# Applied Deep Learning

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## Learning objectives of today

**Goals:** Creating neural networks – from logistic regression to feed-forward networks

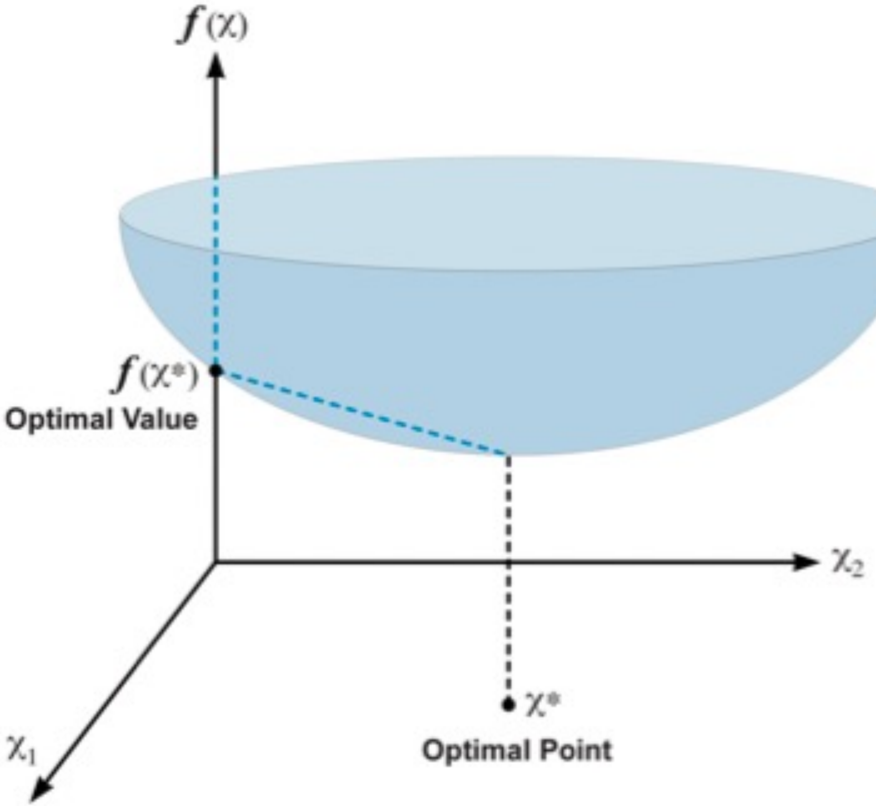
- Use what we have learned about linear algebra and calculus to create a logistic regression algorithm from scratch
- Understand how what we learned generalizes to neural networks in general

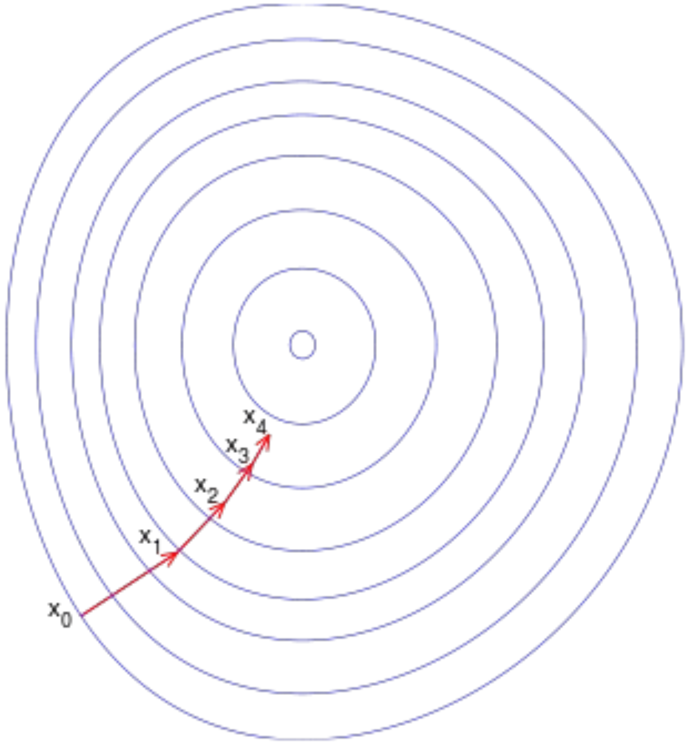
**How will we do this?**

- We discuss the implementation of a logistic regression algorithm with numpy only
- Next, we visualize more general neural networks with the TensorFlow playground, before defining the concepts relevant for running our own networks
- In the tutorial, we will implement more complex networks

# Recap – logistic regression

# Gradient descent – the idea

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# What do we actually do when training a logistic regression model?

- We are given values $(\boldsymbol{x}^{(i)}, y^{(i)})$, where $\boldsymbol{x}^{(i)} \in \mathbb{R}^m$ and $y^{(i)} \in \{0,1\}$
- Our prediction $\hat{y}^{(i)}$ should reflect the probability that $y^{(i)} = 1$: $\hat{y}^{(i)} = P\left(y^{(i)} = 1 \middle| \boldsymbol{x}^{(i)}\right)$

- We model this probability, using the sigmoid function:

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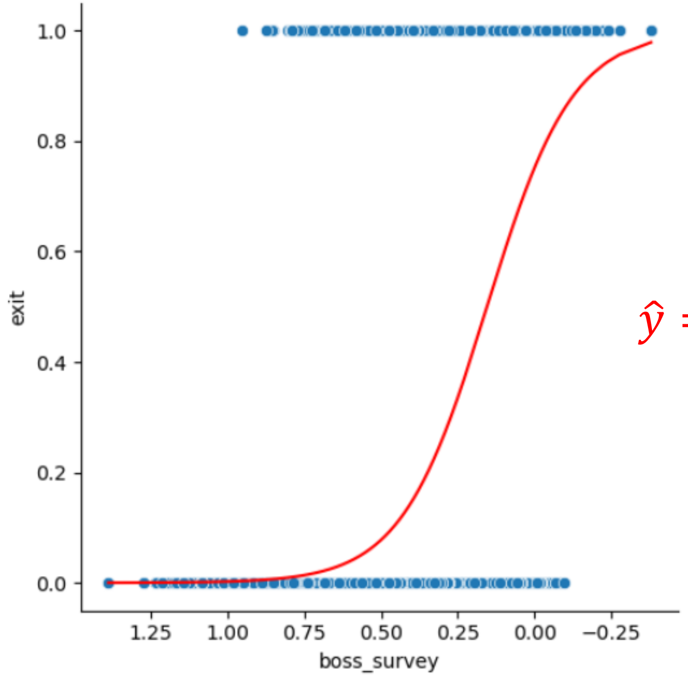


$$\hat{y} = \frac{1}{1 + e^{-(\boldsymbol{x} \cdot \boldsymbol{w} + \boldsymbol{b})}}$$

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# The optimization part

- Remember that $\boldsymbol{w} \in \mathbb{R}^m$ and $b \in \mathbb{R}$
- To get to the "right" model, we optimize our parameters $\boldsymbol{w}, b$ so that the $\hat{y}^{(i)}$s are "as close as possible" to the $y^i$s
- What we do is to minimize the "cost-function" $J(\boldsymbol{w}, b)$, where $\hat{y}^{(i)} = \frac{1}{1+e^{-\left(\boldsymbol{x}^{(i)}\boldsymbol{w}+\boldsymbol{b}\right)}}$ :

$$J(\boldsymbol{w}, b) = -\frac{1}{n}\sum_{i=1}^{n}\left[y^{(i)} \ln \hat{y}^{(i)} + \left(1 - y^{(i)}\right) \ln\left(1 - \hat{y}^{(i)}\right)\right]$$

