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Forecasting Value at Risk with Historical and Filtered Historical Simulation Methods

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Abstract

The dissatisfaction with the previous parametric VaR models in estimating the market values during past few years has put their reliability in question. As a substitute, non-parametric and semi-parametric techniques were created, which are the subjects of this thesis. We study the Historical Simulation and Filtered Historical Simulation as two powerful alternatives to primary models in VaR measurement. In addition, we apply these methods to ten years data of the OMX index, to show how well they work.

Acknowledgements

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Contents

1 Introduction

Value at Risk (VaR) as a branch of risk management has been at the centre of attention of financial managers during past few years, especially after the financial crises in 90's. And now, after the market failure in 2008, the demand for a precise risk measurement is even higher than before. Risk managers try to review the previous methods, as they think one of the most important causes of the recent crisis was mismanagement of risk.

In addition to VaR's fundamental application which is measuring the risk, it also has other usages related to risk, such as controlling and managing it. VaR as a widespread method is applicable in any kind of institutions which are somehow involved with financial risk, like financial institutions, regulators, nonfinancial corporations or asset managers.

There are different approaches to VaR models for estimating the probable losses of a portfolio, which differ in calculating the density function of those losses.

The primary VaR methods were based on parametric approaches and some imposed assumptions, which in real cases did not work. One of the most important assumptions could be mentioned as the normal distribution of the density function of the daily returns. Empirical evidence shows the predicted loss or profit by this distribution underestimates the ones in real world.

So a non-parametric method, based on historical returns of market, called historical simulation (HS) has been introduced as a substitute. But some of the disadvantages of this method (especially its inability to model the most recent volatility of market) make it inefficient.

Therefore Barone-Adesi et al, introduced a number of effective refinements of this method, by mixing it with some parametric techniques i.e. GARCH time series models (as this kind of models are able to reveal volatility clusters), which leads to a new method called filtered historical simulation (FHS).

Investigating how well each of these methods (HS and FHS) works in VaR measurement field is the main purpose of this thesis.

In this thesis, which is based on paper [4], section 2 is allocated to the explanation of the VaR. In section 3 we will explain concepts of HS and FHS as a new generation of VaR measurement methods. In section 4 we will go through the theory behind them, in section 5 we will examine their application for measuring VaR in past few years OMX [1] index daily prices. Finally section 6 is devoted to conclusions.

2 Value at Risk

This section contains interpretation of Value at Risk (VaR), its formula and an introduction of different approaches related to its measurement.

2.1 'Risk' Definition from the Financial Point of View

When we are confronted with the word "Risk", the first definition that comes to our mind is 'loss disaster', but the financial theory take a more comprehensive look at this word. Although there is not any unique definition for this word in finance, we could mention risk as a possibility of losses due to unexpected outcomes caused by the financial market movements. And the probability of the loss occurrence more than expected amount, in a specific time period is the VaR measurement.

The standard deviation of the unexpected outcomes (σ) which is called volatility, is the most common risk measurement tool.

There are four types of financial risks:
- Interest rate risk
- Exchange rate risk
- Equity risk
- Commodity risk

Volatility changes do not have any trend. There will be a higher probability to increase or decrease in value for a more volatile instrument. Increased volatility could occur with any positive or negative unexpected changes in price. The volatility of financial markets is a source of risk, which should be controlled as precise as possible.

When we encounter a volatility diagram and we see a lot of fluctuation over time, an important question arises i.e. whether the risk is unstable or these fluctuations related to our estimation method and they only reflects the "noise" in data.

[1] The OMX Index is a market value-weighted index, that tracks the stock price performance of the most liquid issues traded on the Stockholm Stock Exchange (SSE).[31]

2.2 VaR

VaR could be defined as an easy method for measuring market volatility of unexpected outcomes (risk) with the help of statistical techniques. On the other hand the purpose of VaR is measurement of worst expected loss at a special time period (holding period) and the special given probability under assumption of the normal market condition e.g. if a bank announces that the daily VaR of its trading portfolio is $50 million at the 1% confidence level it means that there is only 1 chance out of 100 for a loss greater than $50 million over a one day period (when the time horizon is 1 day) i.e. the VaR measure is an estimate of more than $50 million decline in the portfolio value that could occur with 1% probability over the next trading day. The two important factors in defining VaR of a portfolio, is the length of time and the confidence level that the market risk is measured. The choice of these two factors completely changes the essence of the VaR model.
The choice of time horizon could differ from few hours to one day, one month, one year etc. For instance for a bank, holding period of 1-day could be effective enough, as banks have highly liquid currencies. This amount could change to 1 month for a pension organization.
About the determination of the confidence level when a company encounters an external regulatory[2], this number should be very small, such as 1% of confidence level or less for banks, but for internal risk measurement modelling in companies it could increase to around 5%. [18]

VaR models are based on the assumption that the components of the portfolio do not change over the holding period. But this assumption is only accurate for the short holding periods, so most of the time the discussion of the VaR rounds about the one-day holding period.

When the predicted VaR threshold is contradicted by the observed asset return, this is called ‘VaR break’, which could be a good VaR approach accuracy criterion.

It’s good to mention that although VaR is a necessary tool for controlling risk, it is not sufficient, because it should be accompanied by the limitations and controls plus an independent risk management functions.[3]

The early VaR methods including Variance-Covariance approach and Simulation, which are also called parametric methods, were based on the linear multiplication of the variance-covariance risk factors estimates.

[2] ‘Regulatory agency: An independent governmental commission established by legislative act in order to set standards in a specific field of activity, or operations, in the private sector of the economy and to then enforce those standards.’ [35].

[3] ‘Independent risk management system as a key component of risk management in an organization, including a strong internal control environment and an integrated, institutionwide system for measuring and limiting risk, is an important strategic and tactical support to management and board of directors.’ [13], [17].

Some of the important shortcomings of these methods, motivated risk managers to look for better estimation of VaR, despite their worldwide reputation.

Number of theoretical assumptions is put on data properties by these methods. One of these assumptions is about the density function of the risk factors which should be adjusted to a one or higher-dimensional Gaussian distribution i.e. multivariate normal distribution (normal distribution is mentioned here because they could be defined by only their first and second moments) and has constant mean and variance. But empirical results show something different which is emphasis on the **non-normality** of the daily asset price changes. They indicate significant and more common occurrences of the losses higher than VaR, caused by excess kurtosis[4] (volatility of volatility) in comparison to the ones predicted by normal distribution.

Another important disadvantage of these methods was the large number of inputs they required to be able to work well. Because, as a matter of fact all data covariance's should be mentioned[5].

And finally lack of good provision of the VaR estimates during the financial crises lead risk managers to search for better ones.

So historical simulation (HS) models based on non-parametric methods and filtered historical simulation (FHS), as a mixture of parametric and non-parametric methods appeared.

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[4]An important benchmark for the future returns of a stock or portfolio, i.e. the probability that the future returns will be significantly large or small depends on the difference of the kurtosis coefficient and number 3, which is the kurtosis of normal distribution (higher kurtosis coefficient from normal distribution kurtosis leads to a more probable too large or too small future returns).
Kurtosis explains the "flatness" degree of a distribution. The Kurtosis of a normal distribution is 3. This measure is a criterion to check whether the sample distribution is close to normal distribution. Left tail of the empirical distribution, which is used to compute VaR, will not fit a normal distribution for the large values of the kurtosis.

[5] By the normal distribution assumption of the portfolio returns of such methods, they need to estimate the expected value and standard deviation of returns of all the assets.

2.3 VaR Formula

If we mention confidence level in VaR measurement as below:

$$Confidence.Level = (1-\alpha)100\%$$

VaR formula will be like this:

$$VaR_{\alpha} = \inf\{\mathrm{L}:\mathrm{Prob}[\mathrm{Loss} > \mathrm{L}] \le 1-\alpha\},$$

where L is the lower threshold of loss, which means that the probability of losing more than L in a special time horizon (e.g. 1 day) is at most $(1-\alpha)$.

In the context of computer simulation, given α, if we make the probability below equal to $1-\alpha$ (by varying L), we will find the value of L as VaR at the specific time horizon.[6]

$$\mathrm{Prob}\ [\mathrm{Loss} > \mathrm{L}] = \frac{No.of.Simulations.with.Value < P - L}{N},$$

where N (multiple of 1000) is a number of simulations we have done with HS or FHS method, and P is the initial amount of investment.

3 Historical Simulation and Filtered Historical Simulation Concepts

In this section we explain fundamental concept of HS and FHS, and the reason behind using these two methods which are accompanied by explanation of other significant definitions and time series models related to the risk management field.

3.1 HS and FHS

As a short and straightforward definition of these two methods we could categorize them by the following structure:
When volatility and correlation are constant, we use classical HS, but as we confront the time-varying volatility then we should change our strategy to the FHS.

[6] α is a number between zero and one ($0 < \alpha < 1$), which usually is chosen equal to 0.99 or 0.95.

For specifying a model to a portfolio the simplest assumption could be standard normal distribution of the portfolio's shock returns, because this distribution has no parameters and the model will soon be ready for forecasting risks. But we know that this assumption is not true for most assets of a portfolio. So an important question arises here about selecting the best substitute distribution. Instead of looking for a special distribution, risk managers rely on resampling methods for modelling such as HS and FHS.

3.1.1 HS

HS which is also known as bootstrapping simulation, gather market raw values of risk in a special past period of time, and calculate their changes over that period to be used in the VaR measurement.

HS could be mentioned as a good resampling alternative method because of its simplicity and lack of distributional assumption about underlying process of returns (finding a distribution, fit to all the assets of a portfolio is not a simple approach).

The main assumptions of HS are:

- Chosen sample period could describe the properties of assets very well,

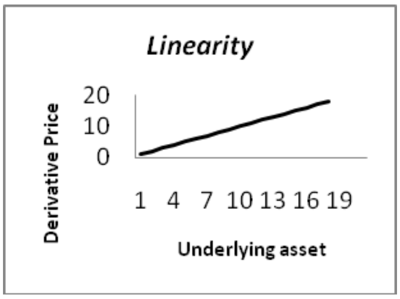
- There is a possibility of repeating the past in the future i.e. the replication of the patterns appeared in the volatilities and correlations of the returns in historical sample, in the future. On the other hand past could be a good criterion of the future forecast.[7]

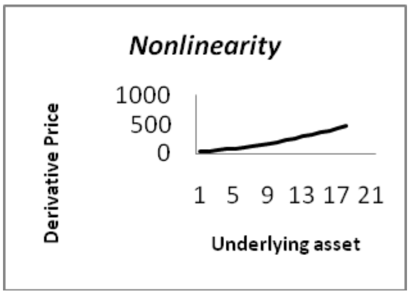
3.1.2 HS Shortcomings

HS named as a simple explained concept method which is based on the important fact in risk management i.e. nonlinearities[8] and non-normal distribution. But its simplicity ends to below disadvantages:

[7] HS is restricted in markets which are developing fast e.g electricity, because of the contradiction of the 'past repeat in the future' assumption.

[8] Nonlinearity means time dependency of the price of a financial instrument such as derivatives.





- The importance of the past returns is as much as the recent returns as observations are equally weighted, which causes lack of good weight allocation to the sudden increase in volatility of the recent past which has significant influence on the near future.[9] It means that HS has neglected the more effective role of the recent past, by equalizing its related weight to weight of the further away past.

- HS needs a long enough series of raw data (found by Vlaar [21], [34] through testing the accuracy of the different VaR models on Dutch interest rate portfolio), and this is the most difficulty of HS technique in estimating VaR of new risks and assets because there is no historical data available in these two cases. [2]

- If the number of observations is too large then almost the last one which might describe the future better, has the same affect of the first observation as they have equal weights (shown by Brooks and Persand [9]: VaR models could end to inaccurate estimates when the length of historical data sample is not chosen correctly, by testing sensitivity of the VaR models with respect to the sample size and weighting methods). [2]

- The assumption of constant volatility and covariance of the raw returns, leads to the fact that the sensitivity of the VaR cannot be checked. This means that the volatility update[10] does not happen in this model, and so the market changes won't be reflected.

- In HS, often, results are based on one of the most recent crises and the other factors won't be tested. So HS causes VaR to be completely backward-looking, i.e. when a company or organization tries to protect its resources by the VaR measurement based on the HS structure, in principal it organizes to protect itself from the passed crisis not the next one.

- As HS most commonly choose market data from the last 250 days, the "window effect" problem could also happen in this process. This means, when 250 days passes from a special crisis, that will be omitted from the window and the VaR will dramatically drop from day to the other. It could cause doubt for traders about the integrity of this method as they know that nothing special has happened during those two days where the drop happened.

[9] As an evidence of the VaR models failure, we could mention the bank experiments of the unusual number of VaR excess days in August and September 1998, which followed by the lack of attention to the returns' joint distribution at that time interval.

[10] Volatility update could be obtained by dividing historical returns by the corresponding historical volatilities (which is called normalizing the historical returns), and multiplying result by the current volatility.

3.1.3 BRW

By the points we mentioned above as HS deficiencies, different researchers such as Boudoukh et al. or Barone-Adesi et al. attempted to decrease them as much as possible.

As closer past return to the present time could forecast future better than the far past one, an exponential weighting technique which is known as BRW, was suggested by Boudoukh, Richardson and Whitelaw [12], [6] as a tool to decrease the effect of chosen sample's size on VaR estimation.

In this modification instead of equally weighted returns, their weight is based on their priority of happening by using factor called 'decay' factor, e.g. if we assume 0.99 for the decay factor and P for the probability weight of the last observation, the one observation before that will receive the 0.99P weight, 0.9801P will goes to the weight of one before and so on. So we could say that the original method of HS is a special case of BRW method with decay equal to 1. Boudoukh et al. examined the accuracy of their method by computing the VaR of stock portfolio, before and after the 19/10/1987(19th of October) when a market crash happened, with set of return for 250 days. They show that the VaR computed by HS method didn't show any changes the day after the crash because all the days had same weights but by BRW, VaR showed the effect of the crash.

Cabado and Moya [12] showed that the better forecasting of VaR could be reached by using the parameters of a time series model which has been fitted to the historical data. They showed the improvement in the VaR result by "fitting an Autoregressive Moving Average (ARMA) model to the oil price data from 1992 to 1998 and use this model to forecast returns with 99% confidence interval for the holdout period of 1999" [12]. One of the reasons of such an improvement could be interpreted as, the measurement of risk has higher sensitivity to the oil prices' variance changes with time series model than HS.

Hull and White [33] suggested another method of adjusting the historical data to the volatility changes. They used GARCH models for obtaining daily estimation of the variance changes over the mentioned period in the past.

3.1.4 FHS

As we notified before the main deficiency of HS as a non-parametric method is its disability in modelling the volatility dynamics of the returns. Also in the parametric method (which is not the subject of this paper and we do not go through it in detail), the choice of right distribution (if it could be found) is critical, so Hull and White [33] and Barone-Adesi et al [4], [33], combined the two previous methods to receive one called FHS, which is a semi-parametric technique[11] and has the below important priorities to the HS:

- Without any attention to the distribution of the observations you could use volatility model,
- Conforming the historical returns (by filtering them) to show current information about security risk.

"Filtered" expression related to the fact that we do not use raw returns in this method, but we use series of shocks (z_t), which are GARCH filtered returns.

Also Barone-Adesi and Giannopoulos [2], [33] discussed, as the current situation of the market is embedded in risk forecast by FHS, it works better than HS.

FHS is a Monte Carlo approach which is the combination of parametric modelling of risk factor volatility and non-parametric modelling of innovations, which has the best usage in 10-day, 1% VaR.

An important assumption of the FHS is that the return vectors is i.i.d, which means that its correlation matrix[12] is constant[13] (this assumption could be unrealistic for the long time series). For making returns i.i.d we should remove serial correlations and volatility clustered from the data. Serial correlation could be removed by adding MA term in conditional mean equation and to remove the volatility clusters we should model the returns as GARCH processes. GARCH models are based on the normal distribution of the residual asset returns assumption. Non-normality of the residual returns will contradict the efficiency of GARCH estimates, although they might still be consistent. So we could say that every time series which generates i.i.d residuals from returns is good enough for our modelling. [4]

In the simulation we only use historical distribution of the return series and we do not use any theoretical distribution. As we mentioned in the previous paragraph any time series model which generates i.i.d residuals

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[11] The parametric part of this technique is the GARCH estimation of residuals.
[12] Correlation matrix is a matrix which its members are correlation coefficients.
[13] In a Mean-Variance portfolio when the correlation of returns between each security pairs is the same, this is called constant correlation model [20].

from our returns is suitable for us, so we introduce ARMA-GARCH which removes the serial correlations[14] by MA and volatility clusters by GARCH.
To apply the FHS, the estimation of the parameters of GARCH (1, 1) is by quasi-maximum likelihood estimation (QMLE). QMLE behaves as the returns are normally distributed, but even if they are not normally distributed, it estimates the parameters and conditional volatilities.

3.2 The Most Suitable Time Series Model for our Method: ARMA-GARCH

3.2.1 Volatility

Volatility which its almost acceptable forecasting is used to measure risk in credit institutes is an important factor at the centre of attention of risk management techniques. Volatility, measure the size of occurred errors in different variables of financial market modelling such as return. For a large number of models, volatility is not constant and is time varying. Volatility as an unobservable fact should be estimated from data.

The predictability of financial market volatility is a considerable property, with vast usages in risk management. VaR will increase as volatility increase.
So investors will try to change the diversification of their portfolio to decrease the number of those assets which their volatility has been predicted to be increased.
By predictable volatility construction, options' value changes[15] (option are kind of assets, strongly dependent to volatility) will have predictable structure.
Also changes in volatility, influence the equilibrium asset prices. Thus whoever could forecast volatility changes more precisely, will have higher ability in controlling the risk of market. So each technique could satisfy this forecasting is really valuable in the financial world.

[14] When the assumption of $corr(\varepsilon_t \varepsilon_{t-1}) = 0$ is contradicted, this is called serial correlation, which means error terms do not follow an independent distribution and are not strictly random.
[15] Volatility is an important benchmark for specifying which kind of options should be bought or sold. Ending options in-the-money is in the direct relation with the underlying contract price fluctuations. So option's value goes up and down with respect to the value of the contract. As volatility soars, the possibility of receiving higher outcomes out of contracts will be higher, so option's value will increase, and vice versa.

3.2.2 Heteroskedasticity

The other factor is heteroskedasticity which represents the non-constant variance of error terms,
(i.e. violation of the following condition

$$var(\varepsilon_i) = var(y_i) = \sigma^2 , \quad \sigma^2 \text{ is constant.})[30]$$

On the other hand 'dependence of the residual variance on the independent variables is termed heteroskedosticity' (we could also define it as 'variable variance of residuals') [26]. This factor most of the time happens by the cross sectional data[16]. As a most important consequence of heteroskedasticity, we could mention its influence on the efficiency of the OLS estimators. Although the OLS linearity and unbiasedness[17] won't be affected by heteroskedasticity, it violates their minimum variance property and by this way they won't be efficient, consistent and therefore best estimators anymore.

There are three main approaches to deal with this problem of heteroskedasticity:
- Changing the model,
- Transforming data for receiving more stable variation,
- Consider the variance as a function of predictor and modelling it with respect to this property.

(For more information see [28])

In finance this factor usually could be found in stock prices. The volatility level of these equities is not predictable during different time intervals.

These two factors in 3.2.1 and 3.2.2 cannot be covered by any linear or nonlinear AR or ARMA processes. So we introduce very well-known type of volatility models which satisfies all the above specializations: autoregressive conditional heteroskedasticity (ARCH) and generalized autoregressive conditional heteroskedasticity (GARCH) models.

3.2.3 ARCH, GARCH and ARMA-GARCH Time Series Models

In economic and financial modelling, the most undetermined part of any event is 'future'. By gathering more new information as time passes, we could modify the effects of future uncertainty in forecasting our models.

[16] In Cross Sectional Data we look at different areas during the same year, in spite of the time series which we look at one area during so many years.
[17] An unbiased estimator of a parameter is the one with the expectation value equal to the value of the estimated parameter.

In finance, asset prices are the best forecast of future benefit of market, so they are too sensitive to each news. ARCH/GARCH models can be named as a measurement tool of the news process intensity (‘News Clustering’ as an interpretation of the ‘Volatility Clustering’). There are many different factors that influence the spreading steps of the news and their affects on the prices. Although macroeconomics models could temperate the affects of such news, ARCH/GARCH modelling are more well-known related to their ability in improving the volatilities which has been made during these processes.

For simplicity we define the ARCH (1) process here and then show the complete extension of such processes in introducing GARCH process.

ARCH (1)

The process ε_t, $t \in Z$ is ARCH (1), if $E\left[\varepsilon_t \mid F_{t-1}\right] = 0$,

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2$$

With $\omega > 0$, $\alpha \geq 0$ and

- $Var\left(\varepsilon_t \mid F_{t-1}\right) = \sigma_t^2$ and $z_t = {\varepsilon_t}/{\sigma_t}$ is i.i.d (strong ARCH)[18]
- $Var\left(\varepsilon_t \mid F_{t-1}\right) = \sigma_t^2$ (semi-strong ARCH)
- $P\left(\varepsilon_t^2 \mid 1, \varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_{t-1}^2, \varepsilon_{t-2}^2, ...\right) = \sigma_t^2$ (weak ARCH). [15]

In ARCH model the volatility is a function of squared lagged shocks (ε_{t-1}^2). In the generalized format of this model i.e. GARCH model, volatility also depends on the past squared volatilities. As a general definition we can call GARCH model as an unpredictable time series with stochastic volatility. In strong GARCH, there is $\varepsilon_t = \sigma_t z_t$ where σ_t is F_{t-1}-measurable, i.e. σ_t (volatility) only depends on the information available till time t-1 and the i.i.d innovations z_t with $E\left[z_t\right] = 0$ and $Var\left(z_t\right) = 1$. For this time series we also have $E\left[\varepsilon_t \mid F_{t-1}\right] = 0$, $Var\left(\varepsilon_t \mid F_{t-1}\right) = \sigma_t^2$, which means that ε_t is unpredictable and except the cases where σ_t is constant, it is conditionally heteroskedastic. [15]

[18] This means in ARCH models the conditional variance of ε_t is a linear function of the lagged squared error terms.

GARCH (p,q)

The process ε_t, $t \in Z$ is GARCH (p,q), if $E\left[\varepsilon_t \mid F_{t-1}\right] = 0$,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

and

- $Var\left(\varepsilon_t \mid F_{t-1}\right) = \sigma_t^2$ and $z_t = \varepsilon_t / \sigma_t$ is i.i.d (strong GARCH)
- $Var\left(\varepsilon_t \mid F_{t-1}\right) = \sigma_t^2$ (semi-strong GARCH)
- $P\left(\varepsilon_t^2 \mid 1, \varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_{t-1}^2, \varepsilon_{t-2}^2, ...\right) = \sigma_t^2$ (weak GARCH)

The sufficient but not necessary condition for $\sigma_t^2 > 0$ a.s. ($P\left[\sigma_t^2 > 0\right]$=1) are
$\omega > 0$, $\alpha_i \geq 0$, i=1,…,q and $\beta_j \geq 0$, j=1,…,p.[15]

ARMA-GARCH Model

ARMA-GARCH model is: [21]

$$\begin{cases} y_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{i=1}^{q} \psi_i \varepsilon_{t-i} + \varepsilon_t \\ \varepsilon_t = \eta_t \sqrt{h_t}, h_t = \alpha_0 + \sum_{i=1}^{r} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{s} \beta_i h_{t-i} \end{cases}$$

where
ε_t is the random residual and equal to $\eta_t \sqrt{h_t}$, (η_t is modelled by $h_t = \alpha_0 + \sum_{i=1}^{r} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{s} \beta_i h_{t-i}$),
η_t is i.i.d random variable with mean zero and variance 1,
and α_0 is constant.

If s=0: ARMA-GARCH changes to ARMA-ARCH,

If q=0, s=0 and r=1: ARMA-GARCH changes to AR-ARCH (1).

There are two types of parameters in the above equations:

- Set of parameters of conditional mean denoted by m,
- Set of parameters of the conditional variance h_t denoted by δ. [21]

In practice, first of all we estimate m, then the residuals from the estimated conditional mean will be calculated, after that δ can be estimated (the method of calculation δ will be mentioned below) and finally we use the estimated h_t to receive more efficient estimator of m. [21]

Least square Estimator (LSE) of m_0 (true value of m) :[21]

Assume $y_1, ..., y_n$ are the given observations. Then the LSE of m_0, i.e. $\widehat{m}$, could be defined as the value in θ, a compact subset of R^{r+1}, which minimize

$$s_n = \sum_{t=1}^{n} \varepsilon_t^2 \ .$$

Weiss [21], showed that $\widehat{m}$ is consistent for m_0 and

$$\sqrt{n}(\widehat{m} - m_0) \to N(0, A)$$

with

$$A = E^{-1}\left[\frac{\partial \varepsilon_t}{\partial m}\frac{\partial \varepsilon_t}{\partial m'}\right] E\left[\varepsilon_t^2 \frac{\partial \varepsilon_t}{\partial m}\frac{\partial \varepsilon_t}{\partial m'}\right] E^{-1}\left[\frac{\partial \varepsilon_t}{\partial m}\frac{\partial \varepsilon_t}{\partial m'}\right]_{m=m_0}$$

Pantula [21] obtained the asymptotic distribution of the LSE for the AR model with ARCH (1) errors, and gave an explicit form for A. [21]

But the results in Weiss and Pantula [21] needs the finite fourth moment[19] condition for the y_t. By now no one have mentioned the LSE of m_0 for the ARMA-GARCH model. However Weiss result for LSE could be extended to ARMA-GARCH model. The LSE is equivalent to the MLE of m_0, when GARCH reduces to an i.i.d white noise process. [21]

If the fourth moment is finite, the LSE is consistent and asymptotically normal[20], but it is inefficient for ARMA-ARCH/GARCH models. So we should use MLE in such a case. The log-likelihood function is:

$$L(m)=\sum_{t=1}^{n} l_t \quad , \quad l_t=-\frac{1}{2}\ln h_t-\frac{1}{2}\frac{\varepsilon_t^2}{h_t}$$

where h_t is a function of m and y_t, and will be calculated by the below recursion:

$$h_t=\alpha_0+\sum_{i=1}^{r}\alpha_i\varepsilon_{t-i}^2+\sum_{i=1}^{s}\beta_i h_{t-i} \quad , \quad h_0=a \quad \text{positive constant}$$

If we define $\widehat{m}=\max_{m\in\theta} L(m)$, as we didn't assume that the η_t is normal, then $\widehat{m}$ called the QMLE of m. Weiss showed for the ARMA-GARCH, QMLE is consistent and asymptotically normal under finite fourth moment condition. Ling and Li showed if the finite fourth moment condition be true then a locally consistent and asymptotically normal QMLE exist for the ARMA-GARCH. [21]

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[19] Fourth standardized moment is $\frac{\mu_4}{\sigma^4}$ where $\mu_4=E\left[\left(X-\mu\right)^4\right]$ the fourth moment around the mean is and σ is the standard deviation.

[20] An asymptotically normal estimator is an estimator which is consistent and its distribution around the true parameter (in our example true parameter is m_0) is a normal distribution with the standard deviation decreasing proportionally to $\frac{1}{\sqrt{n}}$. $\widehat{m}$ is asymptotically normal for some A(A is an asymptotic variance of the estimator).

***Estimation of* δ : [21]**

Considering the following ARCH(r) model:

$$\varepsilon_t = \eta_t h_t^{1/2} \quad , \qquad h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_r \varepsilon_{t-r}^2 \qquad (1)$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$ (i=1,…,r) are adequate for $h_t > 0$ and η_t are i.i.d random variables with mean zero and variance 1.

For estimating the parameters of the model (1) the easiest technique is LSE. So we write the model as below:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_r \varepsilon_{t-r}^2 + \xi_t$$

where $\xi_t = \varepsilon_t^2 - h_t$ and ξ_t can be mentioned as a martingale difference[21].

Let $\delta = (\alpha_0, \alpha_1, \ldots, \alpha_r)'$ and $\widetilde{\varepsilon_t} = (1, \varepsilon_t^2, \ldots, \varepsilon_{t-r+1}^2)'$. Then LSE of δ is equal to

$$\hat{\delta} = \left(\sum_{t=2}^{n} \widetilde{\varepsilon_{t-1}} \widetilde{\varepsilon_{t-1}}' \right)^{-1} \left(\sum_{t=2}^{n} \widetilde{\varepsilon_{t-1}} \widetilde{\varepsilon_t} \right)$$

which Weiss and Pantula [21] showed that $\hat{\delta}$ is consistent and asymptotically normal. (They assume that the 8th moment of ε_t exists).
In general, for estimating the parameter δ, maximum likelihood estimation (MLE) will be used.

