Appendix: A Full Detailed Wind Farm Model

NOMENCLATURE

Roman symbols

A Wind turbine swept surface
C DC bus capacity
I_t One-mass wind turbine aggregated inertia
L_l Inductance of the grid connection impedance
K_{C_p} Constant tip speed ratio speed control law coefficient
P Generator pole pairs
Q_s Generator stator reactive power
R Wind turbine radius
V_{DC} DC bus voltage
V_{bus} Infinite bus voltage
c_p Aerodynamic power coefficient
f System frequency
r_s Resistance of a single phase of the stator windings
r_l Resistance of the grid connection impedance
v_w Wind speed

Greek symbols

Γ_m Generator torque
Γ_t Wind turbine torque
β Pitch angle
λ_m Flux linkage per rotating speed unit
ν Gearbox multiplication ratio
ω_{mn} Nominal generator speed
ω_r Generator electrical angle speed
ω_t Wind turbine speed
ρ Air density
τ Time constant of the pitch angle controller
θ_m Generator shaft angular position
θ_r Generator rotor electric angle

Superscripts

$*$ Reference value
abc Vector of *abc* components
qd Vector of *qd* components
m Variable related to the generator shaft
r Variable related to the generator rotor
s Variable related to the generator stator
t Variable related to the turbine
z Variable related to the grid connection point

A. WIND FARM MODEL

In this section, we provide the full model of the wind turbine used in the simulations. The wind turbine model consists of several sub-blocks. We summarise each block in terms of inputs, outputs and detailed equations. The Park and inverse Park transformation of a rotation angle θ are indicated by $T(\theta)$ and $T^{-1}(\theta)$, respectively.

1) Wind turbine aerodynamics:
Block inputs: θ_m^*, Γ_m and v_w.
Block outputs: θ_m and ω_m.

$$\begin{aligned}
\beta(s) &= \frac{1}{\tau s+1}\theta_m^*(s),\\
c_p(\Lambda,\beta) &= c_1(c_2\frac{1}{\Lambda} - c_3\beta - c_4\beta^{c_5} - c_6)e^{-c_7\frac{1}{\Lambda}},\\
\Gamma_t &= \frac{1}{2}c_p\rho A v_w^3\frac{1}{\omega_t},\\
\omega_t &= \frac{1}{I_t}(\Gamma_t + \nu\Gamma_m),\\
\dot{\theta}_m &= \omega_m = \nu\omega_t,
\end{aligned} \tag{1}$$

where $[c_1 \cdots c_9]$ are wind turbine characteristic parameters and Λ is defined as $\frac{1}{\Lambda} = \frac{1}{\lambda + c_8\beta} - \frac{c_9}{1+\beta^3}$ with $\lambda = \frac{\omega_t R}{v_w}$.

2) Wind turbine speed controller:
Block input: ω_m.
Block outputs: Γ_m^* and θ^*.

$$\begin{aligned}
K_{C_p} &= \frac{1}{2}\rho A R^3\frac{c_1(c_2+c_6c_7)^3 e^{-\frac{c_2+c_6c_7}{c_2}}}{c_2^2 c_7^4}\\
\Gamma_m^* &= K_{C_p}\omega_m^2,\\
\theta_m^*(s) &= \frac{K_p s + K_i}{s}(\omega_m(s) - \omega_{mn}).
\end{aligned} \tag{2}$$

3) Vector controller:
Block inputs: Γ_m^*, i_s^{abc}, ω_m and θ_m.
Block outputs: v_s^{qd} and v_s^{abc}.

$$\begin{aligned}
i_s^{qd} &= T(\theta_r)i_s^{qbc}\\
i_{sq}^* &= \frac{2}{3P}\frac{\Gamma_m^*}{\lambda_m},\\
i_{sd}^* &= \frac{2}{3P}\frac{Q_s^*}{\omega_m\lambda_m},\\
\hat{v}_{sq} &= \frac{K_{pq}s + K_{iq}}{s}(i_{sq}^* - i_{sq}),\\
\hat{v}_{sd} &= \frac{K_{pd}s + K_{id}}{s}(i_{sd}^* - i_{sd}),\\
\hat{v}_s^{qd} &= \begin{bmatrix} r_s & 0\\ 0 & r_s\end{bmatrix} i_s^{qd} + \begin{bmatrix} L_d & 0\\ 0 & L_q\end{bmatrix}\frac{d}{dt}i_s^{qd},\\
v_s^{qd} &= \hat{v}_s^{qd} + \begin{bmatrix} 0 & \omega_r L_d\\ -\omega_r L_q & 0\end{bmatrix} i_s^{qd} + \lambda_m\omega_r\begin{bmatrix}1\\0\end{bmatrix},\\
v_s^{abc} &= T^{-1}(\theta_r)v_s^{qd}.
\end{aligned} \tag{3}$$

where $\omega_r = P\omega_m$, $\theta_r = P\theta_m$, $v_s^{qd} = [v_{sq}, v_{sd}]^{\mathrm{T}}$, $\hat{v}_s^{qd} = [\hat{v}_{sq}, \hat{v}_{sd}]^{\mathrm{T}}$ and $i_s^{qd} = [i_{sq}, i_{sd}]^{\mathrm{T}}$.