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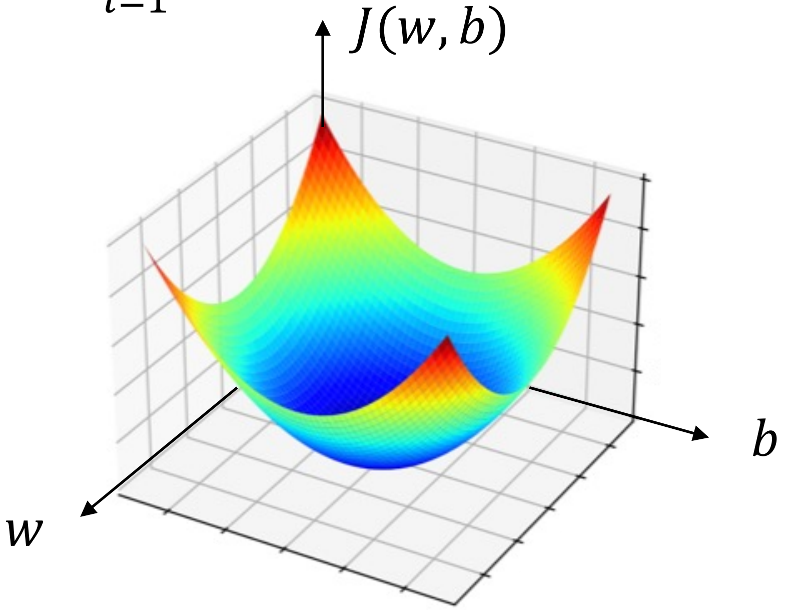
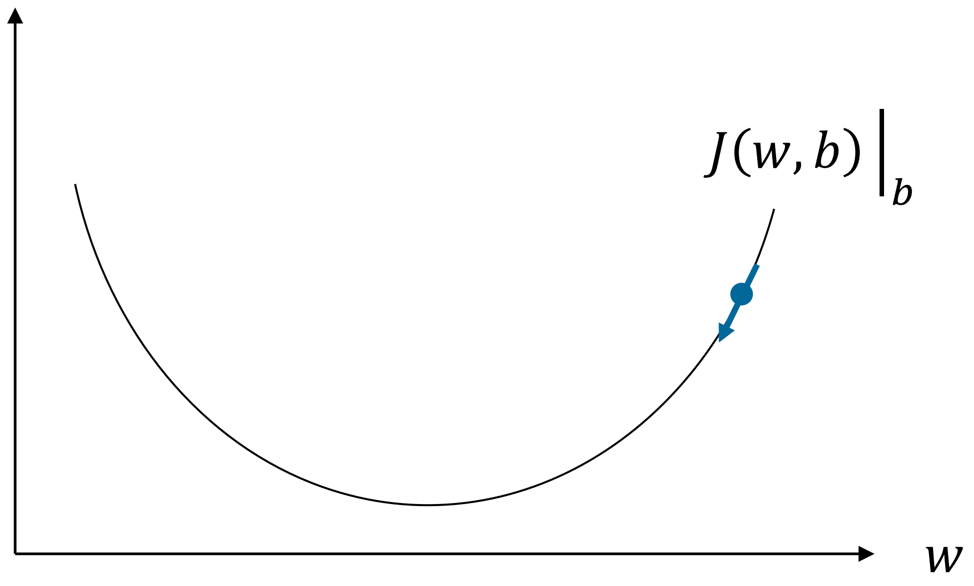


# Solving the optimization problem through gradient descent



# Our first optimization algorithm

1. Decide a “learning rate” $\alpha$
2. Start with some $\boldsymbol{w}$ and $b$ and compute $J(\boldsymbol{w}, b)$
3. Until $J$ “doesn’t change” anymore:
   - Let $w_1 := w_1 - \alpha \frac{\partial J(\boldsymbol{w},b)}{\partial w_1}$
   - Let $w_2 := w_2 - \alpha \frac{\partial J(\boldsymbol{w},b)}{\partial w_2}$
   - …
   - Let $w_m := w_m - \alpha \frac{\partial J(\boldsymbol{w},b)}{\partial w_m}$
   - Let $b := b - \alpha \frac{\partial J(\boldsymbol{w},b)}{\partial b}$
   - Recompute $J(\boldsymbol{w}, b)$
4. Enjoy the fruits of your labor: you have fit a logistic regression model manually!

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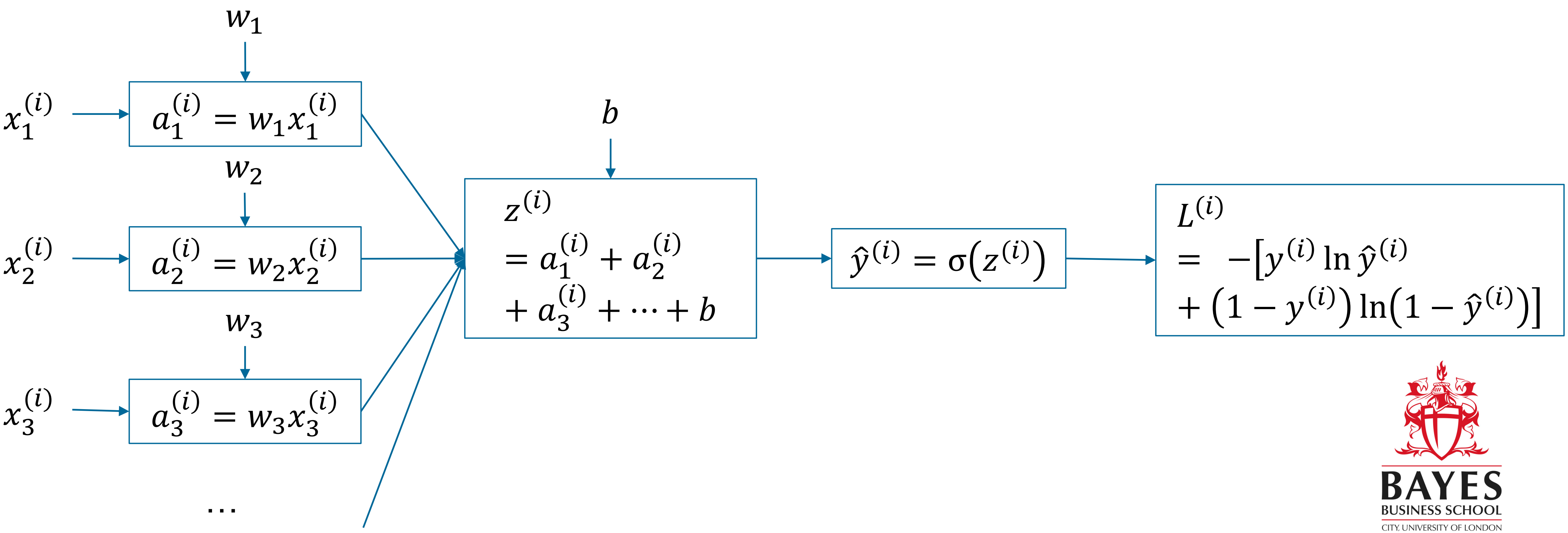
# Wait a second, how do we find all those derivatives?

- We can use again the computation graph!
- Recall that $\hat{y}^{(i)} = \frac{1}{1+e^{-\left(\boldsymbol{x}^{(i)}\boldsymbol{w}+\boldsymbol{b}\right)}} = \sigma\left(\boldsymbol{x}^{(i)}\boldsymbol{w} + b\right)$

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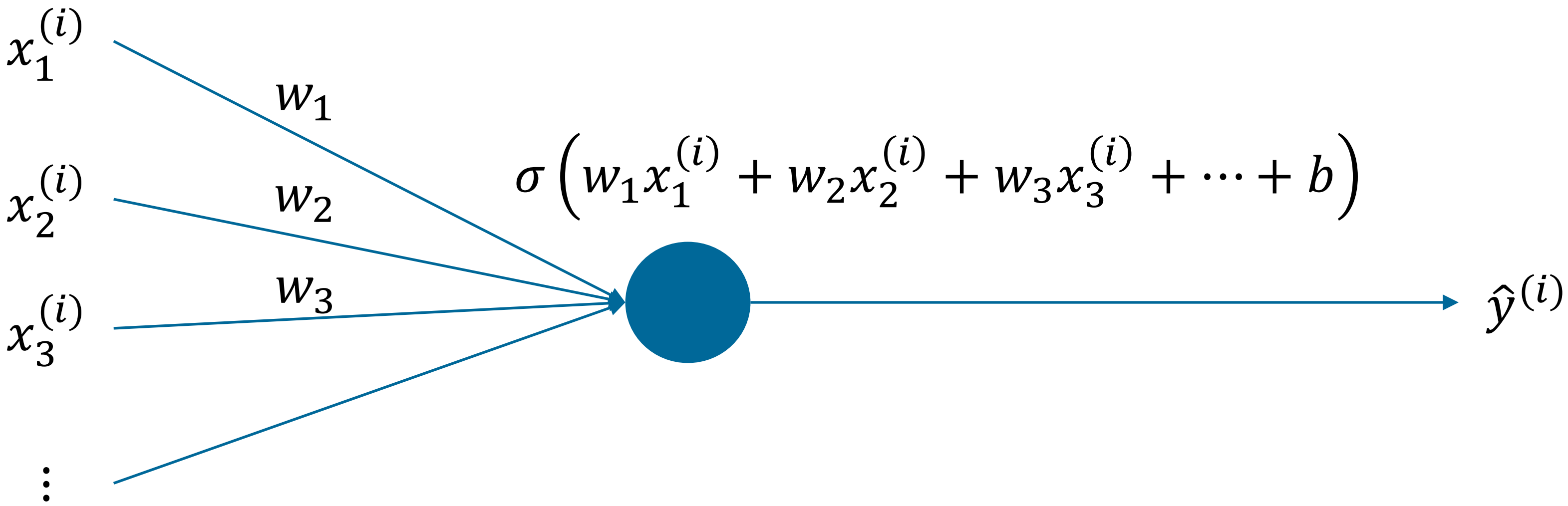
# As the same parameters influence all examples, we have to consider one final step

- Recall that $J(\boldsymbol{w}, b) = -\frac{1}{n}\sum_{i=1}^{n}\left[y^{(i)} \ln \hat{y}^{(i)} + \left(1 - y^{(i)}\right) \ln\left(1 - \hat{y}^{(i)}\right)\right] = \frac{1}{n}\sum_{i=1}^{n} L^{(i)}$
- We have that $\frac{\partial J(\boldsymbol{w},b)}{\partial w_j} = \frac{1}{n}\sum_{i=1}^{n} \frac{\partial L^{(i)}}{\partial w_j}$