Conditional log-likelihood with respect to the ε_t as observations, t=1,…,n can be written as below:

$$L(\delta) = \sum_{t=1}^{n} l_t, l_t = -\frac{1}{2} \ln h_t - \frac{1}{2} \frac{\varepsilon_t^2}{h_t} ,$$

[21] A stochastic series $\{x_i\}$ is a martingale difference sequence with respect to the $\{y_i\}$ if :

$$E(x_{i+1} \mid y_i, y_{i-1}, \ldots.) = 0, \forall i$$

where h_t is a function of ε_t. Assume $\delta \in \theta$, and δ_0 is a true value of δ. If we define

$$\hat{\delta} = \arg\max_{\delta \in \theta} L(\delta),$$

since η_t (conditional error) is not assumed to be normal, $\hat{\delta}$ will be the QMLE.

3.3 ARMA-GARCH Preference in Modelling Market Volatility

During past years' researches about the ARCH/GARCH model, researchers always were interested to use this model to analyze the volatility of financial market data without any respect to the estimation of conditional mean. But it was not reachable because if the conditional mean is not estimated sufficiently**,** then construction of consistent estimates of the ARCH process won't happen and that ends to the failure in the statistical inference and empirical analysis with respect to the ARCH elements. Thus we should not ignore the importance of the estimation of the conditional mean although the most interesting part for us is investigating the volatility of data.
The conditional mean is given by AR or ARMA model. So why we don't use ARMA model and why should we look for the other models such as ARMA-GARCH?
Because the conditional variances of white noise are not constant, so we should generate a new AR or ARMA model completely different from the traditional one which assumes the errors are i.i.d or martingales differences with a constant conditional variance.
So as the statistical properties of the traditional AR or ARMA model could not cover the properties of our case, we introduce another model called ARMA-GARCH model.

4 Methodologies of Historical Simulation and Filtered Historical Simulation

In the following section we will explain the structure of theory behind these two techniques, and VaR computation with the help of them in detail.

4.1 HS

In this technique after gathering at least one year of recent daily historical returns (estimated returns), simulation of returns will be the next step. Thus we choose a small number (compare to the length of our observed daily historical returns) for upper threshold of period we want to forecast, called T, and then we select T-random returns with replacement from our observation data set (simulated returns).

By putting these simulated returns in below formula we form a simulated price series, which is recursively updated up to the last day (T): [2]

$$P^*_{s+T} = P_s(1+r^*_{s+1})(1+r^*_{s+2})...(1+r^*_{s+T})$$

where P_s is our initial price (outcome), r^*_{s+t} is the simulated return of the t-th day of our horizon, which has been chosen (randomly and with replacement) from set of historical returns and P^*_{s+t} is the simulated price of that day.

This simulation should be repeated for N times (N is a multiple of 1000 for receiving more accurate result), which ends to the

$$P_t^*[1], P_t^*[2], ..., P_t^*[N]$$

as our simulated prices in period [t,t+1].

By taking average of these simulated prices related to each day of our horizon and compare it with the actual price, which was received from the exact return of the corresponding day, we could examine accuracy degree of our method.

4.2 FHS

As we explained before for removing shortcomings of HS method, FHS was suggested by Barone-Adesi et al. as a refinement of HS technique.

In this method we fit an ARMA time series model to our data set and then use the parameters of this model for VaR forecasting plus GARCH time series for estimating the time-varying volatility of the model.

So we could write an ARMA-GARCH (1, 1) model as below:

$$r_t = \mu r_{t-1} + \theta\varepsilon_{t-1} + \varepsilon_t \qquad (1)$$

$$h_t = \omega + \alpha(\varepsilon_{t-1} + \gamma)^2 + \beta h_{t-1} \qquad (2)$$

where the equation (1) is ARMA(1,1) modelling of r_t returns with μ as an AR(1) term and θ as MA term, and equation (2) is GARCH (1,1) modelling of random residuals ε_t (noises), which defines volatility of the random residuals ε_t as a function of last period volatility h_{t-1}, closest residual ε_{t-1} and constant ω with $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$ to guarantee that each solution of the equation (2) is positive.

Random residuals are assumed to be unpredictable and conditionally heteroskedastic (except when σ_t is constant).
We could formulate two above properties of the ε_t as below:

$E\left[\varepsilon_t \mid F_{t-1}\right] = 0$ (Unpredictability) (3)

$Var\left[\varepsilon_t \mid F_{t-1}\right] = h_t$ (Conditional heteroskedasticity) (4)

In (3) F_{t-1} is called information set at time t-1.

Also the standardized residual returns

$$e_t = \frac{\varepsilon_t}{\sqrt{h_t}}$$

are i.i.d with mean 0 and variance 1.
(Mention that the only observed data we have are $r_0, r_1, ..., r_s$)

Our aim is to predict the empirical distribution of r_t, which will be obtained by the process we will explain, but first of all, we take a quick look at the theoretical technique:

4.2.1 Theoretical Method of Obtaining Future Returns

If we think of $\lambda = (\mu, \theta, \omega, \alpha, \beta, \gamma)$ as an initial choice of λ we could use below algorithm to find the set of corresponding residuals, $\{e_1, e_2, ..., e_s\}$:

- Assuming initial values of residual and volatility of the residual is equal to 0 and $\dfrac{\omega + \alpha\gamma^2}{1 - \alpha - \beta}$ respectively:

$\varepsilon_0 = 0$,

$h_0 = \dfrac{\omega + \alpha\gamma^2}{1-\alpha-\beta}$ (Unconditional variance of GARCH (1, 1) formula)

- The initial standardized residual return is calculated easily by the above amounts:

$$e_0 = \frac{\varepsilon_0}{\sqrt{h_0}} = 0$$

- For $t = 1, 2, ..., s$ we find ε_t , h_t and $e_t = \dfrac{\varepsilon_t}{\sqrt{h_t}}$ via following process:

  - Put ε_{t-1} in equation (1) and receive ε_t
  - Put ε_{t-1} and h_{t-1} in equation (2) and receive h_t
  - Finding e_t through $e_t = \dfrac{\varepsilon_t}{\sqrt{h_t}}$ formula
  - Repeating these three steps for $t = 1, 2, ..., s$ we receive

$$\{e_1, e_2, ..., e_s\}$$ [22]

4.2.2 Future Returns Estimation

Now, as an initial step in simulating future returns, we need the estimated ones, so:

1. First of all we find the following parameters

$$\hat{\lambda} = (\hat{\mu}, \hat{\theta}, \hat{\omega}, \hat{\alpha}, \hat{\beta}, \hat{\gamma})$$

i.e. the estimation of our ARMA-GARCH (1,1) model parameters $\lambda = (\mu, \theta, \omega, \alpha, \beta, \gamma)$.

For such estimation we use quasi maximum likelihood estimation (QMLE) method as it behaves returns are normally distributed and it could also estimate the parameters when this assumption is contradicted.

Thus $\hat{\lambda} = \arg\max_{\lambda} L(\lambda)$ where L is likelihood function and $\hat{\lambda}$ is QMLE of λ .

[22] These residuals differ with respect to the initial choice of λ changes.

2. When we found $\widehat{\lambda}$, we repeat the algorithm was explained in a theoretical method, for obtaining the estimated residuals and then use them in forming the simulated values:

- Assuming $\widehat{\varepsilon_0}$, $\widehat{h_0}$ as initial values of the estimated residual and estimated volatility of residual respectively.

- So the initial value of the estimated standardized residual return will be obtained by:

$$\widehat{e_0} = \frac{\widehat{\varepsilon_0}}{\sqrt{\widehat{h_0}}}$$

- For $t = 1, 2, ..., s$ we could find $\widehat{\varepsilon_t}$, $\widehat{h_t}$ and $\widehat{e_t} = \frac{\widehat{\varepsilon_t}}{\sqrt{\widehat{h_t}}}$ via following process:

- Put $\widehat{\varepsilon_{t-1}}$ in equation (1) and receive $\widehat{\varepsilon_t}$
- Put $\widehat{\varepsilon_{t-1}}$ and $\widehat{h_{t-1}}$ in equation (2) and receive $\widehat{h_t}$
- Finding $\widehat{e_t}$ through $\widehat{e_t} = \frac{\widehat{\varepsilon_t}}{\sqrt{\widehat{h_t}}}$ formula
- Repeating these three steps for $t = 1, 2, ..., s$ we receive

$$\left\{\widehat{e_1}, \widehat{e_2}, ..., \widehat{e_s}\right\}$$

3. The final process is simulation (which should be repeated thousands of times for reaching the acceptable result, as the number of replications increases, the estimate converges to the true value [18]) and it is based on the following algorithm:

4.2.3 Simulation of the Future Returns

- As the most recent estimated data could forecast future better than the others, so we describe the initial values of the simulated residual and volatility of the residual as below:

$$h_s^* = \widehat{h_s} \quad , \quad \varepsilon_s^* = \widehat{\varepsilon_s}$$

- Select a set

$$\left\{e_{s+1}^{*},\ldots,e_{s+T}^{*}\right\}$$

which has T elements[23], and is constructed randomly but with replacement from the set $\left\{\widehat{e}_1,\widehat{e}_2,\ldots,\widehat{e}_s\right\}$.

- For $t=s+1,\ldots,s+T$, we will find:

$$h_t^{*}=\widehat{\omega}+\widehat{\alpha}(\varepsilon_{t-1}^{*}+\widehat{\gamma})^2+\widehat{\beta}h_{t-1}^{*} \quad ,$$

$$\varepsilon_t^{*}=e_t^{*}\sqrt{h_t^{*}} \quad ,$$

$$\begin{cases} r_t^{*}=\widehat{\mu}r_{t-1}+\widehat{\theta}\varepsilon_{t-1}^{*}+\widehat{\beta}h_{t-1}^{*}, t=s+1 \\ \\ r_t^{*}=\widehat{\mu}r_{t-1}^{*}+\widehat{\theta}\varepsilon_{t-1}^{*}+\widehat{\beta}h_{t-1}^{*}, t=s+2,s+3,\ldots,s+T \end{cases} ,$$

$$\begin{cases} P_t^{*}=P_{t-1}(1+r_t^{*}), t=s+1 \\ P_t^{*}=P_{t-1}^{*}(1+r_t^{*}), t=s+2,\ldots s+T \end{cases}$$

- If we think N as a number of simulations, we obtain N simulated returns and prices for each period such as $\left[t,t+1\right]$, i.e.:

$$r_t^{*}\left[1\right],r_t^{*}\left[2\right],\ldots,r_t^{*}\left[N\right] \quad ,$$

$$P_t^{*}\left[1\right],P_t^{*}\left[2\right],\ldots,P_t^{*}\left[N\right]$$

- We use the above simulated P_t's for finding the predicted empirical probability distribution of P_t, e.g.

$$\frac{1}{N}\sum_{n=1}^{N}P_t^{*}\left[n\right]$$

which is the predicted expected value of P_t.

[23] T as a number of forecasting days is considerably smaller than s, which means that with a large number of historical data we could only forecast short period of time in future.

4.3 Computation of VaR (Same Method in HS and FHS)

By obtaining the distribution of the values

$$P_t^*[1], P_t^*[2], \ldots, P_t^*[N],$$

we could report the VaR at the specified level of confidence e.g. $(1-\alpha)100\%$ and time horizon, by making the probability below equal to $1-\alpha$ (by varying L):

$$\text{Prob [Loss} > \text{L]} = \frac{No.of\ .Simulations.with.Value < P - L}{N}$$

P: Initial investment
N: Total number of simulations

Thus L will be the $(1-\alpha)100\%$ VaR.

5 Empirical Studies

5.1 Daily Returns Plots

As an empirical study we collected 10 years of OMX Index daily closing prices from1999 to 2009 and calculated:
- Daily simple returns:

$$R_t = \frac{P_t}{P_{t-1}} - 1$$

- Daily logarithmic returns:

$$r_t = \ln(\frac{P_t}{P_{t-1}}) = \ln(1 + R_t)$$

- Some of their statistical properties such as Min., Max., Average, Standard deviation, Skewness, Excess Kurtosis,

Also plotted the daily simple and log returns and their empirical distribution for the whole period and some of subintervals such as 2008-2009, 2007-2008, 2006-2007, 2005-2006 and 2003-2005.

1999-2009:

Table 1:

	Simple Returns	Log Returns
Min.	-0.223744292	-0.253273293
Max.	0.482758621	0.393904286
Mean	0.000338233	-0.000668471
St.Dev.	0.045518198	0.044658171
Skewness	1.402108732	0.555878061
Excess Kurtosis	14.8906349	9.7752859

As we know, skewness is a measure of symmetry, which is equal to zero for normal distribution. This number for each symmetric distribution is also zero. Negative skewness (left-skewed distribution) means that the left tail of the distribution is longer and the distribution disposed to the right and vice versa for positive skewness.

Excess kurtosis is a measure of peakedness or flatness of data in comparison to normal distribution. This number for normal distribution is equal to zero. A distribution with negative excess kurtosis is called platykurtic, which has lower peak around mean (than normal distribution) and flat distribution (thin tails). A leptokurtic (distribution with positive excess kurtosis), is a peaked distribution with fatter tails.

By the number we received in table 1, we could say that the distribution of the data between 1999-2009, when we use simple returns, is a peaked distribution with heavier tails than a normal distribution which is also not symmetric. The heavy tail means that if we assume normal distribution for these data, we will underestimate all events in the tails, which could end to a not precise simulation of the future returns.

By take a look at the second column of the table 1, we could conclude that 1999-2009 distribution, using log returns, again ends to a peaked distribution with fatter tails than normal distribution but rather symmetric.

When we compare these results with the histograms in figures 3 and 4, we will find that they conform.

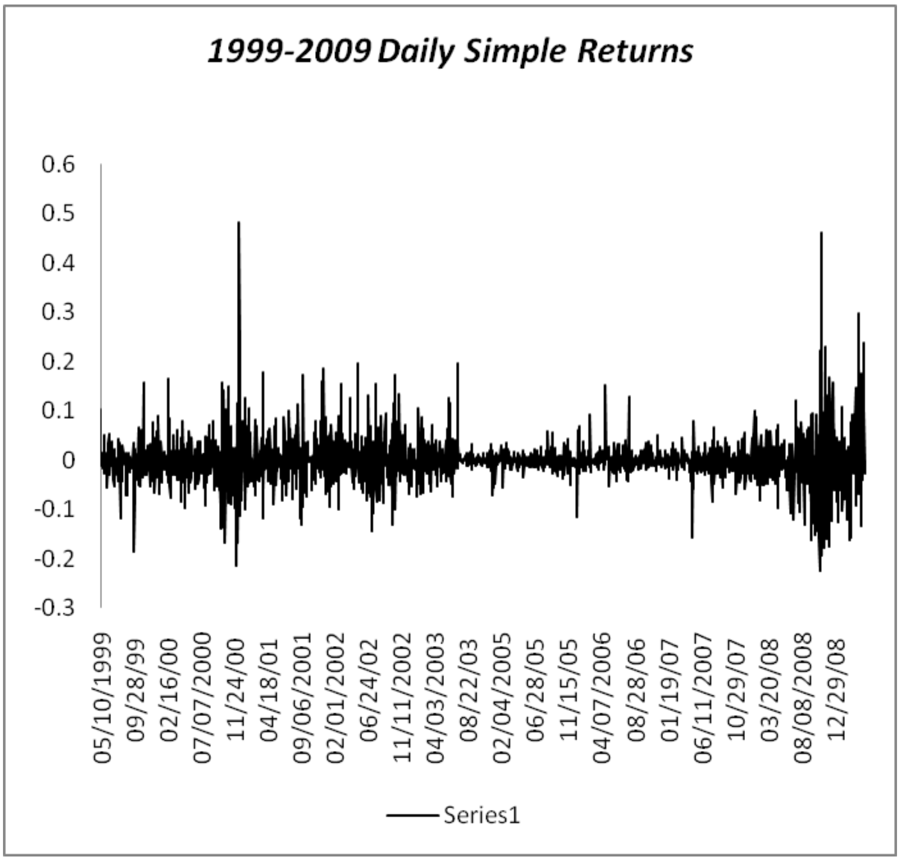

Figure 1

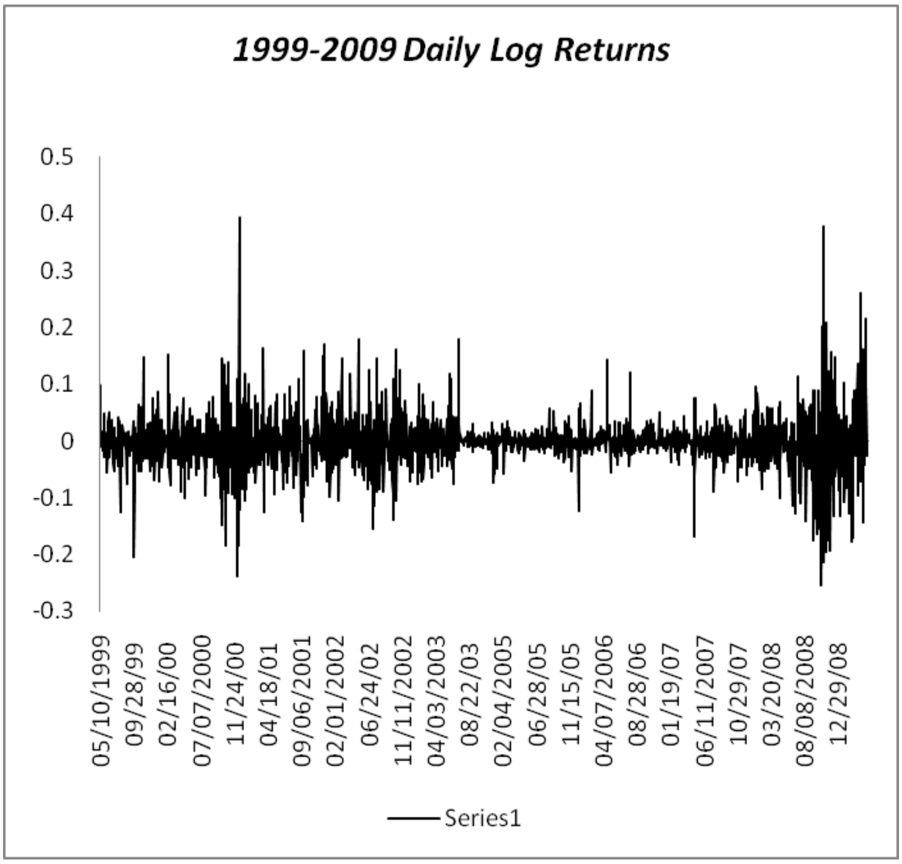

Figure 2

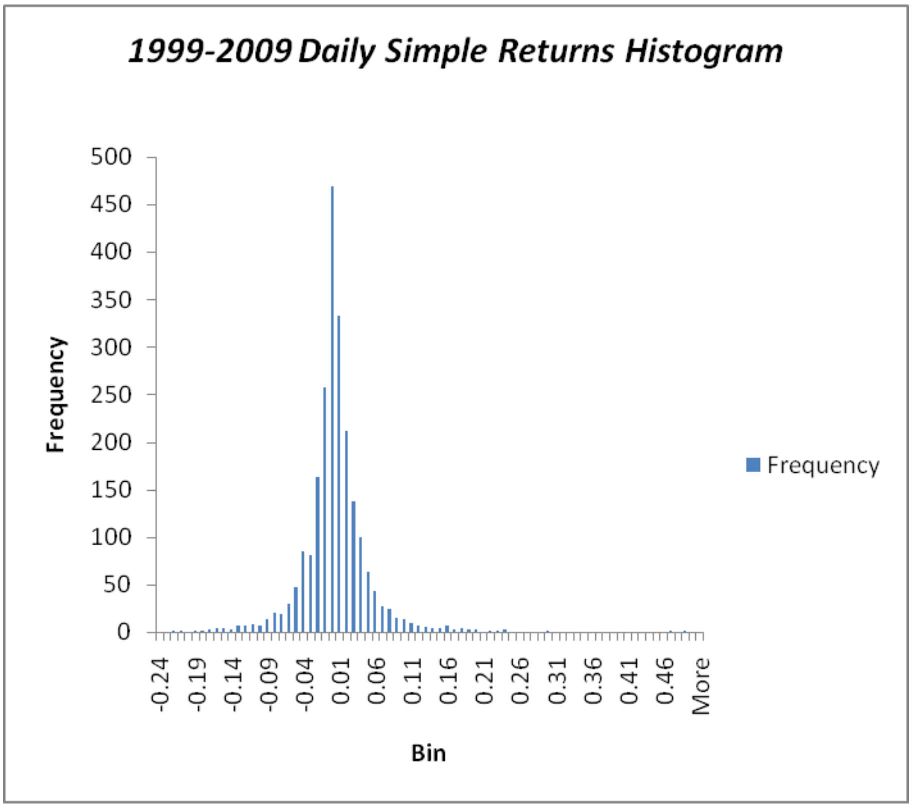

Figure 3

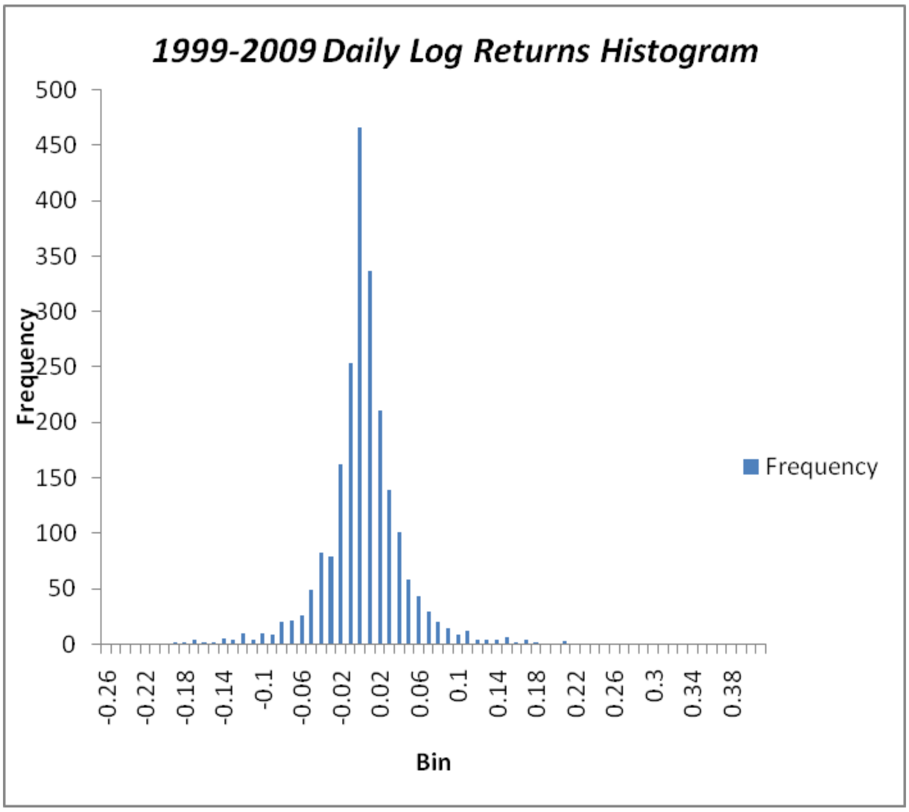

Figure 4

Below we will go through the yearly data ('one year' means 252 days in our samples):

2008-2009:

Table 2:

	Simple Returns	Log Returns
Min.	-0.223744292	-0.253273293
Max.	0.460743802	0.378945759
Mean	-9.93648E-05	-0.003397028
St.Dev.	0.082660705	0.080911797
Skewness	0.94830462	0.376305354
Excess Kurtosis	4.401059008	2.519196237

Again by take a look at the Skewness line in table 2, we could conclude that the distribution of both simple and log returns should be almost symmetric, and both of them have high peaked, heavy tailed distributions.

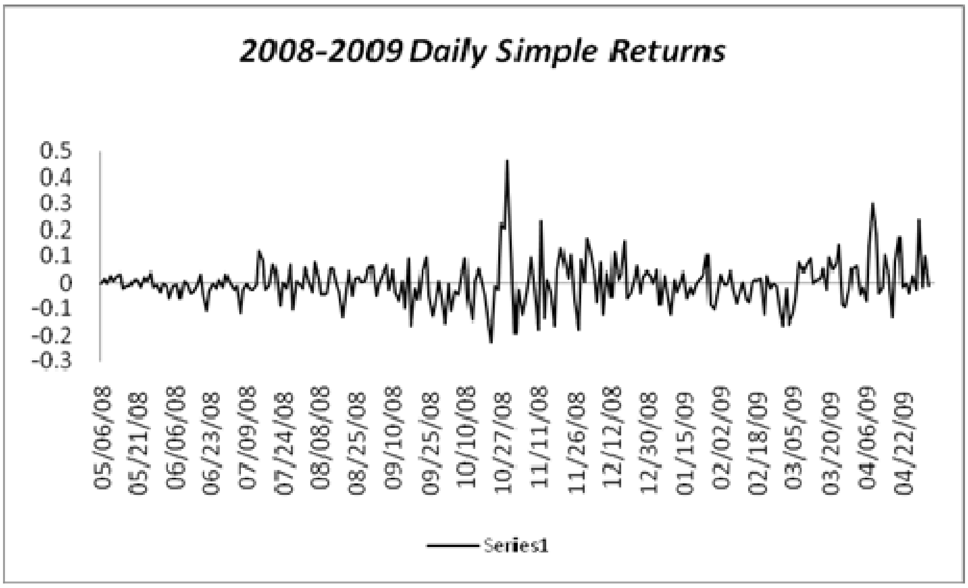

Figure 5

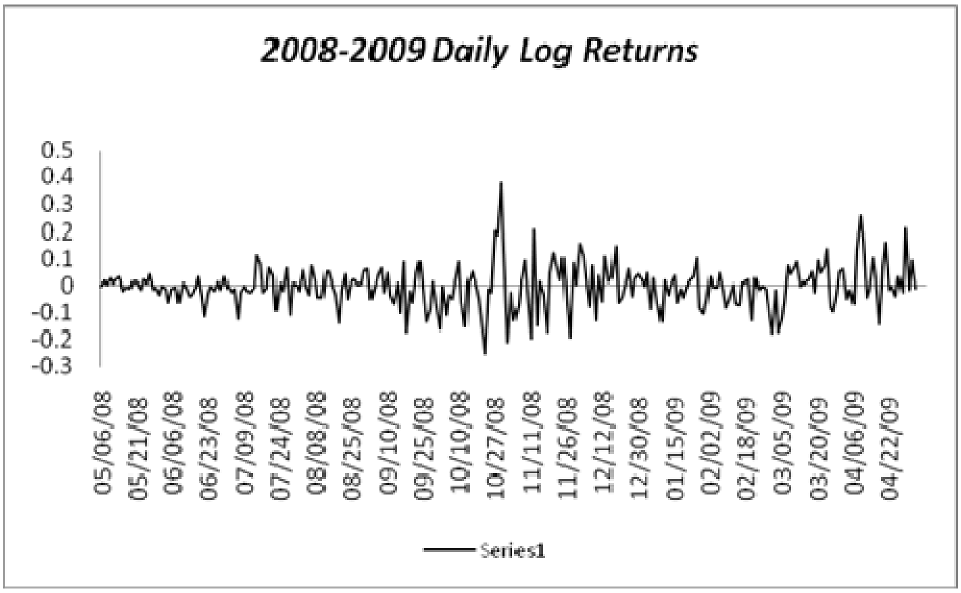

Figure 6

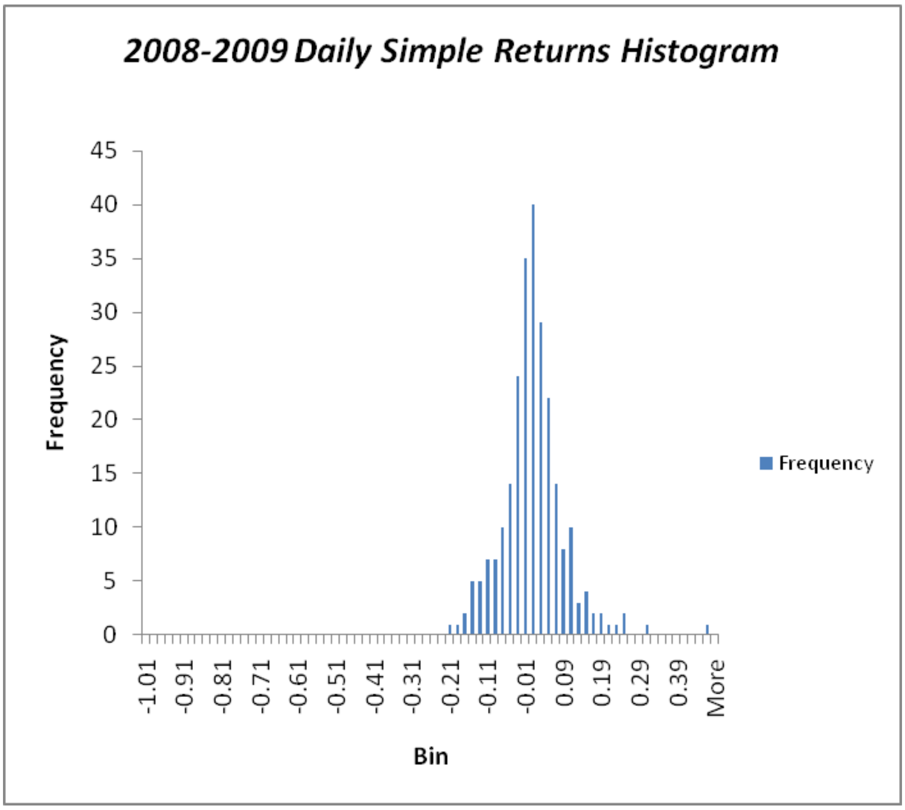

Figure 7

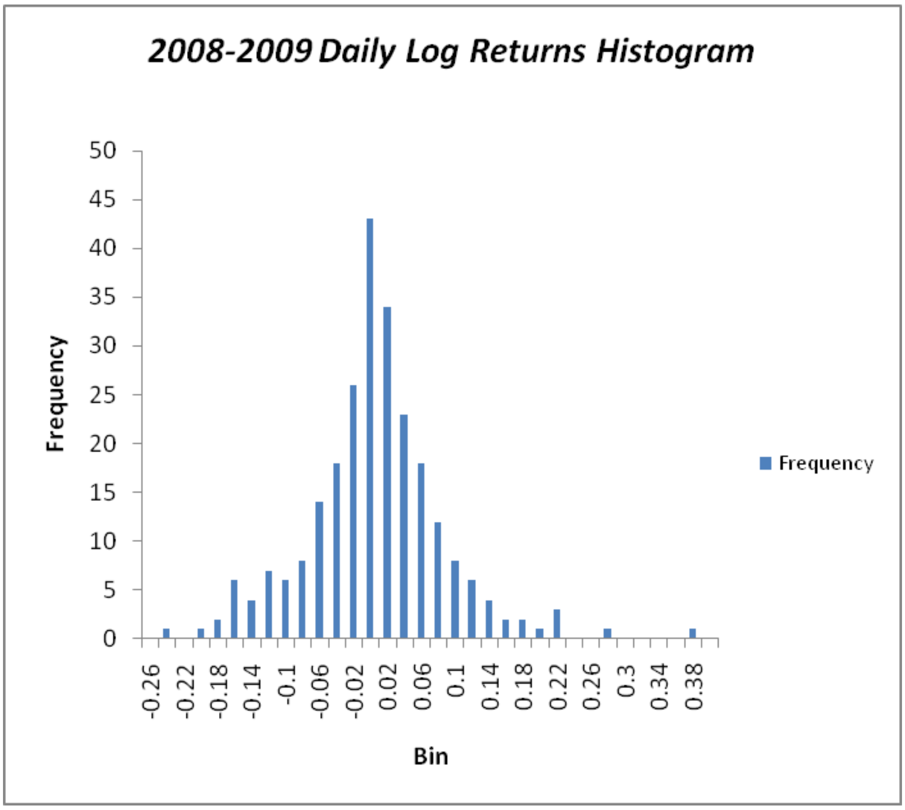

Figure 8

2007-2008:

Table 3:

	Simple Returns	Log Returns
Min.	-0.095544554	-0.100422234
Max.	0.102230502	0.097335856
Mean	-0.002811429	-0.00325175
St.Dev.	0.029608247	0.029560184
Skewness	0.398876565	0.273630295
Excess Kurtosis	0.997149659	0.946366854

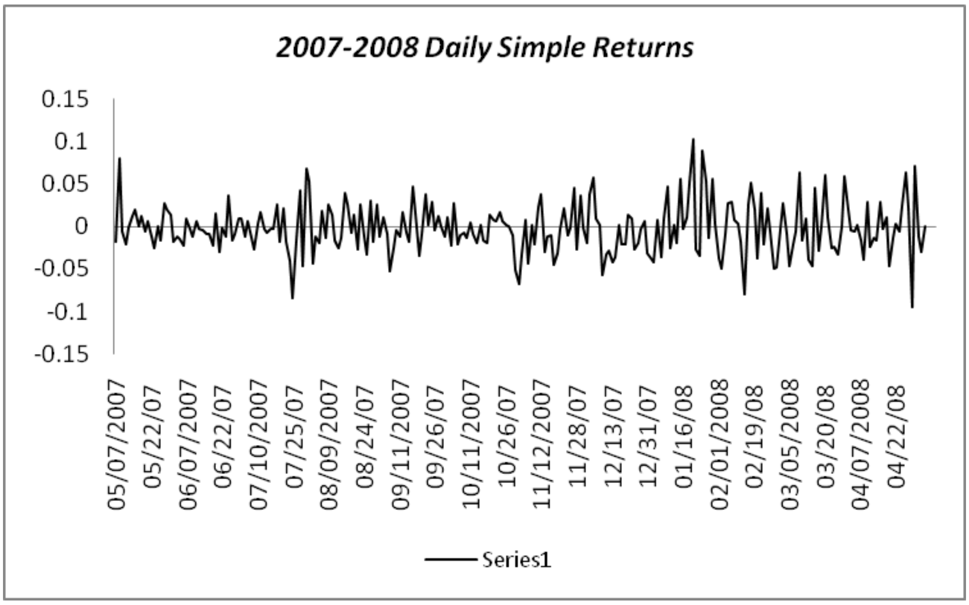

Figure 9

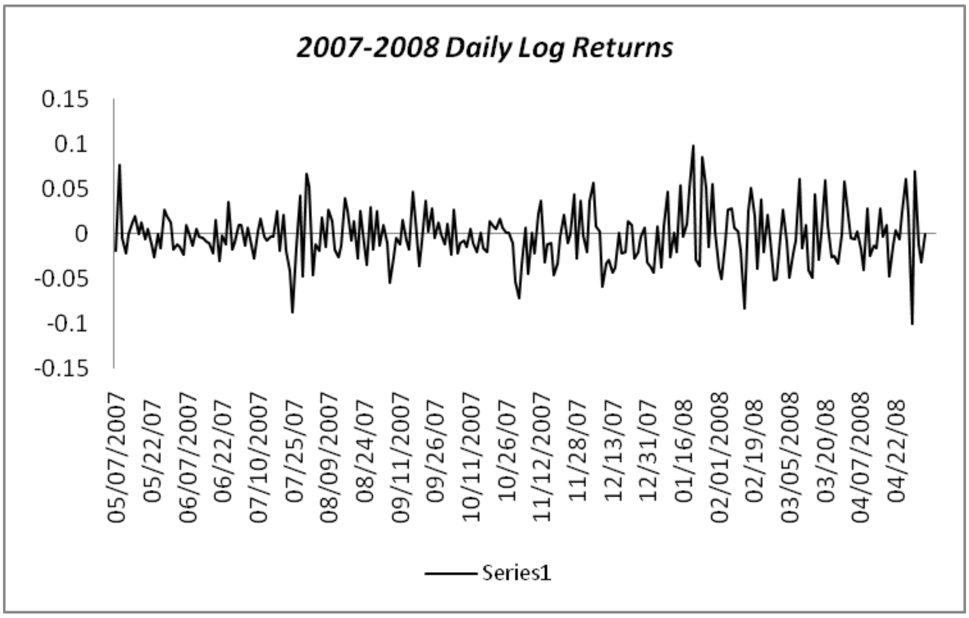

Figure 10

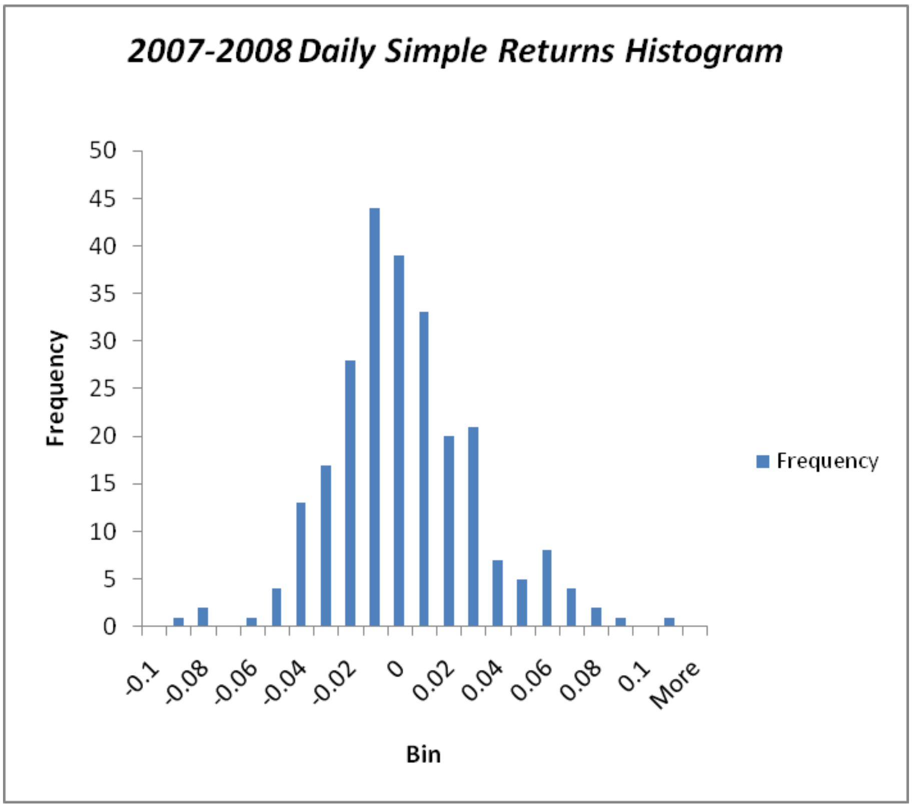

Figure 11

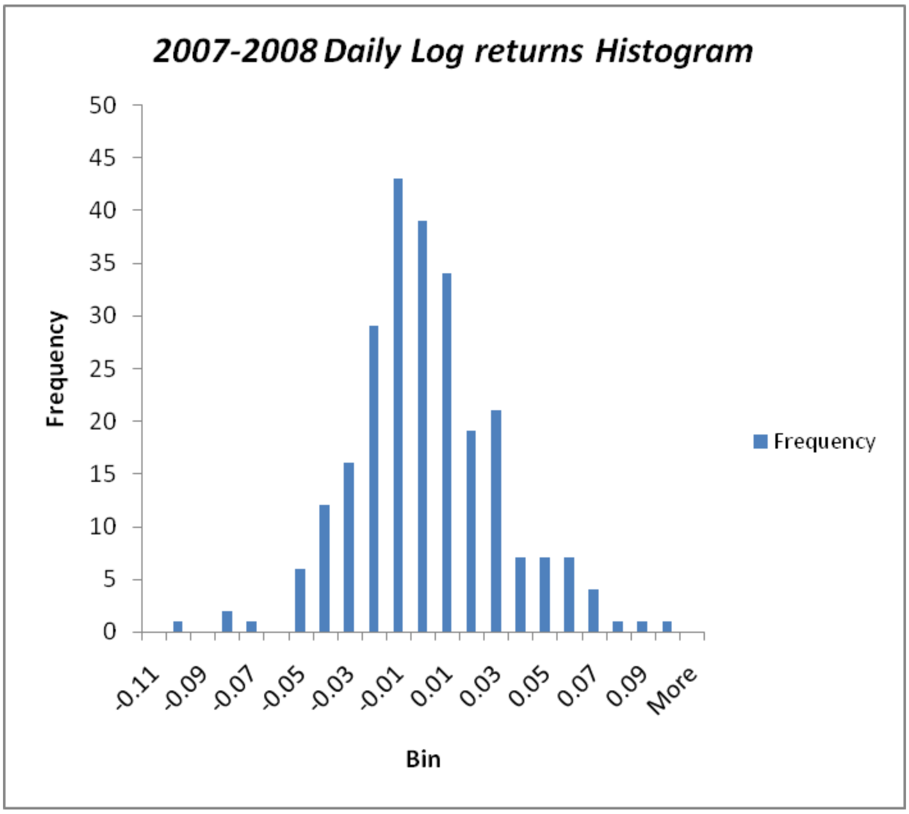

Figure 12

2006-2007:

Table 4:

	Simple Returns	Log Returns
Min.	-0.155074808	-0.168507186
Max.	0.129704985	0.121956523
Mean	0.00062236	0.000405856
St.Dev.	0.02073535	0.020916132
Skewness	-0.491518638	-1.141744009
Excess Kurtosis	18.37675064	21.11350663

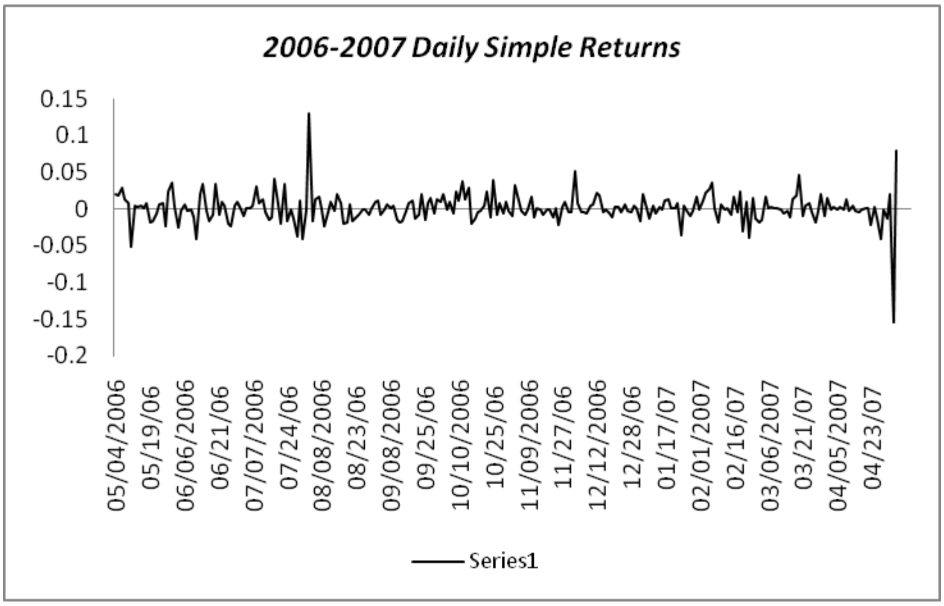

Figure 13

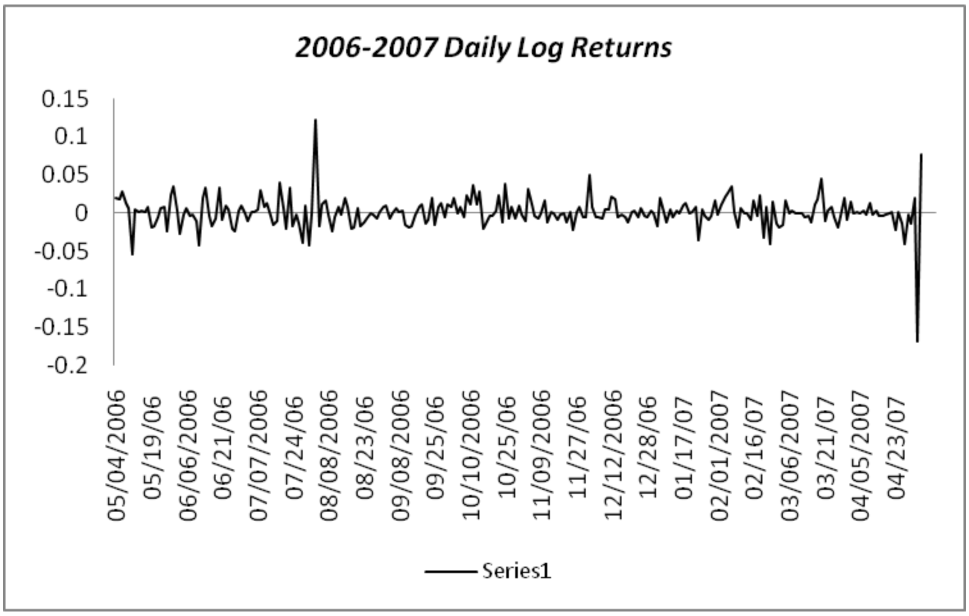

Figure 14

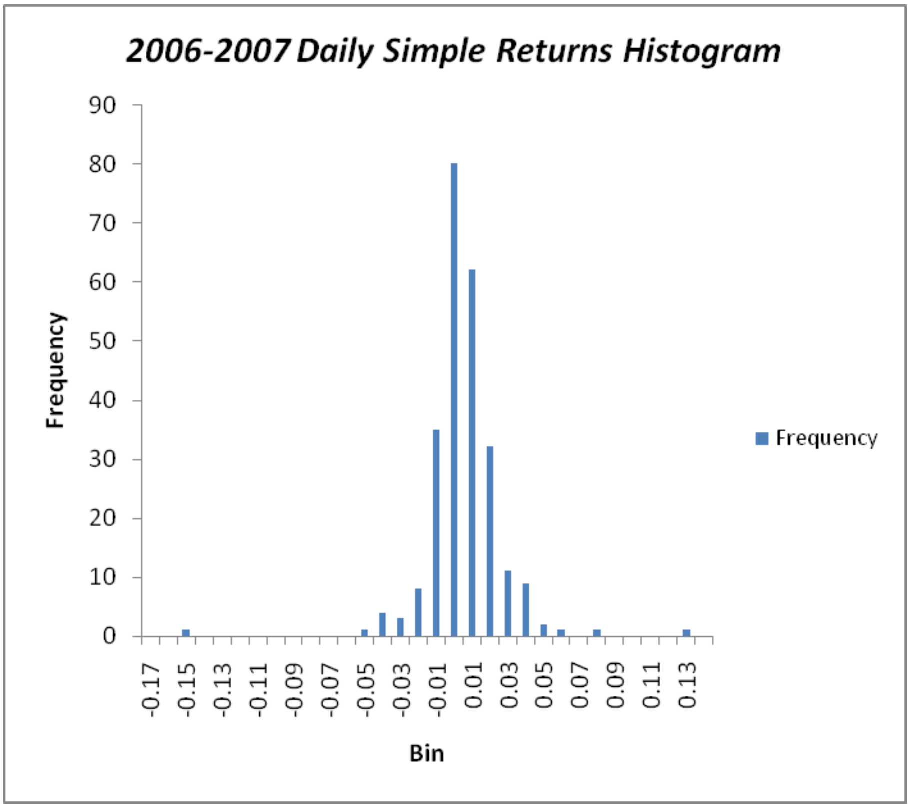

Figure 15

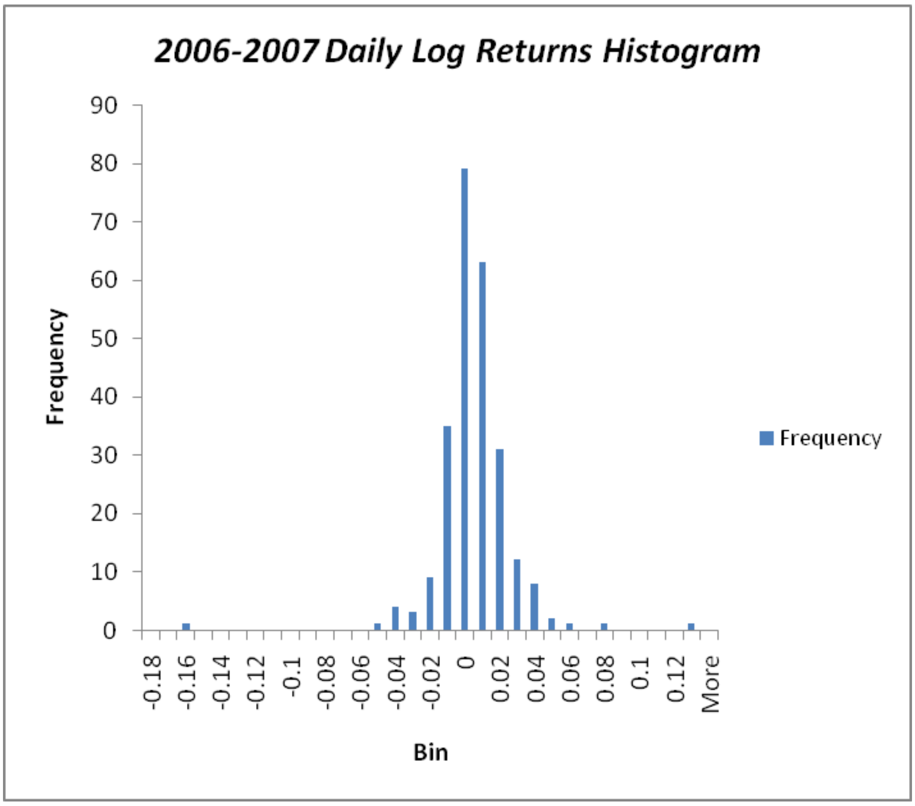

Figure 16

2005-2006:

Table 5:

	Simple Returns	Log Returns
Min.	-0.115188953	-0.122381164
Max.	0.154122939	0.143340695
Mean	0.001078188	0.000870182
St.Dev.	0.020616813	0.020327172
Skewness	1.612040821	1.113877725
Excess Kurtosis	17.23806951	15.88216401

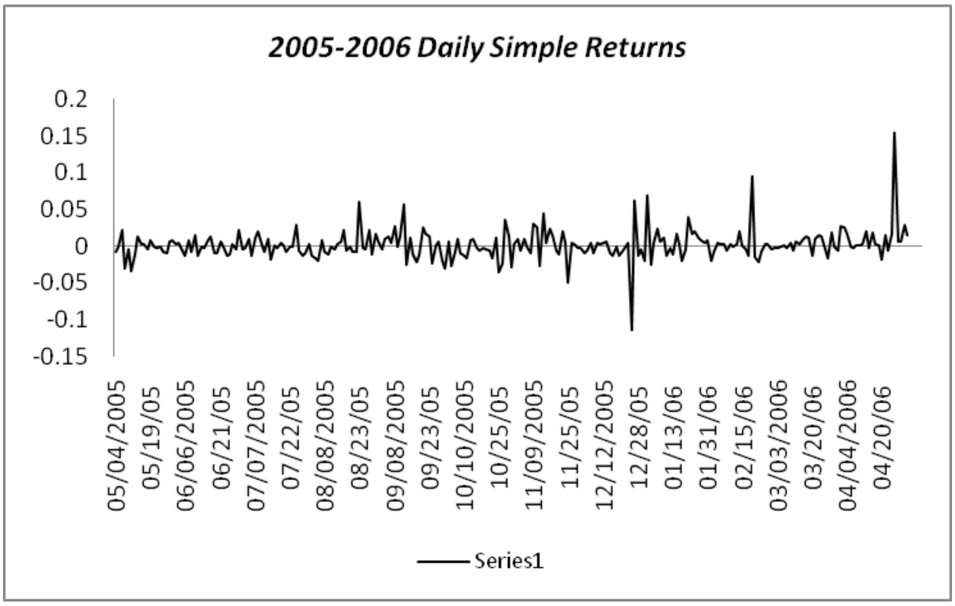

Figure 17

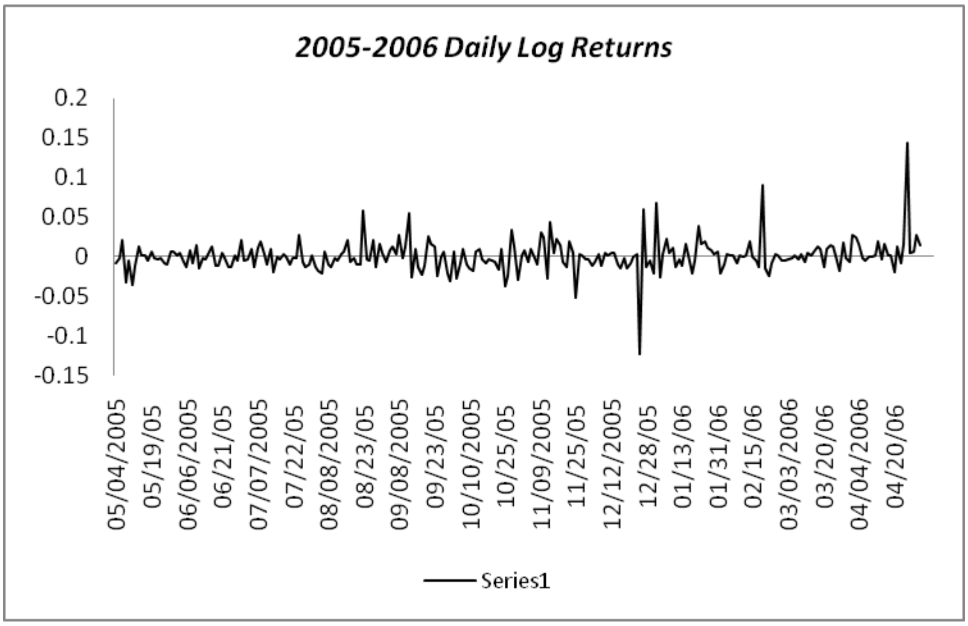

Figure 18

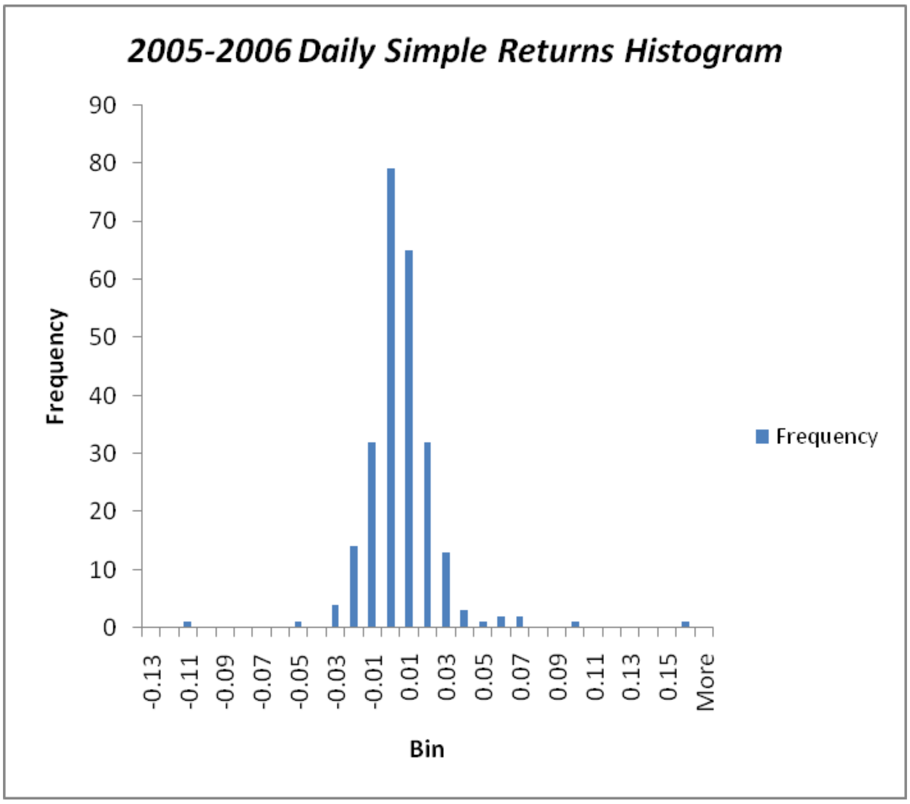

Figure 19

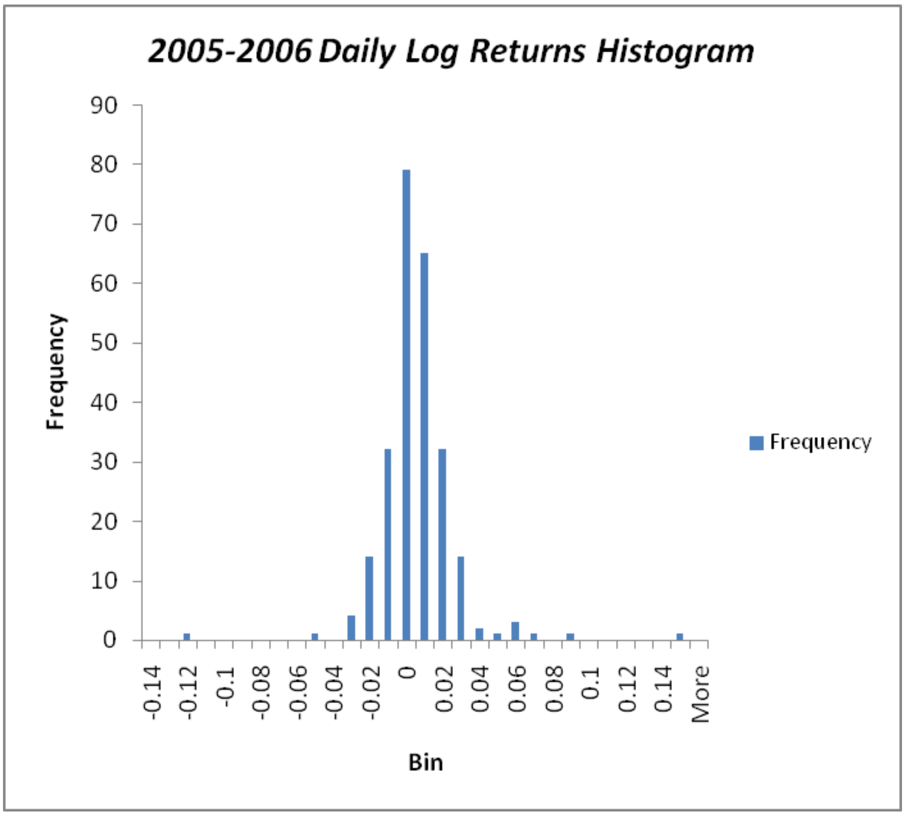

Figure 20

2003-2005:

Because of lack of complete data in 2004 we chose 252 days from 2003 and 2005:

Table 6:

	Simple Returns	Log Returns
Min.	-0.072072072	-0.074801213
Max.	0.197771588	0.18046282
Mean	0.002563105	0.002278934
St.Dev.	0.024252997	0.023546409
Skewness	2.640201053	2.120539526
Excess Kurtosis	20.76461687	16.92553841

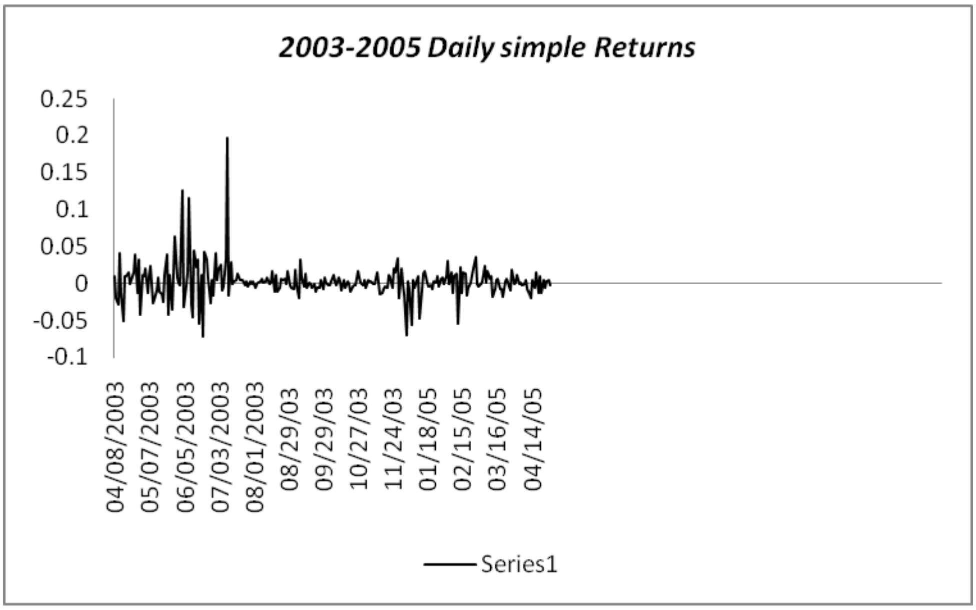

Figure 21

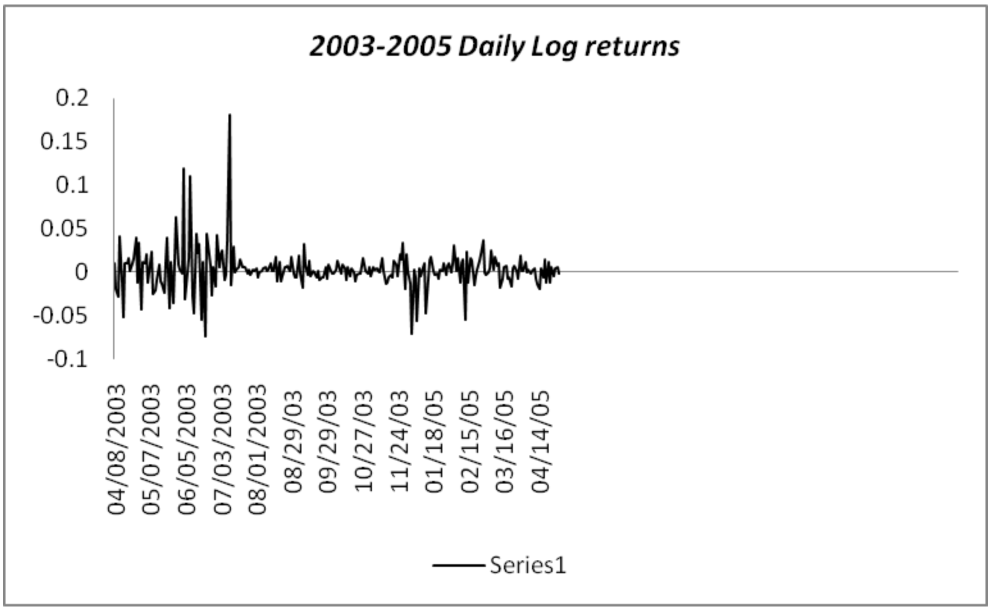

Figure 22

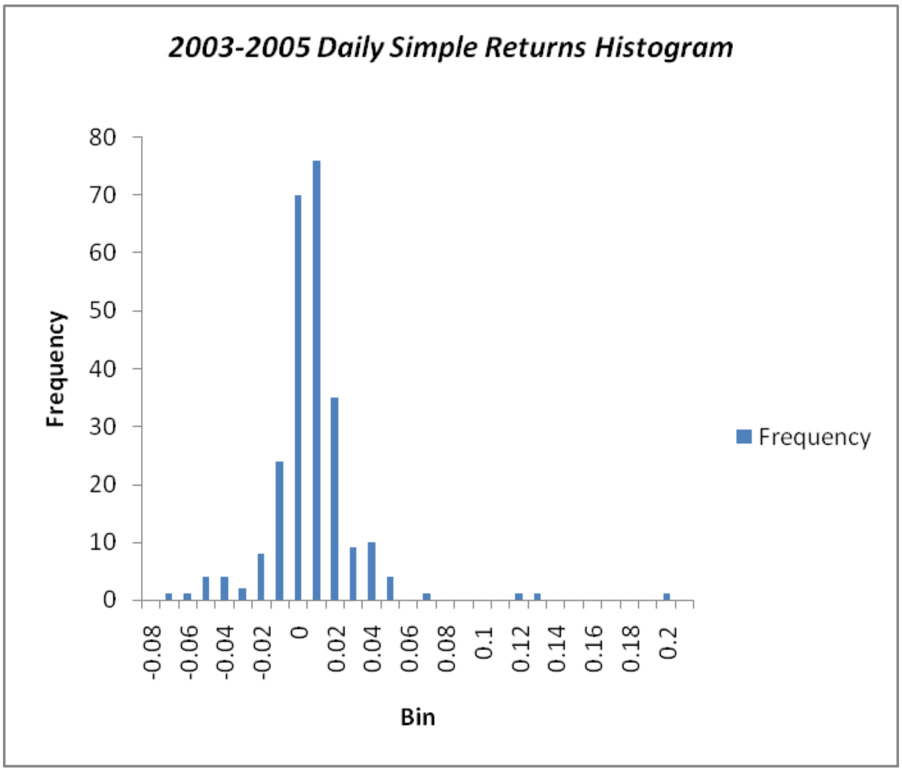

Figure 23

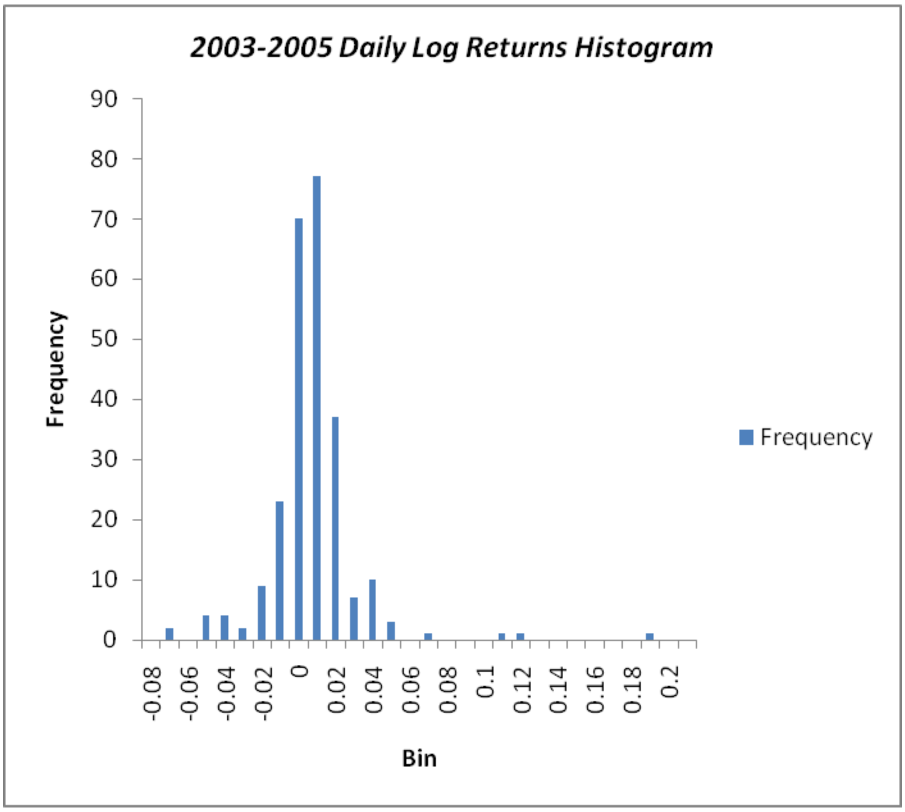

Figure 24

By comparing the above simple and log returns plots in each period separately, it is clear that they are pretty similar to each other, which we expected by the similarity of their formulas.

5.2 Empirical Study of HS and VaR Computation by HS Method

As an empirical study of HS, we simulated[24] the price (outcome) of 12 weeks (here ‘week’ means 5 business days) i.e. 12 series of ‘5-day horizon’ in 2009 and 2008.

In 2008 we started by data ‘from 1-2-2008 to 12-31-2008’ (including financial crisis), simulating price of one week after 12-31-2008, then go one week further in our data i.e. '1-9-2008 to 1-8-2009’ and again simulating one week after the last date of our data and so on, for 5000 times, with the initial price equal to 100.[25]

In 2007 we started by ‘1-3-2007 to 1-3-2008’ and continue by repeating the above process, for simulating prices in 12 weeks.

From these simulated horizons we chose the 5th day simulated prices, took the average of them, to receive the ‘forecasting price’.

Also we computed the actual prices of these days and 1%VaR for each week (5-day horizon).

Here are the results:

[24] By MATLAB coding of HS method, available at appendix A.
[25] MM-DD-YYYY.

5.2.1 Empirical Distributions, Forecasting Prices and 1%VaR for 5-Day Horizon[26]

2009:

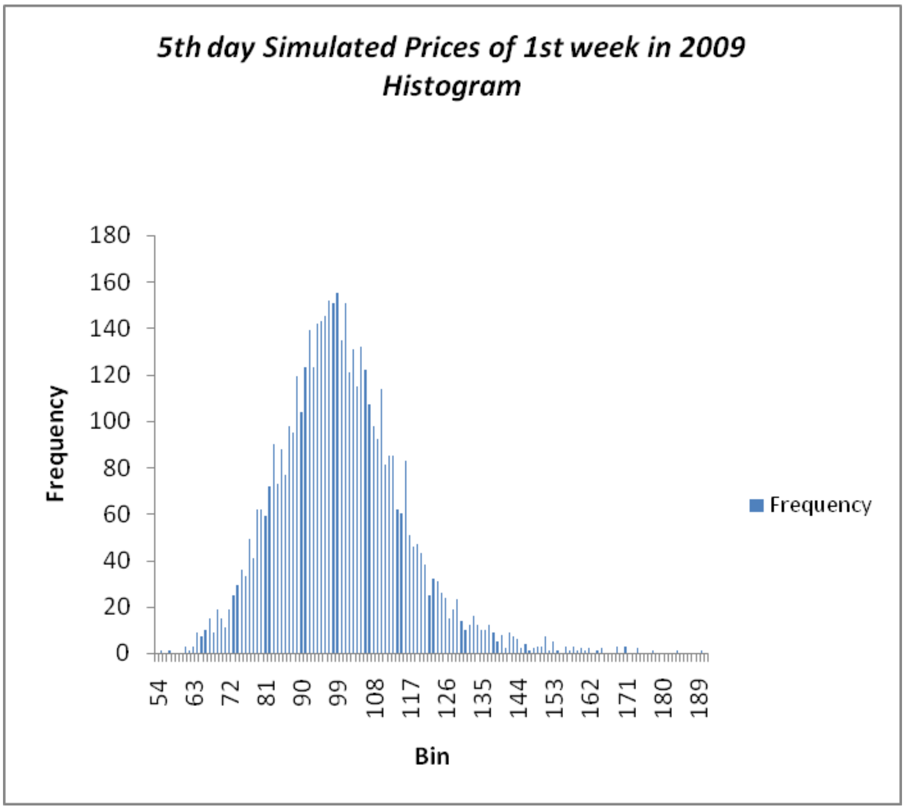

Figure 25

Forecasting price of the 5th day (**1st** week): 99.43682672

1% VaR: 33

[26] As the received prices by using simple returns and log returns are really close to each other , we will only use simple returns in our calculations.

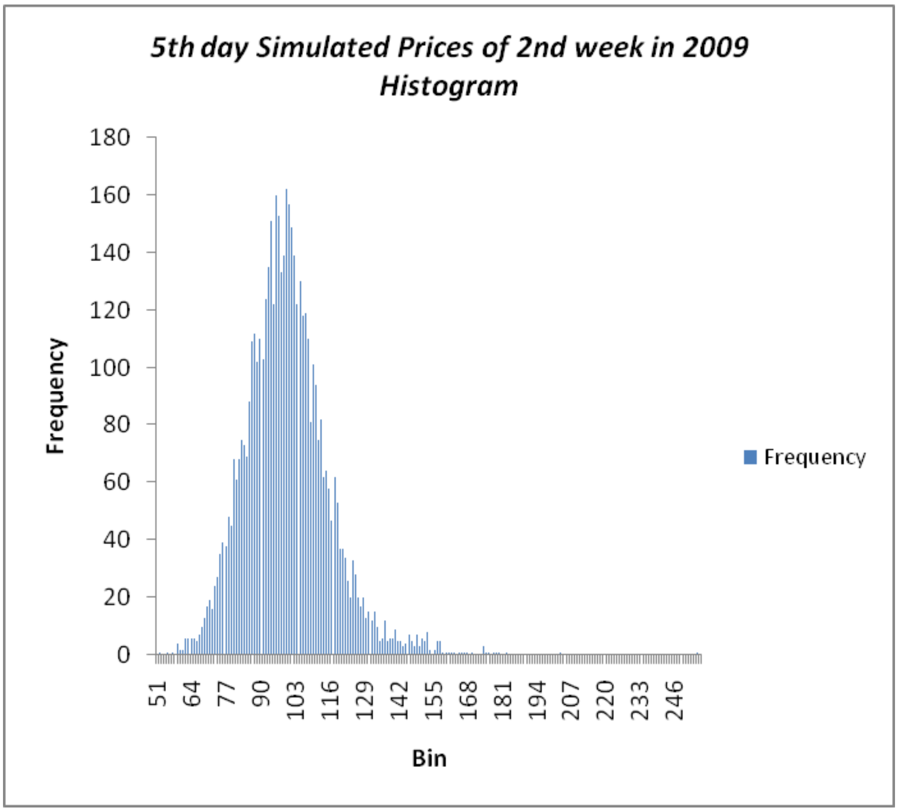

Figure 26
Forecasting Price of the 5th day (**2nd** week): 99.75251536
1% VaR: 32.7

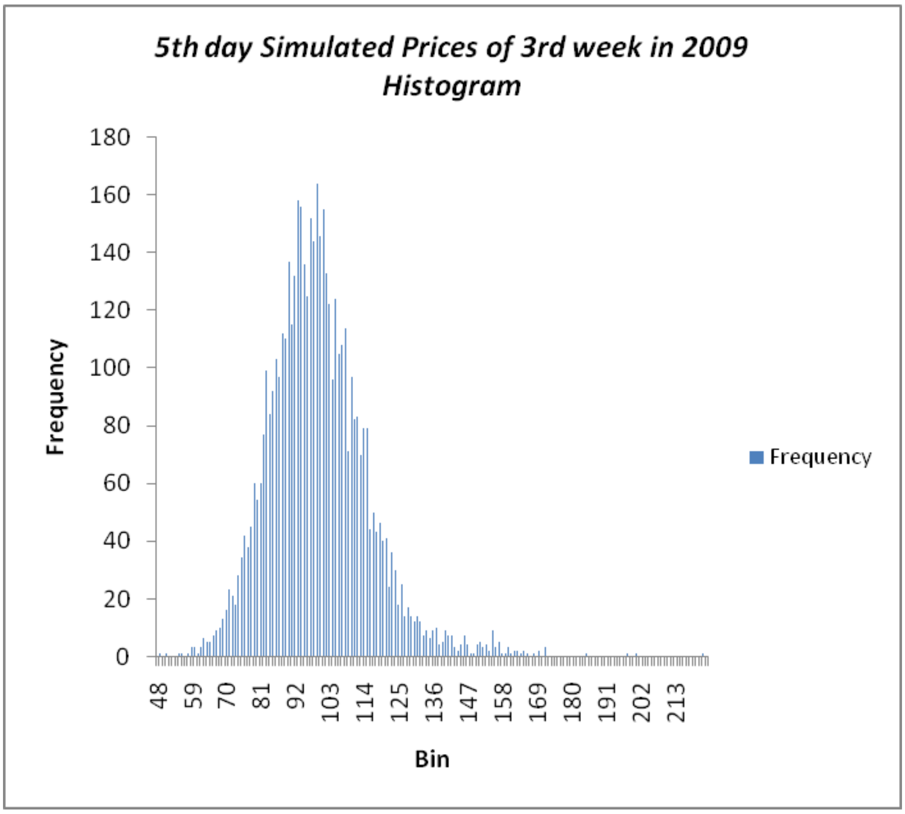

Figure 27
Forecasting Price of the 5th day (**3rd** week): 98.96588739
1% VaR: 32.7

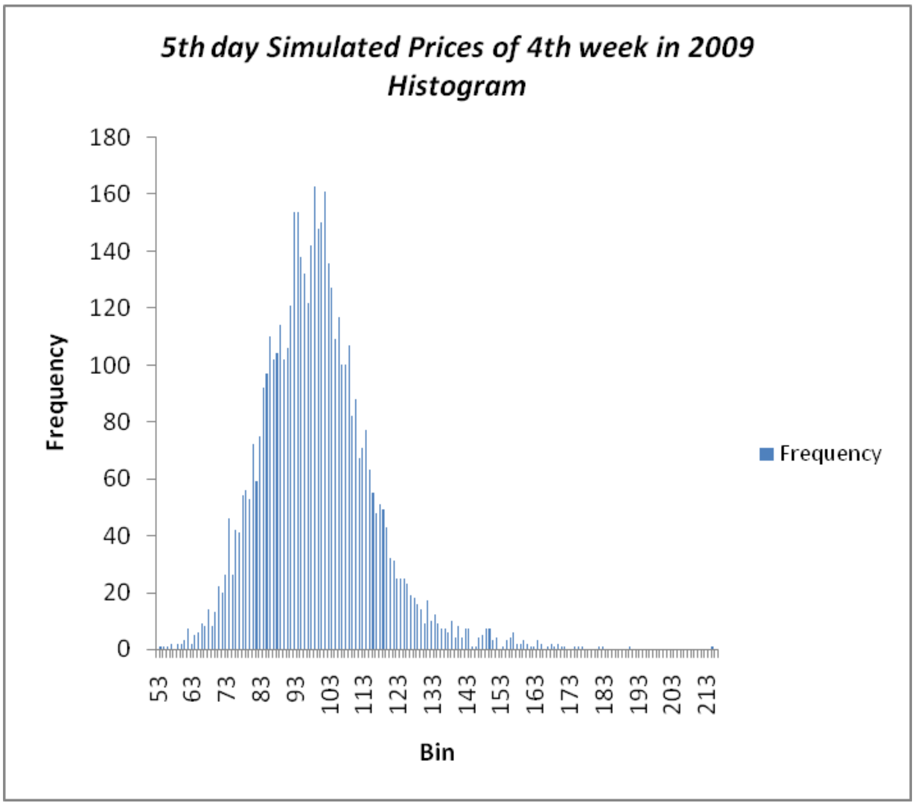

Figure 28
Forecasting price of the 5th day (**4th** week): 99.20445066
1% VaR: 32.93

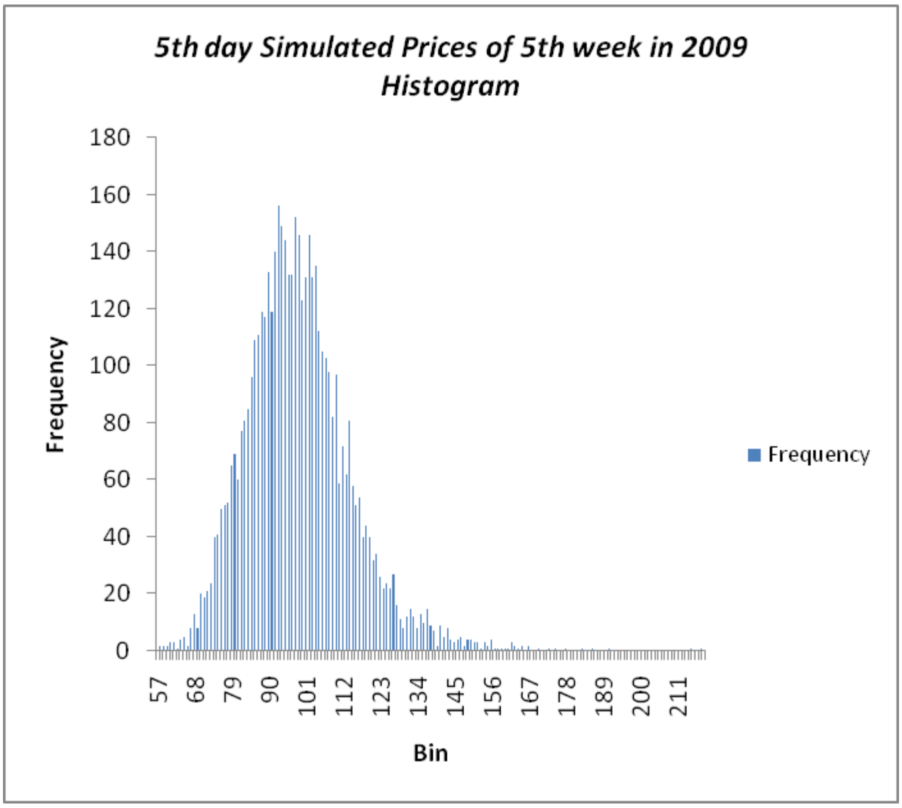

Figure 29
Forecasting price of the 5th day (**5th** week): 98.57989108
1% VaR: 31.38

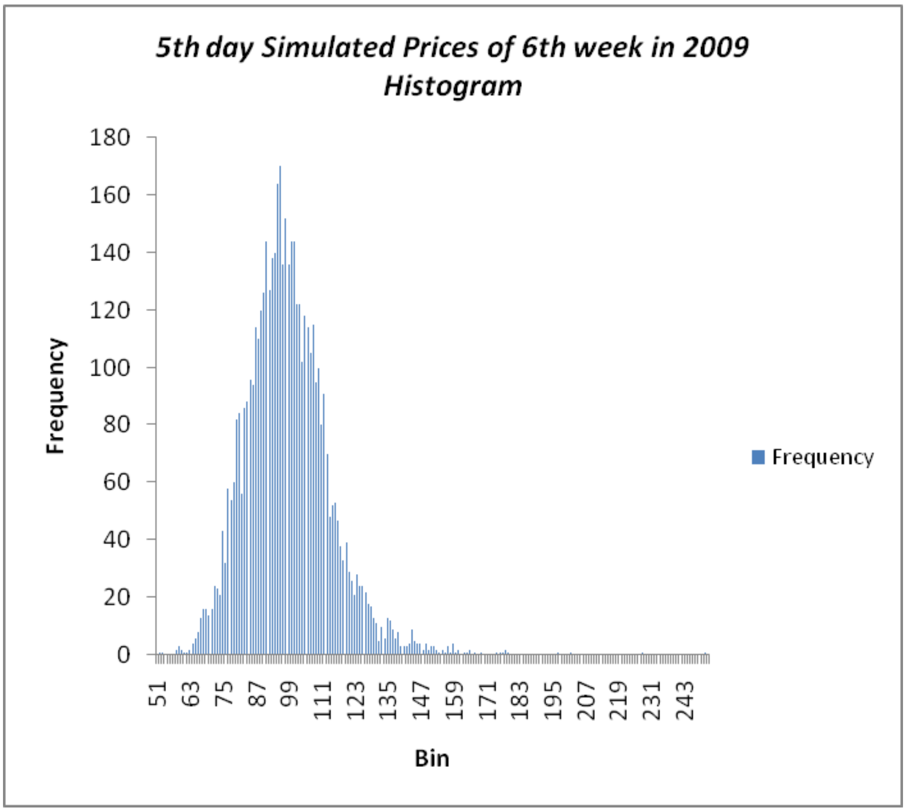

Figure 30
Forecasting price of the 5th day (**6th** week): 98.35036538
1% VaR: 32.63

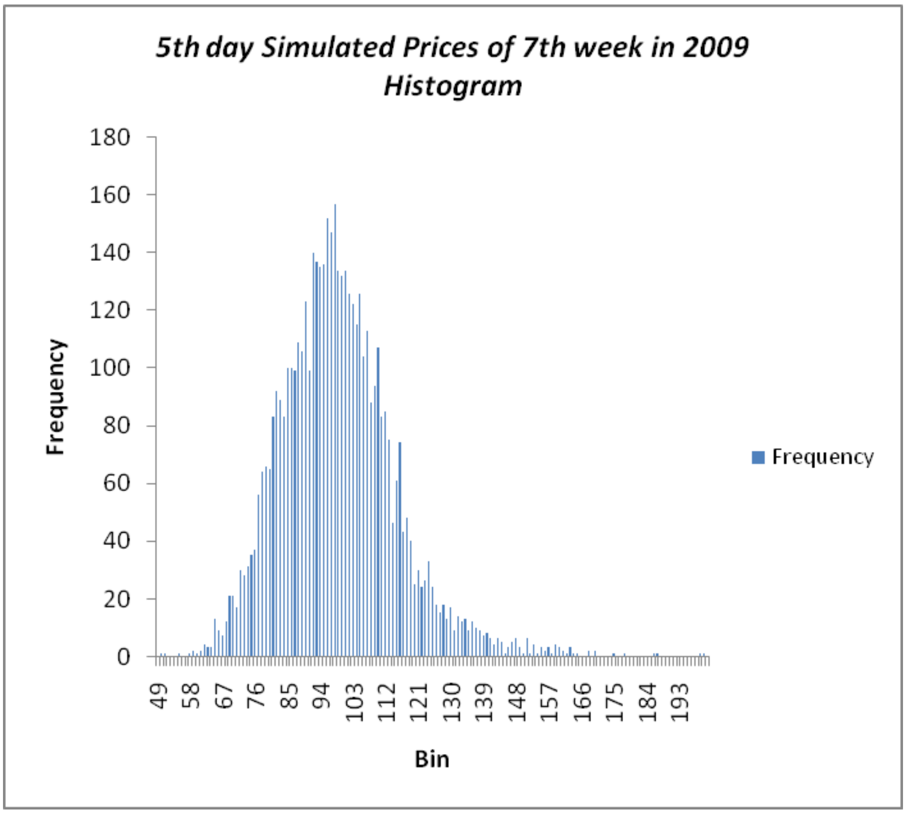

Figure 31
Forecasting price of the 5th day (**7th** week): 98.43740223
1% VaR: 32.84

