Imperial College London

MEng Individual Project

Imperial College London

Department of Computing

TODO

Author:
Shuhao Zhang

Supervisor:
Prof. Nobuko Yoshida
David Castro-Perez

Second Marker:
Dr Iain Phillips

June 16, 2019

Abstract

Parallel code is notorious for its difficulties in writing, verification and maintaining. However, with the end of Moore's law, CPUs evolve by including more cores. Modern programmers are expected to utilize the power of multi-core CPUs and face the challenges brought by parallel programs.

This project builds an embedded framework in Haskell to generate parallel code. Combining the power of multiparty session types with parallel computation, it creates a session typed monadic language as the middle layer and use the Arrow interface as an abstraction layer on top of the language. With the help of Arrow interface, we convert the data flow of the computation to communications and generate parallel codes according to the communication pattern. Thanks to the addition of session types, not only the generated code is guaranteed to be deadlock-free but also we gain a set of local types so that it is possible to reason about the communication structure of the parallel computation.

In order to show the framework is as expressive as usual programming languages, we write a few common parallel computation patterns and three algorithms to benchmarks using our framework. Benchmarks show users can express computation similar to traditional sequential code and gain a high-performance parallel code in low-level target languages for free. Also, the use case of the framework is not limited to a standalone tool for parallel computation; we show the framework can act as a code generation backend for other data-flow based high-level parallel languages with an example.

Contents

Chapter 1

Introduction

1.1 Motivation

Writing parallel software is not a trivial task. Parallel code is hard to write because it is usually written in low level languages with verbose and non-idiomatic decorations, hard to debug because machines, where code is written, are usually different from machines where code is intended to run and hard to maintain and reuse because even though the underlying algorithms are not changed, multiple version of parallel code is needed to tackle various platform and evolution of architectures.

There are many on-going pieces of research aimed at helping programmers write correct parallel programs smoothly. A common approach is to develop a higher level language and compiles programmes in this language to required parallel code. There are many high-level frameworks for parallel programming (e.g. algorithmic skeletons[1], domain-specific languages for parallelism[2] or famous MapReduce parallel model[3]). An example is to use arrow terms (Section 2.1) to describe data flow implicitly and hence generate parallel code.

The workflow of writing parallel code has evolved from writing it directly in the target platform to writing software in a high-level language designed for parallel computation and then compiling to the target platform. In this project, we present a method to improve the backend of parallel code generation by introducing a monadic domain-specific language to act as a bridge between high-level and target low-level parallel languages.

This specific language needs to be general enough so that it supports multiple high-level parallel programming frameworks. It can be used to generate different parallel code, e.g. MPI [1], Cuda. Moreover, it can be interpreted with a simulator to aid debugging parallel programs.

In addition, it couples with multiparty session type (MPST) [5]. It takes advantages of properties of MPST to enable aggressive optimisation but ensuring code correctness and allow more meaningful static analysis; e.g. cost modelling for parallel programming.

[1] Message Passing Interface (MPI) is a standardized and portable message-passing standard designed by a group of researchers from academia and industry to function on a wide variety of parallel computing architectures [4].

1.2 Contributions

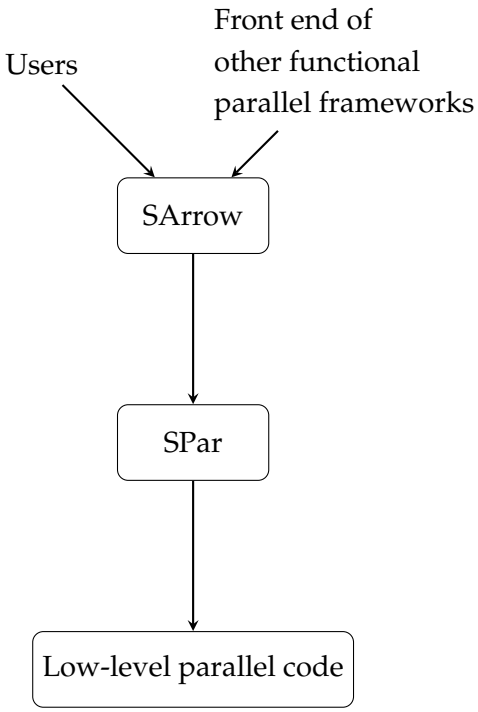


Figure 1.1: Visualization of the workflow

The result of project is a embedded high-level framework in Haskell that is capable of generating low-level parallel code. The major contributions are:

1. **Session-typed intermediate language.** We create an intermediate embedded domain specific language (EDSL): SPar: a session typed free monad EDSL for message passing concurrency. This language can be typed by local types, and hence, we can apply multiple results from the multiparty session types to our framework, especially in terms of safety of generated code and reasoning of communication patterns.

2. **Intuitive user interface.** One innovation of this project is that we apply the mature Arrow interface for users to express parallel computations. We call the interface SArrow: an arrow interface for writing SPar expressions. It is an abstraction layer on top of SPar, which hides communication primitives from users so that users can express parallel algorithms similar to what they would write for sequential programs.

3. **Multiple backends.** We create a backend to generate parallel C code from SPar expressions. The core of the backend is Instr: a low-level EDSL that is independent of target languages. This means that we can support multiple target languages with ease without re-implementing multiple backends. In addition to the code generation backend, we implement an interpreter backend in Haskell for experimenting and fast verification.

4. **Evaluations.** Finally, we show the expressive power of the framework by implementing several common computation patterns and three algorithms using our interface. We evaluate the performance of the generated code from the algorithms on high-performance computers as well as PCs.

The Figure 1.1 summaries the workflow of the framework visually. The main principle supporting the framework is that we convert data flow into communications and from the communication patterns, we gain parallel codes. The results of expressing computation in the framework are 1) compilation to efficient deadlock-free low-level parallel programs and 2) a set of local types to reason the structure of the parallel computation.

At the end of the project, we have discovered two use case of the framework. The primary application is a stand-alone tool to generate parallel C code, and another is a backend for other data-flow based parallel frameworks.

1.3 Report outlines

Chapter 2 gives an overview of the background and related researches. We present the syntax and semantics of SPar in Chapter 4 followed by Chapter 5 introducing the implementation aspect of SPar like session typing and interpreter. Chapter 6 demonstrates the Arrow interface with examples of parallel patterns formed by the interface and justification of the interface satisfying arrow laws. The discussion about some implementation specific issues like role allocation is also contained in Chapter 6. In Chapter 7, we show the code generation backend and discuss our solutions to challenges when compiling to C, i.e. the problem of representing polymorphic algebraic data structure in C. Chapter 8 explains our benchmarks and shows the performance of the generated code. This chapter can also be regarded as a tutorial on how to use the framework. Finally, we conclude with potential future improvements and remarks on this project. We also include the generated C code in the appendix for curious readers.

Chapter 2

Background

This section is an overview of techniques that influence the design choices of our monadic language for parallel computation. First of all, we give an overview of techniques applied in the high-level parallel programming framework: arrows (Section 2.1) and recursion schemes (Section 2.2). We then introduce several techniques for message-passing concurrency: multiparty session types (Section 2.3) and monadic languages for concurrency (Section 2.4). In the end, we introduce free monads (Section 2.5), a technique valuable in implementing embedded domain-specific languages (EDSL).

2.1 Arrows

Arrow is a general interface to describe computation. It can ease the process of writing structured code suitable for parallelising. It also demos a common feature of the frameworks: parallelizability is empowered by underlying implicit but precise data-flow. On the other hand, converting to low-level message-passing code, which requires programmers to define communication using message-passing function and primitives, makes the data-flow explicit.

2.1.1 Definition

Listing 1 shows the Arrow definition in Haskell. Intuitively, an arrow type `y a b`(that is, the application of the parameterised type `y` to the two parameter types `b` and `c`) can be regarded as a computation with input of type `b` and output of type `b`[6]. Visually, arrows are like pipelines (shown in Figure 2.1). In Haskell, an arrow `y` is a type that implements the following interface (type classes in Haskell are roughly interfaces). `arr` converts an arbitrary function into an arrow. `>>>` sequences two arrows (illustrated in Figure 2.1b). Taking two input, `first` apply the arrow to the first input while keeping the second untouched (Figure 2.1a). Conversely, `second` modifies the second input and keeps the first one unchanged. `***` applies two arrows to two input side by side (Figure 2.1d). `&&&` takes one input and applies two separate arrows to the input and its duplications (Figure 2.1c).

The simplest instance of arrow class is the function type (shown in Listing 2). It

is worth noticing that only `arr` and `***` need to be implemented. The reset of function in the arrow type class can be defined in terms of the two functions. For example, `f &&& g = (f *** g) . arr (\b -> (b, b))` and `first = (*** id)`

```
class Arrow y where
  arr :: (a -> b) -> y a b
  first :: y a b -> y (a, c) (b, c)
  second :: y a b -> y (c, a) (c, b)
  (***) :: y a c -> y b d -> y (a, b) (c, d)
  (&&&) :: y a b -> y a c -> y a (b, c)
```

Listing 1: Arrow class in Haskell

```
instance Arrow (->) where
  arr f = f
  (***) f g ~(x,y) = (f x, g y)
```

Listing 2: $(\rightarrow)$ instance of Arrow class

2.1.2 Example: Calculate the mean

Consider the a function to calculate the mean from a list of floating number, we will compare the usual, arrows implementations. Implementation using arrows can be regarded as point-free programming. Point-free programming is programming paradigm where function definitions only involve combinators and function composition without mentioning variables[8].

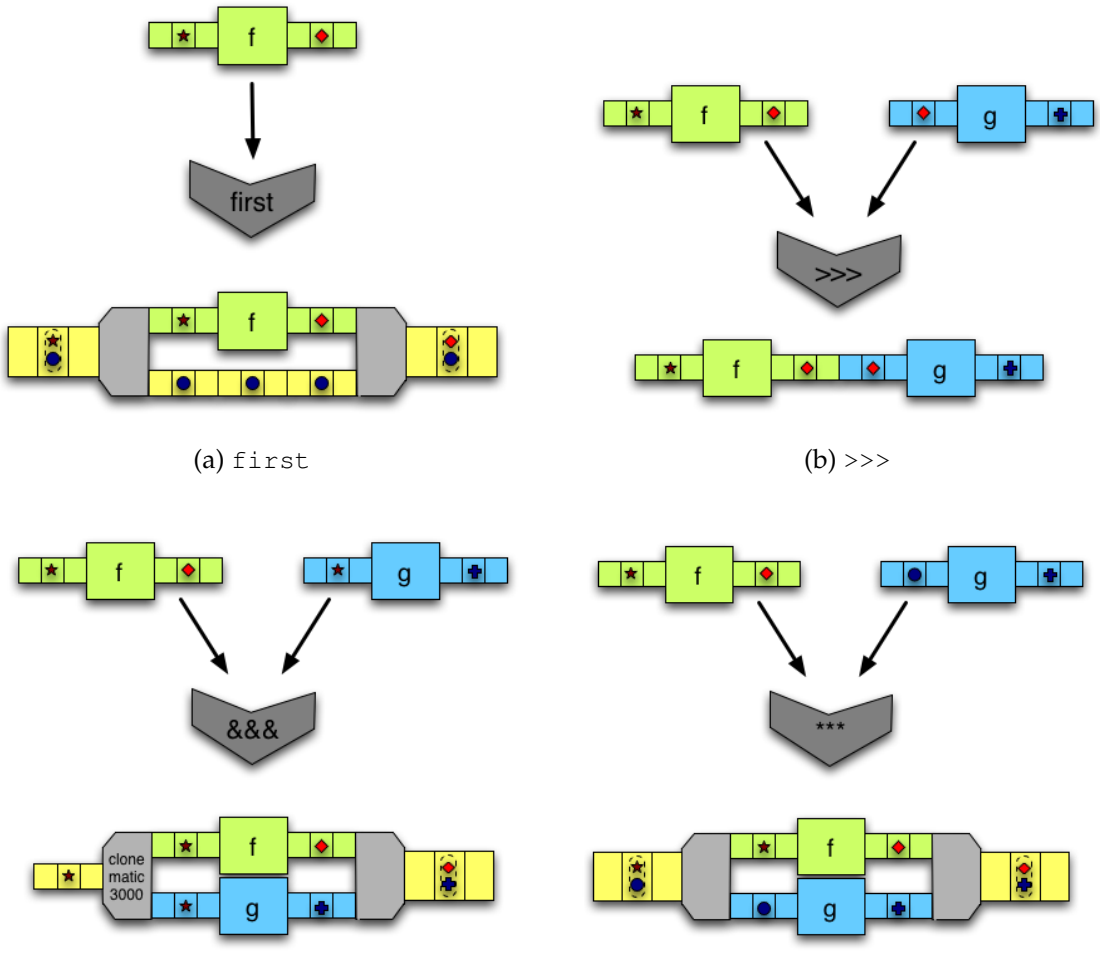
```
mean :: [Float] -> Float
mean xs = sum xs / (fromIntegral . length) xs

mean' :: [Float] -> Float
mean' = (sum &&& (length >>> fromIntegral)) >>> uncurry (/)
```

The arrows implementation can be visualised in Figure 2.2.

```
mean'' :: [Float] -> Float
mean'' = liftM2 (/) sum (fromIntegral . length)
```

Arrows are not the only way to form point-free programs. The above code snippet is the more traditional approach form of point-free mean function in Haskell. We can argue this form of point-free function is more difficult to understand compared to arrows because it involves knowledge of monads (`liftM2`) and does not map to intuitive data-flow.

The simple example demos that arrows combinators make writing point-free programs easier. Arrows union the implementation of algorithm and data-flow in the algorithm.



(a) `first` (b) `>>>`

(c) `&&&` (d) `***`

Figure 2.1: The visual representations of arrow combinators[7]

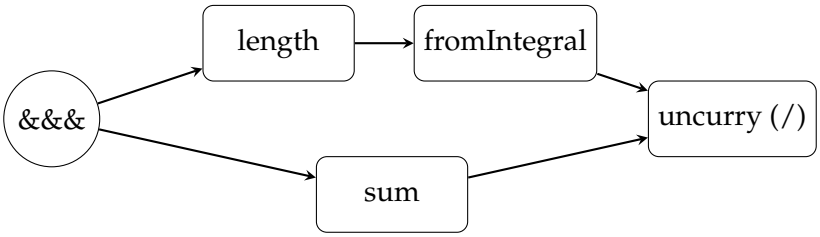
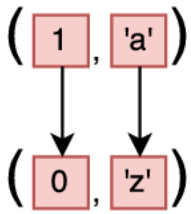


Figure 2.2: Visualization of mean'

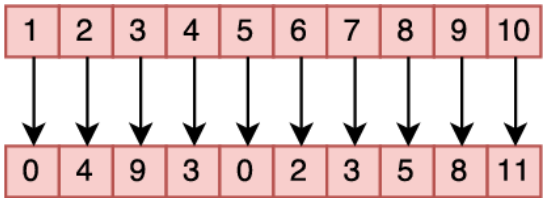
2.1.3 Application in parallel computation

From the previous example, the data flow of programs written regarding arrow combinators can be easily visualised (shown in Figure 2.1). It is intuitive to recognise that the clean separation between the flow of data and actual computation will be useful in generating parallel code. Indeed, arrow describes data flow implicitly, and it is an example of the so-called algebraic pattern. Many works [9, 10, 11] has been done to generate parallel code from algebraic patterns. In particular, details of [11] are introduced in later section.

We will use some figures to explain the idea behind arrows as a framework for parallel computation. For example, as shown in the Figure 2.3a, `f *** g` means computations of `f` and `g` happened in parallel. Figure 2.3b shows that it can be extending to parallel map in terms of arrows, taking an arrow computation `arr ab` and returning a list of computation in parallel (`arr [a] [b]`).



(a) Visualization of parallel `***`[9]



(b) Visualization of parMap [9]

2.2 Recursion Schemes

Recursion schemes are patterns for expressing general computation. In particular, they are like high order function abstracting recursion so that programmer can express any kind of recursion by data structures combined with recursion schemes instead of writing explicit recursive functions.

2.2.1 Definition

We will introduce three typical recursion schemes: catamorphisms, anamorphisms and hylomorphisms (seen in Listing 4). As mentioned before, recursion schemes express recursion with the help of data structures, in particular, the fixed point of data structures (seen in Listing 3)

```haskell
newtype Fix f = Fix { unfix :: f (Fix f) }

data TreeF a =
    Node a a
  | Leaf int
  | Empty
  deriving Functor

type Tree = Fix TreeF
```

Listing 3: Definition of fix point of data structures

Anamorphisms takes a function from a to f a (called the co-algebra) and a value a and return the Fix f. Used Tree as an example, anamorphisms takes a single value a and applies the co-algebra to the value. It continues to apply itself to the branches of the TreeF recursively and finally expands a single value to a complete tree. Intuitively, anamorphism unfolds a single value to a complicated data structure top-down.

Catamorphisms is the reverse of anamorphisms, folding a data structure to a single value bottom-up. It takes a function from f a to a (called the algebra) and Fix f to fold and return a single value a. Catamorphisms and anamorphisms describe the process globally (from a to Fix f and from Fix f to a) while co-algebra and algebra capture what happened locally. The elegant part is while co-algebra and algebra do not involve with any recursion data structure (TreeF is not recursive), catamorphisms consumes recursive data structure while anamorphism builds them.

Hylomorphisms applies anamorphism followed by catamorphisms. It is the most common pattern to use. We will use an example to illustrate its usefulness. It can be thought of as an abstract divide and conquer algorithm.

```haskell
ana :: Functor f => (a -> f a) -> a -> Fix f
ana coalg = Fix . fmap (ana coalg) . coalg

cata :: Functor f => (f a -> a) -> Fix f -> a
cata alg = alg . fmap (cata alg) . unfix

hylo :: (f b -> b) -> (a -> f a) -> b -> a
hylo g f = f . fmap (hylo f g) . g
```

Listing 4: Recursion schemes in haskell

2.2.2 Example: Merge sort

We can write merge sort recursively. First of all, we split the list in half and then apply the merge sort recursively to both parts and finally we merge two lists into a single list.