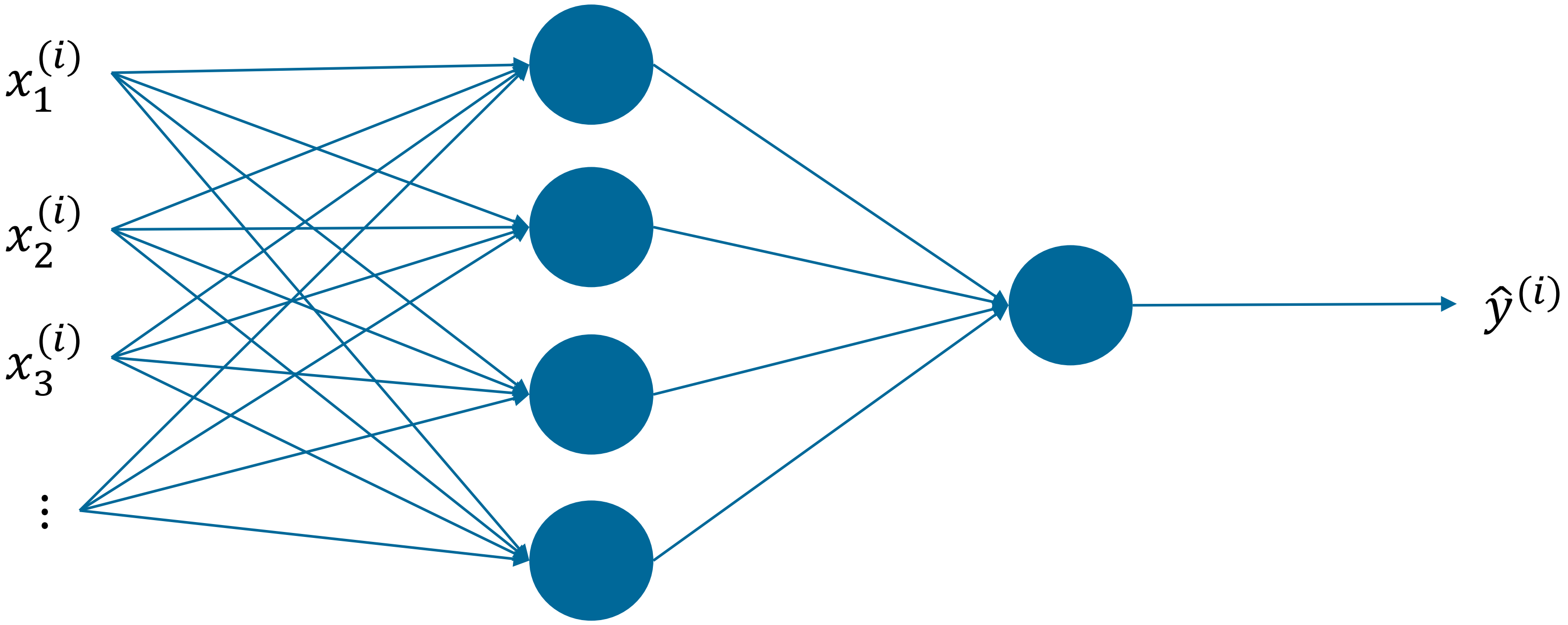
# We can now implement a logistic regression

# From logistic regression to neural network

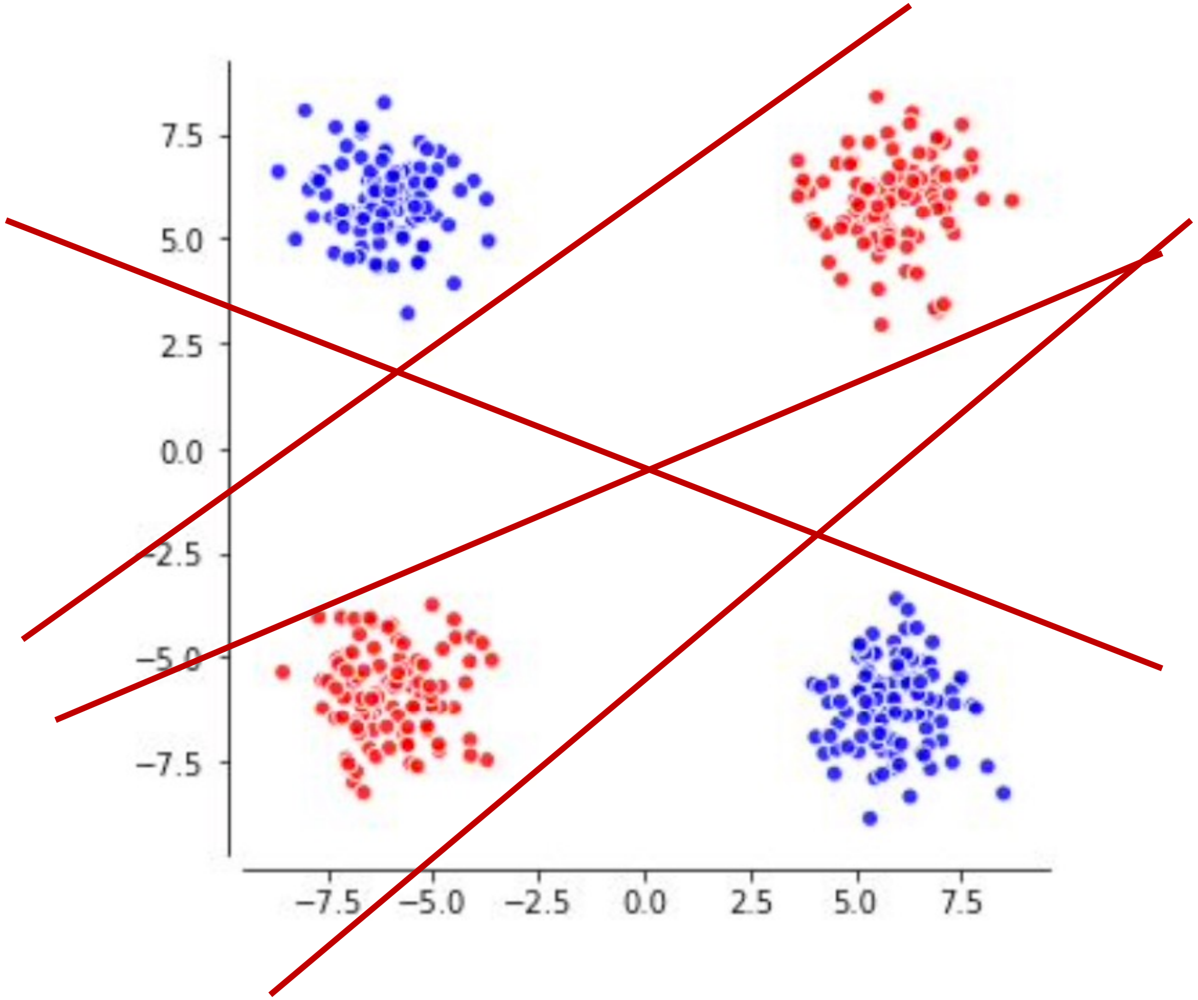
# Schema of a logistic regression



# Putting multiple neurons together



# What neurons learn




Source: Czarnecki

## Training neural networks visually

Open https://playground.tensorflow.org/

1. A simple case of binary classification:
   - Change to the pattern on the lower left
   - Set the level of “Noise” to 50
   - Set “Ratio of training to test data” to 50%
   - Set up the neural network: 1 hidden layer, 1 neuron, then press play
   - Answer the following questions:
     - Did the training eventually find a model that seems to capture the pattern in the data?
     - How would you describe the pattern the model captured?
     - Record the “Training loss” and “Test loss”
   - How do your answers change when you select the pattern at the top right? What about setting the noise to 0?