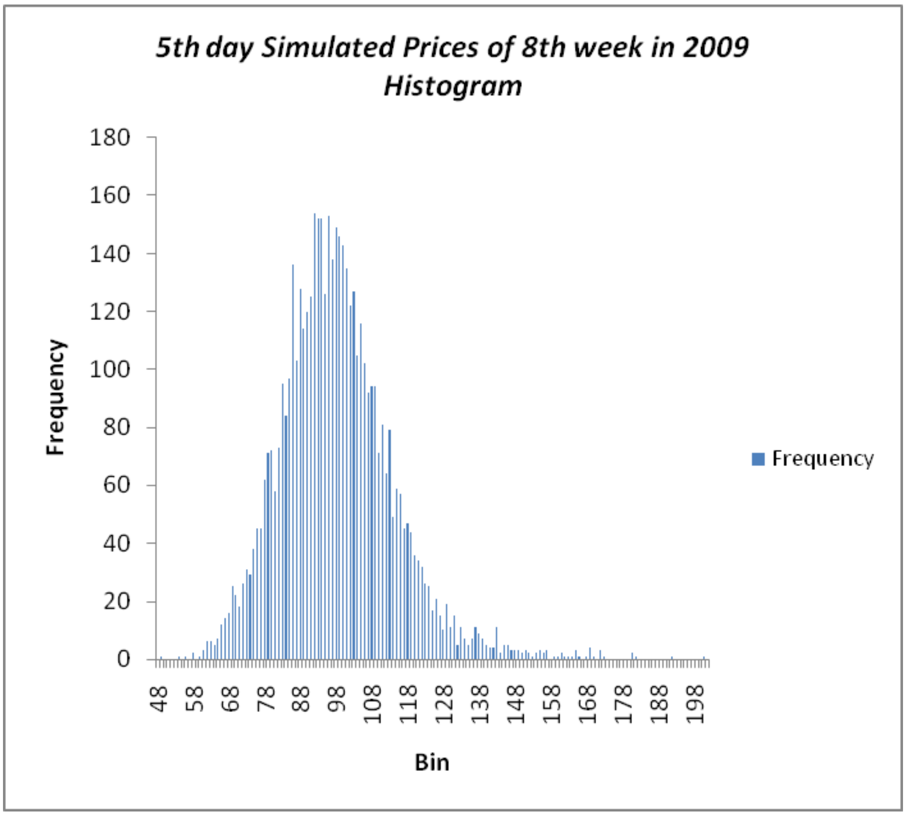

Figure 32
Forecasting price of the 5th day (**8th** week): 97.79964178
1% VaR: 33.65

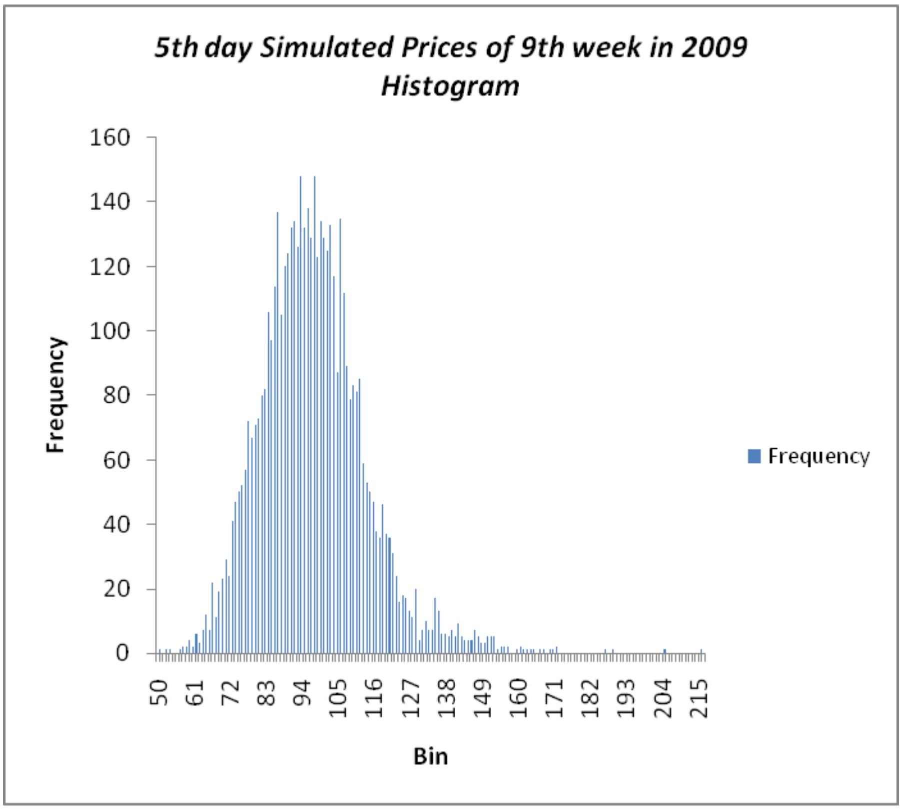

Figure 33
Forecasting price of the 5th day (**9th** week): 97.24849112
1% VaR: 33.96

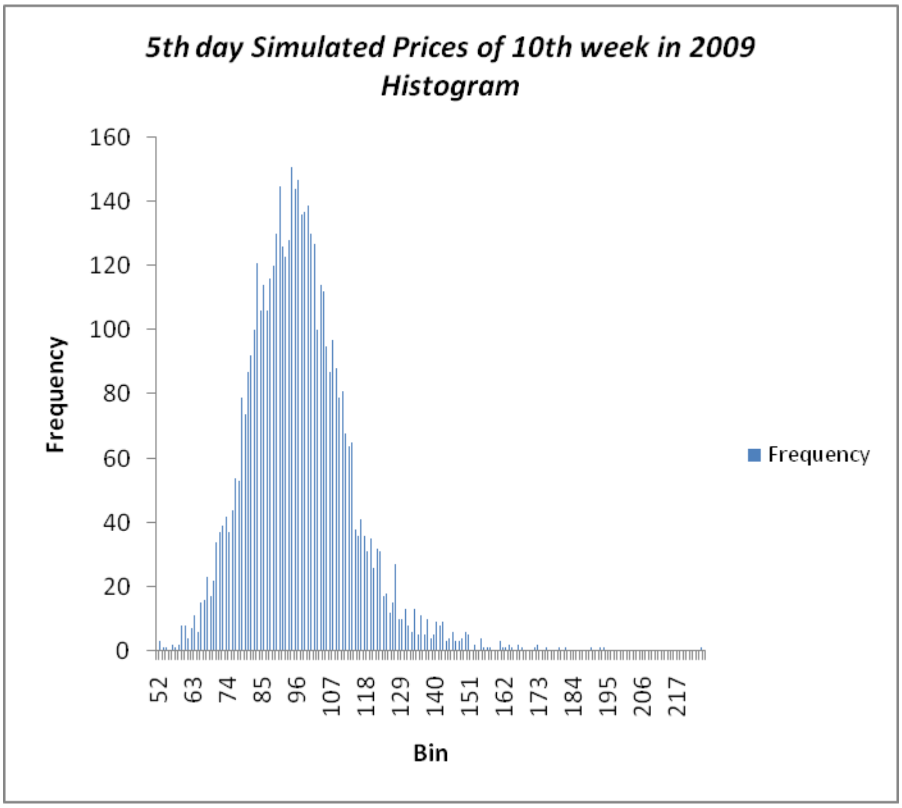

Figure 34
Forecasting price of the 5th day (**10th** week): 96.809153
1% VaR: 51.67

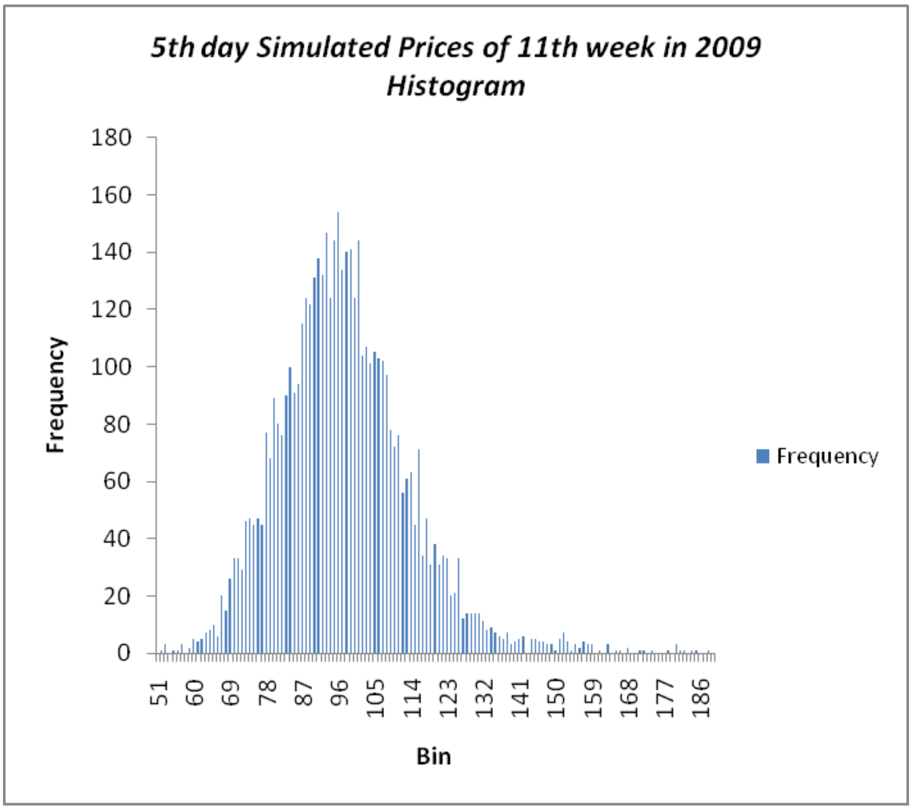

Figure 35
Forecasting price of the 5th day (**11th** week): 97.05947118
1% VaR: 35

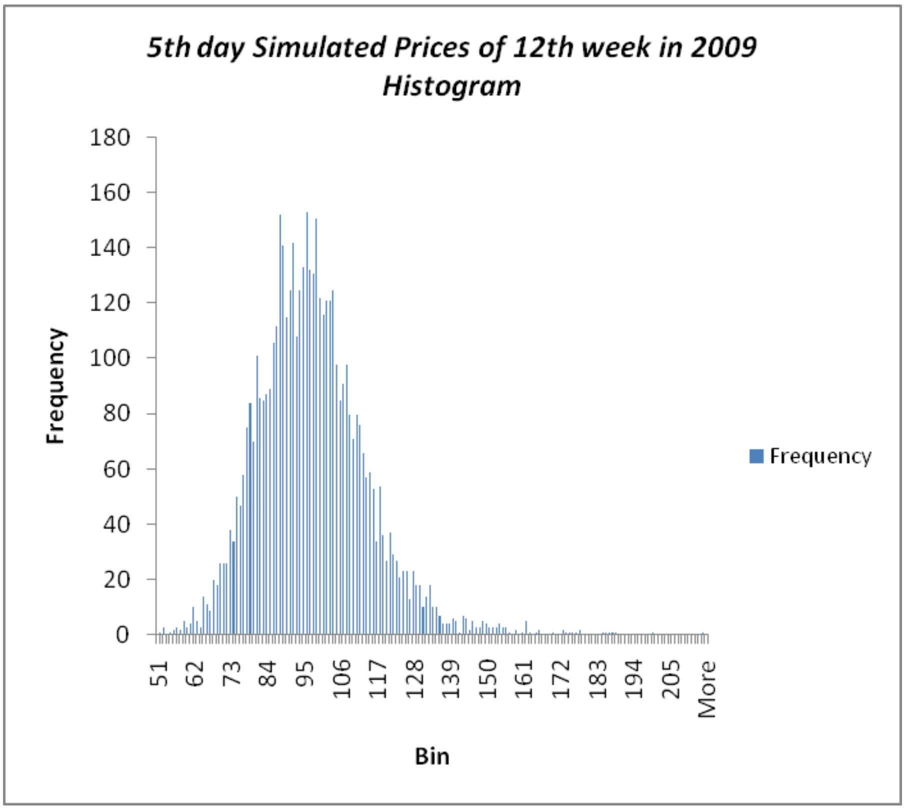

Figure 36
Forecasting price of the 5th day (**12th** week): 97.32013172
1% VaR: 35.43

2008:

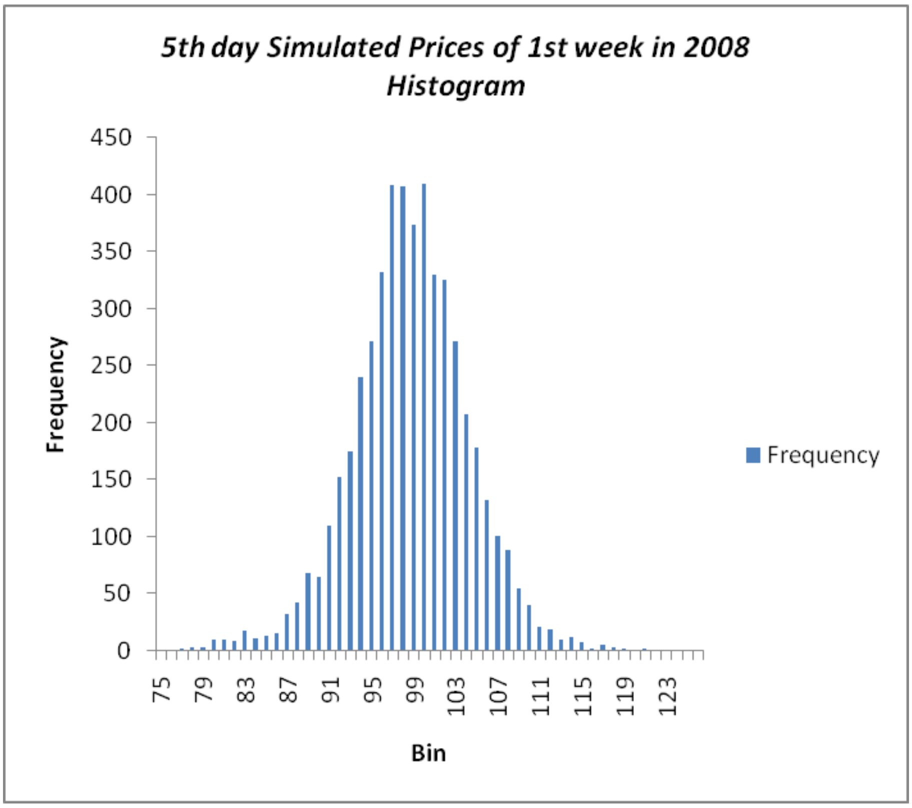

Figure 37

Forecasting price of the 5th day (**1st** week): 98.26123233

1% VaR: 17.34

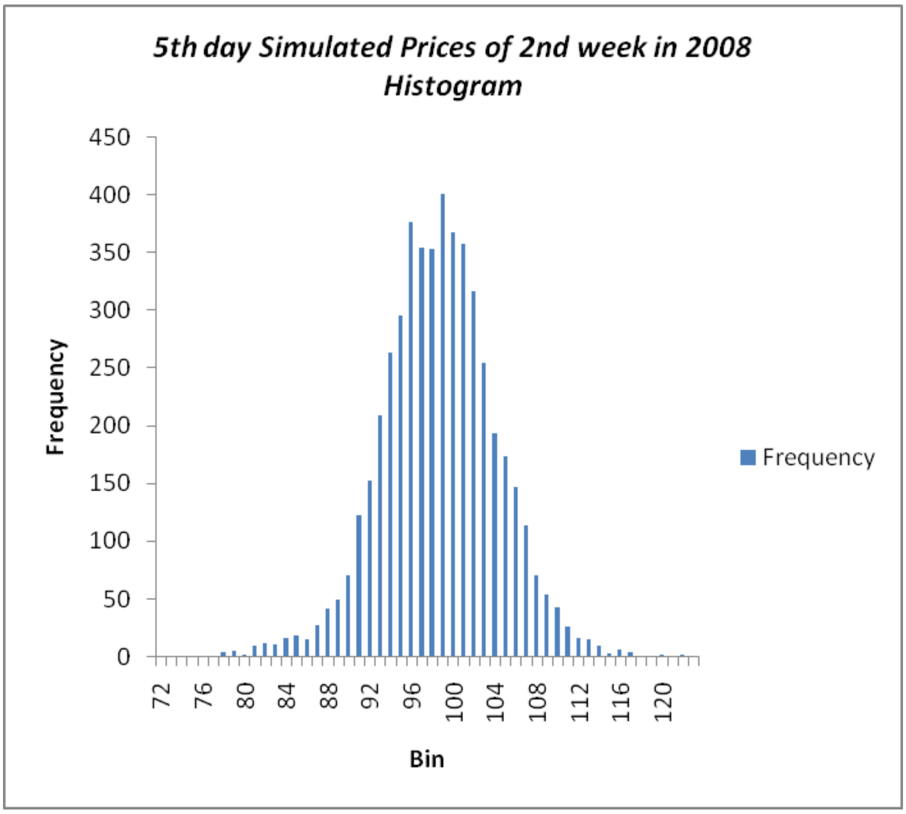

Figure 38

Forecasting price of the 5th day (**2nd** week): 98.18095149

1% VaR: 16.75

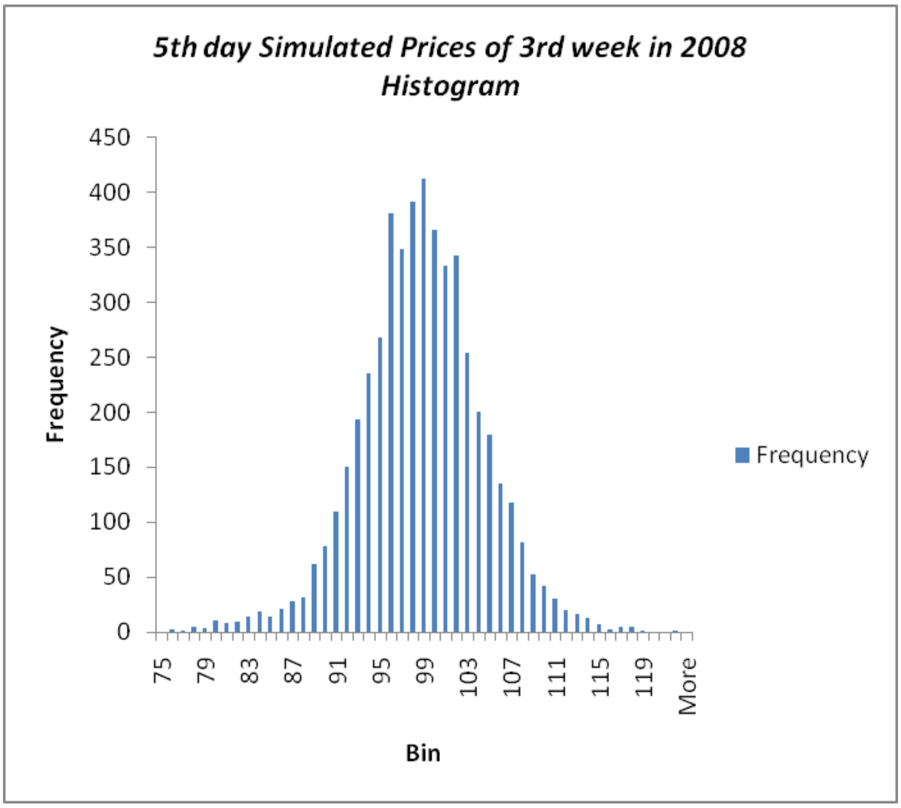

Figure 39

Forecasting price of the 5th day (**3rd** week): 98.28450787

1% VaR: 17.08

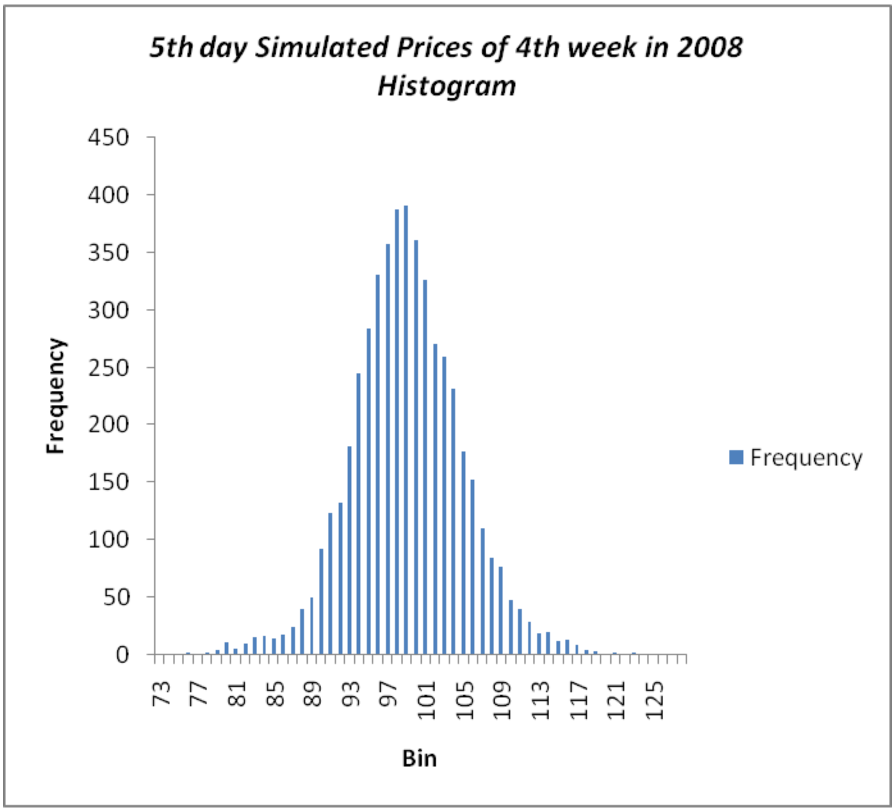

Figure 40

Forecasting price of the 5th day (**4th** week): 98.55682039

1% VaR: 17.19

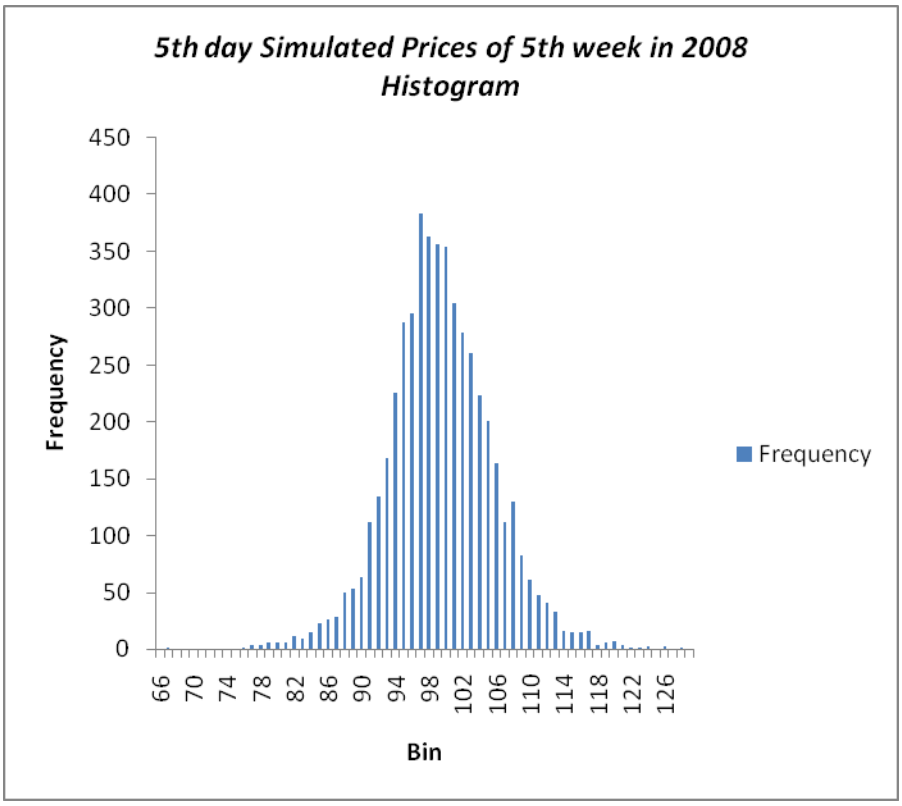

Figure 41

Forecasting price of the 5th day (**5th** week): 98.92792892

1% VaR: 16.6

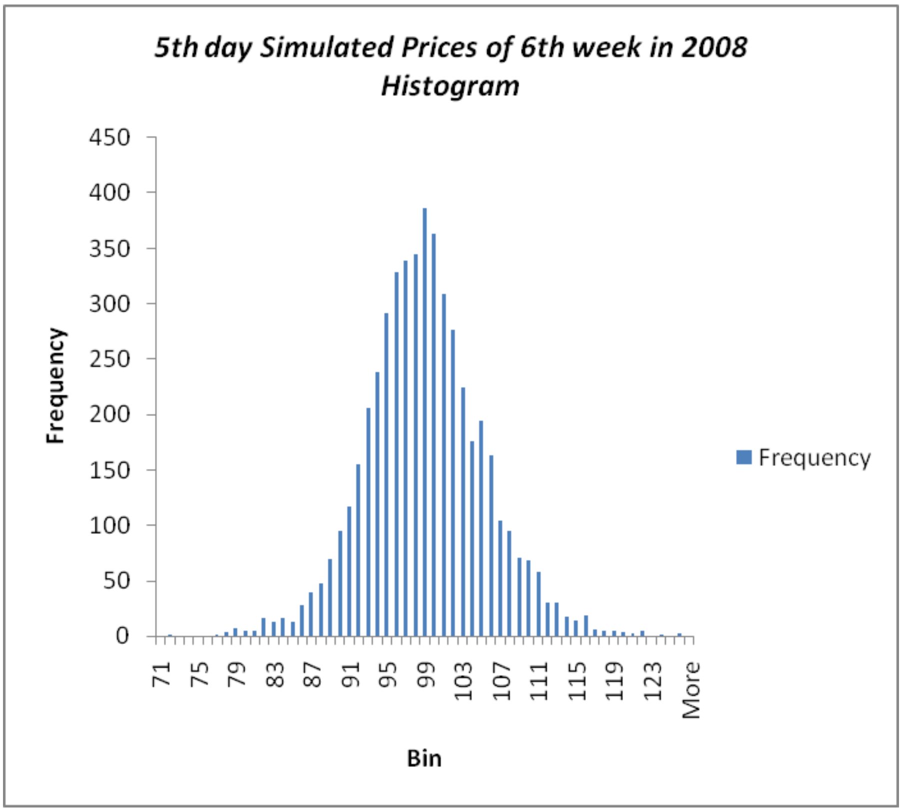

Figure 42
Forecasting price of the 5th day (**6th** week): 98.51487175
1% VaR: 16.94

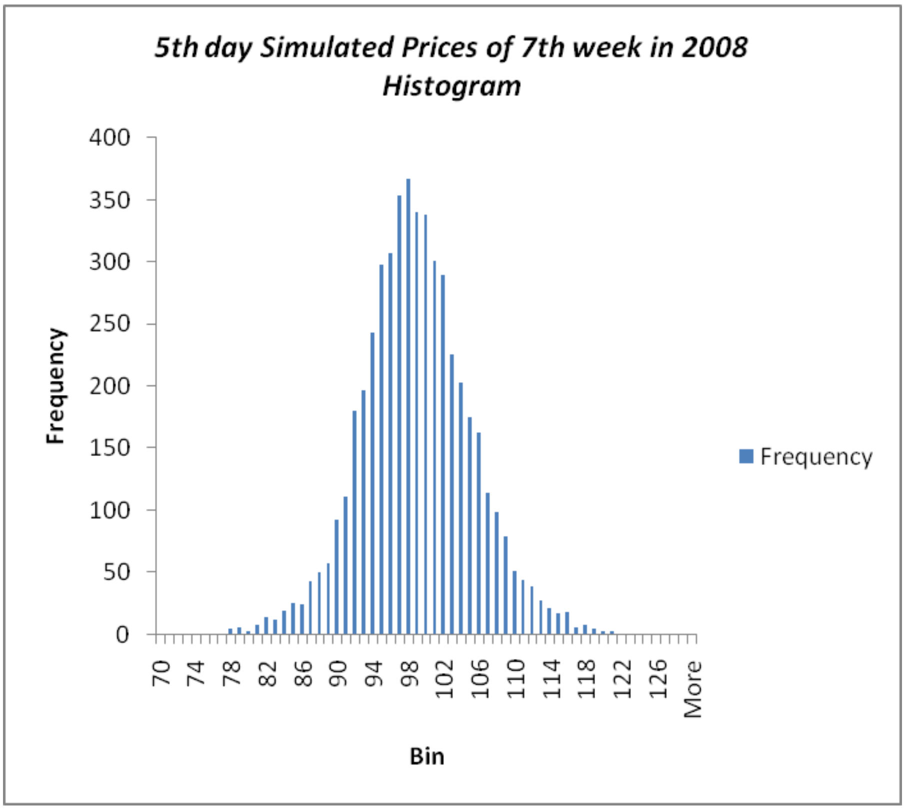

Figure 43
Forecasting price of the 5th day (**7th** week): 98.47073981
1% VaR: 17.09