To write merge sort in terms of recursion scheme, we need to define the recursive structure to represent the control structure. By the definition of merge sort, this structure must have a case with two branches, a base case representing a singleton list and a

base case representing an empty list hence this structure is the TreeF we defined above. Splitting a list is like co-algebra while merging is like algebra. We use hylomorphisms to combine them hence getting a sorted list (seen in Listing 5).

```
mergeSort :: [Int] -> [Int]
mergeSort = hylo merge split where
  merge Empty      = []
  merge (Leaf c)   = [c]
  merge (Node l r) = usualMerge l r

  split []  = Empty
  split [x] = Leaf x
  split xs  = Node l r where
    (l, r) = splitAt (length xs `div` 2) xs
```

Listing 5: Merge sort using hylomorphisms

2.3 Multiparty session types

In complicated distributed systems, participants agree on a protocol, specifying type and direction of data exchanged. Multiparty session types are a branch of behavioural types specifically targeted at describing protocols in distributed systems based on asynchronous communication [5]. They are a type formalism used to model communication-based programming by codifying the structure of communication. The evolution of computing from the era of data processing to the era of communication witnessed the growth and significance of the theory of session types.

The theory of multiparty session type contains three main elements. Global types (seen in Section 2.3.1), local (session) types and processes. Processes are the concrete descriptions of the behaviour of the peers involved in the distributed system [5] using a formal language. Usually, the most used and the original language is π-calculus [12]. However, for the simplicity, we will not introduce π-calculus The coming sections are an intuitive introduction of session types by examples.

2.3.1 Global types and local types

Global type is at the most abstract level, describing a communication protocol from a neutral viewpoint between two or more participants[5]. The syntax of the global types is shown in Table 2.1 and an example of global types is shown in Table 2.3.

Local types or session types characterise the same communication protocol as the global type, but from the viewpoint of each peer [5]. Each process is typed by local type. The syntax of local types is shown in Table 2.2 and an example of local type is shown in Table 2.4.

The relationship between global types and local types are established by the projection operator (seen in the Section 2.3.1.1), and a type system performs syntactic checks,

ensuring that processes are typed by their corresponding local types. Hence, at the compile time, three important properties follow [5].

- **communication safety**: Mismatches between the types of sent and expected messages, despite the same communication channel is used for exchanging messages of different types, do not exist [5].
- **protocol fidelity**: The interactions that occur are accounted for by the global type and therefore are allowed by the protocol [5].
- **progress**: Every message sent is eventually received, and every process waiting for a message eventually receives one [5].

We will learn that these properties are valuable not only in the distributed system but also in the domain of parallel computing in Section 2.3.2.

$G ::=$	Global types
$p \to q : \langle S \rangle.G$	Value exchange
$p \to q : \langle T \rangle.G$	Channel exchange
$p \to q : \{l_i : G_i\}_{i \in I}$	Bracnhing
$\mu \mathbf{t}.G \mid \mathbf{t} \mid \text{end}$	Recursion/End

Table 2.1: Global types

$T ::=$	Session types/local types
$!\langle p, S \rangle.T$	Send value
$!\langle p, T \rangle.T$	Send channel
$?(p, T).T$	Channel Receive
$?(p, S).T$	Sorts Receive
$\oplus \langle p, \{l_i : T_i\}_{i \in I} \rangle$	Selection
$\&(p, \{l_i : T_i\}_{i \in I})$	Bracnhing
$\mu \mathbf{t}.T \mid \mathbf{t} \mid \text{end}$	Recursion/End

$S ::=$	Sorts
bool | nat | string	
$\ldots$	

Table 2.2: Session types/local types

2.3.1.1 Projection between global types and local types

Projection is the formalization of the relationship between global and local types. It is an operation extracting the local type of each peer from the global type [5]. The definition of projection is shown in Table 2.5.

As an example, a projection of global type in Table 2.3 is

$$G \upharpoonright 0 = !\langle 1, \text{string} \rangle; ?(1, \text{string}); \&(1, \{\text{accept} : ?(1, \text{string}); !\langle 2, \text{string} \rangle; ?(2, \text{int}), \text{reject} : \text{end}\})$$

1. Customer(0) sends an order number to Agency(1), and Agency sends back a quote to the customer.
2. If Customer is happy with the price then Customer selects accept option and notifies Agency.
3. If Customer thinks the price is too high then Customer terminate the trade by selecting reject.
4. If accept is selected, Agency notify both Customer and Agency2(2).
5. Customer sends an address to Agency2 and Agency2 sends back a delivery date.

$$
\begin{aligned}
G = \quad & \\
& 0 \to 1 : \quad \langle \text{string} \rangle . \\
& 1 \to 0 : \quad \langle \text{int} \rangle . \\
& 0 \to 1 : \{ \quad \text{accept} : \\
& \qquad\qquad 1 \to \{0,2\} : \langle \text{string} \rangle . \\
& \qquad\qquad 0 \to 2 : \langle \text{string} \rangle . \\
& \qquad\qquad 2 \to 0 : \langle \text{int} \rangle . \text{end}, \\
& \qquad\qquad \text{reject} : \text{end} \}
\end{aligned}
$$

Table 2.3: An example of a protocol described by global types G

$$
\begin{aligned}
S \triangleq \mu t.(\&\{&\text{balance} : ![\text{nat}]; t, \\
&\text{deposit} : ?[\text{nat}]; ![\text{nat}]; t, \\
&\text{exit} : \text{end}\})
\end{aligned}
\qquad\qquad
C \triangleq \oplus\{\text{balance} : ?[\text{nat}]; \text{end}, \quad \text{deposit} : ![\text{nat}]; ?[\text{nat}]; \text{end}\}
$$

Table 2.4: Session types of client and server end point of a ATM service

$$
(\mathtt{p} \to \mathtt{p}' : \langle U \rangle . G') \upharpoonright \mathtt{q} = \begin{cases} !\langle \mathtt{p}', U \rangle . (G' \upharpoonright \mathtt{q}) & \text{if } \mathtt{q} = \mathtt{p}, \\ ?(\mathtt{p}, U) . (G' \upharpoonright \mathtt{q}) & \text{if } \mathtt{q} = \mathtt{p}', \\ G' \upharpoonright \mathtt{q} & \text{otherwise}. \end{cases}
$$

$$
(\mathtt{p} \to \mathtt{p}' : \{l_i : G_i\}_{i \in I}) \upharpoonright \mathtt{q} = \begin{cases} \oplus \langle \mathtt{p}', \{l_i : T_i\}_{i \in I} \rangle & \text{if } \mathtt{q} = \mathtt{p} \\ \&(\mathtt{p}, \{l_i : G_i \upharpoonright \mathtt{q}\}_{i \in I}) & \text{if } \mathtt{q} = \mathtt{p}' \\ G_{i_0} \upharpoonright \mathtt{q} & \text{where } i_0 \in I \text{ if } \mathtt{q} \neq \mathtt{p}, \mathtt{q} \neq \mathtt{p}' \\ & \text{and } G_i \upharpoonright \mathtt{q} = G_j \upharpoonright \mathtt{q} \text{ for all } i, j \in I. \end{cases}
$$

$$
(\mu \boldsymbol{t} . G) \upharpoonright \mathtt{q} = \begin{cases} \mu \boldsymbol{t} . (G \upharpoonright \mathtt{q}) & \text{if } G \upharpoonright \mathtt{q} \neq \boldsymbol{t}, \\ \mathsf{end} & \text{otherwise}. \end{cases} \qquad \boldsymbol{t} \upharpoonright \mathtt{q} = \boldsymbol{t} \qquad \mathsf{end} \upharpoonright \mathtt{q} = \mathsf{end}.
$$

Table 2.5: The definition of projection of a global type G onto a participants q[5]

2.3.1.2 Duality of session types

In binary session types where all protocals are pairwise, duality formalises the relationship between the types of opposite endpoints. For a type T, its dual or co type, written $\bar{T}$ is defined inductively as in Table 2.6.

$$\begin{array}{lcl|lcl|lcl}
\overline{?[\widetilde{S}];\, T} & = & ![\widetilde{S}];\, \overline{T} & \overline{\oplus\{l_i : T_i\}_{i\in I}} & = & \&\{l_i : \overline{T_i}\}_{i\in I} & \overline{?[T];\, T'} & = & ![T];\, \overline{T'} \\
\overline{![\widetilde{S}];\, T} & = & ?[\widetilde{S}];\, \overline{T} & \overline{\&\{l_i : T_i\}_{i\in I}} & = & \oplus\{l_i : \overline{T_i}\}_{i\in I} & \overline{![T];\, T'} & = & ?[T];\, \overline{T'} \\
\overline{\texttt{end}} & = & \texttt{end} & \overline{\mu t.T} & = & \mu t.\overline{T} & \bar{t} & = & t
\end{array}$$

Table 2.6: Inductive definition of duality

Duality is essential for checking type compatibility. Compatible types mean that each common channel k is associated with complementary behavior: this ensures that the interactions on k run without errors.

In order to apply duality into multiparty session types in which more than two participants are allowed, the partial projection operation (seen in [5]) from multiparty session type to binary session type was introduced to allow reusing the definition of duality after applying the partial projection.

2.3.2 Applications in parallel computing

Multiparty session types not only have rich applications in distributed systems but also value in the domain of parallel computation.

Existing work[13] has shown how to generate MPI[1] programs using session types. Users describe the communication topology as a skeleton using a protocol language which is type checked by session types. After that, an MPI program is generated by merging the skeleton and user-provided kernels for each peer. The parallel code obtained in this way is guaranteed to be deadlock-free and progressing.

2.4 Message passing concurrency

This section introduces some interfaces for message passing concurrency from the primitive case: channel to more advanced one: monad for message passing concurrency.

For simplicity, they are represented in Haskell, but in general, most languages can implement similar interfaces.

[1]Message Passing Interface (MPI) is a standardised and portable message-passing standard designed by a group of researchers from academia and industry to function on a wide variety of parallel computing architectures [4].

2.4.1 Primitives for message-passing concurrency

In Section 2.3, channels are bi-directional and used for communication between two parties. In Haskell, channel primitives are represented in Listing 6. However, just using these primitives cannot guarantee progress or communication safety. For example, a program that has one thread writing channel once combined with another thread reading channel twice is type-correct but will cause deadlock. Many kinds of research to encode MPST using Haskell's type system are presented in [14] so that an (MPST) type-correct Haskell program assures progress, communication safety and session fidelity.

```
1  data Chan a
2      newChan :: IO (Chan a)
3      writeChan :: Chan a -> a -> IO ()
4      readChan :: Chan a -> IO a
5      dupChan :: Chan a -> IO (Chan a)
```

Listing 6: Channel primitives in Haskell

2.4.2 Concurrency Monads

The work done by [15] constructs a monad to express concurrent computation. The definition is in Listing 7. `Action` is the algebraic datatype representing basic concurrency primitives. `Atom`, the atomic unit of computation, is a computation (wrapped in the `IO` monad) followed by an action. `Fork` is two parallel action. `Stop` is the termination of an action. type `C` is a special case of the continuation monad. The continuation monad is an encapsulation of computations in continuation-passing style (CPS)[2]. SO `C a` is a CPS computation that produces an intermediate result of type `a` within a CPS computation whose final result type is `Action`. With the help of the monad `C`, sequencing and composing actions can use monadic bind.

```
1  data Action =
2      Atom (IO Action)
3    | Fork Action Action
4    | Stop
5
6  newtype C a = C { runC :: (a -> Action) -> Action } (6)
7
8  instance Monad C where
9      (>>=) :: C a -> (a -> C b) -> C b (9)
10     m >>= f  = C $ \k -> runC m (\v -> runC (f v) k)
11     return :: a -> C a
12     return x = C $ \k -> k x
```

[2]In continuation-passing style function result is not returned, but instead is passed to another function, received as a parameter (continuation)[16]

Listing 7: The definition of concurrency monad

The idea is using continuation to represent the "future" so that computation can pause and resume as well as expressing sequential computation. Atom wraps the actual computation and Fork is responsible for spawning threads. In addition, in order to write programmes in a monadic way easier, some helper functions are defined (shown in Listing 8). `atom` lifts an IO computation to `C`. And `fork` takes a computation in `C` and return a `C` which involves the `Fork` action. Given a `C a`, `action` gives the result of running the CPS computation. We use `\_. Stop` to represent the final continuation (`Stop` action is the last action).

```
atom :: IO a -> C a
atom m = C $ \k -> Atom $ do
    r <- m
    return $ k r

fork :: C () -> C ()
fork m = C $ \k -> Fork (runC m (const Stop)) (k ())

action :: C a -> Action
action m = m (\_. Stop)
```

Listing 8: Helper functions

An example of programme written in the concurrency monad is shown below.

```
example :: C ()
example = do
  atom $ putStrLn "Hello"
  name <- atom getLine
  fork $ atom $ putStrLn "World"
  atom $ putStrLn name
```

We can easily define a round-robin scheduler for programmes in this monad. We can regard a list of action as a queue of threads that are running concurrently. `schedule` will pattern match on the head of the list. If it is `Atom` then the scheduler will run the computation (seen `a <- ioa` at ⑦) and pause its remaining computation and put it at the end of the thread queue (seen at ⑧). If it is `Fork` then the scheduler will spawn the thread and put the new thread and the current thread to the bottom of the queue (seen at ⑨). Finally, If it is `Stop` then it means this thread has finished and the scheduler will resume with the rest of threads in the queue. For example, to run the above example, we call `schedule [action example]`.

```
schedule :: [Action] -> IO ()
schedule [] = return ()
schedule (a:as) = sched as a

sched :: [Action] -> Action -> IO ()
```

```
6  sched as (Atom ioa) = do
7      a <- ioa ⑦
8      schedule $ as ++ [a] ⑧
9  sched as (Fork a1 a2) = schedule $ as ++ [a2, a1] ⑨
10 sched as Stop = schedule as
```

The concurrency monad can be extended to support many features. For example, work done by [17] modifies the definition of Action as well as implements a work-stealing parallel scheduler (seen in Listing 9) to build a monad for parallel computation.

Besides, extending the concurrency monad to monad for message-passing concurrency can be done by adding channel primitives like newChan, writeChan and readChan into the Action. Since channel primitives are possible to represent in this monad, we naturally think of its prospect in connecting with MPST (will be discussed in the later section).

```
1  newtype IVar a = IVar (IORef (IVarContents a)) ①
2  data IVarContents a = Full a | Blocked [a -> Action.]
3
4  data Action .=
5      Fork Action Action
6    | Stop
7    | forall a . Get (IVar a) (a -> Action) ⑦
8    | forall a . Put (IVar a) a Action ⑧
9    | forall a . New (IVar a -> Action)
```

Listing 9: Par Monad

① Parent threads and child threads communicate data via IVar

⑦ Get operation blocks when the underlying IVarContents is Blocked

⑧ Put operation updates the underlying IVarContetns to Full with the result a and resume the list of blocking threads by applying a to the continuation.

In summary, many techniques and ideas like continuation presented in the implementation of this monad afford us inspirations in designing our intermediate language.

2.5 Free monad

Free monad[18] is a concept from category theory. Intuitively, a free monad as a programming abstraction is a technique for implementing EDSLs, where a functor represents basic actions of the EDSL and the free monad of this Functor provides a way to sequence and compose actions. Speaking of the advantages, we are particularly interested in its benefits in flexible interpretations which will be illustrated by an example (Section 2.5.2) and discussed further (Section 2.5.3).

2.5.1 Definition

In practice, a free monad in Haskell can be defined as an algebraic data type(ADT) (shown in Listing 10). `Free f` is the monad produced given a functor `f`. `Free` has two type constructors: `Pure` and `Free`. `Monad (Free f)` is the Haskell implementation of the Monad interface for `Free f`. Many useful helper functions are derived from the simple definition of the free monad (shown in Listing 11). `liftF` lift the functor to its free monad representations. `freeM` maps a natural transformation of functor (`f a -> g a`) to the natural transformation of their free monad versions. Given `m` is a monad, `freeM` is a special case of interpreting `Free m a`: to the `m` monad itself. Finally, `interpret` shows the power of free monad. We can interpret the free monad version of a functor `f` to any monad `m` given a natural transformation from `f` to `m`.

```haskell
data Free f a
  = Pure a
  | Free f (Free f a)

instance Functor f => Monad (Free f) where
  return = pure
  (Pure x) >>= fab = fab x
  (Free fx) >>= fab = Free $ fmap (>>= fab) fx
```

Listing 10: Free monad in Haskell

```haskell
liftF :: Functor f => f a -> Free f a
liftF = Free . fmap Pure

freeM :: (Functor f, Functor g) => (f a -> g a) -> (Free f a) -> (Free g a)
freeM phi (Pure x ) = Pure x
freeM phi (Free fa) = Free $ phi (fmap (freeM phi) fa)

monad :: Monad m => Free m a -> m a
monad (Pure x  ) = pure x
monad (Free mfx) = mfx >>= monad

interpret :: (Functor f, Monad m) => (f a -> m a) -> (Free f a -> m a)
interpret phi = monad . freeM phi
```

Listing 11: Helper functions based on free monad

2.5.2 Example

Free monad is useful in interpreting an abstract syntax tree (AST). In order to apply free monad to a given AST, we can follow a routine [18].

1. Create an AST, usually represented as an ADT

2. Implement functor for the ADT
3. Create helper constructors to Free ADT for each type constructor in ADT by liftF
4. Write a monadic program using helper constructors. It is essentially a program written in DSL operations.
5. Build interpreters for Free ADT by interpreting
6. Interpret the program by the interpreter.

We will demo the above procedure by a made-up example. We would like to build a simple EDSL for getting customers' name and greeting customers. First of all, we build a functor `GreetingF` to represent the basic operations: getting the name and greeting. Then we wrap the functor with `Free` constructor so that a program written in our EDSL can be regarded as a Haskell expression with type `Free GreetingF a`.

```
data GreetingF next
  = Getname (String -> next)
  | Greet String next
  deriving Functor

type Greeting = Free GreetingF
```

Then we create helper functions of Greeting using liftF.

```
getName = liftF $ Getname id
greet str = liftF $ Greet str ()
```

Then we can write a simple program using operations provided by Greeting.

```
exampleProgram :: Greeting ()
exampleProgram = do
    a <- getName
    greet a
    b <- getName
    greet b
```

Then we can easily implement an interpreter for the example program

```
goodMorningInterpreter :: Greeting a -> IO a
goodMorningInterpreter = interpret helper
    where
        helper (Getname next) =
          fmap next getLine
        helper (Greet str next) =
          putStrLn ("Good morning " ++ str) >> return next
```

Finally, execute the program.

```
ghci:> goodMorningInterpreter exampleProgram
Tom
Good morning Tom
Mary
Good morning Mary
```

2.5.3 Applications

As illustrated by the example (Section 2.5.2), free monad decouple the abstract syntax tree of domain specific language (DSL) and the interpreter. Interpreters with different purposes can be implemented without changing the syntax.

