. The communication structure might change but they both represents the same computation. Equation (4) is valid because applying the input value to composition of $f;g$ is the same as applying the input to f followed by a message passing (data communication could be local which means the communication structure for both side of equations are the same in this case) and then applying it to g . One might claim that the left side is more efficient than the right side due to the fusion depending on the role allocation strategy. Equation (5) holds because they both represents a computation that take a pair input and apply the function a to its first position and return the result. Right hand side of equation (7) represents a SArrow expression that applies a on left position of input pair and applies Id on the right position of input pair in parallel, collects the results and applies b on left position of input pair and applies Id on the right position of input pair while left side fused a and b together and apply them in one step hence there is only one join point. Figure 6.6 is a graphical representation of equation (7). Obviously they represents the same computation. We can also derive equation (6) from its Visualization (see in Figure 6.7).

6.5 Conclusions

Arrow interface is the perfect interface to express general computation for this project because not only is it intuitive to understand and visualize but also its combinators $***$ and $\&\&\&$ has built-in parallel natural. Progressing work from [10] can be adapted to show that there exists a global type for an SArrow expression. This means SPar expressions for all participants beneath the SArrow expression are typeable against the projection of this global type and therefore they are dual pair-wise indicating no communication errors. It means every computation composed by SArrow is deadlock-free.

So far, we've used SArrow to help us compile Par-Alg to SPar. The remained chal-

lenge is the code generation from SPar to a target platform. In the next chapter, we will introduce our methods for code generation and specifically code generation to C. Once we achieved this, every computation in SArrow can be transformed into parallel C code in one step.

Chapter 7

Type-safe code generation from SPar

SPar has two components: Core representing the unit of computation and Proc as a skeleton of the computation, describing the communication patterns. Naturally, the process of code generation from SPar should be divided into two parts correspondingly. We choose to make two parts independent of each other so that it's possible to swap the code generation strategy of one component without modifying another one.

The procedure of code generation is standard: transformation. We start our programs in a high level DSL and run a series of transformation to low-level DSL. SPar expressions are converted to a low-level EDSL which is then transformed to an abstract syntax tree (AST) of C (TODO cite the package). The generated code is obtained by pretty printing the AST.

7.1 Instr: A low-level EDSL for channel communication

In Proc, we have high-level actions like select, broadcast and branch abstracting implementation details, i.e variable declaration, variable assignment, channel initialization, channel communication and channel deletion. Hence, we need to define a EDSL containing instructions related to these low-level operations. We name it Instr. A SPar programs will be translated to a sequence of Instr.

When we design Instr, we keep the simplicity in mind so Instr is not coupled with any fancy language feature related to some specific target languages. Any reasonable target language with a channel communication library can be easily converted from Instr.

7.1.1 Syntax and semantic

The definition of Instr is seen in Listing 30. `Channel` is our abstract representation of Channel in Instr. It is indexed by a type `a` from the reified type `ReprType a`. More details of this reified type will be introduced in Section 7.1.2. This type parameter preserves type in channel initialization hence make sure the value to be sent or received in this channel has the same type as this channel. This is necessary because for some target languages, the channel are typed. Similarly, type parameters in `Exp` have the same functionality.

```

data Channel a where
    Channel :: CID -> ReprType a -> Channel a

data Exp a where
    Exp :: Core a -> ReprType a -> Exp a

data Instr where
    CInitChan :: Channel a -> Instr
    CDeleteChan :: Channel a -> Instr
    CSend :: Channel a -> Exp a -> Instr
    CRecv :: Channel a -> Int -> Instr
    CEnd :: Exp a -> Instr
    CDecla :: Int -> ReprType a -> Instr
    CAssgn :: Int -> Exp a -> Instr
    CBranch :: Int -> Seq Instr -> Seq Instr -> Instr
    CSelect :: Int -> Int -> Seq Instr -> Seq Instr -> Instr

```

Listing 30: The syntax of Instr in Haskell with accompanying low-level data types

Exp is just a wrapper of the expression in Core. In later stages, we will take care of code generation of Exp along with Instr. Instr defines the set of statements that will be generated and Exp represents the sequential computation, which is a value that will be generated.

The semantic of Instr operations are similar to what their names suggest. CInitChan represents operations that initialize a channel according to the given type and cid. CDeleteChan will destroy a channel. CSend operation sends the value Exp a through the Channel. CRecv action means the value received in the channel will be assigned to variable whose postfix name is the int field. CEnd means the instruction exits with the value Exp a. CDecla and CAssgn are instructions for variable declaration and assignment. The type of the variable is determined by ReprType a and the value is Exp a. CBranch and CSelect are used to express conditional control flow of the Instr languages. SPar action like broadcast are built on top of these operations. For CBranch, the first field represents the value of Either type to be received via the channel and two Seq Instrs represents the sequence of Instrs in the left branch and the right branch. For CSelect, the first field represents the variable containing Either value and the second field field represents the variable whose value is assigned by the end results of instructions from either the left branch or the right branch. The third and fourth fields represent the instructions in the left branch and the right branch.

7.1.2 Representation types

SPar programs cannot be fully parametric since the target languages of code generation from SPar are usually less expressive, i.e, they do not treat function type $a \rightarrow b$ as a value, and are less efficient when processing with some specific form of data, i.e, languages tar-

```

data ReprType a where
  NumReprType :: NumType a -> ReprType a
  LabelReprType :: ReprType Label
  SumReprType :: ReprType a -> ReprType b -> ReprType (Either a b)
  UnitReprType :: ReprType ()
  ProductReprType :: ReprType a -> ReprType b -> ReprType (a, b)
  ListReprType :: ReprType a -> ReprType [a]

```

Listing 31: The definition of representation types

```

constToCExpr :: ReprType a -> a -> CExpr
constToCExpr (NumReprType numType) v = numTypeToCExpr numType v
constToCExpr LabelReprType          v = case v of
  Le -> cVar "LEFT"
  Ri -> cVar "RIGHT"
constToCExpr s@(ProductReprType a b) v = defCompoundLit
  (show s)
  [ ([], initExp $ constToCExpr a (fst v))
  , ([], initExp $ constToCExpr b (snd v))
  ]

```

Listing 32: An example usage of reified type in the code generation

getting GPUs are usually more productive in dealing with array of floating point number while slow in working with aggregate structures [24]. Hence, we need to restrict the set of types allowed in SPar. We achieve this using the type class `Repr` and corresponding reified type `ReprType` (shown in Listing 31). `Repr` determines the set of type allowed in SPar. Reified type `ReprType` will be used to alter the behaviors of code generation based on the type. This can be simply done by pattern matching because reified types are values in Haskell [25]. To be more concrete, Listing 32 gives a demo. `constToCExpr` is function that handle code generation from constant value to expression in C programming languages. By pattern matching, we vary the behaviors of code generation so that constants with different types has their own way to be represented in C.

In conclusion, we allow the following type: numerical type like `Float` and `Int`, the unit type `()`, the label type which is used in the code generation of select and branch and the aggregate type: list, product and sum that are built recursively, to be expressed in SPar.

7.2 Compilation from SPar to Instr

7.2.1 Transformation from Proc to Instr

As described in the previous section, `Instr` contains a data type called `Exp` which is a wrapper of Core expression. Compiling Core to Instr is hence not difficult. The challenge

of compiling Core is mainly how to compile it to a specific target language. This will be discussed in the next subsection.