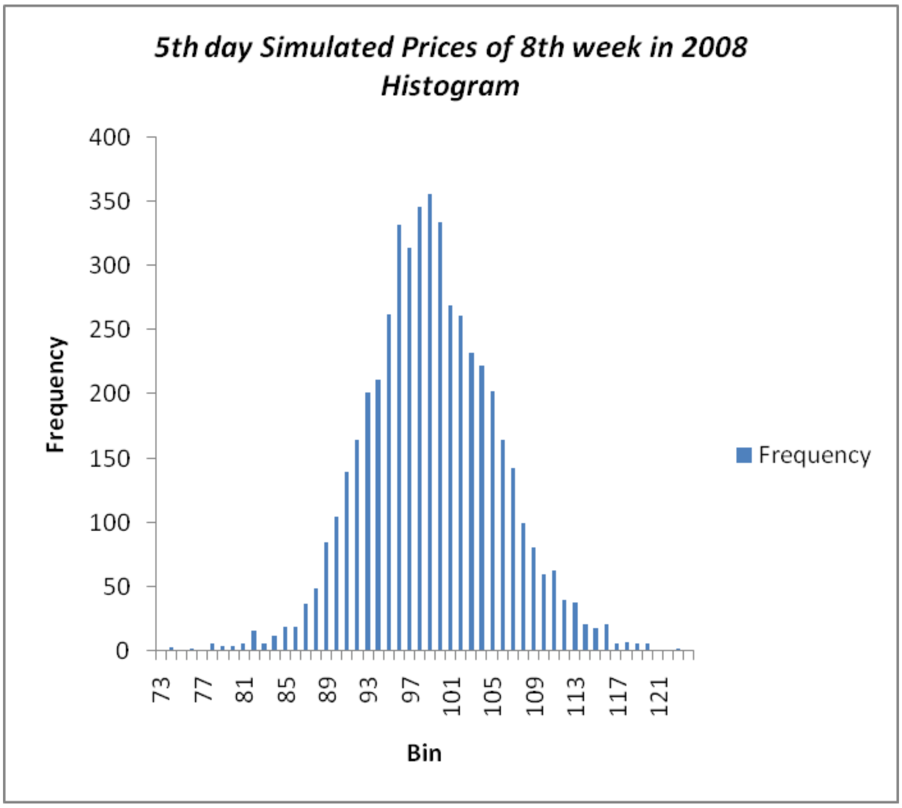

Figure 44

Forecasting price of the 5th day (**8th** week): 98.69789749

1% VaR: 16.84

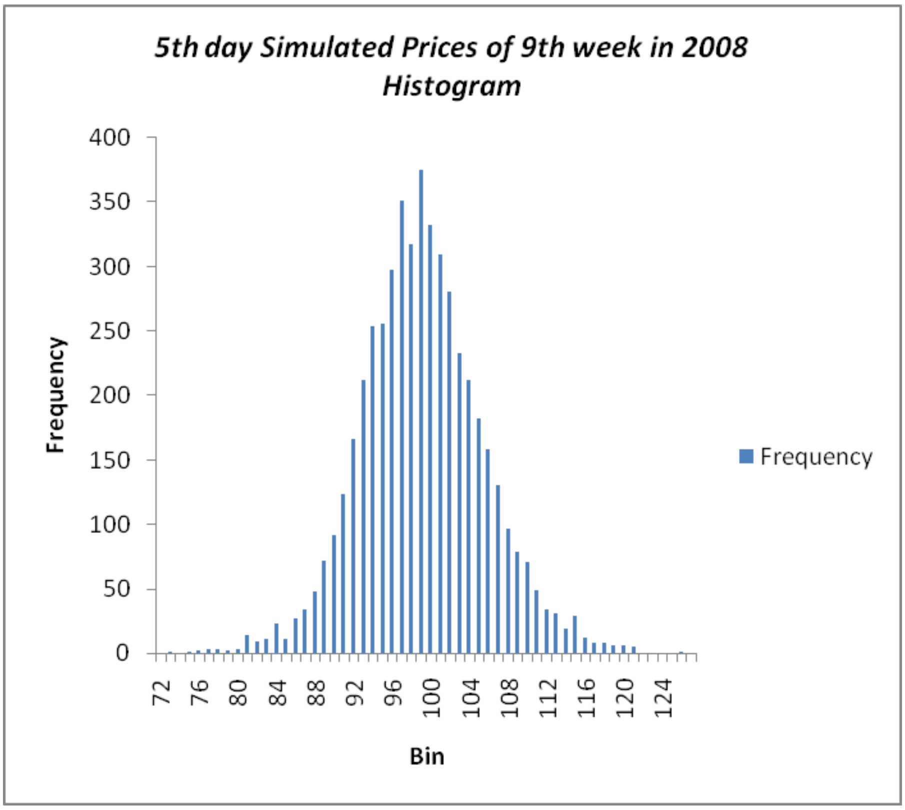

Figure 45

Forecasting price of the 5th day (**9th** week): 98.63028128

1% VaR: 16.96

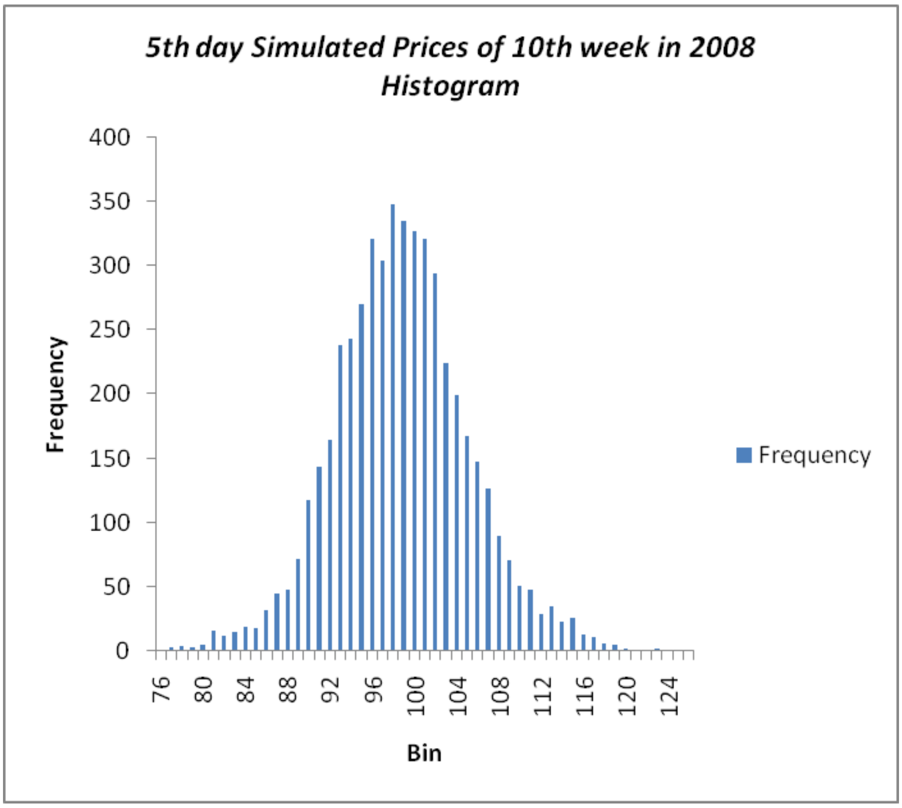

Figure 46
Forecasting price of the 5th day (**10th** week): 98.31307799
1% VaR: 17.54

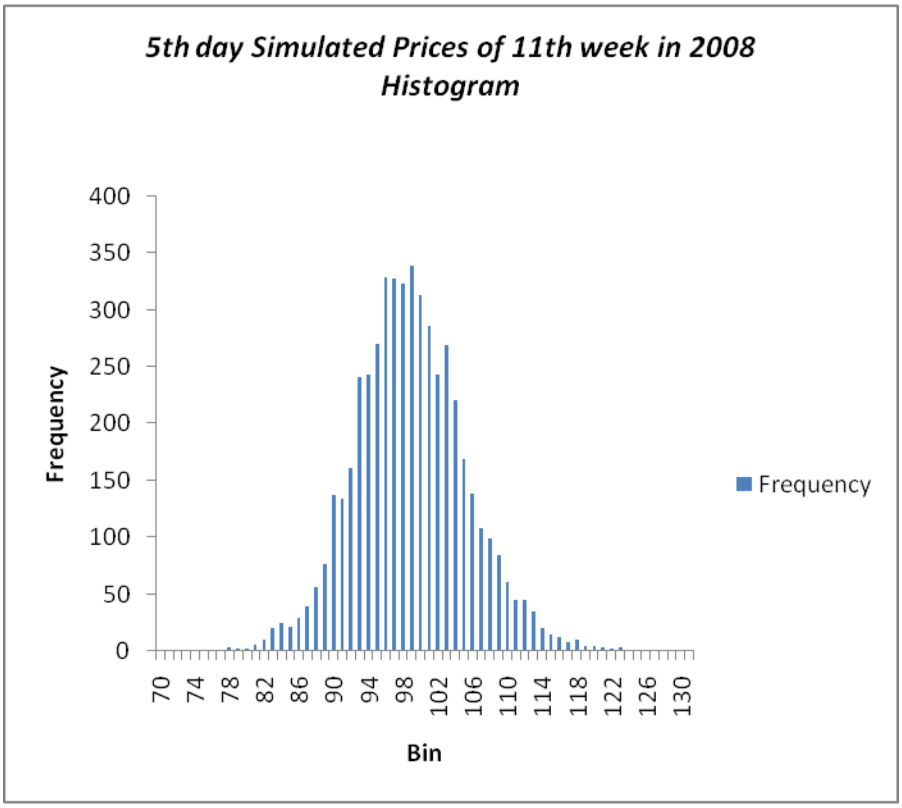

Figure 47
Forecasting price of the 5th day (**11th** week): 98.36598095
1% VaR: 16.88

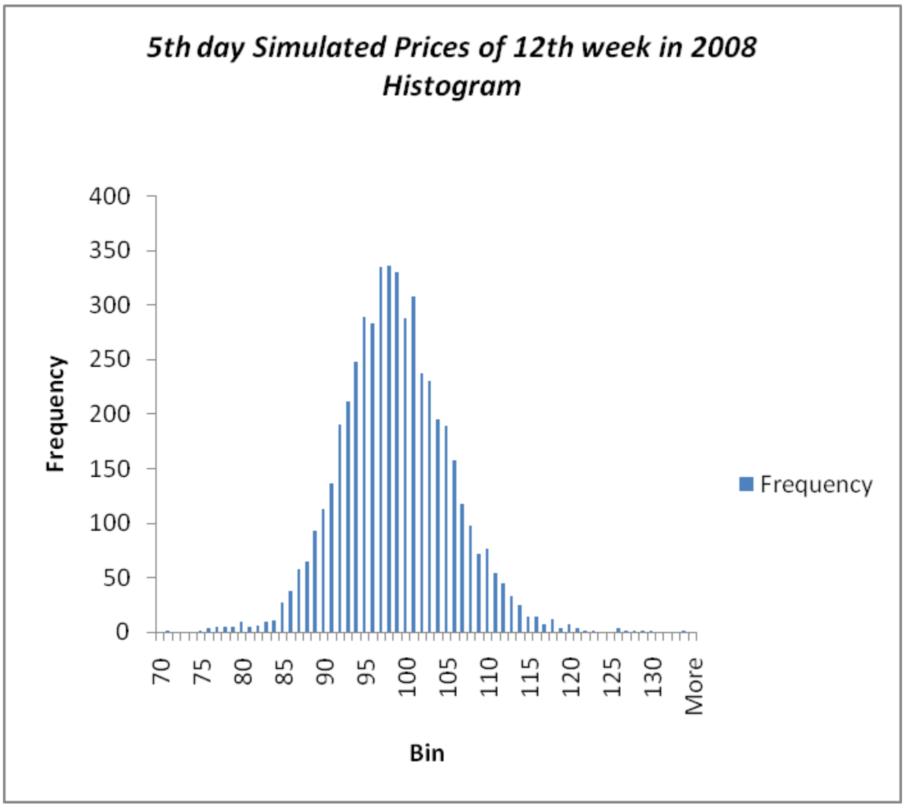

Figure 48
Forecasting price of the 5th day (**12th** week): 98.38021057
1% VaR: 16.6

5.2.2 Actual Prices

For these forecasting prices, we also calculated the actual prices, by the below formula: [2]

$$P_{s+T} = P_s(1+r_{s+1})(1+r_{s+2})...(1+r_{s+T})$$

where P_s is our initial price(outcome), r_{s+t} is the actual return (not the simulated one) of the t-th day of our horizon and P_{s+t} is our actual price of that day.

Here are the results:

- Actual prices for the 5th day of the first 12 weeks in 2009, respectively:

101.178
80.85382
92
95.82609
100.726
80.9009
85.07795
89.00524
57.35294
131.2821
116.0156
107.7441

- Actual price for the 5th day of the first 12 weeks in 2008, respectively:

98.19151
100.3175
109.6519
119.3843
95.60838
93.29962
104.878
91.25754
94.10099
96.33902
108.6934
97.07854

5.2.3 Comparison

In the below table, we gathered the actual, forecasting prices and the VaR corresponding to each, to make clear how well our simulation has worked:

Week	*Actual Prices*	*Forecasting Prices*	*VaR*
1st	98.19151	98.26123233	17.34
2nd	100.3175	98.18095149	16.75
3rd	109.6519	98.28450787	17.08
4th	119.3843	98.55682039	17.19
5th	95.60838	98.92792892	16.6
6th	93.29962	98.51487175	16.94
7th	104.878	98.47073981	17.09
8th	91.25754	98.69789749	16.84
9th	94.10099	98.63028128	16.96
10th	96.33902	98.31307799	17.54
11th	108.6934	98.36598095	16.88
12th	97.07854	98.38021057	16.6

Table 8: 1% VaR Estimates for 5-Day Horizon (HS Method, 2008)

Week	*Actual Prices*	*Forecasting Prices*	*VaR*
1st	101.178	99.43682672	33
2nd	80.85382	99.75251536	32.7
3rd	92	98.96588739	32.7
4th	95.82609	99.20445066	32.93
5th	100.726	98.57989108	31.38
6th	80.9009	98.35036538	32.63
7th	85.07795	98.43740223	32.84
8th	89.00524	97.79964178	33.65
9th	57.35294	97.24849112	33.96
10th	131.2821	96.809153	51.67
11th	116.0156	97.05947118	35
12th	107.7441	97.32013172	35.43

Table 7: 1% VaR Estimates for 5-Day Horizon (HS Method, 2009)

Above tables make clear that the HS method for measuring VaR, in this case i.e. 5-day horizon and 1% confidence level, was approximately successful, except in one item i.e. the 9th week of the 2009, where the break has occurred. (Mention that the break has happened in 2009, which the simulation has made based on the crisis period of the market)

5.3 Empirical Study of FHS and VaR Computation by FHS Method

As we explained in subsection 4.2, in this method we model our returns by an ARMA (1,1), and the random residuals by GARCH (1,1) processes. With the help of these two time series modelling, we will receive the estimated set of parameters mentioned in 4.2.2 i.e. $\hat{\lambda}$ ($\hat{\gamma}$ is assumed to be equal to 0).[27] In the next step, we followed the explained processes in 4.2.2, 4.2.3, 4.3 and obtain the predicted empirical probability distribution of prices and 1% VaR for 5-day horizon.[28](All these steps has been done for the same dates in 2008 and 2009, we did for HS)

Here are the results:

5.3.1 Empirical Distributions, Forecasting Prices and 1%VaR for 5-Day Horizon

2009:

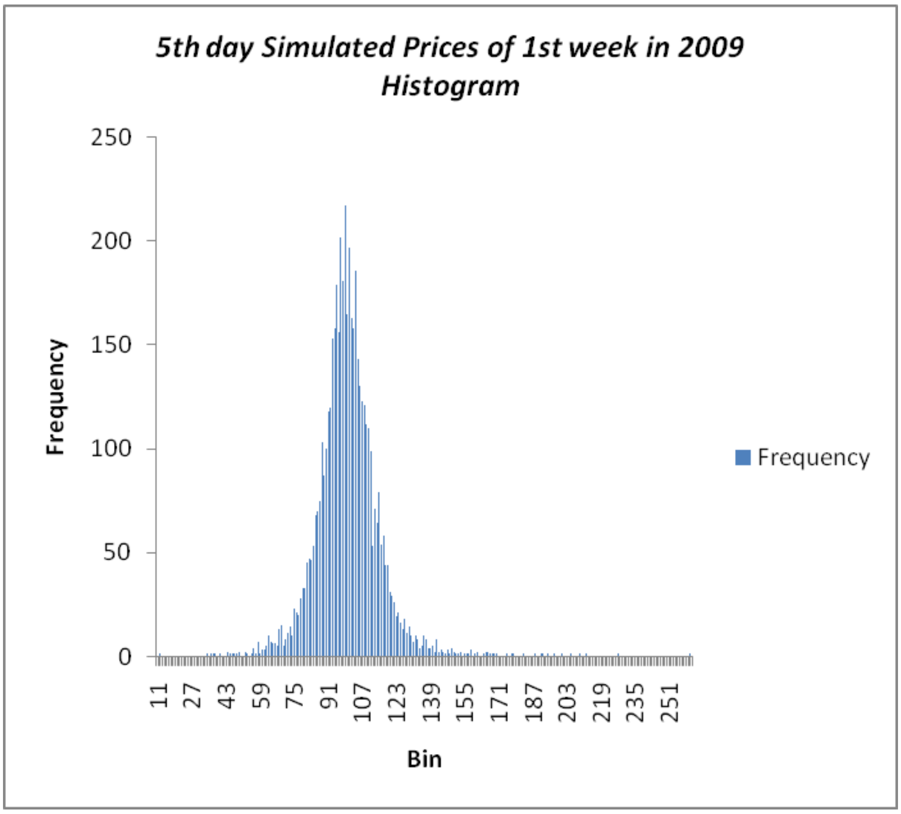

Figure 49
Forecasting price of the 5th day (**1st** week): 99.80834988
1% VaR: 37.2

[27] Using ITSM software (presented by [8])
[28] By MATLAB coding of FHS method, available at appendix A.

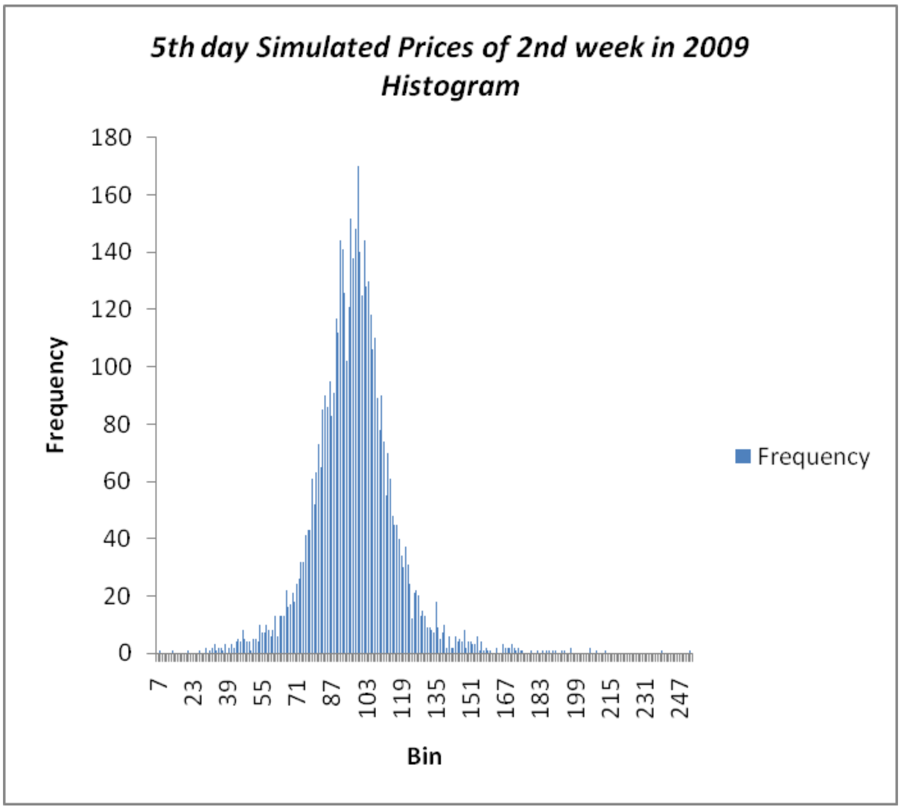

Figure 50
Forecasting price of the 5th day (**2nd** week): 96.92910459
1% VaR: 53.8

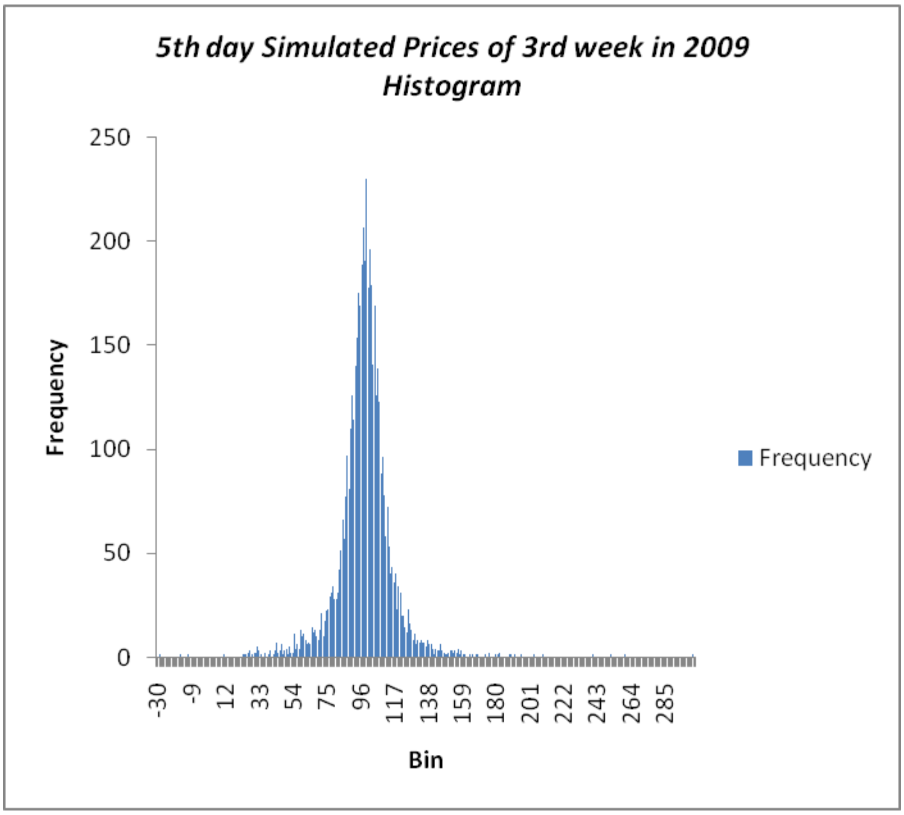

Figure 51
Forecasting price of the 5th day (**3rd** week): 98.4691806
1% VaR: 53.67

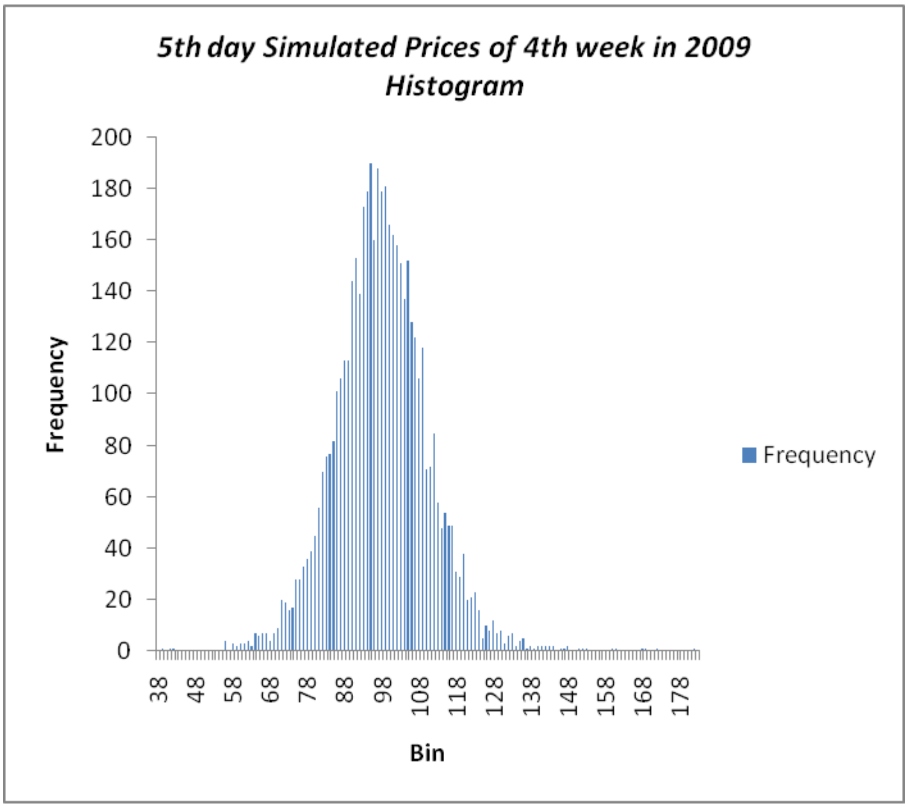

Figure 52
Forecasting price of the 5th day (**4th** week): 97.46526724
1% VaR: 33.15

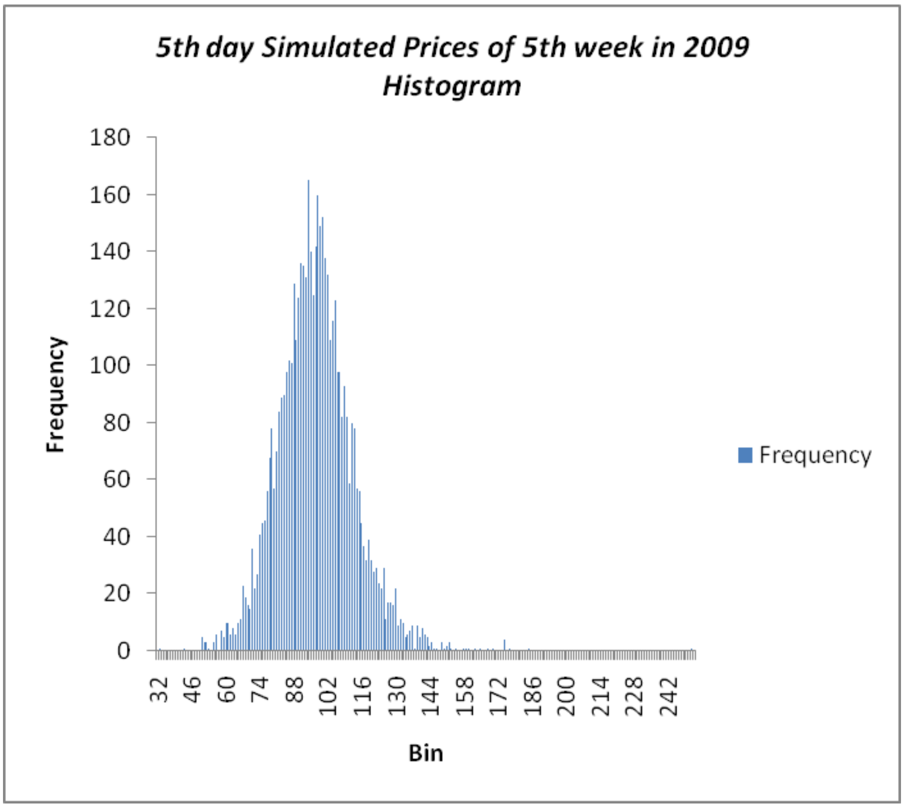

Figure 53
Forecasting price of the 5th day (**5th** week): 96.67541494
1% VaR: 39.5

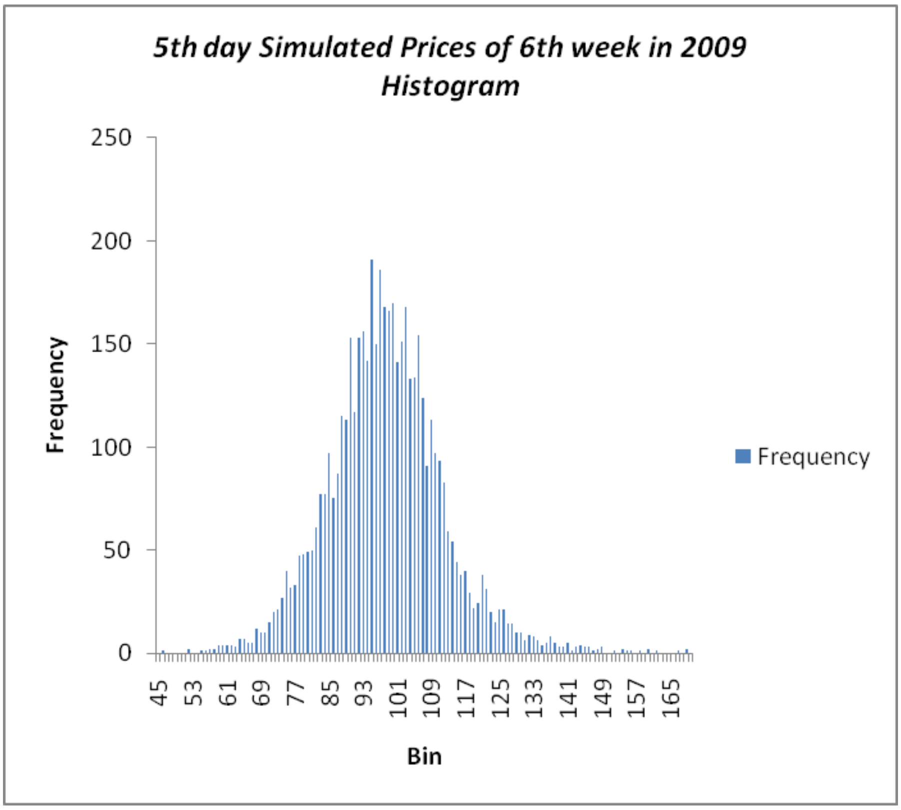

Figure 54
Forecasting price of the 5th day (**6th** week): 97.92280517
1% VaR: 33.4