In the project, we apply free monad to the intermediate language so not only we make the languages monadic for free but also benefits from decoupling the interpreter and the syntax to implement different interpreters, e.g. Simulator, code generators to different platforms easily.

Chapter 3

Alg : Algebraic Functional Language

Algebraic Functional Language (Alg) is an example of the high-level languages to generate parallel code, proposed in the paper [11]. The work done by [11] also proposes a method to do the code generation. Part of this project is about implementing an alternative code generation backend for the PAL. In the evaluation section, we will compare the speed of generated code of our method against the original method. We will give an overview of the language in this section.

3.1 Alg

3.1.1 Syntax

$F_1, F_2 ::=$	
I	Identity functor
Kt	Constant functor
$F_1 + F_2$	Sum functor
$F_1 \times F_2$	Product functor

$t_1, t_2 ::=$	Type
() | int | ...	Primitive types
$a \to b$	Function types
$a + b$	Sum types
$a \times b$	Product types
$F\ t_1$	Functor types
$\mu.F$	Recursive types

$e_1, e_2 ::=$ Expression

$f \mid v \mid \text{const}\ e \mid e_1 \circ e_2 \mid \pi_i \mid e_1 \vartriangle e_2 \mid e_1 \triangledown e_2 \mid l_i \mid F\ e \mid \text{in}_F \mid \text{out}_F \mid \text{rec}_F\ e_1\ e_2$

Table 3.1: Syntax of Alg language

```
newtype L = K () + K Int * I
type List = Rec L
```

Listing 12: Type of integer list in PAL

The syntax of Alg is shown in Table 3.1. In terms of the syntax of expressions, f represents atomic functions which are functions of which we only know their types [11]. The

presence of atomic function is important in the code generation which we will discuss in the later chapter. v is the primitive value like integer 1.F represents functor and a, b are types. Besides primitives type, function types and recursive type constructor μ, Alg use four functors to form more types hence representing complicated data structures by composing them. For example, a list of integer is expressed in Listing 12. One thing we should learn from the syntax is that the Alg and its combinators avoid mentioning variables explicitly and hence Alg programs are known as point-free programs.

$$\frac{f : a \to b \in \Gamma}{\vdash f : a \to b}$$

$$\frac{\vdash e : a}{\vdash \text{cons } e : b \to a}$$

$$\frac{}{\vdash \text{id} : a \to a}$$

$$\frac{}{\vdash \text{in}_F : F\ \mu F \to \mu F}$$

$$\frac{}{\vdash \text{out}_F : \mu F \to F\ \mu F}$$

$$\frac{\vdash e_1 : b \to c, \quad \vdash e_2 : a \to b}{e_1 \circ e_2 : a \to b}$$

$$\frac{i \in [1, 2]}{\pi_i : a_1 \times a_2 \to a_i}$$

$$\frac{i \in [1, 2]}{l_i : a_i \to a_1 + a_2}$$

$$\frac{\vdash e_1 : a \to b, \quad \vdash e_2 : a \to b}{e_1 \vartriangle e_2 : a \to b \times c}$$

$$\frac{\vdash e_1 : a \to c, \quad \vdash e_2 : b \to c}{e_1 \triangledown e_2 : a + b \to c}$$

$$\frac{\vdash e : a \to b}{F\ e : F\ a \to F\ b}$$

$$\frac{\vdash e_1 : F\ b \to b, \quad \vdash e_2 : a \to F\ a}{\text{rec}_F\ e_1\ e_2 : a \to b}$$

Table 3.2: Typing rules for Alg

The typing rule for alg is expressed in Table 3.2 and the semantic for alg is shown in Table 3.3.

An important feature of Alg is the lack of usual control flow like if branch or while loop to build algorithms, instead, it uses the flow of transformation of data structures to replace conventional control flow. For example, $\triangledown$ combinator is the case operation whose types is $a + b \to c$. It can be seen as analogy of if branch in normal programming languages. $\vartriangle$ combinator represents split operation. More importantly, the combinator rec_F uses the idea of recursion schemes (explained in Section 2.2) to build divided-and-conquer algorithms.

To summary, algorithms in Alg are represented as a series of transformations of data, making it easy to transform the Alg programs to programs in arrows mechanically since arrows also express flow of data and their transformations naturally. This property al-

Constant, Identity and Composition $\quad \mathsf{const}\ e = \lambda x.\ e \quad \mathsf{id} = \lambda x.\ x \quad e_1 \circ e_2 = \lambda x.\ e_1\ (e_2\ x)$

Products

$\pi_i = \lambda(x_1, x_2).\ x_i \ \text{ if } i \in [1,2] \quad e_1 \vartriangle e_2 = \lambda x.\ (e_1\ x,\ e_2\ x) \quad e_1 \times e_2 = (e_1 \circ \pi_1) \vartriangle (e_2 \circ \pi_2)$

Coproducts

$\iota_i = \lambda x.\ inj_i\ x \quad e_1 \triangledown e_2 = \lambda x.\ e_i\ y \ \text{ if } x = inj_i\ y \quad e_1 + e_2 = (\iota_1 \circ e_1) \triangledown (\iota_2 \circ e_2)$

Functors

$\mathsf{I}\ a = a$	$\mathsf{K}a\ b = a$	$(F_1 \times F_2)\ a = F_1\ a \times F_2\ a$	$(F_1 + F_2)\ a = F_1\ a + F_2\ a$
$\mathsf{I}\ e = e$	$\mathsf{K}a\ e = \mathsf{id}$	$(F_1 \times F_2)\ e = F_1\ e \times F_2\ e$	$(F_1 + F_2)\ e = F_1\ e + F_2\ e$

Recursion

$\mathsf{in}_F = \lambda x.\ in_F\ x \quad \mathsf{out}_F = \lambda x.\ y \ \text{ where } \ in_F\ y = x \quad \mathsf{rec}_F\ e_1\ e_2 = f \ \text{ where } f = e_1 \circ F\ f \circ e_2$

Table 3.3: Semantics of Alg expression[11]

lows us to generate parallel code without burdens.

3.1.2 Example: Merge sort

$$\begin{aligned}\mathbf{ms} &= \text{rec}_T\ \text{mrg}\ \text{spl} = \text{mrg} \circ T\ (\text{rec}_T\ \text{mrg}\ \text{spl}) \circ \text{spl} \\ &= \text{mrg} \circ (\text{id} + \text{id} + (\text{rec}_T\ \text{mrg}\ \text{spl}) \times (\text{rec}_T\ \text{mrg}\ \text{spl})) \circ \text{spl} \\ &= \text{mrg} \circ (\text{id} + \text{id} + \mathbf{ms} \times \mathbf{ms}) \circ \text{spl}\end{aligned}$$

Listing 13: Merge sort in Alg

By making the hylomorphism as a built-in combinator rec_F, Alg can express merge sort similarly as the example shown in Section 2.2.2, We use the functor $T = K() + Ka + I \times I$ to be substitute the functor F in rec_F and the type Ls to represent a list of elements whose type is a. We also have two atomic function spl : $Ls \to T\ Ls$ and mrg : $T\ Ls \to Ls$. Finally we express merge sort as $\mathbf{ms} = \text{rec}_T$ mrg spl. It is shown in Listing 13. From the example, we observe that $\mathbf{ms}$ can expanded infinitely. Later, we will exploit this property to generate parallel code.

3.2 ParAlg: Alg + role annotations

Alg programs are point-free programs which has an implicit but precise data-flow. For example, $e_1 \circ e_2$ is the function composition so the output of applying e_2 will be used as the input of e_1. We can interpret this as e_2 sends a message to e_1. Parallel Algebraic Language is the formalization of the above idea. In essence, it is Alg with roles annotations, converting implicit data-flow to explicit role communications. Due to the limited contexts, in this section, we will only present the main results and use examples to build intuitions about its principles. More details and proof can be found in [11]. We will briefly mention results that will be useful in our project.

3.2.1 Syntax

$$\rho_1, \rho_2 ::= r \mid \oplus_k^r[\rho] \mid \mathsf{id} \mid \rho_1 \circ \rho_2 \mid \pi_i \mid \rho_1 \vartriangle \rho_2 \mid \iota_i \mid \rho_1 \triangledown \rho_2 \qquad R_1, R_2 ::= r \mid \iota_i\ R \mid R_1 \times R_2$$
$$p_1, p_2 ::= e@r \mid \oplus_k^r[p] \mid \mathsf{id} \mid p_1 \circ p_2 \mid \pi_i \mid p_1 \vartriangle p_2 \mid \iota_i \mid p_1 \triangledown p_2 \qquad A, B ::= a@r \mid \iota_i\ A \mid A \times B$$
$$x \in X \implies x \in \mathcal{T}_X \qquad T_x \in \mathcal{T}_X \wedge T_y \in \mathcal{T}_X \implies T_x \cup^k T_y \in \mathcal{T}_X$$

Table 3.4: Syntax of ParAlg

The syntax of ParAlg is shown in Table 3.4. r is the role identifier representing a unit of computation. $R_1 \times R_2$ specifies that the input is split across roles R_1 and R_2 [11]. More explanation of different constructor can be seen in [11]. In short, ParAlg is just Alg with role annotations at certain points.

Notice that rec_F in Alg does not belongs to constructs of ParAlg. So we need to specify the number of unroll beforehand and unroll the expression in rec_F by the corresponding number and then apply the role annotations to the unrolled expression. The unroll process is shown in Listing 13 and in this example, the number of unroll is 1. The unroll-and-annotate operation will parallelize the recursive functions.

3.2.2 Inferring global types

ParAlg turns implicit data-flow into communication and hence we should be able to describe its communication protocols from a valid ParAlg program. In [11], global types (explained in Section 2.3) are used to represent communication protocols for ParAlg which means for any valid ParAlg program, we can infer its corresponding global types. The inferring rules and typing for ParAlg can be seen in [11].

$G =$
$r_0 \to r_1$: Rec L
$r_1 \to \{r_2, r_3\}$
$\{l_0 :$ $r_1 \to r_3 : () + \text{int}.$
end;
$l_1 :$ $r_1 \to r_2$: Rec L.
$r_1 \to r_3$: Rec L.
$r_2 \to r_3$: Rec L.
end$\}$;
$r_3 \to r_0$: Rec L.
end

Listing 14: Global types for merge sort

An example of the global types is seen in Listing 14. It is the inferred global types of rec_T mrg spl with the number of unroll equal to two. The global types tell us r_1 receives input list from r_0, based on the length of input list, r_1 either send it to r_3 directly when the list is a singleton or empty or split the list into halves and send them to r_2 and r_3 respectively. In the second case, r_2 will process the received list and send it to r_3, r_3 will process its received part and waiting for another half of list to be received from r_2. After both has done, r_3 will process the combined results. Finally r_3 will send the data back to r_0.

3.2.3 Example: Parallel merge sort

```
par_fun msp
    : (Rec L)@r0 -> (Rec L)@r0
    = id@r0
       . (merge@r3
           . (inj[0]
                ||| (inj[1]
                       . (((merge
                            . (inj[0]
                            ||| (inj[1]
                            . ((ms. proj[0])&&& (ms. proj[1, 2])))))
                            . split)@r2
                             . proj[0]@r1)
                            &&& ((merge
                                  . (inj[0]
                                  ||| (inj[1]
                                  . ((ms. proj[0])&&& (ms. proj[1])))))
                                  . split)@r3
                                   . proj[1]@r1))))
           . split@r1)
```

Listing 15: ParAlg for merge sort

The ParAlg expression for merge sort unrolling twice is shown in Listing 15. We use arrow combinator `&&&` and `|||` to replace $\triangledown$ and $\triangle$ in the actual ParAlg expression. They are equivalent while arrow combinator is well-known and easier to be expressed in plain texts. Similar reason also applies to `inj`, the replacement of l_i and `proj`, the replacement of π_i. Its inferred global types is shown in the last subsection.

3.3 Conclusion

A visualization of the pipeline can be seen in the Figure 3.1. The content of the section is a skim of Alg and ParAlg. Curious readers should read about [11] for more details.

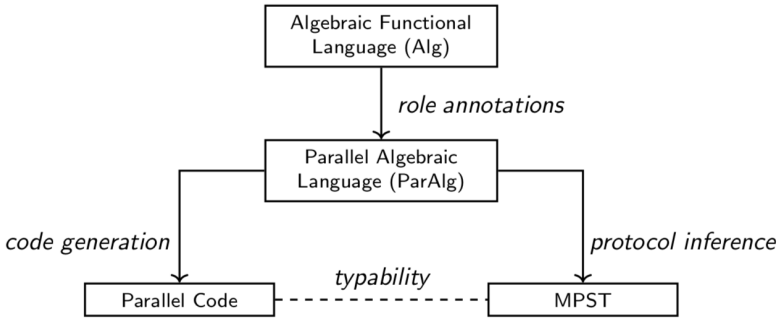


Figure 3.1: Overview of code generating pipeline from Alg[11]

Combinators and the syntax of Alg makes every Alg program to be point-free. Without the use of explicit variables, point-free programs express the underlying data-flow of the computation clearly. Adding role annotations transforms Alg programs to ParAlg programs converting the implicit data-flow to explicit communication. Communication will aid us generate parallel code using message-passing concurrency. We will introduce the our method from next chapter.

Chapter 4

SPar: A session typed free monad EDSL for concurrency

In order to generate parallel code from ParAlg, we first introduce the syntax of our intermediate language, the session-typed free monad EDSL for concurrency hosted in Haskell (SPar). SPar are compromised of two components: Core and Proc. Core is the language expressing sequential computation while Proc is a monadic language with message-passing primitives, communicating Core expression between different roles. We use a group of Proc interacting with each other to represent parallel computations. In addition, session typing the group of Proc ensure the computation is deadlock-free.

4.1 Computation: The Core EDSL

Core is the elemental computation. The syntax of Core is mostly inspired by Alg [11] and FunC, a demo DSL defined in the work done by [19]. For this project, we choose to implement Core syntax as small as possible without sacrificing expressibility.

4.1.1 Syntax

The syntax of Core is shown in Listing 16. Inl and Inr are for the construction of sum type. Pair is responsible for constructing value of product type while Fst and Snd extract value from product type. Supporting sum type, product type and inductive type (see next section) is enough to express any data structure in any computation. In addition to these actions manipulating basic data structure, we have Lit which is used in the Haskell interpreter (see Section 5.2) and benchmarking (see Section 8) and Var, a constructor which is useful when what we do not actually evaluation the Core expression but inspect its static structure. It is used in the code generation (see Section 7.3.2) and session typing (see Section 5.1). Id is similar to identity function and Const is similar to the const function in Haskell. Prim representing user defined functions takes two field name and the haskell implementation. The first field will be useful in the code generation (see Section 7.3.2) when applying user-defined function calls. The second field will be used in the interpreter implementation.

```
data Core a where
    Lit :: a -> Core a
    Var :: Int -> Core a
    Prim  :: String
             -> a
             -> Core a

    Ap :: Core (a -> b) -> Core a -> Core b
    Id :: Core (a -> a)
    Const :: Core a -> Core (b -> a)

    Fst :: Core ((a, b) -> a)
    Snd :: Core ((a, b) -> b)
    Pair :: Core a -> Core b -> Core (a, b)

    Inl :: Core (a -> Either a b)
    Inr :: Core (b -> Either a b)
```

Listing 16: The syntax of Core

4.1.2 Representation of recursive data structures

Core has primitives to operate on sum and product types. Representing recursive type like $\mu\text{list}.() + \text{Int} \times \text{list}$ will be covered in this section. The method are taken from the implementation of Alg language in [11]. First of all, we extend the core with the following two operation. `In` represents the fold operation on iso-recursive types and `Out` represents the unfold operation on iso-recursive types. Consider a recursive type $\mu\alpha.\tau$, the type parameter `t` is equivalent to α, `f :@: t` is equivalent to τ and the typeclass `Data f t` associate two types which is equivalent to $\mu\alpha.\tau$. `:@:` is a type family which is a function acting on types instead of values. `:@:` converts `f` to sum and product type in Haskell which are `(,)` and `Either`.

```
data Core a where
    In   :: Data f t => Core (f :@: t -> t)
    Out  :: Data f t => Core (t -> f :@: t)

type family (:@:) (a :: Poly Type) (b :: Type) :: Type where
    'PId :@: x = x
    'PK y :@: _ = y
    'PProd f g :@: x = (f :@: x, g :@: x)
    'PSum f g :@: x = Either (f :@: x) (g :@: x)

class Data (f :: Poly Type) t | t -> f where
    roll :: f :@: t -> t
    unroll :: t -> f :@: t
```

A concrete example is a list shown below. We know a list has recursive type: $\mu\alpha.() + a \times \alpha$. So the `f` is `('PSum ('PK ()) ('PProd ('PK a) 'PId))` and we use haskell list type `[a]` to present α (equivalent to `t` in `f :@: t`). `f :@: t` is evaluated to the type `Either () (a, [a])`.

```
instance Data ('PSum ('PK ())  ('PProd ('PK a) 'PId)) [a] where
    roll (Left _) = []
    roll (Right (a, b)) = a : b

    unroll [] = Left ()
    unroll (x:xs) = Right (x,xs)
```

However, even though this way of representing recursive types is very smart but later we found out they are seldomly used in the code generation of parallel algorithms. Because functions processing complicated recursive data structure is usually represented by the Prim constructor hiding the implementation details so In and Out are too low-level operations and hence user are difficult to write complicated function by them. So in the conclusion, Core provides basic functionality, i.e Fst or Inl to process data and more importantly, Prim, a black box constructor hiding the implementation details of the sequential computation. Prim turn out to be a very useful abstraction in terms of code generation. More details will be discussed in the Section 7.3.2.

4.2 Communication: The Proc EDSL

Proc will be a free monad EDSL for message passing. As introduced in the free monad section in the background, the first thing to do it to define the algebra of message-passing concurrency: `ProcF` and `Proc` is defined using free monad constructor and `ProcF`. The definition is shown in the Listing 17. Careful reader might notice that `Proc` and `ProcF` are defined mutually with each others in `Branch`, `Select` and `Broadcast`.

The semantics will be defined in the next subsection in terms of a group of Proc programs. A single Proc program is either sequential or deadlock. Operational semantics is only worth discussing when given a group of Proc programs interacting with each others.