In this section, we will explain how we transform operations in Proc to Instr. Generally speaking, each Proc operation is mapped to a sequence of actions in Instr. The transformation algorithm from a Proc expression to a sequence of Instr can be implemented easily by traversing Proc expression, applying the mapping and collecting the results by concatenation. This is an advantage of using free monad technique to build the AST because Proc expression can be treated as a data structures and traversing recursive data structures can be easily done in Haskell. In addition, operations like `Recv` which involves continuation whose type is `Core a -> next` in their constructors are treated differently than those operations whose constructors only have a value type `next`. The latter is easy to implement, we can simply call the traversing function recursively. For the former, we have to pass an expression whose type is `Core a` to the continuation so that we can recursively call the traverse function recursively on the results of applying value to the continuation. The answer to what Core expression should be used is `Var` denoting variable. Passing a unique variable to the continuation gives us `next` inexpensively and we will define where does values of variables comes from for each operations in Proc.

We have introduced the general principle to the readers. Now let us dig into details of translation rules for each operations.

- **Pure.** It is the base case in free monad. Hence it is mapped to the `CEnd` instruction.
- **Send.** It is mapped to a sequence of three instructions. First of all, We declared a temporary variable using `CDecla` and then assign the value that will be sent to this variable using `CAssgn` and send the content of the variable via the specific channel. The problem of how make sure the same channel is used in a send-and-receive pair will be discussed in the next sub chapter.
- **Recv.** It is the reverse of send operation. Firstly, it declares a new variable with `CDecla` and use `CRecv` to assign the value received from the sender to this variable. Notice that the `recv` has a continuation, we will pass the variable declared in the first instr to the continuation to traverse the Proc expression recursively as discussed above.
- **Select.** It is a more complicated operations. Its constructor contains two continuation: one for left branch and one for right branch. Hence for this instruction, we need to declared two variable to passed into continuations. The value of both variables is assigned by the Core expression whose value is a `Either` type. Besides, we need to send label indicating whether the execution of the left branch or the right branch of the receiver is selected. Its value is assigned by the either value as well and the sending operation is done by `CSend`. Finally, we call the transforming function recursively on the left branch and right branch and combine the results using `CSelect`.
- **Broadcast.** The mapping from Broadcast is similar to that of Select. The only difference is that the former sends to a list of receivers while the latter sends to a

single receiver. So in this operation, we will have multiple `CSend` corresponding to each receiver.

- **Branch.** It is the reverse of the Select operation. So it will use `CRecv` to receive a label from the sender and call recursively on two branches and finally use `CBranch` to collect results.

7.2.2 Strategies for channel allocations

Channel allocations is important because correction allocation is essential in making sure the correctness and deadlock-free of generated code. Besides correctness concerns, we are also want to reduce the amount of channels creations hence increase performances.

In the first iteration, inspired by the linearity of channels in the π calculus, we choose to allocate a one-time channel for each send-and-receive pair. All channels' buffer size is one because of the linearity. Send action will initialize a channel and Receive action will destroy this channel once it receive the value. We use the ensure the same channel is used for the pair. During the transformation, we use a map of queue whose key is a pair of sender id and receiver id. When visiting the send action, it will push the channel into the queue and the corresponding received operation will pop the channel from the queue. Because we've ensured the duality of all processes in the system, we can claim the channel is right for each send-and-receiver pair. However, we realize this strategy is complicate to implement and not resource efficient since too many channels are initialized.

In the second iteration, we come up with a more efficient and simple strategy. The buffer size of all channels is still one due to the same reason about linearity. However, we decide to only allocate one channel for a pair of sender and receiver. We will not destroy the channel after the value has received and will reuse the channel for the next communication. When all processes have returned, we will destroy all channels at once. For this strategy, we have simplified the state from a map of queues of channels to a map of channels.

7.2.3 Monad for code generation

From the last two subsections, we need to maintain a number of states during the compilation process. Hence we define a state monad to be used during the traversal. The `CodeGenState` is the collection of states with different purposes and it is shown in Listing 33. `chanTable` is the map we required during the channel allocation. `varNext` represents the next variable id to used. It will increment by one every time we declare a new variable. It helps us make sure the variable names are unique. `chanNext` has the similar functionality ensuring the uniqueness of channel names. `dataStructCollect` is the set of compound type we encounter during the traversal. With this states, we know what kind of data structures whose definitions will be generated before the generation of executing code.

```

data CodeGenState = CodeGenState
{
    chanTable :: Map ChanKey CID,
    varNext   :: Int,
    chanNext  :: CID,
    dataStructCollect :: Set AReprType
}

newtype CodeGen m a =
    CodeGen { runCodeGen :: StateT CodeGenState m a }
    deriving (Functor,
              Applicative,
              Monad,
              MonadState CodeGenState,
              MonadIO)

```

Listing 33: States required during the traversal

7.3 Code generation to C: from Instr to C

The last piece of jigsaw is compilation from a sequence of Instr to C. This process is rather simple but trivial. This is simply done by transforming the sequence of Instr to a C AST. We used an open source library: language-c [26] to represent C AST in Haskell and pretty printing the C AST gives us the generated code. This method can be generalized to any target language. As for the implementation of channel communication in C, we used another open source library: chan [27] whose internal is based on shared memory. In the following subsections, we will present some design choices during this final step.

7.3.1 Representations of Core data type in C

The first challenge we face is how to represent data structure in C. For primitive data type like Int or Float, a simple one-to-one mapping is sufficed. It is hard to deal with compound data type in C. First of all, C does not support polymorphic type. Hence, we choose to generate specific data type for every different compound data type even though they have the same structure. We have a way to name the generated data type to avoid name duplication. The naming simply reflects the structure of the data types with its elemental type. For example, `(Int, (Int, ()))` is converted to `Prod-Int-Prod-Int-Unit` and `Either (Either () (Int, Int)) (Int, Int)` is convert to `Sum-Sum-unit-Prod-int-int-Prod-int-int`. In this project, all compound types are formed by sum type and product type. The product type will be converted to a struct with two field in C. The sum type is represented by the tagged union type. Tagged union is a struct with two field. The value of the first field indicates whether it is a left value or right value and the value of second field is a union type containing either the left value or right value. We also implement a sorting algorithm based on the depth of compound type so that all

```

typedef enum Label {
    LEFT, RIGHT
} Label;

typedef struct Prod_int_int {
    int fst; int snd;
} Prod_int_int;

typedef struct Sum_unit_Prod_int_int {
    Label label;
    union {
        int left; Prod_int_int right;
    } value;
} Sum_unit_Prod_int_int;

```

Listing 34: Compound data type in C

necessary data types have been defined before the definition of the compound types. An example of what `Either Int (Int, Int)` will be converted to is shown in Listing 34.

Another challenge is representation of the recursive type. From the type theory, we learn that a list of `int` can be expressed as $\mu a.() + \text{Int} \times a$. We might reuse the idea from the last paragraph to generated recursive type in terms of sum and product type. Hence a list of `Int` will look like the code below.

```

typedef struct Sum_unit_Prod_int_a {
    Label label;
    union {
        int left;
        Sum_unit_Prod_int_a *right;
    } value;
} Sum_unit_Prod_int_a

```

However, we believe expressing typical recursive data structures like list of `int` in this way is bad for performances. Obviously, C has a more efficient way to represent list of `int` using an array. So we decided to has two ways to represent recursive data structures. For a set of specific recursive data structure, users can write their own representation to exploit the advantages of the target language. For example, a list of `int` are encoded in C using a wrapper of pointer type (shown in Listing 35). This way is not very generic but friendly for performances. For other types of recursive data structures where user do not specify their optimized versions in C, we simply apply the method in the last paragraph to encode them in C. This way is generic but not efficient.


```
typedef struct List_int {
    size_t size; int * value;
} List_int;
```