4) Machine dynamics:
Block inputs: v_s^{qd} and ω_m.
Block outputs: i_s^{qd}, i_s^{abc} and Γ_m.

$$
\begin{aligned}
v_s^{qd} &= \begin{bmatrix} r_s & \omega_r L_d \\ -\omega_r L_q & r_s \end{bmatrix} i_s^{qd} + \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \frac{d}{dt} i_s^{qd} + \lambda_m \omega_r \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
\Gamma_m &= \frac{3}{2} P(\lambda_m i_{sq} + (L_d - L_q) i_{sq} i_{sd}), \\
i_s^{abc} &= T^{-1}(\theta_r) i_s^{qd},
\end{aligned} \tag{4}
$$

5) DC bus dynamics:
Block inputs: i_l^{abc}, v_l^{abc}, i_s^{abc} and v_s^{abc}.
Block outputs: V_{DC} and i_{DCm}

$$
\begin{aligned}
\frac{d}{dt} V_{DC} &= \frac{1}{C}(i_{DCm} - i_{DCl}), \\
\{v_l^{abc}\}^{\mathrm{T}} i_l^{abc} &= V_{DC} i_{DCl}, \\
\{v_s^{abc}\}^{\mathrm{T}} i_s^{abc} &= V_{DC} i_{DCm}.
\end{aligned} \tag{5}
$$

6) Grid side system dynamics:
Block inputs: v_z^{abc} and v_l^{abc}.
Block output: i_l^{abc}

$$
v_l^{abc} = r_l i_l^{abc} + L_l \frac{d}{dt} i_i^{abc} + v_z^{abc}. \tag{6}
$$

7) Grid side controller:
Block inputs: V_{DC} and i_{DCm}.
Block output: i_{DCl}^*

$$
i_{DCl}^* = \frac{K_{pg} s + K_{ig}}{s} (V_{DC}^* - V_{DC}) + i_{DCm} \tag{7}
$$

8) Grid current controller
Block inputs: V_{DC}, i_{DCl}^*, v_z^{abc} and i_l^{abc}.
Block output: v_l^{abc}.

$$
\begin{aligned}
i_{lq}^* &= \frac{2}{3} \frac{V_{DC}}{v_{zq}} i_{DCl}^*, \\
i_{ld}^* &= 0, \\
\hat{v}_{lq} &= \frac{K_{pc} s + K_{ic}}{s} (i_{lq}^* - i_{lq}), \\
\hat{v}_{ld} &= \frac{K_{pc} s + K_{ic}}{s} (i_{ld}^* - i_{ld}), \\
v_{lq} &= v_{zq} - 2\pi f i_{ld} L_l - \hat{v}_{lq}, \\
v_{ld} &= 2\pi f i_{lq} L_l - \hat{v}_{ld}, \\
\hat{\omega} &= \frac{s + 0.129}{s} v_{zd}, \\
i_l^{qd} &= T(2\pi f t - \hat{\omega} t) i_l^{abc}, \\
v_l^{qd} &= T(2\pi f t - \hat{\omega} t) v_l^{abc}, \\
v_l^{abc} &= T^{-1}(2\pi f t - \hat{\omega} t) v_l^{qd}.
\end{aligned} \tag{8}
$$

where $i_l^{qd} = [i_{lq}, i_{ld}]^{\mathrm{T}}$, $v_l^{qd} = [v_{lq}, v_{ld}]^{\mathrm{T}}$ and $v_z^{qd} = [v_{zq}, v_{zd}]^{\mathrm{T}}$.

B. Model parameters

In this section we summarise the parameters used in the simulation.

1) Wind turbine: $c_1 = 1$, $c_2 = 3952$, $c_6 = 204$, $c_7 = 1447$, $c_3 = c_4 = c_5 = c_8 = c_9 = 0$, $R = 40\mathrm{m}$, $A = 5026.5\mathrm{m}^2$, $\rho = 1.125\mathrm{kg/m}^3$, $\nu = 90$, $I_t = 4\mathrm{kg} \cdot \mathrm{km}^2$, $\tau = 0.1\mathrm{s}$ and $v_w = 7\mathrm{m/s}$.
2) Wind turbine speed controller: $\omega_{mn} = 167.7325\mathrm{min}^{-1}$, $K_p = 0.1°/\mathrm{rad} \cdot \mathrm{s}$ and $K_i = 0.02°/\mathrm{rad}$.
3) Machine and vector controller: $P = 2$, $Q_s^* = 10\mathrm{VAr}$, $\lambda_m = 2.35\mathrm{V/rad{\cdot}s}$, $K_{pq} = 0.0637\mathrm{V/A}$, $K_{iq} = 7.5\mathrm{V/(A{\cdot}s)}$, $K_{pd} = 0.0638\mathrm{V/A}$ and $K_{id} = 7.5\mathrm{V/(A \cdot s)}$, $r_s = 15\mathrm{m}\Omega$, $L_q = 0.12732\mathrm{mH}$ and $L_d = 0.12764\mathrm{mH}$.
4) DC bus: $C = 10\mathrm{mF}$.
5) Grid side system: $f = 50\mathrm{Hz}$, $V_{bus} = 66\mathrm{kV}$ (ph-ph), $r_l = 20\mathrm{m}\Omega$ and $L_l = 1\mathrm{mH}$.
6) Grid side controller: $V_{DC}^* = 2.6\mathrm{kV}$, $K_{pg} = 0.6032\mathrm{A/V}$ and $K_{ig} = 14.2122\mathrm{A/(V \cdot s)}$.
7) Grid current controller: $K_{pc} = 0.2803\mathrm{V/A}$ and $K_{ic} = 10\mathrm{V/(A \cdot s)}$.