4.3 Concurrent computation: A group of Proc

We have introduced syntax for computation and communication. We also know that a single Proc expression is meaningless since there does not exist another party to interact with this Proc hence the computation has no progress. Naturally, we use a group of Proc to represent concurrent computations. To be more precise, a collection of Proc with their own role identifiers can be treated as a system of roles executing their own programs concurrently. In most of the cases, in order to make the group of Proc meaningful, we will allocate a start role in the system acting as the original data provider and a end role whose Proc program will receive data from others, process and output the final computation which is wrapped by the Pure constructor at the end of the Proc program.

```
data ProcF next where
    Send :: Nat -> Core a -> next -> ProcF next

    Recv :: Nat -> (Core a -> next) -> ProcF next

    Select :: Nat
           -> Core (Either a b)
           -> (Core a -> Proc c)
           -> (Core b -> Proc c)
           -> next
           -> ProcF next

    Branch :: Nat
           -> Proc c
           -> Proc c
           -> (Core c -> next)
           -> ProcF next

    Broadcast :: [Nat]
              -> Core (Either a b)
              -> (Core a -> Proc c)
              -> (Core b -> Proc c)
              -> next
              -> ProcF next

type Proc a = Free ProcF (Core a)
```

Listing 17: The algebra for message-passing

Readers might find it easy to visualize the group of Proc as a computation graph. The start role is the source node and the end role is the sink node. A pair of nodes are connected if they communicate data with each others.

4.3.1 Operational semantics

Due to the similarities between Proc and multiparty session calculus introduced in [5]. We borrow some syntax and operational semantics rule from multiparty session calculus to define the operational semantics of Proc. P, Q denote Proc programs. A message queue is h which contains messages (q, p, v) meaning that the sender q sends the receiver p with value v. $h \cdot m$ is a message queue whose bottom element is message m. h are runtime syntax to model the asynchronous message communication where the order of the messages are retained [5]. $e \downarrow v$ means the evaluation of the Core expression to the value v.

Table 4.1 shows the small step semantics for Proc. Rule (Init) describes the initializa-

$(P_1, r_1) \mid (P_2, r_2) \mid \ldots \mid (P_n, r_n) \rightarrow$ (Init)

$\quad (P_1, r_1) \mid (P_2, r_2) \mid \ldots \mid (P_n, r_n) \mid \emptyset$

(Free (Send r_j e next), r_i) $\mid \ldots \mid h \rightarrow$ (Send)

$\quad$ (next, r_i) $\mid \ldots \mid h \cdot (r_i, r_j, v) \qquad (e \downarrow v)$

(Free (Recv r_i cont), r_j) $\mid \ldots \mid (r_i, r_j, v) \cdot h \rightarrow$ (Recv)

$\quad (p, r_j) \mid \ldots \mid h \qquad$ (cont $v \downarrow p$)

(Free (Select r_j e cont1 cont2 next), r_i) $\mid \ldots \mid h \rightarrow$ (Sel-Left)

$\quad$ (cont1 $v \gg$ next, r_i) $\mid \ldots \mid h \cdot (r_i, r_j, \mathrm{L}) \qquad (e \downarrow v, \mathrm{label}(v) \downarrow \mathrm{L})$

(Free (Select r_j e cont1 cont2 next), r_i) $\mid \ldots \mid h \rightarrow$ (Sel-Right)

$\quad$ (cont2 $v \gg$ next, r_i) $\mid \ldots \mid h \cdot (r_i, r_j, \mathrm{R}) \qquad (e \downarrow v, \mathrm{label}(v) \downarrow \mathrm{R})$

(Free (Branch r_i next1 next2 cont), r_j) $\mid \ldots \mid (r_i, r_j, \mathrm{L}) \cdot h \rightarrow$ (Branch-Left)

$\quad$ (next1 $\gg\!=$ cont, r_j) $\mid \ldots \mid h$

(Free (Branch r_i next1 next2 cont), r_j) $\mid \ldots \mid (r_i, r_j, \mathrm{R}) \cdot h \rightarrow$ (Branch-Right)

$\quad$ (next2 $\gg\!=$ cont, r_j) $\mid \ldots \mid h$

(Free (Broadcast [r_{k_1}, ..., r_{k_n}] e cont1 cont2 next), r_i) $\mid \ldots \mid h \rightarrow$ (Broadcast-1)

$\quad$ (Free (Select r_{k_1} v c c (Pure ())), r_i) $\mid \ldots \mid h$

$\quad$ where $(e \downarrow v,$ c = Free (Broadcast [r_{k_2}, ..., r_{k_n}] cont1 cont2 next))

(Free (Broadcast [r_{k_1}] e cont1 cont2 next), r_i) $\mid \ldots \mid h \rightarrow$ (Broadcast-2)

$\quad$ (Free (Select r_{k_1} v cont1 cont2 next), r_i) $\mid \ldots \mid h \qquad (e \downarrow v)$

(Pure v, r_i) $\mid (P_i, r_i) \mid \ldots \mid (P_j, r_j) \mid h \rightarrow$ (Pure)

$\quad (P_i, r_i) \mid \ldots \mid (P_j, r_j) \mid h$

Table 4.1: Small step semantics for Proc

tion of a group of Proc programs with an empty message queue at the beginning. Rule (Send) append the value to the message queue. Its complementary rule: (Recv) will recv the value at the top of the message queue. Rule (Branch) is also the complementary rule of the rule (Sel). You might notice that the Rule (Broadcast) is kind of special. Its rule states that Broadcast is expressed in terms of a series of Select operations. This operation broadcasts the label to a group of receivers. Even though its semantics can be expressed in terms of Select, the reason why we still treat broadcast an independent operations in SPar is because 1) this operation is very common in communication, including Broadcast as a primitive operation is beneficial for user to write code 2) In the code generation stages, we can generate more efficient code for Broadcast operation instead of generating a series of Select operations. Broadcast operation can be treated as a syntax sugar in the Proc language.

4.3.2 Session types and duality checking

Immediately, we notice that a Proc program can be typed by session types. `Send` operation in ProcF corresponds to the type !⟨p, S⟩.T, `Select` corresponds to $\oplus\langle \mathrm{p}, \{l_i : T_i\}_{i \in I}\rangle$ and so on. One exception is the `broadcast` operation which does not mapped to any type of the session types. We will discuss how to handle `Broadcast` in Section 5.1.

$$\text{Inst}\frac{\Gamma \vdash e : \forall l.\text{Proc } L\ a}{\Gamma \vdash e : \text{Proc } ([\text{end}/l]L)\ a}$$

$$\text{Gen}\frac{\Gamma \vdash e : \forall l.\text{Proc } L\ a, \quad \text{fresh } l}{\Gamma \vdash e : \text{Proc } ([l/\text{end}]L)\ a}$$

$$\text{Ret}\frac{\Gamma \vdash v : a}{\Gamma \vdash \text{Pure } v : \forall l.\text{Proc } l\ a}$$

$$\text{Abs}\frac{\Gamma, x : a \vdash e : \forall l.\text{Proc } L\ b}{\Gamma \vdash \lambda x.e : a \to \forall l.\text{Proc } L\ b}$$

$$\text{Bind}\frac{\Gamma \vdash m : \forall l_1.\text{Proc } L_1\ a, \quad \Gamma \vdash f : a \to \forall l_2.\text{Proc } L_2\ b}{\Gamma \vdash m \gg\!= f : \forall l_2.\text{Proc } [L_2/L_1]L_1\ b}$$

$$\text{Send}\frac{\Gamma \vdash v : a}{\Gamma \vdash \text{Free (Send } r\ v\ \text{(Pure ()))} : \forall l.\text{Proc } (r!\langle a\rangle.l)\ ()}$$

$$\text{Recv}\frac{}{\Gamma \vdash \text{Free (Recv } r\ (\backslash v \to \text{Pure } v)) : \forall l.\text{Proc } (r?(a).l)\ a}$$

$$\text{Select}\frac{\Gamma \vdash v : a + b, \quad \Gamma \vdash f_1 : a \to \forall l_1 \text{Proc } L_1\ c, \quad \Gamma \vdash f_2 : b \to \forall l_2.\text{Proc } L_2\ c}{\Gamma \vdash \text{Free (Select } r\ v\ f_1\ f_2\ \text{(Pure ()))} : \forall l.\text{Proc } r \oplus \{L : [l/l_1]L_1, R : [l/l_2]L_2\}\ ()}$$

$$\text{Branch}\frac{\Gamma \vdash \text{next1} : \forall l_1 \text{Proc } L_1\ c, \quad \Gamma \vdash \text{next2} : \forall l_2.\text{Proc } L_2\ c}{\Gamma \vdash \text{Free (Branch } r\ \text{next1 next2 } (\backslash x \to \text{Pure } x)) : \forall l.\text{Proc } r\ \&\{L : [l/l_1]L_1, R : [l/l_2]L_2\}\ ()}$$

Table 4.2: Typing rules for Proc expressions

The Table 4.2 shows the session typing rule for a Proc expression. We borrowed some notation from the work done [11]. $\Gamma \vdash e : \text{Proc } L\ a$ is the type of a Proc expression that follows protocol L and returns a value of type a. The types are parameterized by a variable l representing the continuation of a local type L. The typing rules contains all the operations except `Broadcast`. This is because `Broadcast` can be expressed in terms of a series of `Select` as explained in the previous subsection. So its session type $\oplus\{r_j\}_{j\in[1,n]}L : L_1, R : L_2$ can be expanded to $r_1 \oplus \{l : r_2 \oplus \{\ldots \oplus \{r_n \ldots\}\}, \ldots\}_{l\in[L,R]}$.

We have argued that a Proc program can be typed by multi-party session types. In order to utilize this property, we should check the duality of the each pair of Proc in the group. In short, duality check examines whether any pair of Proc in the system are complement with each other. If the duality properties is satisfied, the computation is guaranteed to be deadlock-free. This safety guarantee is useful and powerful in the application of parallel code generation. In this domain, SPar is considered to be the intermediate languages. Passing duality check for the intermediate representation means that as long as we preserve types and communication pattern carefully in later stages of code generation pipeline, the generated code obtained will share the same non-trivial properties: communication safety, protocol fidelity and deadlock-free.

The work done by [5] constructed the theoretic foundation of algorithms for checking

dualities and we will give an overview of the implementation in the Section 5.1.

4.4 Conclusions

In this section, we have introduced our intermediate language. It is friendly to use thanks to the monadic interface. In addition, communication and computation are independent in SPar. We can make Proc to parameterized by the type represent sequential computation so users can simply use their construction for sequential computation if they found Core is limited. More importantly, our strategy for parallelism is clear now. In a nutshell, we achieve parallelism by message-passing concurrency: spawning a group of threads on a multi-core CPU where each thread executes its corresponding Proc program.

Before jumping into the code generation, we will use next chapter to give an overview of some implementation challenges related to SPar first.

Chapter 5

SPar: Implementation

5.1 Session types

Haskell does not have session type natively. Other encodings of session types in Haskell is an overkill to this project since Proc does not support all actions that can be typed by session types i.e. channel delegation. We decide to create our representation of session types in Haskell corresponding to set of actions supported by Proc. We will introduce two method for session typing the group of Proc programs followed by duality checking. One is at the value level and another is at the type level.

5.1.1 Representations of session types in Haskell

Session types belong to the family of behavior types. So the type looks very similar to the value that will be typed. We exploit this similarity to defined our session types in terms of free monad, the same method we have used in defining Proc.

```
data STypeF a next where
    S :: Nat -> a -> next -> STypeF a next
    R :: Nat -> a -> next -> STypeF a next
    B :: Nat -> SType a c -> SType a c -> next -> STypeF a next
    Se :: Nat -> SType a c -> SType a c -> next -> STypeF a next

type SType a next = Free (STypeF a) next
```

Listing 18: Session types in Haskell

The Listing 18 shows the definition in Haskell. `SType` is the session type in Haskell. It is parameterized by a type variable `a` so that the value-level session types and the type-level session types can share the same basic definition of session types. `S` is mapped to $!\langle p, S\rangle.T$. `R` is mapped to $?(p, S).T$. `B` is mapped to Branch type and `Se` is mapped to Select type.

5.1.2 Value-level duality check

For the value-level session types, the type variable `a` is instantiated with type `TypeRep`. TypeRep reifies types to some extent by associating type representations to types [20]. Due to session types are represented as value expressions in Haskell, session typing a Proc program is the same as writing an interpreter which can be easily done since a Proc program is based on free monad.

We traverses Proc programs converting each operation to its corresponding type in `STypeF` and convert the value to its typeRep. For output actions, we recursively call the substructure to build the rest of session types. For input actions like `Recv`, we will apply the continuation with the Core value constructed by`Var` and recursively call the function on the result. The trick to apply `Var` to the continuation make it possible to inspect the static structure of every Proc programs because `b -> Proc a` is not inspectable i.e, we cannot pattern match on i, while `Proc a` can be inspected. We will also use this trick in the code generation which will be introduced in later chapter.

5.1.3 Type-level duality check

The value-level duality check works well in checking duality at runtime but as programmers, we aim to eliminate problems earlier. Hence, we proposed a solution that makes use of Haskell's powerful type systems to check the duality of the system at the compile time. In addition, we will introduce some combinators to helps us build a group of Proc to form parallel computation and this mechanism can act as an extra safety guard to make sure the correctness of these combinators.

The general approach of type-level duality checks can be summarized as the following steps.

1. Create a type-level representation of session types.
2. Modify the algebra of Proc to make it indexed by session types so that we can session type a proc while building it at the same time. Unlike the above method, we can only session type a Proc by interpretation after it has been constructed.
3. Gather the indexed session types of each Proc in the system and check the duality pair-wise at the type level.

```
type family (>*>) (a :: SType * c) (b :: SType * c) where
    'Free ('S r v n) >*> b = 'Free ('S r v (n >*> b))
    'Free ('R r v n) >*> b = 'Free ('R r v (n >*> b))
    'Free ('B r n1 n2 n3) >*> b = 'Free ('B r n1 n2 (n3 >*> b))
    'Free ('Se r n1 n2 n3) >*> b = 'Free ('Se r n1 n2 (n3 >*> b))
    'Pure _ >*> b = b
```

Listing 19: Implementations of type level bind

The first step is achieved by reusing the definition in Listing 18 and use Haskell DataKind extension to promote data constructors of `STypeF` and data constructors of Free monad to type constructors. At this stage, type parameter `a` has been promoted to kind parameter and it will be instantiated with kind `*` representing the kind of all types that have values i.e. Int, List of float. In addition, we should also create type level function that is equivalent to bind in Free monad to help us compose session types. The implementation of type level bind `>*>` can be seen in Listing 19. It is similar to bind in Free monad but defined in type family.

```
data ProcF (i :: SType * *) (j :: SType * *) next where
    Send :: Sing (n :: Nat) -> Core a -> next
            -> ProcF ('Free ('S n a j)) j next
    Recv :: Sing (n :: Nat) -> (Core a -> next)
            -> ProcF ('Free ('R n a j)) j next
    Branch :: Sing (n :: Nat) ->
              Proc' left ('Pure ()) c ->
              Proc' right ('Pure ()) c ->
              next ->
              ProcF ('Free ('B n left right j)) j next
    Select :: Sing (n :: Nat) ->
              Core (Either a b) ->
              (Core a -> Proc' left ('Pure ()) c) ->
              (Core b -> Proc' right ('Pure ()) c) ->
              next ->
              ProcF ('Free ('Se n left right j)) j next

type Proc (i :: SType * *) a =
    forall j . IxFree ProcF (i >*> j) j (Core a)
```

Listing 20: The algebra of Proc indexed by session types and the definition of indexed Proc

The second step is challenging since we need to find a way to make the Proc indexed by our free monad. Obviously, the original definition of Free monad and Proc does not provide any extra type parameters to be indexed by session types. Hence we use the indexed free monad. It is indexed by two parameter `i` and `j`. In the context of this project, you can treat `i` as the session type for the current proc and `j` as the continuation of session types. Accordingly, we will modify the definition of the algebra of Proc as well as Proc (see in Listing 20). The main operations remained unchanged and the main difference is 1) makes the algebra of Proc indexed by its corresponding session type (`i`) and continuation (haskj) 2) For the role identifier, we use the type level identifier: the type whose kind is `Nat` instead of values because in the later stage, we have to check duality at the type level. The definition of Proc use Haskell's RankNType and type family `>*>` to extract its corresponding indexed session types `i` from the continuation. By the definition of `>*>`, session type `i` must end with the Pure type constructor which is mapped to end

in the session type. The basic helper functions for constructing Proc expression are also indexed by session types. Some example can be in seen in Listing 21. We will omit the implementation for some helpers functions. Observing the function signatures is crucial in understanding how it works.

```
liftF :: ProcF i j a -> IxFree ProcF i j a

(>>=) ::  IxFree ProcF i j a
          -> (a -> IxFree ProcF j k b)
          -> IxFree ProcF i k b

send
    Sing n
    -> Core a
    -> Proc ( 'Free ( 'S n a ( 'Pure ()))) a
send role value = liftF $ Send role value value

recv :: Sing n -> Proc ( 'Free ( 'R n a ( 'Pure ()))) a
recv role = liftF (Recv role id)
```

Listing 21: Implementations of helper functions

```
example = do
    x :: Core Int <- recv zero
    send one x

ghci:> :type example
example
  :: Proc
     ('Free
        ('R 0 Int
            ('Free ('S 1 Int (Pure ())))))
     Int
```

Listing 22: An example of session type

We will conclude the implementation of second step with an example. After all, dependent types in Haskell is not an easy topic. An example is worth thousand words to build understanding (see in Listing 22). It represents a simple proc that receives an int from role zero and send the int to role one. Haskell type system infers its session type is `'Free ('R 0 Int ('Free ('S 1 Int (Pure ()))))` which corresponds to the behavior of the example. Session typing can be done simultaneously and automatically while users are building the Proc processes thanks to Haskell's type inference.

For the third step, we can assume we have already gathered a type level list of session types paired with its role identifier. The duality check algorithm is still the same. We

match different proc pair-wise, and check whether the projection of both session types are complementary. The algorithm is easy to implement as the Haskell function but lifting the computation into type level is tricky. One of the obstacles is that type family does not support higher order type-level functions. We divide the problem into sub-problems. We need to implement 1) a type family that converts a list of Session type to a list of Session type pair 2) a type family that maps a function to list and combine the result 3) a type family that that includes projection, checking whether a pair of session types are complementary. We combine the solutions to these problems and encapsulate them into type class constraint. The constraint is satisfied only if the duality checked is passed at the compile time.

5.2 SPar interpreter

5.2.1 Overview

SPar interpreter is an interpreter that interprets a group of Proc programs in Haskell. It can be considered as the simplest backend for evaluating SPar expressions. It records traces of the executions and the final output values of each Proc programs in the system.