Listing 35: Optimized represent of List in C

$\text{Var} \frac{}{\llbracket \text{Var } n \rrbracket = \text{vn}}$	$\text{Lit} \frac{}{\llbracket \text{Lit val} \rrbracket = \text{toC}(\text{val})}$
$\text{Fst} \frac{\llbracket a \rrbracket = c}{\llbracket \text{Fst 'ap' } a \rrbracket = c.\text{fst}}$	$\text{Snd} \frac{\llbracket a \rrbracket = c}{\llbracket \text{Snd 'ap' } a \rrbracket = c.\text{snd}}$
$\text{Inl} \frac{\llbracket a \rrbracket = c}{\llbracket \text{Inl 'ap' } a \rrbracket = \{\text{LEFT}, c\}}$	$\text{Inr} \frac{\llbracket a \rrbracket = c}{\llbracket \text{Inr 'ap' } a \rrbracket = \{\text{RIGHT}, c\}}$
$\text{Pair} \frac{\llbracket a \rrbracket = c_1, \llbracket b \rrbracket = c_2}{\llbracket \text{Pair } a \text{ } b \rrbracket = \{c_1, c_2\}}$	
$\text{Id} \frac{\llbracket a \rrbracket = c}{\llbracket \text{Id 'ap' } a \rrbracket = c}$	$\text{Const} \frac{\llbracket a \rrbracket = c, \llbracket v \rrbracket = b}{\llbracket (\text{Const } v) \text{ 'ap' } a \rrbracket = b}$
$\text{Prim} \frac{\llbracket a \rrbracket = c}{\llbracket (\text{Prim } \text{fname } \text{fimpl}) \text{ 'ap' } a \rrbracket = \text{fname}(c)}$	

Table 7.1: Rules for compilation from Core to C

7.3.2 Compiling from Core to C

Core has a concise syntax so it does not require too much work to write a function that generate C expressions from Core expressions. Not surprisingly, the compilation is a traversal of the Core expression. Pattern matching on the structure of the core expression alters the behaviors of compilation. `ap` (apply) constructor is used with an expression whose type is `Core (a -> b)` and another expression whose type is `Core b`. The code generation for `ap` depends on the what the function expression is. The code generation rule is explained by the inference rules shown in Table 7.1. $\llbracket a \rrbracket$ means the C code generated by Core expression `a`. `toC` is function that convert constant value in Haskell to constant value in C.

- **Var, Lit:** Code generation of `Lit` simply convert Haskell value to its corresponding C value. As for `Var`, it will be converted to a string literal composed of the variable identity prefixed by `'v'`.
- **Fst, Inl, Pair...** For `Inl` and `Inr` and `Pair`, we used C99 style to initialize struct and union. The rule for generating corresponding struct are explained in the previous subsection. For `Fst` and `Snd`, we simply access the specific value using the designator.

- **Id, Const v:** Code generation of Id constructor is same as the code generation of the argument of the Id. Code generation of Const ignores the argument and use code generation of v expression instead.
- **Prim:** Prim constructor represents user defined functions. The code generation for function call of Prim is converted to function call by the name of the primitive function. Users can implement those functions in C and include them in the main generated file.

7.3.3 The structure of generated C code

We have covered the code generation algorithm for each process. We will now tackle how the generated code is structured from a group of interacting processes representing a parallel computation. For each process, we will generate its own C functions that take no argument. We decide to generate an addition C function since function is portable and acts like a black box hiding all the execution details so users can interact with generated code by calling the function. This function will take a parameter as input data and return the computation result. Inside, the function, it will spawn the same number of threads as the number of roles in the system. Each thread will execute the code which is generated from their corresponding Proc expression. In addition to that, the function will send the input parameter to the starting role in the group of Proc to kick off the computation using channel and waiting for the the ending role to send back the computation results. After it receive the result and all the threads have returned, it will return the result. We called this function `proc0`.

An example of the generated code is shown in Listing 36. The code contains one process which received a list of int from `proc0`, sort the input by an user defined function and then send the result to `proc0`. We omit global channels declarations and data type declaration for simplicity.

7.4 Conclusion

With the completion of code generation, we deliver the results we promised in the introduction section. We have implemented a end-to-end process that will generate low-level deadlock-free parallel code from an expressive high-level languages embedded with a flexible backend that can target multiple languages with ease. Now, it is time to evaluate the performances of our achievement with quantitative measurements.

```

void proclRt ()
{
    List_int v0;
    chan_recv_buf(c1, &v0, sizeof(List_int));
    List_int v1;
    v1 = sort(v0);
    chan_send_buf(c2, &v1, sizeof(List_int));
}
void * procl ()
{
    proclRt ();
    return NULL;
}
List_int proc0(List_int v0)
{
    c1 = chan_init(1);
    c2 = chan_init(1);
    pthread_t th1;
    pthread_create(&th1, NULL, procl, NULL);
    chan_send_buf(c1, &v0, sizeof(List_int));
    List_int v1;
    chan_recv_buf(c2, &v1, sizeof(List_int));
    pthread_join(th1, NULL);
    chan_dispose(c1);
    chan_dispose(c2);
    return v1;
}

```

Listing 36: An example of generated code

Chapter 8

Parallel algorithms and evaluation

What we have implemented so far not only can be used as a backend for compiling ParAlg but also a stand-alone tool to express parallel algorithms. In this section, we will give an overview of how to use SPar for parallel algorithms, benchmarking the performance of the generated code for various computation and analyzing the design choices of the project.

8.1 Parallel algorithms

The biggest advantage of writing parallel programs in SPar is that user can express the computation similar to the sequential one without worrying about any low-level primitives for parallel computations. We will give a recommended recipe of expressing computation in SPar to follow and two concrete examples for the explanation.

8.1.1 Four steps to write parallel algorithms in SPar

Usually, divide-and-conquer algorithms are the best candidates for SPar to parallelize. The recipe is below:

1. Understand the algorithm: We recommend programmers to express the algorithms in the recursion schemes (see Section 2.2) Recursion schemes is recommended because it separates the split function, the merge function and the structure of divide-and-conquer from each others so it helps you familiar with the building blocks of the divide-and-conquer algorithms, i.e, the data structures involved, the type signature of the split function, the type signature of merge functions and their implementations.
2. Systematic parallelization: In the first iteration, we can express the split, merge functions and other necessary helper functions in terms of the `SArrow` by combining `arr` constructor and `Prim` constructor. Every computation wrapped by `arr` with `Prim` is consider to be sequential. We will substitute them into high-level parallel patterns provided by SPar. For example, divide-and-conquer algorithms can be

parallelized by `divConquer` (see Listing 27) helper functions. Notice that the parameter for number of ways of parallelized divide-and-conquer algorithms should be set accordingly by the number of cores of the execution machine.

3. Specific parallelization: The above step is generic and can be applied to any divide-and-conquer algorithm. In this stage, what will be done is determined by the specific implementations. Programmer should inspect the implementation of these function wrapped by `arr` with `Prim` to see whether there are any parallelism to exploit. If so, programmer should rewrite these functions by the parallel patterns provided by `SPar` or arrow combinators. For example, the `split` function of the Quickhull [28] to solve convex hull problems will use a for-loop to find the point whose distance to the line is at the maximum. This step can be expressed by the parallel map-and-reduce pattern (see Listing 26). Programmers can apply this step iteratively until all possible parallelisms are exploited or programmers think it is enough. All the sequential computation are left in the form: `arr with Prim`.
4. Wrap up: Before the code generation, programmer need to implement all the `Prim` functions in terms of the target language. In the scope of this project, programmer will write them in C and create a header file. The generated code will include the header file and from then on, programmers obtain the parallelized version of the algorithm that is guaranteed to be deadlock-free.

