Appendix: A Full Detailed Wind Farm Model

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NOMENCLATURE

Roman symbols

A	Wind turbine swept surface
C	DC bus capacity
I_t	One-mass wind turbine aggregated inertia
K_{C_p}	Constant tip speed ratio speed control law coefficient
L_l	Inductance of the grid connection impedance
P	Generator pole pairs
Q_s	Generator stator reactive power
R	Wind turbine radius
V_{DC}	DC bus voltage
V_{bus}	Infinite bus voltage
c_p	Aerodynamic power coefficient
f	System frequency
r_s	Resistance of a single phase of the stator windings
r_l	Resistance of the grid connection impedance
v_w	Wind speed

Greek symbols

Γ_m	Generator torque
Γ_t	Wind turbine torque
β	Pitch angle
λ_m	Flux linkage per rotating speed unit
ν	Gearbox multiplication ratio
ω_{mn}	Nominal generator speed
ω_r	Generator electrical angle speed
ω_t	Wind turbine speed
ρ	Air density
τ	Time constant of the pitch angle controller
θ_m	Generator shaft angular position
θ_r	Generator rotor electric angle

Superscripts

$*$	Reference value
abc	Vector of *abc* components
qd	Vector of *qd* components
m	Variable related to the generator shaft
r	Variable related to the generator rotor
s	Variable related to the generator stator
t	Variable related to the turbine
z	Variable related to the grid connection point

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A. WIND FARM MODEL

In this section, we provide the full model of the wind turbine used in the simulations. The wind turbine model consists of several sub-blocks. We summarise each block in terms of inputs, outputs and detailed equations. The Park and inverse Park transformation of a rotation angle θ are indicated by $T(\theta)$ and $T^{-1}(\theta)$, respectively.

1) Wind turbine aerodynamics:

Block inputs: θ_m^*, Γ_m and v_w.

Block outputs: θ_m and ω_m.

$$\begin{aligned} \beta(s) &= \frac{1}{\tau s+1}\theta_m^*(s), \\ c_p(\Lambda,\beta) &= c_1(c_2\frac{1}{\Lambda} - c_3\beta - c_4\beta^{c_5} - c_6)e^{-c_7\frac{1}{\Lambda}}, \\ \Gamma_t &= \frac{1}{2}c_p\rho A v_w^3\frac{1}{\omega_t}, \\ \omega_t &= \frac{1}{I_t}(\Gamma_t + \nu\Gamma_m), \\ \dot{\theta}_m &= \omega_m = \nu\omega_t, \end{aligned} \tag{1}$$

where $[c_1 \cdots c_9]$ are wind turbine characteristic parameters and Λ is defined as $\frac{1}{\Lambda} = \frac{1}{\lambda + c_8\beta} - \frac{c_9}{1+\beta^3}$ with $\lambda = \frac{\omega_t R}{v_w}$.

2) Wind turbine speed controller:

Block input: ω_m.

Block outputs: Γ_m^* and θ_m^*.

$$\begin{aligned} K_{C_p} &= \frac{1}{2}\rho A R^3\frac{c_1(c_2+c_6c_7)^3 e^{-\frac{c_2+c_6c_7}{c_2}}}{c_2^2c_7^4} \\ \Gamma_m^* &= K_{C_p}\omega_m^2, \\ \theta_m^*(s) &= \frac{K_p s + K_i}{s}(\omega_m(s) - \omega_{mn}). \end{aligned} \tag{2}$$

3) Vector controller:

Block inputs: Γ_m^*, i_s^{abc}, ω_m and θ_m.

Block outputs: v_s^{qd} and v_s^{abc}.

$$
\begin{aligned}
i_s^{qd} &= T(\theta_r) i_s^{qbc} \\
i_{sq}^* &= \frac{2}{3P}\frac{\Gamma_m^*}{\lambda_m}, \\
i_{sd}^* &= \frac{2}{3P}\frac{Q_s^*}{\omega_m \lambda_m}, \\
\hat{v}_{sq} &= \frac{K_{pq}s + K_{iq}}{s}(i_{sq}^* - i_{sq}), \\
\hat{v}_{sd} &= \frac{K_{pd}s + K_{id}}{s}(i_{sd}^* - i_{sd}), \\
v_s^{qd} &= \hat{v}_s^{qd} + \begin{bmatrix} 0 & \omega_r L_d \\ -\omega_r L_q & 0 \end{bmatrix} i_s^{qd} + \lambda_m \omega_r \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
v_s^{abc} &= T^{-1}(\theta_r) v_s^{qd}.
\end{aligned} \tag{3}
$$

where $\omega_r = P\omega_m$, $\theta_r = P\theta_m$, $v_s^{qd} = [v_{sq}, v_{sd}]^{\mathrm{T}}$, $\hat{v}_s^{qd} = [\hat{v}_{sq}, \hat{v}_{sd}]^{\mathrm{T}}$ and $i_s^{qd} = [i_{sq}, i_{sd}]^{\mathrm{T}}$.