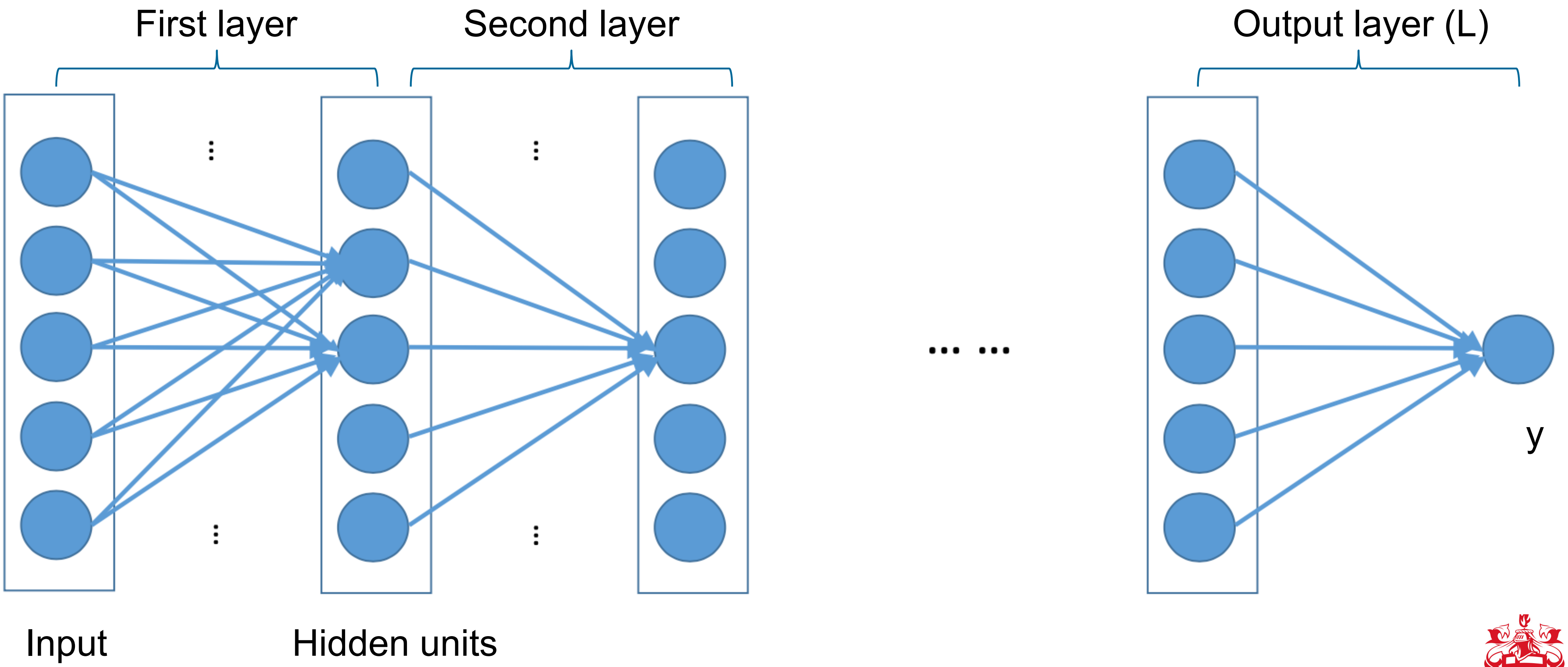
# Training neural networks visually

2. A shallow neural network:
   - Stick with the pattern at the top right, a noise of 0 and a ratio of 50%
   - Now use 3 neurons for your hidden layer
   - Answer the three questions from before:
     - Did the training eventually find a model that seems to capture the pattern in the data?
     - How would you describe the pattern the model captured?
     - Record the “Training loss” and “Test loss”
   - How do your answers change when you use 6 neurons instead?

3. A deep neural network:
   - Use a second hidden layer, with 3 neurons each (and the other setups from 2.)
   - How do your answers change now?

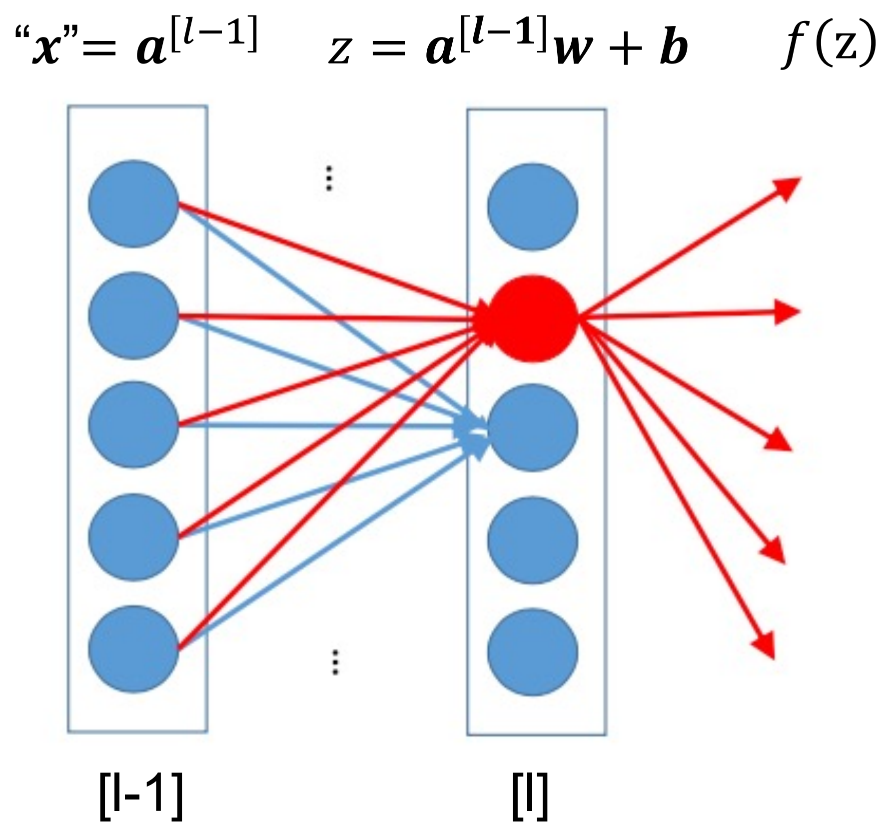
# Key components of a neural network

# Components




Source: Liang

## Hidden layers

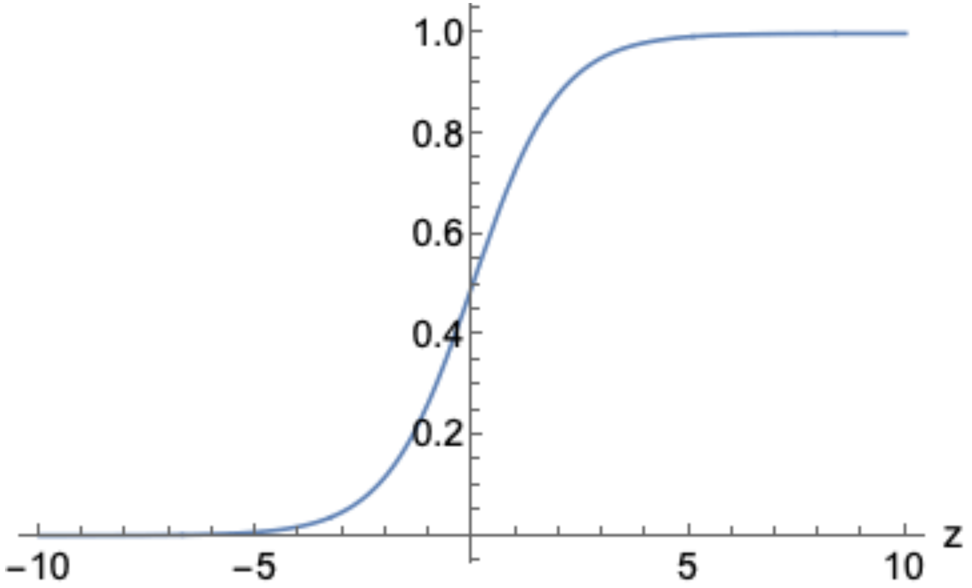



- $f$ is what we call an "activation function"
- There are many activation functions, and new ones are invented all the time
- Many of these functions do just fine, or slightly better than existing ones

Source: Liang

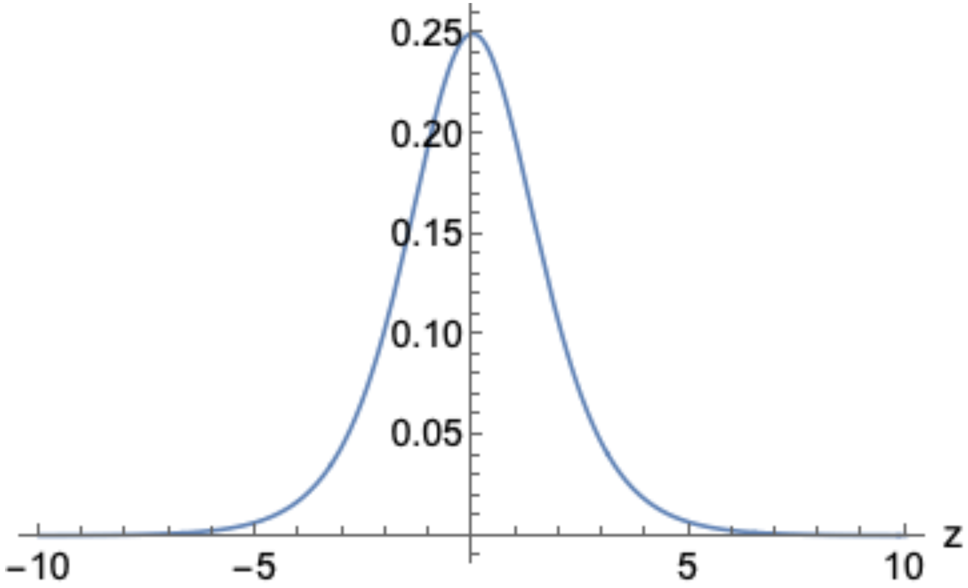
# Typical activation functions: logistic (sigmoid) function