8.1.2 Example: Merge sort

We will show how to use the framework to generate parallel code by expressing merge sort on a list of integer.

```
split :: SArrow [Int] (Either (Either () Int) ([Int], [Int]))
split = arr $ Prim "split" undefined

merge :: SArrow (Either (Either () Int) ([Int], [Int])) [Int]
merge = arr $ Prim "merge" undefined

sort :: SArrow [Int] [Int]
sort = arr $ Prim "sort" undefined
```

Listing 37: The code for atomic functions

1. The first step is to understand algorithms. The merge sort is a quite famous divide-and-conquer algorithm. It splits the list into two halves, apply the algorithm to sub-lists recursively and finally merge the sub-lists by an order. From the above description, we identify three atomic functions `split`, `merge` and the base function `sort`. We need to define the type signatures for these functions to understand what data structures are involved. For the `split` function, it will take a list of integers `[Int]` as an input and output a pair of lists of integers `(Int, Int)` as an output. To deal with the case where the input list can not be split i.e when the list is empty

or a singleton list, we wrap the output pair with a `either` type to deal with these situations. So the output type is `Either (Either () Int) ([Int], [Int])`. Accordingly, the type signature of `merge` is the reverse of the `split`. `sort` type signature will be the same as the merge sort: `[Int] -> [Int]`. Finally, we wrap them using `Prim` and `arr`. The code written for the first step is shown in the Listing 37.

```
mergesort :: Int -> SArrow [Int] [Int]
mergesort = divConq sort split merge

divConq ::
  SArrow a b
-> SArrow a (Either c (a, a))
-> SArrow (Either c (b, b)) b
-> Int
-> SArrow a b
divConq baseFunc _ _ 0 = baseFunc
divConq baseFunc alg coalg x =
  alg
  >>> (   arr Inl
      ||| (   (   (arr Fst >>>
                  divConq baseFunc alg coalg (x - 1))
              &&& (arr Snd >>>
                  divConq baseFunc alg coalg (x - 1))
            )
        >>> arr Inr
      )
  )
  >>> coalg
```

Listing 38: Construction of the algorithm using the parallel pattern and atomic functions

2. The second step is combine these atomic function in the first step in a parallel pattern we define. Since we are using our framework to parallelizing a merge sort, we will use the divide-and-conquer parallel pattern. The result of applying the pattern pattern is an expression whose type is `Int -> SArrow [Int] [Int]` where the first parameter determines the number of level of the divide-and-conquer algorithm. In this case, we will use our refined version of divided-and-conquer parallel pattern that support shortcut. The code written for the second step is shown Listing 38. For the completeness, we also include the implementation of the parallel pattern. The polymorphic parallel pattern can be used to generate non-polymorphic code.
3. The third step is optimizing the atomic function. Since `split` and `merge` are not very intensive computation. We will not modify anything for this step.

4. Up to this step, we have finished everything we need to do in the Haskell side to express computation. First of all, we will show how to generate C code from a SArrow expression using the framework. Secondly, we will talk about how to complete the implementation for the atomic functions. In the library, we have defined a function `codeGen` that takes an SArrow expression as an input and generates three files in the specified path. The first file is called `func.h` which contains declarations of all atomic functions. The second file is `data.h` which includes the definition of data structures involved in C. The third file is `code.c` which includes all the necessary headers like standard library and channel library and generated code. The structure is similar to what we described in Section 7.3.3. The last job to do is to complete the implementation of the functions declared in the `func.h`. After that, we use the generated code in whatever way we want. The structures are shown in Table 8.1.

```

0: !<1, [Int]>.(4, [Int]).end
1: ?(0, [Int]).Br<[2, 3, 4],
    {L: !<4, Either (Either () Int) ([Int], [Int])>.end,
     R: !<3, ([Int], [Int])>.Br<[2],
        {L: !<2, Either (Either () Int) ([Int], [Int])>.end,
         R: !<2, ([Int], [Int])>.!<2, [Int]>.end}>
        .end}>
    .end
2: &(1,
    {L: end,
     R: &(1,
        {L: ?(1, Either (Either () Int) ([Int], [Int])).end,
         R: ?(1, ([Int], [Int])).?(1, [Int]).end} .!<4, [Int]>
        .end})
    .end
3: &(1, {
    L: end,
    R: ?(1, ([Int], [Int])).Br<[4],
        {L: !<4, Either (Either () Int) ([Int], [Int])>.end,
         R: !<4, ([Int], [Int])>.!<4, [Int]>.end}>
        .end})
    .end
4: &(1,
    {L: ?(1, Either (Either () Int) ([Int], [Int])).end,
     R: &(3,
        {L: ?(3, Either (Either () Int) ([Int], [Int])).end,
         R: ?(3, ([Int], [Int])).?(3, [Int]).end}
        .?(2, [Int]).end})
    .!<0, [Int]>.end

```

Listing 39: Inferred session types

In addition to the generated code, we also show the inferred session types of each role in the system for the merge sort at level two in Listing 39. We can simply call the function `runType` on any SArrow expression to get a list of session types. The session types is pretty printed. The original representation of session types is in free-monad as explained in Chapter 4. `br` is a syntax sugar for a series of select (see in Section 4.3.2).

8.2 Benchmarks

```
int main()
{
    int * tmp = randomList(1048576);
    List_int a = (List_int) {1048576, tmp};
    double start = get_time();
    proc0(a);
    double end = get_time();
    printf("%lf\n", end - start);
    return 0;
}
```

Listing 40: The main function for benchmark

We have defined a specific code generation function for benchmarking the parallel algorithms. The main difference from the normal generated code is the main function. The main function will create a random source data by the specified input size, record the execution time and output the execution time. The main file is shown in Listing 40.

For each benchmark of a divide-and-conquer SArrow, we will generate the code for a range of size and for different number of unroll representing the level of the algorithms. We will record the execution time on normal laptop as well as high performance computer.

The compiler we used is gcc (version 9.1.0) with the default optimization. The two platform we run our benchmark on 1) Intel i5-8259u with 4 physical core 2) 32 cores high throughput computing machine provided by Imperial College London ICT services ¹.

We have implemented three benchmark to run.

1. **Merge sort.** It is one of the most classic divide-and-conquer algorithms. Its details are shown in the above subsection.
2. **Dot product.** It computes the inner product of two vectors whose sizes are the same and of the form 2^n .
3. **Int count.** It counts the number of occurrence of integer given a list of integer ranging from 0 to 50. `split` divides the list by halves. `count` will count the occurrence

¹<http://www.imperial.ac.uk/admin-services/ict/self-service/research-support/rcs/computing/high-throughput-computing/>

of integers and output a list of tuple where the integer is the value and the second integer is the number of occurrence of that value. `union` will union the resulting of sub-lists by summing up the occurrence.

8.2.1 Evaluation

We will demonstrate the execution time against different sizes for different levels of divide-and-conquer algorithms as well as the speedup of the parallel algorithms against the sequential algorithm. Figure 8.1 shows the result for different cases running on a 32-core machine. For each level k , we will generate a total number of 2^k threads to execute the parallel computation. The x axis indicates the size of the input where 22 means the input is a list of 2^{20} integers or a pair of list of 2^{20} integers. The execution time is measured in seconds and the speedup is computed by $\frac{t_{\text{sequential}}}{t_{\text{parallel}}}$.