4) Machine dynamics:
Block inputs: v_s^{qd} and ω_m.
Block outputs: i_s^{qd}, i_s^{abc} and Γ_m.

$$
\begin{aligned}
v_s^{qd} &= \begin{bmatrix} r_s & \omega_r L_d \\ -\omega_r L_q & r_s \end{bmatrix} i_s^{qd} + \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \frac{d}{dt} i_s^{qd} + \lambda_m \omega_r \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
\Gamma_m &= \frac{3}{2} P(\lambda_m i_{sq} + (L_d - L_q) i_{sq} i_{sd}), \\
i_s^{abc} &= T^{-1}(\theta_r) i_s^{qd},
\end{aligned} \tag{4}
$$

5) DC bus dynamics:
Block inputs: i_l^{abc}, v_l^{abc}, i_s^{abc} and v_s^{abc}.
Block outputs: V_{DC} and i_{DCm}.

$$
\begin{aligned}
\frac{d}{dt} V_{DC} &= \frac{1}{C}(i_{DCm} - i_{DCl}), \\
\{v_l^{abc}\}^{\mathrm{T}} i_l^{abc} &= V_{DC} i_{DCl}, \\
\{v_s^{abc}\}^{\mathrm{T}} i_s^{abc} &= V_{DC} i_{DCm}.
\end{aligned} \tag{5}
$$

6) Grid side system dynamics:
Block inputs: v_z^{abc} and v_l^{abc}.
Block output: i_l^{abc}.

$$
v_l^{abc} = r_l i_l^{abc} + L_l \frac{d}{dt} i_i^{abc} + v_z^{abc}. \tag{6}
$$

7) Grid side controller:
Block inputs: V_{DC} and i_{DCm}.
Block output: i_{DCl}^*.

$$
i_{DCl}^* = \frac{K_{pg}s + K_{ig}}{s}(V_{DC}^* - V_{DC}) + i_{DCm} \tag{7}
$$

8) Grid current controller
Block inputs: V_{DC}, i_{DCl}^*, v_z^{abc} and i_l^{abc}.
Block output: v_l^{abc}.

$$
\begin{aligned}
i_{lq}^* &= \frac{2}{3}\frac{V_{DC}}{v_{zq}} i_{DCl}^*, \\
i_{ld}^* &= 0, \\
\hat{v}_{lq} &= \frac{K_{pc}s + K_{ic}}{s}(i_{lq}^* - i_{lq}), \\
\hat{v}_{ld} &= \frac{K_{pc}s + K_{ic}}{s}(i_{ld}^* - i_{ld}), \\
v_{lq} &= v_{zq} - 2\pi f i_{ld} L_l - \hat{v}_{lq}, \\
v_{ld} &= 2\pi f i_{lq} L_l - \hat{v}_{ld}, \\
\hat{\omega} &= \frac{s + 0.129}{s} v_{zd}, \\
i_l^{qd} &= T(2\pi f t - \hat{\omega} t) i_l^{abc}, \\
v_l^{qd} &= T(2\pi f t - \hat{\omega} t) v_l^{abc}.
\end{aligned} \tag{8}
$$

where $i_l^{qd} = [i_{lq}, i_{ld}]^{\mathrm{T}}$, $v_l^{qd} = [v_{lq}, v_{ld}]^{\mathrm{T}}$ and $v_z^{qd} = [v_{zq}, v_{zd}]^{\mathrm{T}}$.

B. Model parameters

In this section we summarise the parameters used in the simulation.

1) Wind turbine: $c_1 = 1$, $c_2 = 3952$, $c_6 = 204$, $c_7 = 1447$, $c_3 = c_4 = c_5 = c_8 = c_9 = 0$, $R = 40\mathrm{m}$, $A = 5026.5\mathrm{m}^2$, $\rho = 1.125\mathrm{kg/m}^3$, $\nu = 90$, $I_t = 4\mathrm{kg}\cdot\mathrm{km}^2$, $\tau = 0.1\mathrm{s}$ and $v_w = 7\mathrm{m/s}$.
2) Wind turbine speed controller: $\omega_{mn} = 167.7325\mathrm{min}^{-1}$, $K_p = 0.1°/\mathrm{rad}\cdot\mathrm{s}$ and $K_i = 0.02°/\mathrm{rad}$.
3) Vector controller: $P = 2$, $Q_s^* = 10\mathrm{VAr}$, $\lambda_m = 2.35\mathrm{V/rad}\cdot\mathrm{s}$, $K_{pq} = 0.0637\mathrm{V/A}$, $K_{iq} = 7.5\mathrm{V/(A}\cdot\mathrm{s)}$, $K_{pd} = 0.0638\mathrm{V/A}$ and $K_{id} = 7.5\mathrm{V/(A}\cdot\mathrm{s)}$, $L_q = 0.12732\mathrm{mH}$ and $L_d = 0.12764\mathrm{mH}$.
4) Machine dynamics: $r_s = 15\mathrm{m}\Omega$.
5) DC bus: $C = 10\mathrm{mF}$.
6) Grid side system: $r_l = 20\mathrm{m}\Omega$, $L_l = 1\mathrm{mH}$ and $V_z = 6.6\mathrm{kV}$ (ph-ph).
7) Grid side controller: $V_{DC}^* = 2.6\mathrm{kV}$, $K_{pg} = 0.6032\mathrm{A/V}$ and $K_{ig} = 14.2122\mathrm{A/(V}\cdot\mathrm{s)}$.
8) Grid current controller: $f = 50\mathrm{Hz}$, $K_{pc} = 0.2803\mathrm{V/A}$ and $K_{ic} = 10\mathrm{V/(A}\cdot\mathrm{s)}$.