It focused on providing a reference implementation explaining the semantics of SPar expressions not on performance. In addition, SPar interpreter served as a very useful tool in the development of this project. We use it as prototype to quickly verify whether the computation produces produces the expected results. This feature is useful especially in the early stage of an implementation or during debugging.

5.2.2 Implementation

The implementation of SPar interpreter is standard. It is similar to the implementation of the scheduler we explained in the free monad section of the background chapter. In essence, it is a round robin scheduler for a group of Proc programs. A partial implementation can be seen in Listing 23. It takes a list of Proc as a parameter and it maintain a state which is the combination of message queue, the trace and the list of output values. For the base case, the list is empty which means all processes has exited, it returns the trace and the list of output values. For the recursive case, it pattern matches on the first process at the beginning of the list. If the operation is Pure, it will update the list of output values, then abandon this process, and call itself recursively on the tail of the list. If the operation is an output action i.e, `Send`, `Select` or `Broadcast`, it will update the message queue with the corresponding message containing the sender, the receiver and the value, then pop the head proc and append its next step to the end of list of Procs, and call the list recursively. If the operation is an input action i.e, `Recv` or `Branch`, it will first examine whether sender and receiver pairs match the pair from the message at the top of the message queue. If so, it applies the continuation with the value in the message, removes the message from the queue, removes the head of list, moves the result of applying continuation to the end of the list and calls the list recursively. If not, it simply moves the head of list to the end of list without changing the value of head process and call the

```haskell
data InterpState = InterpState
    {
        outputValues :: [(Nat, String)],
        trace :: [String],
        messageQueue :: [(Nat, Nat, String)]
    }

interpret :: [(Proc (), Nat)] -> State InterpState ()
interpret []       = return ()
interpret (x : xs) = interp x xs

interp :: (Proc (), Nat) -> [(Proc (), Nat)] -> State InterpState ()
interp (Pure value, role) xs = do
    updateOutputValue role value
    interpret xs
interp (Free (Send receiver value next), role) xs = do
    updateMessageQueue (role, receiver, show $ interpCore value)
    updateTrace "send"
    interpret $ xs ++ [(next, role)]
interp p@(Free (Recv sender cont), role) xs = do
    (x, y, v) <- getTopMessage
    if x == sender && y == role
        then do
            popMessageQueue
            updateTrace "recv"
            let value = Lit (read v)
            interpret $ xs ++ [(cont value, role)]
        else
            interpret $ xs ++ [p]
```

Listing 23: Partial implementation of the SPar interpreter

list recursively. Because we have checked the duality of the processes in the system, it guarantee that for any input action, the required message will eventually appear at the head of the message queue in finite steps.

Also, the interpreter has a helper function that interpret Core expression into haskell value. This helper function is easy to write because operation on product or sum type has its own mapping function in Haskell and for the `Prim` and `Lit`, we can get their underlying Haskell implementation directly. As for `Var`, this constructor is not intended to externally so we will not encounter.

Chapter 6

SArrow: An arrow interface for writing SPar expressions

When trying to express more complicated and interesting parallel patterns, e.g map or reduce, We realize SPar is too low-level so that it is difficult to express simple computation because of overheads of expressing communication patterns by hand. In addition, compilation from Par-Alg to SPar is hard since they are very different domain specific languages.

To solve both issues, we draw inspirations from the Arrow interface (in particular, work done by [9] where they use arrow interface to express parallel computation) and introduce SArrow.

SArrow is an arrow interface for writing SPar expressions. Withe the help from SArrow, Users can use canonical arrow combinators to write algorithms in Arrow without writing any explicit communication and gain parallelized algorithms for free. Similarly, SArrow makes hassle-free compilation from Par-Alg to SPar possible because Par-Alg Proto is also an arrow expression and simply interpreting arrow combinators by the SArrow implementations fills the gap between Par-Alg and SPar.

6.1 Syntax

The simplified syntax of SArrow can be found in Listing 24. SArrow is a type synonym of `Nat -> Pipe a b`. It consumes `Nat` which means the identifier of a process and output `Pipe a b`. The reson why we use `Nat` as the only parameter is to ensuring no duplication of processes name since in most of the time, duplication is bad for parallelization. It will be explained more thoroughly in Section 6.3.

`Pipe a b` data structures is the essential component of SArrow. It regards computation as a pipe where data with type `a` goes into the pipe and data with type `b` get out of the pipe. Internally, it's a record type of four fields. `start` field identifies the process where the input data is received. `cont` field has the type `a -> Proc` which is a continuation waiting for the input data produced by the last pipe. `env` represents a group of Procs interacting inside the pipe to produce the output data, in other words, it is the

```
data Pipe a b = Pipe
    { start :: Nat
    , cont  :: a -> Proc
    , env   :: Map Nat Proc
    , end   :: Nat
    }

type SArrow a b = Nat -> Pipe a b

instance Arrow SArrow where
instance ArrowChoice SArrow where
```

Listing 24: Definition of SArrow

parallel computation. `end` indicates the process that produces the output data in the end. We can retrieve the corresponding process by a look up on the env with the key `end`. The returned Proc returns a data with type `b`.

6.1.1 Arrow interface

`SArrow` is an instance of Arrow typeclass as well as ArrowChoice type class. For example, the type signature of the combinators `>>>`, `|||`, `&&&` and `arr` are shown below. The main difference between their type signatures and the usual arrow interface is that in the `arr`, the function is wrapped with Core. In general, it captures the same meaning as the usual arrow interfaces. Implementation details of these combinators will be explained in Section 6.2.

```
(>>>) :: SArrow a b -> SArrow b c -> SArrow a c
arr :: Core (a -> b) -> SArrow a b
(|||) :: SArrow a c -> SArrow b c -> SArrow (Either a b) c
(&&&) :: SArrow b c -> SArrow b c' -> SArrow b (c, c')
(***) :: SArrow b c -> SArrow b' c' -> SArrow (b, b') (c, c')
```

6.1.2 Example: Parallel programming patterns

As an example, we will illustrate some typical computation patterns used in parallel computing.

First of all, the branching pattern illustrated by Figure 6.1 is equivalent to an expression formed by `|||` combinators, where the data constructor `Left` leads to one computation path and the data constructor `Right` leads to another computation path. It seems to be a simple pattern but it is useful when composed with other complicated patterns. We will use an example to illustrate at the end of this section. Secondly, the fundamental building block of parallel pattern, the fork-join pattern illustrated by Figure 6.2a can be expressed by `&&&` combinator. The SArrow produced by `&&&` has the two-ary tuple as the output type collecting the computation result of the main thread and the forked thread

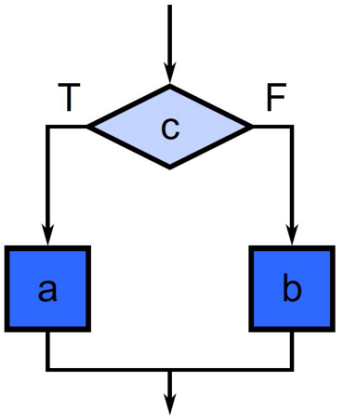


Figure 6.1: Visualization of the branching pattern [21]

and also acts as a synchronization point. Thirdly, the familiar parallel map pattern illus-

```
pmap :: SArrow a b
     -> SArrow a (a, (a, (a, a)))
     -> SArrow (b, (b, (b, b))) b
     -> SArrow a b
pmap f s c = s >>> (f *** (f *** (f *** f))) >>> c
```

Listing 25: Parallel map in SArrow

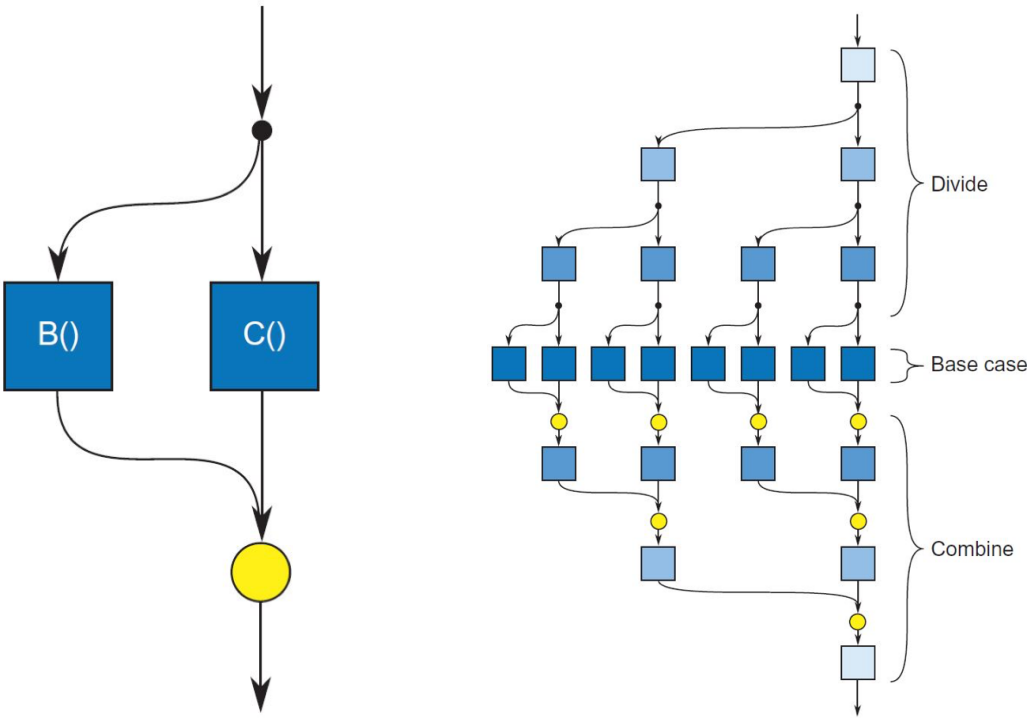
trated in Figure 6.3 is also a candidate to be expressed in SArrow. The code sample is in Listing 25. `pmap` splits the input `a` into 4 chunks using the splitting function `s`, applied the elemental function `f` and the arrow combinator `***` in parallel and finally use the collecting function `c` to collect the results. Usually, the input `a` is a list and `s` splits the list into 4 equal chunks. The number of times where function is `f` applied decides the number of ways of parallelism.

```
preduc :: SArrow (a, a) a -> SArrow (a, (a, (a, a))) a
preduc r = helper >>> (r *** r) >>> r
 where
  helper = (arr Id *** arr Fst) &&& (arr Snd >>> arr Snd)
```

Listing 26: Parallel reduce in SArrow

Fourthly, we can apply the similar logic to express parallel reduce pattern shown in Figure 6.4. The code sample is in Listing 26. The result of parallel reduce has similar type signature as the collecting function in `pmap` so it is often used with the `pmap` function. We use nested tuple `(a, (a, (a,a)))` to represent a sized array of data. The `helper` function transforms the array representation of data into a form so that we apply the reduce function `r` to the elements pair-wise and parallel.

Finally, more complicated pattern can be expressed compositely from simpler pattern expressed in SArrow. We can use a typical divide-and-conquer algorithms implemented with fork-join as an example. Figure 6.2b shows a divide-and-conquer algorithms with



(a) Visualization of the fork-join pattern [21] (b) Fork-Join Pattern for Divide-Conquer [21]

Figure 6.2: Fork-join pattern and divide-and-conquer algorithms

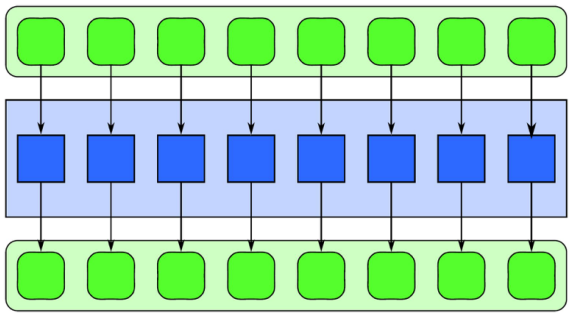


Figure 6.3: Visualization of parallel map [21]

2-ways and 3-levels of fork-join. The algorithm can be expressed in SArrow shown in Listing 27. The divide-and-conquer pattern can be built recursively in Haskell. For the base case, we simply apply the basic computation. Otherwise, we first call split and then call the function recursively with the level decremented by one and, in the end, call the merge to combine the results. Every expressions in the function definition are connected using arrow combinators. A 3-level divided-and-conquer algorithm is constructed by passing 3 to the function resulting a algorithm with $2^3 = 8$-way parallelism.

In addition, the divide-and-conquer parallel pattern can be optimized when combining with branch pattern. The branch pattern allows us to add shortcuts to the pattern (illustrated in the Figure 6.5, redlines represent the alternative computation path provided by branching patterns). The shortcut give us the ability to decide whether to do local computation or split into multiple subtasks depending on the input size. When the input size is small, the overhead of the latter usually outweighs its parallelism. Adding

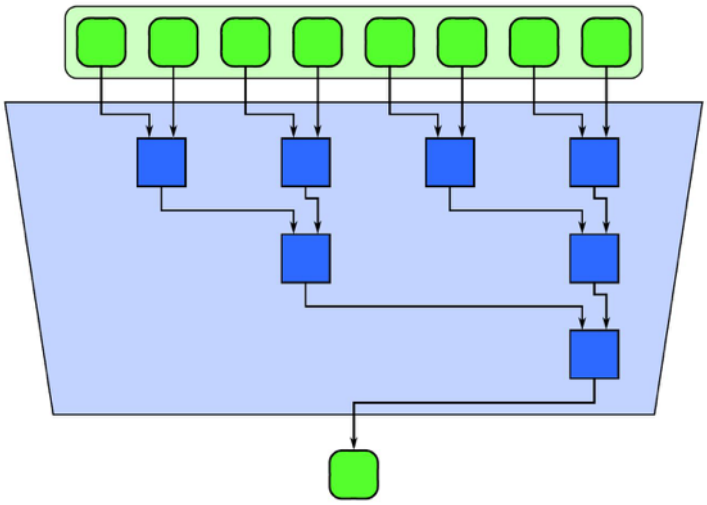


Figure 6.4: Visualization of parallel reduce in SArrow [21]

the simple branching pattern results in a pattern that is adaptive to various input size with better performance.

The implementation demos the power of implementing SArrow as a domain specific language embedded in Haskell. We make full use of Haskell features, i.e high order functions and polymorphic functions to construct expressive, composable and generic computation patterns.

More examples of algorithms formed by SArrow, e.g. dot product or merge sort are shown in the Section 8.

6.2 Implementation of arrow combinators

In this chapter, we will present naive implementation and the optimized solution is introduced in the next section.

The intuition why SArrow is an instance of Arrow comes from the Kleisli arrow of a monad is an instance of Arrow class (shown in Listing 28). The `cont` field in the Pipe has similar type signatures as the `runKleisli` field in the Kleisli arrow. From the previous section, we shown that Proc is a monad so Pipe is just an extended version of Kleisli arrow where computations in Pipe usually finish in one of the process stored in `env` instead of finishing at `cont` like Kleisli arrow. Intuitively, SArrow, a function from the role to the Pipe, should be an instance of Arrow since Pipe looks like an arrow instance and functions are composite.

The essential issue when implementing arrow combinators is how to connect one Pipe by another Pipe. The first problem we need to address is how to deal with the `cont` in the tail Pipe. We know that only one `cont` field exists in the resulting Pipe and it must be that from the head Pipe. Hence the only option is to convert it to a Proc expression and store the converted expression in the updated env in the resulting Pipe. A right way to do it bind the cont with the action: receive from end in the first Pipe. Also, extend the proc related to the end in the head Porc with action: send to start in the second Pipe.

```
divConquer
  :: Int
  -> SArrow a b
  -> SArrow a (a, a)
  -> SArrow (b, b) b
  -> SArrow a b
divConquer 0 baseFunc _split _merge = baseFunc
divConquer level baseFunc split merge =
  split
    >>> (   divConquer (level - 1) baseFunc split merge
        *** divConquer (level - 1) baseFunc split merge
        )
    >>> merge

twoWayThreeLevelDq = divConquer 3
```

Listing 27: 2-ways and 3-levels divided-and-conquer algorithm in SArrow

```
newtype Kleisli m a b = Kleisli { runKleisli :: a -> m b }

instance Monad m => Arrow (Kleisli m) where
    id = Kleisli return
    (Kleisli f) . (Kleisli g) = Kleisli (\b -> g b >>= f)
    arr f = Kleisli (return . f)
    first (Kleisli f) =
        Kleisli (\ ~(b,d) -> f b >>= \c -> return (c,d))
    second (Kleisli f) =
        Kleisli (\ ~(d,b) -> f b >>= \c -> return (d,c))
```

Listing 28: The implementation of arrow instance for Kleisli arrow of a monad

Besides addition of the converted `cont` expression, the new env is formed by merging the env from the head Pipe and the env from the tail Pipe. Merging two envs is trivial. When there are duplication, we simply use monadic bind to combine them so that the actions belonging to head Pipe followed by the actions belonging to the tail Pipe. The `start` field in the resulting Pipe is the same as that from the head Pipe and the `end` field will be set the same as that from the tail Pipe. We can apply the Pipe composing function to implement arrow combinators for SArrow. The implementation is just apply the first SArrow with the input role (don't forget SArrow is a function) to get the Pipe and apply the second SArrow with a new role to get the second Pipe (usually in order to avoid duplication of roles, the new role is set to be maximum role in the first Pipe + 1) and finally apply the Pipe composing functions to both Pipe. A simplified code explanation can be seen in Listing 29. The rest of combinators can be implemented in a similar fashion.

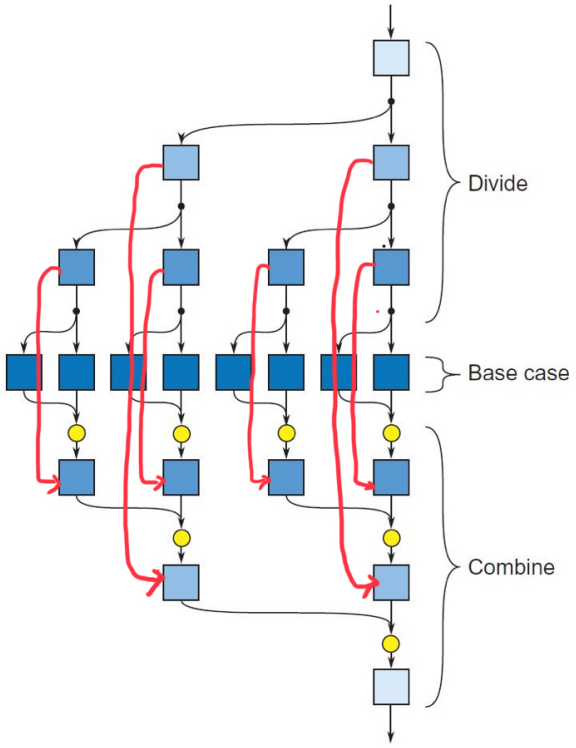


Figure 6.5: Combination of branching pattern and divide-and-conquer pattern

```
(>>>) :: (SArrow a b) -> (SArrow b c) -> (SArrow a c)
(>>>) leftArrow rightArrow start = compose firstPipe secondPipe
 where
  firstPipe  = leftArrow start
  secondPipe = rightArrow (end leftP + 1)
```

Listing 29: The simplified implementation of >>>

6.3 Strategies for optimized role allocations

From the last section, we know the number of roles in the system is directly related to the number of processes in the final generated code. Hence, role allocating is an essential part in generating efficient parallel programs.