For merge sort, the speedup increases as the size increases which is shown by the increasing graph from Figure 8.1a. It is valid for all the level due to all the lines are increasing. Different levels will have different degree of performance boost on increasing sizes. For dot product, this trend holds for up to size 26. When the size is 26, we witness a sudden decrease in speedup at various levels. The reason for this abnormal behavior is yet to be studied. As for the int count, the speedup increases against increasing sizes when the level is big enough. The degree of speedup increase at the large level is also the most obvious for int count among the three benchmark. This can be observed by the two slope when level = 7 and level = 8. Unexpected behavior is the line when level = 6. Its speedup is nearly as small as level = 1. Also, overproduction of threads has positive impacts on the speedup. The best level is 7 which contains 128 threads. It gives a 7.5X speedup for merge sort, 8X speedup for dot product and nearly 12X speed up for int count when the size is 2^{26} . In general, greater number of threads has better speedup for greater sizes. The level that gives the best performance depends on the number of cores of the machine. The relationship is not simply one-to-one and from experiment, we recommend to overproduce the number of threads compared the number of cores. However, the performance will not be increased greatly and even slower if the level is too big (see the comparison of speedup line for level = 7 and 8).

```

typedef enum Label {
    LEFT, RIGHT
} Label;
typedef struct List_int {
    size_t size; int * value;
} List_int;
typedef struct Sum_unit_int {
    Label label;
    union {
        int left; int right;
    } value;
} Sum_unit_int;
typedef struct Prod_List_int_List_int {
    List_int fst; List_int snd;
} Prod_List_int_List_int;
typedef struct Sum_Sum_unit_int_Prod_List_int_List_int {
    Label label;
    union {
        Sum_unit_int left; Prod_List_int_List_int right;
    } value;
} Sum_Sum_unit_int_Prod_List_int_List_int;

```

(a) data.h

```

#include "data.h"
List_int merge(Sum_Sum_unit_int_Prod_List_int_List_int);
Prod_List_int_List_int split(List_int);
List_int sort(List_int);

```

(b) func.h

```

#include<stdint.h>
#include<stdlib.h>
#include<chan.h>
#include<pthread.h>

#include"data.h"
#include"func.h"

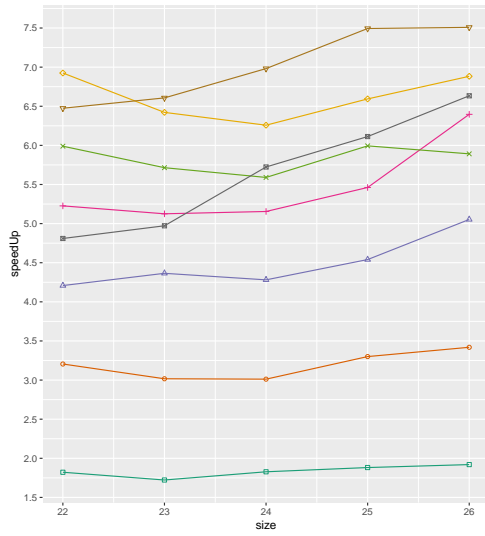
List_int proc0(List_int a) {

}

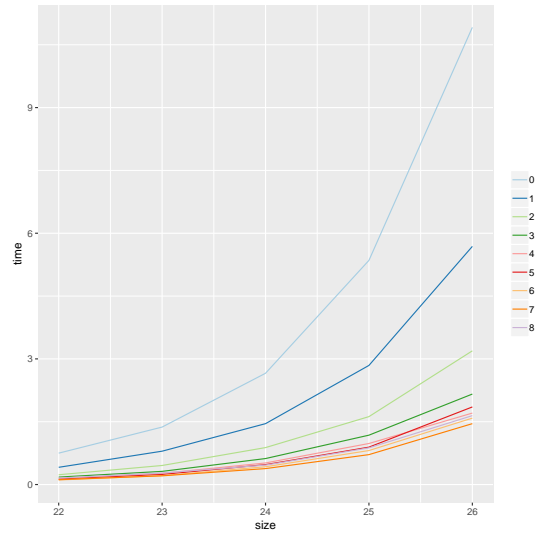
```

(c) code.c

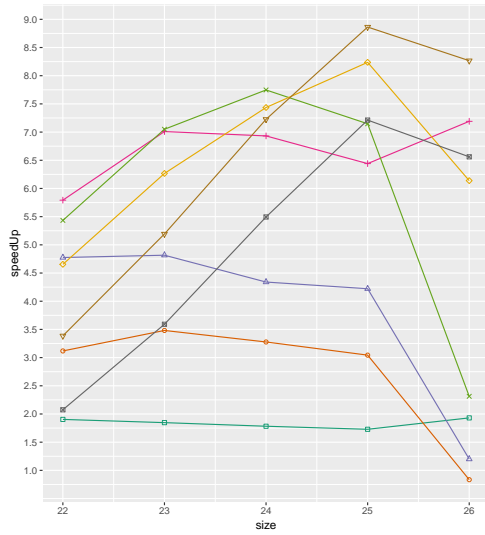
Table 8.1: The complete structure of the generated c code. Omit the implementation of code.c



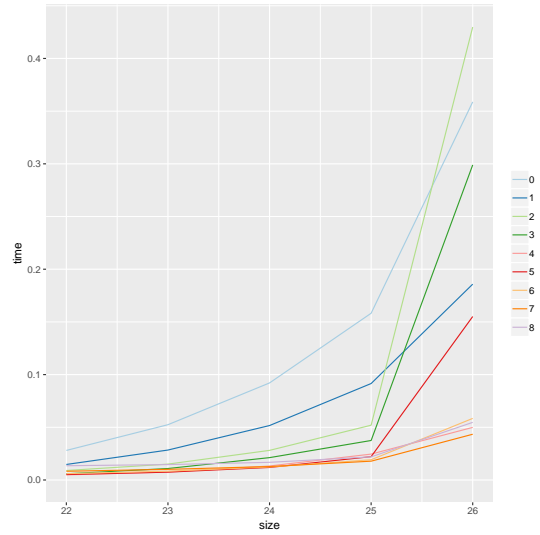
(a) Merge sort: speedup



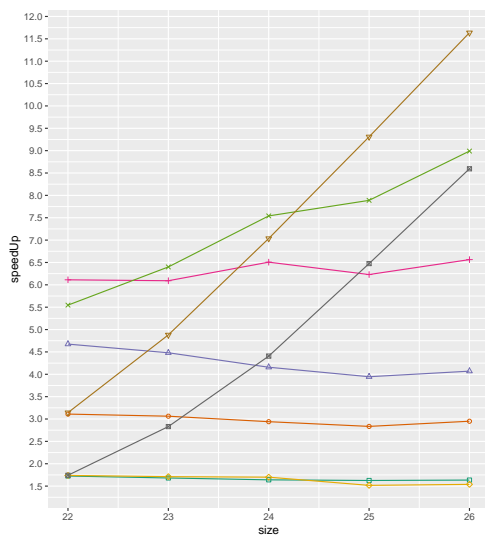
(b) Merge sort: execution time



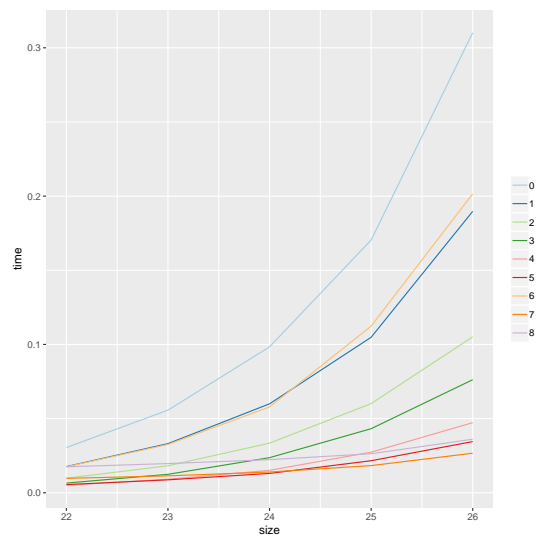
(c) Dot product: speedup



(d) Dot product: execution time



(e) Int count: speedup



(f) Int count: execution time

Figure 8.1: Benchmarks results

Chapter 9

Conclusions and future works

9.1 Conclusions

At the beginning of this project, our goal was to implement a backend for code generation for Alg parallel language using a session-typed intermediate language. The project evolved when we tried to fill the huge gap between Alg computation and SPar communication. It was then we started to design an interface as an abstraction layer on top of SPar for ease of compilation from the high-level language to it. The interface iterated from simple helper functions to arrow interface capturing general computation. Then we realized SArrow should not be limited as a compiler writer tool that hides from users, and it should be exposed to users to express computation. The application as a backend is just a side-product while the project shines as a stand-alone framework for parallel code generation.

