Appendix: A Full Detailed Wind Farm Model

Zilong Gong, Junyu Mao, Adrià Junyent-Ferré, *Senior Member, IEEE*, and Giordano Scarciotti, *Senior Member, IEEE*

NOMENCLATURE

Roman symbols

- A Wind turbine swept surface
- C DC bus capacity
- I_t One-mass wind turbine aggregated inertia
- K_{C_p} Constant tip speed ratio speed control law coefficient
- L_l Inductance of the grid connection impedance
- P Generator pole pairs
- Q_s Generator stator reactive power
- R Wind turbine radius
- V_{DC} DC bus voltage
- V_{bus} Infinite bus voltage
- c_p Aerodynamic power coefficient
- f System frequency
- f^r Rated frequency
- r_s Resistance of a single phase of the stator windings
- r_l Resistance of the grid connection impedance
- U_g^r Rated collection grid voltage
- U_w^r Rated wind turbine output voltage
- v_w Wind speed

Greek symbols

- Γ_m Generator torque
- Γ_t Wind turbine torque
- β Pitch angle
- λ_m Flux linkage per rotating speed unit
- ν Gearbox multiplication ratio
- ω_{mn} Nominal generator speed
- ω_r Generator electrical angle speed
- ω_t Wind turbine speed
- ρ Air density
- τ Time constant of the pitch angle controller
- θ_m Generator shaft angular position
- θ_r Generator rotor electric angle

Superscripts and Subscripts

- $*$ Reference value
- abc Vector of *abc* components
- qd Vector of *qd* components
- m Variable related to the generator shaft
- r Variable related to the generator rotor
- s Variable related to the generator stator
- t Variable related to the turbine
- z Variable related to the grid connection point

Zilong Gong, Junyu Mao, Adrià Junyent-Ferré and Giordano Scarciotti are with the Department of Electrical and Electronic Engineering, Imperial College London, London, SW7 2AZ, UK. {zilong.gong18, junyu.mao18, adria.junyent-ferre, g.scarciotti}@imperial.ac.uk.

A. WIND FARM MODEL

In this section, we provide the full model of the wind turbine used in the simulations. The wind turbine model consists of several sub-blocks. We summarise each block in terms of inputs, outputs and detailed equations. The Park and inverse Park transformation of a rotation angle θ are indicated by $T(\theta)$ and $T^{-1}(\theta)$, respectively.

1) Wind turbine dynamics:
Block inputs: θ_m^*, Γ_m and v_w.
Block outputs: θ_m and ω_m.

$$\begin{aligned}\beta(s) &= \frac{1}{\tau s+1}\theta_m^*(s),\\ c_p(\Lambda,\beta) &= c_1(c_2\frac{1}{\Lambda}-c_3\beta-c_4\beta^{c_5}-c_6)e^{-c_7\frac{1}{\Lambda}},\\ \Gamma_t &= \frac{1}{2}c_p\rho A v_w^3\frac{1}{\omega_t},\\ \omega_t &= \frac{1}{I_t}(\Gamma_t+\nu\Gamma_m),\\ \dot{\theta}_m &= \omega_m = \nu\omega_t,\end{aligned} \tag{1}$$

where $[c_1 \cdots c_9]$ are wind turbine characteristic parameters and Λ is defined as $\frac{1}{\Lambda} = \frac{1}{\lambda + c_8\beta} - \frac{c_9}{1+\beta^3}$ with $\lambda = \frac{\omega_t R}{v_w}$.

2) Wind turbine speed controller:
Block input: ω_m.
Block outputs: Γ_m^* and θ_m^*.

$$\begin{aligned}K_{C_p} &= \frac{1}{2}\rho A R^3\frac{c_1(c_2+c_6c_7)^3 e^{-\frac{c_2+c_6c_7}{c_2}}}{c_2^2c_7^4}\\ \Gamma_m^* &= K_{C_p}\omega_m^2,\\ \theta_m^*(s) &= \frac{K_p s + K_i}{s}(\omega_m(s)-\omega_{mn}).\end{aligned} \tag{2}$$

3) Generator dynamics:
Block inputs: v_s^{qd} and ω_m.
Block outputs: i_s^{qd}, i_s^{abc} and Γ_m.

$$\begin{aligned}v_s^{qd} &= \begin{bmatrix} r_s & \omega_r L_d\\ -\omega_r L_q & r_s\end{bmatrix} i_s^{qd} + \begin{bmatrix} L_d & 0\\ 0 & L_q\end{bmatrix}\frac{d}{dt}i_s^{qd} + \lambda_m\omega_r\begin{bmatrix}1\\0\end{bmatrix},\\ \Gamma_m &= \frac{3}{2}P(\lambda_m i_{sq} + (L_d-L_q)i_{sq}i_{sd}),\\ i_s^{abc} &= T^{-1}(\theta_r)i_s^{qd},\end{aligned} \tag{3}$$

4) Generator vector controller:
Block inputs: Γ_m^*, i_s^{abc}, ω_m and θ_m.
Block outputs: v_s^{qd} and v_s^{abc}.

$$
\begin{aligned}
i_s^{qd} &= T(\theta_r) i_s^{qbc} \\
i_{sq}^* &= \frac{2}{3P} \frac{\Gamma_m^*}{\lambda_m}, \\
i_{sd}^* &= \frac{2}{3P} \frac{Q_s^*}{\omega_m \lambda_m}, \\
\hat{v}_{sq} &= \frac{K_{pq}s + K_{iq}}{s}(i_{sq}^* - i_{sq}), \\
\hat{v}_{sd} &= \frac{K_{pd}s + K_{id}}{s}(i_{sd}^* - i_{sd}), \\
v_s^{qd} &= \hat{v}_s^{qd} + \begin{bmatrix} 0 & \omega_r L_d \\ -\omega_r L_q & 0 \end{bmatrix} i_s^{qd} + \lambda_m \omega_r \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\
v_s^{abc} &= T^{-1}(\theta_r) v_s^{qd}.
\end{aligned} \tag{4}
$$

where $\omega_r = P\omega_m$, $\theta_r = P\theta_m$, $v_s^{qd} = [v_{sq}, v_{sd}]^{\mathrm{T}}$, $\hat{v}_s^{qd} = [\hat{v}_{sq}, \hat{v}_{sd}]^{\mathrm{T}}$ and $i_s^{qd} = [i_{sq}, i_{sd}]^{\mathrm{T}}$.