In this section, we propose strategies for optimizing role allocations. We have two goals in mind when optimizing; The first one is we would like to reduce the number of roles (processes) in the computation since the overhead of thread creation and data transmission has negative impact on performance. The second one is we do not want roles duplication when we try to compose SArrows since role duplications means the different computation must be merged in the same role and computations in the same thread is sequential hence role duplications has negative impact on degree of parallelization.

If we only put the first goal in mind, an easy solution will be setup an upper bound of the number roles, and then we cycle through a fixed bound when allocating new roles. Processes corresponding to duplicated roles can be simply merged using binds since Proc

is a monadic DSL and duality check ensures binding will not cause deadlocks. However, this strategies is not ideal since duplications of roles will decrease the degree of parallelization in the system.

If we only consider the second goal, naive strategies used in the previous section will satisfy the goal. However, the number of channels required and the number of roles in the system will grow exponentially. In a divided-and-conquer algorithm, the number of channels increases from 10 to 120 and the number of roles increases from 6 to 36 when the level is increased from 1 to 3.

$$\text{id}\frac{x : \text{Role}, \quad a : \text{Type}}{id : \text{SArrow a a}, x \Rightarrow x}$$

Listing 30: Role allocation for id

For the purpose of illustration, we use inference rules to explain our proposed strategies for optimized role allocations when composing SArrows. Please see Listing 30 as an example. $x \Rightarrow x$ means the computation start with role x and end with role x.

$$\text{compose}\frac{e1 : \text{SArrow a b}, x \Rightarrow y, \quad e2 : \text{SArrow b c}, y \Rightarrow z}{e1 >>> e2 : \text{SArrow a c}, x \Rightarrow z}$$

$$\text{arr}\frac{f : \text{Core (a} \rightarrow \text{b)}, \quad x : \text{Role}}{\text{arr } f : \text{SArrow a b}, x \Rightarrow x}$$

$$\text{arrow choice: } |||\frac{e1 : \text{SArrow a c}, x \Rightarrow y, \quad e2 : \text{SArrow b c}, x \Rightarrow z}{e1 \ ||| \ e2 : \text{SArrow (Either a b) c}, x \Rightarrow \max(y, z)}$$

$$\text{arrow choice: } +++\frac{e1 : \text{SArrow a c}, x \Rightarrow y, \quad e2 : \text{SArrow b d}, x \Rightarrow z}{e1 \ +++ \ e2 : \text{SArrow (Either a b) (Either c d)}, x \Rightarrow \max(y, z)}$$

$$\text{arrow: } \&\&\&\frac{e1 : \text{SArrow a b}, x \Rightarrow y, \quad e2 : \text{SArrow a c}, (y + 1) \Rightarrow z}{e1 \ \&\&\& \ e2 : \text{SArrow a (b, c)}, x \Rightarrow z}$$

$$\text{arrow: } ***\frac{e1 : \text{SArrow a b}, x \Rightarrow y, \quad e2 : \text{SArrow a' c}, (y + 1) \Rightarrow z}{e1 \ *** \ e2 : \text{SArrow (a, a') (b, c)}, x \Rightarrow z}$$

Listing 31: Rules fo role allocations of different combinators

The rule for the rest of combinators are shown in Listing 31. Notice that for compose, id, arr and ArrowChoice we do not introduce any new roles, in other words, there is no parallelization for these combinators. Reader may find it strange that we do not intent to parallelize arr combinator which lifts a sequential computation represented by Core (a $\rightarrow$ b) into SArrow. It makes sense to introduce a new role to execute the computation

and hence parallelize computational heavy tasks. We use this strategy in the first place but later we found a more suitable strategy exists which will be introduced in the later paragraph. Also, another reason not to introduce new role when encountering arr combinators is that we gained function fusion for free. Simple function i.e. fst, inject left or snd are automatically fused into more complicated user defined functions. Introducing new roles for these simple functions will damage performance.

For the class of combinators belonging to arrow choice, we do not introduce any new role. The expressions at the lhs and at the rhs starts with the same role x because when only one code path will be executed as the name choice suggested so we should not use separate roles for two expressions that will never be executed simultaneously. In the end, we decided the computation end in the role $\max(y, z)$. Max guarantees that there will not be role duplications when we compose expressions formed by ArrowChoice combinators with other combinators. For the implementation, all process in both left and right SArrow expression are wrapped inside a branch operation separately. Assume $\max(y, z) = y$, the process at the role y will be extended with actions that receive data from $\min(y, z) = z$ role at its right branch. Finally, applying inject left and inject right at left and right branches gives us Either type as the output.

Finally, we discovered the right place to allocate new roles is `&&&` combinator. As shown in type signature, product types mean computation at both branches will both be executed and they are independent. In order to make sure both computation are executed simultaneously, we constraint that the right SArrow expression must start with a role greater than the end role of the left SArrow expression. This ensures no role duplications hence maximize parallelism. The combined expression ends in the end role of the right SArrow expression instead of introducing a unnecessary new role. For the implementations, the process corresponded to end role z are extended with actions that receive data from the end role y of the left process and store the computation of SArrow expression at the right side and finally output a pair.

Even though from the implementation point of view, SArrow composition with the optimized role allocations is ad-hoc and less elegant to implement because we need to consider composition by send-and-receive and composition by local monadic bind and more edge cases to be dealt with compared to the naive solution in the last section where composition is done solely by send-and-receive and role allocations is mindless. We believe the effort is worthy because for a n-level divided-and-conquer algorithms, the optimized role allocation strategies allocate 2^n roles in total which is the same as the number of way of parallelization in theory. All the roles are used to maximize parallelism instead of wasting the valuable resources to create roles that merely transmit data.

In conclusion, the optimized strategy presented in this chapter is not the only solution. For example, there is a strategy that only introduce new roles into the computation graph when encountering a specific atomic functions. Different strategies result in different communication structures hence different kind of parallelisms. We should choose the right strategy depending on the specific task.

6.4 Satisfaction of arrow laws

We have provided the implementation of SArrow combinators that is similar to the arrow interface. However, the implementation is not enough to state that SArrow is an arrow interface. Furthermore, we need to prove that SArrow satisfy the arrow laws. Because of the limited context, we will present justification instead of formal proofs. We will focus on arrow laws. The justification for arrow-choice law can be reasoned in a similar way.

$$arr(Id) \ggg a = a \tag{1}$$
$$a \ggg arr(Id) = a \tag{2}$$
$$(a \ggg b) \ggg c = a \ggg (b \ggg c) \tag{3}$$
$$arr(f; g) = arr(f) \ggg arr(g) \tag{4}$$
$$first(a) \ggg arr(Fst) = arr(Fst) \ggg a \tag{5}$$
$$first(a) \ggg arr(\alpha) = arr(\alpha) \ggg first(first(a)) \tag{6}$$
$$first(a \ggg b) = first(a) \ggg first(b) \tag{7}$$

Table 6.1: Arrow laws [22]

Table 6.1 shows the rules of arrow laws. We includes a subset of law that contains the `first` combinator. The other half of laws that contains the `second` combinator can be proved by symmetry. `first` combinator is implemented as `first = (*** id)` and $\alpha :: ((x, y), z) \to (x, (y, z))$.

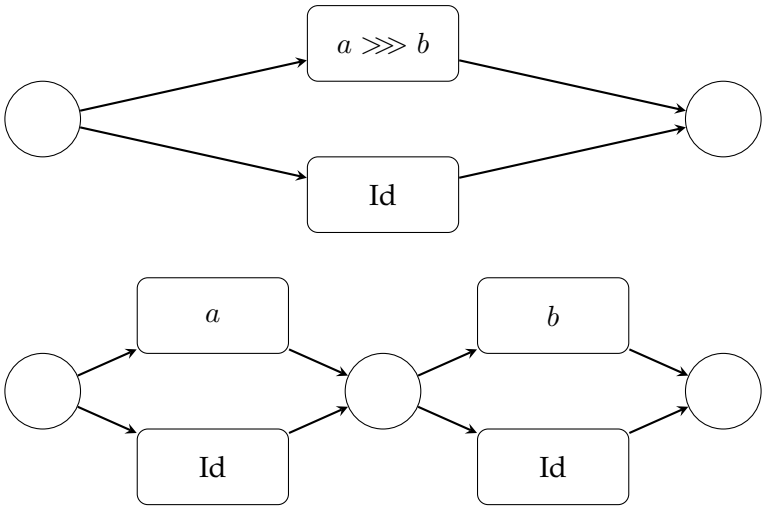


Figure 6.6: Graphical representation of equation (7)

We argue above equation holds if both sides of equation express the same computation. Equation (1), (2) holds because lifting Id from Core to SArrow will not modify the result of computation due to the semantics of Id. Equation (3) is the associativity rule of $\ggg$. Left side of the equation sends the output from $a \ggg b$ to the input of c

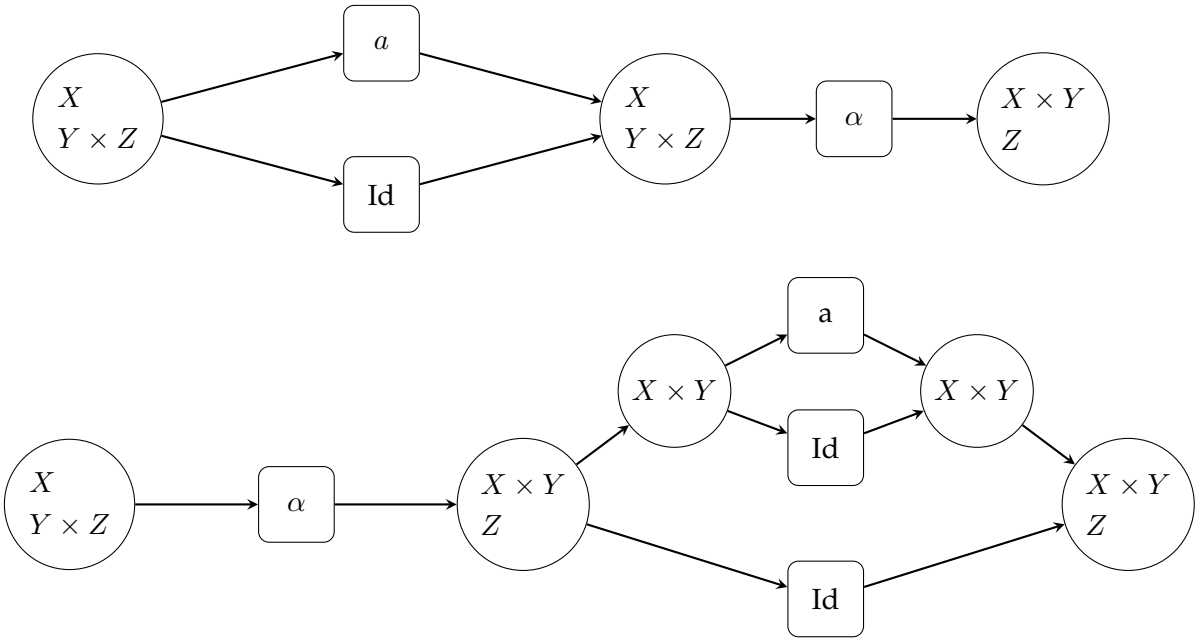


Figure 6.7: Graphical representation of equation (6)

while right side sends the output of a to the input $b \ggg c$. The communication structure might change but they both represents the same computation. Equation (4) is valid because applying the input value to composition of f;g is the same as applying the input to f followed by a message passing (data communication could be local which means the communication structure for both side of equations are the same in this case) and then applying it to g. One might claim that the left side is more efficient than the right side due to the fusion depending on the role allocation strategy. Equation (5) holds because they both represents a computation that take a pair input and apply the function a to its first position and return the result. Right hand side of equation (7) represents a SArrow expression that applies a on left position of input pair and applies Id on the right position of input pair in parallel, collects the results and applies b on left position of input pair and applies Id on the right position of input pair while left side fused a and b together and apply them in one step hence there is only one join point. Figure 6.6 is a graphical representation of equation (7). Obviously they represents the same computation. We can also derive equation (6) from its Visualization (see in Figure 6.7).

6.5 Conclusions

Arrow interface is the perfect interface to express general computation for this project because not only is it intuitive to understand and visualize but also its combinators `***` and `&&&` has built-in parallel natural.

So far, we've used SArrow to help us compile Par-Alg to SPar. The remained challenge is the code generation from SPar to a target platform. In the next chapter, we will introduce our methods for code generation and specifically code generation to C. Once we achieved this, every computation in SArrow can be transformed into parallel C code in one step.

Chapter 7

Type-safe code generation from SPar

SPar has two components: Core representing the unit of computation and Proc as a skeleton of the computation, describing the communication patterns. Naturally, the process of code generation from SPar should be divided into two parts correspondingly. We choose to make two parts independent of each other so that it's possible to swap the code generation strategy of one component without modifying another one.

The procedure of code generation is standard: transformation. We start our programs in a high level DSL and run a series of transformation to low-level DSL. SPar expressions are converted to a low-level EDSL which is then transformed to an abstract syntax tree (AST) of C (TODO cite the package). The generated code is obtained by pretty printing the AST.

7.1 Instr: A low-level EDSL for channel communication

In Proc, we have high-level actions like select, broadcast and branch abstracting implementation details, i.e variable declaration, variable assignment, channel initialization, channel communication and channel deletion. Hence, we need to define a EDSL containing instructions related to these low-level operations. We name it Instr. A Spar programs will be translated to a sequence of Instr.

When we design Instr, we keep the simplicity in mind so Instr is not coupled with any fancy language feature related to some specific target languages. Any reasonable target language with a channel communication library can be easily converted from Instr.

7.1.1 Syntax and semantic

The definition of Instr is seen in Listing 32. `Channel` is our abstract representation of Channel in Instr. It is indexed by a type a from the reified type `ReprType a`. More details of this reified type will be introduced in Section 7.1.2. This type parameter preserves type in channel initialization hence make sure the value to be sent or received in this channel has the same type as this channel. This is necessary because for some target languages, the channel are typed. Similarly, type parameters in `Exp` have the same functionality.

```
data Channel a where
    Channel :: CID -> ReprType a -> Channel a

data Exp a where
    Exp :: Core a -> ReprType a -> Exp a

data Instr where
    CInitChan :: Channel a -> Instr
    CDeleteChan :: Channel a -> Instr
    CSend :: Channel a -> Exp a -> Instr
    CRecv :: Channel a -> Int -> Instr
    CEnd :: Exp a -> Instr
    CDecla :: Int -> ReprType a -> Instr
    CAssgn :: Int -> Exp a -> Instr
    CBranch :: Int -> Seq Instr -> Seq Instr -> Instr
    CSelect :: Int -> Int -> Seq Instr -> Seq Instr -> Instr
```

Listing 32: The syntax of Instr in Haskell with accompanying low-level data types

`Exp` is just a wrapper of the expression in Core. In later stages, we will take care of code generation of `Exp` along with `Instr`. `Instr` defines the set of statements that will be generated and `Exp` represents the sequential computation, which is a value that will be generated.

The semantic of Instr operations are similar to what their names suggest. `CInitChan` represents operations that initialize a channel according to the given type and cid. `CDeleteChan` will destroy a channel. `CSend` operation sends the value `Exp a` through the Channel. `CRecv` action means the value received in the channel will be assigned to variable whose postfix name is the int field. `CEnd` means the instruction exits with the value `Exp a`. `CDecla` and `CAssgn` are instructions for variable declaration and assignment. The type of the variable is determined by `ReprType a` and the value is `Exp a`. `CBranch` and `CSelect` are used to express conditional control flow of the Instr languages. SPar action like broadcast are built on top of these operations. For `CBranch`, the first field represents the value of Either type to be received via the channel and two `Seq Instr`s represents the sequence of `Instr`s in the left branch and the right branch. For `CSelect`, the first field represents the variable containing Either value and the second field field represents the variable whose value is assigned by the end results of instructions from either the left branch or the right branch. The third and fourth fields represent the instructions in the left branch and the right branch.

7.1.2 Representation types

SPar programs cannot be fully parametric since the target languages of code generation from SPar are usually less expressive, i.e, they do not treat function type `a->b` as a value, and are less efficient when processing with some specific form of data, i.e, languages tar-

```haskell
data ReprType a where
    NumReprType :: NumType a -> ReprType a
    LabelReprType :: ReprType Label
    SumReprType :: ReprType a -> ReprType b -> ReprType (Either a b)
    UnitReprType :: ReprType ()
    ProductReprType :: ReprType a -> ReprType b -> ReprType (a, b)
    ListReprType :: ReprType a -> ReprType [a]
```

Listing 33: The definition of representation types

```haskell
constToCExpr :: ReprType a -> a -> CExpr
constToCExpr (NumReprType numType) v = numTypeToCExpr numType v
constToCExpr LabelReprType         v = case v of
  Le -> cVar "LEFT"
  Ri -> cVar "RIGHT"
constToCExpr s@(ProductReprType a b) v = defCompoundLit
  (show s)
  [ ([], initExp $ constToCExpr a (fst v))
  , ([], initExp $ constToCExpr b (snd v))
  ]
```

Listing 34: An example usage of reified type in the code generation

geting GPUs are usually more productive in dealing with array of floating point number while slow in working with aggregate structures [23]. Hence, we need to restrict the set of types allowed in SPar. We achieve this using the type class `Repr` and corresponding reified type `ReprType` (shown in Listing 33). `Repr` determines the set of type allow in SPar. Reified type `ReprType` will be used to alter the behaviors of code generation based on the type. This can be simply done by pattern matching because reified types are values in Haskell [24]. To be more concrete, Listing 34 gives a demo. `constToCExpr` is function that handle code generation from constant value to expression in C programming languages. By pattern matching, we vary the behaviors of code generation so that constants with different types has their own way to be represented in C.