Iterations of the project lead to the current work: A framework generates parallel C code along with local types describing the communication patterns by interpreting data-flow by communication, from Arrow based high-level expressions that can be easily formed, composed and manipulated by users with the help of the hosting function: Haskell.

In conclusion, with the recent release of AMD's latest generation of consumer CPUs featuring a processor with sixteen physical cores, Moore's law will be replaced by the addition of cores. No need to mention the area of high-performance computing where CPU with 64 cores are common. In contrast, most of the programmers have only written sequential code, and most of the algorithms about which students learn are not parallel. We hope this project can contribute its force on parallelism on CPU, encouraging more and more programmer to take advantages of modern computing architectures.

9.2 Future works

There are many interesting future works that we would like to implement. We will select some of them to introduce:

- **Optimization for benchmark.** Because of the time constraint, there are lots of space to optimize the generated code. We should do more fine-grained profiling on the generated code. It is interesting to use tools like EzTrace to trace and visualize the execution of all the threads. More importantly, reducing the size of the generated code by eliminating common sub-expressions will be useful. At the moment, there are many code duplication for communication among different roles. The only difference is that the role of participating in the communication. The size of generated code can be reduced a lot if we can extract the common part to a function parameterized by the roles participating.
- **Integrated user experience.** As demonstrated in the evaluation chapter, users need to write the computation using the EDSL in Haskell and then generate C code. From then on, they need to finish the implementation of their atom functions in C. Finally, they can run the generated code with their data in C. The user experience is isolated when you have to write Haskell first and manually completed the generated code and run them in C. Instead, it will be great if we can provide an integrated user experience where the user does everything in Haskell from writing the high-level expression to collect computation results. This is possible thanks to packages like inline-C and foreign language interface in Haskell. User experience will be greatly improve if we can offer an interface in Haskell that looks like `run :: SArrow a b -> (a -> b)`. This function will take a SArrow expression and produces a function that will convert a Haskell value into C data and execute the computation in C and copy back the C output by foreign language interface to Haskell. From the user pointer of way, it can be used the same as a normal Haskell function with type `a -> b`. Forming a closed loop in Haskell would give us the best user interface and automate a large amount of boilerplate work.
- **Fine-grained control for strategies in role allocation.** We talked about how different role allocation strategies give us different parallel computation. It will be great if we parameterize the SArrow with a different strategy and adding ways to specify what strategy will be used at a different stage of the computation. This also opens the possibilities for users to implement their strategies to customize their parallel computation tasks.
- **More customizations.** Similar to customized role allocation strategies, we can even have customized representation of sequential computation since the separation of the communication EDSL and the sequential computation EDSL. This kind of work requires expert usage of Haskell TypeClass and well-designed interfaces and hence is challenging.

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Appendix A

Examples of generated code

A.1 Merge sort

This is the generated code.c at level 3 for merge sort.

```
1  #include<stdio.h>
2  #include<stdlib.h>
3  #include<chan.h>
4  #include<pthread.h>
5  #include"data.h"
6  #include"func.h"
7  chan_t * c1;
8  chan_t * c2;
9  chan_t * c3;
10 chan_t * c4;
11 chan_t * c5;
12 chan_t * c6;
13 chan_t * c7;
14 chan_t * c8;
15 chan_t * c9;
16 chan_t * c10;
17 chan_t * c11;
18 chan_t * c12;
19 chan_t * c13;
20 chan_t * c14;
21 chan_t * c15;
22 chan_t * c16;
23 chan_t * c17;
24 void proclRt()
25 {
26     List_int v0;
27     chan_rcv_buf(c1, &v0, sizeof(List_int));
28     Sum_Sum_unit_int_Prod_List_int_List_int v1;
29     v1 = split(v0);
30     Label v2;
31     v2 = v1.label;
32     chan_send_int(c2, v2);
33     chan_send_int(c3, v2);
34     chan_send_int(c4, v2);
35     chan_send_int(c5, v2);
36     chan_send_int(c6, v2);
37     chan_send_int(c7, v2);
38     chan_send_int(c8, v2);
39     Sum_unit_int v3;
40     Prod_List_int_List_int v4;
41     int v5;
42     if (v1.label == LEFT)
43     {
44         v3 = v1.value.left;
45         Sum_Sum_unit_int_Prod_List_int_List_int v6;
46         v6 = (Sum_Sum_unit_int_Prod_List_int_List_int) {LEFT, { .left = v3 }};
47         chan_send_buf(c8,
48                     &v6,
49                     sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
50         v5 = 0;
51     }
52     else
53     {
54         v4 = v1.value.right;
55         Prod_List_int_List_int v7;
```



```

56     v7 = v4;
57     chan_send_buf(c5, &v7, sizeof(Prod_List_int_List_int));
58     Sum_Sum_unit_int_Prod_List_int_List_int v8;
59     v8 = split(v4.snd);
60     Label v9;
61     v9 = v8.label;
62     chan_send_int(c2, v9);
63     chan_send_int(c3, v9);
64     chan_send_int(c4, v9);
65     Sum_unit_int v10;
66     Prod_List_int_List_int v11;
67     int v12;
68     if (v8.label == LEFT)
69     {
70         v10 = v8.value.left;
71         Sum_Sum_unit_int_Prod_List_int_List_int v13;
72         v13 = (Sum_Sum_unit_int_Prod_List_int_List_int) {LEFT, { .left = v10 }};
73         chan_send_buf(c4,
74                     &v13,
75                     sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
76         v12 = 0;
77     }
78     else
79     {
80         v11 = v8.value.right;
81         Prod_List_int_List_int v14;
82         v14 = v11;
83         chan_send_buf(c3, &v14, sizeof(Prod_List_int_List_int));
84         Sum_Sum_unit_int_Prod_List_int_List_int v15;
85         v15 = split(v11.snd);
86         Label v16;
87         v16 = v15.label;
88         chan_send_int(c2, v16);
89         Sum_unit_int v17;
90         Prod_List_int_List_int v18;
91         int v19;
92         if (v15.label == LEFT)
93         {
94             v17 = v15.value.left;
95             Sum_Sum_unit_int_Prod_List_int_List_int v20;
96             v20 = (Sum_Sum_unit_int_Prod_List_int_List_int) {LEFT, { .left = v17 }};
97             chan_send_buf(c2,
98                         &v20,
99                         sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
100            v19 = 0;
101        }
102        else
103        {
104            v18 = v15.value.right;
105            Prod_List_int_List_int v21;
106            v21 = v18;
107            chan_send_buf(c2, &v21, sizeof(Prod_List_int_List_int));
108            List_int v22;
109            v22 = sort(v18.snd);
110            chan_send_buf(c2, &v22, sizeof(List_int));
111            v19 = 0;
112        }
113        v12 = 0;
114    }
115    v5 = 0;
116 }
117 }
118 void proc2Rt()
119 {
120     int v24;
121     Label v23;
122     chan_recv_int(c2, &v23);
123     if (v23 == LEFT)
124     {
125         v24 = 0;
126     }
127     else
128     {
129         Label v25;
130         chan_recv_int(c2, &v25);
131         if (v25 == LEFT)
132         {
133         }
134         else
135         {
136             Sum_Sum_unit_int_Prod_List_int_List_int v27;
137             Label v26;
138             chan_recv_int(c2, &v26);
139             if (v26 == LEFT)
140             {
141                 Sum_Sum_unit_int_Prod_List_int_List_int v28;