Logistic (sigmoid) function




$$f(z) = \sigma(z) = \frac{1}{1 + e^{-z}}$$

Derivative




$$f'(z) = \sigma(z)\big(1 - \sigma(z)\big)$$

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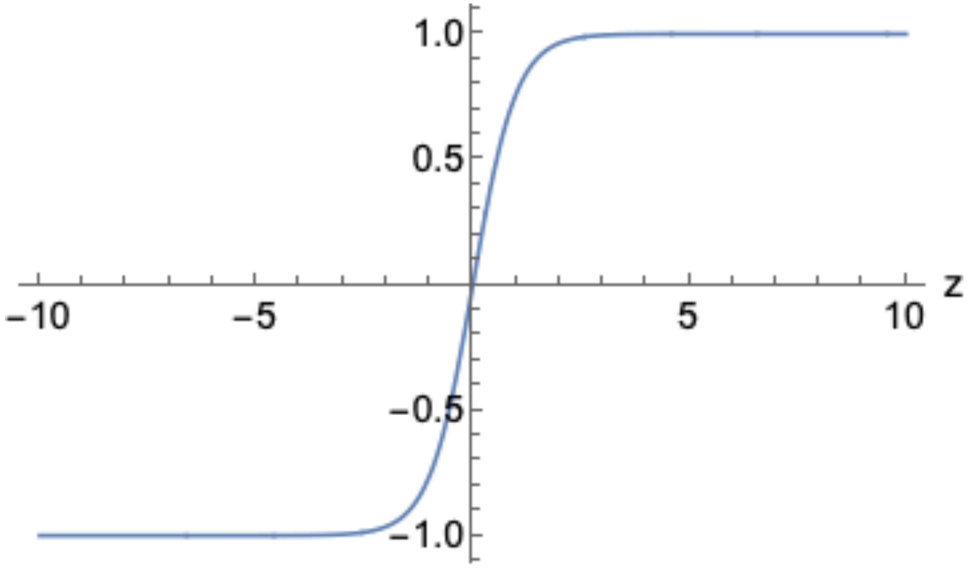
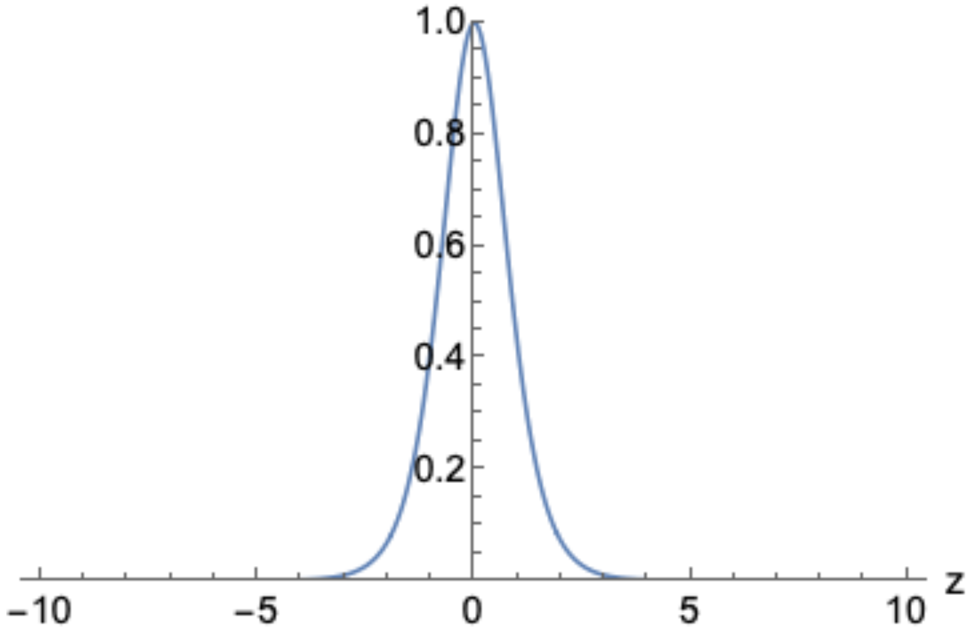
# Typical activation functions: hyperbolic tangent

Hyperbolic tangent




$$f(z) = \tanh(z) = \frac{e^{2z} - 1}{e^{2z} + 1}$$

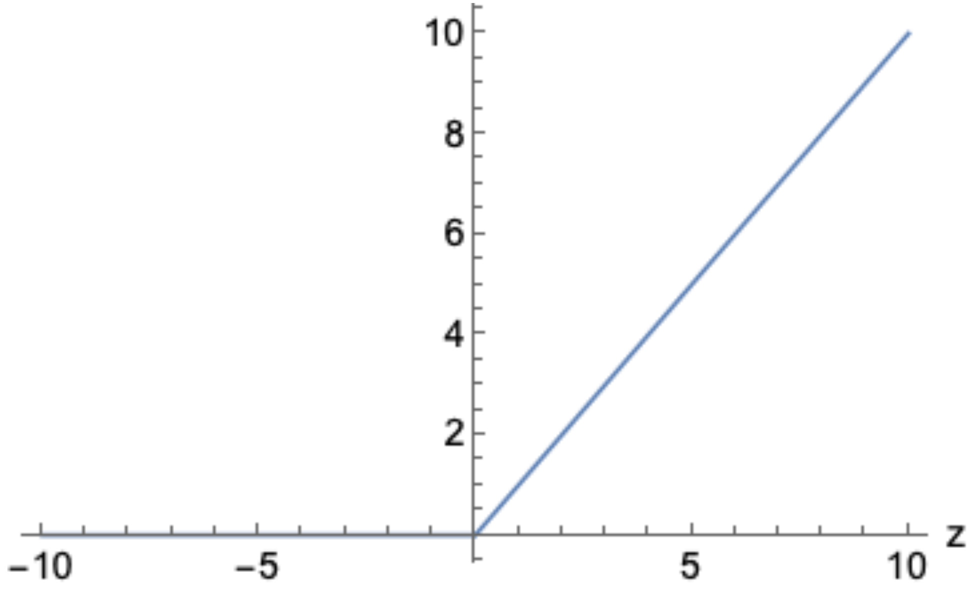
Derivative




$$f'(z) = \text{sech}(z)^2$$

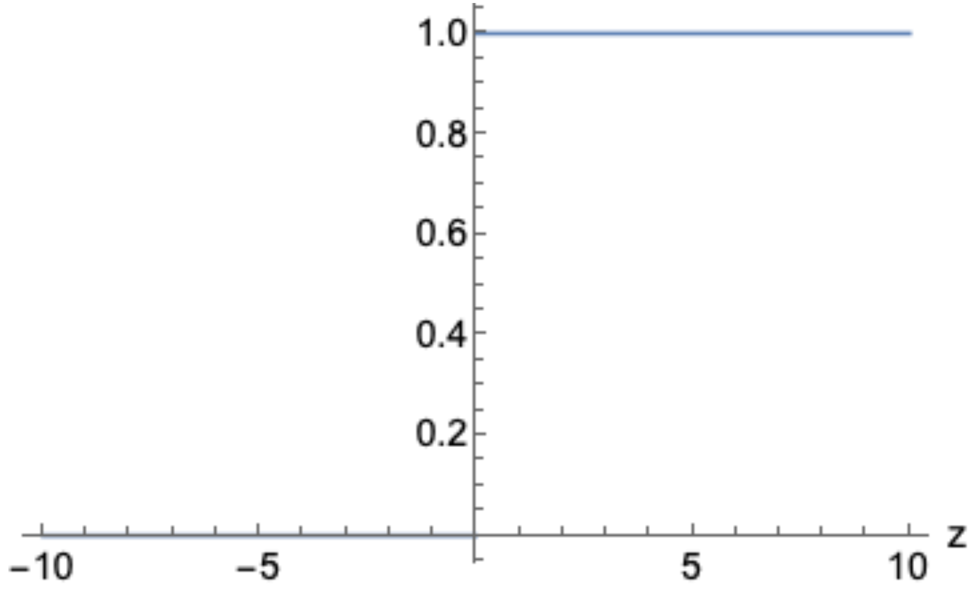
# Typical activation functions: Rectified Linear Unit

Rectified Linear Unit (ReLU)




$$f(z) = \max\{0, z\}$$

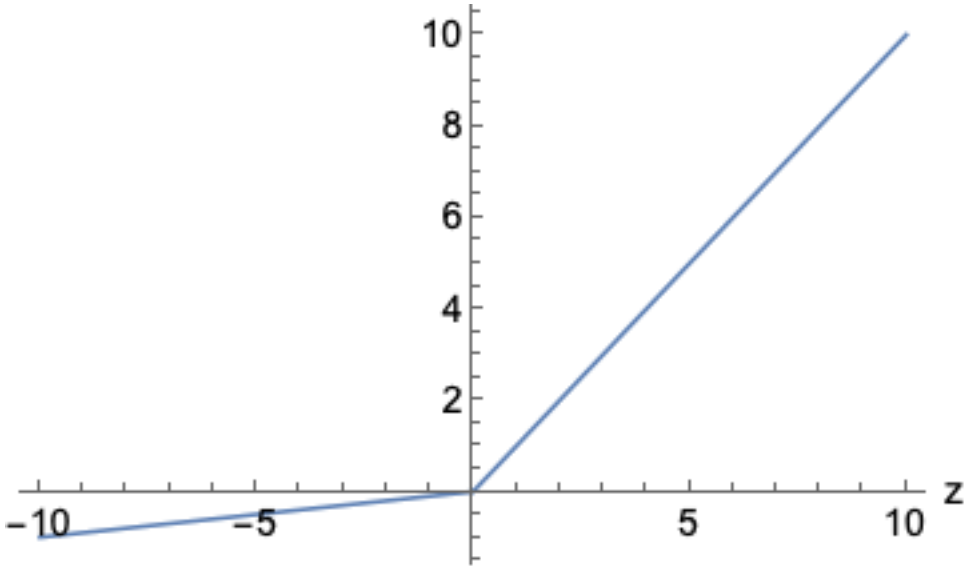
“Derivative”