In conclusion, we allow the following type: numerical type like `Float` and `Int`, the unit type `()`, the label type which is used in the code generation of select and branch and the aggregate type: list, product and sum that are built recursively, to be expressed in SPar.

7.2 Compilation from SPar to Instr

7.2.1 Transformation from Proc to Instr

As described in the previous section, Instr contains a data type called `Exp` which is a wrapper of Core expression. Compiling Core to Instr is hence not difficult. The challenge

of compiling Core is mainly how to compile it to a specific target language. This will be discussed in the next subsection.

In this section, we will explain how we transform operations in Proc to Instr. Generally speaking, each Proc operation is mapped to a sequence of actions in Instr. The transformation algorithm from a Proc expression to a sequence of Instr can be implemented easily by traversing Proc expression, applying the mapping and collecting the results by concatenation. This is an advantage of using free monad technique to build the AST because Proc expression can be treated as a data structures and traversing recursive data structures can be easily done in Haskell. In addition, operations like `Recv` which involves continuation whose type is `Core a -> next` in their constructors are treated differently than those operations whose constructors only have a value type `next`. The latter is easy to implement, we can simply call the traversing function recursively. For the former, we have to pass an expression whose type is `Core a` to the continuation so that we can recursively call the traverse function recursively on the results of applying value to the continuation. The answer to what Core expression should be used is `Var` denoting variable. Passing a unique variable to the continuation gives us `next` inexpensively and we will define where does values of variables comes from for each operations in Proc.

We have introduced the general principle to the readers. Now let us dig into details of translation rules for each operations.

- **Pure.** It is the base case in free monad. Hence it is mapped to the `CEnd` instruction.
- **Send.** It is mapped to a sequence of three instructions. First of all, We declared a temporary variable using `CDecla` and then assign the value that will be sent to this variable using `CAssgn` and send the content of the variable via the specific channel. The problem of how make sure the same channel is used in a send-and-receive pair will be discussed in the next sub chapter.
- **Recv.** It is the reverse of send operation. Firstly, it declares a new variable with `CDecla` and use `CRecv` to assign the value received from the sender to this variable. Notice that the recv has a continuation, we will pass the variable declared in the first instr to the continuation to traverse the Proc expression recursively as discussed above.
- **Select.** It is a more complicated operations. Its constructor contains two continuation: one for left branch and one for right branch. Hence for this instruction, we need to declared two variable to passed into continuations. The value of both variables is assigned by the Core expression whose value is a Either type. Besides, we need to send label indicating whether the execution of the left branch or the right branch of the receiver is selected. Its value is assigned by the either value as well and the sending operation is done by `CSend`. Finally, we call the transforming function recursively on the left branch and right branch and combine the results using `CSelect`.
- **Broadcast.** The mapping from Broadcast is similar to that of Select. The only difference is that the former sends to a list of receivers while the latter sends to a

single receiver. So in this operation, we will have multiple `CSend` corresponding to each receiver.

- **Branch.** It is the reverse of the Select operation. So it will use `CRecv` to receive a label from the sender and call recursively on two branches and finally use `CBranch` to collect results.

7.2.2 Strategies for channel allocations

Channel allocations is important because correction allocation is essential in making sure the correctness and deadlock-free of generated code. Besides correctness concerns, we are also want to reduce the amount of channels creations hence increase performances.

In the first iteration, inspired by the linearity of channels in the π calculus, we choose to allocate a one-time channel for each send-and-receive pair. All channels' buffer size is one because of the linearity. Send action will initialize a channel and Receive action will destroy this channel once it receive the value. We use the ensure the same channel is used for the pair. During the transformation, we use a map of queue whose key is a pair of sender id and receiver id. When visiting the send action, it will push the channel into the queue and the corresponding received operation will pop the channel from the queue. Because we've ensured the duality of all processes in the system, we can claim the channel is right for each send-and-receiver pair. However, we realize this strategy is complicate to implement and not resource efficient since too many channels are initialized.

In the second iteration, we come up with a more efficient and simple strategy. The buffer size of all channels is still one due to the same reason about linearity. However, we decide to only allocate one channel for a pair of sender and receiver. We will not destroy the channel after the value has received and will reuse the channel for the next communication. When all processes have returned, we will destroy all channels at once. For this strategy, we have simplified the state from a map of queues of channels to a map of channels.

7.2.3 Monad for code generation

From the last two subsections, we need to maintain a number of states during the compilation process. Hence we define a state monad to be used during the traversal. The `CodeGenState` is the collection of states with different purposes and it is shown in Listing 35. `chanTable` is the map we required during the channel allocation. `varNext` represents the next variable id to used. It will increment by one every time we declare a new variable. It helps us make sure the variable names are unique. `chanNext` has the similar functionality ensuring the uniqueness of channel names. `dataStructCollect` is the set of compound type we encounter during the traversal. With this states, we know what kind of data structures whose definitions will be generated before the generation of executing code.

```haskell
data CodeGenState = CodeGenState
  {
    chanTable :: Map ChanKey CID,
    varNext   :: Int,
    chanNext  :: CID,
    dataStructCollect :: Set AReprType
  }

newtype CodeGen m a =
  CodeGen { runCodeGen :: StateT CodeGenState m a }
    deriving (Functor,
              Applicative,
              Monad,
              MonadState CodeGenState,
              MonadIO)
```

Listing 35: States required during the traversal

7.3 Code generation to C: from Instr to C

The last piece of jigsaw is compilation from a sequence of Instr to C. This process is rather simple but trivial. This is simply done by transforming the sequence of Instr to a C AST. We used an open source library: language-c [25] to represent C AST in Haskell and pretty printing the C AST gives us the generated code. This method can be generalized to any target language. As for the implementation of channel communication in C, we used another open source library: chan [26] whose internal is based on shared memory. In the following subsections, we will present some design choices during this final step.

7.3.1 Representations of Core data type in C

The first challenge we face is how to represent data structure in C. For primitive data type like Int or Float, a simple one-to-one mapping is sufficed. It is hard to deal with compound data type in C. First of all, C does not support polymorphic type. Hence, we choose to generate specific data type for every different compound data type even though they have the same structure. We have a way to name the generated data type to avoid name duplication. The naming simply reflects the structure of the data types with its elemental type. For example, `(Int, (Int, ()))` is converted to Prod-Int-Prod-Int-Unit and `Either (Either () (Int, Int)) (Int, Int)` is convert to Sum-Sum-unit-Prod-int-int-Prod-int-int. In this project, all compound types are formed by sum type and product type. The product type will be converted to a struct with two field in C. The sum type is represented by the tagged union type. Tagged union is a struct with two field. The value of the first field indicates whether it is a left value or right value and the value of second field is a union type containing either the left value or right value. We also implement a sorting algorithm based on the depth of compound type so that all

```
typedef enum Label {
            LEFT, RIGHT
        } Label;

typedef struct Prod_int_int {
            int fst; int snd;
        } Prod_int_int;

typedef struct Sum_unit_Prod_int_int {
            Label label;
            union {
                int left; Prod_int_int right;
            } value;
        } Sum_unit_Prod_int_int;
```

Listing 36: Compound data type in C

necessary data types have been defined before the definition of the compound types. An example of what `Either Int (Int, Int)` will be converted to is shown in Listing 36.

Another challenge is representation of the recursive type. From the type theory, we learn that a list of int can be expressed as $\mu a.() + \text{Int} \times a$. We might reuse the idea from the last paragraph to generated recursive type in terms of sum and product type. Hence a list of Int will look like the code below.

```
typedef struct Sum_unit_Prod_int_a {
    Label label;
    union {
        int left;
        Sum_unit_Prod_int_a *right;
    } value;
} Sum_unit_Prod_int_a
```

However, we believe expressing typical recursive data structures like list of int in this way is bad for performances. Obviously, C has a more efficient way to represent list of int using an array. So we decided to has two ways to represent recursive data structures. For a set of specific recursive data structure, users can write their own representation to exploit the advantages of the target language. For example, a list of int are encoded in C using a wrapper of pointer type (shown in Listing 37). This way is not very generic but friendly for performances. For other types of recursive data structures where user do not specify their optimized versions in C, we simply apply the method in the last paragraph to encode them in C. This way is generic but not efficient.

```
typedef struct List_int {
        size_t size; int * value;
    } List_int;
```

Listing 37: Optimized represent of List in C

7.3.2 Compiling from Core to C

$$\text{Var}\frac{}{[\![\text{Var n}]\!] = \text{vn}} \qquad \text{Lit}\frac{}{[\![\text{Lit val}]\!] = \text{toC(val)}}$$

$$\text{Fst}\frac{[\![\text{a}]\!] = c}{[\![\text{Fst 'ap' a}]\!] = c.\text{fst}} \qquad \text{Snd}\frac{[\![\text{a}]\!] = c}{[\![\text{Snd 'ap' a}]\!] = c.\text{snd}}$$

$$\text{Inl}\frac{[\![\text{a}]\!] = c}{[\![\text{Inl 'ap' a}]\!] = \{\text{LEFT}, c\}} \qquad \text{Inr}\frac{[\![\text{a}]\!] = c}{[\![\text{Inr 'ap' a}]\!] = \{\text{RIGHT}, c\}}$$

$$\text{Pair}\frac{[\![\text{a}]\!] = c_1, \quad [\![\text{b}]\!] = c_2}{[\![\text{Pair a b}]\!] = \{c_1, c_2\}}$$

$$\text{Id}\frac{[\![\text{a}]\!] = c}{[\![\text{Id 'ap' a}]\!] = c} \qquad \text{Const}\frac{[\![\text{a}]\!] = c, \quad [\![\text{v}]\!] = b}{[\![\text{(Const v) 'ap' a}]\!] = b}$$

$$\text{Prim}\frac{[\![\text{a}]\!] = c}{[\![\text{(Prim fname fimpl) 'ap' a}]\!] = \text{fname}(c)}$$

Listing 38: Rules for compilation from Core to C

Core has a concise syntax so it does not require too much work to write a function that generate C expressions from Core expressions. Not surprisingly, the compilation is a traversal of the Core expression. Pattern matching on the structure of the core expression alters the behaviors of compilation. `ap` (apply) constructor is used with an expression whose type is `Core (a -> b)` and another expression whose type is `Core b`. The code generation for `ap` depends on the what the function expression is. The code generation rule is explained by the inference rules shown in Listing 38. $[\![a]\!]$ means the C code generated by Core expression `a`. toC is function that convert constant value in Haskell to constant value in C.

- **Var, Lit:** Code generation of Lit simply convert Haskell value to its corresponding C value. As for Var, it will be converted to a string literal composed of the variable identity prefixed by 'v'.
- **Fst, Inl, Pair...** For Inl and Inr and Pair, we used C99 style to initialize struct and union. The rule for generating corresponding struct are explained in the previous subsection. For Fst and Snd, we simply access the specific value using the designator.
- **Id, Const v:** Code generation of Id constructor is same as the code generation of the

argument of the Id. Code generation of Const ignores the argument and use code generation of v expression instead.

- **Prim:** Prim constructor represents user defined functions. The code generation for function call of Prim is converted to function call by the name of the primitive function. Users can implement those functions in C and include them in the main generated file.

7.3.3 The structure of generated C code

We have covered the code generation algorithm for each process. We will now tackle how the generated code is structured from a group of interacting processes representing a parallel computation. For each process, we will generate its own C functions that take no argument. We decide to generate an addition C function since function is portable and acts like a black box hiding all the execution details so users can interact with generated code by calling the function. This function will take a parameter as input data and return the computation result. Inside, the function, it will spawn the same number of threads as the number of roles in the system. Each thread will execute the code which is generated from their corresponding Proc expression. In addition to that, the function will send the input parameter to the starting role in the group of Proc to kick off the computation using channel and waiting for the the ending role to send back the computation results. After it receive the result and all the threads have returned, it will return the result. We called this function `proc0`.

An example of the generated code is shown in Listing 39. The code contains one process which received a list of int from `proc0`, sort the input by an user defined function and then send the result to `proc0`. We omit global channels declarations and data type declaration for simplicity.

7.4 Conclusion

With the completion of code generation, we deliver the results we promised in the introduction section. We have implemented a end-to-end process that will generate low-level deadlock-free parallel code from an expressive high-level languages embedded with a flexible backend that can target multiple languages with ease. Now, it is time to evaluate the performances of our achievement with quantitative measurements.

```
void proc1Rt()
{
    List_int v0;
    chan_recv_buf(c1, &v0, sizeof(List_int));
    List_int v1;
    v1 = sort(v0);
    chan_send_buf(c2, &v1, sizeof(List_int));
}
void * proc1()
{
    proc1Rt();
    return NULL;
}
List_int proc0(List_int v0)
{
    c1 = chan_init(1);
    c2 = chan_init(1);
    pthread_t th1;
    pthread_create(&th1, NULL, proc1, NULL);
    chan_send_buf(c1, &v0, sizeof(List_int));
    List_int v1;
    chan_recv_buf(c2, &v1, sizeof(List_int));
    pthread_join(th1, NULL);
    chan_dispose(c1);
    chan_dispose(c2);
    return v1;
}
```

Listing 39: An example of generated code

Chapter 8

Parallel algorithms and evaluation

What we have implemented so far not only can be used as a backend for compiling ParAlg but also a stand-alone tool to express parallel algorithms. In this section, we will give an overview of how to use SPar for parallel algorithms, benchmarking the performance of the generated code for various computation and analyzing the design choices of the project.

8.1 Parallel algorithms

The biggest advantage of writing parallel programs in SPar is that user can express the computation similar to the sequential one without worrying about any low-level primitives for parallel computations. We will give a recommended recipe of expressing computation in SPar to follow and two concrete examples for the explanation.

8.1.1 Four steps to write parallel algorithms in SPar

Usually, divide-and-conquer algorithms are the best candidates for SPar to parallelize. The recipe is below:

1. Understand the algorithm: We recommend programmers to express the algorithms in the recursion schemes (see Section 2.2) Recursion schemes is recommended because it separates the split function, the merge function and the structure of divide-and-conquer from each others so it helps you familiar with the building blocks of the divide-and-conquer algorithms, i.e, the data structures involved, the type signature of the split function, the type signature of merge functions and their implementations.

2. Systematic parallelization: In the first iteration, we can express the split, merge functions and other necessary helper functions in terms of the SArrow by combing `arr` constructor and `Prim` constructor. Every computation wrapped by `arr` with `Prim` is consider to be sequential. We will substitute them into high-level parallel patterns provided by SPar. For example, divide-and-conquer algorithms can be

parallelized by `divConquer` (see Listing 27) helper functions. Notice that the parameter for number of ways of parallelized divide-and-conquer algorithms should be set accordingly by the number of cores of the execution machine.

3. Specific parallelization: The above step is generic and can be applied to any divide-and-conquer algorithm. In this stage, what will be done is determined by the specific implementations. Programmer should inspect the implementation of these function wrapped by `arr` with `Prim` to see whether there are any parallelism to exploit. If so, programmer should rewrite these functions by the parallel patterns provided by SPar or arrow combinators. For example, the split function of the Quickhull [27] to solve convex hull problems will use a for-loop to find the point whose distance to the line is at the maximum. This step can be expressed by the parallel map-and-reduce pattern (see Listing 26). Programmers can apply this step iteratively until all possible parallelisms are exploited or programmers think it is enough. All the sequential computation are left in the form: `arr` with `Prim`.

4. Wrap up: Before the code generation, programmer need to implement all the `Prim` functions in terms of the target language. In the scope of this project, programmer will write them in C and create a header file. The generated code will include the header file and from then on, programmers obtain the parallelized version of the algorithm that is guaranteed to be deadlock-free.

8.1.2 Example: Merge sort

We will show how to use the framework to generate parallel code by expressing merge sort on a list of integer.

```
split :: SArrow [Int] (Either (Either () Int) ([Int], [Int]))
split = arr $ Prim "split" undefined

merge :: SArrow (Either (Either () Int) ([Int], [Int])) [Int]
merge = arr $ Prim "merge" undefined

sort :: SArrow [Int] [Int]
sort = arr $ Prim "sort" undefined
```

Listing 40: The code for atomic functions

1. The first step is to understand algorithms. The merge sort is a quite famous divide-and-conquer algorithm. It splits the list into two halves, apply the algorithm to sub-lists recursively and finally merge the sub-lists by an order. From the above description, we identify three atomic functions `split`, `merge` and the base function `sort`. We need to define the type signatures for these functions to understand what data structures are involved. For the `split` function, it will take a list of integers `[Int]` as an input and output a pair of lists of integers `(Int, Int)` as an output. To deal with the case where the input list can not be split i.e when the list is empty

or a singleton list, we wrap the output pair with a either type to deal with these situations. So the output type is `Either (Either () Int) ([Int], [Int])` Accordingly, the type signature of `merge` is the reverse of the `split`. `sort` type signature will be the same as the merge sort: `[Int] -> [Int]`. Finally, we wrap them using Prim and arr. The code written for the first step is shown in the Listing 40.

```
mergesort :: Int -> SArrow [Int] [Int]
mergesort = divConq sort split merge

divConq ::
     SArrow a b
  -> SArrow a (Either c (a, a))
  -> SArrow (Either c (b, b)) b
  -> Int
  -> SArrow a b
divConq baseFunc _ _ 0 = baseFunc
divConq baseFunc alg coalg x =
  alg
    >>> (   arr Inl
        ||| (   (   (arr Fst >>>
                     divConq baseFunc alg coalg (x - 1))
                &&& (arr Snd >>>
                     divConq baseFunc alg coalg (x - 1))
                )
            >>> arr Inr
            )
        )
    >>> coalg
```

Listing 41: Construction of the algorithm using the parallel pattern and atomic functions

2. The second step is combine these atomic function in the first step in a parallel pattern we define. Since we are using our framework to parallelizing a merge sort, we will use the divide-and-conquer parallel pattern. The result of applying the pattern pattern is an expression whose type is `Int -> SArrow [Int] [Int]` where the first parameter determines the number of level of the divide-and-conquer algorithm. In this case, we will use our refined version of divided-and-conquer parallel pattern that support shortcut. The code written for the second step is shown Listing 41. For the completeness, we also include the implementation of the parallel pattern. The polymorphic parallel pattern can be used to generate non-polymorphic code.

3. The third step is optimizing the atomic function. Since split and merge are not very intensive computation. We will not modify anything for this step.