```

```

142         chan_rcv_buf(c2,
143                     &v28,
144                     sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
145         v27 = v28;
146     }
147     else
148     {
149         Prod_List_int_List_int v29;
150         chan_rcv_buf(c2, &v29, sizeof(Prod_List_int_List_int));
151         List_int v30;
152         v30 = sort(v29.fst);
153         List_int v31;
154         chan_rcv_buf(c2, &v31, sizeof(List_int));
155         v27 = (Sum_Sum_unit_int_Prod_List_int_List_int) {RIGHT, { .right = (Prod_List_int_List_int) {v30, v31} }};
156     }
157     List_int v32;
158     v32 = merge(v27);
159     chan_send_buf(c9, &v32, sizeof(List_int));
160 }
161 v24 = 0;
162 }
163 }
164 void proc3Rt()
165 {
166     int v34;
167     Label v33;
168     chan_rcv_int(c3, &v33);
169     if (v33 == LEFT)
170     {
171         v34 = 0;
172     }
173     else
174     {
175         int v36;
176         Label v35;
177         chan_rcv_int(c3, &v35);
178         if (v35 == LEFT)
179         {
180             v36 = 0;
181         }
182         else
183         {
184             Prod_List_int_List_int v37;
185             chan_rcv_buf(c3, &v37, sizeof(Prod_List_int_List_int));
186             Sum_Sum_unit_int_Prod_List_int_List_int v38;
187             v38 = split(v37.fst);
188             Label v39;
189             v39 = v38.label;
190             chan_send_int(c10, v39);
191             Sum_unit_int v40;
192             Prod_List_int_List_int v41;
193             int v42;
194             if (v38.label == LEFT)
195             {
196                 v40 = v38.value.left;
197                 Sum_Sum_unit_int_Prod_List_int_List_int v43;
198                 v43 = (Sum_Sum_unit_int_Prod_List_int_List_int) {LEFT, { .left = v40 }};
199                 chan_send_buf(c10,
200                             &v43,
201                             sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
202                 v42 = 0;
203             }
204             else
205             {
206                 v41 = v38.value.right;
207                 Prod_List_int_List_int v44;
208                 v44 = v41;
209                 chan_send_buf(c10, &v44, sizeof(Prod_List_int_List_int));
210                 List_int v45;
211                 v45 = sort(v41.snd);
212                 chan_send_buf(c10, &v45, sizeof(List_int));
213                 v42 = 0;
214             }
215             v36 = v42;
216         }
217         v34 = v36;
218     }
219 }
220 void proc4Rt()
221 {
222     Label v46;
223     chan_rcv_int(c4, &v46);
224     if (v46 == LEFT)
225     {
226     }
227     else

```

```

228     {
229         Sum_Sum_unit_int_Prod_List_int_List_int v48;
230         Label v47;
231         chan_rcv_int(c4, &v47);
232         if (v47 == LEFT)
233         {
234             Sum_Sum_unit_int_Prod_List_int_List_int v49;
235             chan_rcv_buf(c4,
236                         &v49,
237                         sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
238             v48 = v49;
239         }
240         else
241         {
242             Sum_Sum_unit_int_Prod_List_int_List_int v51;
243             Label v50;
244             chan_rcv_int(c10, &v50);
245             if (v50 == LEFT)
246             {
247                 Sum_Sum_unit_int_Prod_List_int_List_int v52;
248                 chan_rcv_buf(c10,
249                             &v52,
250                             sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
251                 v51 = v52;
252             }
253             else
254             {
255                 Prod_List_int_List_int v53;
256                 chan_rcv_buf(c10, &v53, sizeof(Prod_List_int_List_int));
257                 List_int v54;
258                 v54 = sort(v53.fst);
259                 List_int v55;
260                 chan_rcv_buf(c10, &v55, sizeof(List_int));
261                 v51 = (Sum_Sum_unit_int_Prod_List_int_List_int) {RIGHT, { .right = (Prod_List_int_List_int) {v54, v55} }};
262             }
263             List_int v56;
264             v56 = merge(v51);
265             List_int v57;
266             chan_rcv_buf(c9, &v57, sizeof(List_int));
267             v48 = (Sum_Sum_unit_int_Prod_List_int_List_int) {RIGHT, { .right = (Prod_List_int_List_int) {v56, v57} }};
268         }
269         List_int v58;
270         v58 = merge(v48);
271         chan_send_buf(c11, &v58, sizeof(List_int));
272     }
273 }
274 void proc5Rt()
275 {
276     int v60;
277     Label v59;
278     chan_rcv_int(c5, &v59);
279     if (v59 == LEFT)
280     {
281         v60 = 0;
282     }
283     else
284     {
285         Prod_List_int_List_int v61;
286         chan_rcv_buf(c5, &v61, sizeof(Prod_List_int_List_int));
287         Sum_Sum_unit_int_Prod_List_int_List_int v62;
288         v62 = split(v61.fst);
289         Label v63;
290         v63 = v62.label;
291         chan_send_int(c12, v63);
292         chan_send_int(c13, v63);
293         chan_send_int(c14, v63);
294         Sum_unit_int v64;
295         Prod_List_int_List_int v65;
296         int v66;
297         if (v62.label == LEFT)
298         {
299             v64 = v62.value.left;
300             Sum_Sum_unit_int_Prod_List_int_List_int v67;
301             v67 = (Sum_Sum_unit_int_Prod_List_int_List_int) {LEFT, { .left = v64 }};
302             chan_send_buf(c14,
303                         &v67,
304                         sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
305             v66 = 0;
306         }
307         else
308         {
309             v65 = v62.value.right;
310             Prod_List_int_List_int v68;
311             v68 = v65;
312             chan_send_buf(c13, &v68, sizeof(Prod_List_int_List_int));
313             Sum_Sum_unit_int_Prod_List_int_List_int v69;

```

```

314         v69 = split(v65.snd);
315         Label v70;
316         v70 = v69.label;
317         chan_send_int(c12, v70);
318         Sum_unit_int v71;
319         Prod_List_int_List_int v72;
320         int v73;
321         if (v69.label == LEFT)
322         {
323             v71 = v69.value.left;
324             Sum_Sum_unit_int_Prod_List_int_List_int v74;
325             v74 = (Sum_Sum_unit_int_Prod_List_int_List_int) {LEFT, { .left = v71 }};
326             chan_send_buf(c12,
327                           &v74,
328                           sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
329             v73 = 0;
330         }
331         else
332         {
333             v72 = v69.value.right;
334             Prod_List_int_List_int v75;
335             v75 = v72;
336             chan_send_buf(c12, &v75, sizeof(Prod_List_int_List_int));
337             List_int v76;
338             v76 = sort(v72.snd);
339             chan_send_buf(c12, &v76, sizeof(List_int));
340             v73 = 0;
341         }
342         v66 = 0;
343     }
344     v60 = v66;
345 }
346
347 void proc6Rt()
348 {
349     int v78;
350     Label v77;
351     chan_recv_int(c6, &v77);
352     if (v77 == LEFT)
353     {
354         v78 = 0;
355     }
356     else
357     {
358         Label v79;
359         chan_recv_int(c12, &v79);
360         if (v79 == LEFT)
361         {
362         }
363         else
364         {
365             Sum_Sum_unit_int_Prod_List_int_List_int v81;
366             Label v80;
367             chan_recv_int(c12, &v80);
368             if (v80 == LEFT)
369             {
370                 Sum_Sum_unit_int_Prod_List_int_List_int v82;
371                 chan_recv_buf(c12,
372                               &v82,
373                               sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
374                 v81 = v82;
375             }
376             else
377             {
378                 Prod_List_int_List_int v83;
379                 chan_recv_buf(c12, &v83, sizeof(Prod_List_int_List_int));
380                 List_int v84;
381                 v84 = sort(v83.fst);
382                 List_int v85;
383                 chan_recv_buf(c12, &v85, sizeof(List_int));
384                 v81 = (Sum_Sum_unit_int_Prod_List_int_List_int) {RIGHT, { .right = (Prod_List_int_List_int) {v84, v85} }};
385             }
386             List_int v86;
387             v86 = merge(v81);
388             chan_send_buf(c15, &v86, sizeof(List_int));
389         }
390         v78 = 0;
391     }
392 }
393
394 void proc7Rt()
395 {
396     int v88;
397     Label v87;
398     chan_recv_int(c7, &v87);
399     if (v87 == LEFT)
400     {