$$f'(z) = \begin{cases} 0, if\ z < 0 \\ 1, if\ z > 0 \end{cases}$$

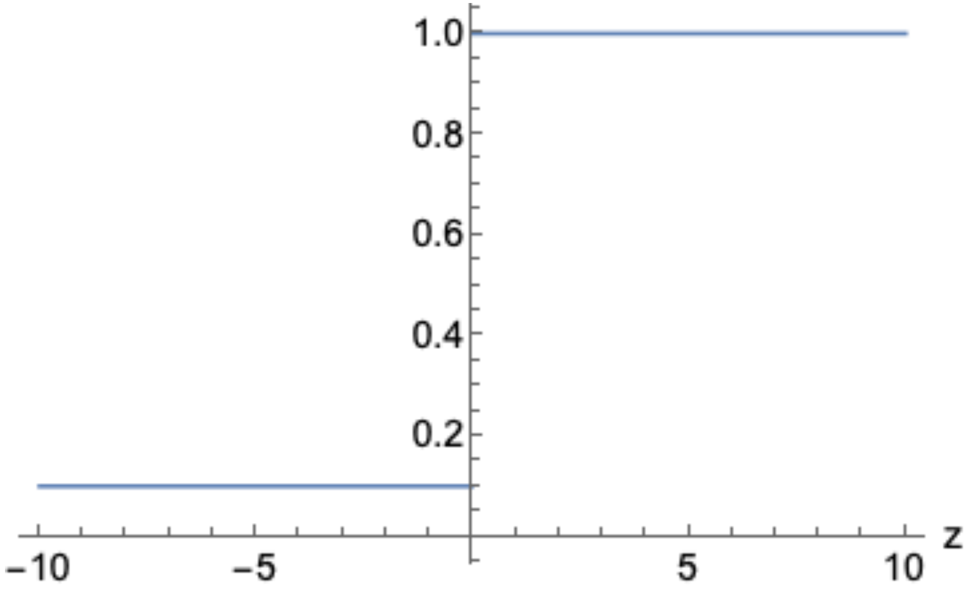
# Typical activation functions: Leaky ReLU

Leaky ReLU




$$f(z) = \max\{0.1z, z\}$$

“Derivative”




$$f'(z) = \begin{cases} 0.1, if\ z < 0 \\ 1, if\ z > 0 \end{cases}$$

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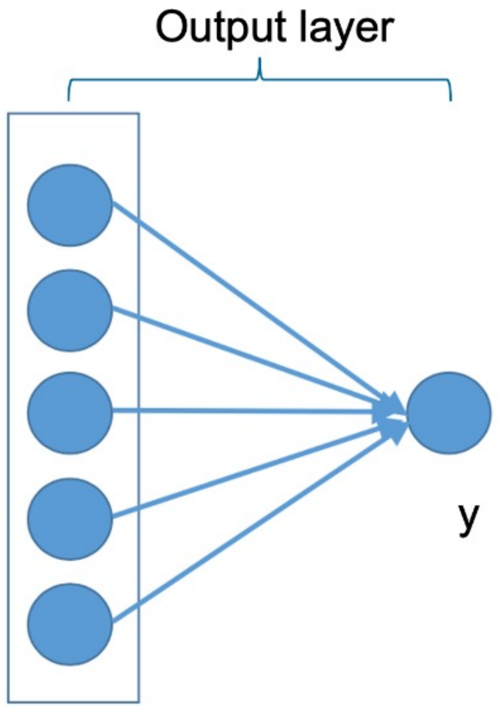
# Binary classification

- Input: $\boldsymbol{a}^{[L-1](i)}$
- As usual, we make a linear transformation:
  $z^{[L](i)} = \boldsymbol{a}^{[L-1](i)}\boldsymbol{w}^{[L]} + b^{[L]}$
- We then use the logistic sigmoid function
  $\hat{y}^{(i)} = f\left(z^{[L](i)}\right) = \sigma(z^{[L](i)}) = \frac{1}{1+e^{-z^{[L](i)}}}$
- We can interpret the output as the probability of $y^{(i)} = 1$




Source: Liang

# Multi-class classification

- We again make a linear transformation with a matrix of weights: $\boldsymbol{z}^{[L](i)} = \boldsymbol{a}^{[L-1](i)}\boldsymbol{W}^{[L]} + \boldsymbol{b}^{[L]}$

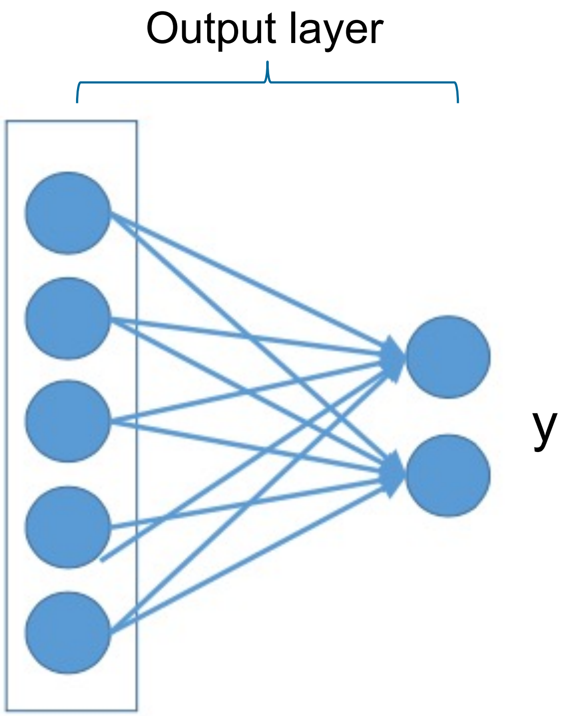
- Note that $\boldsymbol{z}^{[L](i)} = \begin{pmatrix} z_1^{[L](i)} & z_2^{[L](i)} & \cdots & z_K^{[L](i)} \end{pmatrix}$

- We then use the softmax function on each of the outputs:

$$\hat{y}_k^{(i)} = f\left(\boldsymbol{z}^{[L](i)}\right) = \frac{e^{-z_k^{[L](i)}}}{\sum_{k=1}^{K} e^{-z_k^{[L](i)}}}$$

- This implies that $\hat{y}_k^{(i)} \in (0,1)$ and $\sum_{k=1}^{K} \hat{y}_k^{(i)} = 1$

- Hence, we can interpret $\hat{y}_k^{(i)}$ as the probability that $y^{(i)} = k$ ("belongs to class $k$")




Source: Liang

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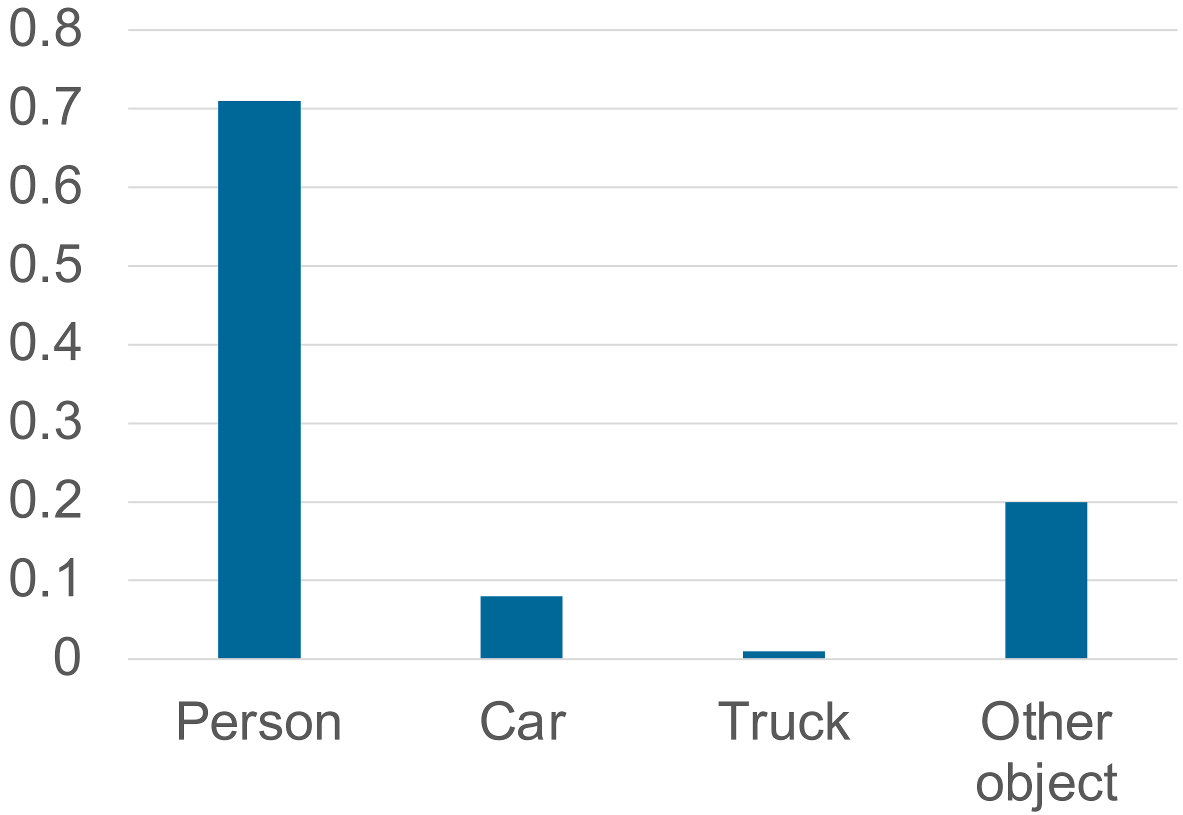
# Softmax output