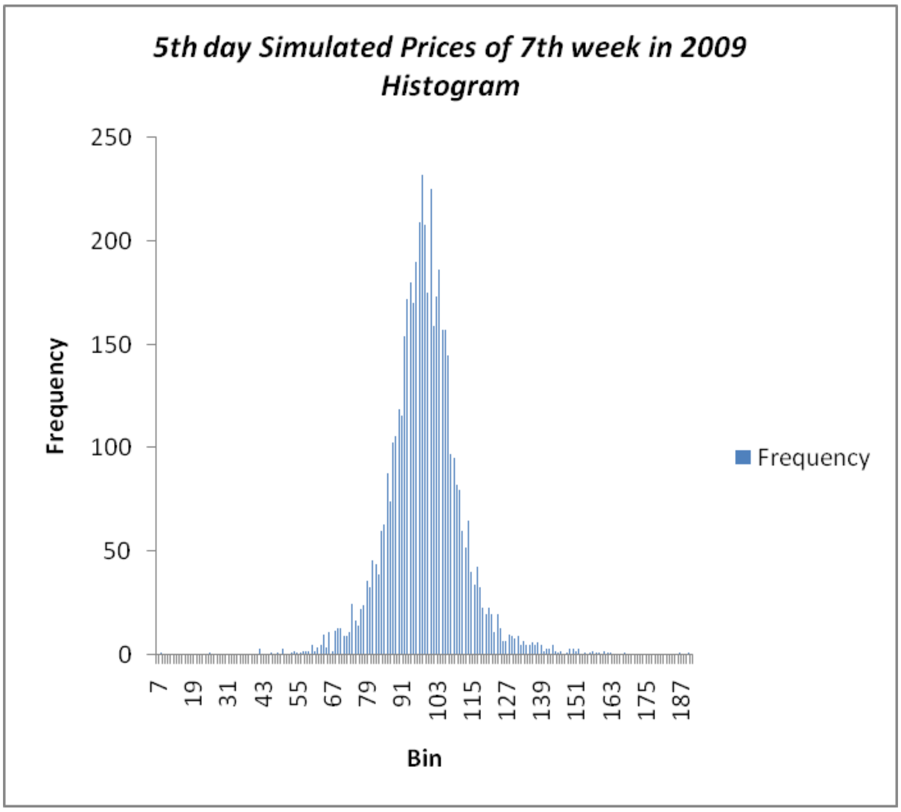

Figure 55
Forecasting price of the 5th day (**7th** week): 98.43802048
1% VaR: 35.25

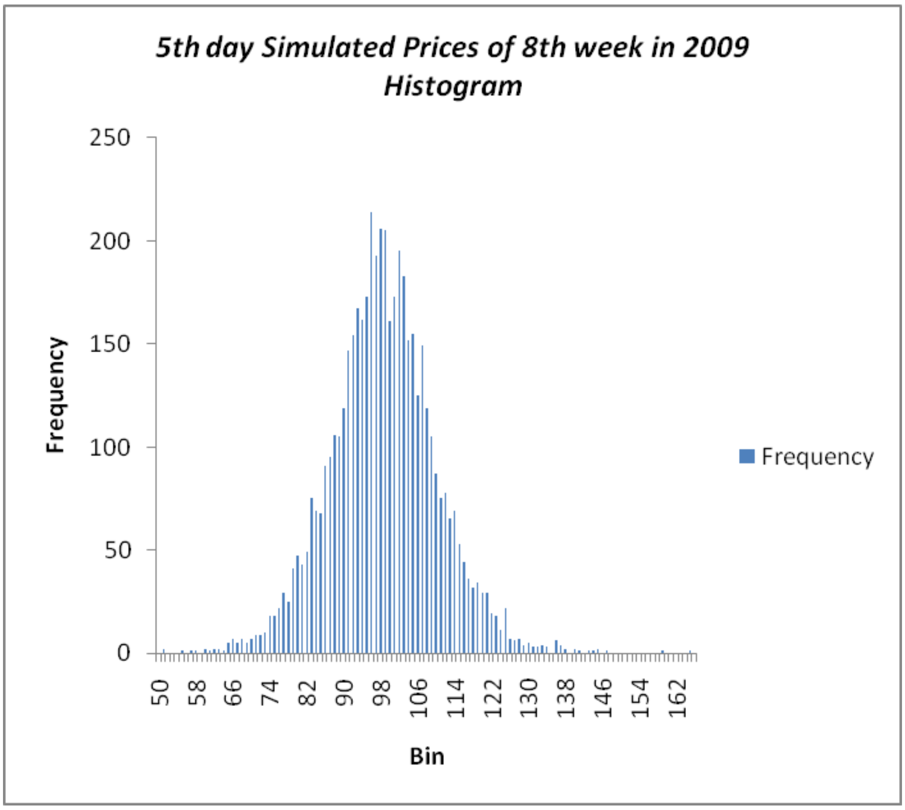

Figure 56
Forecasting price of the 5th day (**8th** week): 98.12439857
1% VaR: 29.89

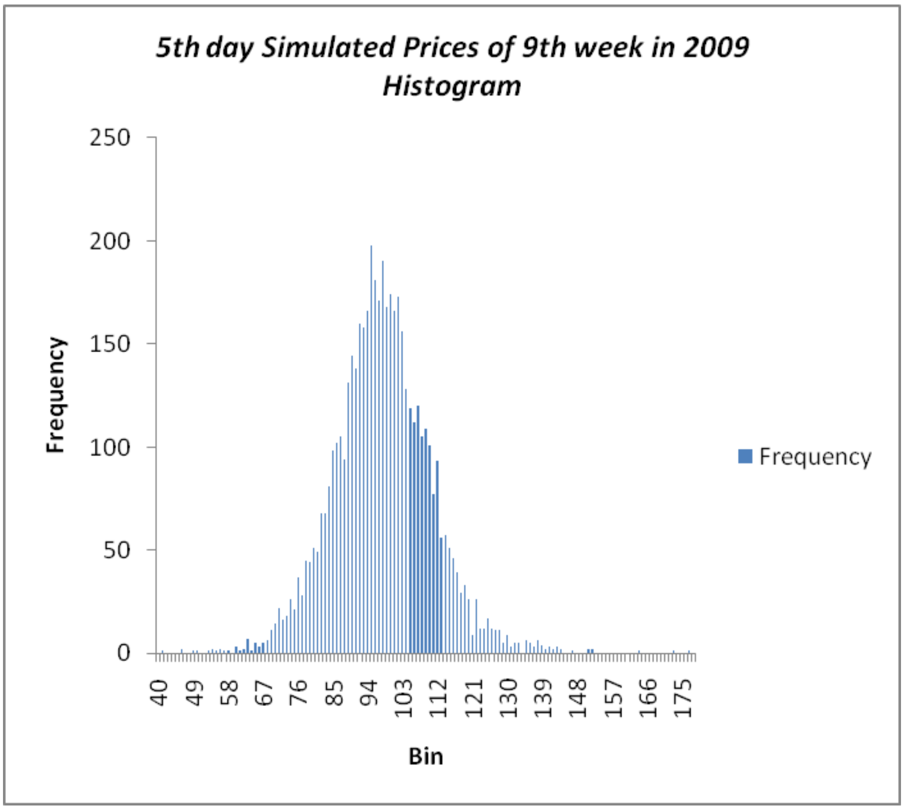

Figure 57
Forecasting price of the 5th day (**9th** week): 97.30601964
1% VaR: 31.64

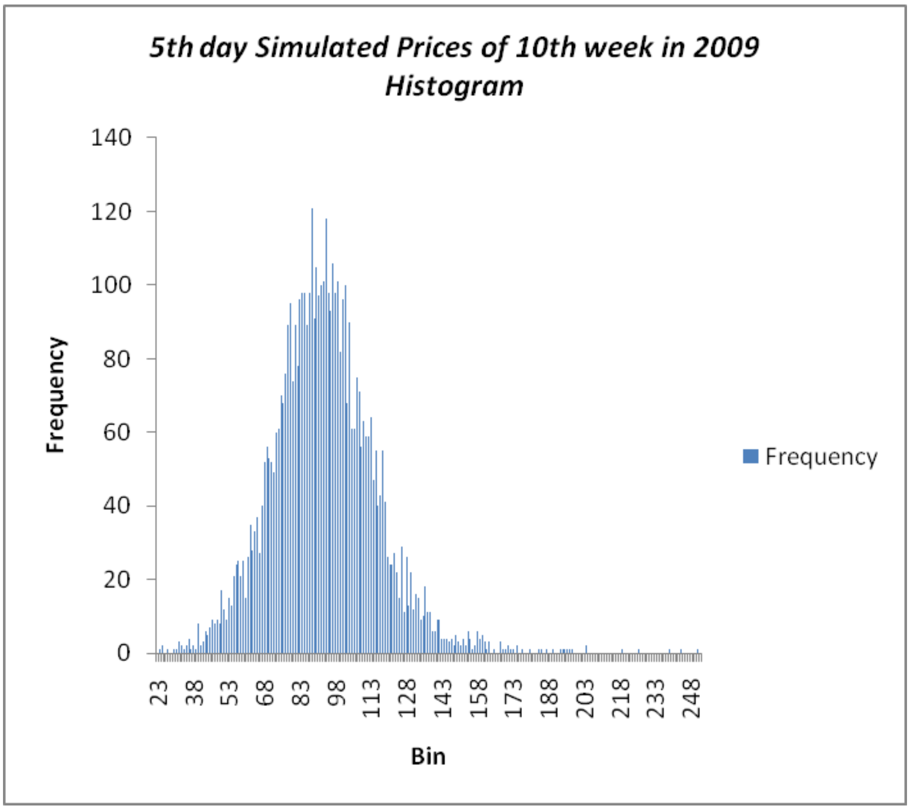

Figure 58
Forecasting price of the 5th day (**10th** week): 92.67344037
1% VaR: 55.43

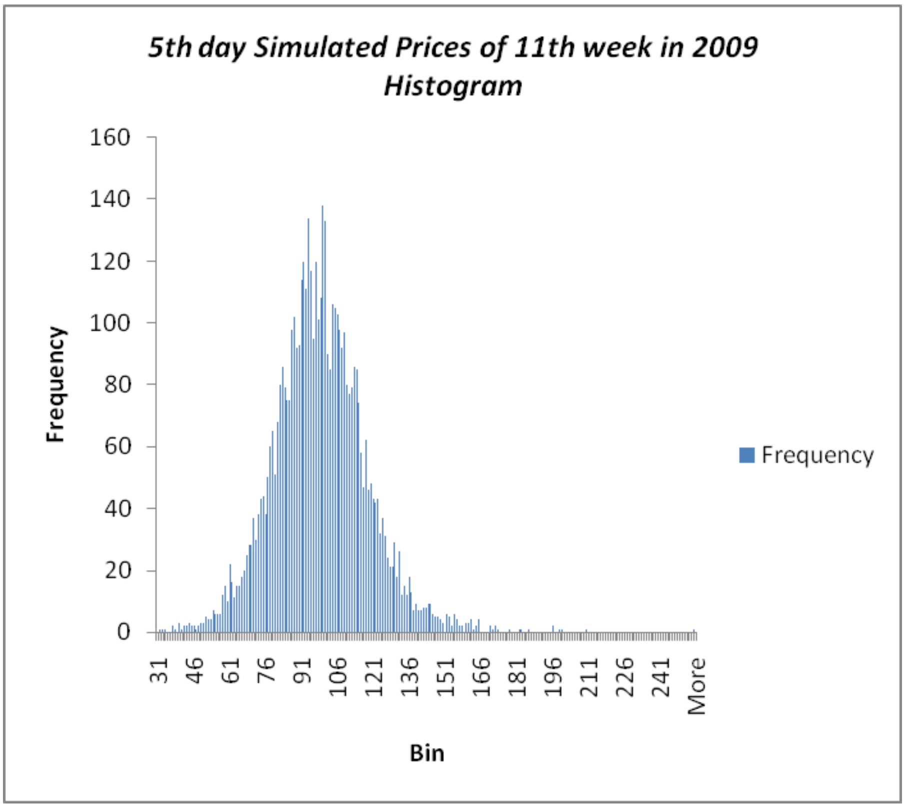

Figure 59
Forecasting price of the 5th day (**11th** week): 98.47018572
1% VaR: 46

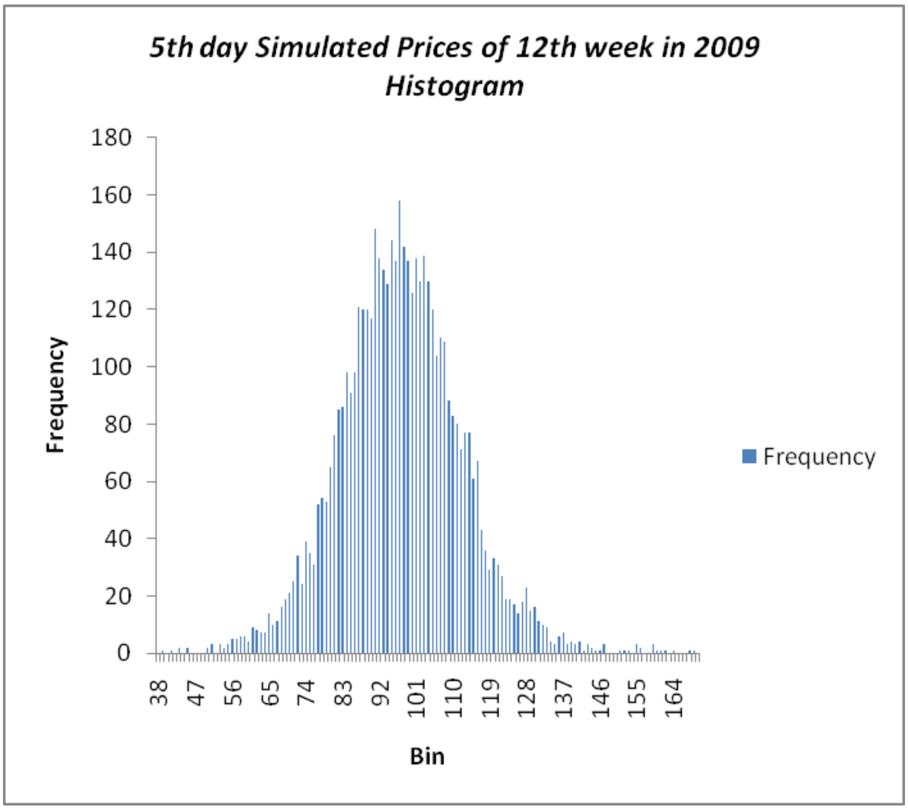

Figure 60

Forecasting price of the 5th day (**12th** week): 96.78321861

1% VaR: 39.45

2008:

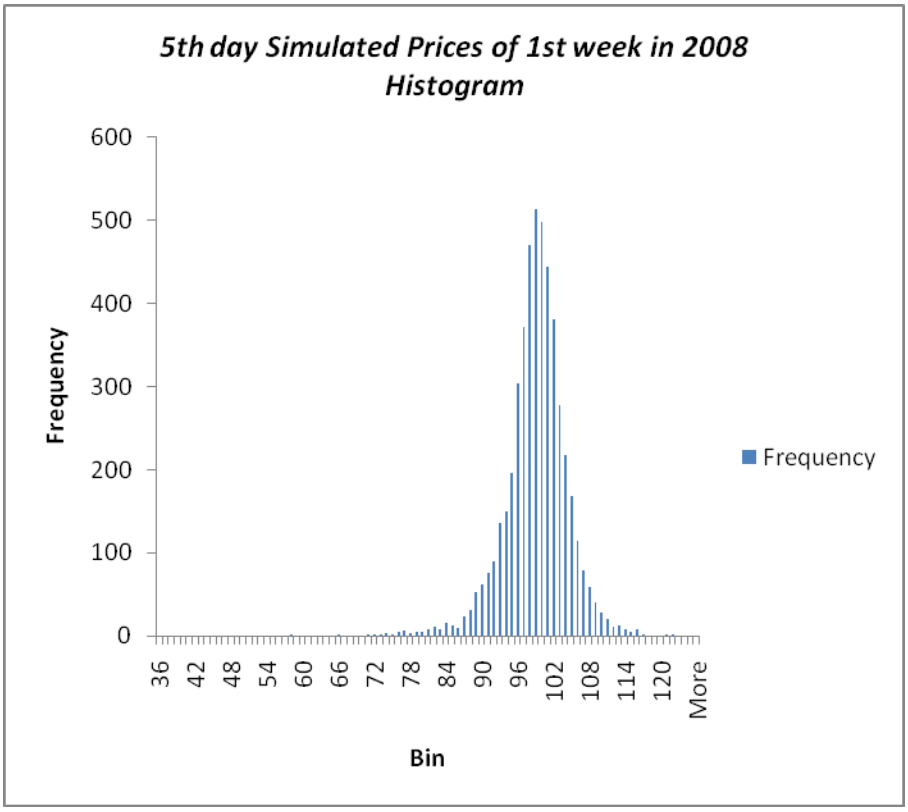

Figure 61
Forecasting price of the 5th day (**1st** week): 98.42203606
1% VaR: 20.84

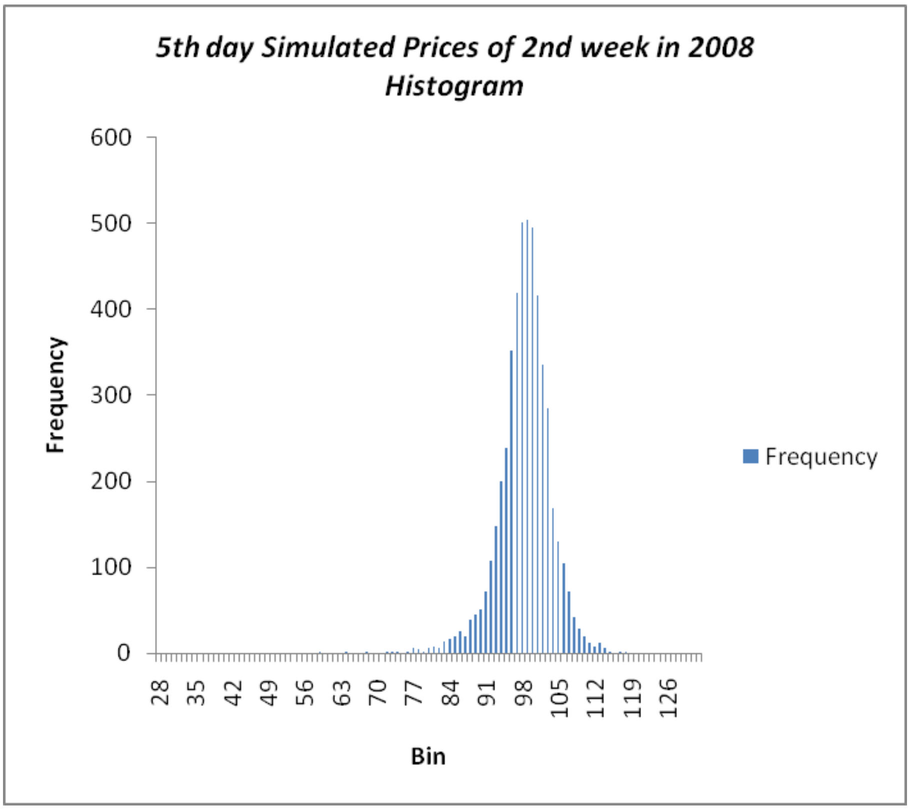

Figure 62
Forecasting price of the 5th day (**2nd** week): 97.97322983
1% VaR: 19.13

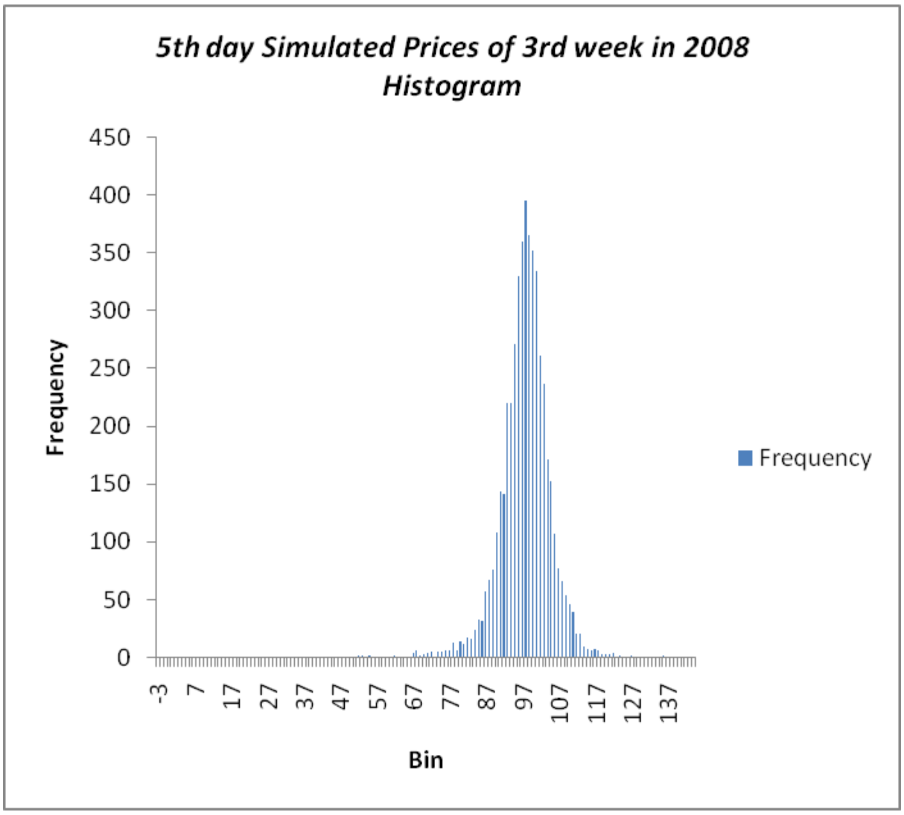

Figure 63
Forecasting price of the 5th day (**3rd** week): 97.30077529
1% VaR: 25.8

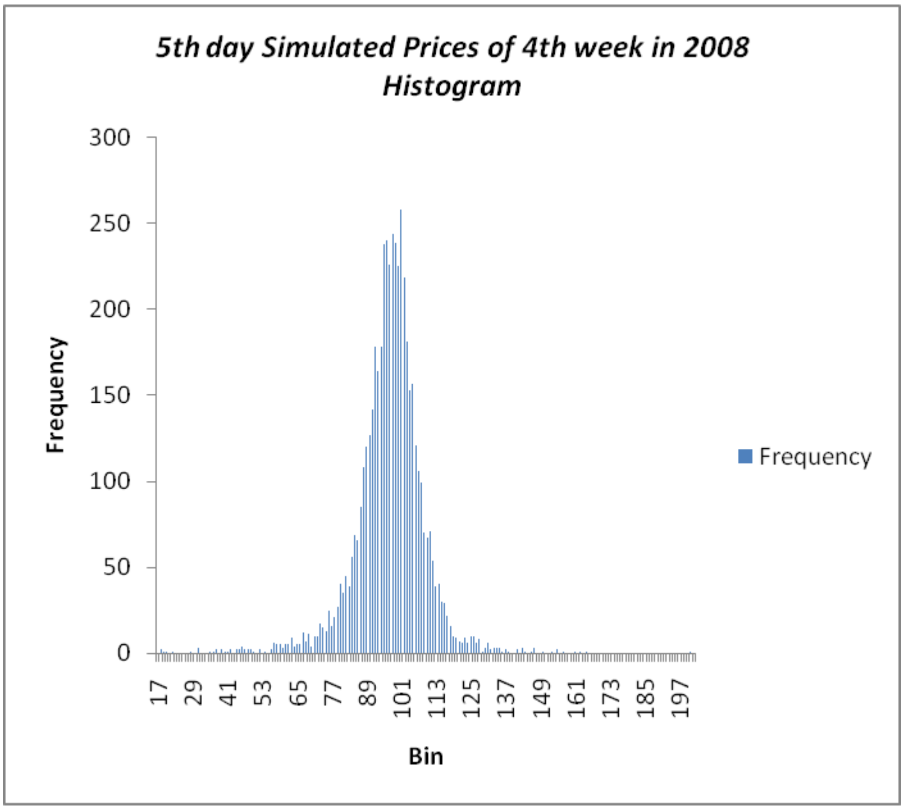

Figure 64
Forecasting price of the 5th day (**4th** week): 96.62444126
1% VaR: 42

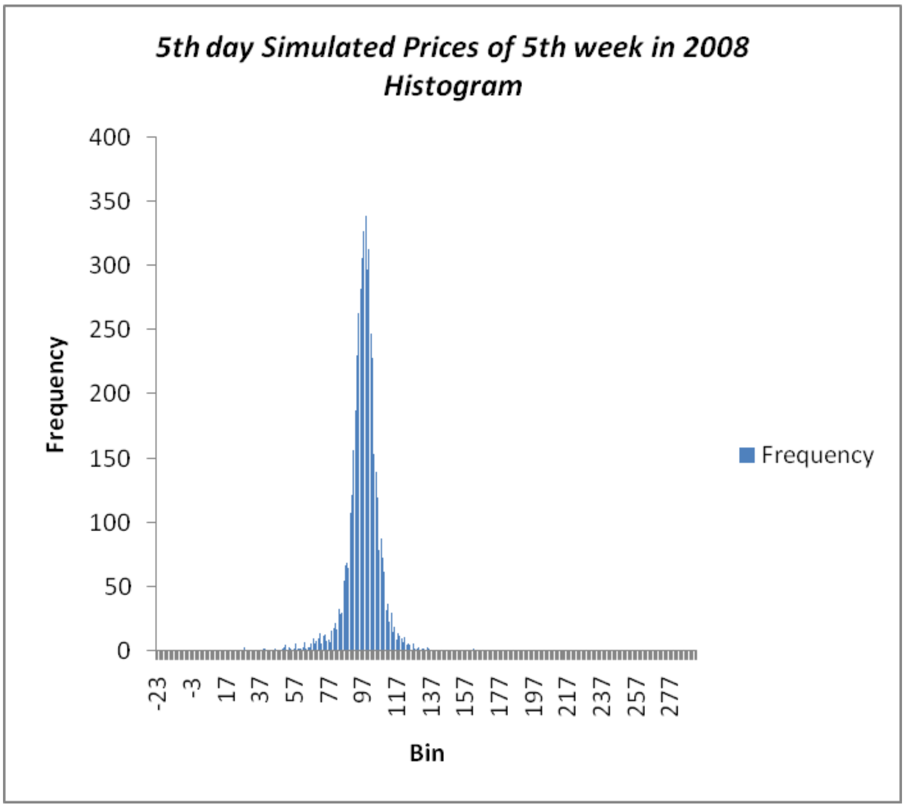

Figure 65
Forecasting price of the 5th day (**5th** week): 97.15862548
1% VaR: 41.5

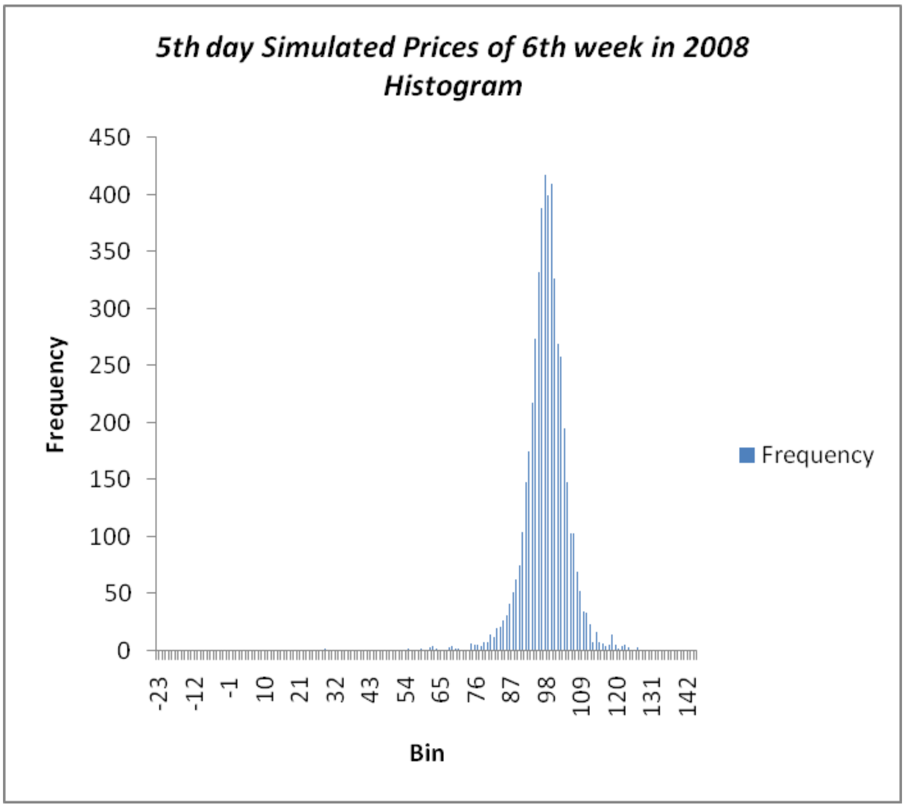

Figure 66
Forecasting price of the 5th day (**6th** week): 97.78903294
1% VaR: 23.6