```

```

400         v88 = 0;
401     }
402     else
403     {
404         int v90;
405         Label v89;
406         chan_rcv_int(c13, &v89);
407         if (v89 == LEFT)
408         {
409             v90 = 0;
410         }
411         else
412         {
413             Prod_List_int_List_int v91;
414             chan_rcv_buf(c13, &v91, sizeof(Prod_List_int_List_int));
415             Sum_Sum_unit_int_Prod_List_int_List_int v92;
416             v92 = split(v91.fst);
417             Label v93;
418             v93 = v92.label;
419             chan_send_int(c16, v93);
420             Sum_unit_int v94;
421             Prod_List_int_List_int v95;
422             int v96;
423             if (v92.label == LEFT)
424             {
425                 v94 = v92.value.left;
426                 Sum_Sum_unit_int_Prod_List_int_List_int v97;
427                 v97 = (Sum_Sum_unit_int_Prod_List_int_List_int) {LEFT, { .left = v94 }};
428                 chan_send_buf(c16,
429                             &v97,
430                             sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
431                 v96 = 0;
432             }
433             else
434             {
435                 v95 = v92.value.right;
436                 Prod_List_int_List_int v98;
437                 v98 = v95;
438                 chan_send_buf(c16, &v98, sizeof(Prod_List_int_List_int));
439                 List_int v99;
440                 v99 = sort(v95.snd);
441                 chan_send_buf(c16, &v99, sizeof(List_int));
442                 v96 = 0;
443             }
444             v90 = v96;
445         }
446         v88 = v90;
447     }
448 }
449 void proc8Rt()
450 {
451     Sum_Sum_unit_int_Prod_List_int_List_int v101;
452     Label v100;
453     chan_rcv_int(c8, &v100);
454     if (v100 == LEFT)
455     {
456         Sum_Sum_unit_int_Prod_List_int_List_int v102;
457         chan_rcv_buf(c8,
458                     &v102,
459                     sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
460         v101 = v102;
461     }
462     else
463     {
464         Sum_Sum_unit_int_Prod_List_int_List_int v104;
465         Label v103;
466         chan_rcv_int(c14, &v103);
467         if (v103 == LEFT)
468         {
469             Sum_Sum_unit_int_Prod_List_int_List_int v105;
470             chan_rcv_buf(c14,
471                         &v105,
472                         sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));
473             v104 = v105;
474         }
475         else
476         {
477             Sum_Sum_unit_int_Prod_List_int_List_int v107;
478             Label v106;
479             chan_rcv_int(c16, &v106);
480             if (v106 == LEFT)
481             {
482                 Sum_Sum_unit_int_Prod_List_int_List_int v108;
483                 chan_rcv_buf(c16,
484                             &v108,
485                             sizeof(Sum_Sum_unit_int_Prod_List_int_List_int));

```

```

486         v107 = v108;
487     }
488     else
489     {
490         Prod_List_int_List_int v109;
491         chan_recv_buf(c16, &v109, sizeof(Prod_List_int_List_int));
492         List_int v110;
493         v110 = sort(v109.fst);
494         List_int v111;
495         chan_recv_buf(c16, &v111, sizeof(List_int));
496         v107 = (Sum_Sum_unit_int_Prod_List_int_List_int) {RIGHT, { .right = (Prod_List_int_List_int) {v110, v111} }};
497     }
498     List_int v112;
499     v112 = merge(v107);
500     List_int v113;
501     chan_recv_buf(c15, &v113, sizeof(List_int));
502     v104 = (Sum_Sum_unit_int_Prod_List_int_List_int) {RIGHT, { .right = (Prod_List_int_List_int) {v112, v113} }};
503 }
504 List_int v114;
505 v114 = merge(v104);
506 List_int v115;
507 chan_recv_buf(c11, &v115, sizeof(List_int));
508 v101 = (Sum_Sum_unit_int_Prod_List_int_List_int) {RIGHT, { .right = (Prod_List_int_List_int) {v114, v115} }};
509 }
510 List_int v116;
511 v116 = merge(v101);
512 chan_send_buf(c17, &v116, sizeof(List_int));
513 }
514 void * proc1()
515 {
516     proc1Rt();
517     return NULL;
518 }
519 void * proc2()
520 {
521     proc2Rt();
522     return NULL;
523 }
524 void * proc3()
525 {
526     proc3Rt();
527     return NULL;
528 }
529 void * proc4()
530 {
531     proc4Rt();
532     return NULL;
533 }
534 void * proc5()
535 {
536     proc5Rt();
537     return NULL;
538 }
539 void * proc6()
540 {
541     proc6Rt();
542     return NULL;
543 }
544 void * proc7()
545 {
546     proc7Rt();
547     return NULL;
548 }
549 void * proc8()
550 {
551     proc8Rt();
552     return NULL;
553 }
554 List_int proc0(List_int v0)
555 {
556     c1 = chan_init(1);
557     c2 = chan_init(1);
558     c3 = chan_init(1);
559     c4 = chan_init(1);
560     c5 = chan_init(1);
561     c6 = chan_init(1);
562     c7 = chan_init(1);
563     c8 = chan_init(1);
564     c9 = chan_init(1);
565     c10 = chan_init(1);
566     c11 = chan_init(1);
567     c12 = chan_init(1);
568     c13 = chan_init(1);
569     c14 = chan_init(1);
570     c15 = chan_init(1);
571     c16 = chan_init(1);

```

```

572     c17 = chan_init(1);
573     pthread_t th1;
574     pthread_create(&th1, NULL, proc1, NULL);
575     pthread_t th2;
576     pthread_create(&th2, NULL, proc2, NULL);
577     pthread_t th3;
578     pthread_create(&th3, NULL, proc3, NULL);
579     pthread_t th4;
580     pthread_create(&th4, NULL, proc4, NULL);
581     pthread_t th5;
582     pthread_create(&th5, NULL, proc5, NULL);
583     pthread_t th6;
584     pthread_create(&th6, NULL, proc6, NULL);
585     pthread_t th7;
586     pthread_create(&th7, NULL, proc7, NULL);
587     pthread_t th8;
588     pthread_create(&th8, NULL, proc8, NULL);
589     chan_send_buf(c1, &v0, sizeof(List_int));
590     List_int v1;
591     chan_rcv_buf(c17, &v1, sizeof(List_int));
592     pthread_join(th1, NULL);
593     pthread_join(th2, NULL);
594     pthread_join(th3, NULL);
595     pthread_join(th4, NULL);
596     pthread_join(th5, NULL);
597     pthread_join(th6, NULL);
598     pthread_join(th7, NULL);
599     pthread_join(th8, NULL);
600     chan_dispose(c1);
601     chan_dispose(c2);
602     chan_dispose(c3);
603     chan_dispose(c4);
604     chan_dispose(c5);
605     chan_dispose(c6);
606     chan_dispose(c7);
607     chan_dispose(c8);
608     chan_dispose(c9);
609     chan_dispose(c10);
610     chan_dispose(c11);
611     chan_dispose(c12);
612     chan_dispose(c13);
613     chan_dispose(c14);
614     chan_dispose(c15);
615     chan_dispose(c16);
616     chan_dispose(c17);
617     return v1;
618 }
619 int main()
620 {
621     int * tmp = randomList(1024);
622     List_int a = (List_int) {1024, tmp};
623     double start = get_time();
624     proc0(a);
625     double end = get_time();
626     printf("%lf\n", end - start);
627     return 0;
628 }

```