- E.g., when performing object recognition, we might represent our prediction $\hat{\boldsymbol{y}}^{(i)}$ as



# Cost functions

- Recall from logistic regression:

$$J(\boldsymbol{w}, b) = \frac{1}{n}\sum_{i=1}^{n} L^{(i)} = -\frac{1}{n}\sum_{i=1}^{n}\left[y^{(i)} \ln \hat{y}^{(i)} + \left(1 - y^{(i)}\right) \ln\left(1 - \hat{y}^{(i)}\right)\right]$$

- Generally, to learn parameters $\boldsymbol{\theta}$, we define the cross-entropy

$$J(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n} L^{(i)} = -\frac{1}{n}\sum_{i=1}^{n} \ln p_{\boldsymbol{\theta}}\left(y^{(i)} \middle| \boldsymbol{x}^{(i)}\right)$$

- Also known as “maximum likelihood estimator”

- Mean square error tends to perform poorly, especially when we have activation functions with $e^z$

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# Learning with gradient descent

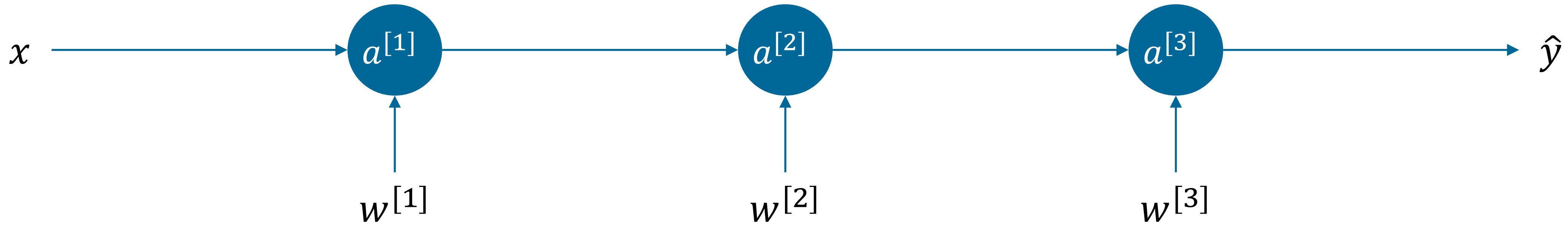
# Generalizing our optimization algorithm

1. Decide a “learning rate” $\alpha$
2. Start with some parameters $\boldsymbol{\theta}$ and compute $J(\boldsymbol{\theta})$ (forward propagation)
3. Until $J$ “doesn’t change” anymore:
   - Let $\boldsymbol{\theta} := \boldsymbol{\theta} - \alpha \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta})$ (back-propagation)
   - Recompute $J(\boldsymbol{\theta})$ (forward propagation)

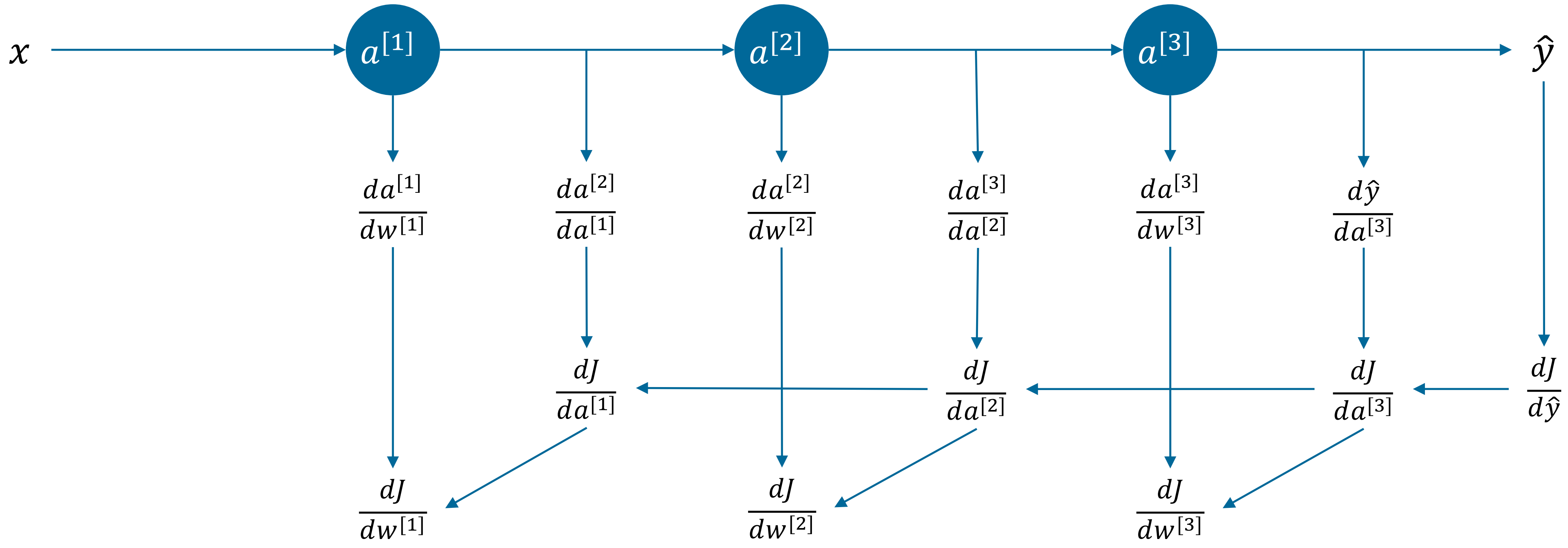
# Initializing parameters

- Initialize weights to small random values ( e.g., *np.random.randn(**shape of W**) * 0.01* )
- Bias terms can be initialized randomly, but can also just be initialized to zero ( e.g., *np.zeros(**shape of b**)* )

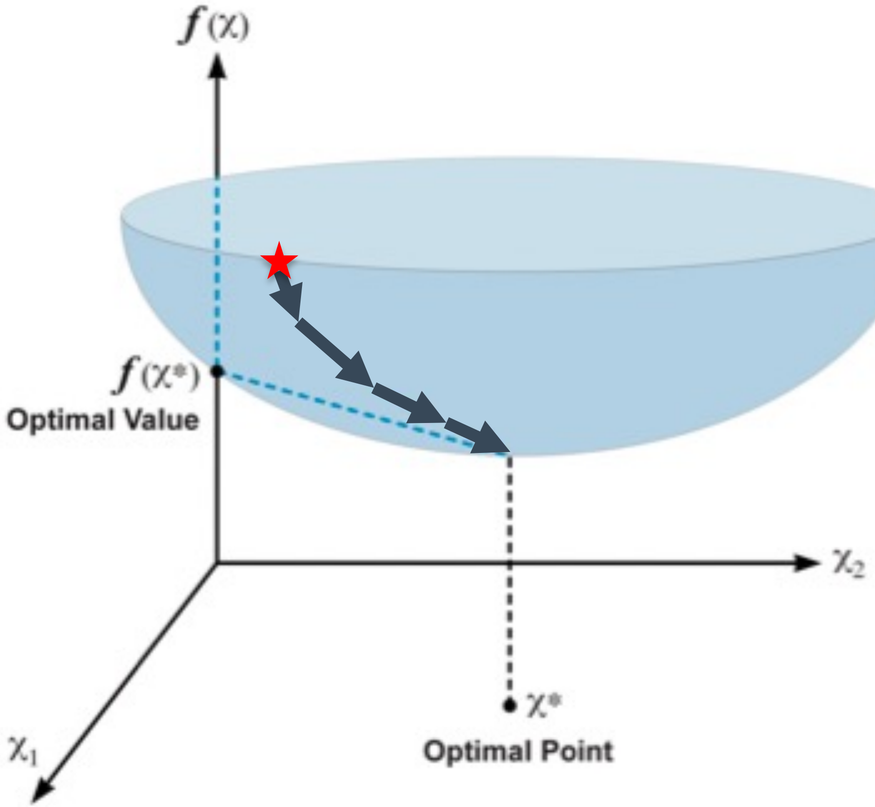
# Step 1: Forward propagation through the computational graph