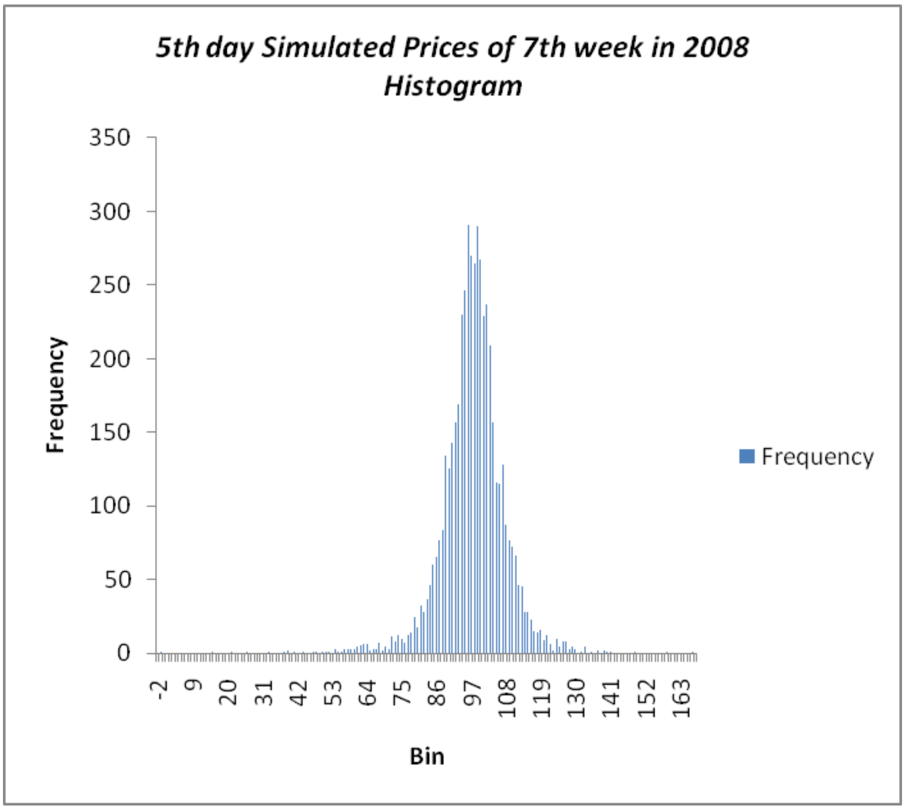

Figure 67
Forecasting price of the 5th day (**7th** week): 97.15876558
1% VaR: 36.5

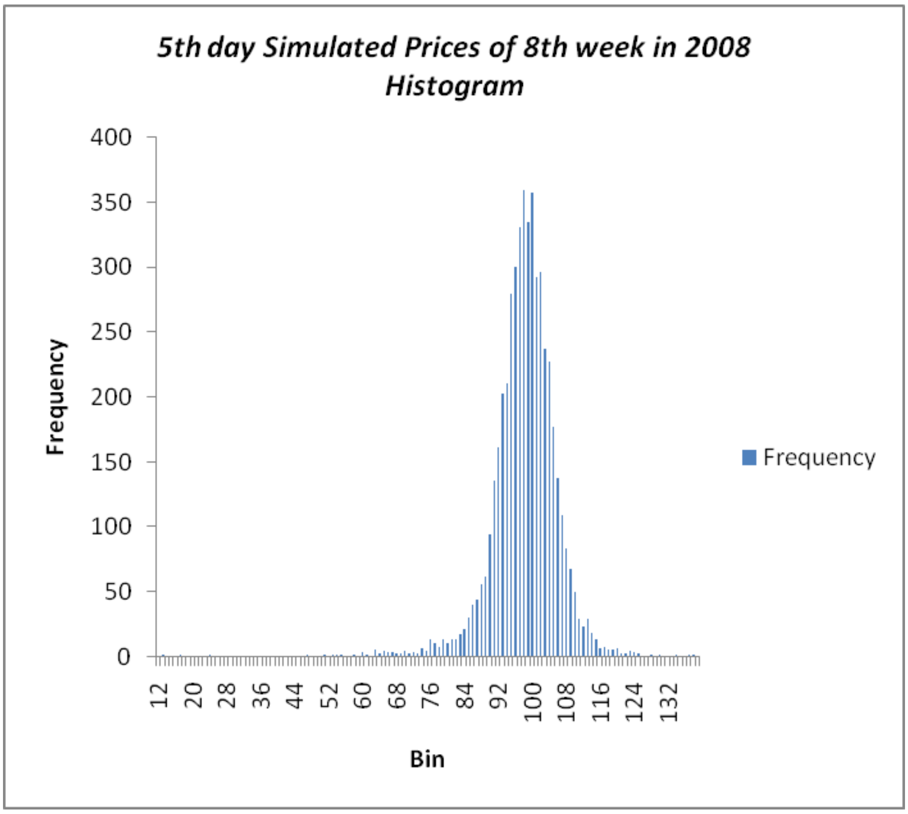

Figure 68
Forecasting price of the 5th day (**8th** week): 97.73772489
1% VaR: 26.17

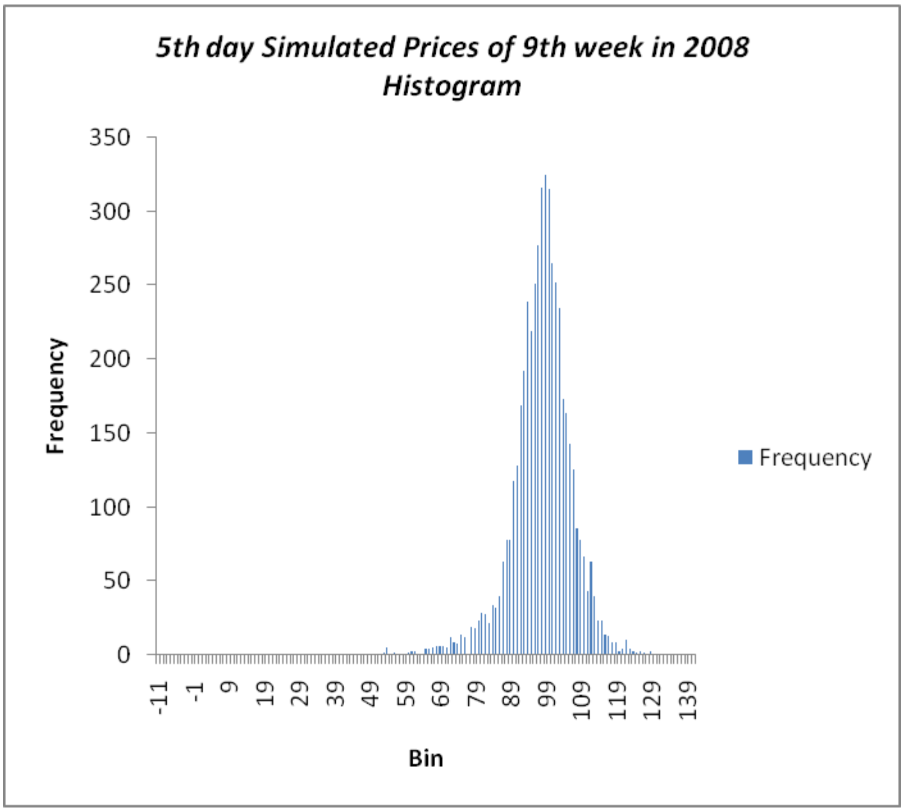

Figure 69
Forecasting price of the 5th day (**9th** week): 97.42594218
1% VaR: 32

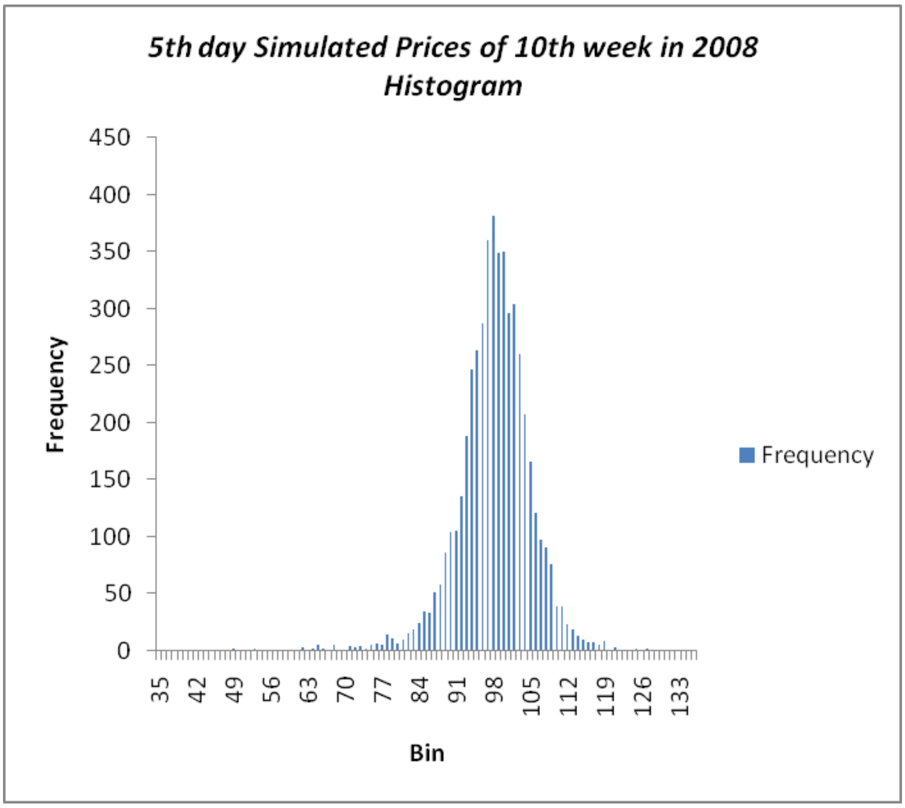

Figure 70
Forecasting price of the 5th day (**10th** week): 97.71668051
1% VaR: 24.29

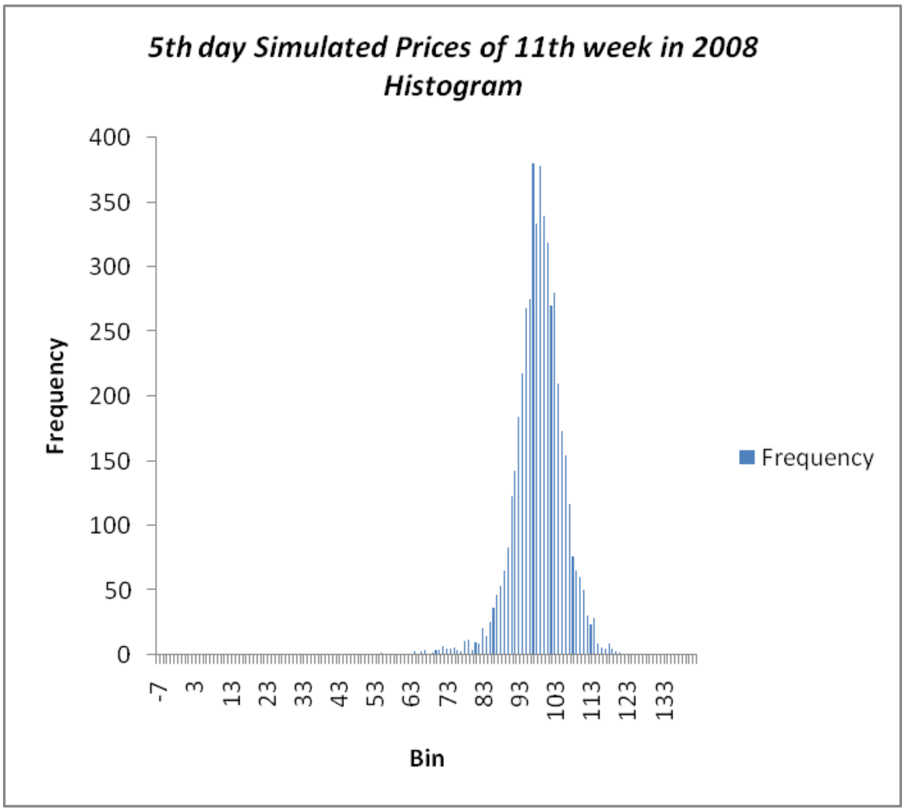

Figure 71
Forecasting price of the 5th day (**11th** week): 98.01741261
1% VaR: 26

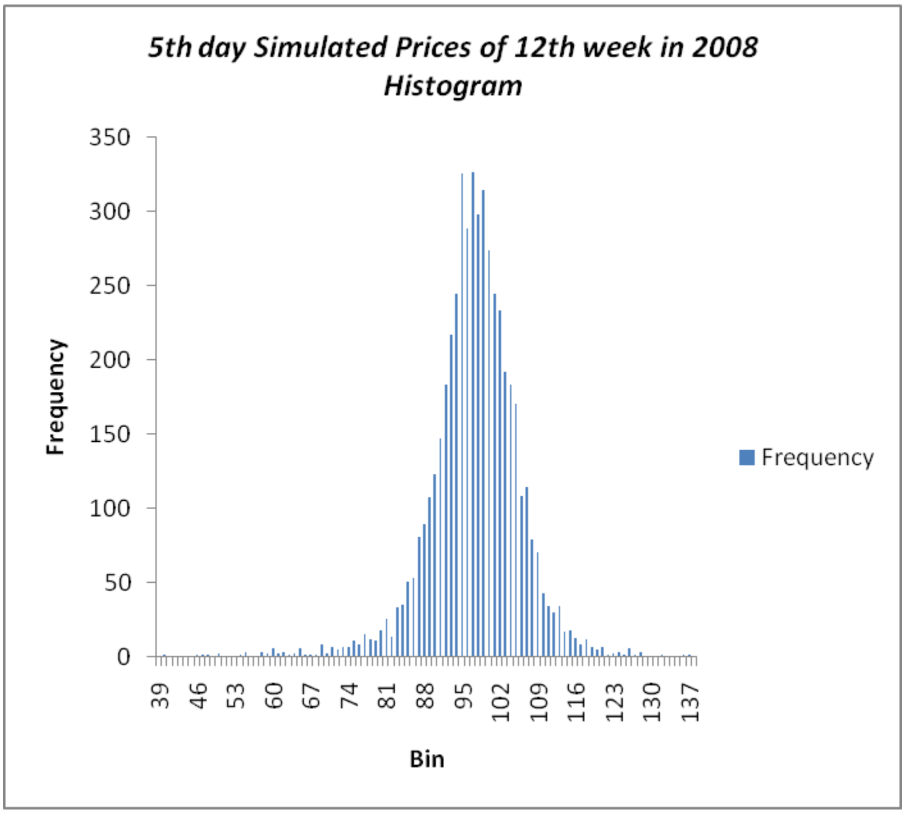

Figure 72

Forecasting price of the 5th day (**12th** week): 96.85088935

1% VaR: 29.34

5.3.2 Comparison

Week	*Actual Prices*	*Forecasting Prices*	*VaR*
1st	98.19151	98.42203606	20.84
2nd	100.3175	97.97322983	19.13
3rd	109.6519	97.30077529	25.8
4th	119.3843	96.62444126	42
5th	95.60838	97.15862548	41.5
6th	93.29962	97.78903294	23.6
7th	104.878	97.15876558	36.5
8th	91.25754	97.73772489	26.17
9th	94.10099	97.42594218	32
10th	96.33902	97.71668051	24.29
11th	108.6934	98.01741261	26
12th	97.07854	96.85088935	29.34

Table 10: 1% VaR Estimates for 5-Day Horizon (FHS Method, 2008)

Week	*Actual Prices*	*Forecasting Prices*	*VaR*
1st	101.178	99.80834988	37.2
2nd	80.85382	96.92910459	53.8
3rd	92	98.4691806	53.67
4th	95.82609	97.46526724	33.15
5th	100.726	96.67541494	39.5
6th	80.9009	97.92280517	33.4
7th	85.07795	98.43802048	32.25
8th	89.00524	98.12439857	29.89
9th	57.35294	97.30601964	31.64
10th	131.2821	92.67344037	55.43
11th	116.0156	98.47018572	46
12th	107.7441	96.78321861	39.45

Table 9: 1% VaR Estimates for 5-Day Horizon (FHS Method, 2009)

By comparing the actual prices and the reported VaR in 5.3.2, we could see that only one break has happened (9th week of 2009), which means that FHS has ended to the same result as HS (in our sample).[29]

5.4 Running Another Empirical Study

For finding biases in VaR forecast more precisely, the choice of time horizon and confidence level play an important role.

Although choosing the time horizon completely depends on the nature of the portfolio, as the assumption of VaR is that portfolio is frozen along the horizon [3], longer horizon might cause reducing the significances of the measure.

Also the other factor, confidence level, is better not to be too high. Higher confidence level decreases the number of observations in the tail of the distribution and this reduces the power of test as well. For instance for a 1% confidence level, we should wait 100 weeks (around 500 days) to find the week that loss is more than the predicted VaR in it, but for a 5% level, waiting period reduces to 20 weeks (around 100 days).

By the above hints we decided to change our previous study, to the below one:

- Decreasing the time horizon to 1 day,
- Increasing the confidence level to 5%,
- 20 repetitions

And here are the results:

[29] The unrealistic factors (confidence level, time horizon, and the repetition of the sample) of a chosen sample, (for reducing the time we have to spend on simulation), lead us to running another simulation, in subsection 5.4.

5.4.1 HS

Day	*Actual Prices*	*Forecasting Prices*	*VaR*
1st	95.80166	99.62882198	4.4
2nd	100.7592	99.61470673	4.33
3rd	96.34015	99.63872677	4.4
4th	100.838	99.57583982	4.44
5th	104.7091	99.65213168	4.34
6th	97.3545	99.61043976	4.45
7th	100.0543	99.67467828	4.37
8th	97.9359	99.66499251	4.44
9th	105.4908	99.70786944	4.35
10th	99.68454	99.65723269	4.35
11th	101.0021	99.6653351	4.44
12th	105.1175	99.68197971	4.27
13th	110.2231	99.65091217	4.49
14th	97.17008	99.65761356	4.43
15th	96.42857	99.73006163	4.35
16th	108.8504	99.71996733	4.5
17th	105.1701	99.70464944	4.36
18th	98.57143	99.73384173	4.42
19th	105.2136	99.73544739	4.43
20th	100.2136	99.77624713	4.35

Table11: 5% VaR Estimates for 1-Day Horizon (HS Method, 2008)

Day	*Actual Prices*	*Forecasting Prices*	*VaR*
1st	102.7487	99.97354285	12.09
2nd	99.87261	99.9472956	12.42
3rd	104.7194	99.8149797	12.6
4th	91.71742	99.90432628	12.26
5th	102.656	99.91933331	12.91
6th	93.79043	99.88177124	12.76
7th	87.58621	99.95599516	12.11
8th	101.5748	99.77297018	12.54
9th	96.43411	99.92058957	12.45
10th	100.4823	99.85936078	12.57
11th	103.84	99.93147342	12.31
12th	93.99076	99.87769959	12.44
13th	98.68852	99.99861493	12.36
14th	95.51495	99.93722976	12.34
15th	100	99.71122756	12.6
16th	101.5652	99.8406286	12.34
17th	103.0822	99.83333714	12.61
18th	110.7973	99.70102081	12.54
19th	91.90405	99.74322408	12.54
20th	89.8858	99.66151271	13.03

Table 12: 5% VaR Estimates for 1-Day Horizon (HS Method, 2009)

5.4.2 FHS

Day	*Actual Prices*	*Forecasting Prices*	*VaR*
1st	95.80166	99.91870474	3.88
2nd	100.7592	99.7558785	5.57
3rd	96.34015	99.71402054	6.22
4th	100.838	99.43746115	4.79
5th	104.7091	99.79589563	5.24
6th	97.3545	99.37803	4.38
7th	100.0543	99.04160985	6.76
8th	97.9359	99.73305002	4.93
9th	105.4908	99.58003617	4.05
10th	99.68454	99.83454177	3.59
11th	101.0021	98.81621136	6.75
12th	105.1175	99.65491722	4.73
13th	110.2231	99.50208946	3.99
14th	97.17008	99.03127557	6.33
15th	96.42857	98.42220421	12.68
16th	108.8504	99.53344419	8
17th	105.1701	99.47069317	7.65
18th	98.57143	98.3149249	12.97
19th	105.2136	99.22325094	9.47
20th	100.2136	99.6203495	5.54

Table13: 5% VaR Estimates for 1-Day Horizon (FHS Method, 2008)

Day	*Actual Prices*	*Forecasting Prices*	*VaR*
1st	102.7487	100.0102428	12.2
2nd	99.87261	100.2900311	11.04
3rd	104.7194	100.3577128	6.87
4th	91.71742	99.91916421	6.46
5th	102.656	100.6416678	7.53
6th	93.79043	98.5272797	15.08
7th	87.58621	100.2158218	10.57
8th	101.5748	98.62091459	11.62
9th	96.43411	97.58802086	19.09
10th	100.4823	99.82091501	12.53
11th	103.84	99.06139055	12.34
12th	93.99076	99.70620455	10.71
13th	98.68852	100.2577074	9.53
14th	95.51495	98.94150809	11.73
15th	100	99.4354164	8.69
16th	101.5652	99.03655216	10.25
17th	103.0822	99.59638345	9.65
18th	110.7973	99.92342577	6.56
19th	91.90405	100.1613179	7.86
20th	89.8858	101.1312528	13.21

Table 14: 5% VaR Estimates for 1-Day Horizon (FHS Method, 2009, First Run)

Day	*Actual Prices*	*Forecasting Prices*	*VaR*
1st	102.7487	100.0578309	8.55
2nd	99.87261	100.4287931	7.95
3rd	104.7194	100.3214719	6.88
4th	91.71742	99.90572649	6.41
5th	102.656	100.4487522	8.12
6th	93.79043	98.45869122	14.5
7th	87.58621	100.1904077	10.32
8th	101.5748	99.04924508	11.4
9th	96.43411	97.87131812	15.26
10th	100.4823	99.76545542	12.42
11th	103.84	99.23313355	12.08
12th	93.99076	99.6944337	10.35
13th	98.68852	100.2992792	9.41
14th	95.51495	98.720458	11.76
15th	100	99.36501084	8.59
16th	101.5652	99.11499852	10.32
17th	103.0822	99.6155951	9.1
18th	110.7973	99.95836158	8.02
19th	91.90405	100.0371922	7.95
20th	89.8858	101.368576	12.18

Table 14: 5% VaR Estimates for 1-Day Horizon (FHS Method, 2009, Second Run)

As the confidence level is 5%, our expectation about the number of breaks in 20 repetitions is at most one out of 20. It is clear from the above tables (11, 12, 13, 14), there has no break happened in 2008 (both HS and FHS) and also in 2009 based on HS method, but the number of breaks in 2009 (by FHS) is 3, which contradict our expectation.

6 Conclusion

The results we have received in the second empirical study (5.4), lead us to the following interesting conclusion:

Although according to [2], [4] and [27] we are expected to observe better result from FHS, but we could see that in the market crisis period HS works more accurately than FHS.

The most significant difference between our initial data sources i.e. year 2008 and year 2007 is absolutely clear. We had a disaster in 2008 called ‘financial crisis’.
Absence of such event in 2007 caused an almost suitable prediction for year 2008 (for both HS and FHS), but this event could be the reason of the unexpected high number of breaks in 2009. The interesting part is, it happened only during FHS method and not HS.

One of the main purposes of choosing year 2008 as one of the sample data, was making comparison between this year and the second arbitrary one without any crisis, and investigating whether the ‘crisis’ could destroy the accuracy of our prediction.

Thus we could conclude that a ‘window’ containing the special crisis, is not a reliable sample data for forecasting the future prices using FHS technique.

Appendix A

Matlab Source Codes

5000 price simulation of the 5th day, of the first week in 2009 (applying HS method to daily simple returns):

```
Data=xlsread('Simple Simulation 2008-2009.xls','Sheet1',sprintf('F%d:F%d',1,252))
for i=1:5000
  C=round((251*rand(1,5))+1)
      Return1 = Data(C(1))
      Price1=100*(1+Return1)

      Return2 = Data(C(2))
      Price2=Price1*(1+Return2)

      Return3 = Data(C(3))
      Price3=Price2*(1+Return3)

      Return4 = Data(C(4))
      Price4=Price3*(1+Return4)

      Return5 = Data(C(5))
      Price5=Price4*(1+Return5)

M(i)=Price5

end
xlswrite('Simple Simulation 2008-2009.xls', M','Related to Sheet1', 'F1')
```

5000 price simulation of the 5th day, of the first week in 2009 (applying FHS method to daily simple returns):

```
r=xlsread('Simple Simulation 2008-2009.xls','Sheet1',sprintf('F%d:F%d',1,252))

  EPSILONHAT(1)=0
  HHAT(1)=(.0005128136)/(1-.5584451-.4382287)
  eHAT(1)=0

   for t=2:251
      EPSILONHAT(t)=r(t)-.1472*r(t-1)-.002085*EPSILONHAT(t-1)

      HHAT(t)=.0005128136+.5584451*(EPSILONHAT(t-1))*(EPSILONHAT(t-1))+.4382287*(HHAT(t-1))

      eHAT(t)=EPSILONHAT(t)/sqrt(HHAT(t))
   end


    HSTAR(1)=HHAT(251)
    EPSILONSTAR(1)=EPSILONHAT(251)

   for i=1:5000

      C=round((250*rand(1,5))+1)
    for t=1:5
       eSTAR(t+1)=eHAT(C(t))
    end


rSTAR(1)=r(251)
PSTAR(1)=100

 for t=2:6
   HSTAR(t)=.0005128136+.5584451*(EPSILONSTAR(t-1))*(EPSILONSTAR(t-1))+.4382287*HSTAR(t-1)

    EPSILONSTAR(t)=eSTAR(t)*sqrt(HSTAR(t))

    rSTAR(t)=.1472*rSTAR(t-1)+.002085*EPSILONSTAR(t-1)+EPSILONSTAR(t)

    PSTAR(t)=PSTAR(t-1)*(1+rSTAR(t))
end

M(i)=PSTAR(6)
```

```
end

xlswrite('Simple Simulation 2008-2009.xls', M','FHS-1', 'F1')
```

As a general code (using parameters instead of the estimated values of them by ARMA-GARCH) of above program we could mention:

5000 price simulation of the 5th day, of the first week in 2009 (applying FHS method to daily simple returns):

```
r=xlsread('Simple Simulation 2008-
2009.xls','Sheet1',sprintf('F%d:F%d',1,252))

EPSILONHAT(1)=0
HHAT(1)=(OMEGAHAT)/(1-ALPHAHAT-BETAHAT)
eHAT(1)=0

for t=2:251
  EPSILONHAT(t)=r(t)-MUHAT*r(t-1)-THETAHAT*EPSILONHAT(t-
1)

  HHAT(t)=OMEGAHAT+ALPHAHAT*(EPSILONHAT(t-
1))*(EPSILONHAT(t-1))+BETAHAT*(HHAT(t-1))

  eHAT(t)=EPSILONHAT(t)/sqrt(HHAT(t))

end


HSTAR(1)=HHAT(251)
EPSILONSTAR(1)=EPSILONHAT(251)

for i=1:5000

C=round((250*rand(1,5))+1)
 for t=1:5
   eSTAR(t+1)=eHAT(C(t))
 end


rSTAR(1)=r(251)
PSTAR(1)=100

for t=2:6
```

```
  HSTAR(t)=OMEGAHAT+ALPHAHAT*(EPSILONSTAR(t-
1))*(EPSILONSTAR(t-1))+BETAHAT*HSTAR(t-1)
 EPSILONSTAR(t)=eSTAR(t)*sqrt(HSTAR(t))

 rSTAR(t)=MUHAT*rSTAR(t-1)+THETAHAT*EPSILONSTAR(t-
1)+EPSILONSTAR(t)

  PSTAR(t)=PSTAR(t-1)*(1+rSTAR(t))
end

M(i)=PSTAR(6)
end

xlswrite('Simple Simulation 2008-2009.xls', M','FHS-1', 'F1')
```

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