5) DC bus dynamics:
Block inputs: i_l^{abc}, v_l^{abc}, i_s^{abc} and v_s^{abc}.
Block outputs: V_{DC} and i_{DCm}.

$$
\begin{aligned}
\frac{d}{dt} V_{DC} &= \frac{1}{C}(i_{DCm} - i_{DCl}), \\
\{v_l^{abc}\}^{\mathrm{T}} i_l^{abc} &= V_{DC} i_{DCl}, \\
\{v_s^{abc}\}^{\mathrm{T}} i_s^{abc} &= V_{DC} i_{DCm}.
\end{aligned} \tag{5}
$$

6) Grid side system dynamics:
Block inputs: v_z^{abc} and v_l^{abc}.
Block output: i_l^{abc}.

$$
v_l^{abc} = r_l i_l^{abc} + L_l \frac{d}{dt} i_i^{abc} + v_z^{abc}. \tag{6}
$$

7) Grid side controller:
Block inputs: V_{DC} and i_{DCm}.
Block output: i_{DCl}^*.

$$
i_{DCl}^* = \frac{K_{pg}s + K_{ig}}{s}(V_{DC}^* - V_{DC}) + i_{DCm} \tag{7}
$$

8) Grid current controller
Block inputs: V_{DC}, i_{DCl}^*, v_z^{abc} and i_l^{abc}.
Block output: v_l^{abc}.

$$
\begin{aligned}
i_{lq}^* &= \frac{2}{3} \frac{V_{DC}}{v_{zq}} i_{DCl}^*, \\
i_{ld}^* &= 0, \\
\hat{v}_{lq} &= \frac{K_{pc}s + K_{ic}}{s}(i_{lq}^* - i_{lq}), \\
\hat{v}_{ld} &= \frac{K_{pc}s + K_{ic}}{s}(i_{ld}^* - i_{ld}), \\
v_{lq} &= v_{zq} - 2\pi f i_{ld} L_l - \hat{v}_{lq}, \\
v_{ld} &= 2\pi f i_{lq} L_l - \hat{v}_{ld}, \\
\hat{\omega} &= \frac{s + 0.129}{s} v_{zd}, \\
i_l^{qd} &= T(2\pi f t - \hat{\omega} t) i_l^{abc}, \\
v_l^{qd} &= T(2\pi f t - \hat{\omega} t) v_l^{abc}.
\end{aligned} \tag{8}
$$

where $i_l^{qd} = [i_{lq}, i_{ld}]^{\mathrm{T}}$, $v_l^{qd} = [v_{lq}, v_{ld}]^{\mathrm{T}}$ and $v_z^{qd} = [v_{zq}, v_{zd}]^{\mathrm{T}}$.

B. Model parameters

In this section we summarise the parameters used in the simulation.

1) Wind turbine: $c_1 = 1$, $c_2 = 39.52$, $c_6 = 2.04$, $c_7 = 14.47$, $c_3 = c_4 = c_5 = c_8 = c_9 = 0$, $R = 40\text{m}$, $A = 5026.5\text{m}^2$, $\rho = 1.225\text{kg/m}^3$, $\nu = 90$, $I_t = 4\text{kg}\cdot\text{km}^2$, $\tau = 0.1\text{s}$ and $v_w = 7\text{m/s}$.
2) Wind turbine speed controller: $\omega_{mn} = 1602\text{rpm}$, $K_p = 0.1^\circ \cdot s/\text{rad}$ and $K_i = 0.02^\circ/\text{rad}$.
3) Generator: 2 pairs of poles, $r_s = 15\text{m}\Omega$, $\lambda_m = 2.35\text{V}\cdot\text{s/rad}$, $L_q = 0.12732\text{mH}$ and $L_d = 0.12764\text{mH}$.
4) Generator vector controller: $Q_s^* = 10\text{VAr}$, $K_{pq} = 0.0637\text{V/A}$, $K_{iq} = 7.5\text{V}/(\text{A}\cdot\text{s})$, $K_{pd} = 0.0638\text{V/A}$ and $K_{id} = 7.5\text{V}/(\text{A}\cdot\text{s})$.
5) DC bus: $C = 10\text{mF}$ and $V_{DC}^* = 2.6\text{kV}$.
6) Grid side system: $r_l = 20\text{m}\Omega$, $L_l = 1\text{mH}$, $U_w^r = 0.97\text{kV}$, $U_g^r = 66\text{kV}$ and $f^r = 50\text{Hz}$.
7) Grid side controller: $K_{pg} = 0.6032\text{A/V}$ and $K_{ig} = 14.2122\text{A}/(\text{V}\cdot\text{s})$.
8) Grid current controller: $K_{pc} = 0.2803\text{V/A}$ and $K_{ic} = 10\text{V}/(\text{A}\cdot\text{s})$.