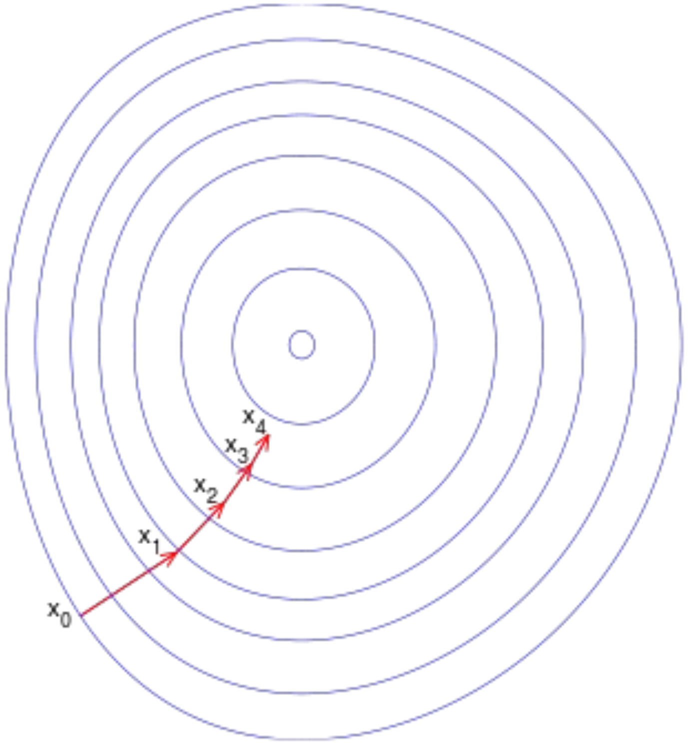


# Step 2: Back-propagation through the computational graph



# Gradient descent

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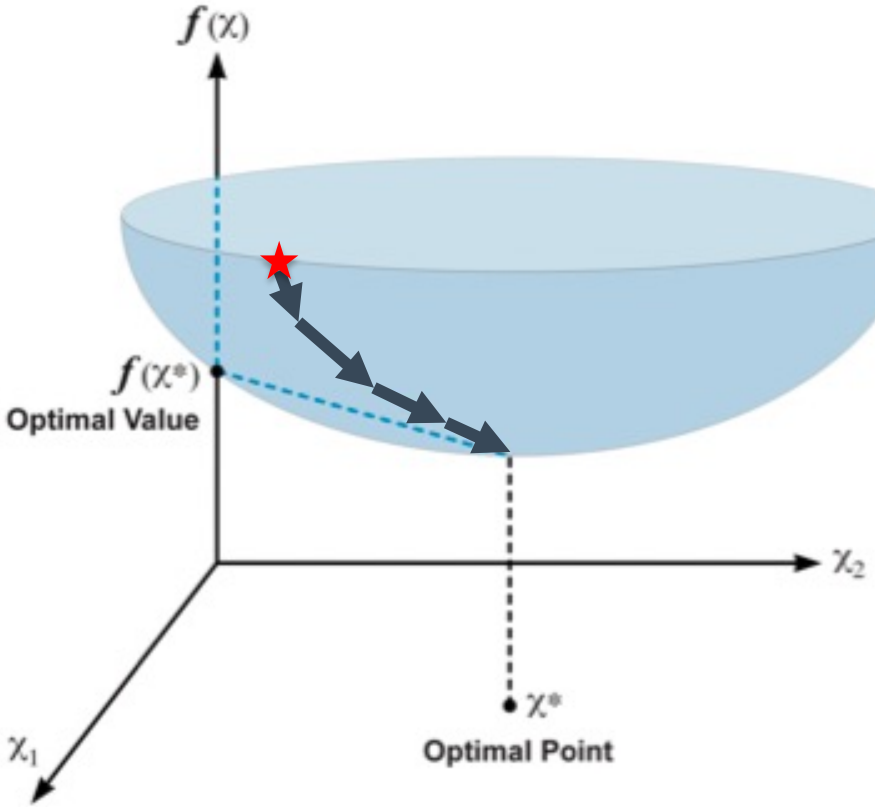
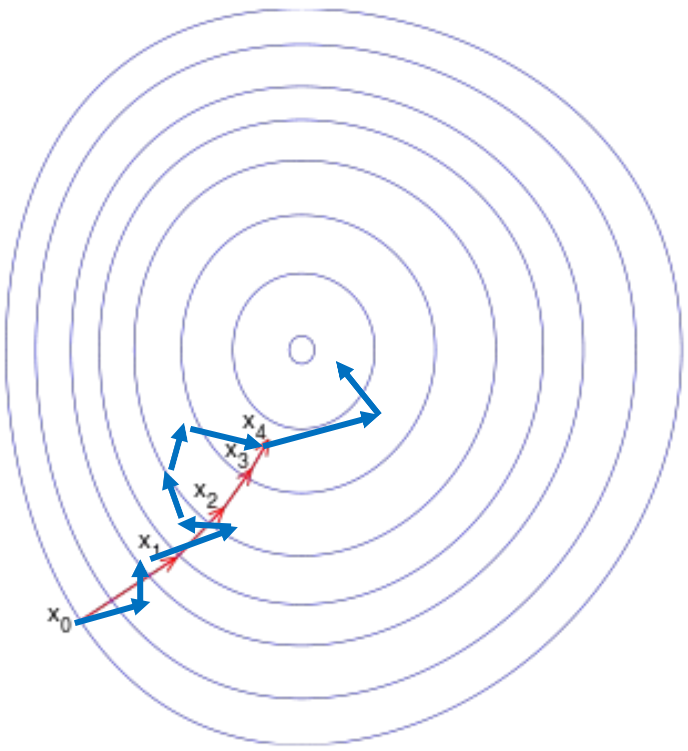
## Stochastic gradient descent: computing the gradient only for one observation




→ No matter which approach, when the function is convex, we will find a global minimum

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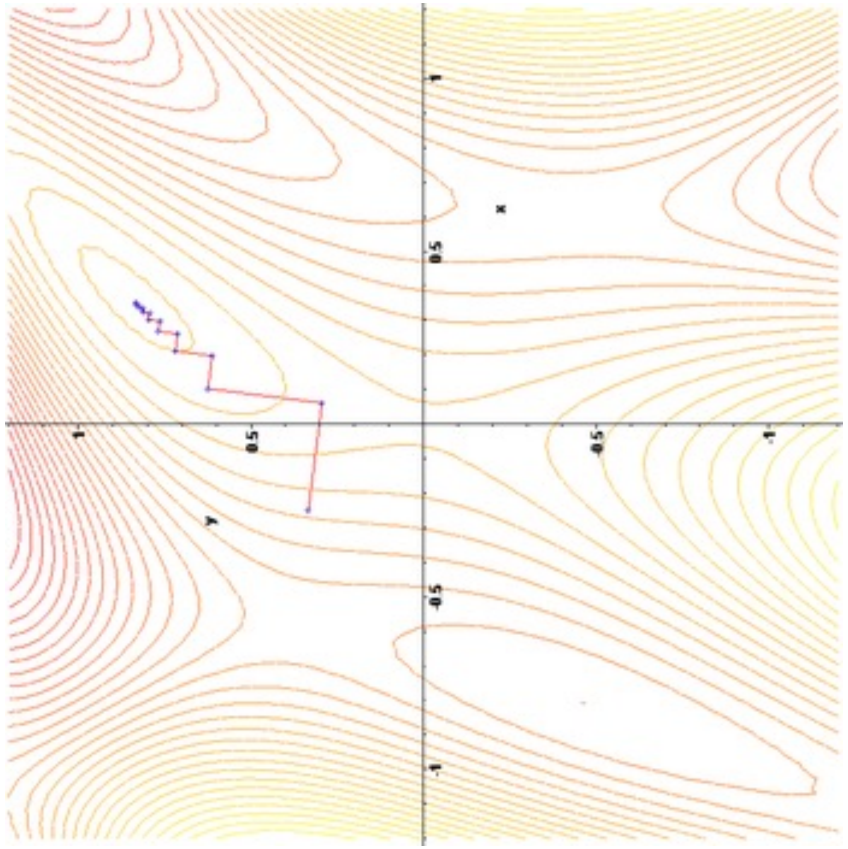
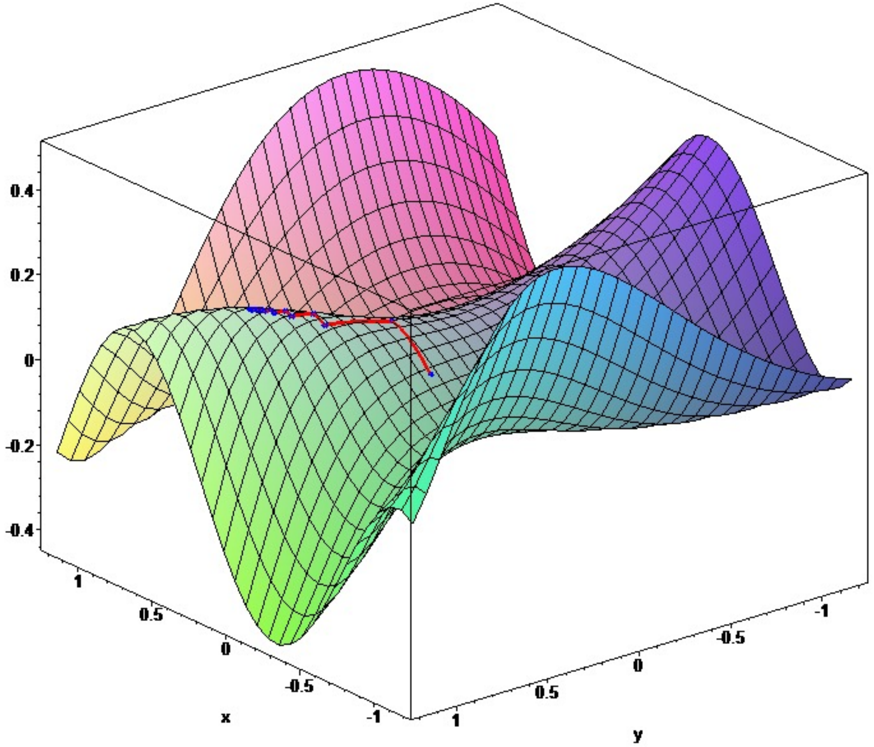
# Pathological non-convex cases




(gradient *ascend*)

Source: Wikipedia

See you next week!

# Sources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