4. Up to this step, we have finished everything we need to do in the Haskell side to express computation. First of all, we will show how to generate C code from a SArrow expression using the framework. Secondly, we will talk about how to complete the implementation for the atomic functions. In the library, we have defined a function `codeGen` that takes an SArrow expression as an input and generates three files in the specified path. The first file is called func.h which contains declarations of all atomic functions. The second file is data.h which includes the definition of data structures involved in C. The third file is code.c which includes all the necessary headers like standard library and channel library and generated code. The structure is similar to what we described in Section 7.3.3. The last job to do is to complete the implementation of the functions declared in the func.h. After that, we use the generated code in whatever way we want. The structures are shown in Table 8.1.

8.2 Benchmarks

```
int main()
{
    int * tmp = randomList(1048576);
    List_int a = (List_int) {1048576, tmp};
    double start = get_time();
    proc0(a);
    double end = get_time();
    printf("%lf\n", end - start);
    return 0;
}
```

Listing 42: The main function for benchmark

We have defined a specific code generation function for benchmarking the parallel algorithms. The main difference from the normal generated code is the main function. The main function will create a random source data by the specified input size, record the execution time and output the execution time. The main file is shown in Listing 42.

For each benchmark of a divide-and-conquer SArrow, we will generate the code for a range of size and for different number of unroll representing the level of the algorithms. We will record the execution time on normal laptop as well as high performance computer.

The compiler we used is gcc (version 9.1.0) with the default optimization. The two platform we run our benchmarks on 1) Intel i5-8259u with 4 physical core 2) 32 cores high throughput computing machine provided by Imperial College London ICT services [1].

We have implemented three benchmarks to run.

[1] http://www.imperial.ac.uk/admin-services/ict/self-service/research-support/rcs/computing/high-throughput-computing/

1. **Merge sort.** It is one of the most classic divide-and-conquer algorithms. Its details are shown in the above subsection.

2. **Dot product.** It computes the inner product of two vectors whose sizes are the same and of the form 2^n.

3. **Int count.** It counts the number of occurrence of integer given a list of integer ranging from 0 to 50. `split` divides the list by halves. `count` will count the occurrence of integers and output a list of tuple where the integer is the value and the second integer is the number of occurrence of that value. `union` will union the resulting of sub-lists by summing up the occurrence.

8.2.1 Evaluation

We will demonstrate the execution time against different sizes for different levels of divide-and-conquer algorithms as well as the speedup of the parallel algorithms against the sequential algorithm. Figure 8.1 shows the result for different cases running on a 32-core machine. For each level k, we will generate a total number of 2^k threads to execute the parallel computation. The x axis indicates the size of the input where 22 means the input is a list of 2^20 integers or a pair of list of 2^20 integers. The execution time is measured in seconds and the speedup is computed by $\frac{t_{\text{sequential}}}{t_{\text{parallel}}}$.

For merge sort, the speedup increases as the size increases which is shown by the increasing graph from Figure 8.1a. It is valid for all the level due to all the lines are increasing. Different levels will have different degree of performance boost on increasing sizes. For dot product, this trend holds for up to size 26. When the size is 26, we witness a sudden decrease in speedup at various levels. The reason for this abnormal behavior is yet to be studied. As for the int count, the speedup increases against increasing sizes when the level is big enough. The degree of speedup increase at the large level is also the most obvious for int count among the three benchmarks. This can be observed by the two slope when level = 7 and level = 8. Unexpected behavior is the line when level = 6. Its speedup is nearly as small as level = 1. Also, overproduction of threads has positive impacts on the speedup. The best level is 7 which contains 128 threads. It gives a 7.5X speedup for merge sort, 8X speedup for dot product and nearly 12X speed up for int count when the size is 2^{26}. In general, greater number of threads has better speedup for greater sizes. The level that gives the best performance depends on the number of cores of the machine. The relationship is not simply one-to-one and from experiment, we recommend to overproduce the number of threads compared the number of cores. However, the performance will not be increased greatly and even slower if the level is too big (see the comparison of speedup line for level = 7 and 8).

```c
typedef enum Label {
            LEFT, RIGHT
        } Label;
typedef struct List_int {
            size_t size; int * value;
        } List_int;
typedef struct Sum_unit_int {
            Label label;
            union {
                int left; int right;
            } value;
        } Sum_unit_int;
typedef struct Prod_List_int_List_int {
            List_int fst; List_int snd;
        } Prod_List_int_List_int;
typedef struct Sum_Sum_unit_int_Prod_List_int_List_int {
            Label label;
            union {
                Sum_unit_int left; Prod_List_int_List_int right;
            } value;
        } Sum_Sum_unit_int_Prod_List_int_List_int;
```

(a) data.h

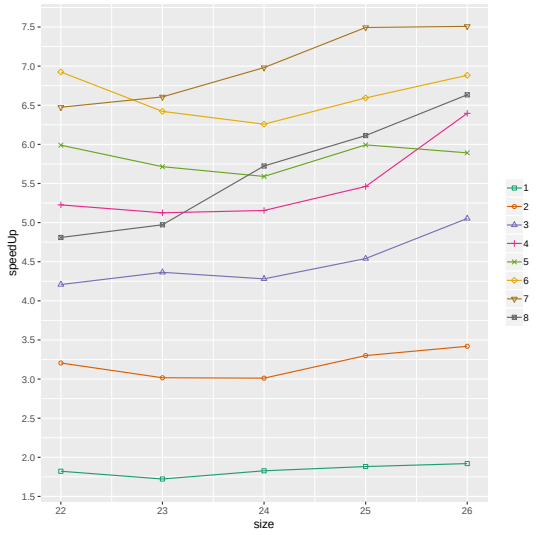
```c
#include "data.h"
List_int merge(Sum_Sum_unit_int_Prod_List_int_List_int);
Prod_List_int_List_int split(List_int);
List_int sort(List_int);
```

(b) func.h

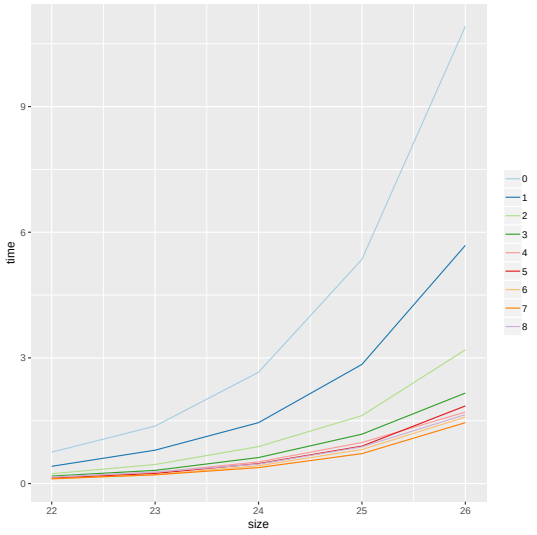
```c
#include<stdint.h>
#include<stdlib.h>
#include<chan.h>
#include<pthread.h>

#include"data.h"
#include"func.h"

List_int proc0(List_int a) {

}
```

(c) code.c

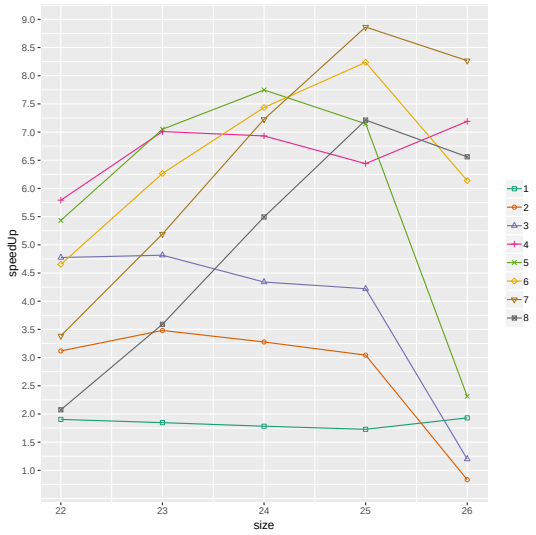
Table 8.1: The complete structure of the generated c code. Omit the implementation of code.c



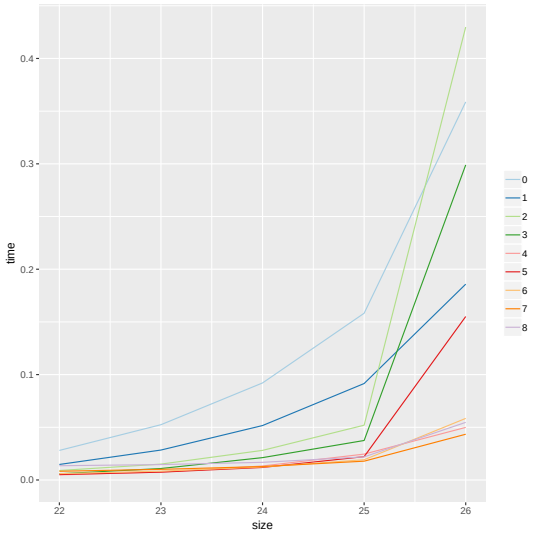
(a) Merge sort: speedup



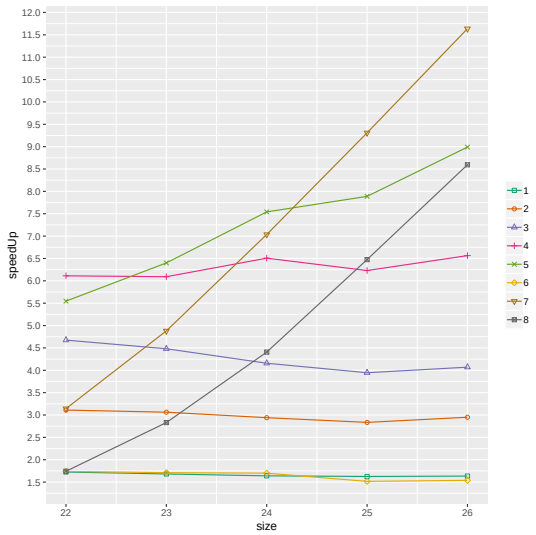
(b) Merge sort: execution time



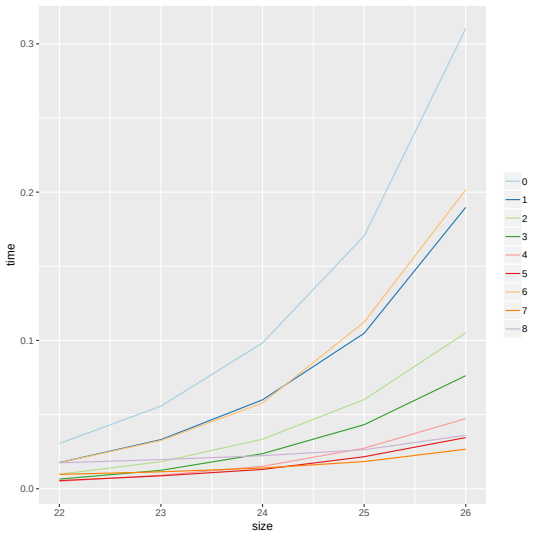
(c) Dot product: speedup



(d) Dot product: execution time



(e) Int count: speedup



(f) Int count: execution time

Figure 8.1: Benchmarks results

Chapter 9

Conclusions and future works

9.1 Conclusions

At the beginning of this project, our goal was to implement a backend for code generation for Alg parallel language using a session-typed intermediate language. The project evolved when we tried to fill the huge gap between Alg computation and SPar communication. It was then we started to design an interface as an abstraction layer on top of SPar for ease of compilation from the high-level language to it. The interface iterated from simple helper functions to arrow interface capturing general computation. Then we realized SArrow should not be limited as a compiler writer tool that hides from users, and it should be exposed to users to express computation. The application as a backend is just a side-product while the project shines as a stand-alone framework for parallel code generation.

Iterations of the project lead to the current work: A framework generates parallel C code along with local types describing the communication patterns by interpreting data-flow by communication, from Arrow based high-level expressions that can be easily formed, composed and manipulated by users with the help of the hosting function: Haskell.

In conclusion, with the recent release of AMD's latest generation of consumer CPUs featuring a processor with sixteen physical cores, Moore's law will be replaced by the addition of cores. No need to mention the area of high-performance computing where CPU with 64 cores are common. In contrast, most of the programmers have only written sequential code, and most of the algorithms about which students learn are not parallel. We hope this project can contribute its force on parallelism on CPU, encouraging more and more programmer to take advantages of modern computing architectures.

9.2 Future works

There are many interesting future works that we would like to implement. We will select some of them to introduce:

- **Optimization for benchmarks.** Because of the time constraint, there are lots of space to optimize the generated code. We should do more fine-grained profiling on the generated code. It is interesting to use tools like EzTrace to trace and visualize the execution of all the threads. More importantly, reducing the size of the generated code by eliminating common sub-expressions will be useful. At the moment, there are many code duplication for communication among different roles. The only difference is that the role of participating in the communication. The size of generated code can be reduced a lot if we can extract the common part to a function parameterized by the roles participating.

- **Integrated user experience.** As demonstrated in the evaluation chapter, users need to write the computation using the EDSL in Haskell and then generate `c` code. From then on, they need to finish the implementation of their atom functions in C. Finally, they can run the generated code with their data in C. The user experience is isolated when you have to write Haskell first and manually completed the generated code and run them in C. Instead, it will be great if we can provide an integrated user experience where the user does everything in Haskell from writing the high-level expression to collect computation results. This is possible thanks to packages like inline-c and foreign language interface in Haskell. User experience will be greatly improve if we can offer an interface in Haskell that looks like `run :: SArrow a b -> (a -> b)`. This function will take a SArrow expression and produces a function that will convert a Haskell value into C data and execute the computation in C and copy back the C output by foreign language interface to Haskell. From the user pointer of way, it can be used the same as a normal Haskell function with type `a -> b`. Forming a closed loop in Haskell would give us the best user interface and automate a large amount of boilerplate work.

- **Fine-grained control for strategies in role allocation.** We talked about how different role allocation strategies give us different parallel computation. It will be great if we parameterize the SArrow with a different strategy and adding ways to specify what strategy will be used at a different stage of the computation. This also opens the possibilities for users to implement their strategies to customize their parallel computation tasks.

- **More customizations.** Similar to customized role allocation strategies, we can even have customized representation of sequential computation since the separation of the communication EDSL and the sequential computation EDSL. This kind of work requires expert usage of Haskell TypeClass and well-designed interfaces and hence is challenging.

Bibliography

[1] M. I. Cole, "Algorithmic Skeletons: Structured Management of Parallel Computation," p. 137.

[2] K. J. Brown, A. K. Sujeeth, H. J. Lee, T. Rompf, H. Chafi, M. Odersky, and K. Olukotun, "A Heterogeneous Parallel Framework for Domain-Specific Languages," in *2011 International Conference on Parallel Architectures and Compilation Techniques*, (Galveston, TX, USA), pp. 89–100, IEEE, Oct. 2011.

[3] R. Li, H. Hu, H. Li, Y. Wu, and J. Yang, "MapReduce Parallel Programming Model: A State-of-the-Art Survey," *International Journal of Parallel Programming*, vol. 44, pp. 832–866, Aug. 2016.

[4] "Message Passing Interface," *Wikipedia*, Oct. 2018. Page Version ID: 863149040.

[5] M. Coppo, M. Dezani-Ciancaglini, L. Padovani, and N. Yoshida, "A Gentle Introduction to Multiparty Asynchronous Session Types," in *Formal Methods for Multicore Programming* (M. Bernardo and E. B. Johnsen, eds.), vol. 9104, pp. 146–178, Cham: Springer International Publishing, 2015.

[6] J. Hughes, "Generalising monads to arrows," *Science of Computer Programming*, vol. 37, pp. 67–111, May 2000.

[7] "Haskell/Understanding arrows - Wikibooks, open books for an open world." https://en.wikibooks.org/wiki/Haskell/Understanding_arrows.

[8] "Tacit programming," *Wikipedia*, Jan. 2019. Page Version ID: 879102751.

[9] M. Braun, O. Lobachev, and P. Trinder, "Arrows for Parallel Computation," *arXiv:1801.02216 [cs]*, Jan. 2018.

[10] C. Elliott, "Generic functional parallel algorithms: Scan and FFT," *Proceedings of the ACM on Programming Languages*, vol. 1, pp. 1–25, Aug. 2017.

[11] "Algebraic Multiparty Protocol Programming," p. 37.

[12] R. Milner, J. Parrow, and D. Walker, "A calculus of mobile processes, I," *Information and Computation*, vol. 100, pp. 1–40, Sept. 1992.

[13] N. Ng, J. G. F. Coutinho, and N. Yoshida, "Safe MPI Code Generation based on Session Types," p. 22.

[14] D. Orchard and N. Yoshida, "Session Types with Linearity in Haskell," p. 24.

[15] K. Claessen, *Functional Pearls: A Poor Man's Concurrency Monad*. 1999.

[16] "Control.Monad.Cont." http://hackage.haskell.org/package/mtl-2.2.2/docs/Control-Monad-Cont.html.

[17] S. Marlow, R. Newton, and S. P. Jones, "A Monad for Deterministic Parallelism," p. 12.

[18] C. contributors, "Cats: FreeMonads." http://typelevel.org/cats/.

[19] J. Svenningsson and E. Axelsson, "Combining deep and shallow embedding of domain-specific languages," *Computer Languages, Systems & Structures*, vol. 44, pp. 143–165, Dec. 2015.

[20] "Data.Typeable." http://hackage.haskell.org/package/base-4.12.0.0/docs/Data-Typeable.html#t:Typeable.

[21] M. D. McCool, A. D. Robison, and J. Reinders, *Structured Parallel Programing: Patterns for Efficient Computation*. Amsterdam: Elsevier, Morgan Kaufmann, 2012.

[22] R. Atkey, "What is a Categorical Model of Arrows?," *Electronic Notes in Theoretical Computer Science*, vol. 229, pp. 19–37, Mar. 2011.

[23] T. L. McDonell, M. M. T. Chakravarty, V. Grover, and R. R. Newton, "Type-safe Runtime Code Generation: Accelerate to LLVM," p. 12.

[24] "Reified type - HaskellWiki." https://wiki.haskell.org/Reified_type.

[25] "Language-c: Analysis and generation of C code." //hackage.haskell.org/package/language-c.

[26] T. Treat, "Pure C implementation of Go channels. Contribute to tylertreat/chan development by creating an account on GitHub," May 2019.

[27] "Quickhull," *Wikipedia*, Jan. 2019. Page Version ID: 879757605.

Appendix A

Examples of generated code

A.1 Merge sort